# A Generalized Quality-Ladder Growth Model with Overlapping Intellectual Property Rights: Quantifying the Effects of Blocking Patents on RD

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**A Generalized Quality-Ladder Growth Model with Overlapping Intellectual Property Rights: Quantifying the Effects of Blocking Patents on R&D**

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**Abstract**

What are the effects of blocking patents on R&D and consumption? This paper develops an R&D-driven endogenous growth model with overlapping intellectual property rights to quantify the inefficiency in the patent system. The analysis focuses on two policy variables: (a) patent breadth that determines the total profit received by a patent pool; and (b) the profit-sharing rule that determines the distribution of surplus between innovators. To quantify the inefficiency arising from blocking patents that are generated by these two policy variables, the model is calibrated to aggregate data of the US economy. Under parameter values that match key features of the US economy and show equilibrium R&D underinvestment, I find that eliminating blocking patents would lead to a conservatively estimated increase in R&D of 12% and long-run consumption of 4% per year. This paper also quantifies the transition-dynamic effects of patent policy and shows implications that are different from previous studies in important ways.

***Keywords*****:** blocking patents, endogenous growth, patent breadth, patent pools, R&D

***JEL classification*****:** O31, O34

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> "Today, most basic and applied researchers are effectively standing on top of a huge pyramid… Of course, a pyramid can rise to far greater heights than could any one person... But what happens if, in order to scale the pyramid and place a new block on the top, a researcher must gain the permission of each person who previously placed a block in the pyramid, perhaps paying a royalty or tax to gain such permission? Would this system of intellectual property rights slow down the construction of the pyramid or limit its heights? … To complete the analogy, *blocking patents* play the role of the pyramid's building blocks." – Carl Shapiro (2001)

## 1. Introduction

What are the effects of blocking patents on research and development (R&D)? In an environment with cumulative innovations, the scope of a patent (i.e. patent breadth) determines the level of patent protection for an invention against imitation and subsequent innovations. This latter form of patent protection, which is known as leading breadth in the literature, gives the patentholders property rights over future inventions, and the resulting overlapping intellectual property rights may dampen the incentives for R&D. This phenomenon is referred to as blocking patents.

The main contribution of this paper is to develop an R&D-driven endogenous growth model to quantify this inefficiency in the patent system. To the best of my knowledge, this paper is the first to perform a quantitative analysis on patent policy by calibrating a dynamic general equilibrium (DGE) model with the following features: (a) overlapping intellectual property rights that are emphasized by the patent-design literature; (b) multiple R&D externalities that are commonly discussed in the growth literature; and (c) endogenous capital accumulation that leads to a dynamic distortionary effect of patent protection on saving and investment and transition-dynamic effects different from previous studies. As Acemoglu (p. 1112, 2007) writes, "… we lack a framework similar to that used for the analysis of the effects of capital and labor income taxes and indirect taxes in public finance, which we could use to analyze the effects… of intellectual property right polices… on innovation and economic growth."

The analysis focuses on two policy variables: (a) patent breadth that determines the total profit received by a patent pool; and (b) the profit-sharing rule that determines the distribution of surplus between innovators. In order to quantify the inefficiency arising from blocking patents and other externalities, the model is calibrated to aggregate data of the US economy.[1] I also show that the key equilibrium condition, which is used to identify the effects of blocking patents on R&D, can be derived analytically without relying on the entire structure of the DGE model. In particular, it can be derived from two conditions: (a) a zero-profit condition in the R&D sector; and (b) a no-arbitrage condition that determines the market value of patents. The DGE model serves the useful purpose of providing a structural derivation and interpretation on the effects of blocking patents.

The main result is the following. Blocking patents have a significant and negative effect on R&D, and eliminating them would lead to a minimum increase in R&D of 12%. This result has important policy implications because given previous empirical estimates on the social rate of return to R&D, the market economy underinvests in R&D relative to the social optimum. To understand this finding, the DGE framework has been made rich enough to be consistent with either R&D overinvestment or underinvestment by combining blocking patents with multiple R&D externalities. Whether the market economy overinvests or underinvests in R&D depends crucially on the degree of externalities in intratemporal duplication and intertemporal knowledge spillovers, which in turn is calibrated from the balanced-growth condition between long-run total factor productivity (TFP) growth and R&D. The larger is the fraction of long-run TFP growth driven by R&D, the larger are the social benefits of R&D; as a result, the more likely it is for the market economy to underinvest in R&D. I use previous empirical estimates for the social rate of return to R&D to calibrate this fraction.

Furthermore, the effects of eliminating blocking patents on consumption in the long run and during the transition dynamics are considered. When blocking patents are eliminated, the balanced-growth level of consumption increases by a minimum of 4% per year. During the transition dynamics, the economy does not always experience a significant fall in consumption in response to the resource

[1] As a robustness check, the model is also calibrated to industry-level data of R&D-intensive industries.

reallocation away from the production sector to the R&D sector. Over a wide range of parameters, upon eliminating blocking patents, consumption gradually rises towards the new balanced-growth path by reducing investment in physical capital and temporarily running down the capital stock. This finding contrasts with Kwan and Lai (2003), whose model does not feature capital accumulation and hence predicts consumption losses during the transition path.

Finally, I identify and analytically derive a *dynamic* distortionary effect of patent protection on saving and investment that has been neglected by previous studies on patent policy, which focus mostly on the *static* distortionary effect of markup pricing.[2] The dynamic distortion arises because the markup in the patent-protected industries creates a wedge between the marginal product of capital and the rental price. Proposition 2 derives the sufficient conditions under which: (a) the market equilibrium rate of investment in physical capital is below the socially optimal level; and (b) an increase in the markup reduces the equilibrium investment rate in physical capital. The numerical exercise also quantifies the discrepancy between the equilibrium capital investment rate and the socially optimal level and shows that eliminating blocking patents helps reducing this discrepancy.

**Literature Review**

This paper relates to a number of studies on R&D underinvestment and provides through the elimination of blocking patents an effective method to mitigate the R&D-underinvestment problem suggested by Jones and Williams (1998) and (2000). Furthermore, the calibration exercise takes into consideration Comin's (2004) critique that long-run TFP growth may not be solely driven by R&D. The current paper also complements the qualitative partial-equilibrium studies on leading breadth from the patent-design

[2] Laitner (1982) is the first study that identifies in an exogenous growth model with overlapping generations of households that the existence of an oligopolistic sector and its resulting pure profit as financial assets creates both the usual static distortion and an additional dynamic distortion on capital accumulation due to the crowding out of households' portfolio space. The current paper extends this study to show that this dynamic distortion also plays an important role and through a different channel in an R&D-driven endogenous growth model in which both patents and physical capital are owned by households as financial assets.

literature,[3] such as Green and Scotchmer (1995), O'Donoghue *et al* (1998) and Hopenhayn *et al* (2006), by providing a quantitative DGE analysis. O'Donoghue and Zweimuller (2004) is the first study that merges the patent-design and endogenous growth literatures to analyze the effects of patentability requirement, lagging and leading breadth on economic growth in a canonical quality-ladder growth model. However, their focus was not in quantifying the effects of blocking patents on R&D. In addition, the current paper generalizes their model in a number of dimensions in order to perform a quantitative analysis on the transition dynamics. Other DGE analysis on patent policy includes Goh and Olivier (2002), Grossman and Lai (2004) and Li (2001).[4] These studies are also qualitatively oriented and do not feature capital accumulation so that the dynamic distortionary effect of patent policy is absent.

In terms of quantitative analysis on patent policy, this paper relates to Kwan and Lai (2003) and Chu (2007). Kwan and Lai (2003) numerically evaluate the effects of extending the effective lifetime of patent in the variety-expanding model originating from Romer (1990) and find substantial welfare gains despite the temporary consumption losses during the transition path in their model. Chu (2007) uses a generalized variety-expanding model and finds that whether or not extending the patent length would lead to a significant increase in R&D depends crucially on the patent-value depreciation rate. At the empirical range of patent-value depreciation rates estimated by previous studies, extending the patent length has only limited effects on R&D and thus social welfare. Therefore, Chu (2007) and the current paper together provide a comparison on the relative effectiveness of extending the patent length and eliminating blocking patents in mitigating the R&D-underinvestment problem. The crucial difference between these two policy instruments arises because extending the patent length increases future monopolistic profit while eliminating blocking patents raises current monopolistic profit for the inventors.

[3] The seminal work on optimal patent length is Nordhaus (1969). Some other recent studies on optimal patent design include Tandon (1982), Gilbert and Shapiro (1990), Klemperer (1990), O'Donoghue (1998), Hunt (1999) and Scotchmer (2004). Judd (1985) provides the first DGE analysis on optimal patent length.

[4] Goh and Olivier (2002) analyze the welfare effects of patent breadth in a two-sector variety-expanding growth model, and Grossman and Lai (2004) analyze the welfare effects of strengthening patent protection in developing countries as a result of the TRIPS agreement using a multi-country variety-expanding model. However, these studies do not analyze patent breadth in an environment with cumulative innovations. Li (2001) analyzes the optimal policy mix of R&D subsidy and lagging breadth in a quality-ladder model with endogenous step size, but he does not consider leading breadth.

The rest of the paper is organized as follows. Section 2 derives the equilibrium condition that is used to identify the effects of blocking patents. Section 3 describes the DGE model. Section 4 calibrates the model and presents the numerical results. The final section concludes. The proofs, derivations, tables and figures are relegated to several appendices.

## 2. An Intuitive Derivation of the Market-Equilibrium Condition for R&D

In this section, I show that the steady-state equilibrium condition for R&D that is crucial for the calibration can be derived from: (a) a zero-profit condition in the R&D sector; and (b) a no-arbitrage condition that determines the market value of patents. Intuitively, given the level of R&D spending in the data, the private benefit of R&D can be inferred from the zero-profit condition. Then, given the private benefit of R&D, the amount of the monopolistic profit received by inventors can be inferred from the no-arbitrage condition. Finally, the discrepancy between the amount of profit received by inventors and the total amount of monopolistic profit is attributed to blocking patents, and the DGE model serves the useful purpose of providing a structural derivation and interpretation of this discrepancy.

The zero-profit condition in the R&D sector implies that

(1) $$\lambda V = R\,\&\,D.$$

$V$ is the market value of a patented invention, and $\lambda$ is the Poisson arrival rate of innovations. $R\,\&\,D$ is the amount of R&D spending. The no-arbitrage condition implies that the market value of a patented invention is the expected present value of monopolistic profit received by the inventor; therefore,

(2) $$V = \frac{\pi_{inventor}}{r + \lambda - g_{\pi}},$$

where $r$ is the real interest rate, and $g_{\pi}$ is the growth rate of monopolistic profit. Because of blocking patents, an inventor may only capture a fraction of the monopolistic profit generated by her invention.

(3) $$\pi_{inventor} = v\,\pi_{monopolistic},$$

where $\nu \in (0,1]$. At this point, the interpretation of $\nu$ is quite vague, and the DGE model provides a structural interpretation of $\nu$ as the backloading discount factor, which captures the effects of delayed reward due to profit-sharing in patent pools.

Substituting (3) and (2) into (1) yields

$$R\ \&\ D = \nu\left(\frac{\lambda}{r+\lambda-g_{\pi}}\right)\pi_{monopolistic}. \tag{4}$$

Finally, the amount of monopolistic profit is given by

$$\pi_{monopolistic} = \left(\frac{\mu-1}{\mu}\right)Y. \tag{5}$$

In the case of industry-level data, $\mu$ is the industry markup and *Y* is the valued-added of R&D-intensive industries. Since empirical estimates for industry markup are known to be imprecise, I will perform the calibration using both industry-level data and aggregate data to ensure the robustness of the numerical results. However, calibration based on aggregate data requires an additional assumption that economic profit in the economy is created by intellectual monopoly. Given this assumption, $\mu$ in (5) becomes the aggregate or average markup in the economy and *Y* becomes the value of gross domestic product (GDP).

## 3. The Model

The model is a generalized version of Grossman and Helpman (1991) and Aghion and Howitt (1992). The final goods, which can be either consumed by households or invested in physical capital, are produced with a composite of differentiated intermediate goods. The intermediate goods are produced with labor and capital, and there are both competitive and monopolistic industries in the intermediate-goods sector. The relative price between the monopolistic and competitive goods leads to the usual *static* distortionary effect that reduces the output of final goods. The markup in the monopolistic industries drives a wedge between the marginal product of capital and the rental price; consequently, it leads to an additional *dynamic* distortionary effect that causes the market equilibrium rate of investment in physical capital to deviate from the social optimum. The R&D sector also uses both labor and capital as factor inputs.

To prevent the model from overestimating the social benefits of R&D and the extent of R&D underinvestment, the long-run TFP growth is assumed to be driven by R&D as well as an exogenous process as in Comin (2004). The class of first-generation R&D-driven endogenous growth models, such as Grossman and Helpman (1991) and Aghion and Howitt (1992), exhibits scale effects and is inconsistent with the empirical evidence in Jones (1995a).[5] In the present model, scale effects are eliminated by assuming decreasing *individual* R&D productivity as in Segerstrom (1998), which becomes a semi-endogenous growth model.[6]

The various components of the model are presented in Sections 3.1–3.7, and the decentralized equilibrium is defined in Section 3.8. Section 3.9 summarizes the laws of motion that characterize the transition dynamics, and Section 3.10 discusses the balanced-growth path. Section 3.11 derives the socially optimal allocations and the dynamic distortionary effect of patent protection.

### 3.1. Representative Household

The infinitely-lived representative household maximizes life-time utility that is a function of per-capita consumption $c_t$ of the numeraire final goods and is assumed to have the iso-elastic form given by

$$U = \int_0^{\infty} e^{-(\rho - n)t} \frac{c_t^{1-\sigma}}{1-\sigma} dt , \tag{6}$$

where $\sigma \geq 1$ is the inverse of the elasticity of intertemporal substitution and $\rho$ is the subjective discount rate. The household has $L_t = L_0 \exp(nt)$ members at time *t*. The population size at time 0 is normalized

[5] See, e.g. Jones (1999) for an excellent theoretical analysis on scale effects.

[6] In a semi-endogenous growth model, the balanced-growth rate is determined by the exogenous labor-force growth rate. An increase in the share of R&D factor inputs raises the *level* of the balanced growth path while holding the balanced-growth rate constant. Since increasing R&D has no long-run growth effect in this model, the calibrated effects on consumption in the numerical exercises are likely to be more conservative than in other fully endogenous growth models.

to one, and $n > 0$ is the exogenous population growth rate. To ensure that lifetime utility is bounded, it is assumed that $\rho > n$. The household maximizes (6) subject to a sequence of budget constraints given by

(7) $$\dot{a}_t = a_t(r_t - n) + w_t - c_t .$$

Each member of the household inelastically supplies one unit of homogenous labor in each period to earn a real wage income $w_t$. $a_t$ is the value of risk-free financial assets in the form of patents and physical capital owned by each household member, and $r_t$ is the real rate of return on these assets. The familiar Euler equation derived from the intertemporal optimization is

(8) $$\dot{c}_t = c_t(r_t - \rho)/\sigma .$$

### 3.2. Final Goods

This sector is characterized by perfect competition, and the producers take both the output price and input prices as given. The production function for the final goods $Y_t$ is a Cobb-Douglas aggregator of a continuum of differentiated quality-enhancing intermediate goods $X_t(j)$ for $j \in [0,1]$ given by

(9) $$Y_t = \exp\left(\int_0^1 \ln X_t(j)dj\right).$$[7]

The familiar aggregate price index is

(10) $$P_t = \exp\left(\int_0^1 \ln P_t(j)dj\right) = 1,$$

and the demand curve for each variety of intermediate goods is

(11) $$P_t(j)X_t(j) = Y_t .$$

[7] To maintain the analytical tractability of the aggregate conditions, a Cobb-Douglas aggregator instead of the more general CES aggregator is adopted. With the CES aggregator, it becomes very difficult to derive the aggregate conditions when there are both competitive and monopolistic industries in the intermediate-goods sector. Furthermore, computation of the transition dynamics becomes possible under the Cobb-Douglas aggregator. Although the arrival rate of innovations varies along the transitional path, a tractable form for the law of motion for aggregate technology can still be derived under the Cobb-Douglas aggregator but not under the CES aggregator.

### 3.3. Intermediate Goods

There is a continuum of industries producing the differentiated quality-enhancing intermediate goods $X_t(j)$ for $j \in [0,1]$. A fraction $\theta \in [0,1)$ of the industries is characterized by perfect competition because innovations in these industries are assumed to be non-patentable. Each of the remaining industries is dominated by a temporary industry leader, who owns the patent for the latest R&D-driven technology for production. Without loss of generality, the industries are ordered such that industries $j' \in [0,\theta)$ are competitive and industries $j \in [\theta,1]$ are monopolistic. The production function in each industry has constant returns to scale in labor and capital inputs and is given by

(12) $$X_t(j) = z^{m_t(j)} Z_t K_{x,t}^{\alpha}(j) L_{x,t}^{1-\alpha}(j)$$

for $j \in [0,1]$. $K_{x,t}(j)$ and $L_{x,t}(j)$ are respectively the capital and labor inputs for producing intermediate-goods $j$ at time *t*. $Z_t = Z_0 \exp(g_Z t)$ represents an exogenous process of productivity improvement that is common across all industries and is freely available to all producers. $z^{m_t(j)}$ is industry *j*'s level of R&D-driven technology, which is increasing over time through R&D investment and successful innovations. $z > 1$ is the exogenous step-size of a technological improvement arising from each innovation. $m_t(j)$, which is an integer, is the number of innovations that has occurred in industry *j* as of time *t*. The marginal cost of production in industry *j* is

(13) $$MC_t(j) = \frac{1}{z^{m_t(j)} Z_t} \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha},$$

where $R_t$ is the rental price of capital. The optimal price for the leaders in the monopolistic industries is a constant markup $\mu(z,\eta)$ over the marginal cost of production given by

(14) $$P_t(j) = \mu(z,\eta) MC_t(j)$$

for $j \in [\theta,1]$. The markup $\mu(z,\eta)$ is a function of the quality step size $z$ and the level of patent breadth $\eta$ (to be defined in Section 3.4). The competitive industries are characterized by competitive pricing so

$$P_t(j') = MC_t(j') \tag{15}$$

for $j' \in [0, \theta)$. The aggregate price level is

$$P_t = \tilde{\mu}(z, \eta, \theta) MC_t, \tag{16}$$

where $\tilde{\mu}(z, \eta, \theta) \equiv \mu(z, \eta)^{1-\theta}$ is the aggregate markup in the economy. The aggregate marginal cost is

$$MC_t = \exp\left( \int_0^1 \ln MC_t(j) dj \right). \tag{17}$$

### 3.4. Patent Breadth

Before providing the underlying derivations, this section firstly presents the Bertrand equilibrium price and the amount of monopolistic profit generated by an invention under different levels of patent breadth, which is denoted by $\eta$.

$$P_t(j) = z^{\eta} MC_t(j), \tag{18}$$

$$\pi_t(j) = (z^{\eta} - 1) MC_t(j) X_t(j), \tag{19}$$ [8]

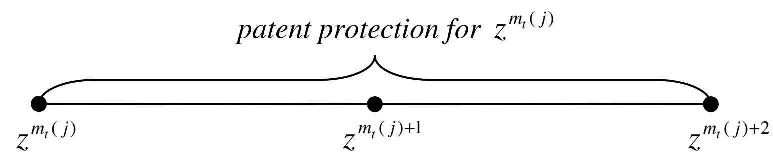
for $\eta \in \{1, 2, 3, ...\}$ and $j \in [\theta, 1]$. The expression for the equilibrium price is consistent with the seminal work of Gilbert and Shapiro's (1990) interpretation of "breadth as the ability of the patentee to raise price." A broader patent breadth corresponds to a larger $\eta$, and vice versa. Therefore, an increase in patent breadth *potentially* enhances the incentives for R&D by raising the amount of monopolistic profit generated by each invention but worsens the distortionary effects of markup pricing.

