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An Endogenous Growth Model with Quality Ladders and Consumers' Heterogeneity[1]

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Abstract

This paper develops an endogenous growth model with quality ladders where consumers heterogeneity is assumed and is modelled through non homothetic preferences. We show that in such a model, unlike mainstream quality ladders models, the steady state equilibrium is characterised by a duopoly were the state of the art technology and the one immediately below it are both able to survive and thrive, under given conditions for the income distribution. In the words of Schumpeter, this model delivers only partial creative destruction. Furthermore, we show that under duopoly, an increase in the degree of income inequality, raises the intensity of research activities and the growth rate of the economy.

Keywords: Industrial Organization, Income Distribution, Technological Change, Innovation, Growth

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1 Introduction

The endogenous growth literature can by now be defined as abundant. More than a decade has passed since the seminal works of Romer, in which for the first time those who engaged in innovative activities with the goal of fostering technological change, were formally modeled (informally, Schumpeter had talked about this long before), as enjoying a degree of market power (Romer, 1990). Thereby, technological change and growth could be determined endogenously as the result of rational decisions taken by economic agents in pursuit of profit incentives.

In the years that followed there were other important contributions, the two most seminal probably being Grossman et al. (1991) and Aghion et al.(1992), These studies split the field of endogenous growth into two branches, with one preoccupying itself with introducing technological change through growth in the number of goods produced (horizontal differentiation), and another which preferred to introduce technological change through quality improvements in production (vertical differentiation, also known as quality ladders). Even the latter stream soon became quite abundant, as the initial studies by Grossman et al. (1991) and Aghion et al.(1992) were followed by many others that tried to use the conceptual framework provided by the quality ladder models to broaden their scope to the study of open economies, trade, foreign direct investment, developed versus developing countries and so on (see Aghion et al., 1998, for an account). The mainstream models with quality ladders all predicted total

obsolescence, or, in the parlance first introduced by Schumpeter, total creative destruction. Put simply, these models predicted that an innovation in the quality goods sector would force lower quality goods out of the market by bringing their sales down to zero. As a result, in these models, the quality goods sector featured a monopoly of the good that embodied the highest quality on the ladder.

However, in the real world there are many examples of markets for quality goods with duopolistic or oligopolistic market structure. Examples include video and radio cassettes vs. digital video disks and compact disks, various generations of computer processors (Pentium II, III, IV, Celeron, etc.) and so on. Perhaps the most telling example is a comparison of the car market in developed vs. developing countries. The former is typically characterized by a monopoly of the latest models, while in the latter one can often see several generations of cars being produced alongside each other and all making a profit. A well known example is the Volkswagen Beetle. While production of this model ceased many years ago in most developed countries, it continued being produced in several developing economies. For instance, production of the Beetle has been continued until very recently in Mexico, and in any case for much longer than in Europe or the U.S. A possible explanation for these phenomena may lie in the different way income is distributed across markets or countries. The main stream models by Grossman et al. (1991) and Aghion et al.(1992) cannot account for income distribution differences.

In Aghion et al.(1992), the consumption good is produced by inputting intermediate goods according to the Cobb Douglas production function: $Y = Ax^{\alpha}, \alpha < 1$. This is a homothetic specification which implies that the rate of substitution between inputs does not change with income.

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In Grossman et al. (1991), household utility is of the form $\ln D_t = \ln \Sigma_m q_m x_m$, where D is a consumption index, and q is an index for quality. Thus products along the quality ladder are perfect substitutes and there is positive demand only for the product that carries the lowest price per unit of quality, which is the highest product on the quality ladder. Again, that occurs regardless of the level of income.

In order to introduce differences in income distributions into the quality ladder framework, we employ the conceptual apparatus produced by that particular branch of industrial organization known as vertical quality differentiation literature. Among the most important contributions to this literature, we cite Gabszewicz et al.(1979), and Shaked et al.(1982). These studies model preferences as follows: $u(I, q_i) = q_i(I - p_i)$, where I is income, q_i is a quality index, and p_i is the price of a good of quality i. Thus consumers problem is not how much to buy of some good (as in Aghion et al., 1992. and in Grossman et al., 1991) But whether to buy one unit of the good, and which quality to buy.

The one feature of the standard quality ladders literature that we do keep, is the so called leapfrogging assumption. By this assumption, the firm that innovates is always an outsider that, by innovating, succeeds in going one step higher on the ladder than the current incumbent (it "leapfrogs" the incumbent). A newer strand of the quality ladders literature works on the different assumption that an outsider innovator must first catch up with the incumbent firm and then, when both firms are level, any of them can make the jump forward on the quality ladder by further innovating. This is the so called "neck-and-neck competition" assumption[3]. Aghion et al. (2005), use the neck-and-neck setting to come closer to our study by analyzing the relationship between the degree of competition in product markets and the rate of innovative activity. As will

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[3] The quality ladder literature with neck-and-neck competition starts with Aghion et al. (1997).

become clear presently through the detailed description of our model, the latter relationship constitutes part of the channel of transmission through which differences in the income distribution affect the rate of innovative activity. However, the link between different income distributions and the degree of competition in product markets can only be made through the adoption of non-homothetic consumer preferences, as those first introduced in the vertical quality differentiation literature.

Non-homothetic consumer preferences such as those just described above, have been employed in the context of a quality ladder model by two other scholars, Li (2003) and Zweimuller et al. (1998, 2005). In the quality ladder literature these are the studies that come closest to the model developed here, but there are some important differences too that distinguish them from the present study.

In Li (2003), income inequality is introduced through assuming that labor income has a uniform distribution with mean preserving spread. This assumption simplifies somewhat the analysis, but it does not allow for a comparison of countries that feature very different mean incomes. Hence, it does not allow for a comparison of the developed world vs. the developing world.

In Zweimuller et al. (1998, 2005), labor income is assumed to be the same across individuals, while income inequality is introduced through heterogeneity in "other wealth", which is endogenous, and whose source is the stake that each individual owns in the firms that produce the quality goods. They too make a simplifying assumption, so that consumers are divided into two categories, the rich and the poor, according to a discrete distribution of other wealth. While we accept that a uniform distribution may fail to give an accurate distribution of income in the real world, we nevertheless feel that it is worthwhile to maintain this assumption,

in order not to lose the rich framework provided by the vertical quality differentiation literature. Crucially, this difference in assumptions leads us to findings that, under duopoly, are the opposite of those of Zweimuller et al. (1998, 2005). In our model, unlike theirs, an increase in income inequality has a positive effect on the rate of research activity and ultimately the growth rate of the economy. Across different market structures, our model predicts, similarly to Li (2003), a U-shaped relationship between inequality and growth, which is negative under monopoly but, as will be amply discussed, turns positive under duopoly. By contrast, Zweimuller et al. (1998, 2005) predict that more inequality is harmful for growth both in a pooling and in a separating equilibrium.

In this study, income inequality is introduced through other wealth which originates from having a stake in the firms that produce the quality goods, like in Zweimuller et al. (1998, 2005). This other wealth is assumed to be uniformly distributed, but without the restrictive mean preserving spread assumption. We also perform some comparative statics on the steady state relationship between inequality, wages and the market structure, as described by the number of qualities, and we give a simple numerical example, to show how the degree of inequality or the size of the technological leap may affect the wealth level and the rate of innovative activity.

Worth mentioning at this point is the fact that technological progress, in its relative novelty, is not the only conceivable channel of transmission in the link between income inequality and growth. The more widely accepted idea has been that income inequality affects growth through physical and human capital accumulation. For instance, Galor et al. (2004) in a recent study, reckon that inequality has long been an important determinant of growth. But they argue that over time the channel through which inequality affected growth has changed. In the early stages of industrialization, inequality was thought to be a positive determinant of growth,

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because physical capital accumulation, in its growth-fostering role, needed a more concentrated, that is unequal, distribution of income to gain sufficient momentum. Later on, as human capital accumulation replaced physical capital accumulation as a prime engine of growth, its characteristic of being embodied in humans implied that in order to display its full potential, human capital accumulation should be widely spread across individuals in society. This time therefore, higher income inequality would be harmful to growth because it would hinder, or at least slow down, such a spread. Galor and Moav attempted to provide a unifying theoretical framework that could reconcile these two effects. However it is interesting to notice that they made no mention of the possibility of existence for a third channel through which inequality could affect growth: technical change.

Another recent addition to the literature that studies the interaction between inequality and growth via demand side effects is Mani (2001). This author too, in order to have income inequality play a role, assumes non-homotheticity when modeling consumers' preferences. However, Mani (2001) focuses entirely on the relationship between income distributions and demand patterns and his model, unlike our study, features no technological progress.

The rest of the paper is organized as follows: Section 2 lays down the model, introduces income heterogeneity and describes the features of the market structures of monopoly and duopoly. Sections 3 to 5 introduce the remaining main building blocks of this model, namely the research and development sector (in short RnD), the labor markets, and how the growth rate of the economy is being computed. Sections 6 and 7 perform steady state equilibrium analysis in monopoly and duopoly respectively. Sections 8 and 9 contain the comparative statics and the numerical example mentioned above. Section 10 comprises the conclusion and some directions for future research. The Appendix has all the remaining mathematical details.

