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Embodiment, Productivity, and the Age Distribution of Capital

An important theme in modern research on productivity has been that technological progress may be embodied in capital in the sense that traditional measures of TFP growth reflect unmeasured improvements in the quality of capital inputs as well as pure disembodied technological progress. It is commonly believed that an implication of this embodiment hypothesis is that there should be a negative relationship between measured TFP and the age of the measured capital stock. This paper presents empirical evidence which suggests that an increase in the age of the capital stock is actually associated with higher TFP growth. This surprising result may be due to the presence of a mis-measurement normally overlooked in this literature: With mis-measured improvements in capital quality, the usual depreciation rates used to construct empirical capital stocks are incorrect for growth accounting. This effect dominates the usual average age effect.

1 Introduction

An important theme in modern research on productivity has been that technological progress may be "embodied" in capital in the sense that traditional growth accounting measures of total factor productivity (TFP) likely reflect unmeasured improvements in the quality of capital inputs as well as pure disembodied technological progress. For example, while the U.S. National Income and Product Accounts (NIPAs) incorporate some attempts to adjust investment data to reflect the higher quality of recent expenditures, it is widely believed that these adjustments do not fully capture improvements in the quality of new capital. This consensus is based in part on Robert Gordon's (1990) extensive study, which suggested substantial biases in NIPA prices for durable equipment; Gordon's alternative price indexes have formed the basis for a number of recent papers aimed at calculating the contribution of embodied technological progress to economic growth.[1] However, Gordon's bias estimates are likely subject to significant sampling and specification uncertainty, and his price series finish in 1983, making them difficult to compare with current U.S. price indices which have incorporated a number of statistical improvements since then. Thus, the likely size of the mis-measurement of capital quality, and its implications for the sources of productivity growth, remains a subject of active debate and research.

One well-known approach to assessing the embodiment hypothesis is based on examining the link between measured TFP and the *age distribution* of capital. This method relies on the idea that, if the embodiment hypothesis is true, then standard growth accounting exercises will underestimate the effect that recent investment has on current productivity relative to older investment. This idea has often been tested empirically using an approximate relationship derived by Richard Nelson (1964), who showed that embodiment implied that measured TFP growth should be negatively correlated with changes in the average age of the measured capital stock. For example, Edward Wolff (1991, 1996) has presented estimates of the relationship between TFP and the average age of capital for the U.S. economy, and argued that these estimates imply that embodiment effects played an important role in the post-1973 productivity slowdown.[2]

This paper uses both aggregate and industry-level U.S. data to re-examine the relation-

[1] See for example, Greenwood, Hercowitz, and Krusell (1997). More recently, Cummins and Violante (2002) calculate updated measures of capital input based on applying a more detailed set of bias estimates from Gordon's study to an updated sample.

[2] Other studies that have estimated the average age effect on TFP include McHugh and Lane (1983, 1987) and Sakellaris (2001)

ship between TFP and the age distribution of the capital stock. I show that the correlation between aggregate TFP growth and changes in the age of the stock is weak, and argue that the causality in this relationship is difficult to interpret. The estimated negative correlation comes from the decline in TFP growth after the mid-1970s coinciding with a slowdown in investment which ended a long period of decline in the average age of the capital stock: The causality in this relationship could run solely from TFP growth to investment to the average age, rather than in the other direction. However, using industry-level data a strong and surprising pattern emerges, which appears to be robust to concerns about causality. Contrary to the usual intuition, an increase in the average age of capital appears be associated with *higher* measured TFP, with the estimated effect being consistent with unmeasured technological *regress* in investment of about 4 percent per year. More generally, measured TFP growth appears to be negatively correlated with recent investment growth.

These empirical results represent something of a puzzle because they appear to question the validity of the embodiment hypothesis of un-measured improvements in capital quality. In the final section of this paper, I provide an explanation that could potentially reconcile these results with the existence of embodiment. This explanation starts from the observation that the traditional result of a negative relationship between measured TFP and the age of capital is derived under the assumption of only one type of mis-measurement, namely mis-measurement of the productive effect of various units of investment. However, as an empirical matter, embodiment turns out to also imply another type of mis-measurement, which is that the traditional depreciation schedules used to construct published capital stocks are not the correct schedules required for the construction of the correct productive capital stock. Specifically, the schedules used to construct published stocks place too little weight on old units of investment. I show that, in general, this latter type of mis-measurement outweighs the traditional average age effect, implying a positive relationship between measured TFP and the age of capital.