The patent-design literature has identified and analyzed two types of patent breadth in an environment with cumulative innovations: (a) lagging breadth; and (b) leading breadth. In a standard quality-ladder growth model, lagging breadth (i.e. patent protection against imitation) is assumed to be complete while leading breadth (i.e. patent protection against subsequent innovations) is assumed to be

[8] Note that the inventor may only capture a fraction of this monopolistic profit because of blocking patents.

zero. The following analysis assumes complete lagging breadth and focuses on non-zero leading breadth, and the formulation originates from O'Donoghue and Zweimuller (2004).[9]

The level of patent breadth $\eta = \eta_{lag} + \eta_{lead}$ can be decomposed into lagging breadth denoted by $\eta_{lag} \in (0,1]$ and leading breadth denoted by $\eta_{lead} \in \{0,1,2,...\}$. In the following, complete lagging breadth is assumed such that $\eta = 1 + \eta_{lead}$. Nonzero leading breadth protects patentholders against subsequent innovations and gives the patentholders property rights over future inventions. For example, if $\eta_{lead} = 1$, then the most recent innovation infringes the patent of the second-most recent inventor. If $\eta_{lead} = 2$, then the most recent innovation infringes the patents of the second-most and the third-most recent inventors, etc. The following diagram illustrates the concept of nonzero leading breadth with an example in which the degree of leading breadth is two.




Therefore, nonzero leading breadth facilitates the new industry leader and the previous inventors, whose patents are infringed, to consolidate market power through licensing agreements or the formation of a patent pool resulting in a higher markup.[10] The Bertrand equilibrium price with leading breadth is

$$P_t(j) = z^{1+\eta_{lead}} MC_t(j) \tag{20}$$

for $\eta_{lead} \in \{0,1,2,...\}$ and $j \in [\theta,1]$. Assumption 1 is *sufficient* to derive this equilibrium markup price.

**Assumption 1:** *An infringed patentholder cannot become the next industry leader while she is still covered by a licensing agreement in that industry.*[11]

[9] See, e.g. Li (2001) for a discussion of incomplete lagging breadth.
[10] See, e.g. Gallini (2002) and O'Donoghue and Zweimuller (2004), for a discussion on market-power consolidation through licensing agreements.
[11] The sufficiency of this assumption in determining the markup price is most easily understood with an example. Suppose leading breadth is one and lagging breadth is complete, the lower bound on the profit-maximizing markup

Then, the amount of monopolistic profit captured by the licensing agreement or patent pool at time *t* is

(21) $$\pi_t(j) = (z^{1+\eta_{lead}} - 1)MC_t(j)X_t(j)$$

for $\eta_{lead} \in \{0,1,2,...\}$ and $j \in [\theta,1]$.

The share of profit obtained by each generation of patentholders in the patent pool depends on the profit-sharing rule (i.e. the terms in the licensing agreement). A stationary bargaining outcome is assumed to simplify the analysis.

**Assumption 2:** *The profit-sharing rule is symmetric across industries and is stationary. For each degree of leading breadth* $\eta_{lead} \in \{0,1,2,...\}$, *the profit-sharing rule is* $\Omega^{\eta_{lead}} = (\Omega_1,...,\Omega_\eta) \in [0,1]$, *where* $\Omega_i$ *is the share of profit received by the i-th most recent inventor, and* $\sum_{i=1}^{\eta} \Omega_i = 1$.

Although the shares of profit and licensing fees eventually received by the owner of an invention are constant overtime, the present value of profit is determined by the actual profit-sharing rule. The two extreme cases are: (a) *complete frontloading* $\Omega^{\eta_{lead}} = (1,0,...,0)$; and (b) *complete backloading* $\Omega^{\eta_{lead}} = (0,0,...,1)$. Complete frontloading maximizes the incentives on R&D provided by leading breadth by maximizing the present value of profit received by an inventor. The opposite effect of blocking patents arises when profit is backloaded, and complete backloading maximizes this damaging effect on the incentives for R&D. The law of motion for the market value of ownership in patent pools for each generation of patentholders will be derived in Section 3.7.

is the square of $z$, which is the limit price from the collusion of the most recent and the second-most recent inventors against the third-most recent inventor, whose patent is not infringed upon by the most recent invention. In this example, the limit-pricing markup would be even larger if the third-most recent inventor happens to be the new industry leader. Continuing this reasoning, the markup could grow without bound; therefore, Assumption 1 is made to rule out this possibility. The empirical plausibility of this assumption is appealed to the existence of antitrust policy.

### 3.5. Aggregation and Static Distortion

Define $A_t \equiv \exp\left(\int_0^1 m_t(j)dj \ln z\right)$ as the aggregate level of R&D-driven technology. Also, define total labor and capital inputs for production as $K_{x,t} = \int_0^1 K_{x,t}(j)dj$ and $L_{x,t} = \int_0^1 L_{x,t}(j)dj$ respectively.

**Lemma 1:** *The aggregate production function for the final goods is*

$$Y_t = \vartheta(\eta) A_t Z_t K_{x,t}^{\alpha} L_{x,t}^{1-\alpha}, \tag{22}$$

*where* $\vartheta(\eta) \equiv (z^{\eta})^{\theta} / (z^{\eta}\theta + 1 - \theta)$ *is decreasing in* $\eta$ *for* $\theta \in (0,1)$.

**Proof:** Refer to Appendix I.

$\vartheta(\eta)$ captures the static distortionary effect of the markup $z^{\eta}$. Markup pricing in the monopolistic industries distorts production towards the competitive industries and reduces the output of the final goods. Also, $\vartheta(\eta)$ is initially decreasing in $\theta$ and subsequently increasing with $\vartheta(\eta) = 1$ for $\theta \in \{0,1\}$. Therefore, at least over a range of parameters, the static distortionary effect becomes increasingly severe as the fraction of competitive industries increases. The distortionary effect is not monotonic in $\theta$ because the relative-price distortion disappears when either all industries are monopolistic or competitive.

The market-clearing condition for the final goods is

$$Y_t = C_t + I_t, \tag{23}$$

where $C_t = L_t c_t$ is the aggregate consumption and $I_t$ is the investment in physical capital. The factor payments for the final goods are

$$Y_t = w_t L_{x,t} + R_t K_{x,t} + \pi_t. \tag{24}$$

$\pi_t = \int_\theta^1 \pi_t(j)dj$ is the total amount of monopolistic profit. Substituting (12) and (13) into (19) and then summing over all monopolistic industries yields

(25) $$\pi_t = (1-\theta)\left(\frac{z^\eta - 1}{z^\eta}\right)Y_t .$$

Therefore, the growth rate of monopolistic profit equals the growth rate of output. The factor payments for labor and capital inputs in the intermediate-goods sector are respectively

(26) $$w_t L_{x,t} = (1-\alpha)\left(\frac{z^\eta \theta + 1 - \theta}{z^\eta}\right)Y_t ,$$

(27) $$R_t K_{x,t} = \alpha\left(\frac{z^\eta \theta + 1 - \theta}{z^\eta}\right)Y_t .$$

(27) shows that the markup drives a wedge between the marginal product of capital and the rental price. As will be shown below, this wedge creates a dynamic distortionary effect on the rate of investment in physical capital. Finally, the correct value of GDP should include R&D investment such that

(28) $$GDP_t = Y_t + w_t L_{r,t} + R_t K_{r,t} .$$ [12]

$L_{r,t}$ and $K_{r,t}$ are respectively the number of workers and the amount of capital for R&D.

### 3.6. Capital Accumulation

The market-clearing condition for physical capital is

(29) $$K_t = K_{x,t} + K_{r,t} .$$

$K_t$ is the total amount of capital available in the economy at time $t$. The law of motion for capital is

(30) $$\dot{K}_t = I_t - K_t \delta$$

[12] In the national income account, private spending in R&D is treated as an expenditure on intermediate goods. Therefore, the values of investment and GDP in the data are $I_t$ and $Y_t$ respectively. The Bureau of Economic Analysis and the National Science Foundation's R&D satellite account provides preliminary estimates on the effects of including R&D as an intangible asset in the national income accounts.

$\delta$ is the rate of depreciation. The endogenous rate of investment in physical capital is

(31) $$i_t = (\dot{K}_t / K_t + \delta) K_t / Y_t$$

for all $t$. The no-arbitrage condition $r_t = R_t - \delta$ for the holding of capital and (27) imply that the capital-output ratio is

(32) $$\frac{K_t}{Y_t} = \frac{\alpha(z^{\eta}\theta + 1 - \theta)}{z^{\eta}(1 - s_{K,t})(r_t + \delta)}.$$

$s_{K,t}$ is the endogenous share of capital in the R&D sector. Substituting (32) into (31) yields

(33) $$i_t = \frac{\alpha(z^{\eta}\theta + 1 - \theta)}{z^{\eta}(1 - s_{K,t})}\left(\frac{\dot{K}_t / K_t + \delta}{r_t + \delta}\right).$$

In the Romer model, (skilled) labor is the only factor input for R&D (i.e. $s_{K,t} = 0$); therefore, the distortionary effect of markup pricing on the *steady-state* rate of investment in physical capital is unambiguously negative (i.e. $\partial i / \partial \eta < 0$). In the current model, there is an opposing positive effect operating through the R&D share of capital. Intuitively, an increase in patent breadth potentially raises the private return on R&D and increases the R&D share of capital. Proposition 2 in Section 3.11 shows that the negative distortionary effect still dominates if the intermediate-goods sector is at least as capital intensive as the R&D sector.

### 3.7. R&D

$V_t(j)$ is the market value of the patent pool created by the most recent invention in industry $j \in [\theta, 1]$ at time $t$ and is determined by the following no-arbitrage condition

(34) $$r_t V_t(j) = \pi_t(j) + \dot{V}_t(j) - \lambda_t V_t(j).$$

The first terms in the right is the flow profit captured by the patent pool at time $t$. The second term is the capital gain due to the growth in the amount of monopolistic profit. The third term is the expected value of capital loss due to creative destruction, and $\lambda_t$ is the Poisson arrival rate of the next invention that creates a new patent pool. However, the incentives for R&D depend on the market value of the shares in patent pools obtained by the next inventor. Denote $V_{i,t}(j)$ for $i \in \{1,...,\eta\}$ as the market value of ownership in patent pools for the *i*-th most recent inventor in industry $j \in [\theta,1]$.

**Proposition 1:** *$V_{i,t}(j)$ for $i \in \{1,2,...,\eta\}$ and $j \in [\theta,1]$ is determined by the following law of motion*

$$r_t V_{i,t}(j) = \Omega_i \pi_t(j) + \dot{V}_{i,t}(j) + \lambda_t (V_{i+1,t}(j) - V_{i,t}(j)), \tag{35}$$

*where $V_{\eta+1,t}(j) = 0$. The no-arbitrage condition for $V_{1,t}(j)$ can be re-expressed as*

$$V_{1,t}(j) = \pi_t(j)\left(\sum_{k=1}^{\eta} \Omega_k \lambda_t^{k-1}\left(\prod_{i=1}^{k} \frac{1}{r_t + \lambda_t - \dot{V}_{i,t}(j)/V_{i,t}(j)}\right)\right). \tag{36}$$

**Proof:** Refer to Appendix I.

The intuition behind (35) is very similar to (34) with two differences. Firstly, the *i*-th most recent inventor in industry *j* only captures a share $\Omega_i$ of the flow profit. Secondly, when the next invention occurs, the *i*-th most recent inventor losses $V_{i,t}(j)$ but gains $V_{i,t+1}(j)$ as she becomes the *i*+1-th most recent inventor in the next patent pool. Once (35) has been derived for $i \in \{1,2,...,\eta\}$, (36) can be derived by recursive substitutions in order to obtain an expression for $V_{1,t}(j)$.

**Assumption 3:** *Innovation successes of the R&D entrepreneurs are randomly assigned to the industries in the intermediate-goods sector.*[13]

The expected present value of an invention obtained by the most recent inventor at time *t* is

[13] A reasonable implication of this assumption is that the equilibrium level of R&D is determined by the amount of monopolistic profits in the economy.

(37) $$V_{1,t}=\int_{\theta}^{1}V_{1,t}(j)dj=(1-\theta)\left(\frac{z^{\eta}-1}{z^{\eta}}\right)Y_t\left(\sum_{k=1}^{\eta}\Omega_k\lambda_t^{k-1}\left(\prod_{i=1}^{k}\frac{1}{r_t+\lambda_t-\dot{V}_{i,t}/V_{i,t}}\right)\right).$$[14]

The arrival rate of an innovation success for an R&D entrepreneur $h\in[0,1]$ is a function of labor input $L_{r,t}(h)$ and capital input $K_{r,t}(h)$ given by

(38) $$\lambda_t(h)=\overline{\varphi}_t K_{r,t}^{\beta}(h)L_{r,t}^{1-\beta}(h).$$

$\overline{\varphi}_t$ is a productivity parameter that the entrepreneurs take as given. The expected profit from R&D is

(39) $$E_t[\pi_{r,t}(h)]=V_{1,t}\lambda_t(h)-w_tL_{r,t}(h)-R_tK_{r,t}(h).$$

The first-order conditions are

(40) $$(1-\beta)V_{1,t}\overline{\varphi}_t(K_{r,t}(h)/L_{r,t}(h))^{\beta}=w_t,$$

(41) $$\beta V_{1,t}\overline{\varphi}_t(K_{r,t}(h)/L_{r,t}(h))^{\beta-1}=R_t.$$

To eliminate scale effects and capture various externalities, I follow Jones and Williams (2000) to assume that the *individual* R&D productivity parameter $\overline{\varphi}_t$ is given by

(42) $$\overline{\varphi}_t=\varphi(K_{r,t}^{\beta}L_{r,t}^{1-\beta})^{\gamma-1}/A_t^{1-\phi},$$

where $K_{r,t}=\int_0^1 K_{r,t}(h)dh$ and $L_{r,t}=\int_0^1 L_{r,t}(h)dh$. $\gamma\in(0,1]$ captures the negative externality in intratemporal duplication or the so-called "stepping-on-toes" effects, and $\phi\in(-\infty,1)$ captures the externality in intertemporal knowledge spillovers.[15] Given that the arrival of innovations follows a Poisson process, the law of motion for R&D-driven technology is given by

---

[14] Note that the second equality is obtained by firstly integrating over (35) and then by recursive substitutions.

[15] This specification captures how semi-endogenous growth models eliminate scale effects as in Jones (1995b). $\phi\in(0,1)$ corresponds to the "standing-on-shoulder" effect, in which the *economy-wide* R&D productivity $A_q\overline{\varphi}$ increases as the level of R&D-driven technology increases (see the law of motion for R&D-driven technology). On the other hand, $\phi\in(-\infty,0)$ corresponds to the "fishing-out" effect, in which early technology is relatively easy to develop and $A_q\overline{\varphi}$ decreases as the level of R&D-driven technology increases.

(43) $$\dot{A}_t = A_t \lambda_t \ln z = A_t \overline{\varphi}_t K_{r,t}^{\beta} L_{r,t}^{1-\beta} \ln z = A_t^{\phi} (K_{r,t}^{\beta} L_{r,t}^{1-\beta})^{\gamma} \varphi \ln z.$$[16]

### 3.8. Decentralized Equilibrium

The analysis starts at $t = 0$. The equilibrium is a sequence of prices $\{w_t, r_t, R_t, P_t(j), V_{1,t}\}_{t=0}^{\infty}$ and a sequence of allocations $\{a_t, c_t, I_t, Y_t, X_t(j), K_{x,t}(j), L_{x,t}(j), K_{r,t}(h), L_{r,t}(h), K_t, L_t\}_{t=0}^{\infty}$ such that they are consistent with the initial conditions $\{K_0, L_0, Z_0, A_0, \overline{\varphi}_0\}$ and their subsequent laws of motions. Also, in each period,

(a) the representative household chooses $\{a_t, c_t\}$ to maximize utility taking $\{w_t, r_t\}$ as given;

(b) the competitive firms in the final-goods sector choose $\{X_t(j)\}$ to maximize profit according to the production function taking $\{P_t(j)\}$ as given;

(c) each industry leader in the intermediate-goods sector chooses $\{P_t(j), K_{x,t}(j), L_{x,t}(j)\}$ to maximize profit according to the Bertrand price competition and the production function taking $\{R_t, w_t\}$ as given;

(d) the competitive firms in the intermediate-goods sector choose $\{K_{x,t}(j'), L_{x,t}(j')\}$ to maximize profit according to the production function taking $\{P_t(j'), R_t, w_t\}$ as given;

(e) each entrepreneur in the R&D sector chooses $\{K_{r,t}(h), L_{r,t}(h)\}$ to maximize profit according to the R&D production function taking $\{\overline{\varphi}_t, V_{1,t}, R_t, w_t\}$ as given;

(f) the market for the final-goods clears such that $Y_t = C_t + I_t$;

(g) the full employment of capital such that $K_t = K_{x,t} + K_{r,t}$; and

---

[16] This convenient expression is derived as $\ln A_t = \left( \int_0^1 m_t(j) dj \right) \ln z = \left( \int_0^t \lambda(\tau) d\tau \right) \ln z$; then, simple differentiation yields $\dot{A}_t / A_t = \lambda_t \ln z$.

(h) the full employment of labors such that $L_t = L_{x,t} + L_{r,t}$.

### 3.9. Aggregate Equations of Motion

The transition dynamics of the decentralized equilibrium is characterized by the following differential equations. The capital stock is a predetermined variable and evolves according to

$$\dot{K}_t = Y_t - C_t - K_t\delta . \tag{44}$$

R&D-driven technology is also a predetermined variable and evolves according to

$$\dot{A}_t = A_t \lambda_t \ln z . \tag{45}$$

Consumption is a jump variable and evolves according to the Euler equation

$$\dot{c}_t = c_t (r_t - \rho) / \sigma . \tag{46}$$

The market value of ownership in patent pools is also a jump variable and evolves according to

$$\dot{V}_{i,t} = (r_t + \lambda_t) V_{i,t} - \lambda_t V_{i+1,t} - \Omega_i \pi_t \tag{47}$$

for $i \in \{1,2,...,\eta\}$ and $V_{\eta+1,t} = 0$.

At the aggregate level, the generalized quality-ladder model is similar to Jones's (1995b) model, whose dynamic properties have been investigated by a number of recent studies. For example, Arnold (2006) analytically derives the uniqueness and local stability of the steady state with certain parameter restrictions. Steger (2005) and Trimborn *et al* (2007) numerically evaluate the transition dynamics of the model. In summary, to solve the model numerically, I firstly transform $\{K_t, A_t, c_t, V_{i,t}\}$ in the differential equations into its stationary form,[17] and then, compute the transition path from the old steady state to the new one using the relaxation algorithm developed by Trimborn *et al* (2007).

[17] Refer to Appendix II for the details.