2 The Model

The economy is populated by L individuals, whose life-span is infinite. The representative individual consumes two types of goods each period. The first type is a good that is subject to quality innovation over time. Each individual consumes at most one unit per period of these goods. The quality good is denoted as q_{it} where t indexes time, and $i=1,2$ indexes quality, in ascending order. An innovation raises the quality of good q_{it} by a constant factor $\gamma > 1$. Therefore $q_1 = \gamma$ denotes the product that sits second from top on the quality ladder, and $q_2 = \gamma q_1 = \gamma^2$ denotes the good that occupies the highest position on the ladder. The second type of good is a homogeneous product, that can be thought of as a composite commodity that comprises all other purchases made beyond the quality goods sector. Let this homogeneous good be denoted by h_t (again t indexes time). In any period t, the utility achieved by the representative individual is given by:

$$u_t = q_{it}h_t \tag{1}$$

In words, this utility function says that the representative individual can buy only one unit of the quality good and an amount h of the homogeneous good. This individual's choice concerns which quality to buy, as represented by the quality index q_{it}, and how much to have of the homogeneous good. We take advantage of a utility functions' property by which these are only defined up to a monotonic transformation, to rewrite the above utility in logs as follows:

$$\ln u_t = \ln q_{it} + \ln h_t \tag{2}$$

The intertemporal utility maximization problem for the representative individual is:

$$Max \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \ln u_t \tag{3}$$

where ρ is the rate of time preference.

We assume that consumers are endowed with assets k, earn a wage w from supplying one unit of labor, and make expenditures C on the two types of goods defined above. Over time, in any given period, assets next period must be equal to the sum of the assets, augmented at the current interest rate, and the wage income earned from working this period, minus consumption expenditure this period, according to the following budget constraint:

$$k_{t+1} = (1+r_t)k_t + w_t - C_t \tag{4}$$

Since consumers purchase one unit of the quality good and spend the rest on the homogeneous good, demand for the latter is given by:

$$h_t = C_t - p_{it} \tag{5}$$

where p_{it} is price in period t of one unit of good of quality $i = 1,2$.

Having stated what the representative individual's intertemporal problem is, we will restrict our attention to the steady state, utility-maximizing, consumption pattern of such an individual[4], which is:

$$C = w + \rho k. \tag{6}$$

2.1 Introducing Income Heterogeneity

In order to introduce income heterogeneity, we shall assume that w, wage income, is the same for everybody who supplies one unit of labor and exogenous, whereas assets k, which we will henceforth refer to as "wealth" (other than wage income), is uniformly distributed on a support $[k_l, k_u]$.

We further define: $\dfrac{v}{L} = \dfrac{k_u + k_l}{2}$ as the mean other wealth (per capita), a measure of the position of the distribution concerning other wealth, and $\dfrac{x}{L} = \dfrac{k_u - k_l}{2}$ as a measure of the spread of the distribution. Given the linear relationship between other wealth and steady state consumption, the latter is also uniformly distributed on support $[C_l, C_u]$. These assumptions

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[4] In order to solve for the steady state, we write down the Bellman equation for this problem:
$V(k_{t-1}) = \max\limits_{C_t, k_t} \left[u_t(C_t) + \dfrac{1}{1+\rho} V(k_t) \right]$, subject to the intertemporal budget constraint:
$C_t = -k_{t+1} + (1+r_t)k_t + w_t$. We recall that $u_t(C_t) = \ln(C_t - p_{it}) + \ln q_{it}$. Substituting from the budget constraint into the Bellman equation, we can rewrite the latter as follows: $V(k_{t-1}) = \max\limits_{k_t} \left[u_t(k_t) + \dfrac{1}{1+\rho} V(k_t) \right]$.
The Euler equation is: $u_t' = \dfrac{1}{1+\rho} u_{t+1}' f'(k_t)$. Here:
$\dfrac{1+r_t}{C_t - p_{it}} = \dfrac{1}{1+\rho} \dfrac{1+r_{t+1}}{C_{t+1} - p_{it+1}} (1+r_t) \Leftrightarrow \dfrac{C_{t+1} - p_{it+1}}{C_t - p_{it}} = \dfrac{1+r_{t+1}}{1+\rho}$. In steady state we set $C_{t+1} = C_t = C$ and
$p_{it+1} = p_{it} = p_i$ to derive: $1 = \dfrac{1+r_{t+1}}{1+\rho} \Rightarrow r_t = r = \rho$. Steady state can be arrived at by setting
$k_t = k, C_t = C, w_t = w, r_t = \rho$ in the budget constraint, to get: $k = (1+\rho)k + w - C \Leftrightarrow C = w + \rho k.$.

imply that individuals have same preferences and wage income, but differ in their other wealth and, as a result, in their levels of consumption. The consumption pattern of individual $j \in [l, ..., u]$ is thus given by $C_j = w + \rho k_j$, and the mean consumption pattern, as a case of special interest, is given by: $C_m = w + \rho \frac{v}{L}$.

2.2 Bertrand Duopoly Game

Each individual on the wealth scale makes a decision regarding the purchase of the quality good, namely whether to buy it, and which quality to buy. This decision depends on the income of the individual in question and, again because of the linear relationship between income and consumption, on the level of consumption that this individual can afford. Let C_{eq} denote the consumption level of the individual who is indifferent between purchasing one unit of good 1 at price p_1 and one unit of good 2 at price p_2. For such an individual, utility derived from these two consumption patterns must be the same:

$$\ln u(q_1) = \ln u(q_2) \Leftrightarrow \ln q_1 + \ln(C_{eq} - p_1) = \ln q_2 + \ln(C_{eq} - p_2) \tag{7}$$

Solving for C_{eq} yields:

$$C_{eq} = \frac{q_2 p_2 - q_1 p_1}{q_2 - q_1} = \frac{\gamma p_2 - p_1}{\gamma - 1} \tag{8}$$

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The individual with consumption pattern C_{eq} divides the population into two groups. There are individuals $C_j \in [C_l, C_{eq}]$ who buy good 1, and individuals $C_j \in [C_{eq}, C_u]$ who buy good 2.[5]

Therefore, demand for goods 1 and 2 is respectively:

$$D_1 = LF(C_{eq}) \tag{9}$$

$$D_2 = L(1 - F(C_{eq})) \tag{10}$$

Where $F(\cdot)$ is the cumulative distribution function (cdf) of consumption levels, which has density $f(\cdot)$. Under duopoly, firms compete for customers by choosing prices (Bertrand competition). Their objective is to maximize the following profits:

$$\pi_1 = D_1(p_1 - wc) = LF(C_{eq})(p_1 - wc) \tag{11}$$

$$\pi_2 = D_2(p_2 - wc) = L(1 - F(C_{eq}))(p_2 - wc) \tag{12}$$

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[5] Underlying this statement, there is an assumption that the market is "covered", that is, everybody buys the quality good, although some people prefer good 1 and some others buy good 2. Another equilibrium scenario, which is ruled out here, is when some consumers prefer not to buy the quality good at all. In this instance, we say that the market is not covered. It is not difficult to show that, in order for the market to be covered, the following condition must hold: $C_{eq} \leq 2C_l - \frac{1}{\gamma - 1} wc$. The following is an interpretation of this condition: The equilibrium household, which is indifferent from buying good 1 at price p_1 and good 2 at price p_2, must have consumption that is less than twice the consumption of the poorest household minus a weighted marginal cost, where the weight $\frac{1}{\gamma - 1}$ is a measure of the quality differential between good 1 and good 2.

where π_i is profit accruing to firm producing good of quality $i = 1, 2$ and selling it at price p_i and wc is a cost per unit produced (assumed to be the same for both qualities, and equal to wage income multiplied by a labor coefficient c<1).

We can use first order conditions for profit maximization ($\frac{\partial \pi_i}{\partial p_i} = 0$) to derive the following two equilibrium conditions:

$$1 - 2F(C_{eq}) = f(C_{eq})(C_{eq} - wc) \tag{13}$$

$$f(C_{eq})\left(B_{eq} - \frac{wc(\gamma + 1)}{\gamma - 1}\right) = 1 \tag{14}$$

where we define $B_{eq} = \frac{\gamma p_2 + p_1}{\gamma - 1}$. Further manipulations yield the equilibrium prices:

$$\begin{aligned}
p_1^e &= (\gamma - 1)\frac{F(C_{eq})}{f(C_{eq})} + wc \\
p_2^e &= \frac{\gamma - 1}{\gamma}\frac{1 - F(C_{eq})}{f(C_{eq})} + wc
\end{aligned} \tag{15}$$

And equilibrium profits:

$$\begin{aligned}
\pi_1^e &= (\gamma - 1)\frac{L[F(C_{eq})]^2}{f(C_{eq})} \\
\pi_2^e &= \frac{\gamma - 1}{\gamma}\frac{L[1 - F(C_{eq})]^2}{f(C_{eq})}
\end{aligned} \tag{16}$$

This formulation of equilibrium relationships offers the advantage of being valid for any distribution of consumption patterns, not just the uniform case.

This consideration enables us to state the following lemma for any distribution of consumption levels:

Lemma: In a duopolistic market where everybody buys one of the two quality goods, the following is always true:

$$F(C_{eq}) \leq \tfrac{1}{2} \Leftrightarrow C_{eq} \leq C_m \tag{17}$$

Proof: Recall that one of the two equilibrium conditions is:

$$1 - 2F(C_{eq}) = f(C_{eq})(C_{eq} - wc) \tag{18}$$

Notice that the RHS of this condition is ≥ 0 by definition, so too must be the LHS. But this implies: $F(C_{eq}) \leq \tfrac{1}{2}$.

From this point onwards, we shall restrict ourselves to the uniform distribution case. For later use, we rewrite equilibrium profits as a function of mean other wealth v and spread x, as follows:

$$\pi_1^e(v) = (\gamma - 1)\rho \frac{\left[x - \frac{v}{3} - \frac{wL(1-c)}{3\rho}\right]^2}{2x}.$$

$$\pi_2^e(v) = \frac{\gamma - 1}{\gamma}\rho \frac{\left[x + \frac{v}{3} + \frac{wL(1-c)}{3\rho}\right]^2}{2x}$$

(19)

2.3 Monopoly

Here we want to describe the conditions that in equilibrium yield a monopoly, that is that market structure, where the quality good sector is characterized by everybody buying the state-of-the-art. Moreover, just as we did when discussing duopoly, we want to determine the monopoly equilibrium price and profit that accrues to the firm which produces the state-of-the-art.

