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# PRICE INDICES AND UNIT VALUE INDICES IN GERMAN FOREIGN TRADE STATISTICS

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## 1. INTRODUCTION

Very few countries are able to provide both, a unit value index (PU) and a true price index (P) on a regular (mostly monthly) basis. Fortunately Germany is one of those countries which offer the opportunity to study the impact of the still not well understood methodological differences of the two tools of measuring the price development in export and import. While a PU-Index is basically resulting from foreign trade statistics as a kind of by-product, the compilation of a true price index is much more demanding. It requires special surveys addressing exporting and importing establishments as well as compliance with some principles of price statistics among which aiming at "pure price comparisons" is most prominent. This implies in turn making adjustments (of reported prices) for quality changes in the traded goods or avoiding changes in the collection of goods, reporting firms or in the countries of origin (in the case of imports) or destination involved. By contrast there is no need of satisfying such requirements in the production of PU-indices. Hence the PU type of index is popular though much less commendable from a theoretical point of view. This gives rise to relate the main empirical differences between the PU-index and the P-index observed using German data to their respective conceptual and methodological characteristics.

Another aspect contributing a lot to the topicality of the issue of the present paper is that it is right now subject of an international discussion in the relevant statistical bodies (of statistical institutes, reserve banks etc.).<sup>1</sup> In this context an influential position is the idea to replace P-indices by PU-indices. In what follows it is shown that this would be rather unwise.

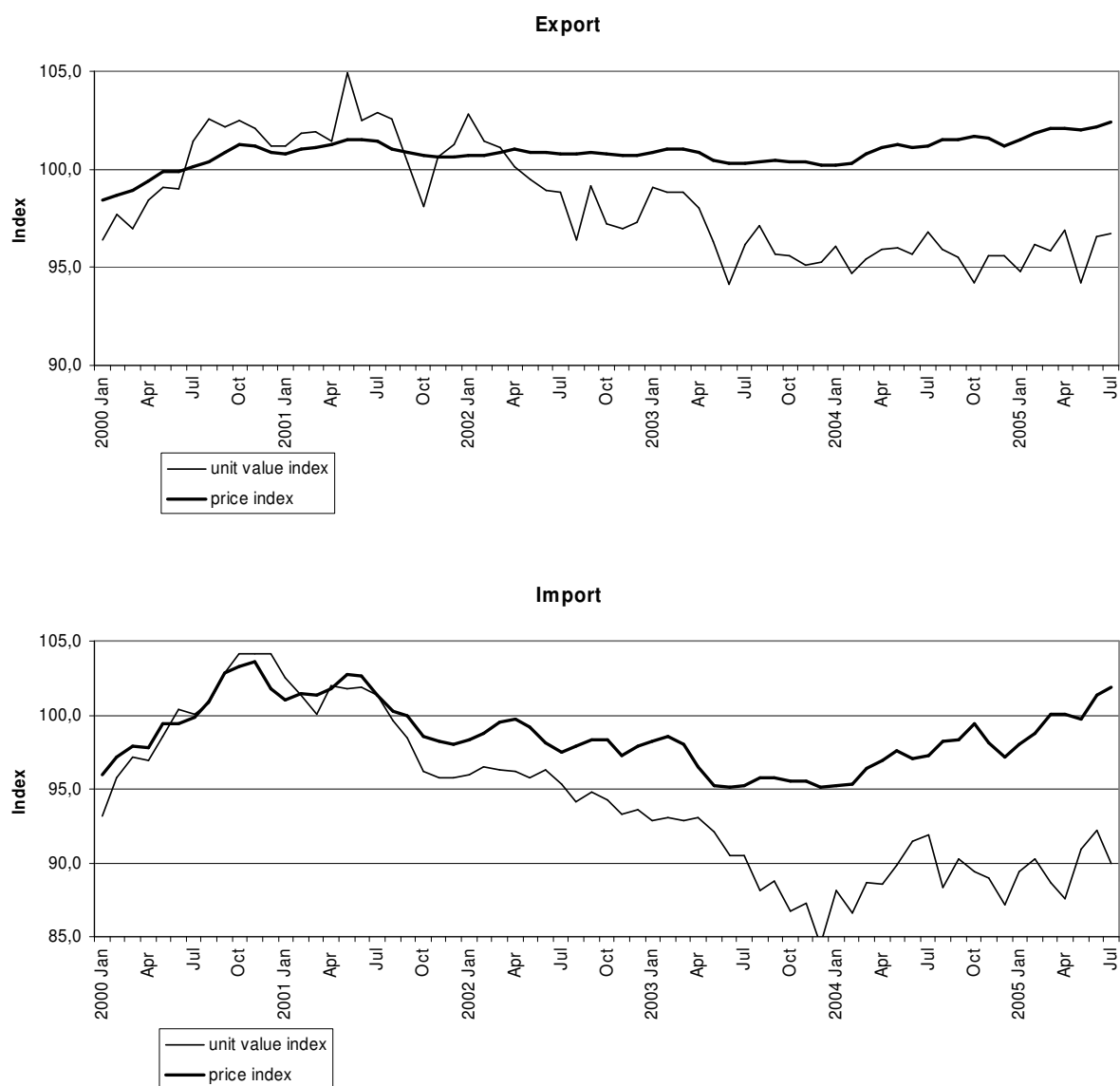
Above all PU- and P-indices of export and import respectively differ with regard to their level and volatility. PU indices tend to display a relative to P-indices more moderate rise of prices combined with more accentuating oscillations as shown in figure 1.

