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Abstract

The Lucas and Romer production functions are currently the two most popular descriptions of the new growth theories in which knowledge is endogenous to economic growth and technological change. However, in both production functions the relationship between technology and human capital is implicit, and human capital is measured as an area. The latter is inconsistent with common understanding by which knowledge can be deep and wide and evolving over and in time, meaning human capital is 3D. This paper takes human capital as being 3D and frames it in Lucas and Romer production functions. Both functions bunch knowledge together, i.e., human capital and technological knowledge are strictly inseparable. Analysis and intuition find that if we take human capital to be a volume, the Lucas production function is most appropriate for use at the micro-economic level where emphasis is on production knowledge. At the macroeconomic level the Romer production function appears more reasonable, as knowledge is far more than just production knowledge. A limitation of the paper is how to actually calculate human capital as a volume and then to estimate the production functions empirically. The limitation represents a fruitful direction for future research.

Keywords: Knowledge, technology, 3D human capital, Lucas production function, Romer production function

JEL Code: Y1, C80, D60, D83, O15, O43

1. Introduction

Standard economic theory holds that the production possibilities of any economy depend on its technical capability. In turn technical capability depends on the current state of technology (A) and productive resources, primary among them land (R) and other natural resources, like the human population (N), labor (L), physical capital (K), human capital (H), and entrepreneurship (E). In classical and neoclassical theories of economic growth, R is fixed in quantity (Malthus, 1960) and quality (Ricardo, 2004), and therefore imposes a cost on variable factors in terms of diminishing marginal products (returns) - see Richard A. Easterlin (1977), Chad Jones (2004), Kosobud and O'Neill (1974), and Prettner (2011).[2] Although economists have known for a long time the role economic organizations play in economic activities as apparent from both Karl Marx (1906) and Max Weber (1947), it was not until Frank Knight's (1921) *Risk, Uncertainty and Profit* that economic theory appreciated fully the importance of E to long-run economic performance. But even then many continued to view E as an embodiment of L and K in alliance with management, ownership, and A. Since for the sole propriatorship the entrepreneur was often the worker, owner, and manager such a view was understandable.

As Eagly (1974) shows, the same blindspot occurred with regard to A despite the fact that A featured prominently in Karl Marx and F. Engels' *Communist Manifesto* (1848). In the classical production function Y depended on K made by L, which grew out N. From the physiocrats and early mercantalists N depended on the fixed supply of R. The Marxist L theory of value then suggested that the presence of diminishing returns implied an unavoidable struggle between K and L, a struggle, which L would win because L has a class consciousness that K did not have. The resulting class revolution would culminate into a 'proletariat dictatorship' (socialism), and begins another political confrontation this time between the State and workers. The worker-State struggle is necessary for a classless society (communism) to emerge according to Marx and Engels's stages (epochs) of historical development.

It took many years of development for neoclassical economic growth theory to introduce the direct role of A in the production function. Even so, the introduction viewed A only as exogenous "manna from the heaven" – so to speak (Solow, 1956, Swan, 1956, 2002). Moreover, while the concept of exogenous A was straightforward, issues surrounding its precise definition, measurement, the specific channels through which it affects Y, and how it is transmitted (transferred and diffused) have remained difficult. These issues became even more pronounced in light of the empirical findings that showed that the effect of A on Y was larger than the effects expected *a priori* of K, L, and R (Solow, 1957). Such findings stimulated research activities into two avenues. In one avenue researchers considered whether A was really more important to the production process than other factors, or its strength was a fluke due in part or whole to some specification bias (e.g., incorrect functional forms), or some mis-specification bias (i.e., a correct functional form that included incorrect variables and/or excluded correct variables). The second research looked into whether neoclassical growth theory

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[2] In the Malthusian sense population is not biologically fixed. In fact, because of the "passion between the sexes" population expands uncontrollably, and it is like weed which can overwhelm crop.

itself neglected to consider the endogenous nature of the growth process (cf. Cass, 1965, Koopmans, 1965).

As Nelson (1996, 2005) tells us the research activities taking place along the avenues just mentioned form the basis of at least two current versions of the new growth theory. One version, deriving from Solow (1960) and Arrow (1962), focuses on human capital (H) as a factor of production that has positive side-effects (externalities) on labor productivity – see, e.g., Solow (1997). Lucas (1988) is a fine example of this, although he has moved away at bit in Lucas (1993, 2009, 2011). The second version treats H, total factor productivity (TFP), and technological change interchangeably. In that case H is "disembodied knowledge as manifested in technology" (Messinis and Ahmed, 2008, p. 1).The view of H as a positive knowledge externality draws upon Schumpeter (1934, 1939, 1942, 2005), among others. Paul Romer (1990), Aghion and Howitt (1992), along with extension that utilize the Nelson and Phelps (1966) channels like Benhabib and Spiegel (1994), provide recent interpretations. Since there is already a huge literature review on that such as Tamura, Dwyer, Devereux, and Baer (2011), Jones (2004), Rogers (2003), Islam (2004), Parente (2001), and McCallum (1996), I use my time and space here to create an impression about the things that I find important [at least to me].[3]

For example, in a Solow economy growth beyond investment and capital is determined by exogenous technical change, the so-called Solow-residual, or technological constant, see, e.g., R.G.D. Allen (1967, Chapter 13), Phelps (1964, 1969), and Frank Hsiao (1968). Learning-by-doing models such as those by Solow (1960), Arrow (1962), Sheshenski (1967), Shell (1967), Kaldor (1966, 1961), and Kaldor and Mirrlees (1962) modified Solow *originale*. However, in these modifications too diminishing returns to net K ultimately occur. Barro and Sala-i-Martin (1990, 1991), and Mankiw, Romer, and Weil (1992), Romer (1986), all attempted to differentiate H from Solow's 'effective labor" and the Arrow learning effect. However, the process of K-accumulation continued to depend on the saving rate, and H-accumulation remained largely unexplained (see Tamura, 1988, Romer, 1989). Consequently the technological spill-overs and "spill-ins" suggested by these models tended to zero with convergence to some steady-state equilibrium.[4]

On the one hand, Neo-Solow growth models emphasized the determinants of technical change, especially where these postponed the onset of diminishing returns to K by way of H externalities,

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[3] The historical and current state of the growth theory is reviewed by prominent economists in the Winter 1994 issue of the *Journal of Economic Perspectives*, the May 1991 issue of the *Quarterly Journal of Economics*, as well as in Volume 32, No. 3 of the *Journal of Monetary Economics*, 1993. A good review of the traditional growth theory is Grossman and Helpman (1991, Chapter 2). Before these recent reviews I strongly recommend Sidney Weintraub's (1977) *Modern Economic Thought*, especially the essays in that book by Daniel Hamberg, Karl Shell, and A. Asimakopulos (see, pp. 329-393).

[4] I am making liberal use of the term "spill-ins" that is due to James Buchanan (1979, pp.115-142). "Spillouts" are outward externalities such as the effects of economics on "its scientific neighbors" and "spillins" are effects of neighboring sciences on economics. This is a nice way of indicating the direction of external effects without giving it an arithmetic sign.

taking place through trade for example or through the general policy environment (Grossman and Helpman, 1991). On the other hand Post-Solow or Neo-Schumpeter models considered technological change, technical progress broadly, to be endogenous with respect to intermediate goods and services (Romer, 1989, 1990, 1992, 1993, 1994, Grossman and Helpman, 1991, Lucas, 1993, 2009, cf. Lucas and Moll, 2011, Aghion and Howitt, 1992). A key difference between Solow and neo-Solow models on one side, and the post-Solow or neo-Schumpeter models on the other side is a matter of both focus and perspective. For instance, given a Solow Cobb-Douglas $Y=Ae^{\omega t}L^{\alpha}K^{\beta}$, traditional theory stresses ωt, whereas endogenous models would emphasize the determination of $Ae^{\omega t}$, the direction of factor bias, if any present, and the implications for factor substitutability and complementarity. So, from Neo-Solow models like Mankiw, Romer and Weil, $Y=F(K,Ae^{\omega t}L,H)$; for Arrow, Jones, and other and Barro, and others $Y=F(K,Ae^{\omega}t(K)L,H)$. Further discussion of these specifications are beyond the scope of this paper.

