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Kelly, Logan

Bryant University

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# Measuring the Economic Stock of Money

Logan J. Kelly\*

*Bryant University, Department of Economics, Smithfield, RI 02917  
e-mail: lkelly@bryant.edu Ph: (401) 232-6897 Fax: (401) 232-6319*

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## Abstract

Aggregation theoretic measures of the economic stock of money (ESM) have been criticized for their dependence on future expectations. I answer some of those objections by using several forecasting methods to generate the expectations needed for calculating the ESM. I find that targeted factor model forecasting improves the accuracy of the measurement of the ESM but also that measurement of the ESM is robust to assumptions about future expectation. These findings suggest that concerns about the dependency of theoretical monetary stock aggregates on forecasted future expectations may have been overstated.

*Key words:* Monetary Aggregation, Money Stock, Targeted Factor Models, Least Angle Regression

*JEL classification codes:* C43, E49

## 1 Introduction

Aggregation theoretic measures of the economic stock of money (ESM) have been criticized for their dependence on forecasted future expectations. I answer some of those objections by using several forecasting methods to generate the expectations needed for calculating the ESM. Factor model forecasts tend to outperform other models (see Stock and Watson, 1999; Bai and Ng, 2002, 2007), but Barnett, Chae, and Keating (2005) found that the calculation of the ESM is robust to assumptions about future expectation. Therefore, the purpose of this study is twofold, to improve the current measurement of the ESM, and to confirm the robustness result of Barnett et al. (2005).

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Stock and Watson (1999) show that approximate factor models tend to outperform other forecasting methods because they allow the use of a large panel of data. Bai and Ng (2002) derive information criteria for determining the number of factors that underlie a large panel of time series data. Boivin and Ng (2006) found that adding additional predictors that bear little information about factor components does not necessarily improve forecasts, but Bai and Ng (2007) examine the problem of which variables should be included in the panel. They use a sequential model selection algorithm, known as least angle regression (LARS), developed by Efron, Hastie, Johnstone, and Tibshirani (2004) to target the panel of explanatory variables to the variable being forecasted.

I find that targeted factor model forecasting to generate the expectations needed to calculate the ESM improves the accuracy of the measurement of the ESM. But distant future service flows are heavily discounted, so I find that calculating the ESM is robust to assumptions about expectations formation, confirming the result of Barnett et al. (2005). These findings suggest that concerns about the dependency of theoretical monetary stock aggregates on forecasted future expectations may have been overstated.

The remainder of this paper is organized as follows. Section 2 reviews the relevant monetary aggregation theory, and explains how the ESM is calculated in practice. Section 3 presents the forecasting methodologies to be tested and how they will be evaluated. Section 4 presents the results of the forecasting evaluation and the results of calculating the ESM using the best performing forecasting methodology to calculate the ESM. Section five concludes.

## 2 The Economic Stock of Money

### 2.1 Definition Under Perfect Foresight

Following Barnett (1978), let period  $t$  be the current time period, and let  $T$  be the length of the planning horizon, possibly infinity, such that the representative consumer plans for all periods,  $s = t, t + 1, \dots, t + T$ . Barnett (1991) defines the ESM,  $V_t$ , as

$$V_t \equiv \sum_{s=t}^T \sum_{n=1}^N \left[ \frac{p_s^*}{\rho_s} - \frac{p_s^* (1 + r_{n,s})}{\rho_{s+1}} \right] m_{ns}, \quad (1)$$

where the discount rate for period  $s$  is

$$\rho_s = \begin{cases} 1 & s = t \\ \prod_{u=t}^{s-1} (1 + R_u) & s \neq t \end{cases}, \quad (2)$$

$R_s$  is the benchmark rate, i.e. rate of return provided by a pure investment asset, at time period  $s$ ,  $r_{n,s}$  is the user cost of monetary asset  $m$  at time period  $s$ ,  $m_{n,s}$  is the quantity of monetary asset  $m$  at held time period  $s$ , and  $p_s^*$  is the true cost of living index. Following Barnett et al. (2005), (1) can be rewritten

as

$$V_t = \sum_{s=t}^{\infty} \frac{TE_s}{\rho_s}, \quad (3)$$

where  $TE_s$  is the total nominal expenditure on monetary services in period  $s$ , and  $T$  is allowed to approach infinity.