The present paper tries to relate the index formulas used in the case of the German PU- and P-index respectively in order to describe the differences in terms of quantifiable "effects". Another motivation may be expressed in questions like

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<sup>1</sup> The discussion is triggered by the preparation of an IMF "Export and Import Price Manual". Hence the present paper should be understood as a contribution to this ongoing project.

- what can we learn from the observed differences between the two gauges of price movement?
- can we make use of the more readily available unit values as building blocks in order to facilitate the complicated compilation of P-indices?



**Figure 1**

Finally a general concern of the paper is to demonstrate the limitations of PU-indices which to date are unfortunately often played down.

The paper is organised as follows. Section 2 defines the notion of "unit values" and the index formulas based on them as opposed to the traditional "true" price and quantity indices usually well known from the relevant statistic textbooks. In addition to different formulas, the indices to be compared, differ also with respect to concepts, data sources and definitions (of prices for example).

Section 3 summarizes some empirical findings, showing in particular the influence of the type of goods in question. Seasonal fluctuations reflected in PU indices but not in P-indices are for example more effective in the case of apparel, than in the case of chemical products.

Section 4 introduces what might be called a formal theory of PU-indices that is in terms of formulas and "axioms" and section 5 concludes

## 2. DEFINITIONS AND FUNDAMENTAL RELATIONSHIPS

### 2.1. Unit values and indices made thereof

Let  $k = 1, \dots, K$  denote the  $k$ -th group of goods (GG for short) a collection of related (preferably homogeneous) goods for which a common unit of quantity (e.g. kilograms) is used and meaningful. The so called unit value of the  $k$ -th GG at period  $t$  (a kind of average price) is given by

$$(1) \quad \tilde{p}_{kt} = \frac{\sum p_{kjt} q_{kjt}}{\sum q_{kjt}} = \frac{\sum p_{kjt} q_{kjt}}{Q_{kt}},$$

where the summation takes place over all  $n_k$  goods included in the  $k$ th GG.<sup>2</sup>

In the case of foreign trade statistics<sup>3</sup> unit values for more or less broadly defined GGs are easily calculated as both "values"  $V_{kt} = \sum p_{kjt} q_{kjt}$  (numerators of unit values) as well as quantities (denominators)  $Q_{kt} = \sum q_{kjt}$  are elements of foreign trade statistics. Note that no separate statistical inquiry of prices of individual goods  $p_{kjt}$  is necessary. Indices of the unit value type designed to measure price movements are comparing present unit values  $\tilde{p}_{kt}$  with base period unit values  $\tilde{p}_{k0}$ . Correspondingly a quantity index of the unit value type is made of aggregated) quantities  $Q_{kt}$  and  $Q_{k0} = \sum q_{kjo}$  as opposed to a (true) quantity index composed of individual quantities.

Assuming, in contradiction to the actual facts of index computations in Germany though, that prices and quantities of all  $n$  commodities enter the formulas of "true" index numbers, summation over  $i = 1, \dots, n$  would be equivalent to a double summation over all  $K$  groups of goods (GGs) and their respective commodities  $j = 1, \dots, n_k$  ( $\sum n_k = n$ ). In theory we would arrive at a system of eight index formulas as presented in table 1 where superscripts L and P stand for Laspeyres and Paasche respectively.

<sup>2</sup> In general unit values as such (rather than unit values as elements of an index formula) are not an object of interest. It sounds rather strange when some Austrian authors (Glatzer et al., 2006, p. 11, 17) state that import prices in Austria amount to about 20 € per kilogram with the explicitly mentioned consequence that a reduction in weight is equivalent to a rise in "prices".

<sup>3</sup> Statistics of wages and salaries is another field in which frequently use is made of unit values. Often wage sums paid for a group of similarly qualified employees in charge of comparable type of work are readily available allowing the calculation of average wages whereas it would be difficult if worthwhile to derive a statistic of wages based on truly comparable qualifications.

