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Jel codes: G00, G30, G31, B40, B41, C00, M2, M4, G12, D46, A12.

A caveat before reading the paper:

As written in the paper, equations (1.3) and (1.6) are consistent each other: Equation (1.3) is the standard way of formally translating the notion of residual income, while equation (1.6) (the EVA model) is just a particular instantiation of equation (1.3). There is no need of using equation (1.8), which is actually incorrect (unless book value of assets coincide with market value of assets). Given that this trivial misprint is totally irrelevant for all arguments, statements, and results in the paper, you may conveniently skip it.

Feel free to contact me (magni@unimo.it).

Zelig and the Art of Measuring Excess Profit

Carlo Alberto Magni[1]

Abstract

This paper tells the story of a student of economics and finance who meets a couple of alleged psychopaths, suffering from the 'syndrome of Zelig', so that they think of themselves to be experts of economic and financial issues. While speaking, they come across the concept of excess profit. The student tells them that the formal way to translate excess profit is to apply Stewart's (1991) EVA model and shows that this model is equivalent to Peccati's (1987, 1991, 1992) decomposition model of a project's Net Present (Final) Value. The 'Zeligs' listen to him carefully, then try to apply themselves the EVA model: Unfortunately, both She-Zelig and He-Zelig seem to feel uneasy with basic mathematics, so they make some mistakes. Consequently, each of them miscalculates the excess profit. Strangely enough, they make different mistakes but both get to the (correct) Net Final Value of the project and, in addition, their excess profits do coincide. Further, the (biased) models presented by the Zeligs, though different from the EVA model, seem to bear strong relations to the latter. The student is rather surprised.
I give my version of this event, arguing that the Zeligs are offering us a rational way of measuring excess profit, alternative to EVA but equally valuable. As I see it, they are only adopting a different cognitive interpretation of the concept of excess profit, which is based on a counterfactual conditional that differs from Stewart's and Peccati's.

Keywords: Excess profit, Economic Value Added, Net Final Value, Systemic Value Added, Counterfactual.

JEL Classification: G00, G30, G31.

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Zelig and the Art of Measuring Excess Profit

1 - Introduction

Excess profit (henceforth, often residual income or economic profit) is an economic concept used in both theoretical economics and finance, and is strictly connected with the concept of Net Present Value (NPV) or Net Final Value (NFV): Actually the problem of formally translating such a concept for evaluation purposes is equivalent to the problem of decomposing a Net Present Value (NPV) in periodic shares. As for the former, Stewart (1991) introduces the EconomicValue Added for valuing firms (or projects) and as a tool for rewarding managers; as for the latter, Peccati (1987, 1991, 1992) introduces a disaggregation model of a project's NPV (NFV) for valuing the periodic performance of the project (see also Luciano and Peccati, 1987, for decomposition under uncertainty).

In this paper I report the story of a student of economics and finance who meets two alleged psychopaths suffering from the 'syndrome of Zelig': 'Zeligs' are ill-minded people, inclined to take on the personality of the person they are dealing with. Although they are nearly always unacquainted with the matter they are disputing on, they have so intense an enthusiasm toward the matter they are coping with and so absolute a lack of cultural and social inhibitions, that their raving ideas often give new insights on the subject. Unfortunately, their way of communicating their ideas is not always clear. Not enough, they enjoy making fun of people so they conceal their ideas in seemingly irrational arguments. Meeting the student, the two Zeligs of this story gradually take on his personality until they believe themselves to be students concerned with economic and financial issues. Coming across the concept of excess profit (and the related problem of evaluating the periodic performance of a project) the student presents the above mentioned models by Peccati and Stewart. After having listened carefully to him, the Zeligs try to repeat the EVA procedure. Due to their mental illness, they make some mistakes. I consider it rather interesting to report their mistakes, for they seem to offer us a new way of thinking of the notion of residual income.

The paper is organized as follows. In section 1 the student explains Peccati's and Stewart's models, showing that they formally translate the concept of excess profit and are equivalent. In section 2 the two Zeligs try to repeat the procedure they have just learned but make irreparable mistakes. Their incorrect procedure is applied to a concrete case in section 3. In section 4 I give my version of the alleged psychopaths' lucubrations showing that

they are offering us a rational tool for measuring residual income: Only, they are adopting a different cognitive perspective and shaping the evaluation process so that their procedure looks like an EVA model full of errors. I conclude the paper with some remarks dealing on the counterfactual features of the notion of economic profit.

2 - Peccati's and Stewart's models

In this section I summarize the presentation of Peccati's and Stewart's models made by the student. Suppose a decision maker aims at evaluating a project *P* that generates equidistant cash flows $a_s \in R$ at time *s*=0, 1, … , *n*. All flows are certain. The investor aims at evaluating the periodic performance of the project. The cash flows released by project *P* are withdrawn from (if negative) and invested in (if positive) an account, say *C*, whose value C_s at time *s* evolves according to the following recurrence equation:

$$C_s = C_{s-1}(1+i) - a_s \qquad s = 1, 2, \ldots, n$$

where *i* is the so-called *opportunity cost of capital*. Following Peccati (1991) let us assume $C_0 = E_0$ where E_0 denotes the value of the evaluator's net worth at time 0, $E_0 \in R$. The decision maker faces two alternative courses of action:

(i) to invest in project *P*
(ii) to keep her wealth in account *C*.