Again, since $\partial Y/\partial A>\partial Y/\partial X$, many in the endogenous growth community have interpreted such a finding as meaning that A can be measured as total factor productivity, the latter itself a function of some stock of unknown knowledge – "measure of our ignorance" (Abramovitz, 1986, cf. Choi, 1983).[5] Because a significant part of A is acquired through investment in knowledge building activities and processes like education, experience, and training, knowledge can be thought of as a function K^*, where $K^*=f(K,H)=H^{\beta_H}K^{\beta_K}$, so that

$$Y=AL^{\alpha}K^{*\beta}=AL^{\alpha}H^{\beta_H}K^{\beta_K}=AL^{\alpha}(s_iY)^{\beta}=AL^{\alpha}(s_HY)^{\beta_H}(s_KY)^{\beta_K},\ s_H=1-s_K, \tag{1}$$

where s is the Keynesian saving rate. In per capita terms, (1) depends mainly on K^*, i.e., $y=Ak^{*\beta}$, and this version is often called an AK model (Galor and Weil, 2000).

We assume that K depreciates at δ_K rate, but H appreciates at a net rate δ_H so that

$$Y=AL^{\alpha}(s_iY-\delta_iK^*)^{\beta}=AL^{\alpha}(s_HY\pm\delta_HH)^{\beta_H}(s_KY-\delta_KK)^{\beta_K}. \tag{2}$$

Normailzing (2) leads to $y=A(s_iy-\delta_ik^*)^{\beta}$, $\Rightarrow$ $\partial y/\partial k^*=\beta As_if_{k^*}-\delta_i=\beta y-\delta_i$. Hence, neoclassically the steady-state growth rate of y (gy) equals to the growth rate of k (gk), i.e.,

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[5] It is to say whether unknown knowledge is knowable or unknowable. However, Arrow (1969) says that "knowledge arises from deliberate seeking, but it also arises from observations incidental on other activities [such that production and investment may lead to increases in productivity without any identifiable allocation of resources to that end" (30).

$$gy=gk=A(s_i-\delta_i)=A(s_H\pm\delta_H)(s_K-\delta_K)=f(A,s), \tag{2'}$$

where δ_i is either according to Harold Hotelling (1925), or according to Trygve Haavelmo (1960). Regarding Haavelmo, K is a fraction of *actual Y* devoted toward the production of *potential Y* – an idea well espoused by Kalecki (1971). As such K has a maintenance cost and a replacement cost, in that its accumulation can increase maintenance cost and/or reduce replacement cost. The net effect may be to reduce the volume or quantity of K available for use in production. Hence, depreciation is really a function of use, and use depends on both technical factors such as resistence to wear and tear (accounting depreciation), and economic considerations such as the price of K, prices of what K produces, and the state of the general economy.[6] This is the foundation of the so-called "perpetual inventory method" by which $K_t=I_t+(1-\delta)K_{t-1}$, implying that

$$\delta_K=\frac{1}{K_{t-1}}(K_{t-1}+I_{t,K}-K_t)=1+i_K^*-k^*\approx\gamma K(t),\ 1=K_{t-1}/K_{t-1},\ i_K^*=I_K/K_{t-1},\ k^*=K/K_{t-1}, \tag{3}$$

where all variables are dated and $\gamma_K K(t)$ is Equation 23.3 in Haavelmo (1960, p.127). This perspective is consistent with H. Hotelling's (1925) theory of depreciation by which

$$K_t=\int_n^t\lambda\Phi\ dt+\rho S(n)\ \Rightarrow\ \delta_K=\frac{d}{dt}\Big[\int_t^n\lambda\Phi\ dt+\rho S(n)\Big], \tag{4}$$

and Φ is the surplus value of K net of its cost, and λ and ρ are discount factors, and $\rho S(n)$ is the salvage value of K. Thus, given the values of λ and ρ, one can demonstrate that $i^*=\frac{d}{dt}\big[\int_n^t\lambda\Phi\ dt\big]$, $\rho S(n)=(1-k^*)$, such that

$$\delta_K=\frac{d}{dt}\Big[\int_t^n\lambda\Phi\ dt+\rho S(n)\Big]=i_K^*+(1-k^*). \tag{5}$$

It is then logically tempting to think that $H_t=I_H+(1-\delta_H)H_{t-1}$, so that $\delta_H=(H_{t-1}+I_H-H_t)/H_{t-1}=1+i_H^*-h^*\approx\gamma_H H_t$. However, while the processes behind investment in physical capital (I_K) and human capital (I_H) are alike, they are not identical. The rate of investment in capital

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[6] This view suggests that depreciation is accounting plus economic depreciation.

K (i_K^*) is mainly private, and the rate of investment in H (i_H^*) can be either private, public (social), or both. Moreover, K largely depreciates, while H appreciates more than it depreciates and that is why we can write $1 \pm \delta_H$ in (2).

Amavilah (2014) shows that from the works of J. Mincer (1958, 1974, 1981, T.W. Schultz (1961, 1981, 1979), W.A. Lewis (1965), M. Blaug (1977, 1976), G.S. Becker (1993), and many others the appropriate foundation for H is N, but we return to that later. Now, let's step back to note that (1) and (2) derive from Frankel's (1962) specification in which A is the "development modifier" (cf. De la Croix and Michel, 2002, p. 124ff, Rebelo, 1991, Romer, 1989). From there Lucas (1988) interchanges A and H, and proxies H with years of schooling. However, since Lucas finds the effects of lower educational levels stronger than those of higher educational levels, the results suggested an inverted-U relationship between economic growth and H. Thus, the solution described in "On the Mechanics of Economic Development" raised more questions than it solved, and these led Lucas to a new and different model in which it is the proportion μH of H actually spend in production that matters rather than H measured as years of schooling.[7] This is comparable to the difference Hanushek and Kimko (2000), and Hanushek and Woessmann (2008) make between cognitive skills and affective skills. In this light the Lucas production function (LPF) would be,

$$Y = AL^{\alpha}K^{\beta_K}(\mu H)^{\beta_H}, \quad \alpha + \beta_K + \beta_H \leq 1, \tag{6}$$

where μH if the fraction of L actually engaged in production. From this understanding Lucas concluded that there was no such thing as miraculous growth in East Asia, because μH had larger and statistically stronger impact on economic growth there than H. In other words miracles were man-made.

The second version of the new growth theory, also deriving from Frankel's (1962) insight, assumes that A = H is driven by innovations. Paul Romer (1990) used the Dixit-Stiglitz (1977) production function to frame that idea by representing innovations as product varieties (cf. Dingel, 2009, Foltyn, 2012). This led him to the following Romer production function (RPF)

$$Y = L_Y^{\alpha}H_Y^{\beta}\int_0^A x(i)^{1-\alpha-\beta}dA \approx (AL_Y)^{\alpha}(AH_Y)^{\beta}(A\eta\bar{x})^{1-\alpha-\beta}, \tag{7}$$

where $(\eta\bar{x}) = K$, $1-\alpha-\beta=\beta_K$, and A modifies all factors of production – in that strict sense it is a true Frankel's "development modifier." Dinopoulos and Thompson (1996) suggested an implementable

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[7] Presumably people with good years of schooling, but who are not currently producing are not part of H. This presumption raises questions about what constitutes production.

version of the RPF, which confirmed that divergence is not only possible, it may be the only outcome of the endogenous growth process. However, Mankiw, Romer, and Weil (1992), and Barro and Sala-i-Martin (1992) found convergence, not divergence, and they concluded that the new growth theory does not work, or does not work as well as advertized.

