Communication and Information Acquisition in Networks

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Abstract

This paper deals with information acquisition and communication in networked organizations. Agents receive private signals about a payoff-relevant parameter and may communicate it to other players to whom they are linked. I derive a key condition that ensures truthful communication. Since the degree of substitution between information acquired and obtained through personal contacts depends on the truthfulness of communication, information acquisition efforts may not be monotonic. Finally, I show that these results hold in a modified version of the game that includes potentially infinite many rounds of communication.

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1 Introduction

Economists have long recognized the acquisition and transmission of information between individuals as one of the key objectives of organizations (Arrow (1974).) Indeed, organizations take over the role of prices when these fail to accomplish their mission of aggregating disperse information and encouraging individuals to take the appropriate actions. Mimicking the role of prices in market transactions, organizational design should enable efficient information transmission within the organization and provide the right incentives to create and maintain information flows from outside. While these two elements have been separately studied in different papers, this paper tries to analyze their interrelation and their implications for organizational design. I argue that this link may explain some of the features of many real-world organizations.

An example of an organization in which information transmission is important is the stock market. Most information is conveyed through prices, but it is also well-known that word-of-mouth communication and other networked activities are ubiquitous in those environments. Shiller and Pound (1986) shows that most trading decisions involved interpersonal communication, and very few agents spend resources in obtaining first-hand information. Similarly, Hong et al. (2005) finds strong correlation in the positions of traders based on the same city, controlling for the location of the assets. This evidence suggests a strong use of personal contacts in information acquisition. This has been neglected in the majority of papers studying financial markets, where the information structure is a reduced-form stochastic process. In particular, no explicit distinction is made about the sources originating the signals[1].

This paper highlights a bidirectional interaction between information acquisition and communication. First, smooth information transmission helps to disseminate relevant information and coordinate behavior, while reducing the duplication of efforts in information acquisition. But differences in the information available to different agents will hamper their (mutual) communication, since they introduce a wedge between their conditional expectations after some signal is observed. Agents communicate their signals before obtaining all the relevant information and use interim beliefs which depend on the amount of information that they expect to receive. For instance, more informed agents rely *less* on every particular

[1]An exception is Ozsoylev and Walden (2011)), who study a rational-expectations equilibrium in which agents communicate truthfully their exogenously received signals through an exogenous random network.

signal than less informed agents. Hence, in general, beliefs about other player's beliefs may fail to be aligned and information transmission will be noisy.

To get a grasp of the implications of this trade-off, I study a standard beauty-contest type of game (Morris and Shin (2002)) where every agent must take a decision facing a trade-off between adaptation to global uncertainty and coordination with the rest of players. Importantly, and in contrast with prior literature, there is no exogenous conflict of interest among the different players, in the sense that, conditional on the true realization of the state of the world, all agents would agree on the same action. I allow them to choose the amount of information they acquire (Hellwig and Veldkamp (2009), Myatt and Wallace (2011)) and to report this information to their peers through a discrete (undirected) network. In the benchmark model, this communication takes the simple form of a round of messages in which every agent chooses a profile of reports to each of his peers conditional on the information he owns.

I first provide a characterization of the networks that induce an equilibrium where every player who receives information communicates it truthfully, taking the profile of information acquisition efforts as given. I show that in these networks, if two players are linked and at least one of them receives some information, the *total* amount of information they both expect to receive must be the same. I then show that such condition is violated for an important class of networks, trees of diameter larger than 2, that have been widely studied in the economics of organization.

I then study the profiles of information acquisition efforts that may emerge in equilibrium. I show that if the technology is linear, then a truthful-revelation equilibrium exists (generically) only if the set of players contains a subset such that no two members of the set are linked to each other and such that all non members are linked to exactly one member.[2] I also show that information acquisition may not be monotonic in centrality. Regarding welfare, I show that star networks are typically efficient in the class of networks inducing a truthfully revealing equilibrium with linear costs but I can also show that truthful-revelation is neither necessary nor sufficient for a given network to attain the second best payoff. The reason is that, under truthfully revealing communication, private signals become public goods and there is, generically, underinvestment in information acquisition. Networks with less specialization increase the total amount of information acquired and increase the expected welfare of the group (as

[2] This definition is more restrictive than maximal independent sets used in Bramoulle and Kranton (2007). See Section 4 for a comparison

measured by the sum of utilities).

Finally, I show that my qualitative results extend to an environment with more rounds of communication, as long as players leave the network once they take their actions. In particular, I identify network structures for which, independently of the number of rounds of communication, information cannot be truthfully revealed between two linked players because they will use it differently. Thus, even if the mechanism highlighted in this paper requires players to use interim beliefs, the assumption of one round of communication is not crucial[3].

Related Literature

This paper contributes to a couple of strands in the literature. First, there is a small but influential literature on communication in networked organizations started by Geanakoplos and Milgrom (1988)[4] (1991) and Radner (1993), within the realm of team theory. There are no strategic issues and the problem is simply to choose the optimal organization of workers to minimize time processing, due to bounded rationality. The typical finding of this literature is that hierarchical organizations are likely to be optimal for information transmission purposes. Adding strategic incentives to the transmission of information, we find that hierarchies are likely to be suboptimal since they yield a very unequal distribution of information and, therefore, weak incentives for truth-telling.

Second, there is a growing literature of game-theoretical views of networked organizations. Calvó-Armengol et al. (2011) study information acquisition and truthful and costly communication in networks. Ozsoylev and Walden (2011) analyze a rational expectations equilibrium in the presence of communication via networks. They assume that communication is truthful and that information acquisition is exogenously given. They show that information acquisition is increasing in centrality in a linear-quadratic model of network formation. However, this paper is the first study addressing information acquisition and strategic communication jointly. Another strand of the literature has analyzed strategic information transmission in networks. Hagenbach and Koessler (2010) analyze a game in which signals are strategic complements and agents differ on their preference relation over outcomes, but there is no information acquisition and the preference divergence is exogenous to the network structure. Galeotti et al. (2013) analyze a similar game, but their focus is on competing signals and analyze

[3]The crucial assumption is that agents may take actions after each round of reports.

