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ABSTRACT. Collusion in auctions affects both revenue and efficiency and are prevalent. Yet, sellers do not use collusion-proof auctions as often as they should. Why is that? We find that one reason for this could be the cost of implementing efficient collusion-proof auctions. We use California highway procurements data, to estimate the cost of implementing collusion-proof auction. Our estimates show that cost must increase by at least 10.8% to ensure efficient outcome. The cost can sometimes be as high as 48.8% (depending on the size of bidding-ring in the data).
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JEL: C1, C4, C7, D4, L4.

1. INTRODUCTION

In recent years, auctions have become synonymous with buying and selling. Timber sales, financial assets, highway procurement and online advertisements, all use auction. A contract with many agents may be susceptible to collusion, which is quite prevalent. See Comanor and Schankerman [1976]; Feinstein, Block, and Nold [1985]; Lang and Rosenthal [1991]; Porter and Zona [1993]; Bajari [2001]; Porter and Zona [1999]; Pesendorfer [2000]; Asker [2008]; Harrington [2008]; Taibbi [2012, 2013] among others.[1] Collusion lowers revenue and leads to inefficient outcome Ausubel and Milgrom [2006]. Yet most auctions that we see in the data are standard auctions, auctions that are vulnerable to collusion. Why aren't auctions that are efficient and less vulnerable to collusion more common, given that collusion has first-order effect on auction outcome?

The answer to this question cannot just be that such collusion-proof auctions are more difficult to run. It cannot also be that the seller can stop

[1] Since bidding rings are secretive, these instances surely underrepresent the real instances of collusion.

collusion by barring those who collude because detecting collusion is difficult, see Bajari and Ye [2003]; Aryal and Gabrielli [2013].[2] Cartels often go to a greater length to keep its existence secret and anonymous, see Asker [2008] and references therein. Even when we know their identity, it might be difficult to prove any wrong doing. For instance in the Libor scandal, the banks implicated in a law suit have asked a federal court in New York to dismiss the lawsuit, arguing that the plaintiffs have failed to prove collusive behavior, see Hou and Skeie [2013]; Taibbi [2013] for more.[3]

The thesis of this paper is that implementing auctions that are robust with respect to collusion is too costly. To reach that conclusion we first estimate the distribution of bidders' cost from the California highway procurement data, then simulate the cost of running an auction proposed by Chen and Micali [2012] (henceforth, CM) that is resilient to collusion and always guarantees efficient allocation. The increase in cost is the price of achieving efficient allocation. We chose CM-auction because it is simple and like Vickery (see below) is dominant strategy solvable and hence is a "detail-free" auction. We find that procurement cost could increase by no less than 10.8%, and sometimes even as high as 48.8%! The cost depends on the size and number of bidding rings, something that is unknown to the researcher and hence must be determined from the data.

To understand why efficiency might be so costly when bidders collude it is important to understand the auction rules. Unlike the standard auction, in CM-auction bidders report a bid and their affiliation to any bidding ring.[4] Once the bidding rings are determined, the auction rule is similar to Vickery auction: the lowest bidder wins the auction but gets a price that is equal to the lowest bid from outside the winner's ring. Since the price received by a ring is not a function of the bids by ring members, truthful bidding is an

[2] Aryal and Gabrielli [2013] implemented their tests and found no evidence of collusion *even though* they focused only on bidders who failed Bajari and Ye [2003]'s tests.

[3] The size of Libor market is large, estimated to be anywhere from \$350 to \$800 trillion.

[4] Truthful reporting of one's affiliation is achieved by off-the-equilibrium punishment phase where bidders pay a fine if their affiliation reports are inconsistent, for example when any two bidders are inconsistent about their ring-identity.

equilibrium. For more on coalition strategy-proofness see Green and Laffont [1979]. Just like Vickery auction, CM-auction is efficient, but it comes at a price - the incentive for the bidder to report their group affiliation.[5]

To run CM-auction we need the distribution of bidders cost and the identity of ring members. We estimate the distribution by following Guerre, Perrigne, and Vuong [2000]. To identify the ring members, we divide bidders into two types: regular bidders (type 1) and fringe bidders (type 2), Jofre-Bonet and Pesendorfer [2003]. Regular bidders are the ones who bid in expensive contracts, valued at $1 million or more and are the only types who can collude. Then we follow Bajari and Ye [2003] to determine bid that are not consistent with competitive bidding. Of the twenty-five type 1 bidders, we focus on only fifteen bidders. If all these bidders form one ring, the worst case for the seller, the CM-auction increases the cost of procurement by 48.8%.

If some bids are inconsistent with competitive bidding Bajari and Ye [2003], then it will affect the distribution of the cost. To correct that we follow Aryal and Gabrielli [2013]. We assume that the coalition is rational, so all bidders maximize the sum of total payoff. Then we only use the lowest bid from coalition members to estimate the bid distribution. Remaining bids are "cover bids," and hence need not be depend on bidders cost. When we also exploit the frequency of simultaneous bids amongst these fifteen bidders, we find that only four of them fail the test. Then when we consider a coalition of only four bidders, the price of implementing CM-auction drops to 10.8%. Since the steps to determine ring members is never full proof, we remain agnostic and say that the cost of procurement could increase by anywhere from 10.8% to 48.8%, but the steps outlined in this paper can be used in any other data.

To estimate the bid distribution, instead of the widely used kernel-smoothed density estimators that are inconsistent at the endpoints of the support, we use the local polynomial estimation (henceforth, LPE) method, see Fan and

[5] See also Laffont and Martimort [1997, 1998, 2000]; Che and Kim [2006] for Bayesian implementation.

Gijbels [1996]. Using LPE automatically takes care of this boundary issue without trimming the data.[6]

This paper is organized as follows: Section (2) outlines the models, identification and estimation; Section (3) proposes the collusion-proof mechanism; Section (4) explains the data and the ways to determine the colluding bidders; Section (5) presents the empirical findings; Section (6) concludes. Choice of bandwidths, tables and figure are collected in the Appendix.

2. Model, Identification and Estimation

In this section we consider a procurement auction, i.e. a low-price sealed bid auction with asymmetric bidders: regular and fringe bidders. The section is divided into two subsections. The first subsection considers the model with and without collusion and covers nonparametric identification. The second subsection deals with estimation.

2.1. **Model and Identification.**

For every auction $\ell = 1, 2, \ldots, L$, a single and indivisible project is procured to $N_\ell \geq 2$ risk neutral bidders using first price sealed bids mechanism. The essential characteristic of the project for each auction is summarized by a random variable $X_\ell \in \mathbb{R}_{++}$, which for us will be the engineer's estimate of the project. We assume that there are two types ($k = 0, 1$) of bidders with $n_{k\ell}$ type k bidders, for auction ℓ, such that $N_\ell = n_{0\ell} + n_{1\ell}$.[7] In every auction ℓ a type k bidder draws his cost, i.i.d. across all other bidders, from $F_k(\cdot|X_\ell, N_\ell)$. Further, we assume that the costs are independent across auctions. Now, we consider two cases: competition and collusion.

2.1.1. *Competition.*

The set of observables W is

$$W := \left\{X_\ell, n_{0\ell}, n_{1\ell}, \{b_{0i}\}_{i=1}^{n_{0\ell}}, \{b_{1i}\}_{i=1}^{n_{1\ell}}, \right\}, \ell = 1, 2, \ldots L.$$

where b_{ki} is the bid by type $k \in \{0, 1\}$ bidder $i \in n_{k\ell}$. Then we make the following assumptions:

Assumption 1. *(A1)*

[6] As far as we know, the only other paper to use LPE in empirical auction is Gabrielli and Vuong [2010], who use it to propose a $\sqrt{n}-$ consistent semiparametric estimation method.

[7] We abuse the notation to use n_k as both the number and set of type k bidders.