One troubling aspect of the RPF was its assumption of unchanging product varieties. Grossman and Helpman (1991), and Aghion and Howitt (1992) modified Romer by allowing A to vary and to improve, and thus make possible the "creative destruction" among varieties, such that the modified RPF becomes

$$Y = L_i^{\alpha} \int_0^1 A(i)^{\beta} x(i)^{1-\alpha-\beta} di. \tag{8}$$

This version was flexible enough to capture both convergence and/or divergence, but it also seemed to imply that economies are simply independent innovation monopolies made possible by the existence of scale effects. For Grossman and Helpman convergence happened because one economy has access to a larger A_i than another that has A_j, i.e., $A_i > A_j$. However, Jones (1995) tested that hypothesis using data for the OECD countries relative to the US data and was not able to confirm convergence. He argued that the hypothesis that $A_i > A_j$ is in fact unrealistic because competition equalizes the A's. Aghion, Akcigit, and Howitt (2013) picked up from Jones by allowing $A_i = A_j$ transfer via trade, for instance, and instead of convergence they found parallel long-run growth. Given this literature Stephen Parente (2001) is justified in questioning the performance record of the endogenous new growth theory, and for concluding that the endogenous theory "... has not proven useful for understanding the most important questions why the whole world is not rich, [even as it] may prove useful for understanding growth in world knowledge over time" (p. 1).

Grossman and Helpman (1991, Chapters 1 and 2) provide an excellent description of the differences between the Lucas and Romer versions of the endogenous new growth theory, especially how each treats A and H. In both models A and H are clearly endogenous, and the H formulations are consistent with empirical work, see, e.g., Bils and Klenow's application (2000, Equation 4, p. 1163). However, while the modeling efforts are novel and even groundbreaking, the ideas about the endogeneity of A and H are not so new. For example, according to Arrow (1969) "knowledge arises from deliberate seeking, but it also arises from observations incidental on other activities" such that "production and investment may lead to increases in productivity without any identifiable allocation of resources to that end" (p. 30) – "learning-by-doing," as shown by Arrow (1962), Kaldor (1961, 1966), Haalvamo (1960), Fellner (1969), and many others. Hence, the production or investment activity generates Y as well as A and H that influence future activities, which to Arrow means that "deliberate ... expenditures [on the production of A and H] are actual steps in the [Y] production process" (p. 30).

In other words, A, H, and physical Y are simultaneously created (cf. Nicholson, 2002).[8] This insight is consistent with the Parente and Prescott's (1994, cf. Amavilah, 2014) characterization of *world knowledge* as *general knowledge* plus *scientific knowledge*.[9]

2. Theoretical Stuff

Using firm-level data Stephen Parente and Edward Prescott (1994) found that technology and technological change are key factors in long-run economic growth and development (cf. Nelson, 1996, 2005). However, technological change depends on investment in technology, and "the amount of investment required by a firm to go from one technology level to a higher technology level depends on two key factors: the level of general and scientific knowledge in the world and the size of barriers to adoption in the firm's country" (p. 299). They dubbed the sum of general knowledge and scientific knowledge "world knowledge" and assumed it to be given and "to grow exogenously."

Characterizing A and H

Assume there exists a Parente-Prescott (1994) *world knowledge* (A), consisting of *general knowledge* (A_g) and *scientific knowledge* (A_s). A_s is made up of technological knowledge (A_τ) and human capital (H). Following Amavilah (2014) we can say that A_g depends on N, which is a function of R and time (t), i.e., $A_g = f(N(R,t))$, so that[10]

$$A = A_g + A_s = A_g + (1 + A_\tau)H. \tag{9}$$

However, we know from Hayek (1937, 1945, 1974) that A_g and A_s are interrelated. To account for the interactions between A_g and A_s, let (9) be multiplicative, i.e.,

$$A = [A_g(1 + A_\tau)H = \Phi e^{nt} H = f(A,H). \tag{9.1}$$

As described in Amavilah (2014) Bowles and Gintis (2000, 2002) have shown conceptually that A has inherited and self-acquired components and that, because A_g growth rate is *n*, (9) can be rewritten as

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[8] Both Haalvamo and Nicholson state that H building is one of those activities that is both consumption and production; one has to consume knowledge in order to build it.

[9] The italics are Parente and Prescott's terms.

[10] All variables are timed, but for convenience often I have ignored the time subscript.

$$A=[(\bar{A}_g+\psi\Delta A_g+e_A)N_0e^{nt}][(1+A_\tau)H]=[(1+A_\tau)(A_gN_0)e^{nt}]H=\Phi e^{nt}H,$$
$$\Phi=(1+A_\tau)A_g=(1+A_\tau)(A_gN_0). \tag{9.2}$$

where $\bar{A}_g$ is the mean initial stock of A_g, ψ is the intergenerational coefficient of A_g inheritance, and $1-\psi$ is the coefficient of the distance $A_g-\bar{A}_g$. An alternative to (9.2) is to say an economy's A is a product of the level of A at t = 0 (A_0) and the cumulative human development index (HDI), or

$$A = A_0e^{\int mdt}=A_0e^{x};\ x=\int mdt \equiv HDI. \tag{10}$$

Then from Anand and Sen (1994), and Sen (1997, cf. Temple and Johnson, 1998) we know that $x\equiv HDI=H^cY^d$, where H is the human capital index of the population, Y is the material conditions of the population, and $c=2/3$ and $d=1/3$ are weights. Algebraically

$$Y=(x\cdot H^{-c})^{1/d},\ c,d>0. \tag{11.1}$$

Now we can think of $x=f(L,K)=AL^\alpha K^\beta$, such that

$$Y=[AL^\alpha K^\beta H^{-c}]^{1/d}. \tag{11.2}$$

From (9.2) $A=\Phi\exp(nt)$, and assuming H = μH, (11.2) becomes the following LPF:

$$Y=[\Phi e^{nt}L^\alpha K^\beta(\mu H)^{1-c)}]^{1/d}. \tag{12.1}$$

Dividing through by L and taking the natural logarithms of both sides of (12.1) we get per worker Y as

$$y=\varphi_0+\eta t+\beta_kk+\beta_h[\mu h], \tag{12.2}$$

where $y=\ln(Y/L)$, $\varphi_0=1/d\ln\Phi$, $\eta=n/d$, $\beta_k=\beta/d$, $\beta_h=(1-c)/d$, $k=\ln(K/L)$ and $\mu h=\ln(\mu H/L)$. Similarly, the RPF would be

$$Y=[(\Phi e^{nt})^{\alpha+\beta-c}L^{\alpha}K^{\beta}H^{\alpha+\beta-2c}]^{1/d}, \tag{13.1}$$

which upon normalizing and natural-logging gives per labor Y as

$$\begin{gathered} y=\varphi_0^*+\eta^* t+\beta_k^* k^*+\beta_h^* h \\ \varphi_0^*=(\alpha+\beta-c)/d\ln\Phi,\ \eta^*=[(\alpha+\beta-c)n]/d,\ \beta_k^*=\beta/d,\ \beta_h^{=}(\alpha+\beta-2c)/d. \end{gathered} \tag{13.2}$$

If we let $k=h=1$, from (12.2) and (13.2) $gy=\eta+\beta_k+\beta_h$ (Lucas), and $gy=\eta^*+\beta_k^*+\beta_h^*$ (Romer), for $\delta = 0$. Alternatively, $gy=\eta+(1-\delta)(\beta_k+\beta_h)$ (Lucas), and $gy=\eta^*+(1-\delta)(\beta_k^*+\beta_h^*)$ (Romer), for $\delta > 0$. Moreover, we can see that $A=\Phi\exp(nt)=A_{Solow}$ is a Hicks neutral Solow technological constant (residual), $A=[\Phi\exp(nt)]^{1/d}=A_{Lucas}$ is a μH-biased Lucas technological constant, and the Romer technological constant is $A=[\Phi\exp(nt)]^{(\alpha+\beta-c)/d}=A_{Romer}$, and it is K^**-leaning*. All this means $gy_{Lucas} \neq gy_{Romer}$. Taken as a technological constant (residual), and without any interpretation $A_{Solow}>A_{Lucas}>A_{Romer}$, *ceteris paribus*.