**Table 1** System of eight indices

	Index - concept	
	(true) Price index concept	Unit value index concept
Price index	$P^L, P^P$	$PU^L, PU^P$
Quantity index	$Q^L, Q^P$	$QU^L, QU^P$

In actuality, "true" formulas are not comprehensive but based on a fixed sample of selected goods. In addition to the value index (or value ratio)  $V_{0t} = V_t/V_0$ , use is made of only 3 out of the 8 formulas of table 1 in German foreign trade statistics, that is

$$(2) \quad PU_{0t}^P = \frac{\sum_k \tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{k0}} = \frac{\sum_k \sum_j^{n_k} p_{kjt} q_{kjt}}{\sum_k Q_{kt} \left( \sum_j^{n_k} \frac{p_{kj0} q_{kj0}}{Q_{k0}} \right)},$$

$$(3) \quad P_{0t}^L = \frac{\sum_k \sum_j p_{jkt} q_{jk0}}{\sum_k \sum_j p_{jk0} q_{jk0}} = \frac{\sum_i p_{it} q_{i0}}{\sum_i p_{i0} q_{i0}}, \text{ and}$$

$$(4) \quad QU_{0t}^L = \frac{\sum_k \tilde{p}_{k0} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{k0}},$$

Under the assumptions made values can be derived in both ways using unit values as well as individual prices

$$(5) \quad V_{0t} = \frac{\sum_k \tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{k0}} = \frac{\sum_k \sum_j^{n_k} p_{kjt} q_{kjt}}{\sum_k \sum_j^{n_k} p_{kj0} q_{kj0}} = \frac{V_t}{V_0}.$$

This leads to the following identities

$$(6) \quad V_{0t} = \frac{\sum p_t q_t}{\sum p_0 q_0} = PU_{0t}^L QU_{0t}^P = PU_{0t}^P QU_{0t}^L = P_{0t}^L Q_{0t}^P = P_{0t}^P Q_{0t}^L$$

serving as our starting point in section 4. It should be noted that the interesting comparison is not between  $P_{0t}^L$  and  $P_{0t}^P$  -where much theory already exists- but between  $P_{0t}^L$  and  $PU_{0t}^P$  where new ground is to be broken.

The observed differences between the time series of unit value indices and true price indices in German exports and imports are not only attributable to (idealized or simplified) differences in the formulas, but also stem, in no small measure, from conceptual and procedural differences in index compilation.

## 2.2. Conceptual differences

Table 2 exhibits some of the most influential conceptual differences between the PU- and the P-index of foreign trade (export and import) in German statistics. They are best understood by considering the type of measurement the two approaches are taking. A price index, such as  $P_{0t}^L$  in particular, intends to achieve a "pure" price comparison where the index reflects the changes of prices only. It therefore should not be "contaminated" by simultaneous changes in the qualities and quantities of goods as well as other price determining characteristics (such as reporting firms, countries involved, etc.). Price indices are compiled on the basis of the selection of preferably identical goods which may, however, with the passage of time, become progressively less relevant or "representative" of all traded goods (requiring the updating of this selection at certain time intervals). In contrast to P-indices a PU-index encompasses all goods and is hence affected from a number of influences and structural changes<sup>4</sup>.

**Table 2:** Comparison of true price and unit value (price) indices

	(True) Price index (P-index)	Unit value price index (PU-index)
What is measured?	How the prices of ideally the same products of a given (fixed) collection of products are developing over time	Unit value (average value) of all products of a certain type (e.g. all exported goods) at two points in time
New and disappearing goods	Price of new goods are included only when a new base period is defined (i.e. the index is rebased); vanishing good should, if possible, be replaced by similar goods.	New goods enter immediately the formula. The price quotation of disappearing goods is simply discontinued. No "corrections" are made in the case of incomparability.
Prices	Prices refer to the time of contracting; they express the valuation agreed upon in the contract.	Prices are implicitly given by cross-border values (at the time of crossing the frontier of a country)
Merits	P-indices guarantee pure price comparison by keeping the selection constant and making adjustments for quality changes	PU-indices satisfy "representativity" by inclusion of <i>all</i> products (complete coverage instead of a selection); no quality adjustments are made.
Demerits	Representativity is said to be impaired; a lot more demanding as far as price collection, empirical foundation of weights and quality adjustment is concerned	PU-indices are influenced by changes in the composition of the products in the group. A structural change may be reflected in the average price rather than in the quantity (volume) dimension*

\* A mere switch from cheaper to more expensive products within a group of commodities for which a unit value is established is producing a rise in the PU-index (and thus in the price dimension which thereby is overstated since prices remained unchanged); using  $PU_{0t}$  (instead of  $P_{0t}$ ) as deflator therefore may overstate price and understate volume change.

<sup>4</sup> From the point of view of pure comparisons such changes should be represented in the quantity dimension rather than price dimension.