We start by writing the demand schedule facing the firm selling good 2:

$$D_2 = L \int_{C_{eq}}^{C_u} f(C)dC = \frac{L}{C_u - C_L}(C_u - C_{eq}) = \frac{L}{C_u - C_l}\left(C_u - \frac{\gamma}{\gamma - 1}p_2 + \frac{1}{\gamma - 1}p_1\right) \quad (20)$$

If both firm producing good 1 and firm producing good 2 were to adopt marginal cost pricing (so that $p_1 = wc$ and $p_2 = wc$), output would be: $\frac{L}{C_u - C_l}(C_u - wc)$. A monopolist facing the above mentioned demand schedule, which is linear in p_2, would therefore choose output that is half of that chosen when both firms adopt marginal cost pricing: $\frac{L}{C_u - C_l}(C_u - wc)/2$. Because every individual buys at most one unit of the quality good, monopoly occurs whenever the size

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of the population is not greater than the number of units produced by a profit-maximizing

monopolist, or in symbols:

$$L \leq \frac{L}{C_u - C_l}(C_u - wc)/2 \Leftrightarrow C_u \leq 2C_l - wc \tag{21}$$

We can rewrite the above condition in terms of mean other wealth v and spread x, as

follows:

$$x \leq \frac{1}{3}\left(v + \frac{wL(1-c)}{\rho}\right) \tag{22}$$

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The economic interpretation of this condition is that for monopoly to obtain in

equilibrium, the spread in the distribution of other wealth, cannot exceed one third of the

expression in brackets. The latter is the sum of wealth v and the discounted flow (discounted at

the rate of time preference ρ) of the total amount of salaries wL earned in sectors other than

manufacturing of the quality good (this exclusion is obtained by multiplying wL by the

coefficient 1-c). Salaries in the manufacturing of quality goods sector are excluded because they

also represent a cost for households/entrepreneurs. The condition for monopoly can therefore be

restated in words as: "the spread in wealth must be at most equal to one third of the total wealth

in the system"[6]. Under these circumstances the market for the quality good is a monopoly. The

profit accruing to the monopolist is: $\pi_M = L(p_M - wc)$. In order to maximize this profit, the

monopolist will set the maximum price compatible with a monopolist market structure. This

amounts to setting the highest price such that $C_{eq} \leq C_l$. Recall that under monopoly, the firm

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[6] Obviously, the condition for duopoly is the complement of this and reads "duopoly obtains as soon as the spread
in other wealth is larger than one third of the total wealth in the system".

producing the inferior good is assumed to set a price equal to marginal cost, $p_1 = wc$ which yields no profit so that the firm does not start production in the first place. As a result, the above inequality may be rewritten as: $\dfrac{\gamma p_m - wc}{\gamma - 1} \leq C_l$. To get the price that maximizes monopoly profits, notice that the above constraint is binding and solve for p_M to obtain:

$$p_M^e = \frac{\gamma - 1}{\gamma} C_l + \frac{wc}{\gamma} \tag{23}$$

Putting this price back into the expression for profit, yields equilibrium profit under monopoly:

$$\pi_M^e = L \frac{\gamma - 1}{\gamma} (C_l - wc) \tag{24}$$

Both monopoly equilibrium price and profit depend on C_l only. Thus these relationships are valid for any distribution of consumption patterns. Nevertheless, in order to maintain a parallel with the duopoly case, we shall restrict our analysis to the uniform distribution case. For later use, we rewrite monopoly price and profit as function of mean other wealth v and spread x:

$$p_M^e = \frac{\gamma - 1}{\gamma} \left[w + \rho(\frac{v}{L} - \frac{x}{L}) \right] + \frac{wc}{\gamma}$$
$$\pi_M^e = \frac{\gamma - 1}{\gamma} [\rho(v - x) + wL(1 - c)] \tag{25}$$

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3 The RnD Sector

In order to close the model, and before passing to the description of steady states with associated equilibrium analyses, we need to introduce two more elements.

In this section we describe the RnD sector, while the next section is dedicated to the labor markets. Both these sectors are crucial building blocks of this model, but, because here nothing novel is added to them that did not appear in the previous quality ladders literature, we provide a concise account of them, and refer the interested reader to that literature (e.g. Grossman and Helpman, 1991) for a more detailed description.

We assume that innovations are random and arrive according to a Poisson arrival rate μ. μdt describes the probability that the next innovation occurs within dt from now, when the innovator research effort is μ. In steady state, the value of an innovation V will be such that the following arbitrage equation holds:

$$\rho V = \pi_2 - \mu V + \mu V^n \tag{26}$$

In turn:

$$\rho V^n = \pi_1 - \mu V^n \tag{27}$$

where V^n is the value of the innovation next period, after it has become second best due to the arrival on the market of a better product (i.e. after it has been pushed one step lower on the

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quality ladder); π_i is profit for the firm producing good of quality $i = 1, 2$, and ρ is the rate of time preference.

After substitution, we get the asset arbitrage equation:

$$V = \frac{\pi_2}{\rho + \mu} + \frac{\mu \pi_1}{(\rho + \mu)^2} \tag{28}$$

Free entry in RnD implies zero profit for the innovator, or:

$$\mu V - \mu w a = 0 \tag{29}$$

where μwa is the cost of doing research, whose components are the intensity of research μ, wage income w, and a labor coefficient a. We can and rewrite this free entry condition as follows:

$$\mu(V - wa) = 0 \tag{30}$$

and conclude that positive but finite research investments can take place only when $V = wa$. In symbols:

$$V = wa, \mu > 0$$
$$V < wa, \mu = 0 \tag{31}$$

Putting together the asset arbitrage and free entry conditions yield the following equilibrium relationship for the RnD sector:

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$$wa = \frac{\pi_2}{\rho + \mu} + \frac{\mu \pi_1}{(\rho + \mu)^2} \tag{32}$$

When wealth and consumption distributions are such that, in equilibrium, the market structure is a monopoly, sales of the good of lower quality are zero and $\pi_1 = 0$. As a result, next period value is $V^n = 0$ and the RnD condition (32) reduces to:

$$wa = \frac{\pi_m}{\rho + \mu} \tag{33}$$

4 The Labor Market

The final element of this model is the labor market. In this market, total demand is the sum of demand for labor in each sector. With research intensity μ and a research sector characterized by a labor coefficient a, demand for labor in research is equal to $a\mu$. In the manufacturing sector, the demand for labor in manufacturing the quality good is cL. Demand for labor in manufacturing the homogeneous good is $\frac{h}{w}$, and it is obtained as follows: The technology for the homogeneous good is assumed to be such that every individual contributes equally to its production. Given perfect competitive settings, we equate marginal product of labor $\frac{h}{L}$ to the going wage w, which, upon solving for L, yields the demand for labor in manufacturing the homogeneous good as stated above. The supply of Labor is simply given by L.

Equating labor demand and labor supply, yields the full employment condition:

$$a\mu + cL + \frac{h}{w} = L \tag{34}$$

Equation (34), as it stands, does not yet have any relationship with the (uniform) distribution of wealth. In order to have the latter play a role in equation (34), several further arrangements are necessary. We work through the case of monopoly in great detail, and only sketch the main passages for duopoly, because the calculations involved follow the same logic. We start by recalling that demand for the homogeneous good h under monopoly is given by $h = C - p_M$, and that C is characterized by a uniform distribution on support $[C_l, C_u]$. Therefore, we can express h as follows[7]:

$$h = \int_{C_l}^{C_u} \frac{L}{C_u - C_l} h(C) dC = \int_{C_l}^{C_u} \frac{L}{C_u - C_l} (C - p_M) dC = L\left[\frac{C_u + C_l}{2} - p_M\right] \tag{35}$$

Replace this expression for h into equation (4.1) and recall that $\frac{C_u + C_l}{2} = C_m$ is the mean consumption, to end up with the following rearranged full employment condition:

$$a\mu + \frac{LC_m}{w} + cL - \frac{Lp_M}{w} = L \tag{36}$$

The next step is to replace p_m with (23) to get :

$$a\mu + \frac{LC_m}{w} + cL - \frac{L\left(\frac{\gamma - 1}{\gamma} C_l + \frac{wc}{\gamma}\right)}{w} = L \tag{37}$$

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[7] The further passages that were omitted in the main text are as follows:

$$h = \int_{C_l}^{C_u} \frac{L}{C_u - C_l}(C - p_M) dC = \frac{L}{C_u - C_l}\left[\int_{C_l}^{C_u} C dC - p_M \int_{C_l}^{C_u} dC\right] = \frac{L}{C_u - C_l}\left[\frac{C_u^2 - C_l^2}{2} - p_M(C_u - C_l)\right];$$

Then, canceling terms yields the result of the main text.