(1) *Exogenous Participation:* $F_k(\cdot|X,N) = F_k(\cdot|X)$ *for* $k = 0,1$.
(2) *For each* ℓ *and each* $k \in \{0,1\}$ *the variables* $C_{ki\ell}, i \in n_{k\ell} \sim$ *iid* $F_k(\cdot|\cdot)$ *with density* $f_k(\cdot|\cdot)$ *conditional on* X_ℓ.
(3) *An auction* ℓ *has* $N_\ell \in \{\underline{n}, \overline{n}\}$ *risk-neutral bidders with* $\underline{n} \geq 2$.
(4) *The three- dimensional vector* $(X_\ell, n_{0\ell}, n_{1\ell}) \sim$ *iid* $Q_m(\cdot,\cdot,\cdot)$ *with density* $q_m(\cdot,\cdot,\cdot)$ *for all* $\ell = 1, 2, \ldots L$.[8]
(5) *The observed type* $k \in \{0,1\}$ *bids* $B_k \sim$ *iid* $G_k(\cdot|X_\ell, N_\ell)$ *with density* $g_k(\cdot|X_\ell, N_\ell)$.

A strategy for bidder i of type k is a strictly increasing, type symmetric bidding strategy $s_k : [\underline{c}, \overline{c}] \to [\underline{c}, \overline{c}]$. Type k bidder i solves

$$\begin{aligned}\max_b \Pi_k(b_i; c_i, X_\ell, N_\ell) &= \max_{b_i}(b_i - c_i) \prod_{j \in n_k \backslash \{i\}} (1 - F_k(s_k^{-1}(b_i)|X_\ell))^j \prod_{j \in n_{k'}} (1 - F_{k'}(s_{k'}^{-1}(b)|X_\ell))^j \\ &= \max_{b_i}(b_i - c_i) \prod_{j \in n_k \backslash \{i\}} (1 - G_k(b_i|X_\ell, N_\ell))^j \prod_{j \in n_{k'}} (1 - G_{k'}(b_i|X_\ell, N_\ell))^j,\end{aligned}$$

where $k \neq k' \in \{0,1\}$ and $G_k(b|X_\ell, N_\ell) = F_k(s_k^{-1}(b)|X_\ell)$ is the probability that bidder $j \in n_{k\ell} \backslash \{i\}$ will bid less than b, and likewise for k'.[9] The first order condition for $i \in n_{k\ell}$ is

$$(b_{ki} - c_{ki}) = \frac{1}{(n_{k\ell} - 1)\frac{g_k(b_{ki}|X_\ell, N_\ell)}{1 - G_k(b_{ki}|X_\ell, N_\ell)} + n_{k'}\frac{g_{k'}(b_{ki}|X_\ell, N_\ell)}{1 - G_{k'}(b_{ki}|X_\ell, N_\ell)}}. \tag{1}$$

This first order condition with the boundary conditions $s_k(\overline{c}) = \overline{c}, k = 0,1$ uniquely characterizes optimal bidding strategy for all bidders. The model structure is the type specific conditional distribution of cost $\{F_k(\cdot|X_\ell)\}$ for $k = 0,1$ given X. But since the data provide information on the characteristics of the project that is being procured, X_ℓ in the ℓ^{th} project, we can consider only the type specific conditional cost distribution $F_k(\cdot|X_\ell), k = 0,1$ as the structural parameter. The question of identification is to ask if there are two pairs of cost distributions $\{F_0(\cdot|X_\ell), F_1(\cdot|X_\ell)\}$ and $\{F_0'(\cdot|X_\ell), F_1'(\cdot|X_\ell)\}$

[8]We abuse the notation to use $n_{k\ell}$ to represent both the random variable and its realization and $Q(\cdot)$ is a product of absolutely continuous measure and a counting measure.

[9] The second equality follows from Assumption (A1.1)- exogenous participation. This mapping between bids and valuation distribution under equilibrium condition is due to Guerre, Perrigne, and Vuong [2000].

that are observationally equivalent. Evaluating (1) at the estimated bid distribution and densities, we see that for each auction ℓ, bid b_{ki} uniquely determines the cost

$$c_{ki} = b_{ki}^{\ell} - \frac{1}{(n_{k\ell}-1)\frac{g_k(b_{ki}^{\ell}|X_\ell,N_\ell)}{1-G_k(b_{ki}^{\ell}|X_\ell,N_\ell)} + n_{k'\ell}\frac{g_{k'}(b_{ki}^{\ell}|X_\ell,N_\ell)}{1-G_{k'}(b_{ki}^{\ell}|X_\ell,N_\ell)}}, \tag{2}$$

thereby identifying $\{F_0(\cdot|X_\ell), F_1(\cdot|X_\ell)\}$ that is consistent with the data.

2.1.2. *Collusion.* Now, we also consider a model where some of the type 1 bidders collude. We maintain all the afore mentioned assumptions. For every auction ℓ, let $M_\ell \subset n_{1\ell}$ be the set of bidders who collude. We focus on efficient collusion where the colluders have access to a centralized mechanism that can control the bids placed by the members in the real auction. So there will be only one serious bid in each auction from the bidders in M_ℓ and the rest will be just cover-bids. We also assume that the bidders outside the ring are unaware about the ring. This means that when we map the observed bids to the underlying cost we consider only the minimum bid $b_{1\ell}^*$ among the bidders in M_ℓ and the bids by everyone outside M_ℓ to estimate the type 1 bid distribution and density $G_1^*(\cdot|\cdot,\cdot)$ and $g_1^*(\cdot|\cdot,\cdot)$. Then, we can use these estimates instead of G_1 and g_1 in (2) to recover the pseudo cost for each non-members. But for the the minimum bid we use

$$\hat{c}_{1\ell} = b_{1\ell}^* - \frac{1}{(n_{1\ell}-(|M_\ell|-1))\frac{g_1^*(b_{1\ell}^*|X_\ell,N_\ell)}{1-G_1^*(b_{1\ell}^*|X_\ell,N_\ell)} + n_0\frac{g_0(b_{1\ell}^*|X_\ell,N_\ell)}{1-G_0(b_{1\ell}^*|X_\ell,N_\ell)}}, \tag{3}$$

where the main difference from the non-colluders is that while the non-colluders think they are competing with $n_{1\ell}$ type-1 bidders and $n_{0\ell}$ type 0 bidders, the ring knows that it is competing with only $n_{1\ell} - (|M_\ell| - 1)$ bidders. For type 0 bidders, just like with the type 1 non-colluders, the only difference is that the appropriate type 1 bid distribution and density are, respectively, $G_1^*(\cdot|\cdot,\cdot)$ and $g_1^*(\cdot|\cdot,\cdot)$.

2.2. **Estimation.** In the first step we estimate the conditional bid distributions $G_k(\cdot|X,N)$ and the bid densities $g_k(\cdot|X,N)$ given the engineer's estimate X and the set of bidders N, using Local Polynomial Estimation (LPE) method, see Fan and Gijbels [1996]; Gabrielli and Vuong [2010].

Consider a bivariate i.i.d. data $\{X_i, Y_i\}_{i=1}^n$. Our interest is the regression function $m(x_0)$ and its derivatives $m'(x_0), m''(x_0)$ and so on till $m^p(x_0)$. Hence, we regard the model $E[Y|X] = m(X)$. Under the assumption that $(p+1)^{th}$ derivative of $m(\cdot)$ at $x = x_0$ exists, LPE can approximate $m(\cdot)$ by a polynomial of order p. Taylor expansion gives

$$m(x) \approx \sum_{j=0}^{p} m^j(x_0) \frac{(x - x_0)^j}{j!},$$

and this polynomial is fitted locally by a weighted least squares regression that minimizes

$$\sum_{i=1}^{n} \{Y_i - \sum_{j=0}^{p} \beta_j (x - x_0)^j\}^2 K_h(X_i - x_0),$$

where h is the bandwidth, $K_h(\cdot) = K\left(\frac{\cdot}{h}\right)/h$ with K a kernel function. If $\hat{\beta}_j, j = 0, \ldots, p$ is the solution to the weighted least squares then it is clear that $j!\beta_j(x_0)$ is the estimator for $m^j(x_0), j = 0, \ldots, p$. For us, Y will be the indicator function and hence $\beta_0(\cdot)$ will be the LPE estimator of the conditional bids distribution and its first derivative $\beta_1(\cdot)$ will be the corresponding density. The exact form used for our estimation is given in Appendix (A-1). We make the following assumptions for estimation.