Notice that the principles of "pure price comparison" on the one hand, and "representativity" on the other are almost antagonistic and difficult to reconcile. It is therefore not surprising that the merits of one approach coincide with the demerits of the other. This observation strongly lends support to our view that both index approaches, P-indices as well as PU-indices are justifiable in their own right. It is not contradictory, however, to consider P-indices as a superior and more refined type of price measurement from an axiomatic point of view<sup>5</sup>. According to table 2 some consequences may readily be hypothesised. So it seems plausible that PU-indices may be lagging behind P-indices because prices at crossing the border of a country are referring to a later point in time than prices agreed upon in contracts. We may also conjecture that the omission of quality adjustments of PU-indices may contribute to their comparatively high volatility. Conclusions of this sort can be derived from table 2, and tested empirically in section 3.

### 3. EMPIRICAL FINDINGS: THE CASE OF GERMANY

#### 3.1. *Sample and descriptive statistics*

Our data taken from the database of the Deutsche Bundesbank<sup>6</sup> cover  $T = 67$  monthly observations of index numbers starting with January 2000. Both indices are structured according to a commodity classification. It permits comparisons of export and import prices between different groups of goods (services are of course not object of foreign trade statistics). A measure of the "discrepancies" between a PU-index and the corresponding P-index is<sup>7</sup>

$$(7) \quad \Delta = \frac{1}{T} \sum_t (PU_{0t} - P_{0t}).$$

#### 3.2. *Relevance of goods and discrepancies between price indicators*

In order to focus on the most significant groups of goods table 3 displays the five divisions (two-digit-codes) of the German commodity classification<sup>8</sup> which contribute most to the export and import values in our sample. Differences between the two measures of price dynamics, using  $\Delta$ , are not spectacular but rather in the vicinity of the overall mean of  $\Delta$  (export:

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<sup>5</sup> Criteria of index theory (in the way of axioms) as well as the justification of the idea "pure price comparison" are discussed in detail in von der Lippe, 2001, pp. 51 - 79.

<sup>6</sup> This paper is summarizing a first pilot study of an ongoing project in cooperation with the Bundesbank, reporting some results gained from calculations one of our students made when he was a trainee of the bank in 2005.

<sup>7</sup> We studied a number of additional measures of deviation, not reported here. The advantage of  $\Delta$  is its ability to show the direction of the deviation, not only the amount.

<sup>8</sup> Güterverzeichnis für Produktionsstatistiken 2002 (GP2002), a German adaptation of the so-called PRODCOM-list a European classification which in turn is closely related to international standard classifications of goods.

2.52, import: 4.37).<sup>9</sup> In accordance with our expectations the statistic  $\Delta$  is overwhelmingly negative, as a consequence of the fact that the PU-index has a tendency to become progressively lower, over time, than the P-index.

**Table 3:** The four most important groups of goods (divisions of GP 2002)<sup>10</sup> and their discrepancies

Division	$S_X$	$S_M$	$\Delta_X$	$\Delta_M$
29 Machinery and equipment	19.78	10.14	- 4.13	- 5.50
24 Chemicals and chemical products	12.85	11.25	- 4.71	- 4.10
15 Food products and beverages	6.90	8.27	- 1.62	- 5.50
31 Electrical machinery and apparatus	6.69	5.06	- 3.62	- 8.87

S = Value share, X denotes export and M import respectively,  $\Delta$  measures of discrepancy between PU- and P-indices in percentage points (see eq. 7 multiplied by 100)

### 3.3. Volatility and seasonality of indices

A striking phenomenon is the much greater volatility of the PU-indices as compared to the P-indices while the corresponding P-indices are in general much smoother. This is due to the fact that they are reflecting changes in the composition of goods (a factor which later will be called the "structural component", or S-effect). We follow the common practice of taking the coefficient of variation, CV as the gauge of dispersion or "volatility". The relation  $CV_U > CV_P$  holds fairly generally, in both, exports as well as imports, and it is due to the fact that PU-indices are based on constantly changing sets of goods that pass the border, while P-indices are compiled using only an invariant sample of those goods, which is kept as constant as possible. By the same token, PU-indices are much more reflecting seasonal variations or changes in other aspects as e.g. the regional composition of exports and imports. An example of the first aspect (seasonal fluctuations) is division 18 "wearing apparel, clothing" (as contrasted with 17 = textiles).<sup>11</sup>

### 3.4. Homogeneity of groups of goods

We conjecture that a sub-index of  $PU_{0t}^P$  (compiled for a division of the commodity classification) should differ more from the corresponding  $P_{0t}^L$  sub index if the division in question is

<sup>9</sup> Sizeable differences in  $\Delta$ , particular in the case of exports, were for example in the case of energy (electricity, gas etc.) amounting to - 33.59, or crude petroleum and natural gas (- 15.51).

