Profit for Marxists

Kakarot-Handtke, Egmont

University of Stuttgart, Institute of Economics and Law

26 March 2014

Profit for Marxists

Egmont Kakarot-Handtke*

Abstract

Marxian economics and standard economics are widely different yet they share a central weakness: the respective profit theories are demonstrably false – each one in its own characteristic way. Roughly speaking, Marx tried to explain profit by objective factors while standard economics cites subjective factors. For different reasons, neither route led to satisfactory results. The conclusion is straightforward: one has to do better. The conceptual consequence is to first reconstruct the profit theory from a solid basis with no regard to either Marxian or standard premises. In order to succeed, objective-structural axioms have to be taken as formal point of departure.

JEL B59, E11, E25

Keywords new framework of concepts; structure-centric; axiom set; profit theory; surplus value; distribution; real shares

*Affiliation: University of Stuttgart, Institute of Economics and Law, Keplerstrasse 17, 70174 Stuttgart, Germany. Correspondence address: AXEC-Project, Egmont Kakarot-Handtke, Hohenzollernstraße 11, 80801 München, Germany, e-mail: handtke@axec.de

1 United in Error

> We have to do better than that. We have to start with foundations from which the phenomena of reality emerge naturally by constructing monetary models of capitalism built on the melded visions of Marx, Schumpeter, Keynes and Minsky. (Keen, 2011, p. 358)

Marxian economics and standard economics are widely different yet they share a central weakness: the respective profit theories are demonstrably false – each one in its own characteristic way. For Marx, the main explanans was the production conditions, for standard economics it is optimizing behavior. Roughly speaking, Marx tried to explain profit by objective factors while standard economics cites subjective factors. For different reasons, neither route led to satisfactory results (the detailed reasons for the standard approach are given in 2014c).

There is no need for further exegesis and interpretation of Marx's approach. All this has been done exhaustively. The conclusion from what is already known is straightforward: one simply has to do better. The conceptual consequence of the present paper is to first reconstruct the profit theory from a solid basis with no regard to either Marxian or standard premises. More specifically, we take objective-structural axioms as the formal point of departure.

What has to be ensured is formal and material consistency of the centerpiece of theoretical economics. From a blending of visions not much is to be expected. There is nothing wrong with visions, to be sure, but at some point they must come down to earth and that means that, in order to be comparable, their cores have to be articulated in a formal rigorous way. There is too much rigmarole in economics. It is beyond dispute that, if the profit theory is formally defective, the rest of a theoretical edifice is unusable. This applies to both the standard and the Marxian approach.

Real theoretical progress seldom comes from the lengthy elaboration of flaws, it consists in developing a superior alternative. This is done in the following.

Section 2 presents the intuitive graphical reminder that profit is the pivotal concept in economics. Section 3 then provides the formal foundations with the set of four structural axioms. These represent the evolving consumption economy as the most elementary economic configuration. Nonentities like equilibrium are excluded from the premises. In Section 4 monetary profit, surplus value, and the real shares of the receivers of wage income and distributed profit are derived. The intricate profit mechanism is entangled in detail. Section 5 clarifies the relation between exploitation and profit. Section 6 concludes.

2 Looking for profit

> Granting that Marx was wrong, what is the correct explanation of aggregate profit? Under what conditions does it arise? Why is aggregate profit now large, now small or negative? (Murad, 1953, p. 7)

Every economist readily confirms that the market system is about profit. However, profit seldom makes an explicit appearance in the standard models. And it is a curious fact that profit is not categorized once in the widely used JEL Classification Code of economic subjects. In Marx's scheme of things profit appears center stage, in the standard plot it stays most of the time in the coulisses. To recall, Walras's original model, which became paradigmatic, represents a zero profit economy.

The most popular definition that relates income and profit is total income = wages plus profits. This definition seems to be rather obvious, yet, like most commonsensical relations it does not hold water. Income and profit are not on the same analytical plane. A closer look at the basics makes this abundantly clear.

2.1 Zero profit as the base line

Figure 1 shows the simplest possible configuration of the pure consumption economy. This absolute formal minimum cannot be outmatched.

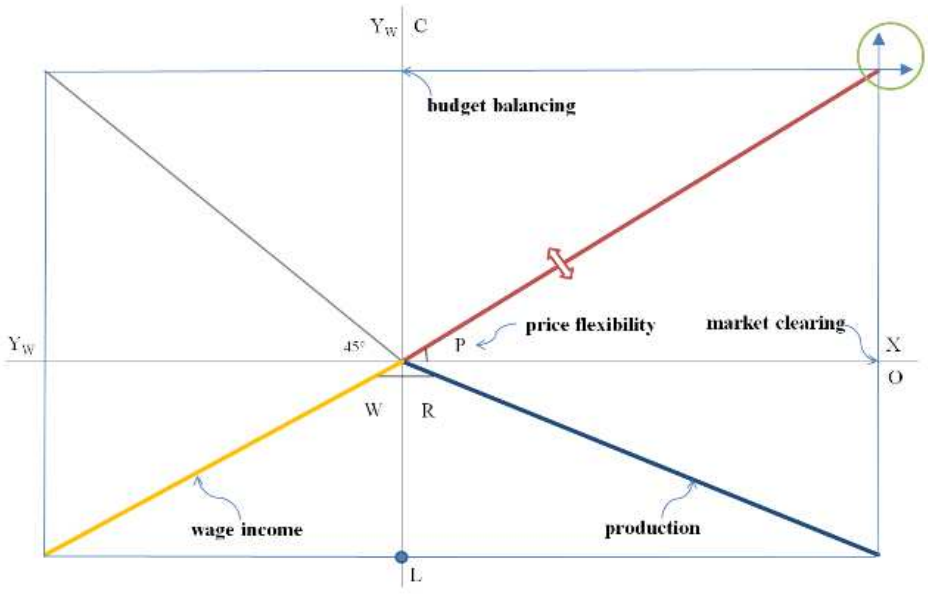


Figure 1: The price in the pure consumption economy in period $t = 1$ is objectively determined by the conditions of market clearing and budget balancing. Legend: P price, L employment, W wage rate, Y_W wage income, C consumption expenditure, R productivity, O output, X quantity bought/sold

At any given level of employment L, the wage income Y_W that is generated in the consolidated business sector follows by multiplication with the (average) wage rate W. On the real side output follows by multiplication with the productivity. Finally, the price follows as the dependent variable under the conditions of budget balancing, i.e. $C = Y_W$, and market clearing, i.e. $X = O$. Note that the ray in the southeastern quadrant is *not* a linear production function; the ray tracks *any* underlying production function. The same holds for the distribution of wage incomes in the southwestern quadrant. All those details are not needed at the moment.