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Further algebra yields the following:

$$au + \frac{LC_m}{w} - \frac{L^{\frac{\gamma-1}{\gamma}}(C_l - wc)}{w} = L \tag{38}$$

Finally, recognition that $L^{\frac{\gamma-1}{\gamma}}(C_l - wc) = \pi_M$ is nothing but (2.24), enables us to express the full employment condition as a function of monopoly profits, as follows:

$$au + \frac{LC_m}{w} - \frac{\pi_M}{w} = L \tag{39}$$

In the duopoly case, the relationship between h and C must account for the fact that now the uniform distribution of C is split into two portions by the indifferent consumer, characterized by consumption pattern C_{eq}. Hence, we can derive the following expression for h:

$$h = \int_{C_l}^{C_{eq}} \frac{L}{C_u - C_l} h(C)dC + \int_{C_{eq}}^{C_u} \frac{L}{C_u - C_l} h(C)dC$$

$$h = \int_{C_l}^{C_{eq}} \frac{L}{C_u - C_l}(C - p_1)dC + \int_{C_{eq}}^{C_u} \frac{L}{C_u - C_l}(C - p_2)dC \tag{40}$$

After following the same procedure as for monopoly, replace the above in the full employment condition, so that the latter can be rewritten in terms of firms' profits:

$$au + \frac{LC_m}{w} - \frac{\pi_2}{w} - \frac{\pi_1}{w} = L \tag{41}$$

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5 Growth Rate

The steady state growth rate of this economy stems from the quality upgrading process in the quality goods sector. As in the earlier quality ladders literature, we recall that innovations arrive at Poisson rate μ, and when they do arrive, the size of the jump up the quality ladder is $\ln\gamma$, to derive the following expression for the steady state growth rate:

$$g = \mu \ln \gamma \tag{42}$$

6 Steady State Analysis - Monopoly

In steady state equilibrium, the model is fully described by the research equation (32) and the full employment condition (34). Under monopoly, these equations take the following form:

$$wa = \frac{\pi_M(v)}{\rho + \mu} \tag{43}$$

$$w\mu a = \pi_M(v) - \rho v \tag{44}$$

Monopoly profit, as a function of wealth v, has in turn been found to be:

$$\pi_m(v) = \frac{\gamma - 1}{\gamma}[\rho(v - x) + wL(1 - c)] \tag{45}$$

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After plugging the profit equation into equations (43) and (44), we get a system of two equations in two unknowns (v, μ) that is amenable to analysis. We state the following:

Proposition 1: If the market structure in the quality goods sector is a monopoly, there exists a unique steady state equilibrium characterized by positive wealth level v, research intensity μ and positive growth rate g. (Proof in the Appendix).

Proposition 2: An increase in the degree of inequality, as measured by an increase in the spread x, is harmful for innovative activity μ and the economy growth rate g. (Proof in the Appendix).

Proposition 1 says that under a monopolistic structure in the market for quality goods, a unique steady state equilibrium can be reached (provided the condition given in the appendix is satisfied) which is characterized by positive rates of innovative research and growth. The graphical representation of proposition 1 is given in Figure 1. In the latter, the line termed (A') represents the locus of points (v, μ) such that the expected value of an innovation is equal to its costs (equation (43)). To understand why this line has a positive slope, notice that higher values of research intensity μ shorten the useful duration of a quality product as innovations occur at a faster pace. On the other hand, a larger wealth v guarantees a higher profitability for a product of top quality because it increases consumers' willingness to pay for higher quality. Therefore, for equation (43) to remain in equilibrium, higher values of μ must be matched by higher values of v, so that the loss of profit due to a shorter useful life is offset by the higher profitability guaranteed by a higher v. The line termed (L') represents the locus of points (v, μ) such that the labor markets clear (equation (44)). This line has a negative slope because higher innovative activity μ results in more demand for labor in the research sector. Since labor supply is fixed at L, for labor markets to clear this increase in labor demand must be offset by a drop in labor

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demand in the manufacturing sector, either for producing the quality good or the homogeneous good. Since labor demand for the quality good is fixed at cL, labor demand for producing the homogeneous good must come down. The latter can occur if less of the homogeneous good is consumed, which is ensured by lower values of wealth v. Proposition 2 is a comparative statics result: it looks at the effect of an exogenous increase in the degree of inequality x on the rate of research μ and the rate of growth g. Both effects are found to be negative. The intuition that lies behind this result is that, as inequality increases, the monopolist of the highest quality starts being under more pressure which stems from threat of a new entry in the market for quality goods by a product of lower quality. This pressure can be thought of as stronger potential competition which translates into decreasing expected returns for future incumbent monopolists and reduced incentives to conduct innovative research. A drop in the latter also results in a fall in the economic growth rate, as it can be seen with a glance at equation (42). Proposition 2 is pictorially explained in the graph of Figure (2). Notice that an increase in the degree of inequality x causes both line (A') and line (L') to shift to their left. The shift is such that at the new equilibrium, the level of wealth v is unchanged while innovative activity μ drops
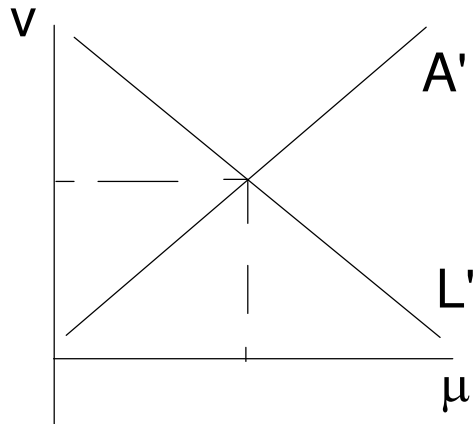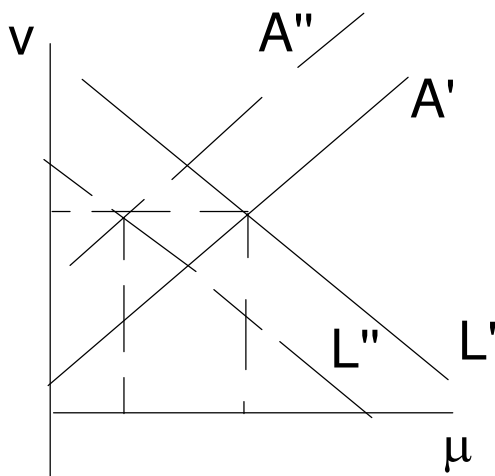
Figure 1 - Monopoly Equilibrium

Figure 2 - Monopoly, Comparative Statics: Increase in Wealth Inequality

7 Steady State Analysis - Duopoly

In equilibrium, the two equations that define the model under duopoly are[8]:

$$wa = \frac{\pi_2(v)}{\rho + \mu} + \frac{\mu \pi_1(v)}{(\rho + \mu)^2} \qquad (46)$$

$$\mu wa = \pi_2(v) + \pi_1(v) - \rho v \qquad (47)$$

Where the two expressions for profits are as follows:

$$\pi_1^e(v) = (\gamma - 1)\rho \frac{\left[x - \frac{v}{3} - \frac{wL(1-c)}{3\rho} \right]^2}{2x}$$

$$\pi_2^e(v) = \frac{\gamma - 1}{\gamma}\rho \frac{\left[x + \frac{v}{3} + \frac{wL(1-c)}{3\rho} \right]^2}{2x} \qquad (48)$$

In the above, the two endogenous variables are μ, which denotes intensity of research, and v, which measures the total value of wealth (other than wage income w). Of great importance in our analysis will also be x, a measure of the spread of the distribution of v. The equilibrium analysis is at first carried out for fixed spread x. Later we shall let it vary and measure the impact of such variation on the endogenous pair (μ, v) in the new steady state equilibrium.

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[8] Equation (43) is exactly as derived in the RnD section, whereas equation (44) is as derived from the labor markets section, with $C_m = w + \rho\frac{v}{L}$, and after multiplying through by w.

Proposition 3: If the market structure in the quality goods sector is a duopoly, there exists a unique steady state equilibrium characterized by positive wealth level v, research intensity μ and positive growth rate g, provided that parameter values are such that the functions $v = v_A(\mu)$ and $v = v_L(\mu)$ implicitly defined by the schedules (46) and (47) satisfy the condition $v_A(\mu) < v_L(\mu)$.

We sketch the proof of proposition 3 here, while the entire proof can be found in the appendix.

Proof: this follows the same methodology that we used to prove proposition 1 regarding monopoly. We note that both (46) and (47) implicitly define functions $v = v_A(\mu)$ and $v = v_L(\mu)$. Our proof is in three steps: 1) prove that the function $v = v_A(\mu)$ has positive slope for all $\mu \geq 0$, $v \geq 0$.

2) prove that the function $v = v_L(\mu)$ has negative slope for all $\mu \geq 0$, $v \geq 0$. This condition holds, provided that the leap in quality brought about by the latest innovation, as measured by γ, is sufficiently small (in particular, the slope is negative for $1 < \gamma < 2$).

3) Compute $v_A(\mu) \mid_{\mu=0}$ and $v_L(\mu) \mid_{\mu=0}$ and notice that a unique equilibrium exists if and only if $v_A(\mu) \mid_{\mu=0} < v_L(\mu) \mid_{\mu=0}$. In words, this latest step consists of showing that the vertical intercept of the function defined by the (46) schedule occurs at a lower point than the vertical intercept of the function defined by the (47) schedule. This fact, together with steps 1 and 2 ensures that the two schedules meet only once in the positive quadrant of the (μ, v) plane and thereby determine a unique and positive pair (μ, v).

The unique equilibrium of proposition 3 can be represented in a graph that looks like that of Figure 1. This is done in Figure 3. In this graph, line A shows the set of paired values (μ, v) for which, under duopoly, the expected value of innovation is equal to its cost. The economic reasoning for the positive slope of A is very similar to that of the positive slope of line A' in Figure 1 and it is not repeated here.

Similarly, the reasoning behind the downward slope of line L, which is the locus of all paired values (μ, v) so that the labor markets clear under duopoly, is the same as that provided to justify the negative slope of line L' in figure 1. Finally, you will notice that Figure 3, unlike Figure 1, does not feature linear schedules, since the slopes of the lines in Figure 1 are constant, while those of the lines of Figure 3 are not (see the calculations in the Appendix).