The contents of the paper are as follows. Section 2 derives a general relationship between observed TFP growth and the age distribution of capital using the traditional assumption that published depreciation rates are the correct rates for the construction of the productive capital stock. Section 3 presents the evidence on this relationship, and Section 4 outlines our potential explanation for the counter-intuitive empirical results obtained, re-deriving the relationship between TFP and the age distribution taking into account how the published depreciation rates have been constructed.

2 Growth Accounting with Measurement Error

This section derives the relationship between measures of TFP growth and the age distribution of capital under the usual assumption that the only mis-measurement due to embodiment relates to the measurement of the true productive effect of new units of investment relative to older units.

Preliminaries: Empirical growth accounting calculates the growth rate of TFP using the formula:

$$z_Y = z_{TFP} + \alpha z_K + (1-\alpha) z_L \tag{1}$$

where z_Y is the growth rate of output, z_{TFP} is the growth rate of TFP, $1-\alpha$ is the labor share of income, and z_K and z_L are the growth rates of capital and labor input. For a particular measure of the growth rate of capital input, z_K^m, TFP growth is constructed as

$$\begin{aligned} z_{TFP}^m &= z_Y - \alpha z_K^m - (1-\alpha) z_L \\ &= z_{TFP} + \alpha \left(z_K - z_K^m \right) \end{aligned} \tag{2}$$

Thus, measured TFP growth depends on both actual TFP growth (true disembodied technological change) as well as a term reflecting the effect of mis-measured growth in capital input.

The true capital stock series is defined as

$$K(t) = I(t) + (1-\delta) I(t-1) + (1-\delta)^2 I(t-2) + ... \tag{3}$$

where the parameter δ measures the decline in the productive capacity of a unit of capital as it ages. The growth rate of this stock can be written as

$$z_K(t) = w_0(t) z_I(t) + w_1(t) z_I(t-1) + w_2(t) z_I(t-2) + ... \tag{4}$$

where $z_I(t)$ is the growth rate of investment and

$$w_n(t) = \frac{(1-\delta)^n I(t-n-1)}{K(t-1)} \tag{5}$$

In other words, capital stock growth is a weighted average of current and past growth rates of investment, where the weights are defined by the share of capital of various ages in last period's stock.

Now we suppose that there is some form of mis-measurement of the true effect of a unit of investment on the productive capital stock. In this case, there exists a separate "measured" version of the capital stock, which we will denote with a superscript m. We will assume for the moment that this measured capital stock can be defined as:

$$K^m(t) = I^m(t) + (1-\delta) I^m(t-1) + (1-\delta)^2 I^m(t-2) + ... \tag{6}$$

Thus, the measured stock differs from the true productive capital stock in being derived from a mis-measured investment series, $I^m(t)$. (Note, however, that for the moment we are assuming that the measured stock uses the same productive decay rate, δ.) As before, the growth rate of the measured capital stock is

$$z_K^m(t) = w_0^m(t) z_I^m(t) + w_1^m(t) z_I^m(t-1) + w_2^m(t) z_I^m(t-2) + ... \tag{7}$$

where

$$w_n^m(t) = \frac{(1-\delta)^n I^m(t-n-1)}{K^m(t-1)} \tag{8}$$

Given these derivations, we can write the difference between the true growth rate of the capital stock and the measured growth rate as

$$z_K(t) - z_K^m(t) = \sum_{n=0}^{\infty} \left[w_n(t) z_I(t-n) - w_n^m(t) z_I^m(t-n) \right] \tag{9}$$

Mis-Measurement and The Age Distribution: Given the formulas derived above, we can consider the relationships between the true and measured stocks under specific functional assumptions concerning the relationship between true and productive investment. Our assumption will be that true productive investment grows γ percent per period faster than measured investment

$$z_I(t-n) = z_I^m(t-n) + \gamma \tag{10}$$

This formulation was chosen to be consistent with the existing empirical evidence concerning the mis-measurement of the quality of new investments in capital goods. Because there is little controversy over the measurement of *nominal* investment spending (these outlays are well captured by various Census Bureau surveys), this debate has focused on the measurement of price deflators for investment. And the results of studies such as those of Robert Gordon (1990) have generally concluded that the nature of such biases is such that the published investment price indexes probably overstate the true quality-adjusted rate of price inflation for investment goods by a certain average amount per year. Assuming that the data on nominal investment expenditures are correct, this translates directly into a statement about the growth rates of the true and measured investment series, as in equation (10).