Characterizing L and K

We do not delve into the processes behind L and K. The latter remains controversial as demonstrated by the socalled "Cambridge K controversy." A number of famous authors outline the controversy in a 2003 issue of the *Journal of Economic Perspectives* (cf. Amavilah and Newcomb, 2004, Taylor, 2000, Eatwell, Milgate, and Newman, 1990). The literature on the nexus between N and economic growth is also as clear as it is plentiful, see, e.g.,Chad Jones (2001), Richard Easterlin (1977), P. Banerjee (2011), K. Prettner (2011), P.F. Peretto (1998), Pitchford (1972), Galor and Weil (2000), and Simon (1977). So, we can assume L and K to increases at rates of growth of N and investment (I) in K, respectively, so that

$$L=\int_0^t L_0 e^{nt}dt,\quad K=\int_0^t K_0 e^{rt}dt, \tag{14}$$

which means $L_0=N_0(1-e^{-\zeta n})$, $\zeta n\neq 0 \Rightarrow L=[N_0(1-e^{t-\zeta})]/n,\ N_0=N\ at\ t=0.$ Hence, at the aggregate level we can plug the above into the LPF and/or RPF. However, doing so would complicate our interest in per capita growth rate; it is easy and practical to assume full L employment, i.e., L = 1, so that given $dI/dt=rI\Rightarrow I=I_0e^{rt}$, at t = 0 $K_0=[I_0(1-e^{-r\zeta}]/r\Rightarrow K=[I_0(1-e^{r(t-\zeta)}]/r,\ r\zeta\neq 0$. Again, theoretically one can plug (14) into (12) and (13), and the result would be Post-Keynesian, more precisely it would be Keleckisque, suggesting that K is *actual Y*, produced by L and devoted towards the production of *potential Y*.[11]

Characterizing H Based on L

Amavilah (2014) illustrates that much of the current H measures are flawed. They over-emphasize the education (years of schooling = S), training, and experience of L, and thereby suggesting that

$$H=e^{\varphi S}L=e^{\varphi S}\int_{0}^{t}L_0e^{nt}dt=e^{\varphi S}[N_0(1-e^{n(t-\zeta)})]/n. \tag{15}$$

However, in that sense (15) views H as just another Solow's "effective labor" – labor is just a smart machine. Hence, it over-estimates the value of H in production and under-states its value in the creation, accumulation, and transfer of all knowledge. Not only is L too narrow a foundation for H, it also overlooks essential interactions and intra-actions between A and H, and among all factors of production. A second problem has been that these measures nearly all represent H as an area.

Characterizing H based on N

The 'true' Mincerian measure of H must have a base larger than L in order to capture both the production and innovation aspects of all knowledge as opposed to just production knowledge. Drawing upon the original theory of H, Amavilah (2008, 2014) has argued that H depends, at the very least, on the quality (*q*) of the working-age population (N*), if not on all N, developed through investment in education, training, experience, health, migration, among many possibilities.[12] Thus, instead of (15)

$$H_t=e^{\varphi_i q_{it}}N_t^*=e^{\varphi_i q_{it}}N_0^*e^{nt}=N_0^*e^{\varphi_i q_{it}+nt}, \tag{16}$$

where N_0^* is N_t^* at t = 0, L < N* < N, and N* is growing at the same rate n over time as N. Now if we insert (16) into (12) and (13) above, and do some careful manipulations, normalizing using N as the numeraire, and taking natural logs, we obtain the intensive forms of LPF and RPF as follows:

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[11] However, doing that is unnecessarily complicated for practical purposes. This is one reason why Amavilah (1997), Amavilah and Newcomb (2004), Newcomb (1976), Harris (1992), and others favor (10) above.

[12] It is a separate issue that investment itself may be of domestic source, FDI, foreign aid, and/or other transfers. If investment is of a foreign source, a variety of H and A transfer models exist such as Javanovic and Nyarko (1997), Arrow (1969), and my personal favorite is Leontief (1966).

$$
\begin{aligned}
&\dot{y}=\dot{\varphi}_0+\dot{\eta}t+\dot{\beta}_k\dot{k}+\dot{\theta}\dot{q},\ \dot{y}=\ln(Y/N),\\
&\dot{\varphi}_0=1/d[\ln\Phi+\ln(\mu\eta^*)]=\ln(A_{Lucas}),\ \dot{\theta}=(-c\theta)/d,\quad \dot{\eta}=-(c(\eta+\delta)-\eta)/d,\\
&\dot{y}=\dot{\varphi}^*+\dot{\eta}^*t+\dot{\beta}^*\dot{k}^*+\dot{\theta}^*,\quad \dot{\varphi}^*=\ln(A_{Romer}),\\
&\dot{\beta}^*=\beta^*/d,\ \dot{\theta}^*=(\varphi(\alpha+\beta-2c))/d, \dot{\eta}=((\alpha+\beta-2c)(\eta+\delta))/d.
\end{aligned}
\tag{17}
$$

In terms of growth rate, assuming *k, h*, and *q* are all constant, $gy_{Lucas}=\dot{\eta}+\dot{\beta}_k+\dot{\varphi}\neq gy_{Romer}=\dot{\eta}^*+\dot{\beta}_k^*+\dot{\varphi}^*$. And again, scrutinizing (9) above would show that A_g is a declining function of time. Since A_g depreciates fast, its level is falling even with positive learning, i.e., the effect of the rate of depreciation are larger than the effect of the rate of learning. On the other hand, $(1+A_\tau)H$ appreciates more than it depreciates, although we understand from Arrow (1969) that its expansion cannot be infinitely exponential.

Characterizing the Interactive and Intra-active Nature A and H

Following W.A. Lewis (1965) and P.M. Romer (1993), Amavilah (2005) shows that resource intra-actions and inter-actions are important to technological change and long-run economic growth. Economies with strong positive resource intra- and inter-actions perform better than those that are weakly interactive and intra-active. This is not entirely new; Becker's (1993) earnings function is clearly interactive (see Equations 76, p. 104, and 2, p. 261). As a further example, Robin Grier (2002, 2005) estimates the impact of K on H to be about 0.48 and that of H on K to be about 0.21 for Sub-Saharan African countries, and in Latin American countries he finds that uneducated people fare worse than educated people at learning, innovating, and assimilating new knowledge. Similarly, Paul Romer (1993) predicts that interactions characterize the flow of ideas from industrialized countries to developing countries via multinational corporations. There are other researchers on this same topic of resource interactions like Philippe Aghion and Howitt (2006), Chad Jones (2006), and Jones and Romer (2009), Graca, Jafarey, and Philippopoulos (1995), and Benhabib and Spiegel (1994, 2002), Gemmell (1995), and so on. However, in all of these brilliant efforts H is measured as an area and the interactions and intra-actions are not explicit. Leaning for support on Harris and Pan (2000, pp. 134-137), Amavilah (2005) claims that technological change can be represented as a *any*-variable correlation coefficient like $\rho_{KHAN}=K\cap H\cap A\cap N$, where any is four, signifying resource interactions and intra-actions such that $A=e^{\int\rho_{KHAN}}dt=e^{\rho_{KHAN}}+Constant$, where $Constant=A_g$, $e^{\rho_{KHAN}}=(1+A_\tau)H$, see Amavilah (2014).

H is a volume, not an area

One incomprehensible fact remains that economists continue to measure H *as an area under the curve of either the quantity of L skills, or the quality of L skills. R*epresenting H by years of schooling is an example of the quantity measure of H, call it X(t), and using cognitive skills is an example of the quality dimension of H, name it Z(t). In this way H is either the area under X(t) or Z(t) , i.e.,

$$
\begin{aligned}
&H=\pi[f(t)]^2=\pi\frac{(L_x^2 e^{2c_1t})}{(a_1e^{c_1t}+b_1)^2} \quad (a)\\
&H=\pi[f(t)]^2=\pi\frac{(L_z^2 e^{2c_2t})}{(a_2e^{c_2t}+b_2)^2}, \quad (b)
\end{aligned}
\qquad (18)
$$

where $a_i > 0$, $b_i \neq 0$, and the subscripts 1 and 2 are for X and Z, respectively.