## 2.2 Extension to Uncertainty

Barnett (1995) and Barnett, Liu, and Jensen (1997) show that, assuming inter-temporal strong separability, all the results on user cost and Divisia aggregation can be extended to the case of risk neutrality by replacing all random variables with their expectations. Thus, applying the consumption-based capital asset pricing model theory (see Blanchard and Fischer, 1989; Cochrane, 2005), the formulas for the ESM under inter-temporal strong separability becomes

$$V_t = E_t \left( \sum_{s=t}^{\infty} \Gamma_s TE_s \right), \quad (4)$$

where

$$\Gamma_s = \beta^{s-t} \frac{\partial u}{\partial C_s} / \frac{\partial u}{\partial C_t} \quad (5)$$

is the subjectively-discounted marginal rate of inter-temporal substitution between consumption in the current period  $t$  and the future period  $s$ .

## 2.3 Calculating the ESM

Following Barnett et al. (2005), the ESM, (3),

$$V_t = \sum_{s=t}^{\infty} \frac{TE_s}{\rho_s},$$

is calculated by assuming perfect foresight using actual future data. The perfect foresight ESM (PF) is not a feasible index number since future data cannot be known ex ante, but as in Barnett et al. (2005), the PF is used to evaluate the performance of measures of the ESM that are based on forecasted data.

A feasible measure of the ESM can be calculated by assuming risk neutrality and using forecasted data in (4),

$$V_t = E_t \left( \sum_{s=t}^{\infty} \beta^{s-t} TE_s \right).$$

The expected value of a nonlinear function is equal to the function evaluated at the expected value of each variable plus covariance terms. Following method 3 of Barnett et al. (2005), I set each of these covariance terms to zero. It is well known from asset pricing theory that

$$i = \frac{1 - \beta}{\beta},$$

where, in our case, the interest rate  $i$  is the benchmark rate. Substituting the benchmark rate and solving for  $\beta$  yields

$$\beta = \frac{1}{1 + R_t}.$$

Thus setting the covariance terms to zero is equivalent to assuming that the covariance between total expenditure on monetary assets and the benchmark rate is zero. Finally, the benchmark rate is assumed to follow a martingale process.

In practice, (3) must be evaluated for a finite number of periods,  $H$ , so that (3) becomes

$$V_t = \sum_{s=t}^H \frac{TE_s}{\rho_s}.$$

To determine the number of iterations,  $H$ , needed to calculate the ESM index number, the smallest  $H$  that satisfies the stopping criterion,

$$\left| \frac{\sum_{s=t}^H \frac{TE_s}{\rho_s} - \sum_{s=t}^{H-1} \frac{TE_s}{\rho_s}}{\sum_{s=t}^{H-1} \frac{TE_s}{\rho_s}} \right| < 10^{-4}, \quad (6)$$

is chosen.

### 3 Forecasting Methodology

Forecasts of total expenditure on monetary services (TE) are used in calculating a feasible ESM index number. In this section, I will evaluate the performance targeted factor models (TFM) relative to martingale and auto regressive (AR) models at the 6 month, 12 month, 24 month and 36 month time horizons. The rest of this section will briefly review the targeted factor model methodology.

#### 3.1 Data Description

This study examines monthly data from 1960:03 - 2004:03 collected from Economic Data - FRED<sup>®</sup> database maintained by the Saint Louis Federal Reserve (<http://research.stlouisfed.org/fred2/>), and the United States Bureau of Labor Statistics (<http://www.bls.gov>). This time period was chosen to remain consistent with Barnett et al. (2005) for comparison purposes.

The variables to be forecasted are total expenditures on monetary services provided by monetary assets included in M1, M2 and M3 monetary aggregates. I test each variable for stationarity using the Augmented Dickey-Fuller test and the Dickey-Fuller GLS test (Elliott, Rothenberg, and Stock, 1996). Each variable is found to be I(1) non-stationary, and so each is differenced once.