In defining the "embodiment hypothesis" in this sense as essentially a measurement issue, the approach taken here may seem somewhat different from other papers which discuss the influence of embodiment in productivity. Such studies often view the measured capital stock series as simply representing "non-quality-adjusted" capital, with all improvements in capital quality showing up in measured TFP. However, it is worth stressing that the BLS and BEA have always made at least *some* attempts to adjust for quality change. For instance, care has always been taken when product specifications change, such as with motor vehicle model changes. And these attempts have been stepped up in recent years. So, ultimately the relevant question is whether these quality adjustments to measured stocks have gone far enough.

Given this assumption about the measurement of investment, the gap between true and measured capital stock growth rates can be written as

$$\begin{aligned} z_K(t) - z_K^m(t) &= \sum_{n=0}^{\infty} \left[w_n(t) \left(z_I^m(t-n) + \gamma \right) - w_n^m(t) \, z_I^m(t-n) \right] & (11) \\ &= \gamma + \sum_{n=0}^{\infty} \left[\left(w_n(t) - w_n^m(t) \right) z_I^m(t-n) \right] & (12) \end{aligned}$$

and measured TFP growth can be written as

$$z_{TFP}^m(t) = z_{TFP}(t) + \alpha\gamma + \alpha \sum_{n=0}^{\infty} \left[\left(w_n(t) - w_n^m(t) \right) z_I^m(t-n) \right] \tag{13}$$

This equation shows how, in addition to depending on disembodied technological progress and the investment measurement error, measured TFP growth will also depend upon the *age distribution of capital.* In particular, measured TFP growth will be positively correlated with recent investment growth as long as $w_n(t) > w_n^m(t)$ for low values of n. This inequality should hold when there is mis-measurement of quality improvement for capital goods. The weights on recent investment growth reflect the relative importance of recent investment in the capital stock; since productive investment has been growing faster than measured investment, the w_n will generally be higher than w_n^m for low values of n.

Steady-State Example: Although the weights w_n and w_n^m will generally be complicated functions of the full history of past investments, they can be derived analytically for the special case in which true productive investment grows at a constant rate, g. Because the growth rate of the true productive capital stock is a weighted average of current and past growth rates of investment, this will also grow at rate g. Using the fact that, by definition, the growth rate of the true capital stock is

$$z_K(t) = \frac{I(t)}{K(t-1)} - \delta \tag{14}$$

we get an expression for the ratio of investment to the lagged capital stock:

$$\frac{I(t)}{K(t-1)} = g + \delta \tag{15}$$

From this, we can derive the weight for current investment growth as

$$w_0 = \frac{I(t-1)}{K(t-1)} = \frac{I(t)}{K(t-1)} \frac{I(t-1)}{I(t)} = \frac{g+\delta}{1+g} \tag{16}$$

More generally, we can write these weights as

$$w_n = \frac{(1-\delta)^n I(t-n-1)}{K(t-1)} = \frac{g+\delta}{1+g} \left(\frac{1-\delta}{1+g} \right)^n \tag{17}$$

The weights for the measured capital stock can be derived in a similar fashion. Measured investment grows at rate $g - \gamma$, as does the measured capital stock because it is a weighted average of current and past investments. Following the same logic as before, we get

$$w_n^m = \frac{g+\delta-\gamma}{1+g-\gamma} \left(\frac{1-\delta}{1+g-\gamma} \right)^n \tag{18}$$

Both sets of weights decline monotonically and sum to one, with the weights for the measured capital stock declining slower. This automatically implies that, for low values of n we have $w_n(t) > w_n^m(t)$ while the higher values of n we have $w_n(t) < w_n^m(t)$. These steady-state calculations are obviously illustrative. However, numerical simulations verify that even when investment growth fluctuates around a steady-state path the weights derived here provide a good approximation to the average weights in measured and true capital growth for the various lags of investment growth. An important implication, then, is that when measured investment has been growing fast recently, the true capital stock will be growing faster than the measured stock.