Clearly (18) is both *imprecise* and *inconsistent with the common understanding and language of knowledge, because knowledge is multidimensional – it is at least 3D.* We commonly speak of solid knowledge, deep knowledge, wide/broad knowledge, and even intimate knowledge.[13]In other words, solid knowledge has depth, width (breadth), and the time *over* and *in* which it evolves. Over time the quantity and quality of knowledge evolve semi- independently. *In time* there exists a negative correlation between the quantity and quality dimensions of H. Hence, *estimating H as an area over-states its importance in production over time, and under-states its role in other related spheres in time.*

The idea of H as a volume is natural, commonsensical, and demonstrable. Building H is just like developing a mineral [ore deposit] (Amavilah, 2008, 2014). In both cases one starts with a stock of inherited knowledge about the existence and extraction of the mineral. We have knowledge inheritance of varying degrees of quantity and quality. Thus, H building is really mining the fraction N^* of N for quality (q), and that quality accumulates into H – not unlike accumulating gold flakes into gold bars. This perspective is consistent with early theories of H like those of Schultz (1961), Blaug (1979), Becker (1993), and others, al of which suggest that the basis for H is population – not L. Again, H as a function of L overestimates its importance in production *over time*, and underestimates its role in H creation and diffusion *in time* (cf. Barro and Lee, 2013, Barro, 1991). In fact, this may be the reason why H is found to be low in developing countries where L is a tiny fraction of N^*. Obviously, just because a large part of L in developing countries is unemployed, should not necessarily mean H is low. It is possible, indeed likely, that H accumulation happens under conditions of high unemployment, even as its rate may slow down under such conditions.[14]

The analogy between mining population and mining land is also appropriate because it makes it possible for us to utilize quantity-quality models familiar to mining and geological engineering as well

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[13] W. Arthur Lewis (1965) saw little economic value in intimate knowledge, because "knowledge does not grow rapidly if it is kept as a secret for the few" (p. 167). However, Leonardi and Meyer (2014) have shown that knowledge can be sticky sometimes so that initimate knowledge facilitates networks which serve as "social lubricant" in the process of knowledge transfer.

[14] It is known that unemployment many people go back to school, retrain, and/or migrate such that they gain additional H via education, training, and migration even as they do not get H through work experience.

as mineral economics (DeVerle P. Harris ,1976, 1984, 1985, 1992a, b, 1993). The Lasky (1950) among such models is particularly commonsensical in associating average quality (grade) with the log of cumulative quantity of a resource . In the case in point,

$$\log H = \varphi q + nt + \log N_0^* \;\Rightarrow\; q = \frac{1}{\varphi}\log H - \frac{n}{\varphi}t - \frac{1}{\varphi}\log N^*. \tag{19}$$

Eq. (19) = (17), and it allows us to view H in its 3D in which on one axis we have X(t), the other axis has Z(t), and X(t) and Z(t) evolve semi-independently *over,* and interact interdependently *in,* time under a constraint imposed by the nature and structure of the economy in line with Arrow (1969, cf. Rogers, 2003, Young, 2004, 2005, 2007, Amavilah, 2003, 2007, 2008, Jovanovic and Nyarko, 1995, Andersen and Jensen, 2013, Mansfiled, 1971, Kamien and Schwartz, 1977, Buchanan, 1979).[15] The constraint itself can be shifted outwards and within it an economy accumulates H over time, but the upper limit still constrains the logistic growth of X(t) and Z(t).

In other words, (19) is still an inadequate description of H, because H grows logistically *over time,* but *in time* X(t) and Z(t) are inversely related in that the higher the average quality (q) of N^*, the smaller the cumulative quantity of N^*, and hence H, is needed for the economic activity. The functional form of the inverse relationship can be either quadratic, linear (Lasky), or power. Here let us say it is of the general form $Z(t)=f(X(t),t)\equiv X(t)=f(Z(t),t)$. Then H is a volume rather than an area, and to compute that volume we assume that X(t) and Z(t) follow logistic growth curves, each with an upper limit, say, $L_x = L_z = 1$, imposed by the economic constraint. Some fraction of H would be inherited (folk = ordinary) quantity of A_g, some would be inherited quality of A_g. Again, unlike conventional theory of H we assume that the *quantity and quality of knowledge evolve semi-independently over time, but interdependently in time.*[16] Using the Lasky set-up, in which

$$Z(X(t)) = 1 - kX(t) = \frac{(a_1 + b_1 e^{-c_1 t} - k)}{(a_1 + b_1 e^{-c_1 t})}, \tag{20.1}$$

and assuming the interactions between X(t) and Z(t), we can find the volume of the area under Z(X(t)) = 1- kX(t) as

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[15] I read the working paper versions of Young's papers, and I advise the reader to find out if newer, perhaps cleaner, versions of the same papers were published since then.

[16] The specific functional forms themselves may not be identical, i.e., one may be a simple logistic such as the Pearl-Reed, others may be Gompertz, or any other in the same family of exponential functions, see, e.g., D.W. Thompson (1992[1942]).

$$VZ(X(t))=\int_t \frac{\pi}{3}\left(\frac{(a_1+b_1e^{-ct}-k)}{(a_1+b_1e^{c_1t})}\right)^2 dt. \tag{20.2}$$

Further V(X(t))- V(Z(t))-V(Z(X(t)) interactions lead to the true H is a volume, $\mathcal{H}$, as

$$\begin{aligned}\mathcal{H}=&\int_t \pi\left(\frac{L_x}{(a_1+b_1e^{-c_1t}}\right)^2 dt+\int_t \pi\left(\frac{L_z}{a_2+b_2e^{-c_2t}}\right)^2 dt+\int_t \frac{\pi}{3}\left(\frac{a_1+b_1e^{-ct}-k}{a_1+b_1e^{-c_1t}}\right)^2 dt\\ &\pm\int_t 3\pi\left[\frac{L_x(a_2+b_2e^{-c_2t})+L_z(a_1+b_1e^{-c_1t})+(a_1+b_1e^{-c_1t}-k)(a_2+b_2e^{-c_2t})}{3(a_1+b_1e^{-c_1t})(a_2+b_2e^{-c_2t})}\right]^2 dt\end{aligned} \tag{21}$$

In brief: The precise nature and magnitude of $\mathcal{H}$ would depend on the volumes of its components, their interactions, functional forms, and whether such forms are independent, identical, and/or symmetrical. Obviously, such things are conceptually easier to imagine than to describe rigorously. Economically thinking, because interactions and intersections are important, it is quite reasonable for the whole volume to be equal to, greater, or smaller than the sum of its parts. However, that should not discourage estimating H in a commonsensical way as the following approximate solution to (21):

$$\begin{aligned}\mathcal{H}=\mathit{Constant}^*&+\left[\frac{\pi((a_1e^{c_1t}+b_1)\log(a_1e^{c_1t}+b_1)+b_1)}{a_1^2c_1(a_1e^{c_1t}+b_1)}\right]\\ &+\left[\frac{\pi((a_2e^{c_2t}+b_2)\log(a_2e^{c_2t}+b_2)+b_2)}{a_2^2c_2(a_2e^{c_2t}+b_2)}\right]\\ &+\left[\frac{\pi}{3}\left(\frac{b_1k^2}{a_1^2c_1(a_1e^{c_1t}+b_1)}+\frac{k(k-2a_1)\log(a_1e^{c_1t}+b_1)+t)}{a_1^2c_1}\right)\right].\end{aligned} \tag{22}$$

Looking at (22) one may ask where the last term of (21) went. The answer is that, whereas X(t) and Z(t) evolve independently *over a common time*, the relationship between X(t) and Z(t) takes place interdependently *in time*. In that case the last two terms of (21) are constants and as such parts of the term Constant*. So, Constant* has three components: (i) the inherited quantity of H, (ii) the inherited quality of H, and (iii) the relevant interactions and intersections between the two.

3. Practical Stuff: Production Functions with H as $\mathcal{H}$

The preceding section mainly showed that $H \neq \mathcal{H}$. Plugging $\mathcal{H}$ into the LPF or RPF would give us

$$Y = [\Phi_i L^{\alpha} K^{\beta} \mathcal{H}^{-c}]^{1/d}, \; i = Lucas, \; Romer. \tag{23}$$

This means that even if we assume that Constant* = 0, H = $\mathcal{H}$ only under two unlikely scenarios. One, in which the second, third, and fourth terms of (22) are zero and $\mathcal{H}$ measures only the quantity dimension of H. The other scenario would require the first, third, and fourth terms of (22) to be zero. I let the reader examine the equilibrium stability of factor marginal productivities in (23). Here I add quickly and in passing that for specific Lucas and Romer technological constants and $\mathcal{H}$, the LPF and RPF with conventional H change to:

$$\begin{aligned} &Y = [A_{Lucas} L^{\alpha} K^{\beta} (\mu\mathcal{H})^{1-c)}]^{1/d} \qquad (a) \\ &Y = [A_{Romer} L^{\alpha} K^{\beta} \mathcal{H}^{\alpha+\beta-2c}]^{1/d} \qquad (b). \end{aligned} \tag{24}$$

Eq.(24) can be estimated with some ease, but there are only a number of empirical issues to consider. First, is finding common units of measurement of say cognitive skills and years of schooling. This is not an impossible issue to deal with as skills can be expressed in years. Second, calculating H would be a little problem for lack of readily available data. Third, at the production level the foundation for H based on L, and the LPF appears more appropriate than the RPF. At the national aggregate level when N is the basis for H, the RPF is more suitable than the LPF. In both cases, though, the precise measure of H is $\mathcal{H}$. Fourth, statistical problems are likely the most challenging, but they are common to all aggregate production functions.