### 3.10. Balanced-Growth Path

Equating the first-order conditions (26) and (40) and imposing the balanced-growth condition on R&D-driven technology

(48) $$g_A = \overline{\varphi}_t L_{r,t}^{1-\beta} K_{r,t}^{\beta} \ln z$$

yield the steady-state R&D share of labor inputs given by

(49) $$\frac{s_L}{1-s_L} = \frac{1-\beta}{1-\alpha}\left(\frac{\lambda}{r+\lambda-g_Y}\right)\frac{(z^{\eta}-1)(1-\theta)}{z^{\eta}\theta+(1-\theta)}\nu(\Omega^{\eta_{lead}}),$$

where $\nu(\Omega^{\eta_{lead}}) \equiv \sum_{k=1}^{\eta} \Omega_k \left(\frac{\lambda}{r+\lambda-g_Y}\right)^{k-1} \in (0,1]$ is defined as the backloading discount factor. For example, in the case of complete frontloading, $\nu(\Omega^{\eta_{lead}}) = 1$. Similarly, solving (27), (41) and (48) yields the steady-state R&D share of capital inputs given by

(50) $$\frac{s_K}{1-s_K} = \frac{\beta}{\alpha}\left(\frac{\lambda}{r+\lambda-g_Y}\right)\frac{(z^{\eta}-1)(1-\theta)}{z^{\eta}\theta+(1-\theta)}\nu(\Omega^{\eta_{lead}}).$$

On the balanced-growth path, $c_t$ increases at a constant rate $g_c$, so that the steady-state real interest rate is

(51) $$r = \rho + g_c\sigma.$$

The balanced-growth rate of R&D technology $g_A$ is related to the labor-force growth rate such that

(52) $$g_A = \frac{(K_{r,t}^{\beta} L_{r,t}^{1-\beta})^{\gamma}}{A_t^{1-\phi}}\varphi \ln z = \left(\frac{\gamma\beta}{1-\phi}\right) g_K + \left(\frac{\gamma(1-\beta)}{1-\phi}\right) n.$$

Then, the steady-state rate of creative destruction is $\lambda = g_A / \ln z$. The balanced-growth rates of other variables are given as follows. Given that the steady-state investment rate is constant, the balanced-growth rate of per capita consumption is

(53) $$g_c = g_Y - n.$$

From the aggregate production function (22), the balanced-growth rates of output and capital are

(54) $$g_Y = g_K = n + (g_A + g_Z)/(1-\alpha)\,.$$

Using (52) and (54), the balanced-growth rate of R&D-driven technology is determined by the exogenous labor-force growth rate $n$ and productivity growth rate $g_Z$ given by

(55) $$g_A = \left(\frac{1-\phi}{\gamma} - \frac{\beta}{1-\alpha}\right)^{-1}\left(n + \frac{\beta}{1-\alpha} g_Z\right).$$

Long-run TFP growth denoted by $g_{TFP} \equiv g_A + g_Z$ is empirically observed. For a given $g_{TFP}$, a higher value of $g_Z$ implies a lower value of $g_A$ as well as a lower calibrated value for $\gamma/(1-\phi)$, which in turn implies that R&D have smaller social benefits and the socially optimal level of R&D spending is lower.

### 3.11. Socially Optimal Allocations and Dynamic Distortion

This section firstly characterizes the socially optimal allocations and then derives the dynamic distortion on capital accumulation. To derive the socially optimal rate of investment in physical capital and R&D shares of labor and capital, the social planner chooses $i_t$, $s_{L,t}$ and $s_{K,t}$ to maximize the representative household's lifetime utility given by $U = \int_0^\infty e^{-(\rho-n)t} \frac{((1-i_t)Y_t/L_t)^{1-\sigma}}{1-\sigma} dt$ subject to: (a) the aggregate production function given by $Y_t = A_t Z_t (1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$; (b) the law of motion for capital given by $\dot{K}_t = i_t Y_t - K_t \delta$; and (c) the law of motion for R&D-driven technology given by $\dot{A}_t = A_t^{\phi}(s_{K,t})^{\beta\gamma}(s_{L,t})^{(1-\beta)\gamma} K_t^{\beta\gamma} L_t^{(1-\beta)\gamma} \varphi \ln z$. After deriving the first-order conditions, the social planner solves for $i^*$, $s_L^*$ and $s_K^*$ on the balanced-growth path.

**Lemma 2:** *The socially optimal steady-state rate of investment in physical capital is*

$$
\begin{aligned}
&i^* = \left(\alpha + \beta \frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A}\right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta} \\
&\neq i = \frac{z^\eta \theta + 1 - \theta}{z^\eta} \left(\alpha + \beta \left(\frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda}\right) \frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + 1 - \theta} \nu(\Omega^{\eta_{lead}})\right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta}
\end{aligned}, \tag{56}
$$

*and the socially optimal steady-state R&D shares of labor* $s_L^*$ *and capital* $s_K^*$ *are respectively*

$$
\begin{aligned}
&\frac{s_L^*}{1 - s_L^*} = \frac{1-\beta}{1-\alpha} \left(\frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A}\right) \\
&\neq \frac{s_L}{1 - s_L} = \frac{1-\beta}{1-\alpha} \left(\frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda}\right) \frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + 1 - \theta} \nu(\Omega^{\eta_{lead}})
\end{aligned}, \tag{57}
$$

$$
\begin{aligned}
&\frac{s_K^*}{1 - s_K^*} = \frac{\beta}{\alpha} \left(\frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A}\right) \\
&\neq \frac{s_K}{1 - s_K} = \frac{\beta}{\alpha} \left(\frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda}\right) \frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + 1 - \theta} \nu(\Omega^{\eta_{lead}})
\end{aligned}. \tag{58}
$$

**Proof:** Refer to Appendix I.

(57) and (58) indicate the various sources of R&D externalities: (a) the negative externality in intratemporal duplication given by $\gamma \in (0,1]$; (b) the positive or negative externality in intertemporal knowledge spillovers given by $\phi \in (-\infty,1)$; (c) the static consumer-surplus appropriability problem given by $(1-\theta)(z^\eta - 1)/z^\eta \in (0,1]$, which is a positive externality; (d) the markup distortion in driving a wedge of $(z^\eta \theta + 1 - \theta)/z^\eta \geq 1$ between the factor payments for production inputs and their marginal products; (e) the positive externality of cumulative innovations together with the negative externality of creative destruction (i.e. the business-stealing effect) given by the difference between $g_A/(\rho - n + (\sigma - 1) g_c + g_A)$ and $\lambda/(\rho - n + (\sigma - 1) g_c + \lambda)$; and (f) the negative effects of blocking patents on R&D through the backloading discount factor $\nu(\Omega^{\eta_{lead}}) \in (0,1]$. Given the existence of

positive and negative externalities, it requires a numerical calibration that will be performed in Section 4 to determine whether the market economy overinvests or underinvests in R&D.

If the market economy underinvests in R&D as also suggested by Jones and Williams (1998) and (2000), the government may want to increase patent breadth to reduce the extent of this market failure. However, the following proposition states that even holding the effects of blocking patents constant, an increase in $\eta$ mitigates the problem of R&D underinvestment at the costs of worsening the dynamic distortionary effect on capital accumulation in addition to worsening the static distortionary effect.

**Proposition 2a:** *The decentralized equilibrium rate of capital investment $i$ is below the socially optimal investment rate $i^*$ if either there is underinvestment in R&D or labor is the only factor input for R&D.*

**Proof:** Refer to Appendix I.

**Proposition 2b:** *Holding the backloading discount factor $v$ constant, an increase in patent breadth leads to a reduction in the decentralized equilibrium rate of capital investment $i$ if the intermediate-goods sector is at least as capital intensive as the R&D sector.*

**Proof:** Refer to Appendix I.

A higher aggregate markup increases the wedge between the marginal product of capital and the rental price. This effect by itself reduces the equilibrium rate of investment in physical capital; however, there is an opposing effect from the R&D capital share. Proposition 2b shows that the intermediate-goods sector being more capital intensive than the R&D sector is a *sufficient* condition for the negative effect to dominate. As for Proposition 2a, the discrepancy between the equilibrium rate of investment in physical capital and the social optimum arises because of: (a) the aggregate markup; and (b) the discrepancy between the market equilibrium R&D capital share $s_K$ and the socially optimal R&D capital share $s_K^*$. Since the equilibrium capital investment rate $i$ is an increasing function of $s_K$, the underinvestment in

R&D in the market equilibrium is sufficient for $i < i^*$. On the other hand, when there is overinvestment in R&D in the market equilibrium, whether $i$ is below or above $i^*$ depends on whether the effect of the aggregate markup or the effect of R&D overinvestment dominates. For the case in which labor is the only factor input for R&D (i.e. $s_K = 0$), only the effect of the aggregate markup is present.

## 4. Calibration

Using the framework developed above, this section provides a quantitative assessment on the effects of blocking patents. Figure 1 shows that private spending on R&D in the US as a share of GDP has been rising sharply since the beginning of the 80's. Then, after a few years, the number of patents granted by the US Patent and Trademark Office also began to increase rapidly as shown in Figure 2. Given the patent policy changes in the 80's, the structural parameters are calibrated using long-run aggregate data of the US's economy from 1953 to 1980 to examine the extent of R&D underinvestment and inefficiency arising blocking patents before these policy changes. The goal of this numerical exercise is to quantify the effects of eliminating blocking patents on R&D, consumption and capital investment. After calibrating the model using aggregate data, an alternative calibration based on industry-level data from R&D-intensive industries will be performed to ensure the robustness of the finding that the negative effect of blocking patents on R&D is significant.

### 4.1. Backloading Discount Factor

The first step is to calibrate the structural parameters and the steady-state value of the backloading discount factor $\nu$. The average annual TFP growth rate $g_{TFP}$ is 1.33%,[18] and the labor-force growth rate $n$ is 1.94%.[19] The annual depreciation rate $\delta$ on physical capital and the household's discount rate are set to conventional values of 8% and 4% respectively. For the aggregate markup $\tilde{\mu} = \mu^{1-\theta}$, Laitner and

[18] Multifactor productivity for the private non-farm business sector is obtained from the Bureau of Labor Statistics.
[19] The data on the annual average size of the labor force is obtained from the Bureau of Labor Statistics.

Stolyarov (2004) estimate that the aggregate markup is about 1.1 (i.e. a 10% markup) in the data; on the other hand, Basu (1996) and Basu and Fernald (1997) estimate that the aggregate production function has constant return to scale and the aggregate profit share is 3%. These estimates imply that the aggregate markup is about 1.03.[20] To be conservative, I will set $\tilde{\mu}$ to the lower value at 1.03.[21] For a given $\tilde{\mu}$, each value of $\theta$ (the fraction of competitive industries in the intermediate-goods sector) corresponds to a unique value for the industry markup $\mu$ in monopolistic industries, and I will consider a wide range of values for $\theta \in \{0, 0.25, 0.5, 0.75\}$. A number of structural studies based on patent renewal models has estimated the arrival rate of innovations $\lambda$, and I will consider a reasonable range of values for $\lambda \in [0.04, 0.20]$.[22] For the capital intensity parameter in the R&D sector, I will set $\beta = \alpha$ as the benchmark case.[23]

For the remaining parameters $\{\nu, \alpha, \sigma\}$, the model provides three steady-state conditions for the calibration: (a) R&D as a share of GDP; (b) labor share; and (c) capital investment rate.

(59)
$$\frac{R\&D}{Y} = \left(\frac{\mu\theta + 1 - \theta}{\mu}\right)\left((1-\alpha)\frac{s_L}{1-s_L} + \alpha\frac{s_K}{1-s_K}\right),$$

(60)
$$\frac{wL}{Y} = \frac{1-\alpha}{1-s_L}\left(\frac{\mu\theta + 1 - \theta}{\mu}\right),$$

(61)
$$\frac{I}{Y} = \frac{\alpha(\mu\theta + 1 - \theta)}{\mu(1-s_K)}\left(\frac{n + g_{TFP}/(1-\alpha) + \delta}{\rho + \sigma g_{TFP}/(1-\alpha) + \delta}\right),$$

---

[20] Cost minimization implies that the return to scale = the markup x (1 – profit share).

[21] As a robustness check, an aggregate markup of 1.1 implies that the negative effect of blocking patents on R&D is even more severe. Intuitively, a higher markup means increased profitability which must be offset by a stronger effect of blocking patents in order for the level of R&D in the data to be constant. In this case, eliminating blocking patents would lead to a more significant increase in R&D and consumption.

[22] For example, Lanjouw (1998) structurally estimate a patent renewal model using patent renewal data in a number of industries from Germany, and the estimated probability of obsolescence ranges 7% for computer patents to 12% for engine patents. Also, a conventional value for the rate of depreciation in patent value is about 15% (e.g. Pakes (1986)). On the other hand, Caballero and Jaffe (2002) estimate a mean rate of creative destruction of about 4%.

[23] I have considered different plausible values for $\beta \in \{0, \alpha, 2\alpha, 3\alpha\}$ as a sensitivity analysis. The extent of R&D underinvestment and the effects of eliminating blocking patent and increasing patent breadth on long-run consumption are robust to these parameter changes.

where $\frac{s_L}{1-s_L} = \frac{s_K}{1-s_K} = \left(\frac{\lambda}{\rho - n + (\sigma-1)g_c + \lambda}\right)\frac{(\mu-1)(1-\theta)}{\mu\theta+1-\theta}\nu(\Omega^{\eta_{lead}})$ for the case of $\alpha=\beta$. The average private spending on R&D as a share of GDP is 0.0115,[24] and the labor share is set to a conventional value of 0.7. The average ratio of private investment to GDP is 0.203.[25]

Table 1 presents the calibrated values for the structural parameters along with the real interest rate $r = \rho + \sigma g_{TFP}/(1-\alpha)$ and the industry markup $\mu = (1.03)^{1/(1-\theta)}$ for $\theta \in \{0, 0.25, 0.5, 0.75\}$ and $\lambda \in [0.04, 0.20]$.

*[insert Table 1 here]*

Table 1 shows that for a given value of $\theta$, the calibrated values for $\{\alpha, \sigma, r\}$ are invariant to different values of $\lambda$. The calibrated value for the elasticity of intertemporal substitution (i.e. $1/\sigma$) is about 0.42, which is closed to the empirical estimates from econometric studies.[26] The implied real interest rate is about 8.4%, which is slightly higher than the historical rate of return on the US's stock market, and this higher interest rate implies a lower optimal level of R&D spending and a higher steady-state value of the backloading discount factor. As a result, the model is less likely to overstate the extent of R&D underinvestment and the degree of inefficiency from blocking patents. Re-expressing (59) into (62) shows that $\nu$ decreases as $\lambda$ increases.

(62) $$\nu = \frac{R\&D}{GDP} \Bigg/ \frac{\lambda(1-\theta)(\mu-1)/\mu}{\rho - n + (\sigma-1)g_{TFP}/(1-\alpha) + \lambda}.$$

Furthermore, the fact that the calibrated values of $\nu \in [0.485, 0.892]$ are smaller than one suggests inefficiency from blocking patents in the economy. Therefore, eliminating blocking patents may be an

[24] The data is obtained from the National Science Foundation and the Bureau of Economic Analysis. R&D is net of federal spending, and GDP is net of government spending. The observations in the data series of R&D spending are missing for 1954 and 1955.

[25] This number is calculated using data obtained from the Bureau of Economic Analysis, and GDP is net of government spending.

[26] It is well-known that there is a discrepancy between the estimated elasticity of intertemporal substitution from dynamic macro models (closed to 1) and econometric studies. Guvenen (2006) shows that this discrepancy is due to the heterogeneity in households' preferences and wealth inequality. In short, the *average investor* has a high elasticity of intertemporal substitution while the *average consumer* has a much lower elasticity. Since my interest is on consumption, it is appropriate to calibrate the value of $\sigma$ according to the average consumer.

effective method to stimulate R&D. After calibrating the externality parameters and computing the socially optimal level of R&D spending, the effects of eliminating blocking patents will be quantified.

### 4.2. Externality Parameters

The second step is to calibrate the values for the externality parameters $\gamma$ (intratemporal duplication) and $\phi$ (intertemporal spillover). For each value of $g_A$, $g_Z$, $n$, $\alpha$ and $\beta$, the balanced-growth condition (55) determines a unique value for $\gamma/(1-\phi)$, which is sufficient to determine the effect of R&D on the balanced-growth level of consumption. However, holding $\gamma/(1-\phi)$ constant, a larger $\gamma$ implies a faster rate of convergence to the balanced-growth path; therefore, it is important to consider different values of $\gamma$. As for the value of $g_A$, I will set $g_A = \xi g_{TFP}$ for $\xi \in [0,1]$. The parameter $\xi$ captures the fraction of long-run TFP growth that is driven by R&D, and the remaining fraction is driven by the exogenous process $Z_t$ such that $g_Z = (1-\xi) g_{TFP}$. Table 2 presents the calibrated values of $\phi$ for $\gamma \in [0.1, 1.0]$ and $\xi \in [0,1]$.

*[insert Table 2 here]*

Table 2 shows that the calibrated values for $\phi$ are very similar across different values of $\theta$ implying that the socially optimal level of R&D spending and the extent of R&D overinvestment or underinvestment are about the same across different values of $\theta$.

To reduce the plausible parameter space of $\gamma$ and $\xi$, I make use of the empirical estimates for the social rate of return to R&D. Following Jones and Williams' (1998) derivation, Appendix III shows that the net social rate of return $\tilde{r}$ can be expressed as

$$\tilde{r} = \frac{1+g_Y}{1+g_A}\left(1+g_A\left(\frac{\gamma}{s_r}+\phi\right)\right)-1. \tag{63}$$

With a lower bound of 0.30 for $\tilde{r}$, (63) pins down a lower bound for $\gamma$ under each value of $\xi$. Table 3 presents the implied social rate of return $\tilde{r}$ for $\gamma \in [0.1, 1.0]$ and $\xi \in [0, 1]$, and the values exceeding 0.30 are highlighted in bold.

*[insert Table 3 here]*

### 4.3. Socially Optimal Level of R&D Spending

This section calculates the socially optimal level of R&D share $(1-\alpha)s_L^* /(1-s_L^*) + \alpha s_K^* /(1-s_K^*)$. Figure 3 plots the socially optimal R&D shares for the range of values for $\gamma$ and $\xi$ that satisfies the lower bound of 0.30 for $\tilde{r}$.

*[insert Figure 3 here]*

Figure 3 shows that there was underinvestment in R&D prior to 1980 over the entire range of parameters. Since it is difficult to determine the empirical value of $\xi$, I will leave it to the readers to decide on their preferred values and continue to present results for this range of parameters.

### 4.4. Eliminating Blocking patents

Given the calibrated structural parameters, this section quantifies the effects of eliminating blocking patents on R&D and consumption. Table 4 shows that eliminating blocking patents (i.e. setting $\nu = 1$) would lead to a substantial increase in the steady-state share of R&D by a minimum of 12% and a maximum of 106%.

*[insert Table 4 here]*

In the following, the effect of eliminating blocking patents is firstly expressed in terms of the percent change in the balanced-growth level of consumption per year. Along the balanced-growth path, per capita consumption increases at an exogenous rate $g_c$. Therefore, after dropping the exogenous growth path and some constant terms and solving for the balanced-growth path of R&D technology and steady-state capital-labor ratio, I derive the expression for the endogenous parts of long-run consumption as a function of the steady-state value of the backloading discount factor $\nu$ through the capital investment rate $i(\nu)$, and the R&D shares of capital and labor (where $s_r(\nu) = s_L(\nu) = s_K(\nu)$ because $\alpha = \beta$).

**Lemma 3:** For $\alpha = \beta$, the expression for the endogenous parts of consumption on the balanced-growth path is

$$(64) \qquad c_0(\nu) = \left( i(\nu)^{\frac{\alpha(1-\phi)+\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1-i(\nu)) s_r(\nu)^{\frac{\gamma}{(1-\alpha)(1-\phi)-\alpha\gamma}} (1-s_r(\nu))^{\frac{(1-\phi)}{(1-\alpha)(1-\phi)-\alpha\gamma}} \right).$$[27]

**Proof:** Refer to Appendix I.

Therefore, in the case of a change in $\nu$, the percent change in long-run consumption can be decomposed into four terms.

$$(65) \qquad \Delta \ln c_0(\nu) = \left( \begin{array}{l} \left( \dfrac{\alpha(1-\phi+\gamma)}{(1-\alpha)(1-\phi)-\alpha\gamma} \right) \Delta \ln i(\nu) + \Delta \ln(1-i(\nu)) + \\ \left( \dfrac{\gamma}{(1-\alpha)(1-\phi)-\alpha\gamma} \right) \Delta \ln s_r(\nu) + \left( \dfrac{(1-\phi)}{(1-\alpha)(1-\phi)-\alpha\gamma} \right) \Delta \ln(1-s_r(\nu)) \end{array} \right).$$[28]

Figure 4 shows that eliminating blocking patents would lead to a substantial increase in long-run consumption by a minimum of 4% and a maximum of 67%. Also, a back-of-the-envelope calculation shows that the change in consumption mostly comes from $(\gamma/((1-\alpha)(1-\phi)-\alpha\gamma))\Delta \ln s_r(\nu)$; in other words, the other general-equilibrium effects only have secondary impacts on long-run consumption.

