An unnoted fair bet in german state run lotteries, a short notice

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This paper is according to the work of Krautmann and Ciecka [1] who made this analysis for state-run lotteries in the U.S.[1].It's common knowledge that gambling markets don't provide fair bets. In fact state-run lotteries in Germany are some of the worst gamblings, because the ToTo-Lotto Block as host of the lotteries keeps 50 percent of the stakes. In other words expected return would be 50 cents on the euro.

As Krautmann and Ciecka [1] have shown it, the chance for a fair bet depends on the size of the pot. Unlike the state-run lotteries in the U.S., lotteries in Germany pay out the winning in a lump sum and lottery winnings are free of tax. Size of the pot depends first on the share of the stakes which is take away by host, second the share of the first prize level in payout, because this money would be sent to the pot. We only take the pot for the first prize level in account, the value of lower-tier prize is uncared in this examination; the reason for doing so is gamblers play to win the first prize and not the second. For a further discussion about the motivation about players interests see also Garrett and Sobel [2]. If, according to the methode described by Krautmann and Ciecka [1] a fair bet was given, it should be possible to apply the same to a game with higher sum of winnings. By this reasons we have to change their analysis in these points.

Pot Size
If B_t is the number of outstanding tickets and q the price of one ticket, the whole sum of bets is S_t. And with $g \in (0,1)$ as the part which is take away by host, the

whole payout PO_t is given by

$$PO_t = (1-g)S_t \qquad (1)$$

So the pot is equal to the share p in payout PO_t, with $p \in (0,1)$.

$$Pot_t = p(1-g)S_t \qquad (2)$$

Like most lotteries, lotteries in Germany roll over the pot whenever there is no winner. So pot size is sum of the last pot and the part of new bets which went to first level prize.

$$Pot_t = Pot_{t-1} + p(1-g)S_t \qquad (3)$$

Upon substitution, we get the sum of bets S_t.

$$S_t = \frac{Pot_t - Pot_{t-1}}{p(1-g)} \qquad (4)$$

Winner or multiple winners
The expected number of winners, EN_t is the probability of winning, $\omega \in (0,1)$, times the number of ticktes, i.e. $EN_t = \omega B_t$. The expected return[2] is the pot divided by the number of winners,

$$ER_t = \omega \frac{Pot_t}{EN_t} = \frac{Pot_t(pq)(1-g)}{Pot_t - Pot_{t-1}} \qquad (5)$$

Like Matheson [3] noted, this equation didn't show the expected return; the expected return depends on the number of gamblers who win, as well. Hence we can use this equation as condition for multiple winners[3].

Fair bet conditions
To provide a fair bet two conditions must be satisfied. The first one is the pot growth condition; the pot must

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grow slow between drawings. If not the stakes increase and thus the number of tickets, and by this chance for multiple winners will rise.

$$Pot_{t-1} > Pot_t(1 + pg - p) \qquad (6)$$

The second condition, is the 'buy all' condition, the value of the pot is to be at least as big as the cost of playing all combination of the lottery.

$$\omega Pot_t \geq q \qquad (7)$$

If the condition of inequality (7) was met once it holds through the pot lifetime, but inequality (6) has be proven at each drawing.

German Lotteries

Now we check the conditions to find a fair bet in one type of state-run lotteries which operates in Germany, "6 aus 45 Auswahlwette". Until first of June 1985 there was an upper limit for winning sum of 3.000.000 DM for this lottery, so survey started on this date.

This lottery is an unpopular game in Germany, gamblers have to choose 6 out of 45 soccer games which end in a tie[4]. It also can be played by randomly picking numbers without knowing that this is not a normal lottery. According to data, part taken by the host is $g = 0.5$, part of the pot is $p = 0.4$, probability $w = 1/8, 145, 060$ and price $q = 0.65$€, so we can estimate the size of Pot_t and Pot_{t-1}.

$$Pot_{t-1} > 4, 235, 431$$
$$Pot_t \geq 5, 294, 289$$

As an example, we take a look at the 2007 super jackpot of Euro 6.3 million. Was this lottery a fair bet? For $g = 0.5, p = 0.4, w = 1/8, 145, 060$ and $q = 0.65$, equations (6) and (7) yield to

$$Pot_{t-1} > 6, 333, 002 \left(1 + 0.2 - 0.4\right) = 5, 066, 401$$
$$\frac{1}{8, 145, 060} 6, 333, 002 = 0.77 > 0.65$$

According to the glüXmagazin 09/04/2007 No.36 [4], the jackpot that turned over from the previous drawing was 5,839,683 € so both inequalities (6) and (7) were satisfied. And so, this was a fair bet.

Conclusion

The result presented in this paper suggest that it is not only theoretically but also practical possible for a fair bet in german state run lotteries.

Notes

[1]When are State Lotteries a Good Bet?

[2]If all possible combinations are played with same probability.

[3]We tested the equation for the jackpots of 6 out 49 Lottery during 1985 to 1991, all of the pots failed this condition and ended up with multiple winners. See althought [5]

[4]The rules of the game are described in detail in [6].

References

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