<sup>10</sup> We take here divisions (two-digit classification units) as groups of goods (GGs). In practice index calculations in terms of unit values are generally making use of more narrowly defined GGs, each of which is comprising still a number of goods (otherwise the difference between P- and PU-indices would fade away).

<sup>11</sup> In division 18  $CV_U = 0.1312$  is about 12 times  $CV_P = 0.0109$  because the type of wearing represented by PU is in summer much different from winter. The same applies to a smaller extent, however, to imports.



less homogeneous. The 31 divisions (two-digit classificatory units) are not only comprising a widely different number of subdivisions (commodity codes) in the classification but also of price quotations (or contracts or "series") in the official P-index ranging from 3 (in the case of metal ores (division 13) to 1067 in machinery and equipment (division 29)).<sup>12</sup> However, there is no formula available for a variable H, the degree of homogeneity of a division. We decided to take the average correlation (over T = 67 months)  $\bar{r}$  between any two series belonging to the same division<sup>13</sup> so that  $H = \bar{r}$ . H was particularly high in imports of division 11 (crude petroleum and natural gas) with an average correlation between the 99 series of + 0.7219 and in exports of other transport equipment (division 35) with H amounting to 0.6594. Regressing H on  $\Delta$  was disappointing from the point of view of our hypothesis, yielding  $R_{adj}^2$  of 0.0742 only.

### ***3.5. Lead of the price index?***

The same possibly applies to the hypothesis that  $P_{0t}^L$  is a leading indicator while  $PU_{0t}^P$  is lagging one or more periods behind due to the different time of recording prices. Correlating  $PU_{0t}^P$  with  $P_{0,t-\lambda}^L$  ( $\lambda > 0$ ) did, however, not result in a systematic improvement of correlations as  $\lambda$  is increasing. Only in some cases, had the shift of the price index a considerable impact on divergence measures (such as  $\Delta$ ).<sup>14</sup>

### ***3.6. The smoothing effect of quality adjustment***

We made the assumption that making quality adjustments (that is reduction of the quoted price in the case of an improvement in quality) will result in a smoother price movement. This could explain the relatively (compared to the true price index P) high volatility of unit value indices (PU-indices) where such adjustments are not made. Fortunately we were in a position to verify (partly at least) this supposition because the Federal Statistical Office (FSO) thankfully carried out a special analysis of their price data in the field of data processing goods. The ordinary user of official statistical data can only make use of data *after* quality adjustment. The raw data are in general not accessible to him. The FSO gave us data concerning four

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<sup>12</sup> Furthermore it should be noticed that each division is represented by a much greater (and basically unknown and varying) number of actual models and varieties being exported or imported. Hence it is difficult if not almost impossible to state the true degree of heterogeneity of the elementary GGs used for the compilation of a PU-index. It should be borne in mind therefore, that the available data is not appropriate (and will continue to be so) to test the notwithstanding highly plausible hypothesis.

<sup>13</sup> Homogeneity is maximum as H approaches +1 and minimum in the case of on average negatively correlating series, where H tends to -1.

<sup>14</sup> In some cases a noticeable "improvement" after lagging P was found in particular in exports of basic non iron metals and imports of electricity, gas etc.

products of the GG “information and communication technology (ICT)”, viz. desktops, notebooks, working storage and hard disk. Each of these products was in turn represented by a number of models of different producers, ranging from 84 to 190. As ICT products are characterised by remarkable price reductions accompanied by quality improvements it was not surprising that the amount of price reduction was uniformly higher after quality adjustment than before<sup>15</sup>. Volatility was also reduced substantially by quality adjustments.

#### 4. PROPERTIES OF UNIT-VALUE-INDICES

##### 4.1. Components of the discrepancy

One might be tempted to explain the fact that  $PU_{0t}^P < P_{0t}^L$ , that is the official German unit value index ( $PU_{0t}^P$ ) is as a rule falling short of the corresponding (time) price index ( $P_{0t}^L$ ) with a recourse to a formula found by Ladislaus v. Bortkiewicz, according to which the covariance  $C$  between  $p_{it}/p_{i0}$  and  $q_{it}/q_{i0}$  respectively, the price and quantity relatives is given by

$$(8) \quad C = V_{0t} - P_{0t}^L Q_{0t}^L = P_{0t}^L (Q_{0t}^P - Q_{0t}^L).$$

Hence the Paasche formula yields lower values than the Laspeyres formula whenever the covariance is negative. As in practice (and for example in the so called "economic theory of index formulas") the situation  $C < 0$  is exclusively considered<sup>16</sup> it is often said that the Laspeyres formula tends to overrate the price movement (much like Paasche is underrating it), which is referred to as Laspeyres- or simply *L-effect*.