[4]See also Bolton and Dewatripont (1994)

the effect of congestion and other network characteristics on the amount of information transmitted.

To conclude, two recent contributions analyze repeated communication in societies. Anderlini et al. (2012) consider an organization composed by one-period-lived agents who send reports to their successors regarding some underlying uncertainty. They show that the existence of an exogenous preference bias impedes common learning of the parameter of interest. Bimpikis et al. (2014) is somewhat closer to the spirit of this paper but they consider the case of large societies transmitting over time information relevant to the decision of whether to undertake or not a project. They highlight an strategic motive to lie to induce agents to transmit their information, but they concentrate mostly on truthful communication.

2 Model

The economy is populated by a set N of $n > 2$ players, who are concerned concerned with the realization of some aggregate uncertainty θ. In the case of financial analysis, θ would be the fundamental value of an asset. I assume that θ follows a normal distribution, $N(0, \tau_\sigma^{-1})$. Each player receives a signal $x_i = \theta + \eta_i$, with η_i normal with zero mean and variance τ_i^{-1}, where τ_i is the precision the signal held by agent i. Notice that $\{x_i\}_{i \in N}$ are independent conditional on θ but may not be identically distributed, since I allow each player to choose $\tau_i \in \Re_+$, by paying some cost $c(\tau_i)$. I assume the cost function to be continuously differentiable, strictly increasing and weakly convex. Let $\tau = (\tau_1, \tau_2, .., \tau_n)$ be the profile of precision choices for each player.

Agents are linked through an undirected and discrete network g, so that i and j have a link if and only if $ij \in g$. A link is interpreted as the existence of a communication channel between any two players. Define a *walk* from i to j as a collection $\{k_1, k_2, .., k_m\}$ such that $k_1 = i$, $k_m = j$ and $k_l k_{l-1} \in g$ for all $l = 2, 3, .., m$. A *path* is the shortest walk between two players, i, j, so that I write $p(i, j)$. Let $|p(i, j)|$ be its length. If such a path does not exist $|p(i, j)| = \infty$. A network is *connected* if and only if for all i, $sup_j |p(i, j)| < \infty$.

A *component* $g^s \subset g$ is a subnetwork of g such that the nodes of g^s, N^s, is a set of players satisfying for all $i, j \in N^s$, $p(i, j) < \infty$ and for all $k \in N \setminus N^s$ we have that $p(i, k) = \infty$.

Players are then allowed to communicate their information with those whom they are linked through g. For most of this paper, communication takes the simple form of a single-round of simultaneous, private messages.

Denote by $N_i(g) = \{j \in N : ij \in g\}$ the set of neighbors of agent i, so that every agent submits a message profile $m_i = \{m_{ij}\}_{j\in N_i(g)}$ and learns a message profile $m^i = \{m_{ji}\}_{j\in N_i(g)}$. For simplicity, I shall assume that $m_{ij} \in \mathcal{R}$, so that a reporting (pure) strategy for agent i is a mapping $m^i : \Re \to \Re^{|N_i(g)|-1}$. I denote with $N_{ij}(g) = N_i(g) \cap N_j(g)$ the set of common neighbors of i and j, and $N_{i-j} = N_i(g) \setminus N_{ij}(g)$ the set of neighbors of i who cannot communicate with j. Finally, let $N_i^*(g) = N_i(g) \cup \{i\}$ be the neighborhood of agent i augmented to himself[5].

Once information is received and transmitted, all players must take an action $a_i \in \Re$, conditional on all the information available to maximize the following loss-function

$$U(a_i, a_{-i}, \tau_i; \theta) = -(a_i - \theta)^2 - \frac{1}{n-1}\sum_{j\neq i}(a_i - a_j)^2 - c(\tau_i) \tag{1}$$

According to (1) every agent wants to match a weighted average of the realization underlying uncertainty and the actions of other players. This loss-function captures the standard trade-off between coordination and adaptation in a simple way. Notice that there is no ex-ante conflict of interest among different agents. Let

$$a_i : \Re \times \Re^{|N_i(g)|-1} \to \Re \tag{2}$$

be the strategy of agent i contingent on her private information and the messages of all other players with whom she is linked.

The Timing of Events is shown in Figure 1. In the first stage, nature draws a state of the world and every agent chooses some information acquisition τ_i. Every player observes the profile of precisions τ. Signals are then drawn conditional on the state of the world according to the chosen distributions. In the second stage, every player communicates to her peers through an undirected network g. Finally, conditional on all her information, every player chooses an action a_i and payoffs are realized.

The rest of the paper is organized as follows. Section 3 contains a characterization of the networked information structures such that a truthful equilibrium exists. In Section 4 I present the results concerning information acquisition in networks admitting a truthfully revealing equilibrium. Section 5 introduces welfare considerations. In Section 6 I extend the benchmark model in order to allow for multiple rounds of communication. Section 7 discusses different potential applications of the theoretical results. Section 8 concludes. All omitted proofs are contained in the Appendix.

[5] I extend analogously the remaining concepts, so that, for instance, $N_{ij}^* = N_j^* \cap N_i^*$

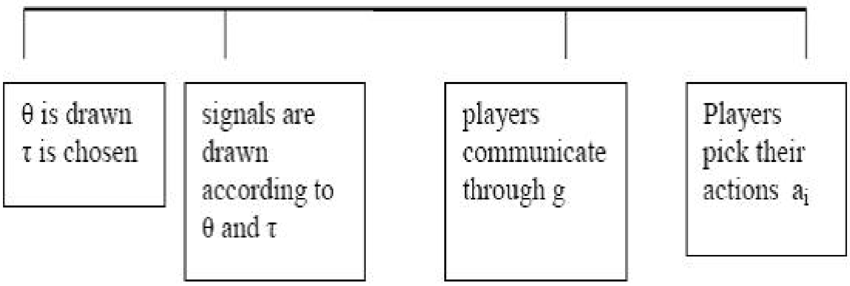

Figure 1: Timing of the Game

3 Truthful Revelation of Information

In this section, I identify the conditions under which there exists an equilibrium in which all signals are credibly revealed. More precisely, I determine the conditions under which there exists a Perfect Bayesian Equilibrium where $m_{ij}(x_i) = x_i$ for all $i \in N$,$j \in N_i(g)$ and $x_i \in \Re$ Notice that, should all information be revealed, there exists a linear equilibrium in the last stage[6], where actions will satisfy.