Let us denote with E_s and E^s, $s \geq 1$, the net worth at time *s* for case (i) and (ii) respectively. Let us define the Net Final Value of *P* as the difference

$$\text{NFV} = E_n - E^n = (E_n - E_0) - (E^n - E_0)$$

which implies

$$\text{NFV}=(E_0+a_0)(1+i)^n+\sum_{s=1}^{n}a_s(1+i)^{n-s}-E_0(1+i)^n=\sum_{s=1}^{n}a_s(1+i)^{n-s}. \tag{1.1}$$

The NPV is obtained by discounting (1.1) at present time. We aim then at decomposing (1.1) in *n* shares G_s so that $G_1+\ldots+G_n=\text{NFV}$. The *outstanding capital* (or *project balance*) of *P* at time *s* at the rate *j* is given by

$$\begin{aligned} w_0 &= -a_0 \\ w_s(j) &= w_{s-1}(1+j)-a_s \qquad\qquad s=1,2,\ldots,n. \end{aligned} \tag{1.2}$$

Assume that *P* belongs to the class of Soper (1959), and let *x* be the internal rate of return. Following Peccati's argument, we focus on a generic period *s*: The investor places the sum $w_{s-1}(x)$ at the beginning of the period and receives $w_s(x)+a_s$ at the end of the period. The gain is $xw_{s-1}(x)$. So doing, she gives up the opportunity of investing $w_{s-1}(x)$ at the return rate *i*, which means that she foregoes the gain $iw_{s-1}(x)$. The latter is then an opportunity cost (a foregone return), and the sum $w_{s-1}(x)(x-i)$ is then the residual income in period *s*, that is the difference between what the investor earns by choosing *P* and what she would earn should she decide to keep funds in *C*. As each such share is money referred to time *s*, we must compound to time *n* before we can sum all shares. We have then

$$G_s=w_{s-1}(x)(x-i)(1+i)^{n-s}. \tag{1.3}$$

In such a way, the model meets both the requirement of finding periodic values for project *P* being significant from an economic point of view (they measure the differential income of period *s*) and the requirement of aggregating these values so as to obtain the NFV (which is the overall excess profit).

If the project is levered (i.e. it is partly financed with debt) an analogous argument is applied so that G_s becomes

$$G_s = \left(w_{s-1}(x)(x-i) + D_{s-1}(\delta)(i-\delta)\right)(1+i)^{n-s} \tag{1.4}$$

where δ is the debt rate and

$$\begin{aligned} D_0 &= f_0 \\ D_s(j) &= D_{s-1}(1+j) + f_s \end{aligned} \tag{1.5}$$

is the outstanding debt at the rate *j*, with $f_s \in R$ denoting the debt's cash flows.

Let us now turn to Stewart's model. The Economic Value Added is a profitability index introduced by Stewart in order to provide a tool for evaluating (projects and) firms as well as for evaluating and compensating managers. The basic objective of EVA is to create a measure of periodic performance based on the concept of excess profit: "Recognized by economists since the 1770s, residual income is based on the premise that, in order for a firm to create wealth for its owners, it must earn more on its total capital invested than the cost of that capital" (Biddle, Bowen and Wallace, 1999, p. 70). To compute it, we calculate the firm's (or project's) total cost of capital, given by the product of the Weighted Cost of Capital (WACC) and the total capital invested (TC). Then the total cost of capital is subtracted from the Net Operating Profit After Taxes (NOPAT). Notationally, we have, for period *s*,

$$\mathrm{EVA}_s = \mathrm{NOPAT} - \mathrm{WACC} * \mathrm{TC} \tag{1.6}$$

where subscripts are omitted for convenience. Summing for *s* and discounting at time 0 (or compounding at time *n*) at a given rate i' we obtain the overall residual income, which Stewart calls
Market Value Added (MVA):

$$\mathrm{MVA} = \sum_{s=1}^{n} \frac{\mathrm{EVA}_s}{(1+i')^s}.$$

It is easy to show that (1.6) is formally equivalent to (1.4). In fact, (1.6) can be rewritten as

$$\mathrm{EVA}_s = \mathrm{ROA} * \mathrm{TC} - \frac{\mathrm{ROD} * \mathrm{Debt} + i * \mathrm{Equity}}{\mathrm{Debt} + \mathrm{Equity}} * \mathrm{TC} \qquad (1.7)$$

whence

$$\begin{aligned} \mathrm{EVA}_s &= \mathrm{ROA} * \mathrm{TC} - \mathrm{ROD} * \mathrm{Debt} - i * (\mathrm{TC} - \mathrm{Debt}) \\ &= \mathrm{TC}(\mathrm{ROA} - i) + \mathrm{Debt}(i - \mathrm{ROD}) \end{aligned} \qquad (1.8)$$

where ROA is the Return on Assets, i is the cost of equity, ROD is the Return on Debt.[2] Applying Stewart's argument to project P, we have TC= $w_{s-1}(x)$, ROA=x, Debt= $D_{s-1}(\delta)$, ROD= δ, and the relation between (1.4) and (1.6) is straightforward:

$$G_s = \mathrm{EVA}_s (1+i)^{n-s}.$$

If $i' = i$ the overall residual income in Stewart's model (MVA) coincides with Peccati's representation of the NPV:

$$\mathrm{NPV} = \frac{\mathrm{NFV}}{(1+i)^n} = \frac{1}{(1+i)^n} \sum_{s=1}^{n} G_s = \sum_{s=1}^{n} \mathrm{EVA}_s (1+i)^{-s} = \mathrm{MVA}$$

The equivalence of the models vanishes only in discounting each EVA_s: Stewart uses the Weighted Average Cost of Capital (i'=WACC), whereas Peccati picks $i' = i$.[3]

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[2] It is here assumed that k_d=ROD, i.e. cost of debt equals return on debt or, which is the same, the market value of debt equals the book value of debt.