Assumption A3:

(i) *The kernels $K_G(\cdot)$, $K_{0g}(\cdot)$ and $K_{1g}(\cdot)$ are symmetric with bounded hypercube supports and twice continuous bounded derivatives with respect to their arguments,*

(ii) $\int K_G(x)dx = 1$, $\int K_{0g}(x)dx = 1$, $\int K_{1g}(b)db = 1$

(iii) $K_G(\cdot)$, $K_{0g}(\cdot)$ *and* $K_{1g}(\cdot)$ *are of order $R - 1$. Thus moments of order strictly smaller than $R - 1$ vanish.*

Assumption A4: *The bandwidths h_G, h_{1g} and h_{2g} satisfy*

(i) $h_G \to 0$ *and* $\dfrac{Lh_G^d}{\log L} \to \infty$, *as* $L \to \infty$,

(ii) $h_{0g} \to 0$, $h_{1g} \to 0$ *and* $\dfrac{Lh_{0g}^d h_{1g}}{\log L} \to \infty$, *as* $L \to \infty$.

From this assumption it is clear that it is possible to choose the optimal bandwidths, i.e. the bandwidths proposed in Stone [1982]. Unlike GPV we do not need to specify a "boundary bandwidth" since the local polynomial method does not require knowledge of the location of the endpoints of the support. Therefore, it is not necessary to estimate the boundary of the support of the bid distribution. This is necessary when one needs to trim out observations, which we do not given that our estimator is not subject to the so–called boundary effect. We have 3 conditioning variables, one that is continuous and two that are discrete. Thus, we have to adapt the definition of the LPE to the present case. However, the discrete variables do not affect the asymptotic properties of the estimator, so in order to choose the optimal bandwidth the relevant number of covariates to consider is the number of continuous variables. [10]

We will denote by p the number of continuous variables and by d the total number of conditioning variables. For our application, $d = 3$ and $p = 1$. Let $\hat{\psi} = \hat{g}(\cdot|\cdot,\cdot)/1 - \hat{G}(\cdot|\cdot,\cdot)$ be the estimator of $\psi(\cdot|\cdot,\cdot,\cdot) = g_k(\cdot|\cdot,\cdot,\cdot)/1 - G_k(\cdot|\cdot,\cdot,\cdot)$. From Proposition 1 in Guerre, Perrigne, and Vuong [2000] we know that $G_k(\cdot|\cdot)$ is $R+1$ times continuously differentiable on its entire support and therefore $g_k(\cdot|\cdot)$ is R times continuously differentiable on its entire support as well.[11] Given the smoothness of each function we propose to use a LPE (R), i.e. a LPE of degree R, for $G_k(\cdot|\cdot,\cdot,\cdot)$ and a LPE ($R-1$) for $g_k(\cdot|\cdot,\cdot,\cdot)$. Following Fan, Gasser, Gijbels, Brockmann, and Engel [1993] we can show that the bid distribution is consistent and following Guerre, Perrigne, and Vuong [2000] it is easy to see that the estimated costs are strongly consistent. The exact econometrics model and the selection of optimal bandwidth are explained in Appendix A-1.

3. Collusion Proof Mechanism

We begin with an example, adopted from Chen and Micali [2012] and for more formal and through treatment we direct the readers to that paper.

[10] Similar observation is made by Abadie and Imbens [2006].

[11] From Proposition 1 by Guerre, Perrigne, and Vuong [2000], we also know that the conditional density $g_0(\cdot|\cdot,\cdot)$ is $R+1$ times continuously differentiable on a closed subset of the interior of the support and thus the degree of smoothness closed to the boundaries and at the boundaries of the support is not $R+1$.

Example 1. *Consider 4 risk neutral bidders with cost* $(1, 1, 100, 100)$ *and the first and second bidders collude together. Furthermore, suppose that they have a wrong belief about the types of the other two bidders. In particular they believe that their respective costs are* $(0.1, 2)$*. Under competitive second price auction one of the two bidders will sell the object at 100. Under collusion they could adopt the following strategy: 1 bids 1 and 2 bids 100. The real bids will then be* $(1, 100, 100, 100)$ *and hence the coalition gets a surplus of 99 which can be shared easily between the two.*

Now, suppose bidders report their bids (the price at which they are willing to supply the public good) and their coalition membership, if any. The winner is the bidder who announces the lowest price and the price is equal to the second highest price outside of the winner's coalition, i.e. the price paid by a coalition is not controlled by them. To see this, consider the following announcement $(1, \{1, 2\}), (1, \{1, 2\})$, $(100, \{3\})$ *and* $(100, \{4\})$*. This ensures strategy proof-ness and the object is sold to coalition for 100.*

The example shows that the mechanism is a slight variation of classic second price auction, now with respect to coalitions rather than a singleton. Let there be $N < \infty$ risk-neutral bidders in an independent private value, low bid auction. Each bidder draws i.i.d cost $C \sim F(\cdot)$.[12] Let $\mathbb{C}$ represent the partition of players such that each element of the partition represents a coalition such that every singleton $\{i\} \in \mathbb{C}$ is an independent bidder, and M is a generic element. The set was $\{\{1, 2\}, \{3\}\}$ in the example. Let $\mathcal{M} = \{N, F(\cdot), \mathbb{C}\}$ be the context of the game and is commonly known by all the bidders. Moreover, we assume that for every coalition $M \in \mathcal{C}$, the $|M|-$tuple cost profile $C_M = \{C_i : i \in M\}$ is common knowledge only amongst the bidders in that coalition. The seller, however, only knows $\{N, F(\cdot)\}$.

Let $\{A_i, P_i\}_{i=1}^{N}$ be an allocation and pricing rule, where $A_i \in \{0, 1\}$ such that $\sum_{i \in N} A_i = 1$ and P_i is the price paid by the bidder i. We assume that all coalitions are efficient and hence when the ex-post utility of a bidder i is $(P_i - C_i)A_i$, the utility of the coalition M is the sum across the members, i.e. $u_M = \sum_{i \in M}(P_i - C_i)A_i$. Each member $i \in M$ acts to maximize $u_M(\cdot)$.

[12]For notational ease, we treat all bidders to be symmetric, extending it to asymmetric bidders is straightforward.

Definition 1. *An auction, for a context $\mathcal{M}$, is directly collusive if the set of pure strategies for $i, s_i(\cdot)$ consist of the set of all mapping from $C \mapsto (C, M)$.*

So, a bidder with cost C reports his cost and the coalition M. Let $u_M(s)$ denote the total utility of coalition M when everyone uses symmetric bidding strategy $s(\cdot)$. Now, we are in a position to define dominant-strategy truthfulness and coalitional rationality.

Definition 2. *An auction is collusively dominant-strategy truthful if, for all coalition $M \in \mathbb{C}$ and all strategy profiles $s_M := \{s_i(\cdot) : i \in M\}$ and $s_{-M} := \{s_j(\cdot) : j \notin \mathcal{C}\backslash\{M\}\}$,*

$$\forall i \in M : u_M((C_i, M), s_{-M}) \geq u_M((C_i', M'), s_{-M}).$$

and is coalitionally rational if $u_M((C_i, M), s_{-M}) \geq 0$.