In order to obtain further insight and an easy way to perform comparative statics with respect to inequality, we merge equations (46) and (47). Such a calculation yields the following relationship between μ and v:

$$\mu = \mu_1(v) = \frac{\pi_2(v) - aw\rho}{2aw - v} \tag{49}$$

Figure 3 - Duopoly Equilibrium
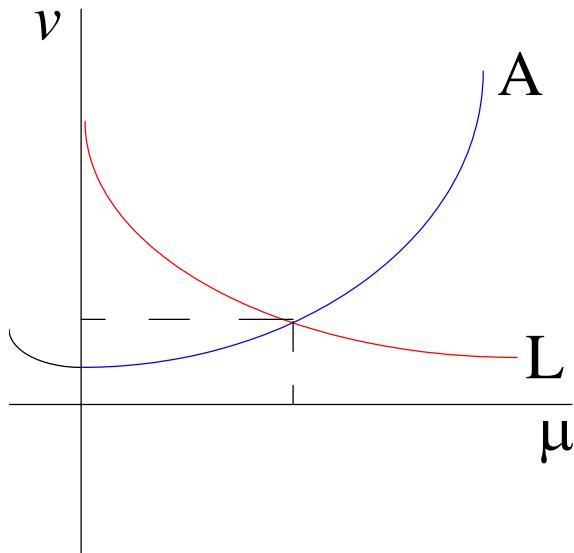
Notice that, upon dividing equation (47) through by *aw*, the latter also provides a direct relationship between μ and *v*:

$$\mu = \mu_2(v) = \frac{\pi_2(v) + \pi_1(v) - \rho v}{aw} \tag{50}$$

Equations (49) and (50) are shown in Figure 4 as lines (*) and (**), which yield another way of looking at the steady state equilibrium under duopoly. We have already seen, earlier on in the text, that under monopoly, the value of wealth is given by $v = wa$ Under duopoly, the remaining value of the good that is second best must be added to the value of the latest

innovation, so that the value of wealth falls in the range $wa < v < 2aw$. Therefore, we want to study the behavior (49) and (50) for values of v falling into this range. Starting with (49), notice that the latter has a vertical asymptote at $v = 2aw$. For $v < 2aw$, μ approaches $+\infty$ as v approaches $2aw$ from below, provided that $\pi_2(v) - aw\rho > 0$. The latter can be ensured for example by taking L to be sufficiently large. For $aw \leq v < 2aw$, μ is a monotonically increasing function of v, because $\pi_2(v)$ in the numerator is monotonically increasing in v, and the denominator goes down as v rises. Figure 4 illustrates the case described. As for line (**), corresponding to equation (50), its slope is given by the sign of the expression $\dfrac{\partial \pi_2}{\partial v} + \dfrac{\partial \pi_1}{\partial v} - \rho$. In the proof of proposition 3, to be found in the appendix, it is shown that a sufficient condition for $\dfrac{\partial \pi_2}{\partial v} + \dfrac{\partial \pi_1}{\partial v} - \rho < 0$ and for (49) to have a negative slope, is that $1 < \gamma < 2$. The two curves will meet once if and only if :

$$\mu_1(aw) = \frac{\pi_2(aw) - aw\rho}{aw} < \mu_2(aw) = \frac{\pi_2(aw) + \pi_1(aw) - aw\rho}{aw} \iff \frac{\pi_1(aw)}{aw} > 0 \qquad (51)$$

Further, in order to ensure that at the point of intersection $\mu > 0$, we need that either $\pi_2(aw) - aw\rho > 0$ on the schedule (49), or that $\pi_2(aw) + \pi_1(aw) - \rho v > 0$ on the schedule (50).

Figure 4 - Duopoly Equilibrium, Alternative Representation
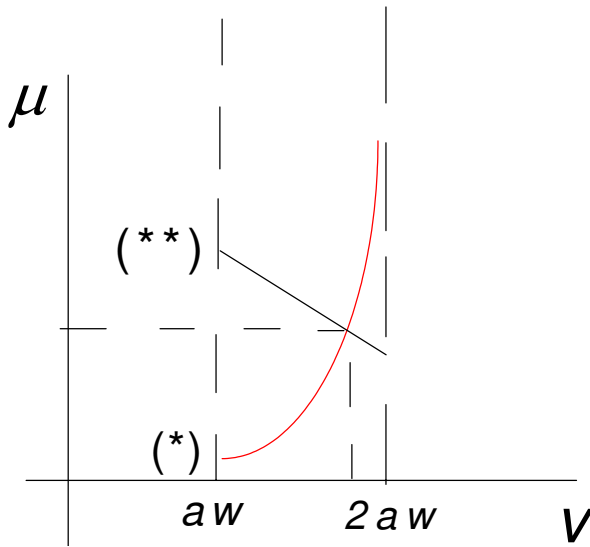
The curves (49) and (50) are also a useful and very simple tool to study how the endogenous variable μ, and thereby the growth rate g, respond to changes in the degree of inequality x.

Proposition 4: Under duopoly, an increase in the degree of inequality x raises the intensity of research μ and thereby the growth rate g.

Proof: Since both in (49) and in (50) inequality x only enters the two profit functions, which in turn enter $\mu_1(v)$ and $\mu_2(v)$ with a positive sign, it is sufficient to show that $\frac{\partial \pi_1}{\partial x} > 0$ and $\frac{\partial \pi_2}{\partial x} > 0$.

We find $\dfrac{\partial \pi_1}{\partial x} = (\gamma - 1)\rho \dfrac{\left(x - \dfrac{v}{3} - \dfrac{wL(1-c)}{3\rho}\right)\left(x + \dfrac{v}{3} + \dfrac{wL(1-c)}{3\rho}\right)}{2x^2}$ and

$\dfrac{\partial \pi_2}{\partial x} = \dfrac{\gamma - 1}{\gamma}\rho \dfrac{\left(x + \dfrac{v}{3} + \dfrac{wL(1-c)}{3\rho}\right)\left(x - \dfrac{v}{3} - \dfrac{wL(1-c)}{3\rho}\right)}{2x^2}$. Both $\dfrac{\partial \pi_1}{\partial x} > 0$ and $\dfrac{\partial \pi_2}{\partial x} > 0$ follow

from the fact that x, under duopoly, must satisfy the condition $x > \dfrac{v}{3} + \dfrac{wL(1-c)}{3\rho}$.

Proposition 4 tells us that duopoly yields a result with respect to changes in income inequality that is the opposite of that obtained under monopoly (see proposition 2). The economic argument is that when the market for quality goods is segmented, rising inequality in wealth weakens competition among qualities, as the incumbent in each segment is able to command a higher price because of the lower threat that consumers in the tails of each segment might turn to the other quality. In the next section we take another look at how inequality in wealth and the number of qualities in the market for quality goods interact.

8 Comparative Statics on the Number of Qualities as a Function of Inequality, Given Low or High Wages, and as a Function of Wages, Given Low or High Inequality

In this section we perform a comparative statics exercise in order to study how the number of qualities in the product markets and hence the structure of the product markets react to changes in wealth inequality x, and to changes in labour income, w. In order to study the behaviour of the threshold levels of inequality and wages that determine the switch from monopoly to duopoly (from one quality to two qualities), the relevant regime to look at is monopoly (this of course

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implies that duopoly would be the relevant regime if we were to study the threshold between two and three qualities). This observation enables us to use the following feature of the monopoly regime and greatly simplify the analysis: under monopoly, other wealth v is equal to the value of the current innovation, V. In order to understand why this should be so, recall that wealth other than wage income stems solely from holding shares in innovating firms. Since innovators' tenure under monopoly lasts for the current period only, the value of such firms coincides with the value of the current innovation V. This in turn is equal to wa because of the free entry into RnD condition (see equation 31). Therefore we can write: $v = wa$. Notice that the equilibrium v in monopoly depends positively on wages, but does not depend on inequality x. We recall (see equation 22) that the threshold level of inequality that determines the passage from a monopoly of the highest quality to a duopoly of two qualities is[9]:

$$x^* \leq \frac{1}{3}\left(v(w) + \frac{wL(1-c)}{\rho}\right)$$

(52)

We can see that:

$$x^*(w_L) < x^*(w_H)$$

(53)

Where w_L and w_H are low wages and high wages, respectively. This situation is represented in Figure 5. We can see that for low wages, the switch occurs at a lower level of inequality. When wages are high, monopoly can persist even for relatively high level of inequality.

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[9] In this section, the superscript "*" indicates equilibrium values of the variable concerned, while the subscripts "L" and "H" stand for "low" and "high".

In terms of wages, the threshold level is obtained simply by rearranging equation (52) to get:

$$w^* \geq \frac{\rho}{L(1-c)}(3x - v(w^*)) \Leftrightarrow w^* \geq \frac{3\rho x}{L(1-c) + a\rho} \tag{54}$$

Here, as x increases, there is only a direct effect on w^*, because the equilibrium v does not depend on x in monopoly. Thus we have:

$$w^*(x_L) < w^*(x_H) \tag{55}$$

This situation is shown in Figure 6.