$$a_i = b_{ii}x_i + \sum_{j \in N_i(g)} b_{ij}m_j = \sum_{j \in N_i^*(g)} b_{ij}x_j \tag{3}$$

for some weights where $b_{ij} \geq 0$ and $\sum_j b_{ij} \leq 1$. In general, the weight that i puts on signal x_j will depend on the total precision of the report of agent j - that is, the accuracy of both its signal and its message -, the total amount of information i has access to and the weight that others put on that signal. Suppose $ij \in g$ and consider the incentives of agent i to truthfully reveal his type to j whenever everybody else does so. Using the envelope theorem, I write his indirect utility in the last stage as

$$-\mathbf{E}\left[V_i\right] = \mathbf{E}\left[\left(\sum_{j \in N_i^*(g)} b_{ij}x_j - \theta\right)^2\right] +$$

[6] See Lemma 13 a proof of existence of linear equilibrium. Notice that if communication is truthful my model reduces to a standard beauty-contests with agents receiving a number of signals equal to their degree and with endogenously determined precision. See Myatt and Wallace (2011) for such a model.

$$\frac{1}{n-1}\sum_{k\neq i}\mathbf{E}\left[(\sum_{h\in N_i^*(g)} b_{ih}x_h - \sum_{l\in N_j(g)\setminus\{i\}} b_{jl}x_l - b_{ji}m_i)^2\right]$$

Assuming that i reports truthfully to all other $k \in N_i(g)$ and that all other players report truthfully, their signals, i will have an incentive to report truthfully to j if and only if.

$$b_{ji}x_i = \mathbf{E}\left[\sum_{h\in N_i(g)} b_{ih}x_h - \sum_{l\in N_j(g)\setminus\{i\}} b_{jl}x_l \mid x_i\right] \tag{4}$$

$$= b_{ii}x_i + \left[\sum_{h\in N_i(g)} b_{ih} - \sum_{l\in N_j^*(g)\setminus\{i\}} b_{jl}\right]\mathbf{E}\left[x_k \mid x_i\right] \tag{5}$$

Definition 1. A networked information structure $\{g,\tau\}$ is **active-regular** if for every i, and for every $j \in N_i(g)$, such that $\tau_i + \tau_j > 0$

$$\sum_{l\in N_i(g)} \tau_l = \sum_{k\in N_j(g)} \tau_k \tag{6}$$

In words, a networked information structure is active-regular if for every two agents in a given component, the total amount of information they have access to is the same. The following Proposition characterizes the set of networks that allow for truthful information transmission.

Proposition 2. *Assume that $\tau_\sigma > 0$. There exists an equilibrium with truthful revelation at every link $ij \in g$ such that $\tau_i + \tau_j > 0$ if and only if the networked information structure is active-regular*

The intuition for the result is simple. Truthful revelation requires that hierarchies of beliefs are ex-ante aligned. This holds if the prior does not convey any information or if information is symmetric (total precision of the signals received by every agent is the same.) The reason is that an informative prior creates a wedge between the expectation of the underlying state conditional on a given signal and the signal. Hence, second order beliefs - the belief of i about the belief of j about θ -conditional on i's signal will differ with the current belief of i. This generates an incentive to i to misrepresent her information and align those beliefs.

Notice that in my model cheap-talk equilibria does not rely on an *exogenous preference bias*. Conditional on the realization of the state of the world, all agents would agree on the best course of action. However, in the interim stage, if the networked communication structure is not active-regular and their neighbors would take their reports at face value, they would have an incentive to misreport their information. In such a case, well-connected agents have an incentive to make *conservative* reports about the state of the world (i.e. to claim that the deviation from the mean is smaller) while badly connected ones have incentives to make *aggressive* reports. In equilibrium, these biases are understood by the receiver and they result in a reduction of the amount of information conveyed. Thus, differences in ex-post information, introduces vagueness in communication and reduces welfare.

Every equilibrium in the information transmission game is characterized by a partition of the set of signals, where a given report m is to be understood simply as $x \in [m_k, m_{k+1}]$. Unfortunately, since signals are normally distributed on the real line, a full characterization of the communication equilibria is not possible. This greatly difficulties the comparison between truthful and non-truthful equilibria in terms of welfare. Because of this, I devote most of my attention to networks for which a truthful equilibrium exists.[7]

3.1 Active-Regular Networks

In order to characterize the set of networked-information structures for which a truthful-revelation equilibrium exists, I will now dig deeper on the nature of active-regular networks. In particular, I am concerned with identifying conditions in the network g such that there exists a profile of information acquisition efforts τ for which (g, τ) is an active-regular network. Moreover I shall impose that the resulting *communication network* (i.e. the subnetwork of g constructed by deleting those links where no information flows because none of the nodes acquire information) is connected.[8]

To this end, for a subset of agents, J, let $p_J(i, j)$ be the length of the shortest path between i and j, such that for all steps of the path $(k_n k_m \in g)$ either $k_n \in J$ or $k_m \in J$ or both. That is a a path in the network resulting from eliminating all links not containing a player in J. Then,

Proposition 3. *Fix g. Then,*

1. *If there exists a set of players J such that for all $i \in N$, $|N_i^*(g) \cup J| = r$ for some $r > 0$ and that $p_J(i, j) < \infty$. Then, there exists τ such that g is active-regular and connected.*

[7]Truthful-revelation networks need are not (first-best) efficient and may be dominated by non-truthful ones. See Section 6.

[8]See Example 7 below.