Let $s(\cdot) = \{(C_1, M_1), \ldots, (C_N, M_N)\}$ be an action profile. Then a disagreement (in $s(\cdot)$) is an ordered pair (i, j) such that $M_i \ni j$ but $M_j \not\ni i$. In other words, we say that (i, j) is disagreement if i claims to be a part of collusion ring M_i that contains j but j does not reciprocate. Given a profile $s(\cdot)$ the outcome (A, P) is computed as follows: First, there is the punishing phase if there is any disagreement, in which case $A_i = 0$ for all i. Then to determine the price we start with $P_i = 0$ and for each disagreement (i, j) charge $P_i = P_i + 2t$ and $P_j = P_j - t$, while keeping t with the seller.[13] Second, when there is no disagreement we initiate the standard phase where from the reported coalitions the coalition partition $\mathbb{C}$ is constructed. Then, the lowest bidder wins the auction and we determine the winning coalition M^* and charge

$$P_i = \begin{cases} 0 & \text{if} \quad A_{M^*} = \sum_{j \in M^*} A_j = 0 \\ C_{1:(N\backslash M^*)} & o/w \end{cases}$$

to every bidder i, where $C_{1:(N\backslash M^*)}$ is the lowest bid from the bidder $j \notin M$.

Theorem 2. *Chen and Micali [2012] The mechanism outlined above is (a) Collusive dominant-strategy truthful; (b) Coalitional rational and ; (c) Efficient.*

[13] This punishment phase is off the equilibrium path and does not affect the estimation results.

Now, in the next section we analyze the data and determine the sets of coalitions that are used for the counterfactual exercise later.

4. Data

The aim of this section is to explain the main features of the data and to explain how we determine the colluding rings. The data consist of the Highway procurements in the state of California between January 2002 and January 2008, where the rights to maintain and construct highways and roads are granted through sealed low-bid auctions (procurements) by the California Department of Transportation (Caltrans).[14] The data include information about the characteristics of the projects that were let, the name of bidders and their bid in each auction. The timing is as follows. First, during the *advertising period* that lasts between three to ten weeks depending on the size of the project, the Caltrans Headquarters Office Engineer announces a project and solicits bids.[15] Potential bidders express their interest by buying the project catalogue. Second, sealed bids are received only from among the potential bidders. Third, on the *letting day*, the received bids are ranked and the project is awarded to the lowest bidder, provided that the bidder fulfills certain responsibility criteria determined by federal and state law. After each letting, the information about all bids and their ranking is made public.

We divide bidders into two broad types of asymmetric bidders: the fringe bidders (type 0) who bid in small projects and are infrequent and the main bidders (type 1) who participate frequently and in bigger projects. The private cost is interpreted as a reduced form of the real cost of production and depends on many unobservable characteristics of the bidder. The data consist of 2,152 projects awarded by Caltrans for a total of $7,645 millions but we focus only on 1,907 projects that had at least two bidders. Of all bidders, only 823 bidders bid at least once. In the remaining subsection we determine the set of bidders who fail the tests proposed by [Bajari and Ye, 2003]:

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[14] The data is publicly available at http://www.dot.ca.gov/hq/esc/oe/awards/bidsum/.

[15] Some examples of projects include asphalt repaving, road paving, bridge reconstruction, striping the highway, constructing, replacing and widening brides, storm damage repair, etcetera.

the colluders. We find that the fifteen type 1 bidders, who bid simultaneously more often on a pairwise basis, fail at least one of the conditions by [Bajari and Ye, 2003] for competitive bidding. Since we do not know the true collusive ring, we further explore some other features in the data in a hope to narrow the set of members in the ring. Looking at the frequency of their bids and their winning patterns we narrow the coalition to only four bidders. The exact process is explained in the reaming of this section. The main difficulty with this exercise is to determine the set of colluders, so the way we determine these two sets should be taken as suggestive and exploratory.[16]

4.1. **Bidding Ring.** To identify the ring members, we focus only on the projects that are worth between \$1 million and \$20 million as smaller projects are unlikely to be worth the risk. There are 724 such projects that worth \$2,408 millions (31% of the total) with 413 bidders out of which 202 win at least once. Furthermore, following the literature we define regular bidders as bidders who have a nontrivial revenue share (at least 1%) in the market. Twenty-five bidders satisfy this criteria and will henceforth be called the type 1 bidders and the remaining bidders are the fringe (type 0) bidders – see Table A-1. The first column is the index of the bidders while the second column gives the number of bids of each of them. To assess the market power of each bidder we define "expected win" (see below) and compare it with the actual number of wins: bidders with consistently higher actual win than the expected win will be termed as those who have higher market power. Expected number of wins is defined as follows: consider A, who bids on a total of 50 projects against a varying number of bidders, n_ℓ for $\ell = 1, \ldots, 50$. Then his expected win is defined to be $\sum_{\ell=1}^{50} 1/n_\ell$. By comparing third and fourth column, we see that with the exception of five bidders, all bidders win more contracts than expected. The fifth column reports the average bid of each bidder and the sixth column reports the revenue share – the total value of the bidder's winning bid as a fraction of the total value of

[16] The main point is, we can either use the tests from the literature and be agnostic about the nature of the collusive ring or try and explore some other features besides the test to determine the ring, in a hope that the exploration helps us find the "true" collusive rings.

winning bids for all contracts. The last column is the participation rate (i.e. the bid frequency rate), and bidder D is the one that stands out at 44%.

Table A-2 contains the summary statistics from which we can conclude: (i) on an average there are slightly more than four bidders; (ii) average winning bid is \$3.33 million, which is less than the average engineers' estimate of \$3.77 million while the average bid is \$3.79 million; (iii) money on the table – the difference between the highest and the second highest bid – is on average \$300,000 suggesting informational asymmetry among bidders. We also find that distance between the bidder's office and the site of project has no bearing on the bids. In general higher valued projects (between \$1 million and \$20 millions) attract relatively fewer bidders, suggesting that it is the main bidders who can gain the most by colluding and moreover, larger projects are more profitable, all else equal.

Now, we follow the tests proposed by [Bajari and Ye, 2003] on this subsample of bids for twenty five type 1 bidders. The basic idea behind the tests is to detect those bidders whose bidding pattern systemically violate competitive and independent bidding. To increase the likelihood of picking a coalition we give more emphasis to bidders who participate in the same auction because as the theoretical literature suggests ring members tend to participate in the same auctions to enforce the bidding agreement. To this end, we consider all combinations of pairs and select those bidders that have at least fifteen simultaneous bids, see Table A-3. There are fifteen bidders who bid frequently together.[17]

First, to test independence we consider the fifteen pairs of bidders bidding frequently described above and estimate the following models for fifteen type 1 bidders and the remaining bidders, respectively

$$BID_{i\ell}/EE_{\ell} = \gamma_0 + \gamma_1 LDIST_{i\ell} + \gamma_2 CAP_{i\ell} + \gamma_3 UTIL_{i\ell} + \gamma_4 LMDIST_{i\ell} + u_{i\ell} \quad (4)$$

$$BID_{i\ell}/EE_{\ell} = \alpha_0 + \alpha_1 LDIST_{i\ell} + \alpha_2 CAP_{i\ell} + \alpha_3 UTIL_{i\ell} + \alpha_4 LMDIST_{i\ell} + \varsigma_{i\ell}. \quad (5)$$

Here $\mathrm{LDIST}_{i\ell}$ is the logarithm of distance, $\mathrm{LMDIST}_{i\ell}$ is the logarithm of the minimum of distances of all bidders (except i) to the project ℓ and $UTIL_{i\ell}$ is the utilization rate of the capacity. We define the utilization rate as $\mathrm{Util}_{\mathrm{it}} =$

[17]This cutoff is based on the data and is big enough to capture the simultaneous bidding but not too big so that we have enough observations left for the test.