Figure 5 - n = f(x/w) - Extension of Tenure by n = 1 Due to a Higher w

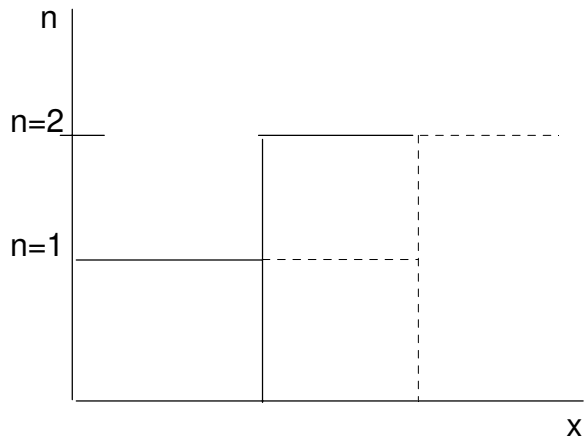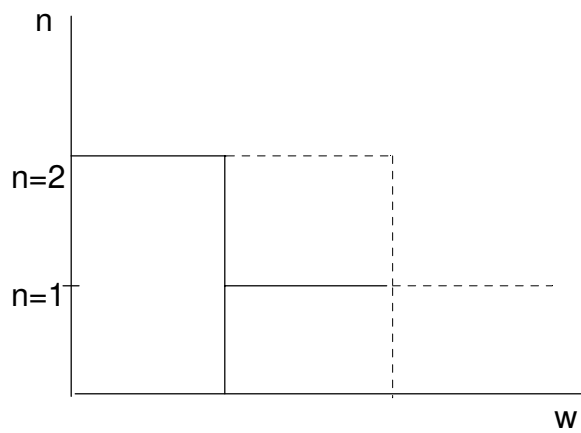*Figure 6 - n=f(w/x) Extension of Tenure by n = 2 Due to a Higher x*

We see there that, when inequality is low, an exogenous rise in wages produces the switch from two qualities to one. When inequality is high, a larger rise in wages is needed for a switch from two qualities to one to take place.

This model would explain why countries were inequality is rather high, but so too are wages (like the U.S.) are characterized by a monopoly of the quality good markets, while some less unequal societies, which also have a lower wage, have duopoly in the markets for quality goods (we may think of Eastern Europe or China).

9 The Case of Duopoly in the Market for Quality Goods: a Numerical Example

At this point we construe a numerical example based on the system of equations (49) and (50) which are repeated below for convenience:

$$\mu = \mu_1(v) = \frac{\pi_2(v) - aw\rho}{2aw - v} \tag{56}$$

$$\mu = \mu_2(v) = \frac{\pi_2(v) + \pi_1(v) - \rho v}{aw} \tag{57}$$

This example may be of interest per se and it may contribute to giving a clearer illustration of the workings of the model. First, we need to calibrate the set of exogenous parameters $\{w, a, \rho, c, L, \gamma, x\}$. Although most of these parameters have been explained when they were first introduced, as a matter of convenience, they will be defined again here as they are assigned the calibrated values. The first parameter to be calibrated is the income from labor, w,

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which, as you may recall, had been assumed to be equal for everybody. For simplicity, we set w = 1. Then we have the labor coefficient in the research sector, a. There is not an immediately available criterion upon which to base the calibration of this parameter. It should be said, however, that in duopoly the product aw constitutes the lower bound for the value of total other wealth v, hence, given the value assigned to w, it seems reasonable to assign a value greater than 1 to the research labor coefficient a. We settle for a = 10. As for the next parameter, the rate of time preference, ρ, it is generally believed to be the same as the long term real interest rate. We think that a reasonable figure for the latter is ρ = 0.03. Table 1 below and Fig. 7 are constructed on the basis of this value for ρ. In Fig. 8, the entire numerical example is re-run after setting ρ = 0.05. This is done in order to check that the graphical output in its substance is not sensitive to small changes in the value of ρ. The case with $\rho = 0.05$ has the additional advantage of yielding graphs that are more clearcut. Another parameter to be calibrated is c, the labor coefficient in manufacturing the quality good, which should be assigned a value less than 1. This is in order to avoid that the labor coefficient in all sectors other than manufacturing the quality good, 1-c, turns negative, which would make no sense. Among all the possible values in the interval [0,1], we decided to pick c = 0.7. We now come to L, the labor force. Perhaps for this parameter, more than for any other, just any value would do. Here we take into account the fact that the product wL, the total wage that the population L is able to earn from labor, should not have a very different order of magnitude than the value of total wealth other than wage income v. The product wa can be thought of as a first approximation for v, since it constitutes its lower bound. In order to have the two quantities v and wL not to be of very different scales, we set L = 10.

Having calibrated the subset of exogenous parameters $\{w, a, \rho, c, L\}$, our strategy is to use the remaining two exogenous variables γ and x to study how the model responds to exogenous shocks.

The choice of γ, which measures the size of the technological leap that occurs with the arrival of a new higher quality product, is motivated by the fact that γ represents the size of the technological advantage that the technological leader has on her closest rival. As such, γ is a very important parameter in determining the incentive to innovate. It is reasonable to expect that, when everything else is held constant, the bigger the technological advantage, the higher will be the rate of innovative activity μ in equilibrium. We imagine two different settings: the first, termed "economy with small technological leaps" is characterized by a $\gamma = 1.3$. The second, which we call "economy with big technological leaps" is characterized by a $\gamma = 2$. Finally, we need to calibrate x, the inequality parameter, in such a way that enables us to study the behavior of the economy as x changes exogenously, in a meaningful way. In order to do so, we have to recall that our aim with the present numerical example is to illustrate how the model works under duopoly. For duopoly to obtain in equilibrium, we must choose a value of x that satisfies the complement to condition (22), as stated in footnote 3, as reported below:

$$x > \frac{1}{3}\left(v + \frac{wL(1-c)}{\rho}\right) \tag{58}$$

Given the values assigned to the parameters so far, this is equivalent to ensuring that $x > \frac{1}{3}\left(10 + \frac{10(1-0.7)}{0.05}\right) \Longleftrightarrow x > 23.33$. Furthermore, to ensure that equation (49) is positively sloped, the numerator of the right hand side of that equation must be positive, which occurs if

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$x > 60$. We consider two different scenarios. In the first, which we call "low inequality economy", we set $x = 70$. In the second, termed "high inequality economy", we set $x = 100$. To recap, given the way we have also classified the possibilities according to different values of γ, we end up with four different possible scenarios: a small leap, low inequality economy; a small leap, high inequality economy; a big leap, low inequality economy; and a big leap, high inequality economy. The following table describes the equilibrium values of the variables "wealth other than wage income", v, and "rate of innovative activity", μ, in these four different situations:

Table 1 - Equilibrium values of v and μ in the four states of the economy, given the following values for the remaining exogenous parameters:: $w = 1$; $a = 10$, $\rho = 0.03$; $c = 0.7$; $L = 10$

	The four possible states of the economy			
	small leap, low inequality $\gamma=1.3$, $x=70$	small leap high inequality $\gamma=1.3$, $x=100$	big leap low inequality $\gamma=2$, $x=70$	big leap high inequality $\gamma=2$, $x=100$
v	11.15	12.32	11.61	13
μ	0.03	0.065	0.111	0.161
$\pi_1(v)$	0.07	0.18	0.23	0.58
$\pi_2(v)$	0.57	0.65	1.23	1.42

The same results can also be viewed in the four panels of Figures 7 and 8, for the two cases that $\rho = 0.3$ and $\rho = 0.5$, respectively. As expected and as predicted by the theory, as inequality increases, in duopoly we observe a rise in the rate of research intensity μ (this is proposition 4 of Section 7). The latter also increases when the technological advantage held by the top-quality is bigger (i.e., when the distance γ between steps on the quality ladder is larger). The economic argument on which proposition 4 rests is that in a segmented, duopolistic market for quality goods, rising income inequality segments that market even further in such a way that the incumbents in the respective segments can enjoy more market power, because of weaker competition from the other segment. There derives an ability to extract higher rents which

motivates people who work in the research lab to increase their innovation effort. Such larger innovation intensity results in faster technical progress and higher rates of growth for the economy. As for the fact that innovation intensity μ and growth rate g are also increasing in the distance between steps on the quality ladder, γ, this depends on the fact that the profit function for the producer of the top quality good increases in γ. The higher profitability of the top quality product results in a higher μ through the usual incentive mechanism. It is interesting to notice that, given the structure of the growth equation (42), an increase in γ raises g both directly, because it explicitly appears in (42), and indirectly, through the increase in the innovation intensity μ.

Given the relationship between the rate of research intensity μ and the rate of economic growth g, as described in equation (42), it is possible to conclude that, as shown by this numerical example and in line with proposition 4, as the degree of income inequality increases, the rate of growth of such an economy should rise.