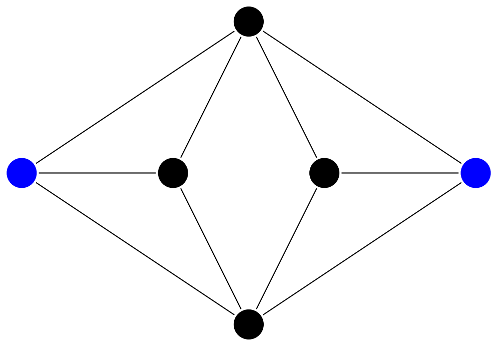

Figure 2: An active-regular networked information structure

2. *If g is a tree with diameter larger than 2, there exists no τ such that g is active-regular and connected.*

The first part of Proposition 2 offers a positive result. If $r = 1$, all networks have at least one such set J, which is called the *maximal independent set* of g. However, not all networks have a maximal independent set which renders the resulting network connected. For instance, a line with four nodes $g = \{12, 23, 34\}$ has only one maximal independent set. Namely, $J = \{1, 4\}$ but $p_J(1, 4) = \infty$. On the other hand, for $r > 1$, then J is not an independent set but the network remains active-regular. In particular, for any regular network there exists a profile of information acquisition efforts such that the network is active-regular.

Figure 2 depicts another active-regular networked information structure. Black nodes acquire some information (belong to J) while blue nodes do not.

The second part of Proposition 2 suggests that truthful communication at every link fails to obtain for a broad class of networks.[9] The following result, formally demonstrated in the Appendix, shows that a contagion-effect may preclude full-revelation of information at almost every link.

Proposition 4. *Let g^T be a line with more than three individuals and assume that $\tau_i > 0$ for all i. For all $i \in g^T$, there exists $j \in N_i(g^T)$, such that i cannot communicate truthfully with j*

[9] Trees are the most common form of organization because it minimizes delay in information processing. See Section 7 for a discussion of the implications of this result for organizational design.

4 Information Acquisition

Even if the communication network ensures that a truthful revelation equilibrium exists for a given τ, there is no guarantee that such profile of information acquisition would arise. Since in most real world organizations individuals have to spend non-trivial resources to obtain information, the communication network must also ensure that they have the appropriate incentives to do so. The aim of this Section is to offer a partial characterization of the classes of networks that induce such equilibria.

To obtain the first positive result I follow the literature on public goods in networks and assume that the technology is linear so that $c(\tau) = c\tau$. As mentioned above, if communication is truthful, information acquisition efforts become local public goods and so specialization arises in equilibrium. As shown by Bramoulle and Kranton (2007) in such a game the set of agents who incur in some positive effort is a *maximal independent set of order r* where no agent in the set is linked to another agent member but non members are linked to at least r members. As it should be obvious there is a close connection between the first part of Proposition 2 and *maximal independent sets of order r*. The difference is that (i) all non-members must have exactly r connections with the members of the set and (ii) those in the set must also have $r - 1$ connections inside. This second difference is erased for the case of $r = 1$. Indeed we have that

Proposition 5. *Suppose that $c(\tau) = c\tau$. g admits an equilibrium with truthful revelation if and only if it has a maximal independent set of order 1 such that for all $i \in N$, $|N_i^*(g) \cup J| = 1$.*

It is straightforward to check that this induces an equilibrium. Indeed, since every agent has access to the same amount of information τ^*, it must be that $V'(\tau^*) = c$. Since the resulting set satisfies Proposition 2, the network induces an equilibrium with truthful revelation. Thus, information acquisition efforts are public goods and the profile of efforts must be a maximal independent set. Notice finally that, if it exists, the set of equilibrium configurations leading to a truthful revelation equilibrium is a selection of those leading to an equilibrium in the public-goods game. For instance, if the network g is a star ($g = \{12, 13, ..., 1n\}$), there are two maximal independent sets of arbitrary order r ($J = 1$ and $J = N \setminus 1$) but a unique maximal independent set of order 1, $J = 1$. In this case, it is also the profile that maximizes the utilitarian's welfare function.[10] Finally notice that

[10] See Section 5 for a discussion of welfare considerations in this model.

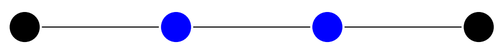

Figure 3: A line with truthful revelation

Theorem 2 in Bramoulle and Kranton (2007) shows that for a stable equilibrium to arise in the public goods game the set of specialists must be a maximal independent set of order $r > 2$. The intuition is that, since efforts are strategic substitutes, a downwards deviation by a member of J triggers an upwards deviation by his neighbors unless the marginal utility of an additional amount of effort is lower than its cost. In the game presented here, however, an equilibrium with a maximal independent set of order 1 may be stable since the degree of depreciation is endogenous. To give a simple example, consider the following equilibrium strategies: Agents in J choose precision τ^* while agents not in J acquire no additional information. If any agent deviates, the babbling equilibrium is played. The equilibrium is stable since a downwards deviation in the amount of information acquired is substituted by other players' effort but the agent is excluded of the resulting information.

For more general cost functions, however, the connection with the public-goods literature becomes more tenuous. For instance, if $c(\tau)$ is strictly convex and the network is regular with degree k, then there exists a symmetric equilibrium where all agents choose $V'(\tau) = c'(\frac{\tau}{k})$ and the profile of information transmission is truthful. More generally, specialization may occur but two different specialists may provide effort in equilibrium.

Finally, is there any pattern in the information acquisition profile that may result in equilibrium? Recent contributions Calvó-Armengol et al. (2011) suggest that information acquisition would typically be monotonic in the position of different players in the network because their marginal return from additional information will typically be larger. In this model, however, the interplay of information transmission and acquisition breaks down this pattern. In particular, consider the following example.

Example 6. Assume that g is a line with and let $c(\tau) = c\tau$. There exists an equilibrium with truthful information transmission and non-monotone pattern of information acquisition

The idea for this example is depicted in Figure 3. By Proposition32, there does not exist an equilibrium with truthful revelation inducing a connected subgraph. There exists, however, an equilibrium with truthful revelation of information, inducing a collection of disconnected components.

In general, different communication structures would lead to different patterns of information acquisition. As already pointed out, if information transmission is truthful, information acquisition efforts become perfect substitutes and the resulting equilibrium has a close connection with the set of equilibria of a standard publics-good game. On the other hand, if the communication equilibrium were characterized by full babbling, information acquisition efforts become strategic complements (see Hellwig and Veldkamp (2009)).