*[insert Figure 4 here]*

After examining the effect on long-run consumption, the next numerical exercise computes the entire growth path of consumption upon eliminating blocking patents. Figure 5a compares the transition path (in blue) of log consumption per capita with its original balanced-growth path (in red) and its new

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[27] The proof in Appendix I also derives the expression for the general case in which $\alpha \neq \beta$.

[28] Note that the coefficients are determined by $\gamma/(1-\phi)$ rather than the individual values of $\gamma$ and $\phi$.

balanced-growth path (in green) for the following parameters $\{\xi, \gamma, \lambda, \theta, \delta\} = \{0.55, 0.55, 0.1, 0, 0.08\}$ to illustrate the transition dynamics. Then, I will discuss the effects of changing these parameter values.

*[insert Figure 5a here]*

Upon setting $\nu = \Omega_1 = 1$, consumption per capita gradually rises towards the new balanced growth path. Although factor inputs shift towards the R&D sector and the output of final goods drops as a result, the possibility of investing less and running down the capital stock enables consumption smoothing. To compare with previous studies, such as Kwan and Lai (2003), Figure 5b presents the transition dynamics for $\{\xi, \gamma, \lambda, \theta, \delta\} = \{0.55, 0.55, 0.1, 0, 1\}$ as an approximation to a model with no capital accumulation. In this case, the result is consistent with Kwan and Lai (2003) that consumption falls in response to the strengthening of patent protection. In this case, consumption falls by about 2% on impact and only recovers to its original growth path after 4 years.

*[insert Figure 5b here]*

A sensitivity analysis has been performed for different values of $\xi$ and $\gamma$. At a larger value of either $\xi$ or $\gamma$, consumption increases by even more on impact. A larger $\xi$ also implies a higher position of the new balanced-growth path. Holding $\xi$ constant, a larger $\gamma$ implies a faster rate of convergence. When both $\xi$ and $\gamma$ are smaller than 0.55, the household suffers small consumption losses during the initial phase of the transition path. For example, Figure 5c presents the transition dynamics for $\{\xi, \gamma, \lambda, \theta, \delta\} = \{0.3, 0.3, 0.1, 0, 0.08\}$.

*[insert Figure 5c here]*

However, Figure 5d shows that when $\xi$ is closed to one, $\gamma$ could be as small as 0.3 without causing any short-run consumption losses.

*[insert Figure 5d here]*

In summary, reallocating resources from the production sector to the R&D sector does not always lead to short-run consumption losses. Finally, at a larger value of $\lambda$, the calibrated value for $\nu$ becomes smaller (see Table 1). This larger magnitude of the policy shock increases slightly the range of parameter values that corresponds to short-run consumption losses.

### 4.5. Dynamic Distortion

Proposition 2a derives the sufficient condition under which the market equilibrium rate of investment in physical capital is below the socially optimal level in (56). The following numerical exercise quantifies this wedge. Figure 6 presents the socially optimal rates of capital investment along with the US's long-run ratio of private investment to GDP of 0.203, and the wedge is about 0.024 on average.

*[insert Figure 6 here]*

The equilibrium rate of investment in physical capital in the long run is increasing in the R&D share of capital; therefore, eliminating blocking patents also increases the rate of capital investment. Table 5 shows that upon eliminating blocking patents, the steady-state capital investment rate increases by 0.0017 on average and moves slightly toward the socially optimal level.

*[insert Table 5 here]*

### 4.6. Robustness Check Based on Industry-Level Data

As mentioned in Section 2, the use of aggregate data relies on the assumption that economic profit in the economy is created by intellectual monopoly. In this section, I will perform a robustness check on the finding that the negative effect of blocking patents on R&D is significant by calibrating the following condition using industry-level data

$$\frac{R\,\&\,D_{industry}}{Y_{industry}} = v\left(\frac{\lambda}{r+\lambda-g_{\pi}}\right)\left(\frac{\mu-1}{\mu}\right). \tag{66}$$

This numerical exercise requires an estimate for the markup in R&D-intensive industries, and I will make use of the empirical estimates for industry-level returns to scale from Basu *et al* (2006). Assuming cost minimization and non-negative economic profit, the estimates for the returns to scale provide a lower bound for the industry markups.[29]

Based on the data on R&D from the National Science Foundation, I choose four R&D-intensive industries that account for 67% of the private R&D spending in manufacturing from 1980 to 1997: chemical products (SIC 28); machinery (SIC 35); electrical equipment (SIC 36); and motor vehicles (SIC 371). I add up the industries' total R&D spending for each year and then divide this number by the value added of these industries. The annual average ratio of R&D over value added is 0.117. The real interest rate is set to 8.4% as before, and $g_{\pi}$ is set to the long-run GDP growth rate of 3.4%. I will consider a range of values for $\lambda \in [0.04, 0.20]$, and the valued-added weighted average return to scale from Basu *et al* (2006) is 1.30 in these R&D-intensive industries. Then, I divide this number by the aggregate profit share of 0.03 from Basu and Fernald (1997) to obtain a conservative estimate of 1.34 for the average markup in these R&D-intensive industries.[30] Figure 7 shows that the calibrated values for $v$ are far below one unless the arrival rate of innovations $\lambda$ is very small.

*[insert Figure 7 here]*

Therefore, the data at both the aggregate and industry levels seems to suggest that blocking patents have a severe and negative effect on R&D.

## 5. Conclusion

This paper has attempted to accomplish three objectives. Firstly, it develops a tractable framework to model the transition dynamics of an economy with overlapping intellectual property rights and patent pools in a generalized quality-ladder growth model. Secondly, it identifies a dynamic distortionary effect of patent policy on capital accumulation that has been neglected by previous studies. Thirdly, it applies the model to the aggregate data of the US economy to quantify the extent of underinvestment in R&D and

[29] See Footnote 20.

[30] This estimate is conservative because the share of economic profits in an R&D-intensive industry should be much higher than in the average industry because of the intellectual monopoly created by patent protection. For example, Comin (2004) argues that the average markup in patent-protected industries should be at least 1.5.

inefficiency arising from blocking patents. The numerical exercise suggests the following findings. If R&D investment is important for TFP growth, then the inefficiency created by blocking patents is a major reason for the underinvestment in R&D in the US. In addition, making the patent system more efficient can have substantial effects on R&D and consumption. From a policy perspective, the patent authority should mitigate the effects of blocking patents through the following policies: (a) compulsory licensing with an upper limit on the amount of licensing fees charged to subsequent inventors of more advanced technology; and (b) making patent-infringement cases in court favorable to subsequent inventors of more advanced technology.

Finally, the readers are advised to interpret the numerical results with some important caveats in mind. The first caveat is that although the quality-ladder growth model has been generalized as an attempt to capture more realistic features of the US economy, it is still an oversimplification of the real world. Furthermore, the finding of eliminating blocking patents having substantial and positive effects on R&D and consumption is based on the assumptions that the empirical estimates for the social rate of return to R&D and the data on private R&D spending are not incorrectly measured by an order of magnitude. The validity of these assumptions remains as an empirical question. Therefore, the numerical results should be viewed as illustrative at best.

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## Appendix I: Proofs

**Lemma 1:** *The aggregate production function for the final goods is*

(a1) $$Y_t = \vartheta(\eta) A_t Z_t K_{x,t}^{\alpha} L_{x,t}^{1-\alpha},$$

*where* $\vartheta(\eta) \equiv (z^{\eta})^{\theta} / (z^{\eta}\theta + 1 - \theta)$ *is decreasing in* $\eta$ *for* $\theta \in (0,1)$.

**Proof:** Recall that the production function for the final goods is given by

(a2) $$Y_t = \exp\left( \int_0^1 \ln X_t(j) dj \right).$$

After substituting $X_t(j)$ for $j \in [0,1]$ into (a2) and assuming cost minimization in the intermediate-goods sector, the aggregate production function becomes

(a3) $$Y_t = A_t Z_t \left( \frac{K_{x,t}}{L_{x,t}} \right)^{\alpha} L_{x,t}^{e},$$

where $L_{x,t}^{e}$ is defined as

(a4) $$L_{x,t}^{e} \equiv \exp\left( \int_0^1 \ln L_{x,t}(j) dj \right) \neq \left( \int_0^1 L_{x,t}(j) dj \right) = L_{x,t}$$

because of the competitive industries. Define $\vartheta(\eta)$ as the ratio of $L_{x,t}^{e}$ and $L_{x,t}$, which is given by

(a5) $$\vartheta(\eta) \equiv \frac{L_{x,t}^{e}}{L_{x,t}} = \frac{(z^{\eta})^{\theta}}{z^{\eta}\theta + 1 - \theta} \in (0,1)$$

for $\theta \in (0,1)$. $\vartheta(\eta)$ represents the static distortionary effect of markup pricing, and it enters the aggregate production function as

(a6) $$Y_t = \vartheta(\eta) A_t Z_t K_{x,t}^{\alpha} L_{x,t}^{1-\alpha}.$$

Finally, simple differentiation shows that for $\theta \in (0,1)$,

(a7) $$\frac{\partial \vartheta(\eta)}{\partial \eta} = -\frac{\theta(1-\theta)(z^{\eta} - 1)\ln z}{z^{\eta}\theta + 1 - \theta} \vartheta(\eta) < 0. \blacksquare$$

**Proposition 1:** $V_{i,t}(j)$ *for* $i \in \{1,2,...,\eta\}$ *and* $j \in [\theta,1]$ *is determined by the following law of motion*

(b1) $$r_t V_{i,t}(j) = \Omega_i \pi_t(j) + \dot{V}_{i,t}(j) + \lambda_t (V_{i+1,t}(j) - V_{i,t}(j)),$$

*where* $V_{\eta+1,t}(j) = 0$. *The no-arbitrage condition for* $V_{1,t}(j)$ *can be re-expressed as*

(b2) $$V_{1,t}(j) = \pi_t(j)\left(\sum_{k=1}^{\eta} \Omega_k \lambda_t^{k-1}\left(\prod_{i=1}^{k} \frac{1}{r_t + \lambda_t - \dot{V}_{i,t}(j)/V_{i,t}(j)}\right)\right).$$

**Proof:** The expected present value of the ownership in patent pools for the *i*-th most recent inventor in industry $j \in [\theta,1]$ is

(b3) $$V_{i,t}(j) = \int_t^{\infty}\left(\int_t^{s} \Omega_i \pi_x(j) \exp\left(-\int_t^{x} r_v dv\right)dx\right) f(s)ds + \int_t^{\infty} V_{i+1,s}(j) \exp\left(-\int_t^{s} r_v dv\right) f(s)ds,$$

where $f(s) = \lambda_s \exp\left(-\int_t^{s} \lambda_x dx\right)$ is the density function of *s* that is a random variable representing the time when the next innovation occurs and follows the Erlang distribution. The first term in $V_{i,t}(j)$ is the expected present value of monopolistic profit captured by the *i*-th most recent inventor in the current patent pool. The second term in $V_{i,t}(j)$ is the expected present value of the ownership in patent pools when the *i*-th most recent inventor becomes the *i*+1-th most recent inventor. Note that $V_{\eta+1,t}(j) = 0$ because the $\eta + 1$-th most recent inventor is no longer in any patent pool. In order to derive (b1), I differentiate (b3) with respect to *t*. To simplify notations, I firstly define a new function such that (b3) becomes

(b4) $$V_{i,t}(j) = \int_t^{\infty} g(t,s)ds,$$

where $g(t,s) = \left( \int_t^s \Omega_i \pi_x(j) \exp\left( -\int_t^x r_v dv \right) dx + V_{i+1,s}(j) \exp\left( -\int_t^s r_v dv \right) \right) f(s)$. Then, using the formula for differentiation under the integral sign,

(b5) $$\dot{V}_{i,t}(j) \equiv \frac{\partial V_{i,t}(j)}{\partial t} = -g(t,t) + \int_t^\infty \frac{\partial g(t,s)}{\partial t} ds,$$

where $g(t,t) = \lambda_t V_{i+1,s}(j)$, and

(b6) $$\frac{\partial g(t,s)}{\partial t} = \left( (\lambda_t + r_t) \left( \int_t^s \Omega_i \pi_x(j) \exp\left( -\int_t^x r_v dv \right) dx + V_{i+1,s}(j) \exp\left( -\int_t^s r_v dv \right) \right) - \Omega_i \pi_t(j) \right) f(s).$$

After a few steps of mathematical manipulation, (b5) becomes

(b7) $$\dot{V}_{i,t}(j) = -\lambda_t V_{i+1,s}(j) + (\lambda_t + r_t) V_{i,t}(j) - \Omega_i \pi_t(j) \int_t^\infty f(s) ds.$$

Finally, after setting $\int_t^\infty f(s) ds = 1$, (b7) becomes (b1).

Upon deriving (b1), each $V_{i,t}(j)$ for $i \in \{1,2,...,\eta\}$ can be rewritten as

(b8) $$V_{i,t}(j) = \frac{\Omega_i \pi_t(j) + \lambda_t V_{i+1,t}(j)}{r_t + \lambda_t - \dot{V}_{i,t}(j)/V_{i,t}(j)},$$

where $V_{\eta+1,t}(j) = 0$. Recursive substitutions show that $V_{1,t}(j)$ can be re-expressed as (b2).■

**Lemma 2:** *The socially optimal steady-state rate of investment in physical capital is*

(c1) $$i^* = \left( \alpha + \beta \frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A} \right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta},$$

*and the socially optimal steady-state R&D shares of labor $s_L^*$ and capital $s_K^*$ are respectively*

(c2) $$\frac{s_L^*}{1 - s_L^*} = \frac{1-\beta}{1-\alpha} \left( \frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A} \right),$$

(c3) $$\frac{s_K^*}{1-s_K^*}=\frac{\beta}{\alpha}\left(\frac{\gamma g_A}{\rho-n+(\sigma-1)g_c+(1-\phi)g_A}\right).$$

**Proof:** To derive the socially optimal rate of capital investment and R&D shares of labor and capital, the social planner chooses $i_t$, $s_{L,t}$ and $s_{K,t}$ to maximize

(c4) $$U=\int_0^\infty e^{-(\rho-n)t}\,\frac{((1-i_t)Y_t/L_t)^{1-\sigma}}{1-\sigma}dt$$

subject to: (a) the aggregate production function expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(c5) $$Y_t=A_tZ_t(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha};$$

(b) the law of motion for capital expressed in terms of $i_t$ given by

(c6) $$\dot{K}_t=i_tY_t-K_t\delta;$$

and (c) the law of motion for R&D technology expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(c7) $$\dot{A}_t=A_t^{\phi}(s_{K,t})^{\beta\gamma}(s_{L,t})^{(1-\beta)\gamma}K_t^{\beta\gamma}L_t^{(1-\beta)\gamma}\varphi\ln z.$$

The current-value Hamiltonian $H$ is

(c8) $$\begin{aligned}H&=(1-\sigma)^{-1}\left(\frac{(1-i_t)A_tZ_t(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}}{L_t}\right)^{1-\sigma}\\&+\upsilon_{K,t}(i_tA_tZ_t(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}-K_t\delta)\\&+\upsilon_{A,t}A_t^{\phi}(s_{K,t})^{\beta\gamma}(s_{L,t})^{(1-\beta)\gamma}K_t^{\beta\gamma}L_t^{(1-\beta)\gamma}\varphi\ln z\end{aligned}.$$

The first-order conditions are

(c9) $$H_i=-\frac{1}{(1-i_t)}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma}+\upsilon_{K,t}Y_t=0,$$

(c10) $$H_{s_L}=-\left(\frac{1-\alpha}{1-s_{L,t}}\right)\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma}-\upsilon_{K,t}\left(\frac{1-\alpha}{1-s_{L,t}}\right)i_tY_t+\upsilon_{A,t}\left(\frac{(1-\beta)\gamma}{s_{L,t}}\right)\dot{A}_t=0,$$

(c11) $$H_{s_K} = -\left(\frac{\alpha}{1-s_{K,t}}\right)\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} - \upsilon_{K,t}\left(\frac{\alpha}{1-s_{K,t}}\right)i_t Y_t + \upsilon_{A,t}\left(\frac{\beta\gamma}{s_{K,t}}\right)\dot{A}_t = 0\,,$$

(c12) $$H_K = \frac{\alpha}{K_t}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}\left(\alpha\frac{i_t Y_t}{K_t} - \delta\right) + \upsilon_{A,t}\left(\beta\gamma\frac{\dot{A}_t}{K_t}\right) = (\rho-n)\upsilon_{K,t} - \dot{\upsilon}_{K,t}\,,$$

(c13) $$H_A = \frac{1}{A_t}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}\left(\frac{i_t Y_t}{A_t}\right) + \upsilon_{A,t}\left(\phi\frac{\dot{A}_t}{A_t}\right) = (\rho-n)\upsilon_{A,t} - \dot{\upsilon}_{A,t}\,.$$

Note that the first-order conditions with respect to the co-state variables $\upsilon_{K,t}$ and $\upsilon_{A,t}$ yield the law of motions for capital and R&D technology. Then, imposing the balanced-growth conditions yields

(c14) $$H_i : \left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} = (1-i)\upsilon_{K,t}Y_t\,,$$

(c15) $$H_{s_L} : \gamma g_A A_t \upsilon_{A,t}\left(\frac{1-s_L}{s_L}\right) = \frac{1-\alpha}{1-\beta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} i Y_t\right),$$

(c16) $$H_{s_K} : \gamma g_A A_t \upsilon_{A,t}\left(\frac{1-s_K}{s_K}\right) = \frac{\alpha}{\beta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_K i Y_t\right),$$

(c17) $$H_K : \alpha\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} i Y_t\right) + \beta\gamma g_A A_t \upsilon_{A,t} = (\rho + g_c\sigma + \delta)K_t \upsilon_{K,t}\,,$$

(c18) $$H_A : \left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} i Y_t = (\rho - n + (\sigma-1)g_c + (1-\phi)g_A)A_t \upsilon_{A,t}\,.$$

Finally, solving (c14)-(c18) yields (c1)-(c3).■

**Proposition 2a:** *The decentralized equilibrium rate of capital investment* $i$ *is below the socially optimal investment rate* $i^*$ *if either there is underinvestment in R&D or labor is the only factor input for R&D.*

**Proof:** The socially optimal capital investment rate $i^*$ is (d1) $$i^* = \left( \alpha + \beta \frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A} \right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta}.$$

The market equilibrium rate of capital investment $i$ is

(d2) $$i = \frac{z^\eta \theta + 1 - \theta}{z^\eta} \left( \alpha + \beta \left( \frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda} \right) \frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + 1 - \theta} \nu(\Omega^{\eta_{lead}}) \right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta}.$$

Therefore, either $\beta = 0$ or the underinvestment in R&D such that $\frac{\gamma g_A}{\rho - n + (\sigma - 1) g_c + (1-\phi) g_A} >$ $\left( \frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda} \right) \frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + 1 - \theta} \nu(\Omega^{\eta_{lead}})$ is *sufficient* for $i^* > i$ because $(z^\eta \theta + 1 - \theta) / z^\eta < 1$ for $\theta \in [0,1)$.■

**Proposition 2b:** *Holding the backloading discount factor $\nu$ constant, an increase in patent breadth leads to a reduction in the decentralized equilibrium rate of capital investment $i$ if the intermediate-goods sector is at least as capital intensive as the R&D sector.*