It can be directly read off from the 4-quadrant scheme that the real wage $\frac{W}{P}$ is always equal to the productivity R, that is, labor gets the whole product. If the wage rate is lowered, the market clearing price falls. If the number of working hours is increased the price remains constant, provided productivity does not change. If productivity decreases the price rises. In any case, labor gets the whole product and profit is zero, or in Walras's terms, there is 'ni bénéfice ni perte', neither profit nor loss.

> The consensus to date has been that it is mathematically impossible for capitalists in the aggregate to make profits. (Keen, 2010, p. 2)

Marx, too, encountered the zero profit economy.

> Our capitalist stares in astonishment. The value of the product is exactly equal to the value of the capital advanced. The value so advanced has not expanded, no surplus-value has been created, and consequently money has not been converted into capital. (Marx, 1906, III.VII.43)

The weak spot in the otherwise impeccable zero-profit argument is that aggregate profit has been greater than zero for most of the time in most of the known market economies up to the present. Clearly, Figure 1 tells not all that is interesting about profit.

2.2 The emergence of profit/loss

There exists no such thing as an immutable law of budget balancing in the *same* period. As a matter of fact, the budget is *never* balanced. Logically, we have three possible cases in the next period: $C_2 < Y_{W2}$, $C_2 = Y_{W2}$, $C_2 > Y_{W2}$. The first case means loss, the second zero profit, and the third profit. Figure 2 shows an example for the third case.

In the pure consumption economy, profit can at first only be greater than zero if consumption expenditure is greater than wage income. This configuration has historically been realized in various ways, the ordinary way is that the household sector takes up credit from the banking industry (for details see 2013, Sec. 18). One pertinent example is the purchase of long-lived consumption goods like family

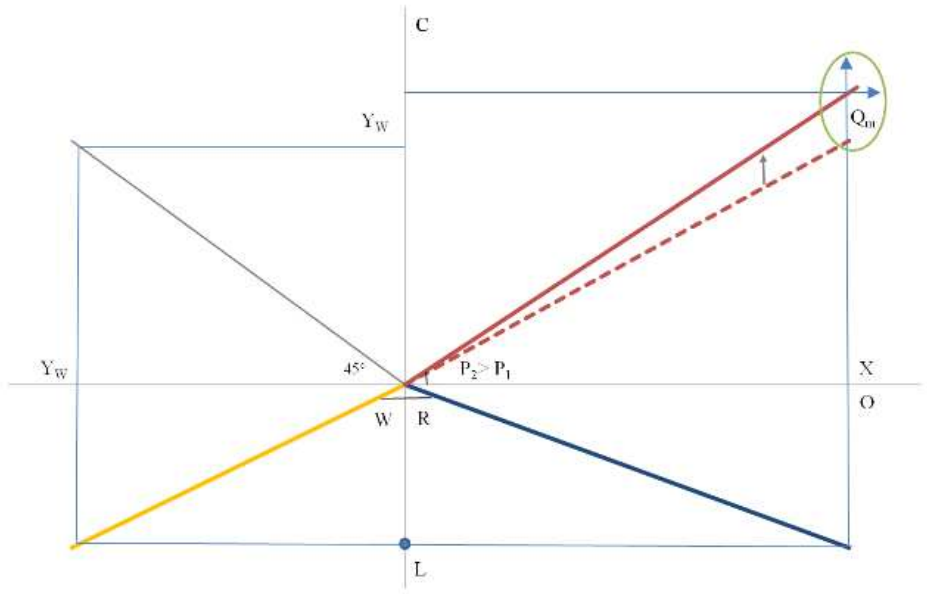


Figure 2: Monetary profit in period $t = 2$ is objectively determined by the difference between consumption expenditure and wage income under the condition of market clearing

homes on credit. The relation between credit expansion of the household sector and profit for the business sector is measurable in principle (Keen, 2011, pp. 337-353).

In the case of Figure 2 monetary profit is given as $Q_{m2} \equiv C_2 - Y_{W2}$ in the northeastern quadrant. Profit takes the form of money in the bank and remains in the business sector in the period under consideration, i.e. profit is fully retained. Monetary as well as retained profit are measurable with an accuracy of two decimal digits. A glance at the statement of bank account suffices. There is not the slightest ambiguity about what monetary profit is.

Note well that by looking at the northeastern quadrant it is quite commonsensical from the viewpoint of the single firm to conclude that, given $P > {}^{\mathrm{W}}/_{\mathrm{R}}$ or price greater than unit wage costs, that profit increases steadily with increasing labor input/product output. This, though, is a manifest fallacy of composition because profit does not depend on overall working hours but on the difference between consumption expenditures and wage income. If this difference remains constant profit for the business sector as a whole remains constant independently of how many workers work how many hours.

Note also that the households have nothing to do with profit until the next logical step has been taken. It consists in the decision to distribute all, part, or nothing of profit. Nondistributed profit is no part, neither factual nor virtual, of the household sector's income. To assume that profit, whether distributed or not, is a component of household income is the fatal blunder of the standard approach. These kind of assumptions cannot be justified and have to be flatly rejected.

2.3 No psycho-sociology of profit

The individual firm is blind to the structural relationships as shown in Figure 2. On the firm's level, profit is therefore subjectively interpreted as a reward for innovation or superior management skills or higher efficiency or toughness on wages or for risk taking or capitalizing on market imperfections or as the result of monopolistic practices or whatever else. These factors can play a role when it comes to the *distribution* of profits *between* firms and these phenomena become visible when similar firms of an industry are compared. Firms do not create profit, they redistribute it.