$\mathrm{Backlog_{it}/Capacity_i}$, where backlog is defined as the past projects that were won but yet to be completed and the capacity is the total capacity of bidder. We find that approximately 60% of bids are explained by capacity, although the effect varies across bidders; for more on the effect of capacity utilization on bidding behavior see [Jofre-Bonet and Pesendorfer, 2003]. For the bidders listed in Table A-3 (those who participate frequently) we estimate (4) with bidder–varying coefficients and for the rest we use (5).

For every pair i, j, from the fifteen type 1 bidders, let $\rho_{ij} = corr(\hat{u}_{i\ell}, \hat{u}_{j\ell})$, be the correlation between estimated residuals. We use the Pearson's correlation test for independence and find that for all but one pair, bidder D and W, we reject the null hypothesis of independence at 5% level; see Table A-4.

Second, we test exchangeability

$$H_0 : (\forall i, j, i \neq j), (\forall s \in \{1,2,3,4\}) \qquad \beta_{is} = \beta_{js},$$
$$H_A : (\exists i, j, i \neq j), (\exists s \in \{1,2,3,4\}) \qquad \beta_{is} \neq \beta_{js},$$

at both market level by pooling the fifteen bidders in one group and on a pairwise basis. Let $T = 3,347$ be the number of observations, m the number of regressors and k the number of constraints implied by H_0. Then under the null the test statistic $F = \frac{(SSR_C - SSR_U)/r}{SSR_U/(T-m)} \Rightarrow^d F(r, T-m)$. At the market level, exchangeability hypothesis imposes that the effect of the four explanatory variables is the same for both potential ring members and the remaining bidders. Since there are fourteen dummies (indexing the bidders) and for each case there are four restrictions (under null), the total number of restrictions imposed under the null is $k = 56$. Here $m = 748$ and $n - m = 2599$ and the estimated $F-$ statistic is 5.934 with the upper tail area equal to 0.0000. Therefore we reject the null of exchangeability when comparing the fifteen bidders (potential cartel members) against the remaining bidders. The assumption thus far is that all fifteen bidders form one single coalition and in our counterfactual exercise of case 1, when we say potential colluders we mean these fifteen bidders.

However, sustaining such a large coalition might be difficult. To see if we can reduce the size of the coalition, we conduct pairwise tests by pooling bidders accordingly and find that the hypothesis of exchangeability is rejected at conventional levels for 13 out of 15 pairs including the pair (D,P), (A,D) and (D,E). See Table A-5 for details. Comparing the "expected win" with the actual win for these pairs, we do see that at least one member of the pair wins often. Comparing Table A-1 and Table A-3 we can conclude that : (i) firm A exclusively bids against firm D; (ii) firm E bids remarkably frequently with both firm A and firm D; (iii) the pairs (D,P) and (A,D) have the highest simultaneous bids. All of these suggest that bidders (A,D,E, P) could be considered as potential collusive ring. Based on the previous analysis all pairs of bidders considered do not pass at least one of the tests for competitive bidding. However, as mentioned above, taking into account the number of simultaneous bids, bidders D and P bid simultaneously more often than others. And since the triplet (A,D,E) also fit the collusive behavior, we consider colluding bidders to be (A, D, E, P).[18]

5. Estimation Results

In this section we present the main findings from estimation of the pseudo-cost and from the counterfactual exercise of implementing CM-auction. As we mentioned in the sections (2) and (4.1), we have two sets of type specific costs and also two sets of colluders. The first case uses all the bids data to recover cost and finds that fifteen out of twenty-five regular bidders could be colluding according to Bajari and Ye [2003]. The second case takes into account the fact that some bidders might already be colluding. To determine the set of colluders we use the same tests as above and some other data features, which allows us to identify a set of four bidders. In every auction we discard all but the minimum bid by these four colluders to estimate the cost.[19]

Let M_ℓ be the set of colluders who are present in auction ℓ and let $i_\ell \in N_\ell$ be the winner and $\tilde{b}_\ell$ the corresponding bid. Let o_ℓ be the smallest cost amongst all bidders participating in auction ℓ who do belong to the coalition M_ℓ. That is if the winner belongs to the coalition, i.e. $i_\ell \in M_\ell$, then $o_\ell =$

[18] Ideally we would have liked to conduct the tests for every subsets of these fifteen bidders not just the pairs, but the amount of data for each case is insufficient making the tests unreliable.

[19] As mentioned earlier we assume that the coalition is rational and maximizes the total payoff. This means only the bid corresponding to the lowest cost will be serious, the rest will only be "cover-bids."

$\min_{j\in n_\ell \setminus M_\ell} \hat{C}_j$ and if the winner is not a member of the coalition then we can set $o_\ell = \tilde{b}_\ell$, the real winning bid. Then the difference between CM-auction and the data is $r_\ell = o_\ell - \tilde{b}_\ell$ if the winner is a cartel member and $r_\ell = 0$ otherwise. Once we compute the change in cost r_ℓ for all auctions the total change in cost of procuring is just $\sum_\ell r_\ell$. For the first case the total set of colluders is the fifteen bidders (see section 4.1) and $M_\ell = M \cap n_{1\ell}$. Likewise for the second case we find that only $M = \{A, B, D, E\}$ bidders are consistent with collusion and hence $M_\ell = n_{1\ell} \cap \{A, B, D, E\}$. Figure 1 shows the empirical CDF of r_ℓ for these two cases.

We find that, in the first case with large coalition, implementing CM-auction increases the total cost by 48.8%, while for the second case the cost increases by 10.8%. This difference in cost is not surprising given the difference in size of the two rings.

6. Conclusion

In this paper we ask why do not sellers choose auction formats that are resilient to collusion? We find that one important reason could be that it could be too expensive to choose such an auction. In particular, we use estimates from California highway procurement data on CM-auction. CM-auction have many advantages: it is not based on any equilibrium notion and uses dominant strategy truthfulness; it is easy to implement and requires minimal common knowledge assumption; and it is also robust with respect to the transfer among ring members.

We find that the procurement cost could increase by anywhere between 10.8%, to 48.8%. The cost could be this high because the auction is always efficient, so one way to reduce the cost could be relax this demand. But to know the exact cost, one needs to estimate the entire revenue-efficiency frontier.

Finally, we acknowledge that the exogenous entry assumption could be a strong assumption. Even though we know much more about how to estimate auctions with costly and selective entry, Gentry and Li [2014], as far as we know, nothing is known about auctions that are both collusive dominant strategy truthful *and* that allow bidders to collude prior to their participation decisions, which is an important area of research.

APPENDIX

A-1. Estimation

In this section we outline the estimation problem and discuss the choice of bandwidths and kernels. To account for the skewness in the bid distribution, a widely observed problem encountered with auction data, we use logarithmic transformation. For notational simplicity we suppress the dependance of the distributions on (X, N), unless otherwise noted. Log transformation of (2) gives

$$c_{kM} = \xi_k(d_k, n) = e^{d_k} - \frac{e^{d_k}}{(n_k - 1)\frac{g_{kd}(d_k|\cdot,\cdot)}{1-G_{kd}(d_k|\cdot,\cdot)} + n_1\frac{g_{1d}(d_k|\cdot,\cdot)}{1-G_{1d}(d_k|\cdot,\cdot)}} \tag{6}$$

where $d_k = \ln(b_k)$ and $G_{kd}(\cdot|\cdot,\cdot), g_{kd}(\cdot|\cdot,\cdot)$ are the distribution and density of $\log(b_k)$ for type k. Define $K_H(u) = |H|^{-1}K(H^{-1}u)$, where H is a nonsingular $d \times d$ matrix, the bandwidth matrix that usually takes the form $H = hI_d$ and $|B|$ denotes its determinant. The observations are given by $\{(Z_i^T, Y_i) : i = 1, \ldots, n\}$ with $Z_i = (X_i, N_{0i}, N_{1i})^T$. Let (x, n_0, n_1) be a point in $\mathbb{R}^3$. The estimators involved are, as mentioned above Local Polynomial Estimators. For our application, $R = 2$ and therefore we implement a LPE(2) for each cdf involved and a LPE (1) for each pdf involved. Let $Y^G_{p\ell} = 1\!\!1(B_{pl} \leq b)$. Using a local quadratic approximation to estimate each cdf implies obtaining the solution to the following least squares minimization problem