The panel of explanatory variables includes 112 series including selected long-term and short-term interest rates, unemployment data, aggregate price data, monetary aggregate data and other macroeconomic time series data. Each

variable is tested for stationarity using the Augmented Dickey-Fuller test and the Dickey-Fuller GLS (Elliott et al., 1996). Each variable is transformed by taking logs, first or second differences as needed.

## 3.2 Forecasting Models

### 3.2.1 Approximate Factor Model

Let  $X_{(T \times N)}$  be a matrix of  $N$  observed variables over  $T$  periods. Then consider the model suggested by Bai and Ng (2002),

$$X = F\Lambda' + e, \quad (7)$$

where  $\Lambda = (\lambda_1 \dots \lambda_N)'$  is a  $(N \times r)$  matrix of loading factors,  $F$  is a  $(T \times r)$  matrix of common factors, and  $e$  is a  $(T \times N)$  matrix of idiosyncratic errors. See Bai and Ng (2002) for the necessary assumptions for consistent estimation of the  $r$  common factors. The factors are estimated by the method of asymptotic principle components.

In order to estimate the number of common factors,  $r$ , minimize, by choosing  $k$ , the following information criterion:

$$IC_{p1}(k) = \ln \left[ V(k, \hat{F}^k) \right] + k \left( \frac{N+T}{NT} \right) \ln \left( \frac{NT}{N+T} \right),$$

where  $\hat{F}^k$  equals  $\sqrt{T}$  times the eigenvectors corresponding to the  $k$  largest eigenvalues of the  $(T \times T)$  matrix  $XX'$  and

$$V(k, \hat{F}^k) = \min_{\Lambda} \left[ (NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T \left( X_{it} - \lambda_i^{k'} F_t^k \right)^2 \right].$$

I use the forecasting equation suggested by Bai and Ng (2007),

$$\hat{y}_{T+h|T}^h = \hat{\alpha}' W_T + \hat{\beta}' \hat{F}_T^k, \quad (8)$$

where  $\hat{y}_{T+h}^h$  is the  $h$ -period forecast of the variable  $y_t$  given the information available as of time period  $T$ ,  $W_T$  is a vector of predetermined variables that could include a constant and/or lags of  $y_{t+h}$ ,  $\hat{F}_T^k$  is a vector of  $k$  common factors of  $X$ , and the parameters  $\alpha$  and  $\beta$  are obtained from the ordinary least squares estimation of

$$y_{t+h}^h = \alpha' W_T + \beta' \hat{F}_T^k + \varepsilon_{t+h}.$$

### 3.2.2 Squared Principle Components

One limitation of the method of principle components is that it presupposes a linear linking function between the data and the latent factors. Bai and

Ng (2007) propose a more flexible approach that allows for rudimentary non-linearity in the factor linking function. Define  $X^*$  to be  $X$  augmented by a subset of the unique cross-products of  $X$ . Specifically, consider  $X^* = \{X_{tn}, X_{tn}^2\}$ , which Bai and Ng referred to as squared principle components (SPC). Estimation of the common factors of  $X^*$  proceeds by the usual principle components method. In the case of SPC, there are  $2N$  predictors, and the dimension  $X^*$  could be much larger if other cross-products are included (Bai and Ng also experimented with the incorporation of cross-products,  $X_{ti}X_{tj}$  where  $i \neq j$ , but they found that this was computationally demanding and did not significantly improve forecasting performance).

### 3.2.3 Least Angle Regression

Let  $\hat{\mu}_k$  be the current estimate of  $y$  with  $k$  predictors and define

$$\hat{c} = X'(y - \hat{\mu}_k)$$

to be the “current correlation.” Note that it is assumed that each column of  $X$  has been standardized. Choose  $j$  to maximize  $|\hat{c}_j|$  and consider the updating rule

$$\hat{\mu}_{k+1} = \hat{\mu}_k + \hat{\gamma} \text{sign}(\hat{c}_j) X_j. \quad (9)$$

At each step, the  $\hat{\gamma}$  is chosen endogenously so that the algorithm proceeds equiangularly between the variables in the most correlated set until the next variable is found. After  $k$  steps,  $k$  variables will have been selected; thus, determining the optimal set of predictors becomes a problem of determining a stopping rule for  $k$ .