> The enterprise which has better management, better luck, superior resources, a better product, no competitors, and so on, is likely to make more profit than the enterprise without these advantages. Not much more can be said about the sources of particular profit without elaborating the obvious. (Murad, 1953, pp. 6-7)

The case is perfectly clear when there is only one firm. It is a matter of indifference whether the firm's management thinks that it needs profit to cover risks or to finance growth or whether it realizes the profit maximum or not. If consumption expenditures are equal to wage income, profit will invariably be zero, no matter what the agents want or plan. Hence there is no need to speculate about it. Profit for the business sector as a whole is a systemic property. Psychologism, as ever, explains nothing. Whether Marx's assertion

> The restless never-ending process of profit-making alone is what he [the capitalist] aims at. (Marx, 1906, II.IV.19)

is true or false does not matter. Whether profit-making is considered as good or bad does not matter either. Moralizing, as ever, explains nothing. Profit is not determined by trivial psychology but by the Profit Law (19).

From the elementary graphical analysis follows:

- The business sector's revenues can only be greater than costs if, in the simplest of all possible cases, consumption expenditures are greater than wage income.
- In order that profit comes into existence for the first time in the pure consumption economy the household sector must run a deficit at least in one period.
- Profit is, in the simplest case, determined by the increase and decrease of household sector's debt.
- Wage income is the factor remuneration of labor input L. Profit is not a factor income, neither is loss. Since capital is nonexistent in the pure consumption economy profit is not functionally attributable to capital.

- Profit has no real counterpart in the form of a piece of the output cake. Profit has a monetary counterpart.
- The existence and magnitude of overall profit does not depend on profit maximizing behavior of the business sector but solely on the relation of consumption expenditure to wage income.
- The value of output is, in the general case, different from the sum of factor incomes. This is the defining property of the monetary economy.

From the graphics follow some essentials about profit intuitively. With regard to Marxian theory the most important point is that there is no relationship between profit and capital. With regard to standard theory the most important point is that there is no relationship between individual optimizing behavior and overall profit. What is needed next is a rigorous formal underpinning.

3 Simpler than you thought

> Others, the inexperienced students, make guesses that are very complicated, and it sort of looks as if it is all right, but I know it is not true because the truth always turns out to be simpler than you thought. (Feynman, 1992, p. 171)

The formal foundations of theoretical economics must be nonbehavioral and epitomize the interdependence of the real and nominal variables that constitutes the monetary economy.

3.1 Axioms

The first three structural axioms relate to income, production, and expenditure in a period of arbitrary length. The period length is conveniently assumed to be the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product. Axiomatization is about ascertaining the *minimum* number of premises.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N. Nothing is implied at this stage about who owns the shares.

$$Y = WL + DN \quad |t \tag{1}$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function; as a matter of fact, it tracks any production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment, no foreign trade, and no government.

The period values of the axiomatic variables are formally connected by the familiar growth equation, which is added as the 4th axiom.

$$\begin{gathered} Z_t = Z_{t-1}\left(1+\dddot{Z}_t\right) \\ \text{with} \quad Z \leftarrow W, L, D, N, R, P, X, \ldots \end{gathered} \tag{4}$$

The path of the representative variable Z_t is then determined by the initial value Z_0 and the rates of change $\ddot{Z}_t$ for each period.

For a start it is assumed that the elementary axiomatic variables vary at random. This minimalistic assumption produces an evolving economy. The respective probability distributions of the change rates are given in general form by:

$$\begin{array}{cc} Pr\left(l_W \leq \dddot{W} \leq u_W\right) & Pr\left(l_R \leq \dddot{R} \leq u_R\right) \\ Pr\left(l_L \leq \ddot{L} \leq u_L\right) & Pr\left(l_P \leq \ddot{P} \leq u_P\right) \\ Pr\left(l_D \leq \ddot{D} \leq u_D\right) & Pr\left(l_X \leq \ddot{X} \leq u_X\right) \\ Pr\left(l_N \leq \ddot{N} \leq u_N\right) & |t. \end{array} \tag{5}$$

The four axioms, including (5), constitute a simulation. The simulation replaces the inoperative set of equations as analytical tool. There is no need at this early stage to discus the merits and demerits of different probability distributions, which, by the way, need not be fix over time. It is, of course, also possible to switch to a completely deterministic rate of change for any variable and any period. The structural formalism does not require a preliminary decision between determinism and indeterminism. If, for instance, the upper (u) and lower (l) bounds of the respective intervals are symmetrical around zero this produces a stationary economy as a limiting case of the growing economy.

The economic content of the four axioms is absolutely transparent: they constitute the evolving consumption economy. One point to mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit.

As will become clear in the following, this distinction makes all the difference between true or false economics.

Not admitted as foundational concepts are, for example, utility, optimization, rational expectation, and equilibrium. The first rule of theory building says: never put a behavioral assumption into the premises. No way leads from the understanding of individual behavior to the understanding of how the monetary economy works. This is why all psycho-sociological approaches miss the target.

3.2 Definitions

Income categories

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (6) wage income Y_W and distributed profit Y_D is defined:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{6}$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

Key ratios

We define the sales ratio as:

$$\rho_X \equiv \frac{X}{O} \quad |t. \tag{7}$$

A sales ratio $\rho_X = 1$ indicates that the quantity bought/sold X and the quantity produced O are equal or, in other words, that the product market is cleared.

We define the expenditure ratio as:

$$\rho_E \equiv \frac{C}{Y} \quad |t. \tag{8}$$

An expenditure ratio $\rho_E = 1$ indicates that consumption expenditures C are equal to total income Y, in other words, that the household sector's budget is balanced.

Stock of money

Money follows consistently from the given axiom set. If income is higher than consumption expenditures the household sector's stock of money increases. The change in period t is defined as:

$$\Delta\bar{\mathsf{M}}_{\mathbf{H}} \doteq Y - C \doteq (1-\rho_E)\,Y \quad |t. \tag{9}$$

The alternative identity sign $\doteq$ indicates that the definition refers to the monetary sphere. An alternative wording of (9) is: depending on the actual expenditure ratio the change of the stock of money can either be positive or negative or zero.