[3] The discussion among our three evaluators has not dealt with this delicate issue. However, if we assume that all cash flows are certain, then i'=WACC=i= r_f and the two models coincide. The reader can refer to Peccati (1996) against the use of the WACC. Further, taking the point of view of an equity-holder, we have EVA_s = Profit After Taxes $- i *$ Equity, which amounts to Peccati's numerator (see Magni, 2005, for further relations between the two models, and Fernández, 2002, for the related notion of Total Shareholder Return).

3 - The Zeligs' Mistakes

As we have seen, Peccati and Stewart (henceforth PS) offer equivalent translations of the notion of residual income, that is they offer different sides of the same medal. The Zeligs illustrate the procedure they have just learned. They firstly begin with the case D_s=0 for all *s*, then generalize their arguments to include debt.

3.1 She-Zelig

Following PS's model, the residual income of an unlevered project is given by

$$\mathrm{EVA}_s = w_{s-1}(x)(x-i) = xw_{s-1}(x) - iw_{s-1}(x) \tag{2.1.1}$$

where $iw_{s-1}(x)$ represents the foregone return, the well-known opportunity cost. But She-Zelig is so unacquainted with finance and mathematics that she misunderstands the concept of opportunity cost. She is convinced that the foregone return is $iw_{s-1}(i)$, i.e. she replaces *x* with *i*. She then computes project *P*'s differential income, unaware of such a mistake. Denoting with EVA_s^S her vitiated Economic Value Added,[4] she calculates

$$\mathrm{EVA}_s^S = xw_{s-1}(x) - iw_{s-1}^S \tag{2.1.2}$$

where we let $w_{s-1}^S := w_{s-1}(i)$. Afterwards, our evaluator assumes that the project is partly financed by a loan contract whose cash flows are f_s with δ being the contractual rate. In this case, we know that we should have

$$\begin{aligned}\mathrm{EVA}_s &= w_{s-1}(x)(x-i) + D_{s-1}(\delta)(i-\delta) \\ &= x\,w_{s-1}(x) - \delta D_{s-1}(\delta) - i(w_{s-1}(x) - D_{s-1}(\delta))\end{aligned} \tag{2.1.3}$$

[4] *S*=She-Zelig, *H*=He-Zelig.

where $i(w_{s-1}(x) - D_{s-1}(\delta))$ is the opportunity cost. Again, She-Zelig seems to fall prey to hallucinations and thinks that the opportunity cost is $i(w_{s-1}(i) - D_{s-1}(i))$, i.e. she replaces both *x* and δ with *i*, so that her 'hallucinated' EVA is now

$$\mathrm{EVA}_s^S = x\, w_{s-1}(x) - \delta D_{s-1}(\delta) - i(w_{s-1}^S - D_{s-1}^S) \qquad (2.1.4)$$

where we let $D_{s-1}^S := D_{s-1}(i)$. She-Zelig is willing to verify whether such a result is consistent with the Net Final Value, as it should. Due to her liberty of conscience, typical of Zeligs, she refuses to follow basic rules of financial calculus, and does not compound the shares so found, but sum all of them as such. Strangely enough, she finds that

$$\sum_{s=1}^{n} \mathrm{EVA}_s^S = \mathrm{NFV}\,.$$

She is therefore satisfied and reinforced in her conviction that she is doing right.

3.2 He-Zelig

He-Zelig too tries to expound PS's model. He states that he will measure the residual income of *P* using PS's arguments but the student notices that He-Zelig does not correctly recognize *P*'s cash flows. Our alleged psychopath is actually convinced that *P* consists of the sequence

$$P^H = (a_0^H, a_1^H, \ldots, a_n^H)$$

where $a_0^H = a_0$ and

$$a_s^H = a_s + \sum_{k=0}^{s-1} a_k[\Phi_k(i) - \Phi_k(x)] \qquad s \geq 1\,. \qquad (2.2.1)$$

with

$$\Phi_k(y) := y(1+y)^{s-1-k} \qquad k \le s-1.$$

Not enough, he casts a sidelong glance at what his mate is doing with the same evaluation process. He realizes that she replaces *x* with *i* in expressing the opportunity cost and he finds it a commendable idea. So he takes $w_{s-1}(i)$ instead of $w_{s-1}(x)$. Letting now $w_{s-1}^H := w_{s-1}(i)$ he accomplishes then (what the student thinks is) a correct argument with incorrect values: "Let us focus on period *s*," He-Zelig says "The capital invested in the project at the beginning of the period is w_{s-1}^H, and at the end of the period I will receive the sum $w_s^H + a_s^H$". Denote with x_s^H the internal rate of return of this one-period project. This means, for all *s*,

$$w_{s-1}^H(1+x_s^H) = w_s^H + a_s^H \tag{2.2.2}$$

or, which is the same,

$$w_s^H = w_{s-1}^H(1+x_s^H) - a_s^H \tag{2.2.3}$$

So w_s^H is the project balance at time *s*. "In period *s*," he argues "I invest money (w_{s-1}^H) at the rate x_s^H and so doing I forego the return iw_{s-1}^H." Then the residual income he finds, i.e. his 'hallucinated' Economic Value Added, is

$$\mathrm{EVA}_s^H = x_s^H\, w_{s-1}^H - i\, w_{s-1}^H = w_{s-1}^H(x_s^H - i). \tag{2.2.4}$$

Afterwards, he assumes that the project is partly financed by a loan contract whose cash flows are f_s, the contractual rate being δ. Again, he falls prey to his illness and does not recognize the debt's cash flows. He is convinced that the cash flows are $f_0^H = f_0$ and

$$f_s^H = f_s + \sum_{k=0}^{s-1} f_k [\Phi_k(i) - \Phi_k(\delta)] \qquad s \geq 1. \qquad (2.2.5)$$