The LARS algorithm begins with  $\hat{\mu}_0 = 0$ . Let  $\hat{\mu}$  be the current estimate of  $y$ , let  $\hat{c} = X'(y - \hat{\mu})$ , and define  $K$  to be the set of indices corresponding to the variables in  $X$  with the largest absolute “current correlation,” i.e. the “in set,”

$$\hat{C} = \max_j |\hat{c}_j| \quad K = \{j : |\hat{c}_j| = \hat{C}\}.$$

Let  $s_j = \text{sign}(\hat{c}_j)$  and thus the active matrix corresponding to  $K$  is

$$X_K = (\dots s_j x_j \dots)_{j \in K}.$$

Let

$$G_K = X_K' X_K \quad \text{and} \quad A_K = (1_K' G_K 1_K)^{-\frac{1}{2}},$$

where  $1_K$  is a vector of ones of length equal to the size of  $K$ . The unit equiangular-vector with the columns of the active matrix is

$$u_K = X_K w_K, \quad w_K = A_K G_K^{-1} 1_K,$$

so that

$$X_K u_K = A_K 1_K \quad \text{and} \quad \|u_K\|^2 = 1.$$

LARS then updates  $\hat{\mu}$  using the LARS variant of (9),

$$\hat{\mu}^{new} = \hat{\mu} + \hat{\gamma}u_K,$$

where

$$\hat{\gamma} = \min_{j \in A_K^c}^+ \left\{ \frac{\hat{C} - \hat{c}_j}{A_K - a_j}, \frac{\hat{C} + \hat{c}_j}{A_K + a_j} \right\},$$

where  $a_K = X'u_K$ ,  $\min^+$  indicates that the minimum is taken over only positive components within each choice of  $j$ , and  $A_K^c$  is the set of indices corresponding to the variables not yet in the “in set.” If LARS is repeated  $N$  times, it returns an ordering of the  $N$  predictors from best to worst.

### 3.2.4 Targeted Factor Model

Boivin and Ng (2006) found that adding additional predictors that bear little information about factor components does not necessarily improve forecasts. They found that when the data panel is too noisy, it is better to eliminate some of the data. The optimal panel of predictors could be determined by the use of an information criteria, such as BIC. However, with  $N$  possible predictors, there are  $2^N$  possible sets to consider. Hence, this method is impractical. Bai and Ng examine the use of several methods by which the panel of predictors can be targeted to the variable being forecasted. They found that the method of least angle regression (LARS) developed by Efron et al. (2004) was the most successful at forecasting inflation, thus LARS is used to target the panel. Following Bai and Ng (2007), I use LARS to select the 30 best variables to include in the targeted panel. Then I use principle components to estimate the common factors.

## 3.3 Comparison Forecasting Models

I evaluate the performance of targeted factor model forecasts as compared to the following models. The first comparison model is a simple auto-regressive process of  $p$  lags using the Bayesian information criterion to select  $p$ . An AR( $p$ ) model is selected as a model for comparison because of its long standing usefulness in forecasting of all types. In many instances, the AR( $p$ ) model has been shown to outperform much more complicated models. Thus, the AR( $p$ ) is a natural benchmark for comparing the performance of any new forecasting methodology. I will refer to this model as AR in all following tables and figures.

The second comparison model is a martingale forecast. The martingale forecast model is chosen as a model for comparison because of the long tradition of modeling interest rates as martingale processes. Arguments supporting martingale expectations date back to Sargent (1976) and Pesando (1979). Elliott and Baier (1979) found empirical evidence for the use of martingale forecasts of interest rates. The martingale forecast model is also chosen as a comparison model because it is a common assumption in the calculation of theoretical monetary stock aggregates, such as the currency equivalent index (Barnett, 1991). I will refer to this model as martingale in all following tables and figures.