**Proof:** Differentiating $i$ with respect to $\eta$ yields

(d3) $$\frac{\partial i}{\partial \eta} = -\frac{(1-\theta) \ln z}{z^\eta} \left( \alpha - \beta \frac{\lambda}{\rho - n + (\sigma - 1) g_c + \lambda} \nu \right) \frac{g_K + \delta}{\rho + g_c \sigma + \delta}.$$

Since $\rho > n$ and $\sigma \geq 1$, $\alpha \geq \beta$ is a *sufficient* condition for $\partial i / \partial \eta < 0$.■

**Lemma 3:** For $\alpha = \beta$, the expression for the endogenous parts of consumption on the balanced-growth path is

(e1) $$c_0(\nu) = \left( i(\nu)^{\frac{\alpha(1-\phi)+\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1 - i(\nu)) s_r(\nu)^{\frac{\gamma}{(1-\alpha)(1-\phi)-\alpha\gamma}} (1 - s_r(\nu))^{\frac{(1-\phi)}{(1-\alpha)(1-\phi)-\alpha\gamma}} \right).$$

**Proof:** The following derivation applies to the more general case in which $\alpha$ and $\beta$ can be different. Without loss of generality, time is re-normalized such that time 0 is the first-period in which the economy reaches the balanced-growth path. The balanced-growth path of per capita consumption (in log) can be written as

(e2) $$\ln c_t = \ln c_0 + g_c t,$$

where $g_c t$ represents the exogenous growth path of consumption because of the semi-endogenous growth formulation. The balanced-growth level of per capital consumption at time 0 is

(e3) $$c_0 = \tilde{\vartheta}(1-i)(1-s_K)^{\alpha}(1-s_L)^{1-\alpha} A_0 Z_0 \left(\frac{K_0}{L_0}\right)^{\alpha},$$

where $Z_0$ is normalized to one. The capital-labor ratio $K_0 / L_0$ and the level of R&D-driven technology $A_0$ at time 0 are respectively

(e4) $$\left(\frac{K_0}{L_0}\right)^{\alpha} = \left(\frac{\tilde{\vartheta} i\,(1-s_K)^{\alpha}(1-s_L)^{1-\alpha} A_0}{g_K+\delta}\right)^{\alpha/(1-\alpha)},$$

(e5) $$A_0 = \left(s_K^{\beta} s_L^{1-\beta}\left(\frac{K_0}{L_0}\right)^{\beta}\right)^{\gamma/(1-\phi)} \left(\frac{\varphi \ln z}{g_A}\right)^{1/(1-\phi)}.$$

After dropping the exogenous growth path and some constant terms, the expression for the endogenous parts of per capita consumption on the balanced-growth path that depends on $\tilde{\vartheta}(\eta)$, $i(\eta,\nu)$, $s_K(\eta,\nu)$ and $s_L(\eta,\nu)$ is

(e6) $$c_0(\eta,\nu) = \left(\begin{array}{l} \tilde{\vartheta}(\eta)^{1/(1-\alpha)}\, i(\eta,\nu)^{\frac{\alpha(1-\phi)+\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}}(1-i(\eta,\nu)) \\ s_K(\eta,\nu)^{\frac{\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}}(1-s_K(\eta,\nu))^{\frac{\alpha(1-\phi)}{(1-\alpha)(1-\phi)-\beta\gamma}} \\ s_L(\eta,\nu)^{\frac{(1-\beta)\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}}(1-s_L(\eta,\nu))^{\frac{(1-\alpha)(1-\phi)}{(1-\alpha)(1-\phi)-\beta\gamma}} \end{array}\right).$$

Finally, after setting $\alpha=\beta$ and dropping $\tilde{\vartheta}(\eta)$, (e6) becomes (e1).■

## Appendix II: Transition Dynamics

This appendix provides the details of transforming the variables in equations (44) – (47) into their stationary forms for the purpose of computing the transition dynamics numerically. To simplify the analysis, the transformation is performed for the special case of $\alpha = \beta$. The Euler equation is

(f1) $$\dot{c}_t = c_t(r_t - \rho)/\sigma .$$

Define a stationary variable $\tilde{c}_t \equiv c_t/(A_t Z_t)^{1/(1-\alpha)}$, and its resulting law of motion is

(f2) $$\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = \frac{1}{\sigma}(r_t - \rho) - \frac{1}{1-\alpha}(\lambda_t \ln z + g_Z) .$$

The law of motion for physical capital is

(f3) $$\dot{K}_t = Y_t - C_t - K_t\delta .$$

Define a stationary variable $k_t \equiv K_t/(L_t(A_t Z_t)^{1/(1-\alpha)})$, and its resulting law of motion is

(f4) $$\frac{\dot{k}_t}{k_t} = (1-s_{r,t})\vartheta(\eta)k_t^{\alpha-1} - \frac{\tilde{c}_t}{k_t} - (\delta + n) - \frac{1}{1-\alpha}(\lambda_t \ln z + g_Z) .$$

The law of motion for R&D-driven technology is

(f5) $$\dot{A}_t = A_t\lambda_t \ln z .$$

Define a stationary variable $a_t \equiv k_t^{\alpha\gamma} A_t^{\alpha\gamma/(1-\alpha)-(1-\phi)} Z_t^{\alpha\gamma/(1-\alpha)} L_t^{\gamma}\varphi$, and its resulting law of motion is

(f6) $$\frac{\dot{a}_t}{a_t} = \alpha\gamma(1-s_{r,t})\vartheta(\eta)k_t^{\alpha-1} - \alpha\gamma\frac{\tilde{c}_t}{k_t} - (1-\phi)\lambda_t \ln z + (n\gamma - \alpha\gamma(\delta+n)) .$$

The law of motion for the market value of ownership in patent pools is given by

(f7) $$\dot{V}_{i,t} = (r_t + \lambda_t)V_{i,t} - \lambda_t V_{i+1,t} - \Omega_i \pi_t$$

for $i \in \{1,2,...,\eta\}$ and $V_{\eta+1,t} = 0$. Define a stationary variable $\tilde{v}_{i,t} \equiv V_{i,t}/(L_t(A_t Z_t)^{1/(1-\alpha)})$, and its resulting law of motion is

(f8) $$\frac{\dot{\tilde{v}}_{i,t}}{\tilde{v}_{i,t}} \equiv (r_t + \lambda_t) - \lambda_t\frac{\tilde{v}_{i+1,t}}{\tilde{v}_{i,t}} - (1-s_{r,t})\vartheta(\eta)\Omega_i(1-\theta)\left(\frac{\mu-1}{\mu}\right)\frac{k_t^{\alpha}}{\tilde{v}_{i,t}} - n - \frac{1}{1-\alpha}(\lambda_t \ln z + g_Z)$$

for $i \in \{1,2,...,\eta\}$ and $\tilde{v}_{\eta+1,t} = 0$. To close this system of differential equations, the endogenous variables $\{r_t, s_{r,t}, \lambda_t\}$ are also expressed in terms of the four newly defined stationary variables. The interest rate is

(f9) $$r_t = \alpha\vartheta(\eta)k_t^{\alpha-1}(\mu\theta + 1 - \theta)/\mu - \delta.$$

From the first-order condition of the R&D sector, the share of factor inputs in R&D is

(f10) $$s_{r,t} = \frac{1}{k_t^{\alpha/(1-\gamma)}}\left(\frac{a_t\tilde{v}_t}{\vartheta(\eta)}\right)^{1/(1-\gamma)}\left(\frac{\mu}{\mu\theta + 1 - \theta}\right)^{1/(1-\gamma)}.$$

From the law of motion of R&D-driven technology, the Poisson arrival rate of innovations is

(f11) $$\lambda_t = s_{r,t}^{\gamma} a_t.$$

Finally, the steady-state values of the variables are

(f12) $$\lambda = g_A / \ln z,$$

(f13) $$\frac{s_r}{1 - s_r} = \frac{\lambda\nu(\mu - 1)(1-\theta)/(\mu\theta + (1-\theta))}{\rho - n + (\sigma - 1)g_c + \lambda},$$

(f14) $$a = \lambda / s_r^{\gamma},$$

(f15) $$k = \left(\frac{\alpha\vartheta(\eta)(\mu\theta + 1 - \theta)}{\mu(\delta + \rho + \sigma(g_A + g_Z)/(1-\alpha))}\right)^{1/(1-\alpha)},$$

(f16) $$\tilde{c} = (1 - s_r)\vartheta(\eta)k^{\alpha} - k\left(\delta + n + \frac{g_A + g_Z}{1-\alpha}\right),$$

(f17) $$\tilde{v}_i = \frac{k^{\alpha}(1 - s_r)\vartheta(\eta)\Omega_i(1-\theta)(\mu - 1) + \lambda\,\tilde{v}_{i+1}}{\alpha\vartheta(\eta)k^{\alpha-1}(\mu\theta + 1 - \theta) + \mu\left(\frac{1}{\ln z} - \frac{1}{1-\alpha}\right)g_A - \mu\left(\frac{g_Z}{1-\alpha} + \delta + n\right)}$$

for $i \in \{1,2,...,\eta\}$ and $\tilde{v}_{\eta+1} = 0$. Note that upon eliminating blocking patents, the backloading discount factor $\nu$ and the share of profit captured by the most recent inventor $\Omega_1$ become one.

## Appendix III: The Social Rate of Return to R&D

Jones and Williams (1998) define the social rate of return as the sum of the additional output produced and the reduction in R&D that is made possible by reallocating one unit of output from consumption to R&D in the current period and then reducing R&D in the next period to leave the subsequent path of technology unchanged. To conform to their notations, I rewrite the law of motion for R&D technology as

$$\dot{A}_t = G(A_t, R_t) \equiv A_t^{\phi} R_t^{\gamma} \varphi \ln z \,, \tag{g1}$$

where $R_t \equiv K_{r,t}^{\alpha} L_{r,t}^{1-\alpha}$. The aggregate production function is rewritten as

$$Y_t = F(A_t, X_t) \equiv \vartheta(\eta) A_t Z_t X_t \,, \tag{g2}$$

where $X_t \equiv K_{x,t}^{\alpha} L_{x,t}^{1-\alpha}$. Using the above definition, Jones and Williams (1998) show that the gross social return is

$$1 + \tilde{r} = \left(\frac{\partial G}{\partial R}\right)_t \left(\frac{\partial F}{\partial A}\right)_{t+1} + \frac{(\partial G / \partial R)_t}{(\partial G / \partial R)_{t+1}} \left(1 + \left(\frac{\partial G}{\partial A}\right)_{t+1}\right). \tag{g3}$$

After imposing the balanced-growth conditions, the net social return becomes

$$\tilde{r} = \frac{1 + g_Y}{1 + g_A} \left(1 + g_A \left(\frac{\gamma}{s_r} + \phi\right)\right) - 1 \,. \tag{g4}$$

## Appendix IV: Tables and Figures

| Table 1a: Structural Parameters for θ = 0 | | | | | | Table 1b: Structural Parameters for θ = 0.25 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| λ | ν | α | σ | r | μ | λ | ν | α | σ | r | μ |
| 0.04 | 0.847 | 0.287 | 2.359 | 0.084 | 1.030 | 0.04 | 0.852 | 0.288 | 2.363 | 0.084 | 1.040 |
| 0.06 | 0.696 | 0.287 | 2.359 | 0.084 | 1.030 | 0.06 | 0.700 | 0.288 | 2.363 | 0.084 | 1.040 |
| 0.08 | 0.620 | 0.287 | 2.359 | 0.084 | 1.030 | 0.08 | 0.624 | 0.288 | 2.363 | 0.084 | 1.040 |
| 0.10 | 0.575 | 0.287 | 2.359 | 0.084 | 1.030 | 0.10 | 0.578 | 0.288 | 2.363 | 0.084 | 1.040 |
| 0.12 | 0.545 | 0.287 | 2.359 | 0.084 | 1.030 | 0.12 | 0.548 | 0.288 | 2.363 | 0.084 | 1.040 |
| 0.14 | 0.523 | 0.287 | 2.359 | 0.084 | 1.030 | 0.14 | 0.526 | 0.288 | 2.363 | 0.084 | 1.040 |
| 0.16 | 0.507 | 0.287 | 2.359 | 0.084 | 1.030 | 0.16 | 0.510 | 0.288 | 2.363 | 0.084 | 1.040 |
| 0.18 | 0.495 | 0.287 | 2.359 | 0.084 | 1.030 | 0.18 | 0.497 | 0.288 | 2.363 | 0.084 | 1.040 |
| 0.20 | 0.485 | 0.287 | 2.359 | 0.084 | 1.030 | 0.20 | 0.487 | 0.288 | 2.363 | 0.084 | 1.040 |

| Table 1c: Structural Parameters for θ = 0.5 | | | | | | Table 1d: Structural Parameters for θ = 0.75 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| λ | ν | α | σ | r | μ | λ | ν | α | σ | r | μ |
| 0.04 | 0.862 | 0.288 | 2.372 | 0.084 | 1.061 | 0.04 | 0.892 | 0.288 | 2.397 | 0.085 | 1.126 |
| 0.06 | 0.708 | 0.288 | 2.372 | 0.084 | 1.061 | 0.06 | 0.732 | 0.288 | 2.397 | 0.085 | 1.126 |
| 0.08 | 0.631 | 0.288 | 2.372 | 0.084 | 1.061 | 0.08 | 0.652 | 0.288 | 2.397 | 0.085 | 1.126 |
| 0.10 | 0.585 | 0.288 | 2.372 | 0.084 | 1.061 | 0.10 | 0.604 | 0.288 | 2.397 | 0.085 | 1.126 |
| 0.12 | 0.554 | 0.288 | 2.372 | 0.084 | 1.061 | 0.12 | 0.572 | 0.288 | 2.397 | 0.085 | 1.126 |
| 0.14 | 0.532 | 0.288 | 2.372 | 0.084 | 1.061 | 0.14 | 0.549 | 0.288 | 2.397 | 0.085 | 1.126 |
| 0.16 | 0.515 | 0.288 | 2.372 | 0.084 | 1.061 | 0.16 | 0.532 | 0.288 | 2.397 | 0.085 | 1.126 |
| 0.18 | 0.503 | 0.288 | 2.372 | 0.084 | 1.061 | 0.18 | 0.519 | 0.288 | 2.397 | 0.085 | 1.126 |
| 0.20 | 0.492 | 0.288 | 2.372 | 0.084 | 1.061 | 0.20 | 0.508 | 0.288 | 2.397 | 0.085 | 1.126 |

| Table 2a: Calibrated Values of φ for θ = 0 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ξ / γ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 1.00 | 0.81 | 0.63 | 0.44 | 0.25 | 0.07 | -0.12 | -0.31 | -0.49 | -0.68 | -0.86 |
| 0.90 | 0.79 | 0.59 | 0.38 | 0.17 | -0.04 | -0.24 | -0.45 | -0.66 | -0.86 | -1.07 |
| 0.80 | 0.77 | 0.53 | 0.30 | 0.07 | -0.17 | -0.40 | -0.63 | -0.86 | -1.10 | -1.33 |
| 0.70 | 0.73 | 0.47 | 0.20 | -0.07 | -0.33 | -0.60 | -0.86 | -1.13 | -1.40 | -1.66 |
| 0.60 | 0.69 | 0.38 | 0.07 | -0.24 | -0.55 | -0.86 | -1.18 | -1.49 | -1.80 | -2.11 |
| 0.50 | 0.63 | 0.25 | -0.12 | -0.49 | -0.86 | -1.24 | -1.61 | -1.98 | -2.36 | -2.73 |
| 0.40 | 0.53 | 0.07 | -0.40 | -0.86 | -1.33 | -1.80 | -2.26 | -2.73 | -3.19 | -3.66 |
| 0.30 | 0.38 | -0.24 | -0.86 | -1.49 | -2.11 | -2.73 | -3.35 | -3.97 | -4.59 | -5.21 |
| 0.20 | 0.07 | -0.86 | -1.80 | -2.73 | -3.66 | -4.59 | -5.53 | -6.46 | -7.39 | -8.32 |
| 0.10 | -0.86 | -2.73 | -4.59 | -6.46 | -8.32 | -10.19 | -12.05 | -13.92 | -15.78 | -17.64 |
| 0.00 | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ |

| Table 2b: Calibrated Values of $\phi$ for $\theta$ = 0.25 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\xi$ / $\gamma$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 1.00 | 0.81 | 0.63 | 0.44 | 0.25 | 0.07 | -0.12 | -0.31 | -0.49 | -0.68 | -0.86 |
| 0.90 | 0.79 | 0.59 | 0.38 | 0.17 | -0.04 | -0.24 | -0.45 | -0.66 | -0.86 | -1.07 |
| 0.80 | 0.77 | 0.53 | 0.30 | 0.07 | -0.17 | -0.40 | -0.63 | -0.86 | -1.10 | -1.33 |
| 0.70 | 0.73 | 0.47 | 0.20 | -0.07 | -0.33 | -0.60 | -0.86 | -1.13 | -1.40 | -1.66 |
| 0.60 | 0.69 | 0.38 | 0.07 | -0.24 | -0.55 | -0.86 | -1.18 | -1.49 | -1.80 | -2.11 |
| 0.50 | 0.63 | 0.25 | -0.12 | -0.49 | -0.86 | -1.24 | -1.61 | -1.98 | -2.36 | -2.73 |
| 0.40 | 0.53 | 0.07 | -0.40 | -0.86 | -1.33 | -1.80 | -2.26 | -2.73 | -3.20 | -3.66 |
| 0.30 | 0.38 | -0.24 | -0.86 | -1.49 | -2.11 | -2.73 | -3.35 | -3.97 | -4.59 | -5.22 |
| 0.20 | 0.07 | -0.86 | -1.80 | -2.73 | -3.66 | -4.59 | -5.53 | -6.46 | -7.39 | -8.32 |
| 0.10 | -0.86 | -2.73 | -4.59 | -6.46 | -8.32 | -10.19 | -12.05 | -13.92 | -15.78 | -17.65 |
| 0.00 | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ |

| Table 2c: Calibrated Values of $\phi$ for $\theta$ = 0.5 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\xi$ / $\gamma$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 1.00 | 0.81 | 0.63 | 0.44 | 0.25 | 0.07 | -0.12 | -0.31 | -0.49 | -0.68 | -0.87 |
| 0.90 | 0.79 | 0.59 | 0.38 | 0.17 | -0.04 | -0.24 | -0.45 | -0.66 | -0.87 | -1.07 |
| 0.80 | 0.77 | 0.53 | 0.30 | 0.07 | -0.17 | -0.40 | -0.63 | -0.87 | -1.10 | -1.33 |
| 0.70 | 0.73 | 0.47 | 0.20 | -0.07 | -0.33 | -0.60 | -0.87 | -1.13 | -1.40 | -1.66 |
| 0.60 | 0.69 | 0.38 | 0.07 | -0.24 | -0.55 | -0.87 | -1.18 | -1.49 | -1.80 | -2.11 |
| 0.50 | 0.63 | 0.25 | -0.12 | -0.49 | -0.87 | -1.24 | -1.61 | -1.98 | -2.36 | -2.73 |
| 0.40 | 0.53 | 0.07 | -0.40 | -0.87 | -1.33 | -1.80 | -2.26 | -2.73 | -3.20 | -3.66 |
| 0.30 | 0.38 | -0.24 | -0.87 | -1.49 | -2.11 | -2.73 | -3.35 | -3.97 | -4.60 | -5.22 |
| 0.20 | 0.07 | -0.87 | -1.80 | -2.73 | -3.66 | -4.60 | -5.53 | -6.46 | -7.39 | -8.33 |
| 0.10 | -0.87 | -2.73 | -4.60 | -6.46 | -8.33 | -10.19 | -12.06 | -13.92 | -15.79 | -17.65 |
| 0.00 | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ |

| Table 2d: Calibrated Values of $\phi$ for $\theta$ = 0.75 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\xi$ / $\gamma$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 1.00 | 0.81 | 0.63 | 0.44 | 0.25 | 0.07 | -0.12 | -0.31 | -0.49 | -0.68 | -0.87 |
| 0.90 | 0.79 | 0.59 | 0.38 | 0.17 | -0.04 | -0.24 | -0.45 | -0.66 | -0.87 | -1.07 |
| 0.80 | 0.77 | 0.53 | 0.30 | 0.07 | -0.17 | -0.40 | -0.63 | -0.87 | -1.10 | -1.33 |
| 0.70 | 0.73 | 0.47 | 0.20 | -0.07 | -0.33 | -0.60 | -0.87 | -1.13 | -1.40 | -1.67 |
| 0.60 | 0.69 | 0.38 | 0.07 | -0.24 | -0.56 | -0.87 | -1.18 | -1.49 | -1.80 | -2.11 |
| 0.50 | 0.63 | 0.25 | -0.12 | -0.49 | -0.87 | -1.24 | -1.61 | -1.99 | -2.36 | -2.73 |
| 0.40 | 0.53 | 0.07 | -0.40 | -0.87 | -1.33 | -1.80 | -2.27 | -2.73 | -3.20 | -3.67 |
| 0.30 | 0.38 | -0.24 | -0.87 | -1.49 | -2.11 | -2.73 | -3.35 | -3.98 | -4.60 | -5.22 |
| 0.20 | 0.07 | -0.87 | -1.80 | -2.73 | -3.67 | -4.60 | -5.53 | -6.46 | -7.40 | -8.33 |
| 0.10 | -0.87 | -2.73 | -4.60 | -6.46 | -8.33 | -10.20 | -12.06 | -13.93 | -15.80 | -17.66 |
| 0.00 | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ | -∞ |