The stock of money $\bar{\mathsf{M}}_{\mathbf{H}}$ at the end of an arbitrary number of periods $\bar{t}$ is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\bar{\mathsf{M}}_{\mathbf{H}t} \equiv \sum_{t=1}^{t} \Delta\bar{\mathsf{M}}_{\mathbf{H}t} + \bar{\mathsf{M}}_{\mathbf{H}0}. \tag{10}$$

The changes in the stock of money as seen from the business sector are symmetrical to those of the household sector:

$$\Delta\bar{\mathsf{M}}_{\mathbf{B}} \doteq C - Y \doteq (\rho_E - 1)\,Y \quad |t. \tag{11}$$

The business sector's stock of money at the end of an arbitrary number of periods is accordingly given by:

$$\bar{\mathsf{M}}_{\mathbf{B}t} \equiv \sum_{t=1}^{t} \Delta\bar{\mathsf{M}}_{\mathbf{B}t} + \bar{\mathsf{M}}_{\mathbf{B}0}. \tag{12}$$

The development of the stock of money follows without further assumptions from the axioms and is ultimately determined by variations of the elementary variables.

Quantity of money

In order to reduce the monetary phenomena to the essentials it is supposed that all financial transactions are carried out without costs by the central bank. The stock of money then takes the form of current deposits or current overdrafts. Initial endowments can be set to zero. Then, if the household sector owns current deposits according to (10) the current overdrafts of the business sector are of equal amount according to (12) and vice versa if the business sector owns current deposits. Money and credit are symmetrical. The current assets and liabilities of the central bank are equal by construction. From its perspective the quantity of money at the end of an arbitrary number of periods is given by the absolute value either from (10) or (12):

$$\bar{\mathsf{M}}_t \equiv \left| \sum_{t=1}^{t} \Delta \bar{\mathsf{M}}_t \right| \quad \text{with} \quad \bar{\mathsf{M}}_0 = 0. \tag{13}$$

While the stock of money can be either positive or negative the quantity of money is always positive. It is assumed at first that the central bank plays an *accommodative* role and simply supports the autonomous market transactions between the household and the business sector. For the time being, money is the *dependent* variable.

From pure transaction medium to store of value

The stock of overdrafts is the *initial* form of financial liabilities and can be replaced at any time by other forms, for instance longer term mortgage loans. In other words, overdrafts represent here the complete portfolio of household sector's debt. The structure of this portfolio is certainly influenced by the varying structure of short term and long term interest rates. At the moment we are not interested in the composition of this portfolio.

The stock of deposits is the *initial* form of the household sector's portfolio of financial assets. Deposits can be replaced at any time by other forms, for example longer term savings accounts. In the following, the endless variety of forms is ignored and we deal exclusively with plain deposits and overdrafts. The rate of interest is left out for the moment.

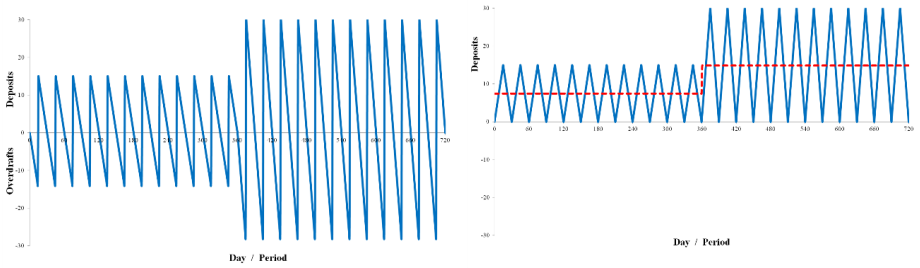
The household sector can freely switch from a positive stock of money (=deposits) to a negative stock of money (=overdrafts) and vice versa. The household sector's stock is at any time exactly mirrored by the business sector's stock. The development of the stocks depends alone on the overall expenditure ratio ρ_E if the household sector consists of a uniform population of agents who either save or dissave. If the population is composed of both savers and dissavers things are different (for details see 2014b).

The transition of the pure medium of transaction to a store of value is formally marked by the switchover from $\rho_E = 1$ to $\rho_E \neq 1$.

Transaction money

By sequencing the initially given period length of one year into months the idealized transaction pattern that is displayed in Figure 3a results.

It is assumed that the monthly income $\frac{Y}{12}$ is paid out at mid-month. In the first half of the month the daily spending of $\frac{Y}{360}$ increases the current overdrafts of the households. At mid-month the households change to the positive side and have current deposits of $\frac{Y}{24}$ at their disposal. This amount reduces continuously towards the end of the month. This pattern is exactly repeated over the rest of the year. At



(a) Transaction pattern over two periods **(b)** Average stock of transaction money $\hat{\mathsf{M}}_T$

Figure 3: Household sector's transaction pattern for different nominal incomes in two periods

the end of each sub-period, and therefore also at the end of the year, both the stock of money and the quantity of money is zero. Money is present and absent depending on the time frame of observation.

In period 2 the wage rate, the dividend and the price is doubled. Since no cash balances are carried forward from one period to the next, there results *no* real balance effect provided the doubling takes place exactly at the beginning of period 2.

From the perspective of the central bank it is a matter of indifference whether the household or the business sector owns current deposits. Therefore, the pattern of Figure 3a translates into the average amount of current deposits in Figure 3b. This average stock of transaction money depends on income according to the transaction equation:

$$\hat{\mathsf{M}}_T \equiv \kappa Y \quad |t \tag{14}$$

For the regular transaction pattern that is here assumed as a idealization the index is $\frac{1}{48}$. Different transaction patterns are characterized by different numerical values of the transaction pattern index.