### 3.4 Criterion for Evaluation of Forecasting Performance

To evaluate the forecasting performance of each model, I calculate root mean squared error and Theil's  $U$  statistic. Let  $TE_{t+h}$  be the observed value of  $TE$  in period  $t+h$ , and let  $\hat{TE}_{t+h|t}$  be the  $h$ -period ahead forecast of  $TE$  conditional on information available in period  $t$ . Then

$$RMSE_H(model) = \sqrt{\frac{1}{H} \sum_{t=1}^H (TE_{t+h} - \hat{TE}_{t+h|t})^2} \quad (10)$$

and

$$U_H(model) = \frac{\sqrt{\frac{1}{H} \sum_{t=1}^H (TE_{t+h} - \hat{TE}_{t+h|t})^2}}{\sqrt{\frac{1}{H} \sum_{t=1}^H (TE_{t+h} - TE_t)^2}} \quad (11)$$

are calculated, where  $h$  is the forecasting horizon and  $H$  is the total number of forecasts. Theil's  $U$  statistic compares a model's forecasting performance to that of the no change model. When  $U$  is less than one, the model forecast performs better than the no change forecast. When  $U$  is greater than one, the model performs more poorly than the no change forecast.

### 3.5 Out of Sample Simulation Methodology

The experiment in this study is performed in two stages. Stage one: Total expenditure on monetary assets is iteratively forecasted by restricting the data set to a rolling window of 240 observations, i.e. in period  $T$  we restrict the data set to periods  $t \in \{T-240, \dots, T\}$  for all periods  $T$  starting in period 1980:03 and ending in period 2002:02. Thus, each forecast is made using only data before the forecast period. For example, the 12 month forecast of  $TE_{t+12}$  uses only data available in time period  $t$ . After removing unit roots from the data, the adjusted data set contains 540 observations. This procedure is implemented for each forecasting method. Stage two: Forecasted  $TE$  from each forecasting method is used to calculate a feasible measure of the ESM for the time period 1980:03 through 1997:12 and are then compared to PF. The time interval 1980:03 through 1997:12 is chosen because the data set allows for simulated out of sample forecasts to begin in 1980:03, and the data set allows for the calculation of PF through 1997:12. The results of each stage of the experiment are reported in the next section.

## 4 Results

In this section, I will report the results of the forecasting comparison and use the best of the forecasting models, AR and TFM, to generate the expectations need to calculate a feasible ESM index number. I will then compare the performance of the two index numbers I calculate to that of the three index numbers

calculated by Barnett et al. (2005) and the official aggregates, which use simple sum methodology.

## 4.1 Forecasting Results

In order to evaluate each of the forecasting models in question, I compare each to actual observed values. The results are generated using Ox version 4.00 (Doornik, 2006). See <http://www.doornik.com> for further information. Table 1 reports the performance of each forecast by measuring the root mean squared error (RMSE) and Theil's  $U$  statistic.

Targeted factor model forecasts outperform all other models, based on Theil's  $U$  statistic, at the M2 and the M3 levels of aggregation and at a forecasting horizons of six and 12 months and targeted factor models are also highly competitive at the M2 and M3 levels of aggregation at longer time horizons. At the M1 level of aggregation, martingale forecasts outperform all other models at all forecasting horizons .

## 4.2 ESM Calculation Results

In order to evaluate how well each of the indices in question is able to measure the ESM, I compare each of index to the PF described above. The results are generated using Ox version 4.00 (Doornik, 2006). Figures 2, 4 and 6 plot the best fitting index calculated in this paper, the best fitting index calculated by Barnett et al. (2005) and the PF at the M1, M2 and M3 levels of aggregation. Figures 1, 3 and 5 present a box plot of the percent deviations from PF of each index. All figures will use the following abbreviations: AR is the auto regression based index, TFM is the targeted factor model based index and B1, B2 and B3 refer the index number calculated using Barnett et al. method 1, 2 and 3 respectively.