$$\begin{aligned}
&\sum_{\{\ell: I_\ell = i\}}^{L} \sum_{p=1}^{i} \Big\{ Y^G_{p\ell} - \Big[\beta_0 + \beta_1(X_{p\ell} - x) + \beta_2(N_{1\ell} - n_1) + \beta_3(N_{0\ell} - n_0) \\
&+\beta_{11}(X_{p\ell} - x)^2 + \beta_{12}(X_{p\ell} - x)(N_{1\ell} - n_1) + \beta_{13}(X_{p\ell} - x)(N_{0\ell} - n0) \\
&+\beta_{23}(N_{0\ell} - n_0)(N_{1\ell} - n1) + \beta_{22}(N_{1\ell} - n_1)^2 + \beta_{33}(N_{0\ell} - n_0)^2\Big]\Big\}^2 K_H(Z - z)
\end{aligned}$$

with respect to $\beta_G = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_{11}, \beta_{12}, \beta_{13}, \beta_{23}, \beta_{22}, \beta_{33})$. In particular we are interested in $\beta_0 = G(b|x, n_0, n_1)$; see Fan and Gijbels [1996]. Then, we know from the least squares theory that $\hat{\beta}_G = (Z_G^T W_G Z_G)^{-1} Z_G^T T_G Y$, where the design matrix Z_G for the local quadratic case (what we use) is

$$Z_G = \begin{pmatrix} 1 & (X_{1,1}-x) & (N_{0,1}-n_0) & (N_{1,1}-n_1) & (X_{1,1}-x)^2 & (X_{1,1}-x)(N_{0,1}-n_0) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & (X_{1,n_i}-x) & (N_{0,n_i}-n_0) & (N_{1,n_i}-n_1) & (X_{1,n_i}-x)^2 & (X_{1,n_i}-x)(N_{0,n_i}-n_0) \end{pmatrix.$$

$$\begin{matrix} (X_{1,1}-x)(N_{1,1}-n_1) & (N_{0,1}-n_0)(N_{1,1}-n_1) & (N_{0,1}-n_0)^2 & (N_{1,1}-n_1)^2 \\ \vdots & \vdots & \vdots & \vdots \\ (X_{1,n_i}-x)(N_{1,n_i}-n_1) & (N_{0,n_i}-n_0)(N_{1,n_i}-n_1) & (N_{0,n_i}-n_0)^2 & (N_{1,n_i}-n_1)^2 \end{matrix}\Bigg)$$

For the densities involved define $Y^g_{p\ell} = \frac{1}{h_{2g}} K_{2g}\left(\frac{B_{p\ell}-b}{h_{2g}}\right)$. We use a local linear estimator, i.e. LPE(1) which, as before, is obtained as the solution to the following least squares problem

$$\sum_{\{\ell:I_\ell=i\}}^{L} \sum_{p=1}^{i} \left\{ Y^g_{p\ell} - \beta_0 + \beta_1(X_{p\ell}-x) + \beta_2(N_{1\ell}-n1) + \beta_3(N_{0\ell}-n0) \right\}^2 K_H(Z-z)$$

It is well known that $\hat{\beta}_g = (Z^T T_g Z)^{-1} Z^T T_g Y$. The design matrix Z for the local linear case is

$$Z = \begin{pmatrix} 1 & (X_{1,1}-x) & (N_{0,1}-n_0) & (N_{1,1}-n_1) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & (X_{1,n_i}-x) & (N_{0,n_i}-n_0) & (N_{1,n_i}-x) \end{pmatrix}.$$

The corresponding weighting matrix for each estimation procedure are $T_G = \text{diag}\{K_H(Z_i - z)\}$ and $T_g = \text{diag}\{K_H(Z_i - z)\}$, respectively. The bandwidths and kernels involved for distributions and densities are different.

A-1.1. **Choices of Kernels and Bandwidths.** Since the exact choice of the Kernels is not crucial for inference, we use product of univariate kernels to represent the multivariate kernel, i.e.

$$K_m\left(\frac{a-A_k}{h_g}, \frac{b-B_k}{h_g}, \frac{n-N_k}{h_{gn}}\right) = K_a\left(\frac{a-A_k}{h_g}\right) K_b\left(\frac{b-B_k}{h_g}\right) K_n\left(\frac{n-N_k}{h_{gn}}\right).$$

Here, $K_m(\cdot,\cdot,\cdot)$ is the multivariate Kernel, $K_a(\cdot)$ and $K_b(\cdot)$ denote the univariate Kernels corresponding to the continuous variables A and B, respectively, and $K_n(\cdot)$ is the kernel for the discrete variables such that $K_n(\cdot) :=$

$K_{n_0}(\cdot) \cdot K_{n_1}(\cdot) \cdot K_{n_2}(\cdot)$. The kernels for continuous variables should be symmetric with bounded supports, so we decided to use the Epanechnikov Kernel function $K(u) = 3/4(1-u^2)\mathbb{I}(|u| \leq 1)$, as it is an optimal Kernel in the sense that it minimizes the asymptotic mean squared error over all nonnegative functions Fan, Gasser, Gijbels, Brockmann, and Engel [1993]. For the discrete variables, we use Gaussian Kernel because, as there is less variation in the number of bidders it is desirable to give less weight to observations farther from the point at which estimation takes place and is best achieved with a kernel with unbounded support.[20] We assume the smoothness parameter $R = 2$ for the cost distribution. To ensure uniform consistency at the optimal rates, the bandwidths for the continuous variables are chosen to be $h_g = 1.06 \times 2.214 \times \hat{\sigma} \times (T)^{-1/(2R+1)}$, $h_G = 1.06 \times 2.214 \times \hat{\sigma} \times (T)^{-1/(2R+3)}$. The constant term comes from the so–called rule of thumb and the factor 2.978 is the one corresponding to the use of Epanechnikov Kernels instead of Gaussian Kernels; see [Hardle, 1991].

[20]There are no theoretical restrictions to the kernels applied to discrete variables.

A-2. TABLES AND FIGURES

TABLE A-1. Revenue Shares and Participation of Main Firms

Firm ID	Number of Bids	Number of wins	Exp. Number of wins	Average bid (Mill. $)	Revenue Share	Participation rate
A	50	9	10.34	4.83	0.020	0.07
B	34	13	10.51	3.21	0.012	0.05
C	43	9	10.46	5.32	0.013	0.06
D	319	97	87.32	3.61	0.145	0.44
E	46	11	10.15	4.49	0.015	0.06
F	42	15	10.70	3.63	0.016	0.06
G	25	12	5.84	4.09	0.027	0.03
H	26	6	5.16	5.03	0.011	0.04
I	21	7	4.27	4.54	0.012	0.03
J	20	9	4.69	3.84	0.015	0.03
K	34	4	6.90	8.44	0.019	0.05
L	35	16	7.95	4.32	0.020	0.05
M	29	13	6.94	3.69	0.016	0.04
N	9	3	1.55	6.33	0.012	0.01
O	31	5	6.82	6.37	0.011	0.04
P	50	16	12.95	4.03	0.027	0.07
Q	33	9	6.31	3.35	0.017	0.05
R	28	10	8.10	3.48	0.012	0.04
S	47	12	8.82	4.37	0.021	0.06
T	25	13	5.99	3.75	0.021	0.03
U	68	16	15.22	4.77	0.026	0.09
V	26	7	4.78	5.75	0.025	0.04
W	41	11	7.18	2.92	0.019	0.06
X	41	7	10.27	4.50	0.021	0.06
Y	11	4	1.89	6.04	0.012	0.02
Total	1148	351	282		0.57	

Only bidders with revenue shares $\geq 1\%$ are reported.