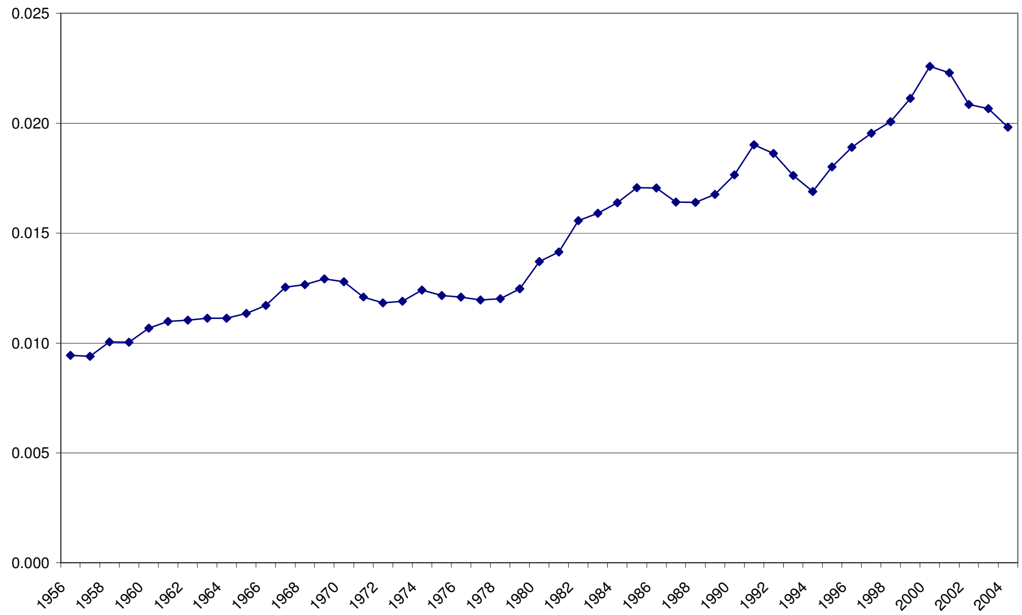
| Table 3a: The Implied Social Rates of Return for θ = 0 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ξ / γ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 1.00 | 0.15 | 0.27 | **0.38** | **0.49** | **0.61** | **0.72** | **0.83** | **0.95** | **1.06** | **1.18** |
| 0.90 | 0.14 | 0.24 | **0.34** | **0.45** | **0.55** | **0.65** | **0.75** | **0.86** | **0.96** | **1.06** |
| 0.80 | 0.13 | 0.22 | **0.31** | **0.40** | **0.49** | **0.58** | **0.67** | **0.76** | **0.86** | **0.95** |
| 0.70 | 0.12 | 0.20 | 0.28 | **0.35** | **0.43** | **0.51** | **0.59** | **0.67** | **0.75** | **0.83** |
| 0.60 | 0.11 | 0.17 | 0.24 | **0.31** | **0.38** | **0.44** | **0.51** | **0.58** | **0.65** | **0.71** |
| 0.50 | 0.09 | 0.15 | 0.21 | 0.26 | **0.32** | **0.37** | **0.43** | **0.49** | **0.54** | **0.60** |
| 0.40 | 0.08 | 0.13 | 0.17 | 0.22 | 0.26 | **0.30** | **0.35** | **0.39** | **0.44** | **0.48** |
| 0.30 | 0.07 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.27 | **0.30** | **0.33** | **0.36** |
| 0.20 | 0.06 | 0.08 | 0.10 | 0.12 | 0.14 | 0.16 | 0.18 | 0.21 | 0.23 | 0.25 |
| 0.10 | 0.05 | 0.06 | 0.07 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 |
| 0.00 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| **Table 3b: The Implied Social Rates of Return for θ = 0.25** | | | | | | | | | | |
| ξ / γ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 1.00 | 0.15 | 0.27 | **0.38** | **0.49** | **0.61** | **0.72** | **0.83** | **0.95** | **1.06** | **1.18** |
| 0.90 | 0.14 | 0.24 | **0.35** | **0.45** | **0.55** | **0.65** | **0.75** | **0.86** | **0.96** | **1.06** |
| 0.80 | 0.13 | 0.22 | **0.31** | **0.40** | **0.49** | **0.58** | **0.67** | **0.76** | **0.86** | **0.95** |
| 0.70 | 0.12 | 0.20 | 0.28 | **0.35** | **0.43** | **0.51** | **0.59** | **0.67** | **0.75** | **0.83** |
| 0.60 | 0.11 | 0.17 | 0.24 | **0.31** | **0.38** | **0.44** | **0.51** | **0.58** | **0.65** | **0.71** |
| 0.50 | 0.09 | 0.15 | 0.21 | 0.26 | **0.32** | **0.37** | **0.43** | **0.49** | **0.54** | **0.60** |
| 0.40 | 0.08 | 0.13 | 0.17 | 0.22 | 0.26 | **0.30** | **0.35** | **0.39** | **0.44** | **0.48** |
| 0.30 | 0.07 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.27 | **0.30** | **0.33** | **0.36** |
| 0.20 | 0.06 | 0.08 | 0.10 | 0.12 | 0.14 | 0.16 | 0.18 | 0.21 | 0.23 | 0.25 |
| 0.10 | 0.05 | 0.06 | 0.07 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 |
| 0.00 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| **Table 3c: The Implied Social Rates of Return for θ = 0.5** | | | | | | | | | | |
| ξ / γ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 1.00 | 0.15 | 0.27 | **0.38** | **0.49** | **0.61** | **0.72** | **0.84** | **0.95** | **1.06** | **1.18** |
| 0.90 | 0.14 | 0.24 | **0.35** | **0.45** | **0.55** | **0.65** | **0.75** | **0.86** | **0.96** | **1.06** |
| 0.80 | 0.13 | 0.22 | **0.31** | **0.40** | **0.49** | **0.58** | **0.67** | **0.76** | **0.86** | **0.95** |
| 0.70 | 0.12 | 0.20 | 0.28 | **0.36** | **0.43** | **0.51** | **0.59** | **0.67** | **0.75** | **0.83** |
| 0.60 | 0.11 | 0.17 | 0.24 | **0.31** | **0.38** | **0.44** | **0.51** | **0.58** | **0.65** | **0.71** |
| 0.50 | 0.09 | 0.15 | 0.21 | 0.26 | **0.32** | **0.37** | **0.43** | **0.49** | **0.54** | **0.60** |
| 0.40 | 0.08 | 0.13 | 0.17 | 0.22 | 0.26 | **0.30** | **0.35** | **0.39** | **0.44** | **0.48** |
| 0.30 | 0.07 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.27 | **0.30** | **0.33** | **0.36** |
| 0.20 | 0.06 | 0.08 | 0.10 | 0.12 | 0.14 | 0.16 | 0.18 | 0.21 | 0.23 | 0.25 |
| 0.10 | 0.05 | 0.06 | 0.07 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 |
| 0.00 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |

| Table 3d: The Implied Social Rates of Return for θ = 0.75 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ξ / γ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 1.00 | 0.15 | 0.27 | **0.38** | **0.49** | **0.61** | **0.72** | **0.84** | **0.95** | **1.06** | **1.18** |
| 0.90 | 0.14 | 0.24 | **0.35** | **0.45** | **0.55** | **0.65** | **0.76** | **0.86** | **0.96** | **1.06** |
| 0.80 | 0.13 | 0.22 | **0.31** | **0.40** | **0.49** | **0.58** | **0.67** | **0.77** | **0.86** | **0.95** |
| 0.70 | 0.12 | 0.20 | 0.28 | **0.36** | **0.43** | **0.51** | **0.59** | **0.67** | **0.75** | **0.83** |
| 0.60 | 0.11 | 0.17 | 0.24 | **0.31** | **0.38** | **0.44** | **0.51** | **0.58** | **0.65** | **0.72** |
| 0.50 | 0.09 | 0.15 | 0.21 | 0.26 | **0.32** | **0.37** | **0.43** | **0.49** | **0.54** | **0.60** |
| 0.40 | 0.08 | 0.13 | 0.17 | 0.22 | 0.26 | **0.30** | **0.35** | **0.39** | **0.44** | **0.48** |
| 0.30 | 0.07 | 0.10 | 0.14 | 0.17 | 0.20 | 0.23 | 0.27 | **0.30** | **0.33** | **0.37** |
| 0.20 | 0.06 | 0.08 | 0.10 | 0.12 | 0.14 | 0.16 | 0.18 | 0.21 | 0.23 | 0.25 |
| 0.10 | 0.05 | 0.06 | 0.07 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 |
| 0.00 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |

| Table 4: R&D Shares without Blocking Patents | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| θ / λ | 0.04 | 0.06 | 0.08 | 0.10 | 0.12 | 0.14 | 0.16 | 0.18 | 0.20 |
| 0.00 | 0.0136 | 0.0165 | 0.0185 | 0.0200 | 0.0211 | 0.0219 | 0.0226 | 0.0232 | 0.0237 |
| 0.25 | 0.0135 | 0.0164 | 0.0184 | 0.0199 | 0.0210 | 0.0218 | 0.0225 | 0.0231 | 0.0236 |
| 0.50 | 0.0133 | 0.0162 | 0.0182 | 0.0196 | 0.0207 | 0.0216 | 0.0223 | 0.0228 | 0.0233 |
| 0.75 | 0.0129 | 0.0157 | 0.0176 | 0.0190 | 0.0201 | 0.0209 | 0.0216 | 0.0221 | 0.0226 |

| Table 5: Capital Investment Rates without Blocking Patents | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| θ / λ | 0.04 | 0.06 | 0.08 | 0.10 | 0.12 | 0.14 | 0.16 | 0.18 | 0.20 |
| 0.00 | 0.204 | 0.204 | 0.205 | 0.205 | 0.205 | 0.205 | 0.206 | 0.206 | 0.206 |
| 0.25 | 0.204 | 0.204 | 0.205 | 0.205 | 0.205 | 0.205 | 0.206 | 0.206 | 0.206 |
| 0.50 | 0.204 | 0.204 | 0.205 | 0.205 | 0.205 | 0.205 | 0.205 | 0.206 | 0.206 |
| 0.75 | 0.204 | 0.204 | 0.204 | 0.205 | 0.205 | 0.205 | 0.205 | 0.205 | 0.206 |

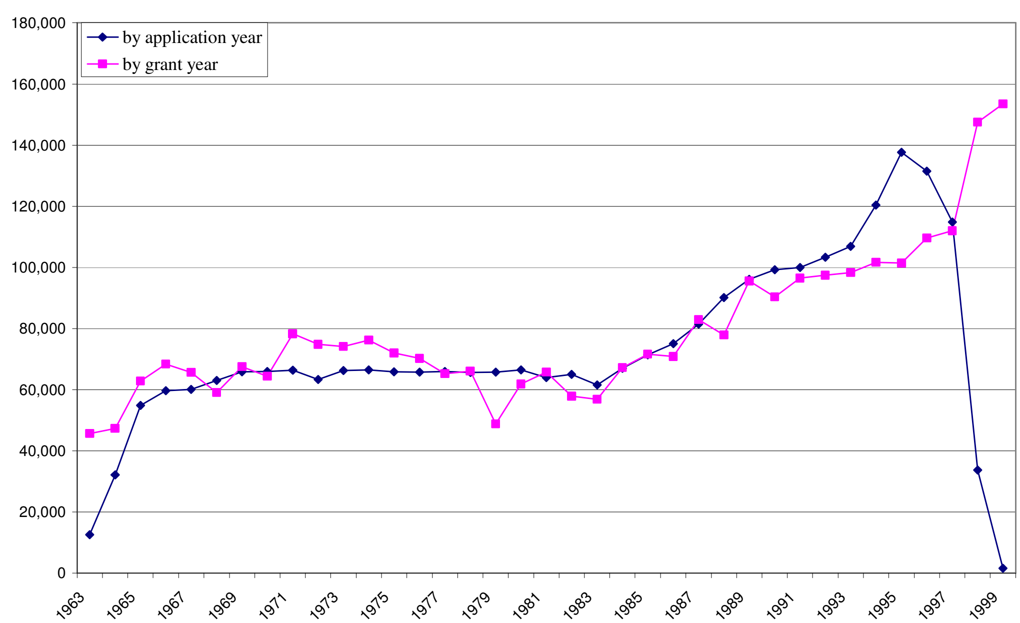
**Figure 1: Private Spending on R&D as a Share of GDP**




Data Sources: (a) Bureau of Economic Analysis: National Income and Product Accounts Tables; and (b) National Science Foundation: Division of Science Resources Statistics.

Footnote: R&D is net of federal spending, and GDP is net of government spending.

**Figure 2: Number of Patents Granted**




Data Source: Hall, Jaffe and Trajtenberg (2002): The NBER Patent Citation Data File.