By taking (14), (7) and (8) together one gets the explicit transaction equation for the limiting case of market clearing and budget balancing:

$$\begin{gathered} \text{(i)}\ \hat{\mathsf{M}}_T \equiv \kappa \frac{\rho_X}{\rho_E} RLP \quad \text{(ii)}\ \frac{\hat{\mathsf{M}}_T}{P} = \kappa O \\ \text{if} \quad \rho_X = 1, \rho_E = 1 \quad |t. \end{gathered} \tag{15}$$

We are now in the position to substantiate the notion of accommodation as a money-growth formula. According to (i) the central bank enables the average stock of transaction money to expand or contract with the development of productivity, employment, and price. In other words, the real average stock of transaction money,

which is a statistical artifact and not a physical stock, is proportional to output (ii) if the transaction index is given and if the ratios ρ_E and ρ_X are unity. Under these *initial* conditions money is endogenous and neutral in the structural axiomatic context. Money emerges from *autonomous* market transactions and has three aspects: stock of money ($\bar{\mathsf{M}}_{\mathbf{H}}$, $\bar{\mathsf{M}}_{\mathbf{B}}$), quantity of money (here $\bar{\mathsf{M}} = 0$ at period start and end because of $\rho_E = 1$) and average stock of transaction money (here $\hat{\mathsf{M}}_T > 0$).

Monetary profit

Total profit consists of monetary and nonmonetary profit. Here we are at first concerned with monetary profit. Nonmonetary profit is treated at length in (2011).

The business sector's monetary profit/loss in period t is defined with (16) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditure C – and costs – here identical with wage income Y_W:

$$Q_m \equiv C - Y_W \quad |t. \tag{16}$$

Because of (3) and (6) this is identical with:

$$Q_m \equiv PX - WL \quad |t. \tag{17}$$

This form is well-known from the theory of the firm.

The Profit Law

From (16) and (1) follows:

$$Q_m \equiv C - Y + Y_D \quad |t \tag{18}$$

or, using the definitions (7) and (8),

$$\begin{aligned} Q_m &\equiv \left(\rho_E - \frac{1}{1+\rho_D}\right) Y \\ \text{with} \quad \rho_D &\equiv \frac{Y_D}{Y_W} \quad |t. \end{aligned} \tag{19}$$

The four equations (16) to (19) are formally equivalent and show profit under different perspectives. The Profit Law (19) tells us that total monetary profit is zero if $\rho_E = 1$ and $\rho_D = 0$. Profit or loss for the business sector as a whole depends on the expenditure and distributed profit ratio and nothing else. Total income is the scale factor.

The Profit Law holds, independently of the definition of property rights, in a capitalist as well as in a communist economy. There is nothing subjective, psychological, or political in it. The Profit Law expresses the pivotal structural property of the monetary economy, that is, of all actual economies.

Individual monetary profits

For firm 1 eq. (17) reads in the case of market clearing:

$$Q_{m1} \equiv P_1 R_1 L_1 \left(1 - \frac{W_1}{P_1 R_1}\right) \quad \text{if} \quad \rho_{X1} = 1 \quad |t. \tag{20}$$

Monetary profit of firm 1 is zero under the condition that the quotient of wage rate, price, and productivity is unity. This holds independently of the level of employment or the size of the firm. From the zero profit condition follows:

$$P_1 = \frac{W_1}{R_1} \quad |t. \tag{21}$$

The price of product 1 is, in the simplest case, equal to unit wage costs.

In the same way one gets the individual profits and the zero profit market clearing prices for all other firms. With this, the structure of relative prices is determined for the most elementary case. This case is the structural-axiomatic counterpart to Walras's 'ni bénéfice, ni perte' general equilibrium. In the non-zero profit economy the derivation of the price vector is a bit more involved.

For an economy with two firms the exchange value is then given by:

$$\frac{P_1}{P_2} = \frac{\frac{W_1}{R_1}}{\frac{W_2}{R_2}} = \frac{R_2}{R_1} \tag{22}$$

$$\text{if} \quad W_1 = W_2,\, Q_{m1} = 0,\, Q_{m2} = 0 \quad |t.$$

In the most elementary case, with equal wage rates and zero profit in both lines of production, the exchange value between product 1 and 2 is determined by the productivities. Eq. (22) provides an implicit refutation of the labour theory of value.

Retained profit

Once profit has come into existence for the first time (that is: logically – a historical account is an entirely different matter) the business sector has the option to distribute

or to retain it. This in turn has an effect on profit. This effect is captured by (18) but it is invisible in (16). Both equations, though, are formally equivalent.

Retained profit Q_{re} is defined for the business sector as a whole as the difference between profit and distributed profit in period t:

$$Q_{re} \equiv Q_m - Y_D \quad \Rightarrow \quad Q_{re} \equiv C - Y \quad |t. \tag{23}$$

Retained profit is, due to (18), equal to the difference of consumption expenditures and total income. As can be seen in comparison with (11), retained profit increases *uno actu* the business sector's stock of money at the central bank.

Monetary saving

The household sector's monetary saving is given as the difference of income and consumption expenditures (for nonmonetary saving see 2011):

$$S_m \equiv Y - C \quad |t. \tag{24}$$

In combination with (23) follows:

$$Q_{re} \equiv -S_m \quad |t. \tag{25}$$

Monetary saving and retained profit always move in opposite directions. This is the Special Complementarity. It says that the complementary notion to saving is negative retained profit; positive retained profit is the complementary of dissaving. There is no such thing as an equality of saving and investment in the consumption economy, nor, for that matter, in the investment economy.

4 Profit, surplus, real shares

> Marx' demonstration of the origins of capitalist profit in his theory of surplus value became his key to the dynamics of capitalism. (Kirkenfeld, 1948, p. 33)

Marx's description of the dynamic economy has always been acknowledged as superior to that of equilibrium economics. If there is something that will never be found in the real economy then it is an equilibrium. A much better metaphor is to think of an explosion in slow motion. Marx's description is more of this sort. Nevertheless, his demonstration of the origins of profit is false. This section provides the correct formal framework.

4.1 The market clearing price

From (3), (7), and (8) follows the price as dependent variable:

$$P = \frac{\rho_E}{\rho_X} \frac{W}{R} \left(1 + \frac{Y_D}{Y_W}\right) \quad |t. \tag{26}$$

This is the general structural axiomatic law of supply and demand for the pure consumption economy with one firm (for the generalization see 2014a). In brief, the price equation states that the market clearing price, i.e. $\rho_X = 1$, is equal to the product of the expenditure ratio, unit wage costs, and the income distribution. Note that the quantity of money is *not* among the determinants. This rules the commonplace quantity theory out. The structural axiomatic price formula is testable in principle.