TABLE A-2. Summary Statistics

	No. observations	Mean	SD
No. Bidders	724	4.62	2.37
Winning bid	724	3.33	3.11
Money on the table	724	0.30	0.46
Engineers' Estimate	724	3.77	3.49
All Bids	3347	3.79	3.51
Backlog	3347	4.30	9.76
Distance (miles)	3347	123.98	162.93
Capacity (across bidders)	413	2.30	5.69
Utilization rate	3347	0.20	0.32

All dollar figures are expressed in millions. Utilization rate is the ratio of backlog to capacity.

TABLE A-3. Summary of Simultaneous Bids

Bidder Pair	# of Simultaneous Bids	# of Expected Wins	First bidder of the Pair Wins	Second bidder of the Pair Wins
(A,D)	44	9.03	9	5
(A,E)	20	4.05	3	6
(B,D)	29	9.51	12	10
(C,D)	17	5.65	5	9
(D,E)	41	8.67	8	9
(D,F)	26	7.46	5	9
(D,H)	19	3.92	7	3
(D,I)	18	3.68	1	7
(D,O)	25	5.16	7	5
(D,P)	44	11.08	13	14
(D,R)	27	7.96	10	10
(D,V)	22	4.20	5	6
(D,W)	19	2.97	2	3
(M,X)	22	4.91	11	2
(W,X)	15	2.81	5	2

TABLE A-4. Conditional Independence Test

Bidder Pair	Test Statistic	p-value	n	deg. freedom
(A,D)	0.7660	0.0000	44	42
(A,E)	0.7427	0.0002	20	18
(B,D)	0.7331	0.0000	29	27
(C,D)	0.9239	0.0000	17	15
(D,E)	0.6530	0.0000	41	39
(D,F)	0.7570	0.0000	26	24
(D,H)	0.4734	0.0406	19	17
(D,I)	0.7121	0.0009	18	16
(D,O)	0.7643	0.0000	25	23
(D,P)	0.8538	0.0000	44	42
(D,R)	0.8555	0.0000	27	25
(D,V)	0.6877	0.0004	22	20
(D,W)	0.4305	0.0658	19	17
(M,X)	0.6529	0.0010	22	20
(W,X)	0.6271	0.0123	15	13

TABLE A-5. Exchangeability Test on Pairwise Basis

PAIR	F	UTA	k	m	n-m
(A,D)	5,2001	0,0000	8	796	2551
(A,E)	2.3540	0.0161	8	796	2551
(B,D)	5.9354	0.0000	8	796	2551
(C,D)	8.4271	0.0000	8	796	2551
(D,E)	7.9549	0.0000	8	796	2551
(D,F)	6.8441	0.0000	8	796	2551
(D,H)	5.2001	0.0000	8	796	2551
(D,I)	4.1670	0.0001	8	796	2551
(D,O)	5.5088	0.0000	8	796	2551
(D,P)	7.1682	0.0000	8	796	2551
(D,R)	6.1147	0.0000	8	796	2551
(D,V)	3.7384	0.0002	8	796	2551
(D,W)	4.6217	0.0000	8	796	2551
(M,X)	0.7984	0.6040	8	796	2551
(W,X)	0.3509	0.9458	8	796	2551

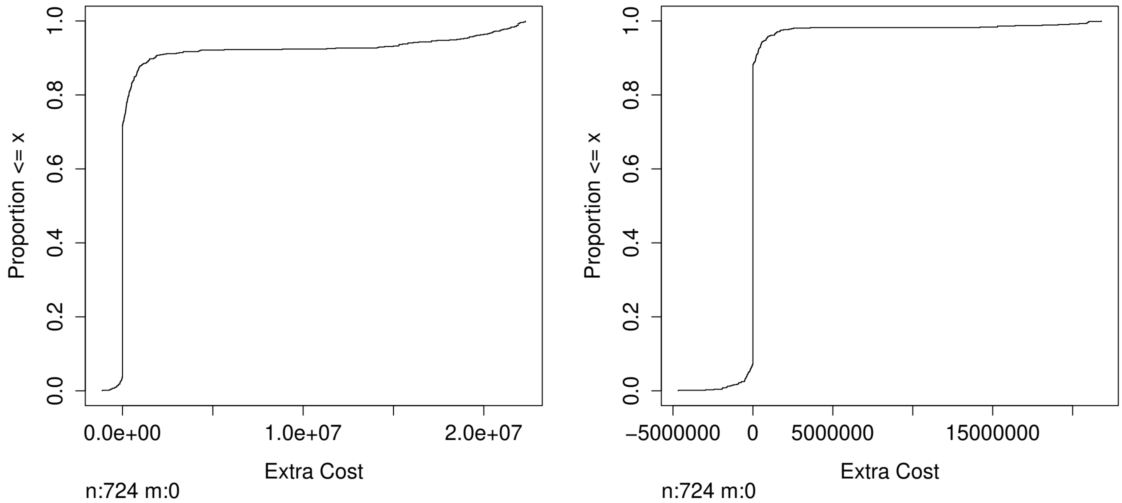

FIGURE 1. ECDFs of extra cost under competition and collusion, respectively.

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Supplementary Appendix: Not for Publication

Steps For Estimation

To make the estimation procedure transparent we outline the steps required to estimate the (pseudo) costs:

(1) Case 1: Competition case where collusion comes only for counterfactual.
   (a) Choose appropriate Kernels and determine the optimal bandwidths, see Section (A-1.1).
   (b) Use $\{\{b_{0i}\}_{i=1}^{n_{0\ell}}, X_\ell\}_{\ell=1}^{L}$ to estimate $\hat{g}_0(b|X_\ell n_{0\ell})$ and $\hat{G}_0(b|X_\ell, n_{0\ell})\}$ on $[\min b_0, \max b_0]$ such that $\hat{g}_0(b|\cdot,\cdot) = 0$ if $b \notin [\min b_0, \max b_0]$.
   (c) Suppressing the conditioning variables, define $\hat{g}_0(b) = max(0, \hat{g}_0(b))$ and $\hat{G}_0(b) = min(1, \hat{G}_0(b))$.
   (d) Repeat Steps (3) and (4) for $\{\{b_{1i}\}_{i=1}^{n_{0\ell}}, X_\ell\}_{\ell=1}^{L}$ to estimate $\hat{g}_1(b|X_\ell, n_{1\ell})$ and $\hat{G}_1(b|X_\ell, n_{1\ell})\}$.
   (e) Use the estimates $\{\hat{G}_0, \hat{g}_0\}$ evaluated at b_0 and $\{\hat{G}_1, \hat{g}_1\}$ evaluated at b_1 to interpolate:
      (i) For every b_{0i} from the type 0 bids, find the highest lower bound of bid $\underline{b}_{1i}(b_{0i})$ and the lowest upper bound $\overline{b}_{1i}(b_{0i})$from type 1 bids data such that $\underline{b}_{i1}(b_{0i}) \le b_{0i} \le \overline{b}_{i1}(b_{0i})$.
      (ii) Determine the weight $w_{0i} = \frac{b_{0i} - \underline{b}_{1i}(b_{0i})}{\overline{b}_{1i}(b_{0i}) - \underline{b}_{1i}(b_{0i})}$.
      (iii) Then define $\hat{g}_1(b_{0i}) = w_{0i}\hat{g}_1(\underline{b}_{1i}(b_{0i})) + (1 - w_{0i})\hat{g}_1(\overline{b}_{1i}(b_{0i}))$.
      (iv) Similarly, determine $\hat{G}_1(b_{0i}) = w_{0i}\hat{G}_1(\underline{b}_{1i}(b_{0i})) + (1 - w_{0i})\hat{G}_1(\overline{b}_{1i}(b_{0i}))$.
      (v) Repeat (a) - (d) for all bids b_{0i} in the domain of observed range of bids b_{1i}. For bids b_{0i} not in the domain put the interpolation density to zero.
      (vi) Repeat (a) - (e) for bids b_{1i}.[21]
   (f) The corresponding estimates of the (pseudo) costs are

[21]We have now 4 sets, the cdf/pdf for both types evaluated at their corresponding data and then the remaining 2 sets that are determined from interpolation of the estimated cdf/pdf.