It should be borne in mind, however, that the comparison in question is not between  $P_{0t}^P$  and  $P_{0t}^L$  but rather between  $PU_{0t}^P$  and  $P_{0t}^L$ . Under such conditions a second component of the discrepancy is coming into play which may well reinforce but would also counteract the L-effect. This factor is called structural component or *S-effect* for short and refers to changing quantities within a GG (for example switching from a high-priced to a relatively cheap good belonging to the same GG). The two effects, L and S will both show up in

$$(9) \quad D = \frac{PU_{0t}^P}{P_{0t}^L} - 1 = \left( \frac{C}{QU_{0t}^L P_{0t}^L} \right) + \left( \frac{Q_{0t}^L}{QU_{0t}^L} - 1 \right) = L + S$$

an equation easily derived from eq. 6 using eq. 8. Hence although  $C < 0$  and therefore necessarily  $P_{0t}^P < P_{0t}^L$  (negative L-effect) the unit value index  $PU_{0t}^P$  (instead of  $P_{0t}^P$ ) may still equal

<sup>15</sup> For example in the case of working storage prices fell by 2.3 % after quality adjustment, compared to 0.9 % before. In this group of goods reduction of volatility also topped the other goods. The coefficient of variation was, after quality adjustment, only nearly one sixth of its value before.

<sup>16</sup> A negative covariance may arise from rational substitution among goods in response to price changes on a given (negatively sloped) demand curve.

or exceed  $P_{0t}^L$  (that is  $D \geq 0$ ) simply because a negative  $L$  is offset or outstripped by a positive  $S$ <sup>17</sup>. The problem of eq. 9 is, however, that we are lacking data as there is no  $Q_{0t}^L$  index compiled in practice, which prevents us from carrying out empirical studies. We therefore have to confine ourselves to a numerical example which will also serve as an illustration of what is meant by  $L$  and  $S$ .

#### 4.2. A fictitious numerical example

Assume two groups of goods (GGs), A and B, each composed of two goods, 1 and 2 in the case of A and 3 and 4 in the case of B. Total quantities in both GGs are kept constant such that  $Q_{k0} = Q_{kt} = 10$ . The quantities  $q_A$  and  $q_B$  are introduced in order to simulate structural changes with in the GGs. Table 4 is putting together all figures needed to calculate the index formulas of eq. 9 (or 9a) as functions of  $q_A$  and  $q_B$ .

**Table 4:** Numerical example

	$p_0$	$p_t$	$q_0$	$q_t$
1 (A)	8	10	5	$q_A$
2 (A)	4	7	5	$10 - q_A$
3 (B)	7	9	5	$q_B$
4 (B)	6	4	5	$10 - q_B$

Note that things rapidly will become more complex and intricate once we abandon the simplifying assumptions  $K = 2$ ,  $n_1 = n_2 = 2$ , and in particular  $Q_{k0} = Q_{kt}$  which yields  $QU_{0t}^L = 1$  and therefore<sup>18</sup>

$$(9a) \quad \underbrace{Q^P - 1}_{=D} = \underbrace{Q^P - Q^L}_{=L} + \underbrace{Q^L - 1}_{=S}$$

is taking the part of eq. 9.

As a consequence we get two straight lines for  $q_B$  as a linear function of  $q_A$ , a positively sloped line (left part of fig. 2) separating positive from negative  $C$ -values (and thus positive from negative  $L$ -effects), and a negatively sloped line delimiting positive  $D$ -values (upper right area) and negative  $D$ -values (lower left are).

When putting both lines together a configuration with a left (L) and right (R) "wedge" is created, in which the effects  $L$  and  $S$  are acting in opposition to one another. In the two trapezoid

<sup>17</sup> This is in fact a situation which is taking place in the right wedge of fig. 2 included to illustrate the numerical example presented in section 4.2.

<sup>18</sup> Here and in what follows it may be more convenient to drop the subscripts 0 for the base period, and  $t$  for the actual observation period.

areas, however, not highlighted in the right part of fig. 2, the effects are tending both in the same direction