As before, he glances his mate's evaluation and considers it a good idea to replace $D_{s-1}(\delta)$ by $D_{s-1}(i)$ .Letting $D_{s-1}^H := D_{s-1}(i)$, the period rate of cost for the loan contract is then that rate δ_{s-1}^H such that

$$D_{s-1}^H (1 + \delta_s^H) = D_s^H - f_s^H \qquad (2.2.6)$$

so that

$$D_s^H = D_{s-1}^H (1 + \delta_s^H) + f_s^H . \qquad (2.2.7)$$

Then he argues as before while taking into account that, at time $s-1$, he invests $w_{s-1}^H - D_{s-1}^H$, whereby he receives, at time s, the sum $w_s^H - D_s^H + a_s^H + f_s^H$. "With such a levered project," he thinks "I forego the return $i(w_{s-1}^H - D_{s-1}^H)$." Hence, the excess profit is now

$$\begin{aligned} \mathrm{EVA}_s^H &= x_s^H \, w_{s-1}^H - \delta_s^H D_{s-1}^H - i(w_{s-1}^H - D_{s-1}^H) \\ &= w_{s-1}^H (x_s^H - i) + D_{s-1}^H (i - \delta_s^H). \end{aligned} \qquad (2.2.8)$$

"This guy is a raving lunatic," the student thinks. He-Zelig is then willing to verify that such a residual income is consistent with the Net Final Value of the project. As his mate, he does not trust basic rules of financial calculus and sums all residual incomes with no compounding. Strangely enough, he finds that

$$\sum_{s=1}^{n} \mathrm{EVA}_s^H = \mathrm{NFV}.$$

"Well done" he says "There is no mistake in my evaluation," so he is reinforced in his conviction that he is doing right.

4 - Rational and Irrational EVAs

Our Zeligs decide to show the student the application of their own procedures to a levered project consisting of the sequence (−100, 30, 38, 84) along with the debt's cash flows (40, −26, −3, −23) at time 0, 1, 2, 3 respectively. The internal rate of return is x=0.2 and the debt's rate is δ=0.15. The net cash flows are then (−60, 4, 35, 61). Assuming i=0.1 the NFV of the levered project is

$$\text{NFV} = -60(1.1)^3 + 4(1.1)^2 + 35(1.1) + 61 = 24.48.$$

Let us then follow the Zeligs in their calculations. The data collected by She-Zelig are:

$$\begin{aligned} w_0(x) &= 100 & D_0(x) &= 40 \\ w_1(x) &= 100(1.2) - 30 = 90 & D_1(x) &= 40(1.15) - 26 = 20 \\ w_2(x) &= 90(1.2) - 38 = 70 & D_2(x) &= 20(1.15) - 3 = 20 \\ w_3(x) &= 70(1.2) - 84 = 0 & D_3(x) &= 20(1.15) - 23 = 0 \end{aligned}$$

$$\begin{aligned} w_0^S &= 100 & D_0^S &= 40 \\ w_1^S &= 100(1.1) - 30 = 80 & D_1^S &= 40(1.1) - 26 = 18 \\ w_2^S &= 80(1.1) - 38 = 50 & D_2^S &= 18(1.1) - 3 = 16.8 \\ w_3^S &= 50(1.1) - 84 = -29 & D_3^S &= 16.8(1.1) - 23 = -4.52 \end{aligned}$$

In the meanwhile, the diligent student computes the excess profit by applying (2.1.3):

$$\mathrm{EVA}_1 = 0.2 * 100 - 0.15 * 40 - 0.1(100 - 40) = 8$$
$$\mathrm{EVA}_2 = 0.2 * 90 - 0.15 * 20 - 0.1(90 - 20) = 8$$
$$\mathrm{EVA}_3 = 0.2 * 70 - 0.15 * 20 - 0.1(70 - 20) = 6$$

and finds back the NFV by just compounding each EVA_s and then summing for *s*:

$$\mathrm{EVA}_1(1.1)^2 + \mathrm{EVA}_2(1.1) + \mathrm{EVA}_3 = 8(1.1)^2 + 8(1.1) + 6 = 24.48 = \mathrm{NFV}$$

Conversely, She-Zelig applies her (2.1.4), so that

$$\mathrm{EVA}_1^S = 0.2 * 100 - 0.15 * 40 - 0.1(100 - 40) = 8$$
$$\mathrm{EVA}_2^S = 0.2 * 90 - 0.15 * 20 - 0.1(80 - 18) = 8.8$$
$$\mathrm{EVA}_3^S = 0.2 * 70 - 0.15 * 20 - 0.1(50 - 16.8) = 7.68.$$

She is even more convinced of such a result when she sums the three shares with no compounding process:

$$\mathrm{EVA}_1^S + \mathrm{EVA}_2^S + \mathrm{EVA}_3^S = 8 + 8.8 + 7.68 = 24.48 = \mathrm{NFV}.$$

As for He-Zelig, he uses the following values:

$$w_0^H = 100 \quad (= w_0^S) \qquad D_0^H = 40 \quad (= D_0^S)$$
$$w_1^H = 80 \quad (= w_1^S) \qquad D_1^H = 18 \quad (= D_1^S)$$
$$w_2^H = 50 \quad (= w_2^S) \qquad D_2^H = 16.8 \quad (= D_2^S)$$
$$w_3^H = -29 \quad (= w_3^S) \qquad D_3^H = -4.52 \quad (= D_3^S).$$