Table 2 reports the performance of each index calculated in this paper and the indexes calculated by Barnett et al. (2005) by measuring the mean percent error (MPE), mean absolute percent error (MAPE), root mean squared error (RMSE) and normalized root mean squared error (NRMSE) of each index relative to PF. NRMSE is calculated by dividing the RMSE of each index number by the RMSE of the simple sum aggregates. NRMSE can be interpreted similarly to Theil's  $U$  statistic.

At the M1 level of aggregation, B1 dominates based on RMSE but B2 dominates based on MAPE. The TFM has slightly higher RMSE and MAPE, but seem to be unbiased with MPE near zero. See also figure 1. At the M2 level of aggregation, the TFM outperforms all other models based on RMSE, but the AR based index number has the lower MAPE. Moreover, the RMSE calculated for AR and B2 is only slightly larger than TFM. Similar results are observed at M3 level of aggregation. One clear advantage of the TFM is that the TFM appears to be unbiased as measured by MPE. All of the indices calculated by Barnett et al. (2005) exhibit bias as measured by MPE, with the bias observed at the M2 and M3 levels of aggregation being over ten percent. It should be

Table 1: Forecasting Model Comparison Results

<i>Variable</i>	<i>Method</i>	<i>6 Month</i>		<i>12 Month</i>	
		<i>Forecasting Horizon</i>	<i>U Stat.</i>	<i>Forecasting Horizon</i>	<i>U Stat.</i>
		<i>RMSE</i>		<i>RMSE</i>	
TE (M1)	Martingale	19.58065	1.00000	36.77069	1.00000
	AR	23.13588	1.18157	38.79432	1.05503
	TFM	21.46643	1.09631	40.06596	1.08962
TE (M2)	Martingale	528.68123	1.00000	1217.54731	1.00000
	AR	532.18815	1.00663	1163.32466	0.95547
	TFM	513.46344	0.97122	1158.32468	0.95136
TE (M3)	Martingale	1450.73554	1.00000	3342.65402	1.00000
	AR	1159.16294	0.79902	2908.17955	0.87002
	TFM	1068.20332	0.73632	2766.15834	0.82753
<i>Variable</i>	<i>Method</i>	<i>24 Month</i>		<i>36 Month</i>	
		<i>Forecasting Horizon</i>	<i>U Stat.</i>	<i>Forecasting Horizon</i>	<i>U Stat.</i>
		<i>RMSE</i>		<i>RMSE</i>	
TE (M1)	Martingale	54.38643	1.00000	80.35107	1.00000
	AR	60.17125	1.10637	86.14545	1.07211
	TFM	61.03253	1.12220	93.98284	1.16965
TE (M2)	Martingale	2142.22520	1.00000	3325.83595	1.00000
	AR	1928.17834	0.90008	2940.10811	0.88402
	TFM	1937.90244	0.90462	2960.97874	0.89030
TE (M3)	Martingale	6150.45427	1.00000	9813.96862	1.00000
	AR	5355.93840	0.87082	8457.48247	0.86178
	TFM	5440.54167	0.88458	8815.59455	0.89827

noted that although the Barnett indices exhibit bias, this bias is small when compared to the bias observed in the official measures of money stock.

The TFM appears to perform better overall at higher levels of aggregation and is unbiased at all levels of aggregation. To determine the magnitude of this improvement, I calculate NRMSE. According to NRMSE, TFM perform best at the M2 and M3 levels of aggregation, but the difference in NRMSE is very small. Moreover, Barnett's methods perform better at the M1 level of aggregation, but again the difference in NRMSE is very small. This finding is consistent with the robustness result of Barnett et al. (2005). Because service flows occurring in the distant future are more heavily discounted than service flows occurring in the present or near future, forecasting methods that provide good predictions over one to 24 month time horizons work well when calculating the ESM. The ESM is robust to the forecasting method chosen because while forecasting error grows as the time horizon increases, the effect of that forecasting error on the ESM diminishes as the time horizon increases.