5 Welfare

For a network g, and an equilibrium of that network (a^*, m^*, τ^*), let W be the corresponding utilitarian social welfare function. Namely,

$$\max W = -\sum_{i \in N} \frac{1}{N} \int U(a^*(s_i, m(s_{-i})), \tau^*, \theta) d\Phi(\theta) \tag{7}$$

Let $W^* < 0$ be its value. The following proposition characterizes the set of optimal networks for linear costs for equilibria with truthful revelation.

Proposition 7. *Suppose $c(\tau) = c\tau$. $W^* = \sqrt{c}$ and it is achieved by any network g if there exists $i \in N$ such that for all $j \neq i$, $ij \in g$.*

The intuition is straightforward. Proposition 4 shows that a network induces a truthful revelation equilibrium with linear costs only if every agent observes at most one signal. All networks in that class yield the same incentives for information acquisition but only a star avoids costly duplication of efforts. It should be noticed, however, that the equilibrium information acquisition effort is independent of n, and, therefore the outcome is inefficient.

5.1 Welfare and Noisy Information Transmission

Up to now I have devoted all my attention to networks inducing truthful revelation of information. However, since information acquisition efforts are perfect substitutes, individual incentives are not aligned with social welfare and the outcome is typically inefficient. Example 8 below shows that truthful revelation is neither necessary nor sufficient for a network to induce a second-best equilibrium outcome. More precisely, it shows that that welfare may be higher in a network that does not induce truthful revelation as compared with the star network.

r	V_1	V_2	V_3	V_4	W
1	$0,5$	$0,5$	0	0	$0,5$
3	$0,5$	$0,51063$	$0,5002$	$0,334$	$0,5835$
5	$0,5$	$0,33016$	$0,5$	$0,3998$	$0,4756$
6	$0,5$	$0,32982$	$0,49992$	$0,39975$	$0,47545$

Figure 4: Welfare and Information Transmission

In order to show this I first have to characterize the equilibrium when individuals do not reveal truthfully all their information. As discussed in Section 3 this is a challenging task since the distribution of signals is Gaussian and the support infinite. I circumvent this problem by focusing on a simple class of equilibria: Individuals report only whether their signals exceed their prior mean or not. It is easy to show that, irrespectively of the network, such communication profile is incentive compatible.

Example 8. Consider a set of 40 players linked through the network g where a number r of hubs are linked two $40 - r - 1$ other (peripheral) agents and there are no links between hubs. Further assume that the information acquisition choice is binary so that c is the cost of acquiring a signal with precision 1. Clearly, for $r > 0$, there is no truthful revelation of information. I consider the following equilibrium outcome: All hubs acquire information and report whether their signal is positive or negative.[11]

Below I display the results of the simulation for different number of hubs. The first column captures the residual variance of the hubs. The second column captures the residual variance of the peripheral agents. The third column measures the expected loss in coordination between hubs and non-hubs, while the fourth measures the coordination loss between hubs. The last column is the Welfare Loss.[12]

Notice that adding more hubs has the immediate effect of decreasing welfare since (i) information transmission becomes noisy and (ii) hubs and peripheral agents fail to coordinate on the same action. As the number of hubs increase, though, the increase in information acquisition efforts overcomes the resulting noise in communication and total welfare increases.[13]

[11]This is an equilibrium for c low enough

[12]If communication is not truthful, optimal policy functions are approximated by a linear function of the posteriors. Reported estimates from 100000 simulations.

[13]For a fixed n, the optimal number of hubs is bounded because hubs fail to coordinate.

6 Repeated Communication

One of the main driving forces of the results presented above is the use of interim beliefs and one round of communication. That is, since agents only communicate once, they rely heavily on the beliefs they hold after observing their signal, when making their reports. This is the source of their *intrinsic bias*. I shall explore now this assumption by constructing a dynamic environment in which the game presented above is (potentially) infinitely repeated. I construct the game following the ideas in Bimpikis et al. (2014)).

The game is as follows. At time zero, every agent makes some investment in information. Then, both θ and the signals are drawn from the appropriate distributions. At time $t = 1$, agents report through the network $g^1 = g^T$ some messages conditional on their signals and their positions in the network $m_{ij}^1(x_i) \in \mathbf{R} \cup \{\emptyset\}$ [14]. Let m^{i1} be the profile of reports received by agent i in period 1. Then, agents take actions $a_{i,1} \in \mathbf{R} \cup \emptyset$ where $a_i = \emptyset$ is defined as inaction. After that, actions are realized and agents who took an action leave the game[15]. Agents who decided to stay inactive keep move into the next period by losing $\delta > 0$. At time $t = 2$, $g^2 = g^1 \cap \{i \in N : a_{i,1} = \emptyset\}$ and again chose a report $m_{ij}(x_i; m^{i1}) \in \mathbf{R} \cup \{\emptyset\}$ where $ij \in g^2$ and an action $a_{i,2} \in \mathbf{R} \cup \emptyset$. Whenever at the end of time t, the set $\{i \in N : a_{i,t} = \emptyset\} = \emptyset$ the game ends and every agent receives his payoff according to the original payoff net of the corresponding loss for delay δt_i, where $a_{i,t_i} \neq \emptyset$.

It is straightforward to realize than any equilibrium of the stage-game analyzed in Section 3 will remain an equilibrium here. Indeed, if every other player is expected to leave the network after making one round of communication only, it is in the best interest of the remaining agent to do so, independently of the amount of information she has received and for every $\delta > 0$. Thus,

Proposition 9. *Suppose that $\{\tau, g^1\}$ is active-regular. Then there is an equilibrium of the repeated game in which every agent reports truthfully and makes an action in the first period.*

This implies that the positive results in the previous section survive into this extended game. However, do the negative ones survive? A qualified

[14] I allow for explicit witholding of information.

[15] This is the main restriction of the framework since it will not be optimal for them to leave (for sufficiently small δ) and clearly their information is still valuable for others. Nevertheless, the incentives to provide information will be unambiguously biased after the action has been made.

answer would be yes. In particular, there are some networked information structures at which information cannot be transmitted at *any* round.