He now needs find, for all *s*, the values of x_s^H and δ_s^H. To this end, he uses (2.2.3) and (2.2.6) so that

$$x_s^H = \frac{w_s^H + a_s^H}{w_{s-1}^H} - 1 \tag{3.1}$$

$$\delta_s^H = \frac{D_s^H - f_s^H}{D_{s-1}^H} - 1. \tag{3.2}$$

Using (2.2.1) and (2.2.5), eqs. (3.1) and (3.2) boil down to

$$\begin{aligned} x_s^H &= \frac{w_{s-1}^H(1+i) - a_s + a_s + \sum_{k=0}^{s-1} a_k\left[\Phi_k(i) - \Phi_k(x)\right]}{w_{s-1}^H} - 1 \\ &= i + \frac{\sum_{k=0}^{s-1} a_k\left[\Phi_k(i) - \Phi_k(x)\right]}{w_{s-1}^H} \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} \delta_s^H &= \frac{D_{s-1}^H(1+i) + f_s - f_s - \sum_{k=0}^{s-1} f_k\left[\Phi_k(i) - \Phi_k(\delta)\right]}{D_{s-1}^H} - 1 \\ &= i + \frac{\sum_{k=0}^{s-1} f_k\left[\Phi_k(\delta) - \Phi_k(i)\right]}{D_{s-1}^H} \end{aligned} \tag{3.4}$$

respectively. Finally, using (3.3) and (3.4) he gets to

$$x_1^H = 0.1 + \frac{10}{100} = 0.2 \qquad \delta_1^H = 0.1 + \frac{2}{40} = 0.15$$

$$x_2^H = 0.1 + \frac{10}{80} = 0.225 \qquad \delta_2^H = 0.1 + \frac{1.2}{18} = 0.16666$$

$$x_3^H = 0.1 + \frac{9}{50} = 0.28 \qquad \delta_3^H = 0.1 + \frac{1.32}{16.8} = 01785714.$$

He is now ready to apply (2.2.8):

$$\mathrm{EVA}_1^H = 100(0.2 - 0.1) + 40(0.1 - 0.15) = 8$$
$$\mathrm{EVA}_2^H = 80(0.225 - 0.1) + 18(0.1 - 0.1667) = 8.8$$
$$\mathrm{EVA}_3^H = 50(0.28 - 0.1) + 16.8(0.1 - 0.1785714) = 7.68,$$

then sums the three shares with no compounding:

$$\mathrm{EVA}_1^H + \mathrm{EVA}_2^H + \mathrm{EVA}_3^H = 8 + 8.8 + 7.68 = 24.48 = \mathrm{NFV},$$

and rubs his hands. The student is rather surprised: "This is a very whimsical result," he says. The Zeligs remark the student that, for all *s*, their EVAs coincide:

$$\mathrm{EVA}_s^S = \mathrm{EVA}_s^H$$

They ask the student: "What about this?" and he answers: "You have applied the EVA model in an absurd way. Your economic profits differ from the correct ones. Yet, I do not understand why your
economic profits coincide and why you actually get to the NFV. All this is so irrational." The Zeligs just reply: "No, this is just what you should have expected. There is no objective excess profit at all."

5 - Systemic Value Added

In this section I would like to expose my opinion on this episode. I am convinced that the Zeligs are far from being irrational evaluators; they are

only adopting a different interpretation of the notion of excess profit. I will unmask the way of reasoning the two have followed and describe the perspective from which their seemingly irrational EVAs have been drawn. We will be left with a sound index, which I shall name *Systemic Value Added* (SVA). I will show that the SVA is a rational tool for measuring residual income, with a significant economic meaning and capable of acting as an alternative to the EVA. As we shall see, the two Zeligs' irrational EVAs are just two ways of computing the SVA. Let us now begin:

The evaluation process starts at time 0, when two lines of action are compared:

(i) undertaking the project
(ii) investing funds at the rate *i*.

If we regard the investor's wealth as a dynamic system, we have that (i) and (ii) give rise to different paths of the system. As for (i) at time *s* the net worth E_s can be seen as structured in three accounts: The account *C*, whose value is denoted by C_s; the project, whose value is the outstanding capital $w_s(x)$; and the loan contract, whose outstanding debt is $D_s(\delta)$; if (ii) is instead chosen, the decision maker's wealth E^s at time *s* will be composed of the only account *C*, whose value I denote with C^s, given by C^0 plus the interest yielded at the rate *i*. Therefore, the following recurrence equations hold:[5]

$$\begin{aligned} C_0 &= E_0 - w_0 + D_0 \\ w_0 &= -a_0 \\ D_0 &= f_0 \\ C_s &= C_{s-1}(1+i) + a_s + f_s && s \geq 1 \\ w_s(x) &= w_{s-1}(x)(1+x) - a_s && s \geq 1 \\ D_s(\delta) &= D_{s-1}(\delta)(1+\delta) + f_s && s \geq 1 \end{aligned} \tag{4.1}$$

[5] As for the value of C at time 0, the relation shown is to be intended ex post (after a_0 has been spent). Prior to investment, the value of C equals $E_0 + D_0$.

for (i) and

$$\begin{aligned} C^0 &= E_0 \\ C^s &= C^{s-1}(1+i) \qquad s \geq 1 \end{aligned} \tag{4.2}$$

for (ii). Graphically, we can conveniently depict the situations by means of double-entry sheets where uses and sources of funds are pointed out: At time *s* we have