Average Age Approximation: Richard Nelson (1964) derived a compact approximation to the relationship between measured TFP and the age distribution of capital. This is derived as follows. The true productive capital stock can be expressed as

$$K(t) = \sum_{v=-\infty}^{t} (1-\delta)^{t-v} I(v) \tag{19}$$

which can be re-written in terms of measured investment as[3]

$$K(t) = \sum_{v=-\infty}^{t} (1-\delta)^{t-v} (1+\gamma)^{v} I^{m}(v) \tag{20}$$

This can be approximated as

$$\begin{aligned} K(t) &\approx (1+\gamma)^{t} K^{m}(t) \sum_{v=-\infty}^{t} \frac{(1-\delta)^{t-v} I^{m}(v)}{K^{m}(t)} (1+\gamma v - \gamma t) \\ &= (1+\gamma)^{t} K^{m}(t) \left(1 - \gamma \sum_{v=-\infty}^{t} \frac{(1-\delta)^{t-v} I^{m}(v)\,(t-v)}{K^{m}(t)}\right) \end{aligned} \tag{21}$$

Note that the term represented by the summation is the average age of the measured capital stock, i.e. it is a weighted sum of integers with the weights determined by the share of capital of each age in the measured capital stock. So, this equation can be re-written as

$$K(t) \approx (1+\gamma)^{t} K^{m}(t)\,(1-\gamma a(t)) \tag{22}$$

Taking log-differences and assuming that γ is relatively small, this yields the following approximation for the growth rate of the true productive capital stock

$$z_K(t) \approx z_K^m(t) + \gamma - \gamma \Delta a(t) \tag{23}$$

Thus, from equation (2) we see that a regression of measured TFP growth on the change in the average age of capital should yield a negative coefficient, and the size of the coefficient should approximately equal $\alpha\gamma$.

[3]This equation assumes that the growth rate of true investment is

3 Empirical Evidence

We now present some empirical work which assesses the embodiment hypothesis in line with the analysis just presented, starting with aggregate data and then moving on to examine industry-level data.

3.1 Aggregate Data

Figure 1 displays the aggregate U.S. data for TFP growth and the average age of the capital stock. The figure shows a pattern previously discussed by Wolff (1991, 1996): The period prior to the mid-1970s saw strong TFP growth on average and a steady decline in the average age of the capital stock, while the period since has seen weaker TFP growth on average and a relatively flat profile for the age of the stock.[4] However, the actual relationship suggested by the embodiment hypothesis—between aggregate TFP growth and $\Delta a(t)$—is quite weak. The correlation between these series is only -0.16 and, as can be seen from the first column in Table 1, when TFP growth is regressed on $\Delta a(t)$ the estimated coefficient, although negative, is not statistically significantly different from zero.[5]

From the point of view of testing the embodiment hypothesis, an obvious concern about this regression is that the correlation it describes may not even come from a causality running from changes in the age of capital to TFP growth. The productivity slowdown associated with slower TFP growth after the mid-1970s likely had important general equilibrium effects, with the slower growth in potential output leading to a slowing pace of capital investment which, in turn, led to the flattening of the average age of the capital stock. Given the simultaneity of all aggregate variables, it is hard to think of convincing instruments that would allow an IV-based solution to this causality problem. However, the second column of Table 1 does show that once we add a dummy variable for the post-1973 period, the estimated coefficient on $\Delta a(t)$ switches from negative to positive (although again not

[4] The series for TFP growth is for the nonhousing nonfarm business sector; this is widely agreed to be the most appropriate series for aggregate productivity analysis. The series was downloaded from the Bureau of Labor Statistics website at http://www.bls.gov/mfp/home.htm. The average age of the capital stock was downloaded from the Bureau of Economic Analysis website at http://www.bea.doc.gov.

[5] One necessary aside about data issues: The BLS series on TFP growth are constructed on a *year-average* basis. However, the BEA's average age series refer to *year-end* capital stocks. To produce comparable series for our regressions, I used averages of the current and previous year-end average ages to measure $a(t)$; however, for this and subsequent regressions the use of the unaveraged BEA series produces results that are very similar to those reported here.

statistically significant). While admittedly a crude way of assessing the potential causality problem, this regression does at least show that there is little evidence of the hypothesized negative relationship between TFP growth over and above the correlation related to the post-1973 productivity slowdown.