Proposition 10. *Suppose that* $\{g^1, \tau\}$ *is a line, and assume that* $\tau_i > 0$ *for all* i*. Then, if there exists an equilibrium where all agents exit at (or before)* $\bar{t}$*, then for every* $t = 1, 2, ..\bar{t}$*, there exists an equilibrium in which all agents leave at period* t*. Further, no equilibrium involves perfect communication.*

The intuition is simple. Suppose that a given agent (i) has only one neighbor (j). Suppose further that his neighbor has at least one additional informed neighbor. j takes the action (and leaves) whenever he has acquired enough information, and therefore does not transmit the last piece of information to i. The interim beliefs remain misaligned and there is no truthful revelation of information. Thus, in this extension of the game, as long as players leave the network after taking their actions with positive probability the qualitative features of the static equilibrium remains, even if they hinge on agents using interim beliefs.

In other words, whenever communication takes place between agents who ”expect to learn more on the future”, the results presented above are likely to hold. However, in most studies, the assumption is that players are either informed ex-ante or ex-post but they never get ”some information” in the interim. Repeated communication games, for instance, assume that players have acquired all the relevant information at stage zero. I argue that this assumption has deep implications in the results, and it is not clear why this possibility should be ruled out.

An exception in this literature is Bimpikis et al. (2014) They study strategic communication using interim beliefs. The main difference is that in their model there are no payoff externalities and the only motive for withholding information is to retain other agents in the networks. To manage so, she is willing to misreport her true signal in order to ”confuse” her peer and make him stay. This is not possible in my model since the residual variance does not depend on the ”content” of the reports.

Proposition 11. *Suppose* $\{g^1, \tau\}$ *is not active-regular, and assume that* $\tau_i > 0$ *for all* i*. If there exists* $i \in N$ *such that* $N_i(g) \subset N_j(g)$*,* $N_i(g) \neq N_j(g)$*, then either both agents hold the same information (and take the same action) or there is no equilibrium with perfect information transmission between them.*

This result shows that poorly informed individuals (in that they have access to a subset of the sources of their neighbors) will also have problems to communicate with those sources, independently of how many rounds of communications are allowed. This may seem counterintuitive, since an agent who talks to a poorly informed agent has a very good posterior belief over the belief of his neighbor. The problem is that this second order belief may be far away from the belief he holds! This yields a novel intuition that was not present in the static game. Namely, since agents are willing to wait only if waiting yields new and useful information, communicating with agents who have access to that information is not only more useful, but easier (in the sense that it reduces the vagueness in communication) than to those who have no new information.

Equilibrium behavior depends (discontinuously) in the discounting of players, as it is the case in many dynamic games. If the cost of continuing in the game is very high, the equilibrium unravels and players leave. If players leave earlier, perfect communication breaks down and thus both coordination and information sharing decreases sharply. Absent any cost, the network structure is not relevant since information would travel in a frictionless manner and eventually coordination would be achieved. In this sense, it is the cost of time that gives a specific content to the network itself.

Proposition 12. *Fix a cost δ, a networked information structure $\{g, \tau\}$ and a truthful equilibrium profile (m^*, a^*). There exists a networked information structure $\{g', \tau\}$ such that (m^*, a^*) is an equilibrium in the static game.*

This result clarifies the main assumption of the model. Namely, that there is some cost of communicating that precludes information to be transmitted fast enough throughout the network that the result is equivalent to one in which information is publicly shared instantaneously. Since the reduction of costs in information processing and transmission are one of the main objectives of organizations (Arrow (1974)), this assumption seems the most natural one.

7 Applications

The model presented in this paper has two main features: a beauty-contest-type payoff function and a bilateral communication network. Beauty-contest games have been widely applied to the study of financial markets (Allen et al. (2006)) and complex organizations (Dessein and Santos (2006)). They provide a simple way to introduce strategic complementarities, disperse information and decentralized decision-making. On the other hand, bilateral communication networks have received increasing attention in the economics literature, providing a successful framework for the study of financial markets (Ozsoylev and Walden (2011)).

More generally, the model may be applied to many different settings in which communication is strategic and unverifiable. For instance, it may be useful to understand information sharing between firms operating in similar markets where strategic complementarities are present (Raith (1996).) Pairwise communication is potentially less costly and more difficult to detect but may create problems in terms of credibility. Similarly, this framework may be applied to the study of complex organizations, where information is disperse and and coordination is key for performance. In these organizations, decisions must be taken rapidly and communication is informal. For instance, coordination, information acquisition and good communication are the key factors underlying the design of Intelligence Agencies (Garicano and Posner (2005)). The results of this model suggest that complex organizations (with multiple layers and partial specialization) may not provide adequate incentives for information transmission while smaller, closely connected teams may outperform them both in terms of coordination and use of the available information.

Hierarchies and Information Sharing

Most of the previous literature on communication in organizations agreed that hierarchies are an efficient way to transmit and process information. Radner (1993) shows that a hierarchical structure (a tree in the jargon of graph theory) is the most efficient structure for an organization that tries to process and summarize a large amount of disseminated information. Bolton and Dewatripont (1994) extended this logic to environments with an infinite stream of signals that have to be processed with minimal delay. Geanakoplos and Milgrom (1988) showed that, under bounded rationality of managers, hierarchical organizations are the most efficient way to use a group to overcome the limitations of its members. Garicano (2000) shows that a hierarchy is the natural organization for a firm that must solve problems in order to produce if workers cannot identify those problems they cannot solve.

A common feature of all these models, however, is that individual members of these organizations are not strategic. In particular, they acquire the information they are told to acquire, they transmit it truthfully and they take the action that the organizations wants them to take, conditional on the information available. In my model, however, agents are rational, strategic players who try to maximize their payoffs in a coordination game under uncertainty. Uncertainty creates a wedge in the way agents with different locations update their beliefs and, therefore, incentives to misreport their information. In hierarchies (or trees) agents at the top are bound to receive more information than agents at the bottom, and thus, information transmission fails to be efficient.

In the real world this problem is solved in the following way. Information is acquired by lower-ranked agents who communicate it upwards to managers. These managers aggregate information and pass it back to the periphery in the form of ”recommendations” or ”commands”. Therefore, although hierarchies are efficient in terms of information handling they require some source of ”power relation” among agents in order to conveniently achieve the organizational goal. In the model presented here, however, decision-making is decentralized and thus, hierarchical information processing fails to induce truthful communication.