Uses	Sources
C_s	$D_s(\delta)$
$w_s(x)$	E_s

(4.3)

for (i),

Uses	Sources
C^s	E^s

(4.4)

for (ii). We have $E_s + D_s(\delta) = C_s + w_s(x)$ and $E^s = C^s$ for all *s*. Using (4.1) and (4.2) the two alternative dynamic systems are then expressed by the following recurrence equations:

$$\begin{aligned} E_s &= E_{s-1} + xw_{s-1}(x) - \delta D_{s-1}(\delta) + iC_{s-1} \\ E^s &= E^{s-1} + iE^{s-1} \end{aligned} \tag{4.5}$$

for cases (i) and (ii) respectively. The second addends of both equalities in (4.5) represent the incomes in period *s* for cases (i) and (ii).[6] This means that for period *s* we have two alternative incomes, depending on the choice made. The excess profit is the incremental profit of (i) over (ii), that is the difference between the two alternative profits drawn from the alternative dynamic systems. In other words, $E_s - E_{s-1}$ is the periodic gain associated with case (i), $E^s - E^{s-1}$ is the gain associated with case (ii). I define *Systemic Value Added* (SVA_s) the differential gain

$$\mathrm{SVA}_s = (E_s - E_{s-1}) - (E^s - E^{s-1}). \tag{4.6}$$

Summing for *s* we obtain what I shall name the *overall Systemic Value Added* (SVA), which is just the NFV of the project seen with our 'systemic eyes':

$$\begin{aligned} \mathrm{SVA} &= \sum_{s=1}^{n} \left((E_s - E_{s-1}) - (E^s - E^{s-1}) \right) \\ &= E_n - E^n = \mathrm{NFV}. \end{aligned} \tag{4.7}$$

PS's models and the SVA model give rise to different partition. From (4.5) and (4.6) we have

$$\mathrm{SVA}_s = xw_{s-1}(x) - \delta D_{s-1}(\delta) - i(C^{s-1} - C_{s-1}) \tag{4.8}$$

so that

$$\mathrm{EVA}_s \neq \mathrm{SVA}_s$$

as well as

$$G_s \neq \mathrm{SVA}_s .$$

[6] The terms profit, gain, return, interest, income are to be intended as synonyms.

The SVA model is grounded on a *systemic way of reasoning*: The net worth is a *system* structured in accounts whose value evolves diachronically following different laws. The algebraic sum of the accounts constitutes the value of the whole net worth. This enables us to avoid compounding, whereas PS's model rests on the concept of Net Final (Present) Values and on capitalization processes. In a sense, by using a systemic perspective we are able to sum cash regardless of its maturity. This result, far from being illicit, suggests that we can create a cognitive outlook where there is no need of capitalization factors: Time dimension is grasped by means of the system's diachronic evolution.

5. Zeligs' EVA and SVA

In the light of what we have seen, the Zeligs are rational agents following a systemic outlook for measuring excess profit. To show it, consider She-Zelig's EVA in (2.1.4), where $\left(w_{s-1}(x) - D_{s-1}(\delta)\right)$ is replaced by $\left(w_{s-1}^{S} - D_{s-1}^{S}\right)$. Just note that

$$w_{s-1}^{S} = -\sum_{k=0}^{s-1} a_k (1+i)^{s-1-k} \tag{5.1}$$

$$D_{s-1}^{S} = \sum_{k=0}^{s-1} f_k (1+i)^{s-1-k} \tag{5.2}$$

whence, using (4.1) and (4.2),

$$C^{s-1} - C_{s-1} = w_{s-1}^{S} - D_{s-1}^{S}.$$

Therefore, we have, from (4.8),

$$\mathrm{SVA}_s = \mathrm{EVA}_s^{S}.$$

As for He-Zelig, he is an arrant rascal, for he just disguises the SVA_s as an Economic Value Added. Using (5.1) and (3.3), we have[7]

$$\begin{aligned} x_s^H &= i + \frac{\sum_{k=0}^{s-1} a_k \left[\Phi_k(i) - \Phi_k(x)\right]}{w_{s-1}^H} \\ &= i + \frac{x w_{s-1}(x) - i w_{s-1}^H}{w_{s-1}^H} \\ &= x \frac{w_{s-1}(x)}{w_{s-1}^H}. \end{aligned} \tag{5.3}$$

Analogously, using (5.2) and (3.4) we easily get to

$$\delta_s^H = \delta \frac{D_{s-1}(\delta)}{D_{s-1}^H}. \tag{5.4}$$

Hence, simple substitutions provide us with

$$\mathrm{EVA}_s^H = \mathrm{EVA}_s^S$$

whence

$$\mathrm{SVA}_s = \mathrm{EVA}_s^H.$$

The Zeligs are then *systemic*-minded agents: They translate the concept of excess profit by adopting a systemic perspective. Such a perspective is not, in my opinion, less significant than the one subsumed by the EVA model, it is just different. But they do not only offer us a new way of treating the concept of excess profit; by showing us their seemingly irrational

[7]Remember that $w_{s-1}^S = w_{s-1}^H$ and $D_{s-1}^S = D_{s-1}^H$.

EVAs they are trying to communicate us some intriguing relations between the SVA model and the EVA model. To investigate the issue thoroughly, I have interviewed the two Zeligs some time ago. I herewith report a summary of what they told me that day.

She-Zelig: "I aim at measuring economic profit for project *P*. Speaking in differential terms, at time *s* my net return from the project is $xw_{s-1}(x) - \delta D_{s-1}(\delta)$. What is the foregone return (the so-called opportunity cost)? Well, if I select opposite case (ii), my account *C*'s value at time *s* will be reduced by the sum $C^{s-1} - C_{s-1}$ $(= w^S_{s-1} - D^S_{s-1})$ with respect to the alternative case. Such value is the sum to which I renounce in order to receive the just mentioned return from the project. On this sum I would have earned interest at the rate *i*. Hence, the foregone return in period *s* is $i(C^{s-1} - C_{s-1})$ $(= i(w^S_{s-1} - D^S_{s-1}))$. What am I doing then? I am just calculating a modified Economic Value Added where the foregone return is not the project's outstanding capital times the rate *i* but the latter times the differential value of account *C*."