**Figure 3a: Socially Optimal R&D Shares for θ = 0**

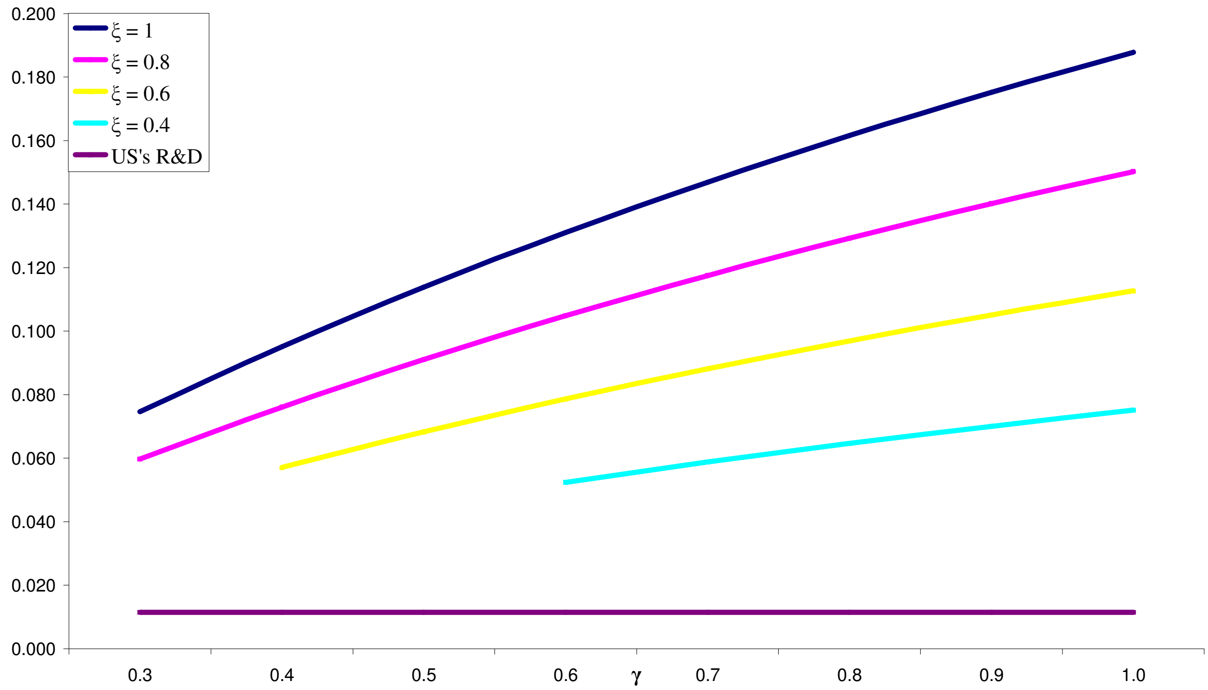



**Figure 3b: Socially Optimal R&D Shares for θ = 0.25**

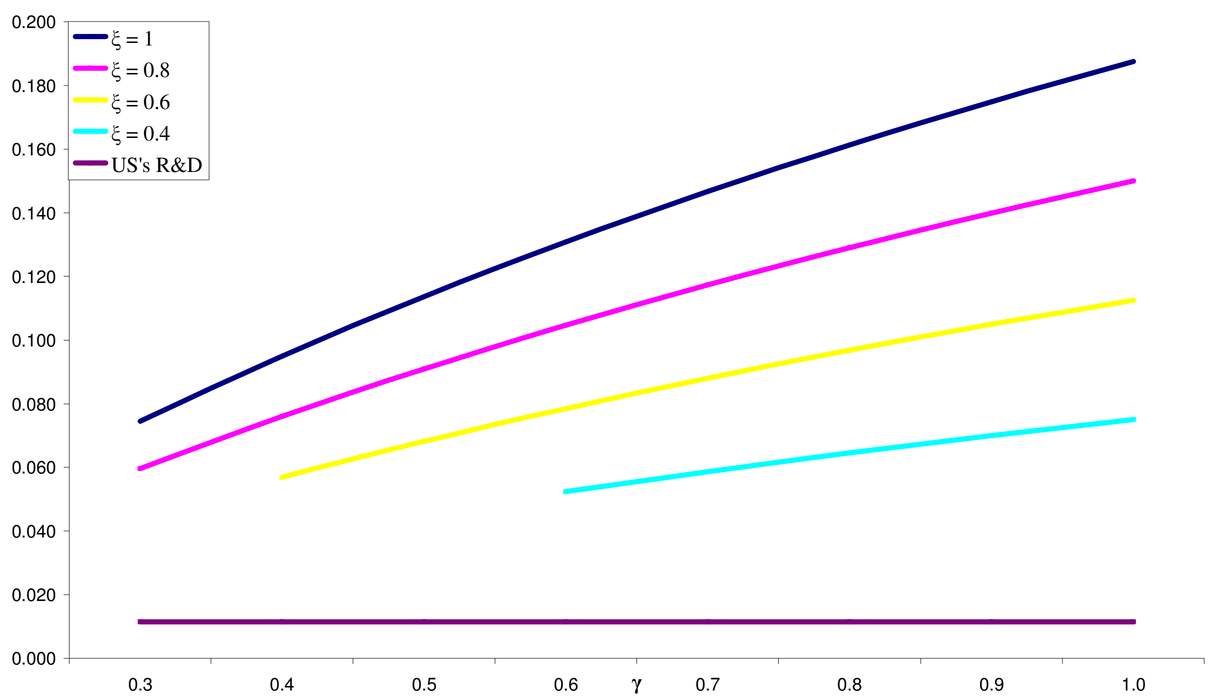


**Figure 3c: Socially Optimal R&D Shares for θ = 0.5**

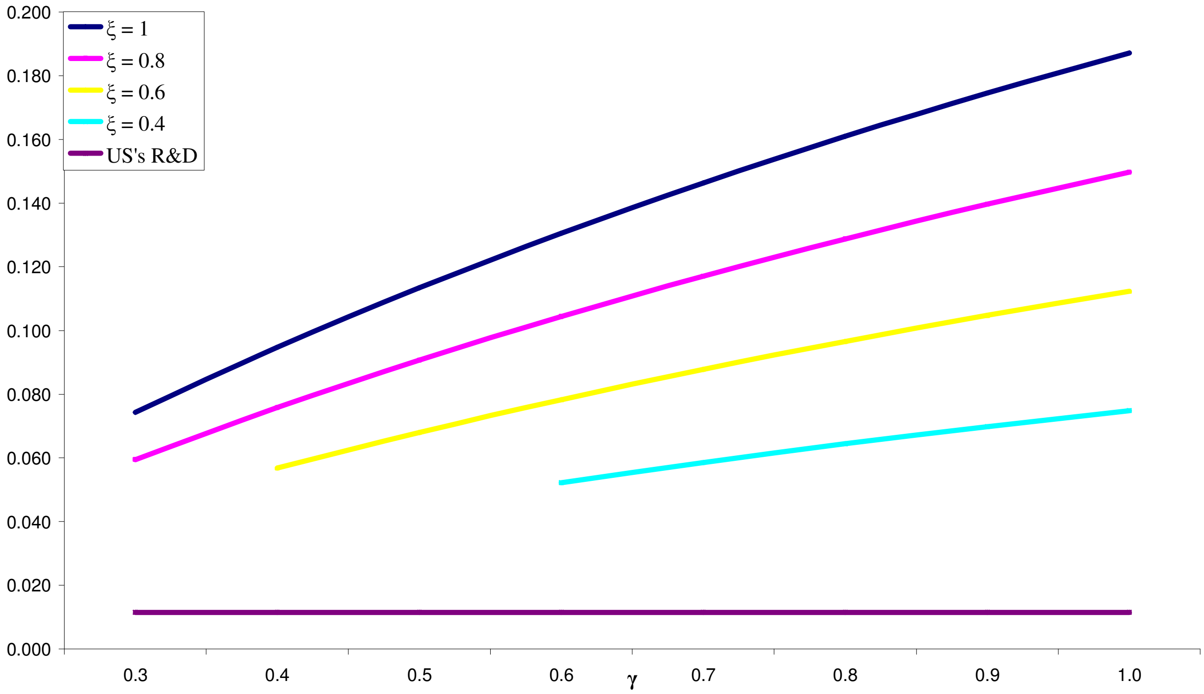



**Figure 3d: Socially Optimal R&D Shares for θ = 0.75**

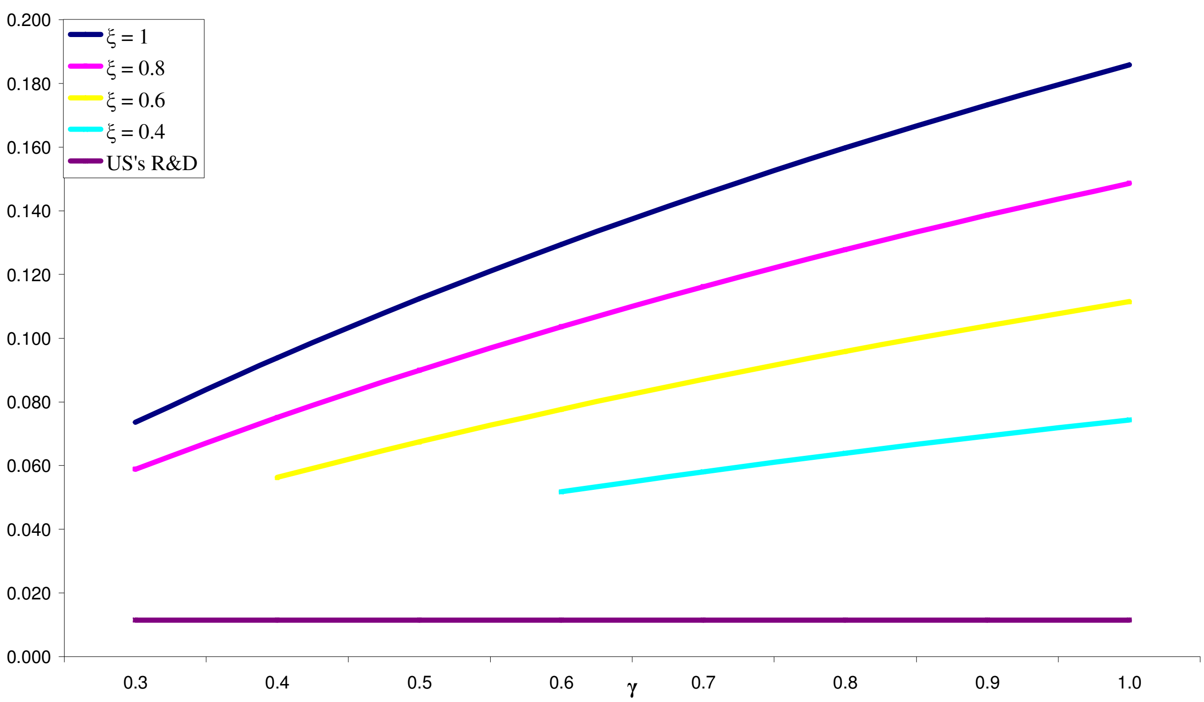


**Figure 4a: Percent Changes in Long-Run Consumption from Eliminating Blocking Patents for θ = 0**

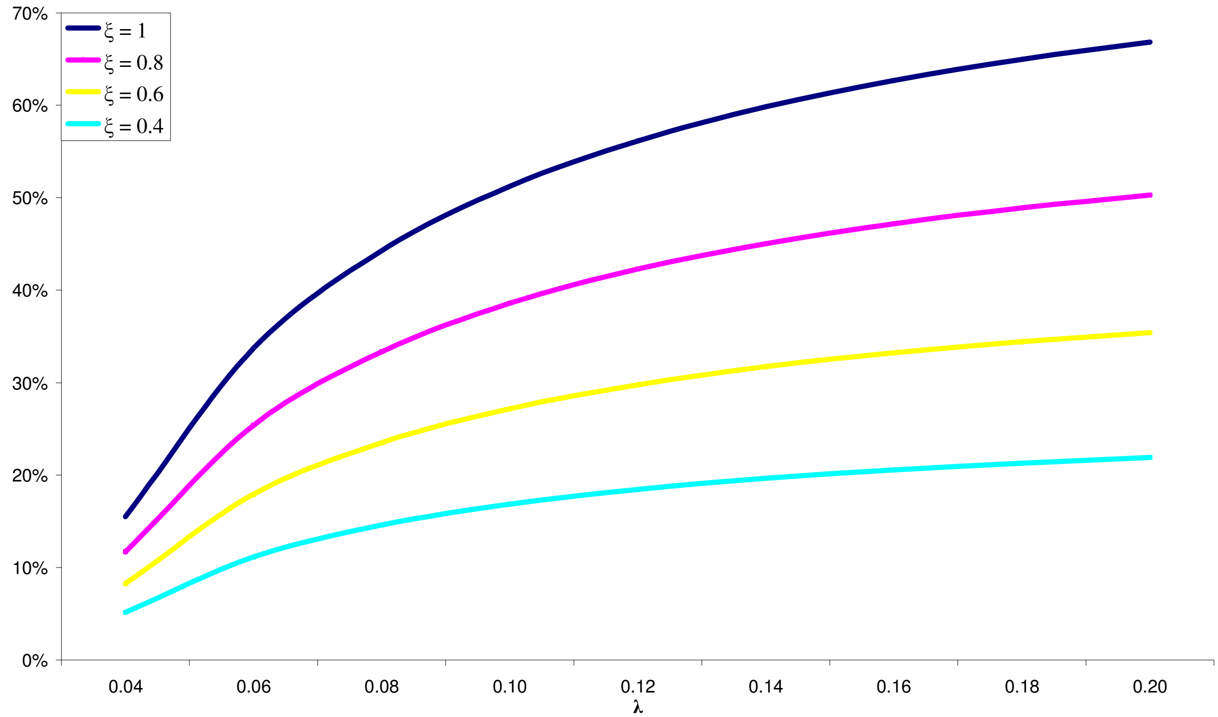



**Figure 4b: Percent Changes in Long-Run Consumption from Eliminating Blocking Patents for θ = 0.25**

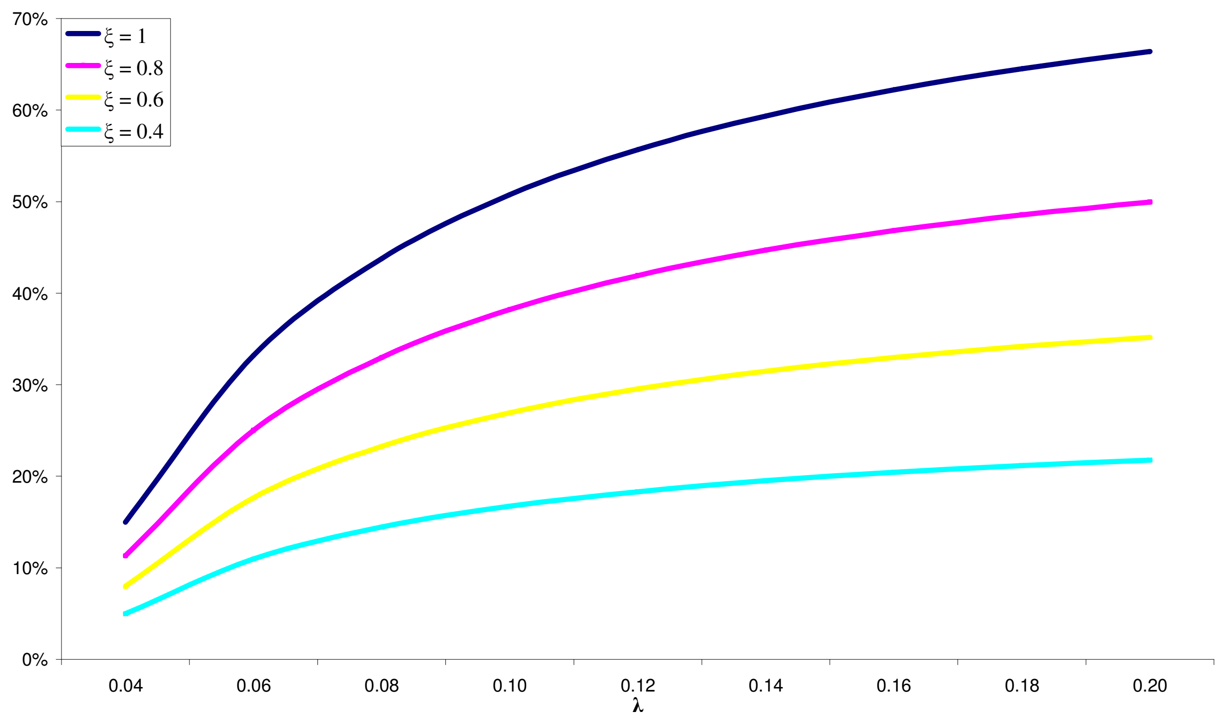


**Figure 4c: Percent Changes in Long-Run Consumption from Eliminating Blocking Patents for θ = 0.5**

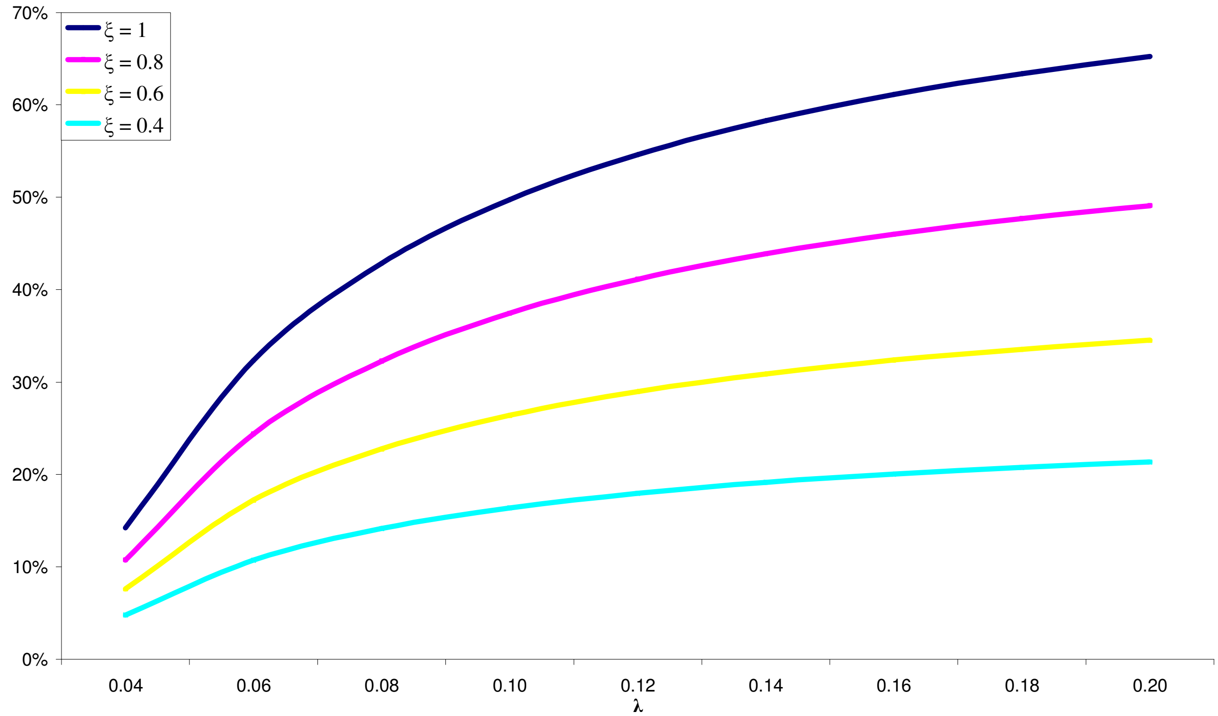



**Figure 4d: Percent Changes in Long-Run Consumption from Eliminating Blocking Patents for θ = 0.75**

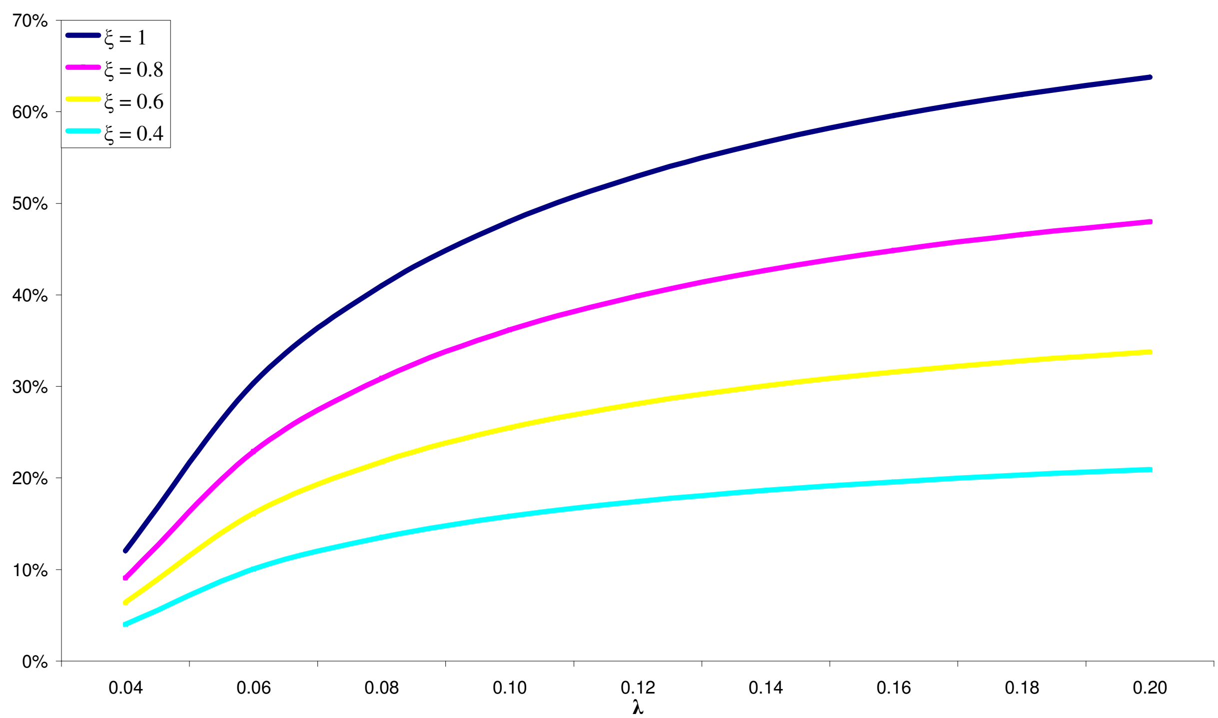


**Figure 5a: Transition Dynamics of Consumption for ξ = γ = 0.55 with Partial Capital Depreciation**

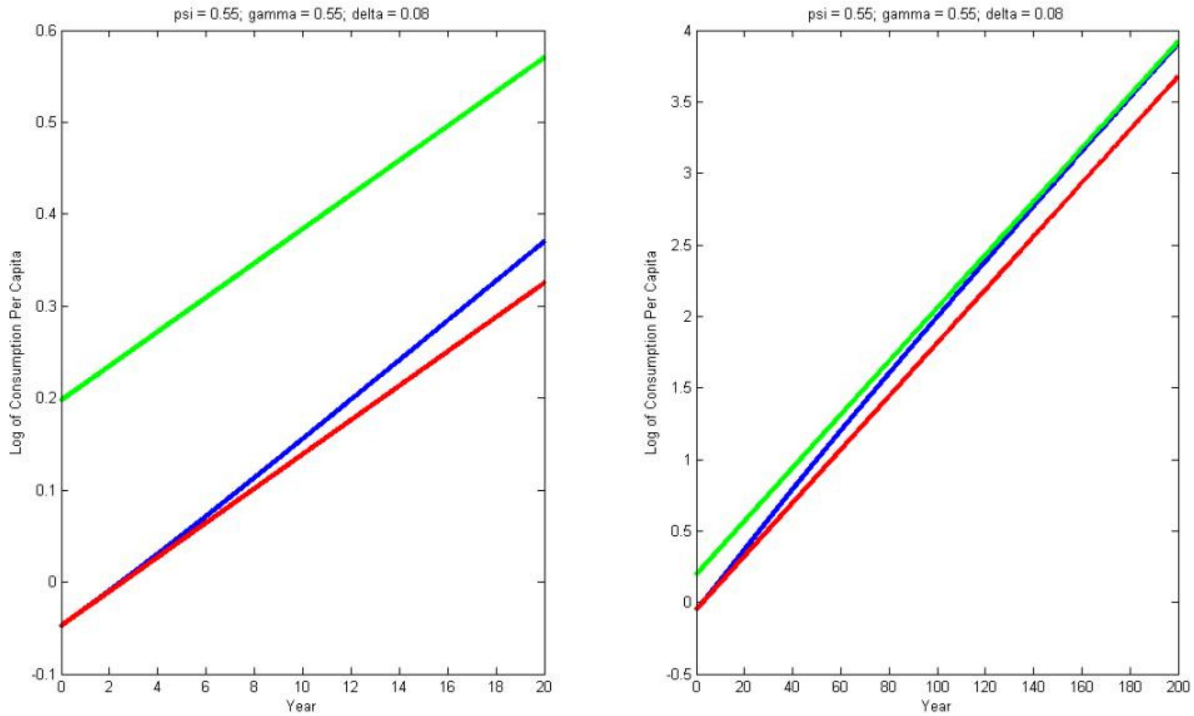