5. Concluding Remark

Measuring H as an area is a mistake. The proper way of measuring H is to think of it as a volume, with its own depth, breadth/width, and the time over and in which depth and width evolve. This paper explores H as a volume and frames it in the Lucas and Romer production functions. We conjecture from analysis that numerically H as a volume may be equal to, smaller, or greater than H as an area. However, the implications are quite different. The Lucas framework stresses H primarily as production knowledge, whereas in Romer H is more than just technological knowledge, although knowledge is clearly endogenous to the processes of economic growth and technological change in both cases. A weakness of all of this that the validity of the conjecture remains untested empirically, and that is the direction for further research.

References[17]

Abramovitz, M 1986 Catching-up, forging ahead, and falling bek=hind, Journal of Economic History 46(2):385-406.

Aghion, P, Akcigit, U, and Howitt, P 2013 What did we learn from Schumpeterian growth theory? http://scholar.harvard.edu/files/aghion/files/what_do_we_learn_0.pdf.

Aghion, P. and P. Howitt 1992 A model of growth through creative destruction, *Econometrica* 60 (2): 323-351.

Aghion, P. and P. Howitt 2006 Joseph Schumpeter lecture: appropriate growth policy: a unifying framework, *Journal of European Economic Association* 4(2-3): 269-314.

Allen, RGD 1968 *Macroeconomic Theory: A Mathematical Treatment*. New York: St Martin's Press.. Chapter 13.

Amavilah, VH 2008a The inhibited (exhibited) spread of innovations. http://mpra.ub.uni-muenchen.de/8993/1/MPRA_paper_8993.pdf.

Amavilah, VH 2007 Innovations spread more like wildfires than like infections. http://mpra.ub.uni-muenchen.de/3958/1/MPRA_paper_3958.pdf.

Amavilah, VH 2008b *Modeling Determinants of Embedded Economies*. New York: Nova Science Publishers.

Amavilah, VH 2005 Resource intra-actions and interactions: Implications for technological change and economic growth. http://129.3.20.41/eps/ge/papers/0508/0508004.pdf.

Amavilah, VH 2009 Knowledge of African countries: Production and value of doctoral dissertations, *Applied Economics* 41(7-9): 977-989.

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[17] This list of references is in no particular style or format. It may also be incomplete in some areas or over-complete in others. If anyone's idea is excluded that must be included, or included that must be excluded, my sincere apologies along with all due acknowledgment and the usual caveat.

Amavilah, Voxi Heinrich and Newcomb, Richard T. 2004 Economic Growth and the Financial Economics of Capital Accumulation under Shifting Technological Change: http://ideas.repec.org/p/wpa/wuwpge/0404001.html.

Amavilah, VH 1996 *Resources, Technology, and Mineral Trade in the Economic Growth of Namibia*. Ann Arbor: UMI (PhD Dissertation).

Anand, S. and AK Sen 1994 Human Development Index: methodology and Measurement. UNDP.

Andersen, TB and PS Jensen 2013 Institutions and growth acceleration. Discussion Papers on Business and Economics No. 7/2013.

Arrow, KJ 1974 Higher education as filter, *Journal of Public Economics* 2: 193-216.

Arrow, KJ 1969 Classificatory notes on the production and transmission of technological knowledge, *American Economic Review* 59(2): 29-35.

Arrow, KJ 1962 The economic implications of learning by doing, *Review of Economic Studies* 29(June):155-173.

Banerjee, R. 2011 Population growth and endogenous technological change: Australian economic growth in the long run. MPRA Paper No. 30892 (March). http://mpra.ub.uni-muenchen.de/30892/.

Barro, R. 1991 Economic growth in a cross-section of countries, *Quarterly Journal of Economics*, CVI(2): 407-443.

Barro, R and J. Lee 2013 A new data set of educational attainment in the world, 1950-2010. NBER Working Paper 15902 (April). http://www.nber.org/papers/w15902

Barro, R and Sala-i-Martin 1992 Convergence, *Journal of Political Economy* 100(2): 223-51.

Becker, GS 1993 *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education*, Third Edition. Chicago: University of Chicago Press.

Becker, GS., EL. Glaeser and KM. Murphy 1999 Population and economic growth, *American Economic*

Review, Papers and Proceedings (May):145-149.

Becker, GS 1981 *A Treatise on the Family*. Cambridge: Havard Univesity Press. Chapter 6.

Becker, G. 1992. Habits, addictions and traditions, *Kyklos* 45: 327-345.

Benhabib, J. and M. Spiegel 1994 The role of human capital in economic development: Evidence from aggregate cross country data, *Journal of Monetary Economics* 34(2): 143-173.

Benhabib, J and M. Spiegel 2002 Human capital and technology diffusion. FRBSF Working Paper #2003-02. http://www.frbsf.org/economic-research/files/wp03-02bk.pdf.

Bils, M and PJ Klenow 2000 Does schooling cause growth? *American Economic Review* 90(5): 1160-1183.

Blaug, M 1976 The empirical status of human capital theory: A slightly jaundiced survey, *Journal of Economic Literature* 14(3): 827-855.

Blaug, M 1979 The quality population in developing countries, with particular referecne to education and training, In *World population and Development: Challenges and Prospects*. Philip M. Hauser (Editor). New York: Syracuse University Press, pp. 361-402

Bowles, S. and H Gintis 2000 The inheritance of economic status: Education, class and genetics. http://www.santafe.edu/media/workingpapers/01-01-005.pdf.

Bowles, S and H. Gintis 2002 The inheritance of inequality, *Journal of Economic perspectives* 16(3): 3-30.

Buchanan, JM 1979 *What Should Economists Do?* Indianapolis: Liberty Fund.

Barro, Robert J., and Xavier Sala-i-Martin (1995): Economic Growth.
McGraw-Hill, New York.

Cass, D 1965 Optimum growth in an aggregative model of capital accumulation, *Review of Economic Studies*, 32: 233–240.

Choi, K 1983 *Theories of Comparative Economic Growth*. Ames: Iowa State University. Chapters 3 and, pp. 62-119.

De la Croix, D, and Michel, P 2002 *A Theory of Economic Growth – Dynamics and Policy in Overlapping Generations*. Cambridge (UK): Cambridge University Press

Dingel, JI 2009 The basics of "Dixit-Stiglitz lite" . http://www.columbia.edu/~jid2106/td/dixitstiglitzbasics.pdf.

Dinopoulos, E., and P Thompson 1996 A contribution to the empirics of endogenous growth, *Eastern Economic Journal* 22(4): 389-400.

Dixit, A and JE Stiglitz 1977 Monopolistic competition and optimum product variety, *American Economic Review* 67(3): 297-308.

Eagley, RV 1974 *The structure of classical economic theory*. New York: Oxford University Press.

Easterlin, RA 1977 Population: The economics of the long run, in *Modern Economic Thought*, Sidney Weintraub (ed). University of Pennsylvania Press. pp. 485-499.

Eatwell, J, Milgate, M and Newman, P (eds) 1990 *The New Palgrave Capital Theory*. New York: W.W. Roton & Company.

Eicher, T 1996 Interaction between endogenous human capital and technological change, *Review of Economic Studies* 63: 127-144.

Fellner, W 1969 Specific interpretations of learning by doing, *Journal of Economic Theory* 1(2): 119-140.