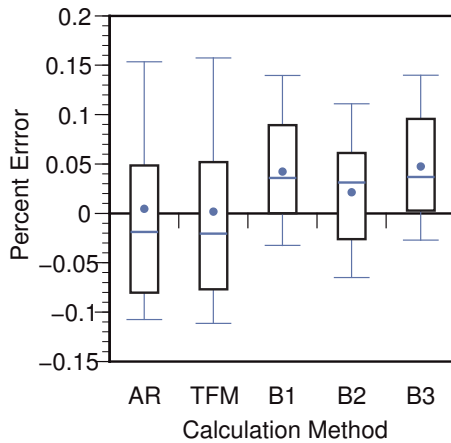


Figure 1: Box plot of percent errors relative to PF across calculation methods (M1)

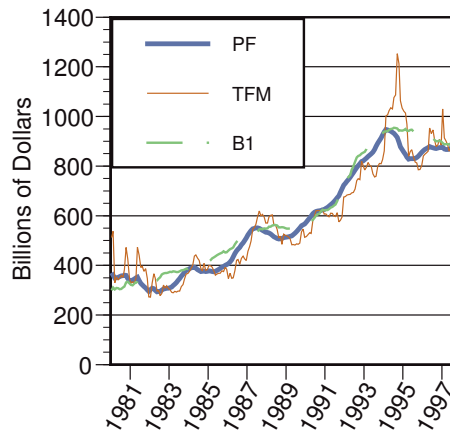


Figure 2: Plot of the best fitting index calculated in this paper, the best fitting calculated by Barnett et al. and PF (M1)

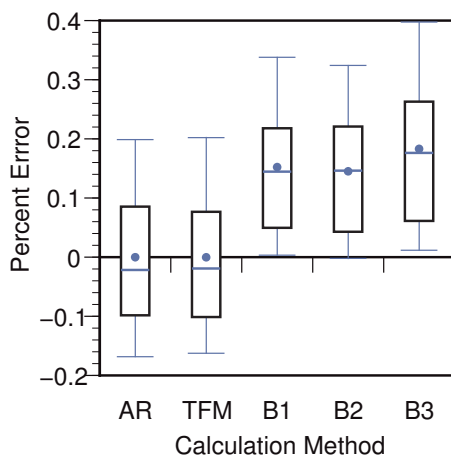


Figure 3: Box plot of percent errors relative to PF across calculation methods (M2)

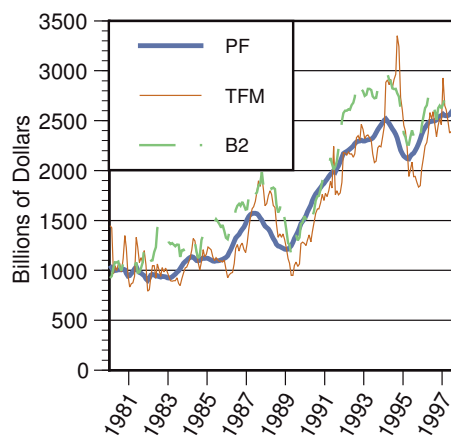


Figure 4: Plot of the best fitting index calculated in this paper, the best fitting calculated by Barnett et al. and PF (M2)

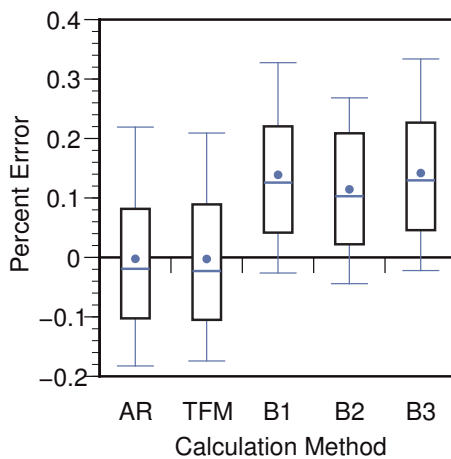


Figure 5: Box plot of percent errors relative to PF across calculation methods (M3)