He-Zelig: "I aim at measuring economic profit for project *P*. The net profit is $xw_{s-1}(x) - \delta D_{s-1}(\delta)$ and, like my mate, I regard as foregone return the differential gain $i(C^{s-1} - C_{s-1})$ $(= i(w^H_{s-1} - D^H_{s-1}))$. Since $xw_{s-1}(x) - \delta D_{s-1}(\delta) = x^H_s w^H_{s-1} - \delta^H_s D^H_{s-1}$ the systemic perspective I rely on leads me to an Economic Value Added of a 'shadow' project whose net cash flows are $a^H_s + f^H_s$. In other words, in period *s* I can invest capital $w_{s-1}(x)$ at the rate x^H_s financing such an investment by borrowing D^H_{s-1} at the rate δ^H_s. In this case that part of the project which is equity-financed is only $w^H_{s-1} - D^H_{s-1}$. The latter sum could alternatively be invested in asset *C*, whose rate of return is *i*. What am I doing then? I am just calculating the EVA of a 'shadow' project."

On the basis of these words, we can say that She-Zelig's EVA is just a *modified* EVA of the *original* project and He-Zelig's EVA is, so to say, the *original* EVA of a *modified* (shadow) project.

6. Remarks and Conclusions

Our alleged psychopaths have been shown to be perfectly rational agents. In my opinion, their provocation must be taken seriously. I find their model rather interesting in more than one sense. They offer a new way of looking at the concept of excess profit. As far as I know, the literature provides us with Stewart's and Peccati's models which represent the two sides of the same medal. It is taken for granted that residual income is an unambiguous concept and the way of reasoning of Stewart and Peccati seems to be the natural one for translating such a concept in a formal way. But there is no natural way of mathematically representing it, for what is exactly excess profit? It formally translates, via a mathematical subtraction, the comparison between two courses of action. But which one is the correct pair of alternatives? Stewart's and Peccati's or the SVA's? In my opinion, it is a conventional matter. It depends on which piece of information the evaluator is willing to draw from the concept of excess profit. That is, it depends on the cognitive perspective the evaluator adopts or, in other terms, the cognitive outlook with which he/she frames the evaluation process. The issue at hand is a subtle one. The hub lies in the fact that the concept of economic profit derives from a counterfactual conditional of the kind:[8]

'If it were not A, then it would be B'

or

'If it had not been A, then it would have been B.'

The question is: What is A and what is B in our case? A and B are evidently two mutually exclusive situations, but it is not obvious how to interpret A and B. Think of PS's model (assuming, for convenience, $D_s = 0$ for all *s*). PS-minded evaluators say:

"We can invest $w_{s-1}(x)$ either at the rate *x* or at the rate *i*."

Now think of the systemic approach. Systemic-minded evaluators say:

[8] It is worthwhile noting that Buchanan (1969), writing about opportunity cost (which is the basis of the concept of excess profit), uses the terms "might be" (p. 46) and "might have beens" (p. vii).

"We can either invest $w_{s-1}(x)$ at the rate *x* or $w_{s-1}(i)$ at the rate *i*."

In terms of the counterfactual conditional above mentioned we have:

A="investing $w_{s-1}(x)$ at the rate *x*" B="investing $w_{s-1}(x)$ at the rate *i*." (6.1)

for PS-minded evaluators, and

A="investing $w_{s-1}(x)$ at the rate *x*" B="investing $w_{s-1}(i)$ at the rate *i*." (6.2)

for systemic-minded evaluators. It is my opinion that there is no unique economically significant interpretation so as to decide which one is the correct one, for A and B are *intrinsically* ambiguous. The excess profit is a *residual* income but it is unclear what 'residual' mean in terms of alternatives of action. PS's interpretation is, so to say, project-oriented, since it focuses on the outstanding capital of the project assuming for it alternative rates of return, disregarding the investor's wealth; conversely, the systemic interpretation is wealth-oriented, for it describes two different stories of the investor's wealth and recognizes that the foregone return depends on the differential value in account *C*: In fact, the two alternative net worths at the beginning of period *s* can be reframed as

$$\begin{aligned} E_{s-1} &= C_{s-1} + w_{s-1}(x) \\ E^{s-1} &= C_{s-1} + (C^{s-1} - C_{s-1}). \end{aligned} \tag{6.3}$$

Now, the term C_{s-1} is shared by both alternatives, so the differential terms are represented by the second addends. In the first case $w_{s-1}(x)$ yields profit at a rate *x*; in the second case $(C^{s-1} - C_{s-1})$ yields profit at a rate *i*. Therefore, $i(C^{s-1} - C_{s-1})$ is the opportunity cost, since, in a systemic perspective, if the investor invested (had invested) in account *C* (at time 0) rather than undertaking the project, she would have (would have had), at time $s-1$, a $(C^{s-1} - C_{s-1})$ surplus in his/her account *C*. Hence, the SVA.