**Figure 5b: Transition Dynamics of Consumption for ξ = γ = 0.55 with Complete Capital Depreciation**

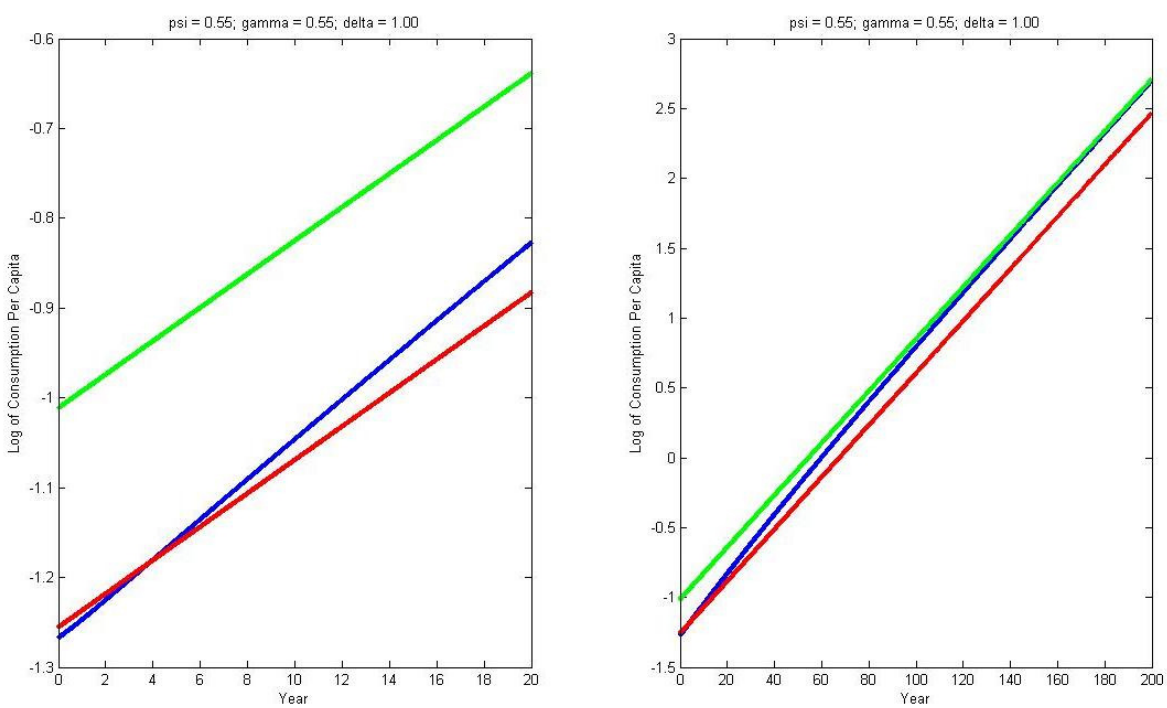


**Figure 5c: Transition Dynamics of Consumption for ξ = γ = 0.3 with Partial Capital Depreciation**

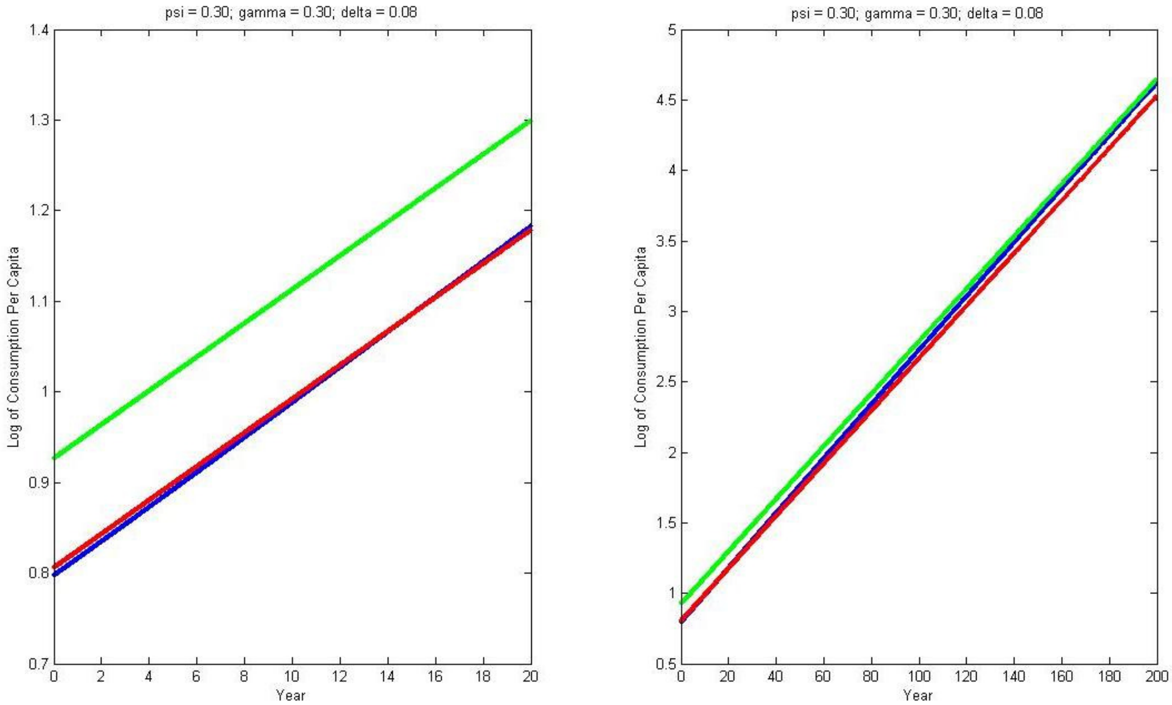



**Figure 5d: Transition Dynamics of Consumption for ξ = 0.95 and γ = 0.3 with Partial Capital Depreciation**

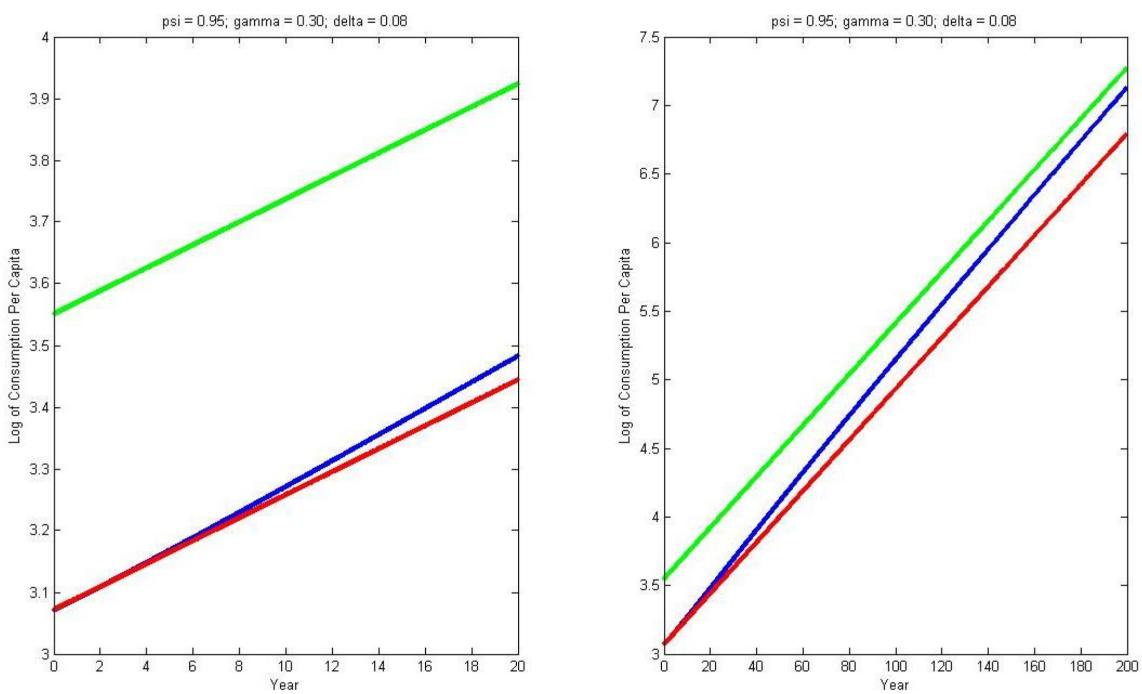


**Figure 6a: Socially Optimal Rates of Capital Investment for θ = 0**

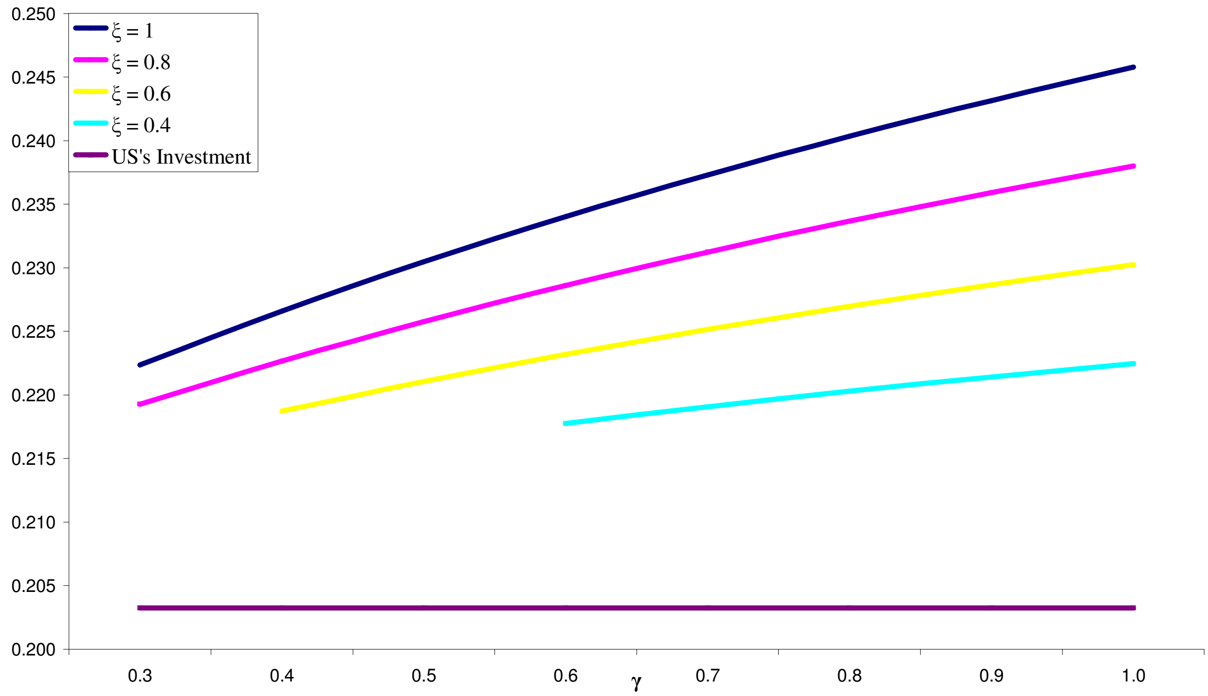



**Figure 6b: Socially Optimal Rates of Capital Investment for θ = 0.25**

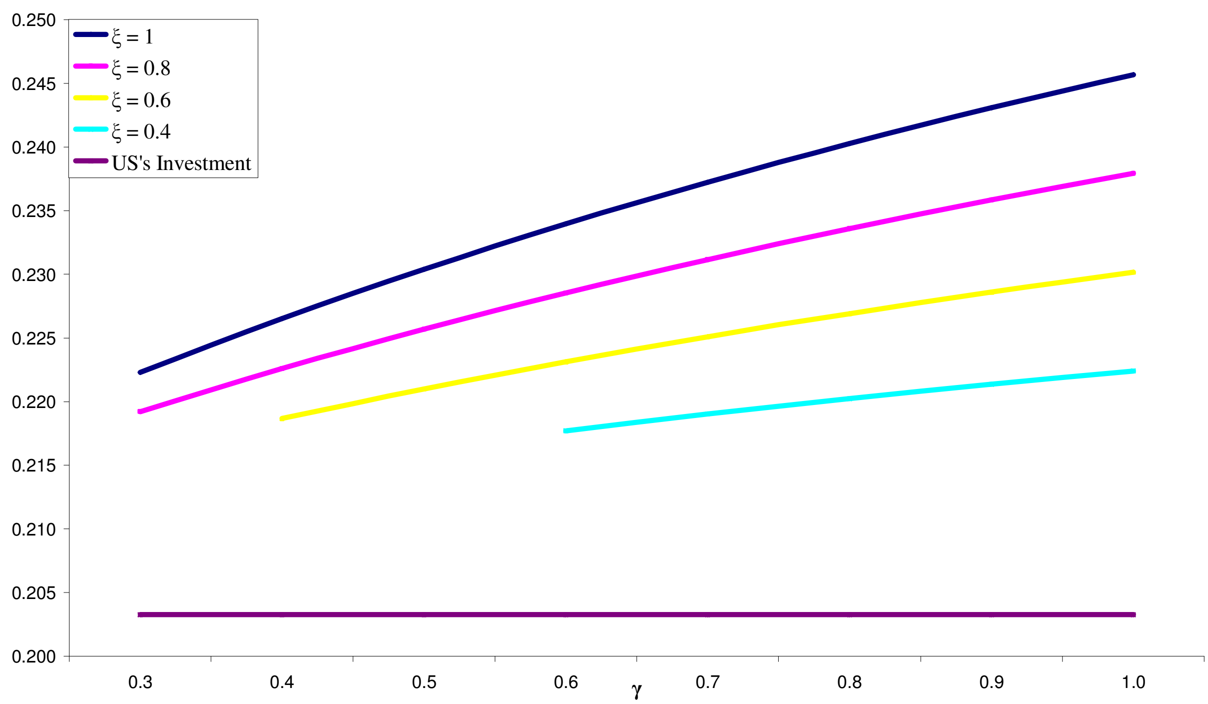


**Figure 6c: Socially Optimal Rates of Capital Investment for θ = 0.5**

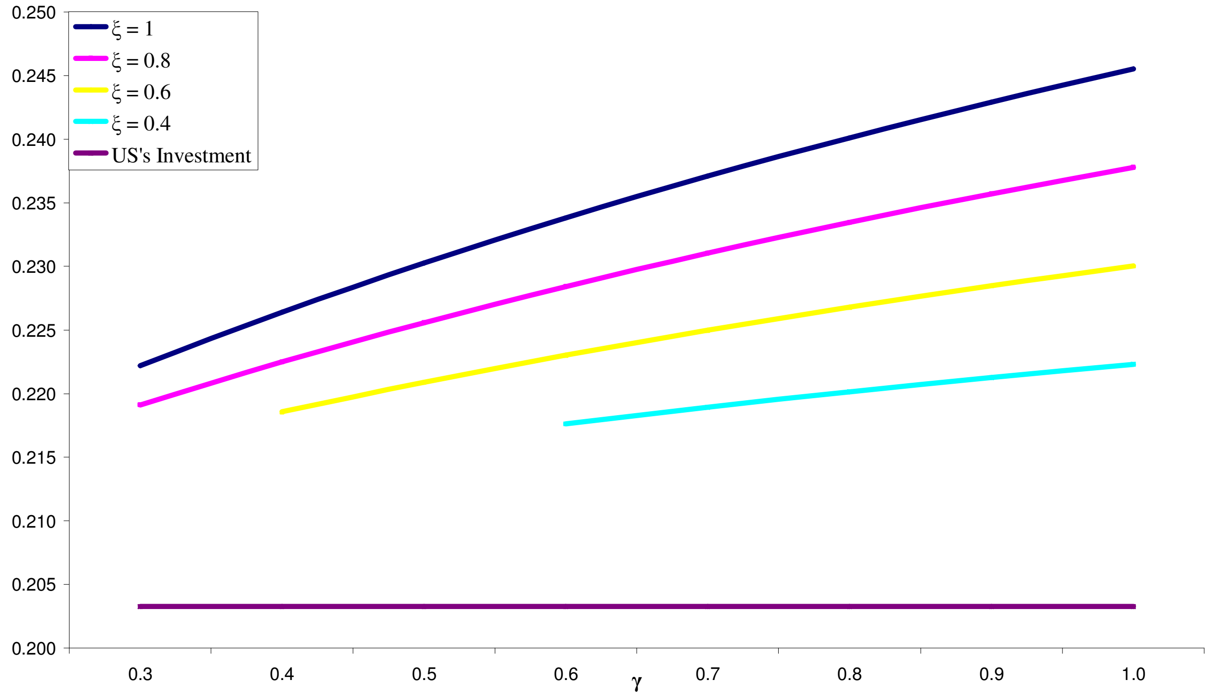



**Figure 6d: Socially Optimal Rates of Capital Investment for θ = 0.75**

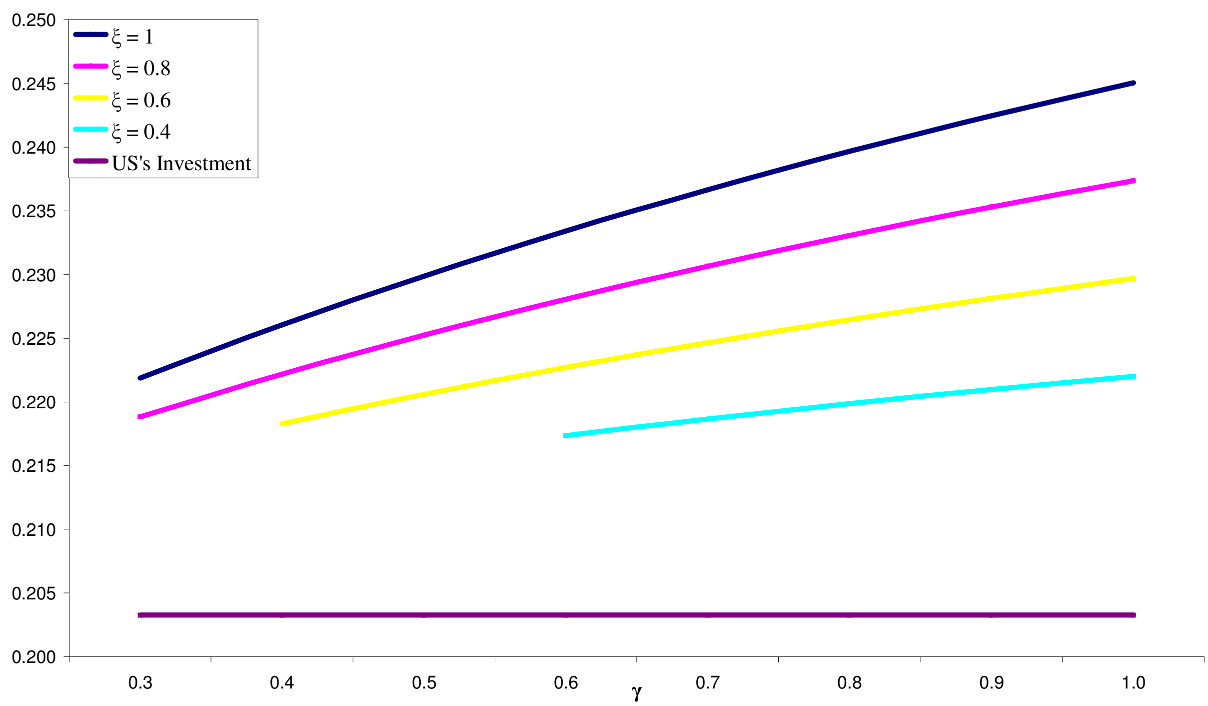


**Figure 7: Calibrated Values for the Backloading Discount Factor Based on Industry-Level Data**