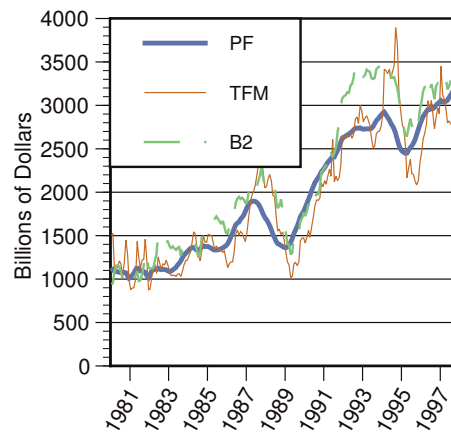


Figure 6: Plot of the best fitting index calculated in this paper, the best fitting calculated by Barnett et al. and PF (M3)

Table 2: Comparison of Methods of Calculating the ESM

<i>Variable</i>	<i>Forecasting Method</i>	<i>RMSE</i>	<i>NRMSE</i>	<i>MAPE</i>	<i>MPE</i>
ESM (M1)	Simple Sum	207.32	1.00	36.46%	36.43%
	Auto Regression Model	68.76	0.33	8.58%	0.46%
	Targeted Factor Model	67.39	0.33	8.39%	0.18%
	Barnett et al. (2005) Method 1	40.66	0.20	6.47%	4.23%
	Barnett et al. (2005) Method 2	42.38	0.20	5.98%	2.13%
	Barnett et al. (2005) Method 3	41.62	0.20	6.76%	4.75%
ESM (M2)	Simple Sum	1259.78	1.00	82.35%	82.35%
	Auto Regression Model	235.41	0.19	10.85%	0.00%
	Targeted Factor Model	232.03	0.18	10.96%	-0.01%
	Barnett et al. (2005) Method 1	287.45	0.23	15.82%	15.24%
	Barnett et al. (2005) Method 2	279.54	0.22	15.48%	14.52%
	Barnett et al. (2005) Method 3	322.19	0.26	18.77%	18.32%
ESM (M3)	Simple Sum	1769.31	1.00	97.93%	97.93%
	Auto Regression Model	295.93	0.17	11.59%	-0.23%
	Targeted Factor Model	289.63	0.16	11.74%	-0.25%
	Barnett et al. (2005) Method 1	341.80	0.19	15.22%	13.88%
	Barnett et al. (2005) Method 2	324.32	0.18	13.50%	11.46%
	Barnett et al. (2005) Method 3	344.25	0.19	15.39%	14.20%

## 5 Conclusion

Aggregation theoretic measures of the ESM require forecasting future monetary service flows. This has produced reluctance to accept aggregation theoretic measures of the ESM. The currency equivalent (see Barnett, 1991; Rotemberg, Driscoll, and Poterba, 1995), for example, makes simplifying assumptions that result in a biased measure of the ESM. This paper attempts to answer some of those objections by using several forecasting methods to generate the expectations needed for calculating the ESM, and then compares the accuracy of the ESM index number generated using each forecasting method.

Using targeted factor model forecasting improves the measurement of the ESM at the M2 and M3 levels of aggregation but not at the M1 level aggregation based on RMSE, but the magnitude of that improvement is small as measured by the NRMSE. One clear advantage of the TFM, however, is that the TFM based appears to be unbiased as measured by MPE.

While the TFM is a slightly better fit to PF, each of the other measures fit nearly as well. Although, B1, B2 and B3 appear to be biased. Thus, measurement of the ESM appears to be robust to the forecasting methodology chosen. Even the currency equivalent index that uses martingale forecasts of future monetary service flows exhibits only a small bias when compared to that of the inferior simple sum indices. This robustness stems from the fact that distant future monetary service flows bear little impact on the ESM measurement.

I conclude that while more sophisticated forecasting methodology may lead to slightly improved measures of the money stock, index numbers based on simple forecasting models, such as the modified currency equivalent index (see Barnett, Keating, and Kelly, 2007), are easily calculated, internally consistent and relatively accurate measures of the ESM as compared to the atheoretic simple sum indices.

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