Under the condition of market clearing and zero distributed profit follows:

$$\begin{gathered} P = \rho_E \frac{W}{R} \\ \text{if} \quad \rho_X = 1, Y_D = 0 \quad |t. \end{gathered} \tag{27}$$

The market clearing price depends now alone on the expenditure ratio and unit wage costs. All changes of the wage rate, of the productivity, and of the average expenditure ratio affect the market clearing price in the period under consideration. We refer to this formal property as *conditional* price flexibility because (27) involves no assumption about human behavior, only the purely formal condition $\rho_X = 1$.

Under the additional conditions of budget balancing follows:

$$\begin{gathered} P = \frac{W}{R} \\ \text{if} \quad \rho_E = 1, \rho_X = 1, Y_D = 0 \quad |t. \end{gathered} \tag{28}$$

The market clearing price is equal to unit wage costs if the expenditure ratio is unity and distributed profit is zero. In this elementary case, profit per unit is zero and by consequence total profit is zero. All changes of the wage rate and the productivity affect the market clearing price in the period under consideration.

With (28) the real wage $\frac{W}{P}$ is *uno actu* given; it is under the enumerated conditions invariably equal to the productivity R. The workers get the whole product. The real wage is determined by the production conditions and *not* in the labor market.

4.2 Profit and real wage (I)

The market clearing price (26) is now inserted into the profit equation (17):

$$\begin{aligned} Q_m &\equiv (\rho_E - 1)\, WL \\ \text{if} \quad \rho_X &= 1, Y_D = 0 \quad |t. \end{aligned} \tag{29}$$

Before profit distribution is taken into account, profit depends alone on the expenditure ratio. If it is greater than unity there is profit, if it is less than unity there is loss. Wage income is the nominal scale factor, labor input is the real scale factor. Eq. (29) is the formal underpinning of the northeastern quadrant of Figure 2.

From the price equation (26) follows for the real wage:

$$\begin{aligned} \frac{W}{P} &= \frac{R}{\rho_E} \\ \text{if} \quad \rho_X &= 1, Y_D = 0 \quad |t. \end{aligned} \tag{30}$$

The real wage is less than the hourly output of the workers if the expenditure ratio is greater unity. In the case of saving, the real wage is higher than the productivity.

The real wage cannot be interpreted as some socially given minimum which is necessary for the reproduction of the working class. The real wage rises by and large with productivity and this explains why the 'capitalist mode of production' does not lead to (absolute or relative) immiserization if the process goes along with a productivity growth. Eq. (30) defines a systemic property of the monetary economy.

While the real wage is below productivity the household sector as a whole, which consists only of wage earners, absorbs the whole output. Dissaving, which is formally represented by $\rho_E > 1$, effects a redistribution of total output among the workers. Neither profit nor the lower real wage is an indicator of exploitation.

4.3 Profit and real wage (II)

For the general case, the market clearing price (26) is again inserted into (17):

$$\begin{aligned} Q_m &\equiv (\rho_E\,(1+\rho_D) - 1)\, WL \\ \text{if} \quad \rho_X &= 1, \quad \text{with} \quad \rho_D \equiv \frac{Y_D}{Y_W} \quad |t. \end{aligned} \tag{31}$$

This equation is formally equivalent to the Profit Law (19). Both tell us that profit depends on the expenditure ratio and the distributed profit ratio in combination.

Wage income is here the scale factor. In the limiting case $\rho_E = 1$ monetary profit is equal to distributed profit, i.e. $Q_m = Y_D$.

The real wage follows from (26) as:

$$\begin{gathered} \frac{W}{P} = \frac{R}{\rho_E\,(1+\rho_D)} \\ \text{if} \quad \rho_X = 1 \quad |t. \end{gathered} \tag{32}$$

The real wage rises with productivity and falls with an increase of the expenditure ratio and/or the distributed profit ratio. Since there is no capital the real wage cannot have anything to do with the marginal productivity of capital. It has nothing to do with the marginal productivity of labor either. The real wage is a *structural* fact. It is neither dependent on a production function with decreasing returns nor on the behavioral hypothesis of profit maximization. And, finally, it has absolutely nothing to do with the reproduction costs of labor.

4.4 Surplus value

Let us now define the surplus value consistently in structural axiomatic terms. We start with the *surplus product* of one hour's work which is defined as difference between productivity and the real wage:

$$\sigma \equiv R - \frac{W}{P} \quad |t. \tag{33}$$

From the surplus product per hour we arrive at the total surplus product by multiplication with total working hours:

$$\sigma L \equiv RL - \frac{WL}{P} \equiv O - \frac{Y_W}{P} \quad |t. \tag{34}$$

To get the total *surplus value s* the total surplus product is valued with the market price P and this gives under the condition of market clearing $\rho_X = 1$ in conjunction with (19):

$$\begin{gathered} s \equiv \sigma LP \equiv C - Y_W \equiv Q_m \\ \text{if} \quad \rho_X = 1 \quad |t. \end{gathered} \tag{35}$$

In the structural axiomatic context total surplus value is by definition identical with monetary profit. It is therefore misleading to say that 'surplus value takes the money form of profits' (Desai, 2008, p. 4). Both, profit and surplus value, are appearances of the actual configuration of the expenditure ratio and the distributed profit ratio.

As joint appearances they are *uno actu* given *by definition*. Surplus value is not produced on the shop floor and then transformed into profits in the product market. As Marx put it in volume III: 'Profit, as we are originally faced with it, is thus the *same thing* as surplus-value, ...' (Marx, 1990, p. 127, italics added). This applies in any case to the consumption economy as constituted by the structural axiom set.

4.5 Real shares

The share of the total quantity bought that wage earners absorb with a given expenditure ratio at a given price is defined as:

$$\delta_W \equiv \frac{\rho_E \frac{Y_W}{P}}{X} \equiv \frac{1}{1+\rho_D} \tag{36}$$

$$\text{with} \quad \rho_X = 1 \quad |t.$$

Since the quantity bought X, which by assumption $\rho_X = 1$ is equal to output O, the share δ_W is identical with the share of output. This share depends solely on the distributed profit ratio ρ_D.