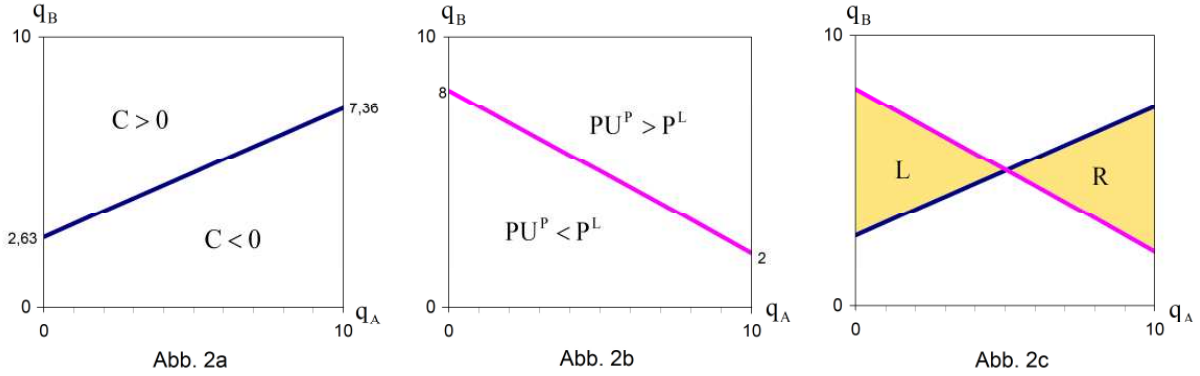


Figure 2

### 4.3. Interpretation of $L$ and $S$

Situations in which  $S$  vanishes are firstly each commodity group (GG) consists of one good only ( $m_k = 1$  for all  $k$ , the maximum possible homogeneity of GGs) or secondly the structure of goods within a CG remains constant. This easily follows from

$$(10) \quad QU_{0t}^L = \frac{\sum_k Q_{kt} \tilde{p}_{k0}}{\sum_k Q_{k0} \tilde{p}_{k0}} = \frac{\sum_k Q_{kt} \sum_j m_{kj0} p_{kj0}}{\sum_k \sum_j q_{kj0} p_{kj0}} \text{ as opposed to}$$

$$(11) \quad Q_{0t}^L = \frac{\sum_k Q_{kt} \sum_j m_{kjt} p_{kj0}}{\sum_k \sum_j q_{kj0} p_{kj0}}$$

where  $m_{kj0} = q_{kj0} / \sum_j q_{kj0}$ , and  $m_{kjt} = q_{kjt} / \sum_j q_{kjt}$  so that

$$(12) \quad Q_{0t}^L - QU_{0t}^L = \frac{\sum_k Q_{kt} \sum_j p_{kj0} (m_{kjt} - m_{kj0})}{V_0}.$$

Hence a constant structure  $m_{kjt} = m_{kj0}$  for all  $k, j$  results in  $S = 0$  which may justify the term structural component (a phenomenon, by definition, non-existent in  $Q^L$ ).<sup>19</sup>

In the absence of a structural change we also have  $PU_{0t}^P = P_{0t}^P$  and  $D$  boils down to  $D = P_{0t}^P / P_{0t}^L - 1 = L$ .

If on the other hand  $L = 0$  eq. 9 shows that  $PU^P$  can differ from  $P^L$  although  $P^P = P^L$  (as  $C = 0$ ), which in turn is possible only if  $QU^L$  differs from  $Q^L$ , that is, if  $S$  is effective. A point on the left straight line with a positive slope, for example on the lower boundary of the left wedge, is given by  $q_A = 10/3$ ,  $q_B = 80/19$ . The reader may easily verify that we then get  $C = L = 0$  and  $PU^P = 1.128$  while  $P^L = P^P = 1.2$ , furthermore  $Q^L = Q^P = 0.94$  while  $QU^L = 1$ .

As the covariance vanishes, the fact that  $PU^P$  is falling short of  $P^L$  by 6 percent is only because of the  $S$ -effect, by virtue of which  $Q^L$  is 6 % less than  $QU^L$ . In the example  $S$  amounts

<sup>19</sup> Also perfectly homogeneous GGs ( $n_k = 1$  so that  $m_{kj0} = m_{kjt} = 1$ ) result in  $S = 0$ . For more detail see sec. 4.5.

to  $Q^L-1$  (as  $Q_{kt} = Q_{k0}$ ). The structural changes  $m_{kjt} - m_{kj0}$  are given by  $-1/6$ ,  $1/6$ ,  $-3/38$ , and  $3/38$ . In combination with prices  $p_{kj0}$  they are generating a negative difference amounting to  $Q^L - 1 = -0.06$ .