Foltyn, R 2012 Results from the Dixit/Stiglitz monopolistic competition model. http://www.foltyn.net/wp-content/uploads/2009/12/dixitstiglitz.pdf. .

Foray, D 2004 *The Economics of knowledge*. Cambridge: MIT Press.

Frankel, M 1962 The production function in allocation and growth: A synthesis. *American Economic*

Review LII(5): 966-1022.

Galor, O., and D. Weil 2000 Population, technology, and growth: From the Malthusian regime to the demographic transition and beyond, *American Economic Review* 90(4): 806-828.

Gemmell, N. 1995 Endogenous growth, the Solow model and human capital, *Economics of Planning* 28(2-3): 169-183.

Graca, J., S. Jafaray, and A. Philippopoulos, 1995 Interaction of human and physical capital in a model of endogenous growth, *Economics Planning* 28(2-3): 93.-118.

Grier, R 2002 On the interaction of human and physical capital in Latin America, *Economic Development and Cultural Change* 50(4): 891-913.

Grier, R 2005 The interaction of human and physical capital accumulation: Evidence from Sub-Saharan Africa, *Kyklos* 58(2): 195-211.

Grossman, G and E Helpman 1991 *Innovation and growth in global economy.* Cambridge (MA): MIT Press.

Haavelmo, T 1960 *A study in the theory of investment.* Chicago: University of Chicago Press.

Hall, RE and CI Jones 1999 Why do some countries produce so much more output per worker than others? *Quarterly Journal of Economics* 114(1): 83-116.

Hanushek, EA and DD. Kimko 2000 Schooling, labor-force quality, and the growth of Nations, *American Economic Review* 90(5): 1184-1208.

Hanushek, E and L. Woessmann 2008 The role of cognitive skills in economic development, *Journal of Economic Literature* 46(3): 607-668.

Harris, DP 1984 *Mineral Resource Appraisal.* Oxford: Clarendon press.

Harris, DP 1992a Long-term forecasting for mineral industries: Innovative methods and case studies, Unpublished Lecture Notes, Mineral Economics Program, Department of Mining and

Geological Engineering, University of Arizona (January).

Harris, DP 1992b Technology, Productivity, and Learning. Unpublished Lectures. Tucson AZ:University of Arizona.

Harris, DP 1993 Mineral resource stocks information in Allen V. Kneese and Jones L. Sweeny (Eds) *Handbook of Natural Resource and Energy Economics*, Volume III, North Holland.

Harris, DP 1976 Quantitative Economic Analysis Applied to the Mineral Industries: Computer Applications. In *Economics of the Mineral Industries*, 3rd Edition, William A. Vogel (Ed.): New York: American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc.: 345-360.

Harris, DP 1985 Mineral Resource Information, Supply, and Policy Analysis. In *Economics of the Mineral Industries*, 4rd Edition, William A. Vogel (Ed.). New York: American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc.: 181-224.

Harris, DP and G Pan 2000 *Information Synthesis for Mineral Exploration.* New York: Oxford University Press.

Hayek, FA 1945 The use of knowledge in society, *American Economic Review*, 35, 4, pp. 519-530.

Hayek, FA1974 The pretense of knowledge. Nobel Prize Lecture (December 11). http://www.nobelprize.org/nobel_prizes/economics/laureates/1974/hayek-lecture.html.

Hayek, FA 1937 Economics and Knowledge, *Economica*, 4(February): 386-405.

Hotelling, H 1925. A general mathematical theory of depreciation, *Journal of American Statistical Association* 20: 340-353.

Hsiao, Frank (1968) Technical progress and the prouction function, a synthesis," In Papers in Quantitative Economics !. James P. Quirk and Arvis M. Zarley (Editiors). Lawrence: University Press of Kansas, pp. 381-398.

Islam, N. 2004 New growth theories: What is in there for developing countries? *Journal of Developing Areas* 38(1): 171-210.

Javanovic, B. and Y. Nyarko 1997 The transfer of human capital, *Journal of Economic Dynamics and Control* 19: 1033-1064.

Jones, CI 2004 Population and ideas: A theory of endogenous growth. NBER Working Paper No. 6285. http://www.nber.org/papers/w6285population. See a recent version at http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.61.4939&rep=rep1&type=pdf.

Jones, CI and PM Romer 2010 The new Kaldor facts: Ideas, institutions, population, and human capital, *American Economics Journal: Macroeconomics* 2(1): 224-245.

Jones, CI 1995 R&D-based models of economic growth, *Journal of Political Economy*, 103(4): 759-84.

Jones, CI 1997 Convergence revisited, *Journal of Economic Growth* 2(June): 131-153.

Jones, CI 2006 Knowledge and the theory of economic development, Version 0.25. http://www.chicagobooth.edu/research/workshops/AppliedEcon/archive/WebArchive20052006/ChadJonesPaperApril19Spring06.pdf.

Kaldor, N.1961 Capital accumulation and economic growth. In *The Theory of Capital*, F.A. Lutz and D.C. Hague (Eds.) New York: St. Martin's Press.

Kaldor, N. and J. Mirrlees 1962 A new model of economic growth, *Review of Economic Studies*, 39(2):174-192.

Kaldor, N. 1966 *Causes of the Slow Rate of Growth in the United Kingdom: An Inaugural Lecture*. London: Cambridge University Press.

Kalecki, M 1971 *Selected Essays on the Dynamics of the Capitalist Economy, 1933-1970*.Cambridge: Cambridge University Press. Chapter 10.

Kamien, MI and NL Schwartz 1977 Technology: More or less in *Modern Economic Thought*, Sidney Weintraub (ed). University of Pennsylvania Press. pp. 501-515.

Keynes, John (1936) *The General Theory of Employment, Interest, and Money*. San Diego/New York: Harvest/HBJ Book. Book 7, Chapter 24, p. 383.

Keynes, JM 1936 *The general theory of employment, interest, and money.* New York: Harcourt, Brace and Company.

Knight, FH 1921 *Risk, Uncertainty, and Profit*. Boston (Mass): Hart, Schaffner & Marx (Houghlin Mifflin Company).

Koopmans, TC 1965 On the concept of optimal economic growth, in (Study Week on the) *Econometric Approach to Development Planning*, chap. 4, pp.
225–87. Amsterdam: North-Holland Publishing Co.

Kosobud, RF. and WD. O'Neil 1974 A Growth Model with Population endogenous,
American Economic Review, (May): 27-32.

Lasky, SG 1950 How Tonnage and Grade Relations Help Predict Ore Reserves, *Engineering and Mining Journal*, 151, 81-85.

Leonardi, PM and SM Meyer 2014 Social Media as Social Lubricant: How Ambient Awareness Eases Knowledge Transfer (May 5). *American Behavioral Scientist*, Forthcoming. Available at SSRN: http://ssrn.com/abstract=2433286

Leontief, W. 1966 *Essays in Economics: Theories and Theorizing*. New York: Oxford University Press. Chapter XVII: 200-222.

Lewis, WA 1965 *The Theory of Economic Growth*. New York: Harper Torchbooks.

Lucas, RE 2009 Ideas and growth, *Economica* 76(301): 1-19.

Lucas, RE and .B. Moll 2011 Knowledge growth and the allocation of time. NBER Working Paper No. 17495 (October).

Lucas Jr., R 1988 On the mechanics of economic development, *Journal of Monetary Economics* 22(1) 2-42.

Lucas Jr., R 1993 Making a Miracle, *Econometrica* 61(2) 251-272

Malthus, TR 1960 *On Population* (Gertrude Himmelfarb, ed). New York: Modern Library.

Mankiw, NG, D Romer and D Weil 1992 A contribution to the emperics of economic growth, *Quarterly Journal of Economics* 107(2): 407-437.

Mansfield, E 1971 *Technological change*. New York: W.W. Norton & Company Inc.

Marx, K 1906 *Capital: A Critique of Political Economy*. New York: Modern Library.

Marx, and F. Engels 1848 *The Communist Manifesto.* New York: Vintage (Classics) Books.

McCallum, BT 1996 Neoclassical vs. endogenous growth analysis: An overview, *Federal Reserve Bank of Richmond Economic Quarterly*, 82($): 41-71.