8 Conclusion

In this paper I have argued that modeling explicitly the information acquisition and transmission may be important to understand the functioning of many organizations and markets. I have shown how the topology of the communication network and information acquisition technology affect the quality of information transmission within the organization. I have fully characterized the set of networked information structures that support perfect communication as an equilibrium and the pattern of information acquisition they generate.

These results highlight the role of information asymmetries in communication and the way in which different network topologies generate those asymmetries endogenously. I have also shown that, whenever information revelation is not truthful, the pattern of information acquisition effort may change dramatically. For instance, in the line with sufficiently many players who hold some information, no agent can communicate truthfully with all of her neighbors. Moreover, if the information acquisition is endogenous an equilibrium may exist with truthful communication but the information acquired in equilibrium will fail to be monotonic in the centrality of the players. This results are in sharp contrast with the previous literature, which highlighted the role of centrality in the intensity of the effort.

Future research may provide a better understanding of those networks where truthful revelation fails and study the incentives of individual agents to establish communication channels with others. In this regard, example 8 suggests that imperfect communication may be preferable if it encourages information acquisition, and overcomes the public-good externality. The question remains, however, whether such networked information structures would naturally emerge if the network itself is endogenous.

A Appendix

In this Appendix I show first that there exists a Linear Equilibrium under Perfect Information Transmission as long as the Economy is large enough compared with the maximum degree of a given player. The rest of the Appendix contains omitted proofs

Lemma 13. *Assume that the network is active-regular. Then, a Linear Equilibrium exists. The weight that a given player puts on his neighbor's signal is decreasing in the amount of information he has access to and increasing in the information this player provides and in the centrality measure of his neighbor.*

Proof. The argument is standard. Assume everyone else follows a linear strategy, and let agent i have a neighborhood $N_i(g)$. He solves

$$\begin{aligned} & \min E_i\left(\theta - a_i\right)^2 + \frac{1}{n-1}\sum_{j\neq i} E_i(a_i - a_j) \\ s.t.\mathbf{E}_i(a_j) \;&=\; \sum_{k\in N^*_{ij}} b_{jk}x_k + \sum_{k\in N_{i-j}} b_{jk}E_i(x_k) \end{aligned}$$

First Order Condition is

$$a_i = \frac{1}{2}E_i(\theta) + \frac{1}{2(n-1)}\left\{\begin{array}{c} \sum_{j\in N_i(g)}\left[\sum_{k\in N_{ij}} b_{jk}x_k + E_i(\theta)\sum_{k\in N_{i-j}} b_{jk}\right] + \\ \sum_{j\notin N^*_i(g)}\left[\sum_{k\in N_{ij}} b_{jk}x_k + E_i(\theta)\sum_{k\in N_{j-i}(g)} b_{jk}\right] \end{array}\right\} \tag{8}$$

We can rewrite this expression as

$$a_i = \frac{1}{2}\left\{1 + \frac{1}{n-1}\sum_{j\neq i}\sum_{k\in N^*_{j-i}} b_{jk}\right\}E_i(\theta) + \frac{1}{2(n-1)}\sum_{j\neq i}\sum_{k\in N^*_{ij}} b_{jk}x_k \tag{9}$$

Since

$$E_i(\theta) = \sum_{k\in N^*_i(g)} \frac{\tau_k x_k}{\sum_{k\in N_i(g)}\tau_k + \tau_\sigma} \tag{10}$$

we can write

$$a_i = \sum_{k\in N^*_i(g)} b_{ik}x_k \tag{11}$$

where the vector $\mathbf{b}$ may be identified through matrix algebra. For instance, if the network is regular we have $b_{ik} = b$ for all $i, k, i \in N_i^*(k)$. Thus, letting m be the degree of the network we have

$$\begin{aligned} a_i &= mb \sum_{k \in N_i(g)} x_k \\ b &= \left[1 - \frac{K - M}{2(n-1)}\right]^{-1} \frac{1}{2} \frac{\tau}{m\tau + \tau_\sigma} \left[1 + \frac{M}{n-1}\right] \end{aligned}$$

where M is the number of links in the network and K is the number of links not in the network. More generally, we can write

$$b_{ij} = \frac{\mu_{ij}}{\sum_{j \in N_{i^*}(g)} \mu_{ij} + \mu_0} \tag{12}$$

for some non-negative weight vector μ □

Proof of Proposition 2. If $\tau_\sigma = 0$, $E[x_k \mid x_i] = x_i$ and $\sum_{j \in N_i(g)} b_{ij} = 1$ for all $i \in N$. Hence condition (4) holds. Otherwise it is needed that both $b_{ji} = b_{ii}$ and

$$\sum_{h \in N_i(g)} b_{ih} = \sum_{h \in N_{j^*}(g) \setminus i} b_{ih} \tag{13}$$

Clearly, if the networked information structure is active-regular the condition holds because $b_{ji} = b_{ii}$ and $\sum_{l \in N_j(g)} b_{jl} = \sum k \in N_i(g) b_{ik}$. Now, assume that the network is not regular so that there exists a pair $ij \in g$, $|N_i(g)| > |N_j(g)|$, To see that it never holds if the condition holds does not hold notice that b_{ji} the only source of discrepancy between players is the amount of information received. In particular, i and j agree about the degree of agent j so that there is no bias generated in asymmetric networks per se. However, if i holds more information than j

$$\frac{\tau_i}{\sum_{k \in N_j(g)} \tau_k} > \frac{\tau_i}{\sum_{k \in N_i(g)} \tau_k} \tag{14}$$

and so $b_{ii} \neq b_{ji}$. But then

$$\sum_{h \in N_i(g)} b_{ih} - \sum_{l \in N_j^*(g) \setminus \{i\}} b_{jl} \neq 0 \tag{15}$$

so that truthful revelation will not be part of any equilibrium. □

Proof of Proposition 3. For the first part, let $\tau_i = 0$ for all $i \in N \setminus J$ and $\tau_i = \tau^*$ for $i \in J$. Clearly, for all $i \in N$, $\sum_{j \in N_i(g)} \tau_i = r * \tau^*$. Further because $p_J(i,j) < \infty$, g_τ is connected. For the second part, notice that trees are minimally connected networks, so that all links are essential. Because the network is minimally connected there must exist at least one player i who is only linked to some other player j. Clearly, it must be that $\tau_j > 0$ for otherwise the network would be disconnected. If they have the same amount of information it must be that no other player k in $N_j(g) \setminus N_i(g)$ acquires information. But because the network is minimally connected and it is not a star, k must have at least some other neighbor $k' \neq j$, such that $\tau'_k > 0$. Thus, $\sum_{l \in N_j^*(g)} \tau_l < \sum_{l \in N_k^*(g)} \tau_l$ and the networked information structure is not active-regular. □