Analogously, the real share of the receivers of distributed profit is given by:

$$\delta_D \equiv \frac{\rho_E \frac{Y_D}{P}}{X} \equiv \frac{\rho_D}{1+\rho_D} \quad |t. \tag{37}$$

Both shares add up to unity:

$$\delta_W + \delta_D = 1 \quad |t. \tag{38}$$

The division of output between the two categories of income depends solely on the distributed profit ratio ρ_D. Profits do not have any impact. An increase of profits without a simultaneous increase in distributed profits therefore has no effect on the real situation of the wage earners taken as a whole. If profits are always retained in full, i.e. $\rho_D = 0$, then total output goes to the wage income recipients.

The distributed profit ratio has already been defined as:

$$\rho_D \equiv \frac{DN}{WL} \equiv \frac{Y_D}{WL} \equiv \frac{Y_D}{Y_W} \tag{39}$$

If distributed profits Y_D stay the same and the wage rate or employment increases then the distributed profit ratio falls and the distribution of the output changes

according to (36) in favor of the wage earners. The real shares of output correspond to the relation of the nominal magnitudes distributed profit income Y_D and wage income Y_W that is expressed by the ratio ρ_D if the expenditure ratio ρ_E for both income categories is identical. This, however, is normally not the case.

In general, the expenditure ratio is not identical for spending out of wage income and spending out of distributed profit income. When the recipients of wage income and distributed profits belong to two separate groups with different spending behavior the general definition of the expenditure ratio is given as the weighted average of the groups' individual expenditure ratios:

$$\rho_E \equiv \rho_E^W \frac{Y_W}{Y} + \rho_E^D \frac{Y_D}{Y} \quad |t. \tag{40}$$

The definition of the real share of the wage income recipients changes accordingly when in (36) the average expenditure ratio ρ_E is replaced by the group-specific expenditure ratio:

$$\delta_W \equiv \frac{\rho_E^W \frac{Y_D}{P}}{X} \equiv \frac{\rho_E^W}{\rho_E^W + \rho_E^D \rho_D} \quad |t. \tag{41}$$

Analogously, the real share of the receivers of distributed profit is then given by:

$$\delta_D \equiv \frac{\rho_E^D \rho_D}{\rho_E^W + \rho_E^D \rho_D} \quad |t. \tag{42}$$

Both shares add up to unity:

$$\delta_W + \delta_D = 1 \quad |t. \tag{43}$$

Hence in general the real shares are determined by the distributed profit ratio ρ_D and the spending pattern of both income groups. With a higher distributed profit ratio and more spending out of distributed profits the real share of the wage earners shrinks. And vice versa, a higher ratio of retained profit and more saving out of distributed profit increases the real share of wage earners. If the spending out of distributed profit is zero the wage earners absorb the whole output independently of what the distribution of nominal incomes looks like. Therefore the income distribution alone cannot tell much about the real distribution. The real distribution is as a rule better – from the perspective of the wage income recipients – than the nominal appearances because it is the received wisdom that saving out of distributed profit income is relatively higher than saving out of wage income.

If, in the limiting case, the expenditure ratio for distributed profit is zero, i.e. $\rho_D^E = 0$, then the wage earners get the whole product no matter whether profit or distributed

profit is greater than zero. For the economy as a whole, the distribution of output does not depend on what happens in production and in particular on what happens with wages and profits. This is certainly against common sense. The paradox resolves itself when we differentiate the business sector.

5 Exploitation is real, but economic classes are not

> Further, Ricardo discussed at considerable length the tension between the workers and the capitalists, in that he claimed consistently that the rate of wages and the rate of profit varied inversely. (Vickers, 1995, p. 62)

The business sector now consists of two firms that produce different consumption goods. To simplify matters profit distribution is excluded; the 1st axiom (1) then turns to:

$$Y = W_1 L_1 + W_2 L_2 + \underbrace{D_1 N_1}_{Y_{D1}=0} + \underbrace{D_2 N_2}_{Y_{D2}=0} \quad |t. \tag{44}$$

With (3), (7), and (8) the market clearing price of firm 1 is given by:

$$P_1 = \frac{\rho_{E1}\left(W_1 + W_2 \frac{L_2}{L_1}\right)}{R_1} \tag{45}$$

$$\text{if} \quad \rho_{X1} = 1, Y_D = 0 \quad |t.$$

The first thing to notice is that the market clearing price of firm 1 is not independent from what happens in firm 2. In the general case, the markets are entangled. Analogously we have for the market clearing price of firm 2:

$$P_2 = \frac{\rho_{E2}\left(W_2 + W_1 \frac{L_1}{L_2}\right)}{R_2} \tag{46}$$

$$\text{if} \quad \rho_{X2} = 1, Y_D = 0 \quad |t.$$

Let us now assume that firm 1 lowers the wage rate W_1 by half. From (45) and (46) then follows that the market clearing prices in *both* firms decline if all other variables are unchanged. Firm 2 is affected because total income falls and with it the nominal demand C_2. The respective expenditure ratios remain unchanged.

From (16) and (8) follows for the profit of firm 1:

$$Q_{m1} \equiv \rho_{E1} Y - W_1 L_1 \quad |t. \tag{47}$$

In more detail this gives after substitution of (1) and rearrangement

$$Q_{m1} \equiv \rho_{E1} W_2 L_2 - (1 - \rho_{E1}) W_1 L_1 \quad |t \tag{48}$$

and analogous for firm 2

$$Q_{m2} \equiv \rho_{E2} W_1 L_1 - (1 - \rho_{E2}) W_2 L_2 \quad |t. \tag{49}$$

According to (48), the reduction of the wage rate W_1 increases the profit of firm 1 and according to (49) it decreases the profit of firm 2. When we look alone at firm 1 we see what Ricardo and Marx have seen before, to wit, wages down – profit up. This fits the time-honored stereotype of wages and profits as antagonists.

However, this situation cannot last for long if profit has been zero in the initial period. In this limiting case firm 2 makes a loss as a consequence of the wage rate reduction in firm 1. This loss is given by (49) and exactly equal to firm 1's profit. If nothing else changes the bankruptcy of firm 2 is only a question of time. An obvious remedy is a wage cut that restores the initial zero profit configuration. Both firms then end up with lower wage rates and lower market clearing prices. The arbitrary wage rate cut does not increase the profit for the business sector as a whole but only redistributes it.