Messinis, G., and AD Ahmed 2008 Valuable skills, human capital and technology diffusion. Working Paper No. 38. Centre for Strategic Economic Studies, Victoria University, Melbourne (April). http://www.cfses.com/documents/wp38.pdf.

Mincer, J 1974 *Schooling, Experience, and Earnings*. New York: Columbia University Press.

Mincer, J 1958 Investment in human capital and personal income distribution, *Journal of Political Economy* 66(4): 281-302.

Mincer, J 1981 Human capital and economic growth. NBER Working Paper No. 803. Cambridge: National Bureau of Economic Research.

Nelson, R and ES Phelps 1966 Investments in humans, technological diffusion, and economic growth, *American Economic Review* 56 (1/2): 69-75.

Nelson, R 2005 *Technology, Institutions and Economic Growth*. Cambridge: Harvard University Press.

Nelson, RR 1996 *The sources of economic growth.* Cambridge (Mass): Harvard University Press.

Newcomb, RT 1976 Modeling growth and change in the American coal industry, Growth and Change, 10(1):111-127.

Newcomb; Richard, and Tsuji, Karl (1990) "The price of gold", *Mineral Processing and Extractive Metallurgy Review: An International Journal*, 1547-7401, 6 (1 & 4), pp. 1– 42.

Nicholson, W 2002 *Microeconomic Theory.* 8th Edition,. New York: South-Western Thomson Learning.

Parente, S. and E. Prescott 1994 Barriers to technology adoption and development, *Journal of Political Economy* 102: 298-321.

Parente, S. 2001 The failure of endogenous growth, *Knowledge, Technology, and Policy* 13(4): 49-58.

Peretto, P. 1998 Technological change and population growth, *Journal of Economic Growth* 3(4): 283-311.

Phelps, E. 1964 Models of technical progress and the golden rule of research. Cowles Foundation Discussion Paper No. 176. Yale University. https://www.researchgate.net/publication/4743289_Models_of_Technical_Progress_and_the_Golden_Rule_of_Research.

Phelps, Edmund [Editor] (1969) *The Goal of Economic Growth: An introduction to a current issue of public policy.* New York: W.W. Norton and Company, Inc.

Pitchford, J. 1972 Population and optimal growth. *Econometrica* 40(1): 109-136.

Prettner, 2012 Population and aging endogenous growth. *Journal of Population Economics*: DOI: 10.1007/s00148-012-0441-9. http://link.springer.com/article/10.1007/s00148-012-0441-9#page-1

Rebelo, ST 1991 Long-Run Policy Analysis and Long-Run Growth, *Journal of Political Economy*, 99(3): 500–521.

Ricardo, D 2004 *The Principles of Political Economy and Taxation* (F.W. Kolthammer). New

York: Dover Publications, Inc.

Rogers, M 2003 A survey of economic growth, *Economic Record*, 79(244):112-135).

Rogers, E 2003 *Diffusion of Innovations*, 5th Edition. London: Free Press.

Romer, Paul (1993) Idea gaps and object gaps in economic development, *Journal of Monetary Economics*, 32(3), pp. 543-573.

Romer, PM 1986 Increasing Returns and Long Run Growth, *Journal of Political Economy*, 94: 1002–37.

Romer, PM 1989 Human Capital and Growth: Theory and Evidence (November). NBER Working Paper No. w3173. Available at SSRN: http://ssrn.com/abstract=227284

Romer, PM 1994 The origins of endogenous growth, *Journal of Economic Perspectives* 8(1): 3-22.

Romer, PM 1990 Endogenous technological change, *Journal of Political Economy* 98(5), S71-S102.

Rosen, S 1983 Specialization and human capital, *Journal of Labor Economics* 1(1): 43-49.

Schultz, TW 1981 *Investing in People: The Economics of Population Quality*. Berkeley: University of California press.

Schultz, TW 1961 Investment in human capital, *American Economic Review* LI(1): 1-17.

Schultz, Theodore W. (1979) "Investing in population quality throughout low-income countries," In *World population and Development: Challenges and Prospects*. Philip M. Hauser (Editor). New York: Syracuse University Press, pp. 339-360

Schumpeter, JH 1934 The theory of economic development. New York: Transaction Publishers.

r

Schumpeter, JH 2005 Development, *Journal of Economic Literature*, 43(1): 108-120.

Schumpeter, JH 1939 *Business cycle*. New York: McGraw-Hill.

Schumpeter, JH 1942 *Capitalism, Socialism and Democracy*. New York: Harper & Row.

Sequeira, TN, and A. Ferreira-Lopes 2011 An endogenous growth model with human and social capital interactions, *Review of Social Economy* 4(December): 465-493.

Shell, K 1967 A Model of Inventive Activity and Capital Accumulation, in *Essays on the Theory of Optimal Economic Growth* (K. Shell, ed.), Cambridge, Massachusetts: MIT Press, 1967, Chapter IV, 67-85.

Sheshenski, E. 1967 Optimal Accumulation with Learning By Doing, in Karl Shell (ed.) *Essays on the Theory of Optimal Economic Growth.* Cambridge (Mass): MIT Press.

Simon, JL 1977 *The Economics of Population Growth*. Princeton: Princeton University.

?Smith, A 1952[1937] *An Inquiry into the Nature and Causes of the Wealth of Nations* (ed. Edwin Cannan; introducer Max Lerner). New York: Modern library. Book II, pp. 259-269.

Solow, RM 1994 Perspectives on growth theory, *Journal of Economic Perspectives* 8(1): 45-54.

Solow, Robert (1997) *Learning from 'Learning by Doing': Lessons for economic growth.* Stanford: Stanford University Press.

Solow, RM 1960 Investment and Technical Progress, in Mathematical Methods in the Social Sciences, (Eds.) K. J. Arrow, S. Karlin and P. Suppes, Stanford University Press, pp.89-104.

Solow, RM 1956 A contribution to the theory of economic growth, *Quarterly Journal of Economics* 70 (1): 65-94.

Solow, RM 1957 Technical change and the aggregate production function, *Review of Economics and Statistics*, 39() 312-320.

Sowell, T 1996 *Knowledge and Decisions*. New York: Basic Books.

Swan, T 1956 Economic growth and capital accumulation, *Economic Record* 32: 334-61.

Swan, T 2002 Economic growth, *Economic Record* 78: 375-80.

Tamura, Dwyer, 2011 Economic growth in the long-run.
http://www.iser.osaka-u.ac.jp/seminar/2011/Tamura_Jan_17.pdf.

Tamura, R 1988 Fertility, Human Capital and the 'Wealth of Nations.' Ph.D. dissertation, Univ. Chicago.

Taylor, LD 2000 *Capital, accumulation, and money*. New York: Springer.

Temple, J. And PA Johnson 1998 Social capability and economic growth, *Quarterly Journal of Economics* 113(3): 965-990.

Thompson, DW 1992[1942] *On Growth and Form*. The Complete Revised Edition. New York: Dover Publications, Inc. Chapter III.

UNDP (2007/2008) Technical Note 1: Calculating the human development indices:
http://hdr.undp.org/en/media/HDR_20072008_Tech_Note_1.pdf. .

Weber, M 1947 *The theory of social and economic organization*. The Free Press of Glencoe.

Weintraub, S 1977 *Modern Economic Thought*. University of Pennsylvania Press.

Young, HP 2005 The spread of innovations by social learning, Working Paper (February 17, 2006 Version). Available at: http://www.econ.jhu.edu/People/Young/Spread28march.pdf

Young, HP 2004 Innovation diffusion and population heterogeneity, Preliminary Draft, Johns Hopkins University (October 20, 2006 Version). Available at: http://www.econ.princeton.edu/seminars/SEMINARS/Fall06Seminars/young.pdf

Young, HP 2007 Innovation diffusions in heterogeneous populations. Discussion Paper Series, No. 303. Department of Economics, University of Oxford.

http://www.economics.ox.ac.uk/Research/wp/pdf/paper303.pdf

Young, Allyn A 1928 Increasing returns and economic progress, *Economic Journal*, 38: 527-542.

Young, A 1991 Learning by doing and the dynamic effects of international trade, *Quarterly Journal of Economics*, CVI(2): 369-405.