Proof of Proposition 4. A line is such that there exists exactly one path linking any two agents and such there exist two agents 1 and n who have only one link. If an agent i communicates truthfully with both of her neighbors, it must be that their residual variances are the same. Since they are communicating truthfully, the residual variance of i equals the inverse of the sum of their precisions. The residual variance of, say, $i-1$ is affected by her communication with $i-2$. If $i-2$ fails to communicate truthfully with $i-1$, it cannot be that i communicates truthfully with $i-1$ since her residual variance would be larger [16]. Notice finally that for $i-2$ to communicate truthfully with $i-1$, it must again be that their residual variances are equal. Repeating the argument in succession we arrive to agent 1 who does not have additional links. Thus, 1 and 2 will not communicate truthfully and the result follows. □

Proof of Proposition 5. The if part is trivial. For the only if part notice that, in a truthfully revealing equilibrium, information acquisition effort is a public good. Because the technology is linear, Theorem 1 in Bramoulle and Kranton (2007) implies that no two individuals who acquire information may be linked to each other. By Proposition 2, no individual can be linked to more than one informed agent if one of its neighbors is linked to only one. Thus, the result obtains. □

Proof of Proposition 7. By Proposition 4, if the network induces a truthful-revelation equilibrium, every agent must have access to at most one signal. Among all networks of this class, the star avoids wasteful duplication and improves coordination. Thus, it cannot be dominated. To see the bound on the payoff notice that, for the individual who acquires information, the

[16] all that matters is that the updating of the posterior that $i-1$ makes will always be different from $i's$ posterior because the information revealed would be a partition of the real line

marginal variance reduction, $V'(\tau)$ equals the cost of acquiring information c. In particular

$$V'(\tau^*) = \frac{1}{\tau^2} = c \tag{16}$$

Thus, $\tau^* = \frac{1}{\sqrt{c}}$. Since all agents would agree on the same action, we get

$$W = -V(\tau^*) - \frac{1}{N} c\tau^* = \sqrt{c} \tag{17}$$

□

Proof of Proposition 10. There are two cases. First assume that no information is withheld. Then, assume for a contradiction that information revelation is perfect. Then, it must be the case that every two individuals obtain the same amount of information at time t_i^* where t_i^* is such that i takes her action at t_i^*. Notice that in a pure strategy equilibrium t_i^* is deterministic (in particular, it does not depend on the realizations of the signals). Clearly, 1 should leave in the same period as 2, since in the following period 1 will not receive new information[17]. However, at the period in which 2 leaves, if optimal, he shall get at least one more signal. Hence, 1 is always less informed than 2 and results in Proposition 3 apply.

It is also straightforward to see that every agent leaving at period $t = 1, 2..., \bar{t}$ it is an equilibrium, provided sufficiently many signals are obtained in each round. In particular $\bar{t}$ would be the earliest period at which the value of two additional signals (on top of $2(\bar{t}-1)+1$) to the most central agent is lower than δ if that period is lower than $\frac{n+1}{2}$ and $\bar{t} = \frac{n+1}{2}$ otherwise. In this later case, all information would eventually spread through the network.

To conclude, I show that withholding of information does not change the results . First notice that withholding information to j for less than t_j^* periods is never optimal (i would just reduce his own influence on other players obtaining the same amount of information.) Now, suppose that agent i conceals his information until period t_j^* - that is, the period at which j leaves, and assume that i and j have access to the same information, then I show that $i-1$ must have access to less information than them. If i and j have access to the same information and $t = t_j^*$ they both leave. Then, if $i-1$ is to have the same amount of information as them, it must be that he receives a report of the same precision at period t_j^* (or later), and then leave. However, in the line, this requires that there exists another agent

[17]If the equilibrium involves mixed strategies, the strategy of player 1, conditional on observing that player 2 left is to leave in the following period, but no more information is revealed to him.

$i - t_j^*$ who originated that report and now gets to $i-1$. Now, if that is the case, then at period t_j^*, j must have received another report coming from agent $i - 1 - t_j^*$, and thus, j has access to more information than i. □

Proof of Proposition 11 . The idea is similar as in Proposition 10. Suppose the result does not hold. Then, there exists $j \in N_i(g)$ such that i can communicate truthfully with j. We know that it must be the case that their residual variances are equal, and thus they have received the same number of signals. Since i's neighborhood is a subset of that of j, this can happen if and only if in the last round of communication whatever j learns also i does. Hence, it must be that j i) does not receive information that was not held by another agent in the neighborhood of i in the previous period and ii) decides not to leave until he gets to that stage. If δ is sufficiently large, he will leave before. If δ is sufficiently small, however, he will stay until all information is received. Since this must happen for all $j \in N_i(g)$ in order for i to communicate truthfully, it must be the case, that at time $t_i^* = t_j^*$ for all $j \in N_i(g)$, no new information arrives to the neighborhood of i so that all information must be aggregated before everyone leaves, thus establishing the claim. □

Proof of Proposition 12. First notice that we do not have to vary the pattern of information acquisition, only the links between agents. Take a network g and a (truthful) equilibrium for that network, the result is established by constructing g' such that incentives are unaltered in the equivalent static game. First of all, it is obvious that if a given agent i eventually obtains information generated at node $k,$ then $ik \in g'$. The converse is also true, so that if i does not obtain information generated at node k', $ik' \notin g'$. This defines the only candidate for g'. Notice that the equilibrium is truthful, and so apply Proposition 7. The result follows directly. □

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