It is worthwhile noting that the formal differences between the two approaches are linguistic and cognitive differences and strictly depend on which notion of opportunity cost we adopt: In terms of our counterfactual conditional, the opportunity cost is the apodosis of the conditional (i.e. B). Now, the student interprets the term "investing" in B as meaning "putting money in an asset", the Zeligs interpret B as a synonym of "renouncing to money". The student tells us that both Stewart and Peccati are using B in the former sense, the Zeligs tell us that we might also interpret B in the latter sense. So, both $iw_{s-1}(x)$ and $i(C^{s-1} - C_{s-1})$ are opportunity costs: As for the former, the student tells us that we might alternatively "put the same amount of money" in account *C*; as for the latter, the Zeligs tell us that if the investor selected (had selected) alternative (i) at time 0, then account *C* would be (would have been), at time *s*, richer by the sum $(C^{s-1} - C_{s-1})$, so that the investor is "renouncing" to the relative return he/she might have earned if he/she had selected alternative (ii).

So we have now two valuable models capable of formally grasping the notion of excess profit. Are there other interpretations? It might be, maybe reframing the evaluation process in another different way (see Magni, 2002, for a third way). However, in my conventionalist view, there is no 'best model', but just different ways to translate the same concept. It is just a matter of convention to choose one or the other, a convention regarding the way we are willing to shape the evaluation process. Actually, there is no way of attaching objectivity to either model. The project-oriented evaluator focuses on the two alternative rates, the wealth-oriented evaluator focuses not only on the alternative rates but also on the alternative values of the net worth: That is, he/she considers that the situation of his/her wealth (in particular, of account *C*) will depend on the choice made at time 0. The model to be used has to be selected on the basis of our subjective interpretation of the notion of excess profit. The fact that only one interpretation has been adopted in the literature is just due, in my opinion, to the fact that no other interpretation has been searched for. Poincaré (1902) writes that the reason why a convention is adopted relies on its simplicity and easiness of applicability. In this way, Poincaré implies that a convention is *deliberately* (even when implicitly) adopted by scientists. Conversely, it is my conviction that a convention is sometimes adopted unconsciously: This is what happens when it is widely accepted that there is one only (objective) way of taking account of some

events or concepts and this is in my opinion what has happened with the concept of excess profit in both theoretical economics and finance.[9]

Future researches can be addressed to investigate more thoroughly these two interpretations: From a financial point of view, some whimsicalities seem to arise in the description of the investor's financial system if PS's approach is adopted (see Magni, 2003). From a theoretical point of view it could be worth re-analysing the concept of residual income focusing on its ambiguities as well as investigating the interrelations between such a concept and the Net Final (Present) Value, which I have here just mentioned while showing the equivalence between Peccati's model and Stewart's. Further, other interpretations of the notion of excess profit could be searched for and tests could be conducted to see whether either interpretation is more natural for decision makers (see Magni, 2002).

Furthermore, another aspect is worth taking account for, in my opinion. He-Zelig has shown us an interesting way of measuring excess profit. He introduces the concept of 'shadow project' to which he applies the EVA procedure obtaining the SVA_s of the original project. He-Zelig is not only willing to making fun of the student, he is actually informing us that the SVA model can be seen as an EVA model. In terms of the counterfactual conditional above mentioned he frames A and B so that they coincide with those in (6.2) but disguises them in such a way that they seem to mirror (6.1), for the capital invested is the same for both alternatives, only the rates differ:

A="investing w_{s-1}^H at the rate x_s^H" B="investing w_{s-1}^H at the rate i." (6.3)

In other words, a 'wealth-oriented' excess profit can be seen as a 'project-oriented' excess profit if we shift from the project to its shadow project. Or, which is the same, He-Zelig, who interprets apodosis B in the sense of "renouncing to money", is able to conceal his interpretation and pretend that he is interpreting B as "putting money in an asset". He then provides us with a SVA_s disguised as an EVA_s. We are therefore tempted to ask whether such a connection may be iterated backward, that is: May the original project be seen as the 'shadow project' of some other project? The

[9] I agree with Duhem (1914) who is perfectly aware of this aspect in scientific research: More than one passage of his *La théorie physique* is devoted to show that in science the same 'practical fact' may be described by more than one 'theoretical fact', i.e. more than one symbolic proposition.

answer is positive and the EVA_S of the original project is the SVA_S of a project whose shadow is the very original one (see Magni, 2004, 2005). Hence, the EVA model can be seen in turn as a systemic model. SVA and EVA, though alternative, seem to bear strong relations to each other and these aspects deserve, in my opinion, careful attention. Other possible developments are the introduction of variable rates (in the sense of Teichroew, Robichek and Montalbano, 1965a, 1965b) and multiple accounts, so enriching the structure of the financial system of the decision-maker and the complexity of the evaluation process (see Pressacco and Stucchi, 1997; Magni, 2003, 2004, 2005). Finally, implications for corporate governance are straightforward: If managers are rewarded on the basis of the SVA, then their past performance is not "erased" as it is in the EVA model. The choice between EVA or SVA as a compensation scheme will then depend on whether shareholders are willing to take account or not of the fact that a higher/lower excess profit in the past have caused capital to be higher/lower today (for difference in value and sign between EVA and SVA see Ghiselli Ricci and Magni,2006).

I think the challenge elicited by the Zeligs is intriguing: Why are there (at least) two mathematical translation of the same economic concept? Should we revisit the notions of excess profit and opportunity cost? Do linguistic categories shape our thought about economic notions? When do linguistic differences entail corresponding cognitive differences? How do the latter relate to mathematical modelling? What are the implications for business and financial decision-making? The issue at hand is not only an economic or a financial one, but also a cognitive and mathematical one. So, the final hope is that this episode will attract attention not only by economists and financial analysts but also by decision-theory scholars and cognitive psychologists as well as financial mathematicians.

P.S. After this episode, the Zeligs were interned into a psychiatric hospital. This paper is intended to pay homage to some of their ideas by giving voice to their alternative way of cognizing some basic economic and financial concepts. Will they be rehabilitated?

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