(i) The cost c_{0i} of bidder $i \in n_{0\ell}$ who bids b_{0i}^{ℓ} is

$$c_{0i}^{\ell} = \begin{cases} b_{0i} - \frac{1}{(n_0-1)\frac{\hat{g}_0(b_{0i}|X_\ell,n_{0\ell},n_{1\ell})}{1-\hat{G}_0(b_{0i}|X_\ell,n_{0\ell},n_{1\ell})} + n_1\frac{\hat{g}_1(b_{0i}|X_\ell,n_{0\ell},n_{1\ell})}{1-\hat{G}_1(b_{0i}|X_\ell,n_{0\ell},n_{1\ell})}}, & \text{if } b_{0i} \in [\min_i b_{1i}, \max_i b_{1i}], \\ b_{0i} - \frac{1}{(n_0-1)\frac{\hat{g}_0(b_{0i}|X_\ell,n_{0\ell},n_{1\ell})}{1-\hat{G}_0(b_{0i}|X_\ell,n_{0\ell},n_{1\ell})}}, & \text{o/w}, \end{cases}$$

(ii) Similarly, cost c_{1i} of bidder $i \in n_{1\ell}$ is

$$c_{1i}^{\ell} = \begin{cases} b_{1i} - \frac{1}{(n_{1\ell}-1)\frac{\hat{g}_1(b_{1i}|X_\ell,n_{0\ell},n_{1\ell})}{1-\hat{G}_1(b_{1i}|X_\ell,n_{0\ell},n_{1\ell})} + n_0\frac{\hat{g}_0(b_{1i}|X_\ell,n_{0\ell},n_{1\ell})}{1-\hat{G}_0(b_{1i}|X_\ell,n_{0\ell},n_{1\ell})}}, & \text{if } b_{1i} \in [\min_i b_{0i}, \max_i b_{0i}], \\ b_{1i} - \frac{1}{(n_{1\ell}-1)\frac{\hat{g}_1(b_{1i}|X_\ell,n_{0\ell},n_{1\ell})}{1-\hat{G}_1(b_{1i}|X_\ell,n_{0\ell},n_{1\ell})}}, & \text{o/w}, \end{cases}$$

(g) Then we have $\{\{c_{0i}\}_{i=1}^{n_{0\ell}}, \{c_{1i}\}_{i=1}^{n_{1\ell}}, \ell = 1, 2, \ldots, L\}$ pseudo-cost vectors.

(h) To implement the counterfactual, we do the Bajari and Ye [2003] tests on the type 1 bidders. We determine 15 bidders whose bidding is at-odds with independent bidding and competition.

(2) Case 2: Estimating with Collusion:

(a) We begin with $n_{0\ell}$ type-0 and $n_{1\ell}$ type-1 bidders.

(b) We follow Aryal and Gabrielli [2013] and identify four bidders $M = \{A, B, D, E\}$ who are treated as colluders and take this into account while estimating pseudo-costs.

(c) Determine $M_\ell = M \cap n_{1\ell}$, the set of bidders in auction ℓ, for every auction.

(d) Since type-0 case is unaltered, repeat 1(a)-1(d) to determine $\{\hat{G}_0, \hat{g}_0\}$

(e) Determine the collusive bids for every auction, i.e. the set $\{b_{1i} : i \in M_\ell\}, \ell = 1, \ldots, L$, and for every auction determine $\{b_{1\ell}^*\}_{\ell=1}^{L}$. Then, the effective type-1 bidder from the point of view of colluding bidders is $n_{1\ell}^* = n_{1\ell} - (|M_\ell| - 1)$ as the coalition M_ℓ is effectively treated as a single bidder, while the number of type-1 bidders for those outside M_ℓ is still $n_{1\ell}$. So we consider two sub-cases: [22]

(f) Colluders:

[22] It is possible that in an auction, $M_\ell = 1$, in which case the minimum is just the bid and everything is the same. This means, we have two sets of $\{G_1, g_1\}$ one for M_ℓ and the other for the rest.

(i) In every auction we discard all but the minimum bid of the ring. Let $\{\{b^*_{1i}\}_{i=1}^{n^*_{1\ell}} : \ell = 1, \dots, L\}$ be the type 1 bids.

(ii) Using this set repeat 1(e) to estimate the pair $\{\hat{G}^*_1, \hat{g}^*_1\}$.

(iii) Using the estimates from 2(d) and 2(f-ii), repeat the steps 1(f) and 1(g) to determine the pseudo-cost of the cartel:

$$c^*_{1\ell} = \begin{cases} b^*_{1\ell} - \dfrac{1}{(n^*_{1\ell}-1)\frac{\hat{g}^*_1(b^*_{1\ell}|X_\ell, n_{0\ell}, n^*_{1\ell})}{1-\hat{G}^*_1(b^*_{1\ell}|X_\ell, n_{0\ell}, n^*_{1\ell})} + n_0 \frac{\hat{g}_0(b^*_{1\ell}|X_\ell, n_{0\ell}, n_{1\ell})}{1-\hat{G}_0(b^*_{1\ell}|X_\ell, n_{0\ell}, n_{1\ell})}}, & \text{if } b^*_{1\ell} \in [\min_i b_{0i}, \max_i b_{0i}], \\ b^*_{1\ell} - \dfrac{1}{(n^*_{1\ell}-1)\frac{\hat{g}^*_1(b^*_{1\ell}|X_\ell, n_{0\ell}, n^*_{1\ell})}{1-\hat{G}^*_1(b^*_{1\ell}|X_\ell, n_{0\ell}, n^*_{1\ell})}}, & \text{o/w}, \end{cases}$$

(g) Non-colluders: Since they still think they compete with $n_{1\ell}$ bidders, so like Case 1 we get

$$c^\ell_{1i} = \begin{cases} b_{1i} - \dfrac{1}{(n_{1\ell}-1)\frac{\hat{g}^*_1(b_{1i}|X_\ell, n_{0\ell}, n^*_{1\ell})}{1-\hat{G}^*_1(b_{1i}|X_\ell, n_{0\ell}, n^*_{1\ell})} + n_0 \frac{\hat{g}_0(b_{1i}|X_\ell, n_{0\ell}, n_{1\ell})}{1-\hat{G}_0(b_{1i}|X_\ell, n_{0\ell}, n_{1\ell})}}, & \text{if } b_{1i} \in [\min_i b_{0i}, \max_i b_{0i}], \\ b_{1i} - \dfrac{1}{(n_{1\ell}-1)\frac{\hat{g}^*_1(b_{1i}|X_\ell, n_{0\ell}, n^*_{1\ell})}{1-\hat{G}^*_1(b_{1i}|X_\ell, n_{0\ell}, n^*_{1\ell})}}, & \text{o/w}, \end{cases}$$

(h) Then we have $\{\{c_{0i}\}_{i=1}^{n_{0\ell}}, \{c_{1i}\}_{i=1}^{n_{1\ell}\setminus\{M_\ell\}}, \{c^*_{1\ell}\}; \ell = 1, 2, \dots, L\}$ pseudo-cost vectors.[23]

(i) Then we can implement the collusion-proof mechanism.

[23] To estimate the type 0 cost, we use $\{\hat{G}^*_1, \hat{g}^*_1\}$ and $\{\hat{G}_0, \hat{g}\}$.