Figure 7 - A Numerical Example with Duopoly in the Market for Quality Goods. The Case with
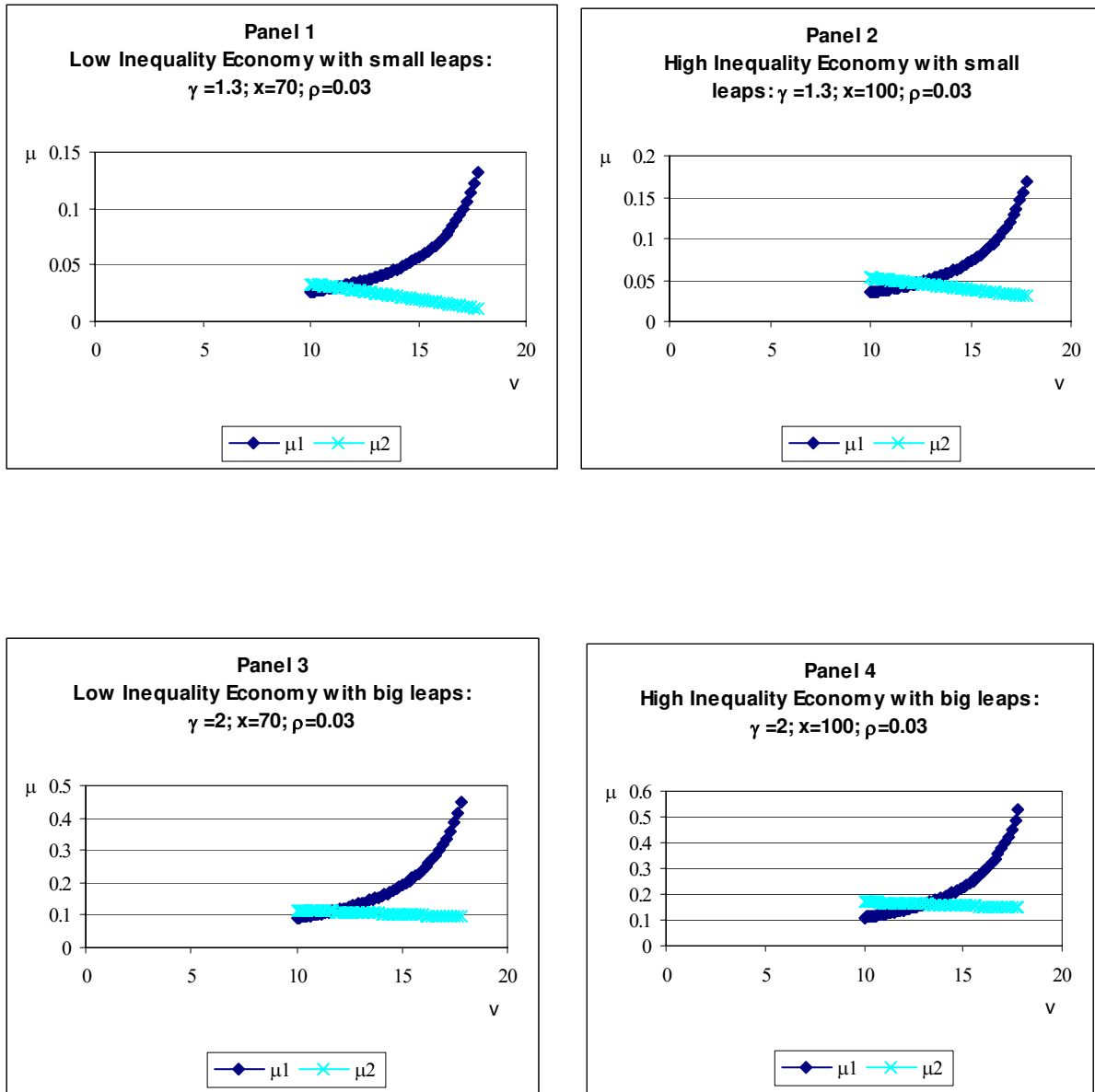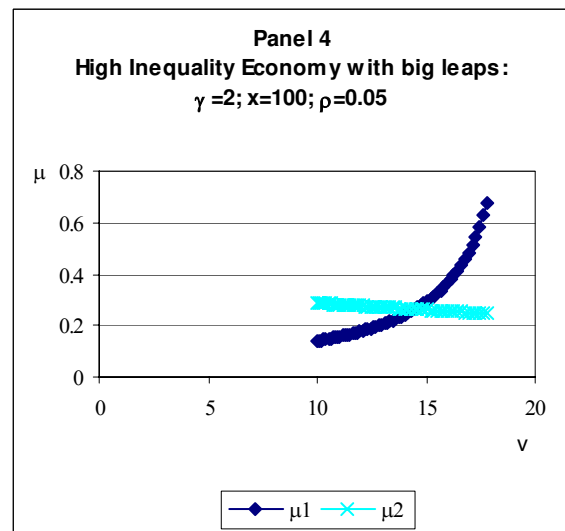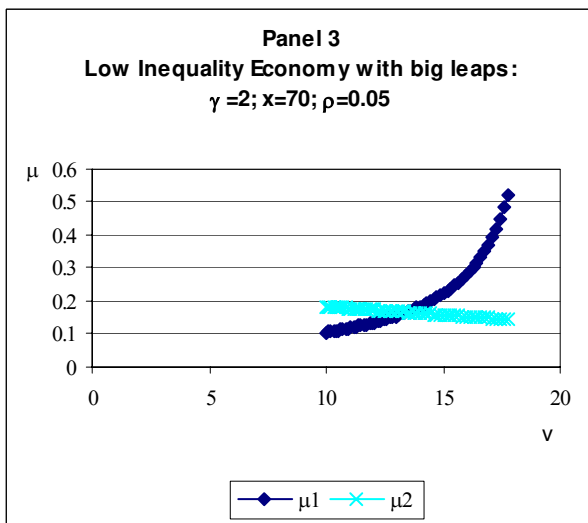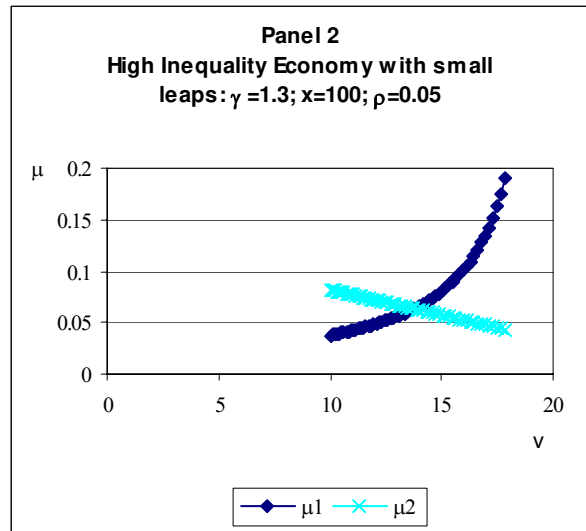
$\rho = 0.3$

Figure 8 - A Numerical Example with Duopoly in the Market for Quality Goods. The Case with

$\rho = 0.5$

Panel 1
Low Inequality Economy with small leaps:
$\gamma = 1.3$; x=70; ρ=0.05

Panel 2
High Inequality Economy with small leaps: $\gamma = 1.3$; x=100; ρ=0.05

Panel 3
Low Inequality Economy with big leaps:
$\gamma = 2$; x=70; ρ=0.05

Panel 4
High Inequality Economy with big leaps:
$\gamma = 2$; x=100; ρ=0.05

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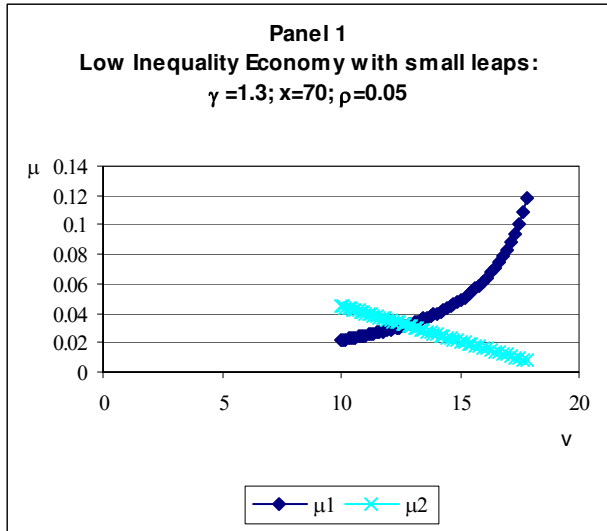
10 Conclusion

This chapter has developed a quality ladder model characterized by non homothetic consumer preferences, in line with the vertical quality differentiation literature. Such modeling of consumer preferences in the context of a quality ladder model is the novel element of this study, and it is an attempt to explain real world phenomena like the survival of older generations of goods along with the state-of-the-art, something which was not accounted for in the mainstream quality ladder models of Grossman et al. (1991) and Aghion et al. (1992).

The next step in this line of research might be to introduce such preferences in open economy quality ladder models, such as that of Grossman et al.(1991, chp.12). Indeed, in an open economy framework, the possibility of having monopoly of the best quality in a richer North with a higher and less unequal distribution of wealth, and duopoly in a poorer South with a lower and more widespread distribution of wealth, might be obtained as an endogenous outcome of the model.

Such a model would then make it possible to study equilibrium outcomes and comparative statics involving changes in the spread of the wealth distribution, much like has been done in the closed economy model developed in the present chapter.

Appendix A.1

Proof of proposition 1:

We plug monopoly profits into equations (43) and (44) so to have them in explicit form:

$$wa = \frac{\frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)]}{\rho+\mu} \tag{43}$$

$$w\mu a = \frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)]-\rho v \tag{44}$$

It is useful to rewrite (43) and (44) as follows:

$$A'(\mu,v)=0 \Leftrightarrow \frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)]-wa(\rho+\mu)=0$$
$$\tag{A.1}$$
$$L'(\mu,v)=0 \Leftrightarrow \frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)]-\rho v-w\mu a=0$$

Proof of proposition 1 will be done in three steps:

Step 1: we shall prove that $\frac{dv}{d\mu}\big|_{A'}>0$,

Step 2: we shall prove that $\frac{dv}{d\mu}\big|_{L'}<0$,

Step 3: we shall write down the condition under which $v_{A'}(\mu)\big|_{\mu=0}<v_{L'}(\mu)\big|_{\mu=0}$.