#### 4.4. Axiomatic defects of unit value indices

Structural changes can also be responsible for unit values showing a price movement although no price has changed, an awkward result, not tolerable from an axiomatic point of view. Assume that no price changes that is  $p_{kjt} = p_{kj0}$  ( $\forall k, j$ ). Nonetheless, as the equation

$$(13) \quad d_k = \tilde{p}_{kt} - \tilde{p}_{k0} = \sum_j p_{kj0} \left( \frac{q_{kjt}}{Q_{kt}} - \frac{q_{kj0}}{Q_{k0}} \right) = \sum_j p_{kj0} (m_{kjt} - m_{kj0})$$

shows unit values need not remain constant, that is  $d_k$  may differ from 0. Thus unit values do not reflect price movements only. They violate the identity axiom of price index theory. For a similar reason they do not necessarily satisfy the mean value property. Expressed as weighted sum of price relatives  $p_{jkt}/p_{kj0}$   $PU^L$  is given by

$$(14) \quad PU_{0t}^L = \sum_k \sum_j \frac{p_{kjt}}{p_{kj0}} \left( p_{kj0} q_{kj0} \frac{m_{kjt}}{m_{kj0}} \right) / \sum_k \sum_j p_{kj0} q_{kj0},$$

and a similar formula applies to  $PU^P$ . The weights (in brackets) in the numerator will in general not add up to  $V_0 = \sum p_{kj0} q_{kj0}$  in the denominator of the right hand side of eq. 14 unless we have no structural change ( $S = 0$  because  $m_{kjt} = m_{kj0}$ ).

These defects of indices in terms of unit values are already well understood, at least on the part of price indicators such as  $PU^P$  and  $PU^L$ . The System of National Accounts (SNA)<sup>20</sup> therefore rightly made some reservations as to the use of unit value (price) indices as an alternative to  $P^P$  or  $P^L$ . They were rightly rejected with the argument

Unit value indices are "affected by changes in the mix of items as well as by changes in their prices. Unit value indices cannot therefore be expected to provide good measures of average price change over time" (SNA 1993, paragraph 16.13).<sup>21</sup>

#### 4.5. Homogeneous groups of goods

PU- and QU-indices may generally be viewed as crude measures of price and quantity levels as they are based on average prices and sums of quantities rather than individual prices and

<sup>20</sup> The System of National Accounts 1993 is a bulky manual prepared by the "Inter-Secretariat Working Group", the members of which were the Commission of the European Communities, the IMF, OECD, World Bank and the UN in order to harmonize National Accounts worldwide.

<sup>21</sup> However the SNA apparently (and amazingly) did not realise that the same type of objections can also be raised against chain indices, advocated with great vigour by the SNA.

quantities. Assume that each GG is consisting of one good only or the structure of the GG is remaining constant. Then eq. 9 "reduces" to

$$(15) \quad D^* = \frac{P_{0t}^P}{P_{0t}^L} - 1 = \frac{C}{Q_{0t}^L P_{0t}^L} = L^* .$$

or equivalently  $S = 0$ ,  $PU^P = P^P$  and  $QU^L = Q^L$ . Note how  $D^*$  and  $D = PU_{0t}^P / P_{0t}^L - 1$  in eq. 9 differ. Moreover  $L^*$  is related to  $L$  as follows  $L = L^*(1+S)$  where  $S$  is given by

$$(16) \quad S = \frac{O_{0t}^L}{QU_{0t}^L} - 1 = \frac{PU_{0t}^P}{P_{0t}^P} - 1 = \frac{\sum_k Q_{kt} \sum_j p_{kj0} (m_{kjt} - m_{kj0})}{\sum_k Q_{kt} \sum_j p_{kj0} m_{kj0}} ,$$

where the denominator can also be expressed as  $V_t / PU_{0t}^P = V_0 QU_{0t}^L = \sum_k Q_{kt} \tilde{p}_{k0}$ .

## 5. CONCLUSION

Unit value indices in foreign trade are not amenable to the "normal" or usual interpretation of price indices. They differ from the latter by a number of reasons not only the formula but also concepts and data collection procedures. The difference between the two approaches to price measurement is hitherto not well understood. Notably an integrating theory of the combined effect of various positively as well as negatively correlated influences remains to be developed. Moreover, many "effects" that probably could explain the difference, are difficult to capture empirically. The present paper, therefore, is only an attempt to improve our understanding of the nature of the two index designs and to exhibit some empirical findings.

In no small measure the paper can only make suggestions for a more thorough analysis. To name but a few of such issues: first more emphasis should be laid on a clear-cut list of determinants of  $L$  and  $S$ , preferably without overlaps and identifiable empirically. Secondly the difference between the two types of indices should be explained in terms of microeconomic theory, that is, by tracing decisions back to utility maximizing behaviour. This should be useful in order to assess the relative strength of factors influencing  $L$  and  $S$ . Thirdly for the most part arguments advanced to explain specific traits of unit value indices are not well suited to understand their volatility in particular. In other words, there must be some other reasons than the omission of quality adjustments to explain volatility, for example possibly the frequent change in the composition of the traded goods.<sup>22</sup> Finally, there is no doubt that homogeneity of the commodity codes matters but it proved difficult to measure homogeneity, and thus to demonstrate this empirically.

<sup>22</sup> In this case, by the way, the analysis of chain indices would benefit from a closer look at unit value indices since we have in both situations a reduced comparability over time due to a changing basis of observations.

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