Seen from the perspective of a single firm the antagonism of wages and profits is real. This, though, is parochial realism. The complete picture reveals that firm 1 is better off to the disadvantage of firm 2 and the workers of firm 2 are better off to the disadvantage of the workers of firm 1 because at lower market clearing prices they absorb a bigger share of output with their unaltered income. The situation of the business sector *as a whole* is unchanged and the same is true for the household sector. If there is exploitation it happens *within* the sectors. A partial wage rate change leads only to a redistribution of profits between the firms and of output between the workers.

Profit has no effect on the distribution of output, only profit distribution and the spending out of distributed profit has as we have seen in Section 4.5. As long as nothing is spent out of distributed profits the workers get the whole product. Neither the length of the working day nor the wage rate plays any role, except for the redistribution of profit within the business sector and of output within the household sector.

From the structural axiom set follows that profit (19) is determined by the expenditure ratio and the distributed profit ratio. The real shares of output are determined in the spheres of income and expenditure and *not*, as classical, Marxian and neoclassical economists in conspicuous unanimity maintain, in the sphere of production.

For the economy as a whole, the classical antagonism of wages and profits is an optical illusion. This has a bearing on the *political* notion of classes. There is *no* distributional conflict about output between profits and wages. When economic classes are defined according to these categories the actual conflict materializes *within* the classes in the form of competition.

When, in the limiting case, there are two groups of workers and two groups of capitalists and the first group of capitalists exploits the first group of workers, then the exploiters objectively act in the interest of the second group of workers whatever their own subjective motives may be. The second group of workers has no economic interest to overcome the wage discrimination of the first group, yet the second group of capitalists has indeed because of (49). On a deeper level, the relation between the two groups of capitalists is antagonistic. The same holds for the two groups of workers.

The myopic agents, workers and capitalists alike, are blind to these interdependencies and therefore prone to the fallacy of composition. The generalization of partial effects has the compelling logic of the profit and loss account and the irrefutable empirical evidence of firm 1 on its side. Indeed, what could be more convincing? Wages down, profits up. It works. The *invisible* redistribution of profit and output is anonymously effected behind the agents' back by the market clearing price. The market turns common sense on its head.

6 Conclusion

> How can they continually draw 600 p.st. out of circulation, when they continually throw only 500 p.st. into it? From nothing comes nothing. The capitalist class as a whole cannot draw out of circulation what was not previously in it. (Marx, 1909, II.XVII.51)

Compared to standard economics, Marx posed the right question. The market system is primarily about profit and not about allocation. It can easily survive with an inefficient allocation but not without positive overall profit. The determination of overall profit is therefore the first and foremost task of theoretical economics. Starting with the labour theory of value, though, Marx could not untangle the intricacies of the profit mechanism.

Both Marxian and standard economics are based on indefensible premises which are in the present paper replaced by objective-structural axioms.

The main results of the structural axiomatic assessment of the profit mechanism are:

- Monetary profit is, in the pure consumption economy, the same as surplus value and depends on the expenditure ratio and the distributed profit ratio.

- Profit is not determined by trivial psychology but by the objective Profit Law.
- Profit is a phenomenon of the monetary economy, in a real economy it does not exist.
- Since Marx, Marxists were unable to come to grips with the pivotal economic concept profit.
- Like parts of standard economics, parts of Marxian economics can be reformulated as limiting cases of the general structural axiomatic framework.

Modern Marxists could end the secular stagnation of their research program if they could bring themselves to base it on a sound formal foundation. With this, they could at long last advance from the proto-scientific stage to the scientific stage.

References

Desai, M. (2008). Profit and Profit Theory. In S. N. Durlauf, and L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics Online*, pages 1–11. Palgrave Macmillan, 2nd edition. URL http://www.dictionaryofeconomics.com/article?id=pde2008_P000213.

Feynman, R. P. (1992). *The Character of Physical Law*. London: Penguin.

Kakarot-Handtke, E. (2011). Primary and Secondary Markets. *SSRN Working Paper Series*, 1917012: 1–26. URL http://ssrn.com/abstract=1917012.

Kakarot-Handtke, E. (2013). Understanding Profit and the Markets: The Canonical Model. *SSRN Working Paper Series*, 2298974: 1–55. URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2298974.

Kakarot-Handtke, E. (2014a). Exchange in the Monetary Economy. *SSRN Working Paper Series*, 2387105: 1–19. URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2387105.

Kakarot-Handtke, E. (2014b). Loanable Funds vs. Endogenous Money: Krugman is Wrong, Keen is Right. *SSRN Working Paper Series*, 2389341: 1–17. URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2389341.

Kakarot-Handtke, E. (2014c). The Logic of Value and the Value of Logic. *SSRN Working Paper Series*, 2399550: 1–20. URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2399550.

Keen, S. (2010). Solving the Paradox of Monetary Profits. *Economics E-Journal*, 4(2010-31). URL http://dx.doi.org/10.5018/economics-ejournal.ja.2010-31.

Keen, S. (2011). *Debunking Economics*. London, New York, NY: Zed Books, rev. edition.

Kirkenfeld, T. (1948). The Paradox of Profit. *Science & Society*, 12(1): 33–41. URL http://www.jstor.org/stable/40399873.

Marx, K. (1906). Capital: A Critique of Political Economy, Vol. I. The Process of Capitalist Production. Library of Economics and Liberty. URL http://www.econlib.org/library/YPDBooks/Marx/mrxCpA.html.

Marx, K. (1909). Capital: A Critique of Political Economy, Vol. II. The Process of Circulation of Capital. Library of Economics and Liberty. URL http://www.econlib.org/library/YPDBooks/Marx/mrxCpB.html.

Marx, K. (1990). *Capital*, volume III. London: Penguin Classics. (1894).

Murad, A. (1953). Questions for Profit Theory. *American Journal of Economics and Sociology*, 13(1): 1–14. URL http://www.jstor.org/stable/3484955.

Vickers, D. (1995). *The Tyranny of the Market. A Critique of Theoretical Foundations.* Ann Arbor; MI: University of Michigan Press.