In step 1:

$$\frac{dv}{d\mu}\Big|_{A'} = -\frac{-wa}{\frac{\gamma-1}{\gamma}\rho} > 0 \qquad (A.2)$$

In step 2:

$$\frac{dv}{d\mu}\Big|_{L'} = -\frac{-wa}{-\frac{1}{\gamma}\rho} < 0 \qquad (A.3)$$

In step 3: to compute $v_{A'}(\mu)\,|_{\mu=0}$ we set $\mu = 0$ in (A.1) and solve for v:

$$\frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)]-wa\rho = 0 \Leftrightarrow v_{A'}(\mu)\,|_{\mu=0} = x + \frac{\gamma}{\gamma-1}wa - \frac{wL(1-c)}{\rho} \quad (A.4)$$

We compute $v_{L'}(\mu)\,|_{\mu=0}$ in the same way:

$$\frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)]-\rho v = 0 \Leftrightarrow v_{L'}(\mu)\,|_{\mu=0} = (\gamma-1)\left[\frac{wL(1-c)}{\rho}-x\right] \quad (A.5)$$

A unique equilibrium with a positive pair (v, μ) exists if and only if:

$$x + \frac{\gamma}{\gamma-1}wa - \frac{wL(1-c)}{\rho} < (\gamma-1)\left[\frac{wL(1-c)}{\rho}-x\right] \Leftrightarrow x < \frac{wL(1-c)}{\rho} - \frac{1}{\gamma-1}wa. \quad (A.6)$$

Proof of Proposition 2: This proposition can be proved in two ways.

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Proof 1: an increase in inequality x, as measured by: $\dfrac{\partial \mu}{\partial x}\mid_{A'} = -\dfrac{-\frac{\gamma-1}{\gamma}\rho}{-wa} < 0$, and by:

$\dfrac{\partial \mu}{\partial x}\mid_{L'} = -\dfrac{-\frac{\gamma-1}{\gamma}\rho}{-wa} < 0$, moves to the left both the (43) and the (44) schedule. As a result, we

have a new equilibrium with less research intensity μ and growth g (recall that $g = \mu \ln \gamma$).

Furthermore, since the effect of an increase in x on both schedules is of the same magnitude, at

the new equilibrium, the level of other wealth v is unchanged.

Proof 2: In a monopoly equilibrium, the value of other wealth v is determined by the value of the

most recent innovation. As shown in the text, the latter is equal to the costs faced by the most

recent innovator: $v = wa$. We plug this result into (43) and (44), to get the same relationship:

$$\frac{\gamma-1}{\gamma}[\rho(wa-x)+wL(1-c)]-wa(\rho+\mu)=0 \qquad\qquad (43, 44)$$

We then solve for μ to get:

$$\mu = \frac{\gamma-1}{\gamma}\left[\frac{L(1-c)}{a}-\frac{\rho x}{wa}\right]-\frac{1}{\gamma}\rho \qquad\qquad (A.7)$$

From the above it is obvious that μ decreases in x.

Proof of Proposition 3:

The proof is done in three steps. Schedules (46) and (47) implicitly define functions $v = v_A(\mu)$

and $v = v_L(\mu)$.

Step 1: prove that $\frac{dv}{d\mu}|_A > 0$.

Rewrite (46) as follows:

$$G(\mu, v) = 0 \iff -wa + \frac{\pi_2(v)}{\rho + \mu} + \frac{\mu \pi_1(v)}{(\rho + \mu)^2} = 0 \qquad (A.8)$$

Then, by the implicit function theorem:

$$\frac{dv}{d\mu}|_A = -\frac{\frac{\partial G}{\partial \mu}}{\frac{\partial G}{\partial v}} \qquad (A.9)$$

We find that

$$\frac{\partial G}{\partial \mu} = -\frac{\pi_2}{(\rho + \mu)^2} - \frac{\pi_1(\rho - \mu)}{(\rho + \mu)^3} \qquad (A.10)$$

We set $\frac{\partial G}{\partial \mu} = 0$ and solve for μ to get $\hat{\mu} = \rho \frac{\pi_1 - \pi_2}{\pi_1 + \pi_2}$. Notice that, in duopoly, $\hat{\mu} < 0$ since

$\pi_1 < \pi_2$.

As $\frac{\partial G}{\partial \mu} < 0$ for $\mu > \hat{\mu}$ and as we are only interested in non-negative values of μ, we conclude that

$\frac{\partial G}{\partial \mu} < 0$ in the relevant range.

Next, we compute

$$\frac{\partial G}{\partial v} = \frac{\frac{\partial \pi_2}{\partial v}}{\rho + \mu} + \frac{\mu \frac{\partial \pi_1}{\partial v}}{(\rho + \mu)^2} = \frac{1}{\rho + \mu}\left[\frac{\partial \pi_2}{\partial v} + \frac{\mu \frac{\partial \pi_1}{\partial v}}{\rho + \mu}\right] \qquad (A.11)$$

which takes the sign of $\dfrac{\partial \pi_2}{\partial v} + \dfrac{\mu \frac{\partial \pi_1}{\partial v}}{\rho + \mu}$

We find that

$$\frac{\partial \pi_2}{\partial v} = \frac{\gamma - 1}{\gamma}\rho\frac{x + \frac{v}{3} + \frac{wL(1-c)}{3\rho}}{3x} > 0 \qquad (A.12)$$

and

$$\frac{\partial \pi_1}{\partial v} = (\gamma - 1)\rho\frac{x - \frac{v}{3} - \frac{wL(1-c)}{3\rho}}{x}\left(-\frac{1}{3}\right) < 0 \qquad (A.13)$$

However

$$\frac{\partial \pi_2}{\partial v} + \frac{\partial \pi_1}{\partial v} = x + \frac{v}{3} + \frac{wL(1-c)}{3\rho} + \gamma\left(x - \frac{v}{3} - \frac{wL(1-c)}{3\rho}\right) > 0 \qquad (A.14)$$

in duopoly, as the expression in brackets is guaranteed to be positive with such a market structure.

$\dfrac{\partial \pi_2}{\partial v} + \dfrac{\mu \frac{\partial \pi_1}{\partial v}}{\rho + \mu} > 0$ follows from $\dfrac{\mu}{\rho + \mu} < 1$ Hence $\dfrac{\partial G}{\partial v} > 0$. The latter enables us to conclude that

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$$\frac{dv}{d\mu} \,|_A = -\frac{\dfrac{\partial G}{\partial \mu} < 0}{\dfrac{\partial G}{\partial v} > 0} > 0 \tag{A.15}$$

Step 2: prove that $\dfrac{dv}{d\mu}\,|_L < 0$

Rewrite (47) as

$$H(\mu, v) = 0 \iff \frac{\pi_2(v) + \pi_1(v) - \rho v}{\mu} - wa = 0 \tag{A.16}$$

By the implicit function theorem: $\dfrac{dv}{d\mu}\,|_L = -\dfrac{\dfrac{\partial H}{\partial \mu}}{\dfrac{\partial H}{\partial v}}$

First, we compute $\dfrac{\partial H}{\partial \mu} = -\dfrac{\pi_2(v) + \pi_1(v) - \rho v}{\mu^2}$

$\dfrac{\partial H}{\partial \mu} < 0$ follows from $\pi_2(v) + \pi_1(v) - \rho v = \mu wa > 0$ (see (47)).

Next, we compute $\dfrac{\partial H}{\partial v} = \dfrac{\dfrac{\partial \pi_2}{\partial v} + \dfrac{\partial \pi_1}{\partial v} - \rho}{\mu}$

We need to determine the sign of $\dfrac{\partial \pi_2}{\partial v} + \dfrac{\partial \pi_1}{\partial v} - \rho$

A few calculations reveal that

$$\frac{\partial \pi_2}{\partial v} + \frac{\partial \pi_1}{\partial v} - \rho < 0 \tag{A.17}$$

provided that

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$$x > \frac{1-\gamma}{2-\gamma}\left[\frac{v}{3} + \frac{wL(1-c)}{3\rho}\right] \tag{A.18}$$

Since x is always positive, this condition certainly holds if its RHS is negative. That is indeed the

case if $1 < \gamma < 2$.

Then we have that $\dfrac{\partial H}{\partial v} < 0$.

Therefore, we conclude that for $1 < \gamma < 2$,

$$\frac{dv}{d\mu}\Big|_L = -\frac{\dfrac{\partial H}{\partial \mu} < 0}{\dfrac{\partial H}{\partial v} < 0} < 0 \tag{A.19}$$

Step 3: a unique equilibrium with a positive pair (μ, v) and a positive growth rate g exists if and

only if

$$v_A(\mu)\,|_{\mu=0} < v_L(\mu)\,|_{\mu=0} \tag{A.20}$$

Where

$$v_A(\mu)\,|_{\mu=0} = 3\left(wa\frac{\gamma}{\gamma-1}2x\right)^{\frac{1}{2}} - 3x - \frac{wL(1-c)}{\rho} \tag{A.21}$$

and $v_L(\mu)\,|_{\mu=0}$ is implicitly determined in (47) with $\mu = 0$.

Appendix A.2 - List of symbols

Y = output
A = technology
x = intermediate input to production (quantity)
D = consumption index
q = quality good
$i = \{1, 2\}$ = quality index
I = income
p_i = price of a good of quality i
L = population
γ = distance between two steps on the quality ladder
h = homogeneous, composite good
u_t = utility in period t
r_t = interest rate at time t
ρ = rate of time preference
k = assets
w = wage
C = consumption
$j = \{l, .., eq, .., m, .., u\}$ = index of consumption patterns, where l is poorest, eq is equilibrium, m is mean, and u is richest consumption pattern respectively
v = total wealth, defined as $\frac{k_u + k_l}{2} L$
x = spread in wealth, defined as $\frac{k_u - k_l}{2} L$
D_i = demand for good of quality i
$F(\cdot)$ = cumulative distribution function of consumption patterns
$f(\cdot)$ = density function of consumption patterns
π_i = profit accruing to firm producing good of quality i
c = labor coefficient in manufacturing sector
e (superscript) = indicates an equilibrium quantity
M (subscript) = indicates monopoly
n (superscript) = indicates next period
μ = arrival rate of innovations, innovation intensity
V = value of an innovation
a = labor coefficient in the research sector
A (subscript) = indicates that subscripted symbol refers to the research sector

In section 3.8:
n = number of qualities
L (subscript) = low quality
H (subscript) = high quality

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