Finally, the third column of Table 1 adds GDP growth to the regression to check whether this sign-switching result is robust to controlling for the well-known pattern of procyclicality of TFP growth; the results show that it is, with the coefficient on $\Delta a(t)$ again being positive, although still not significantly so. Together, these results suggest that it is difficult to interpret even the weak relationship between aggregate TFP and the age of the capital stock as being consistent with the causal link suggested by the embodiment hypothesis.

3.2 Industry-Level Data

We now turn to the industry-level data. Specifically, we examine the relationship between TFP and the age distribution of capital for a range of U.S. manufacturing industries at the two-digit SIC code level. The use of industry-level data is likely to have a number of advantages in this context. The first advantage is the additional number of datapoints. Measured TFP growth is affected by actual disembodied technological progress and other forms of mis-measurement in addition to mis-measured capital input, so we would not necessarily expect to find a very strong statistical relationship between any single set of time series for TFP and the age of capital. In light of this problem, the sizeable number of observations provided by the industry-level data is helpful.

The second advantage relates to the causality problem discussed above for aggregate data. For the aggregate economy, there is likely to be a strong causal relationship running from TFP to investment, and thus to the average age of the stock. However, given the important role that aggregate conditions play in determining the capital investment of any specific industry, the causal link between industry-specific TFP and industry-specific investment is likely to be significantly weaker. This factor likely explains why, as will be discussed below, the relationships estimated from industry-level data do not suffer from the same problems of interpretation of causality as the aggregate regressions, with the results being robust to the inclusion of the dummy variable for the productivity slowdown.

Finally, it is worth noting that the exercise reported here shares some similarities with the recent work of Stiroh (2002), who relates the same set of series on industry-level TFP growth to measures of the intensity of computer usage. As with these tests, Stiroh's focus is on testing a hypothesis of the existence of a type of mis-measurement not allowed for in the growth accounting exercises, in his case the existence of production externalities due to computer usage.

Data: The industry-level TFP figures used here come from the Bureau of Labor Statistics (BLS) multifactor productivity program and were downloaded from the BLS website.[6] The restriction of our focus to manufacturing industries reflects the limited availability of estimates of TFP for other sectors. Industry-level investment data were obtained from the website of the Bureau of Economic Analysis (BEA); these same data were used to construct the measures of capital input underlying the BLS measures of multifactor productivity.[7] Matching these two data sources, there are 17 industries for which we have both TFP and investment data. The average age of industry capital stocks, used in some of the regressions below, also comes from the BEA website. Finally, the sample is 1950-1999.[8]

Average Age Regressions: Table 2 presents the results from regressions relating industry-level TFP growth to changes in the average age of the industry's capital. These regressions were estimated using the Seemingly Unrelated Regressions (SUR) technique, with each equation containing a constant fixed-effect term which allows for differences in the average rate of TFP growth across industries. The first column reports the coefficients on the change in the average age, both for the case where each industry has a separate coefficient on the average age and for the pooled case in which the coefficient is estimated to be the same across industries (the bottom row). As noted above, we wouldn't necessarily expect to find a very strong statistical relationship in these regressions. Nonetheless, if the embodiment hypothesis is correct we should expect to find a pattern of negative coefficients, implying that increases in the average age of the capital stock reduce TFP growth.

Strikingly, however, the regressions reveal the exact opposite pattern. Fifteen of the seventeen industry-specific regression coefficients are positive, with a number being highly

[6] At the time of writing, the URL for this site is http://www.bls.gov/mfp/home.htm

[7] These data were downloaded from http://www.bea.doc.gov/bea/dn/faweb/

[8] One potential concern here comes from the fact that the capital stocks used in the BLS calculations are not identical to the BEA capital stocks underlying the average age series. Specifically, the BEA uses geometric depreciations while the BLS uses a slightly non-geometric pattern for its "productive decay" schedules. In practice this makes little difference. The BLS calibrates its schedules to be roughly consistent with the BEA depreciation rates, so the BLS and BEA stocks are very similar. See http://www.bls.gov/web/mprcaptl.htm for a description of the BLS methodology.

statistically significant and most being significant at least at the 15 percent level. The bottom row reports the results from restricting the coefficients on the change in the average age to be the same across all industries. Perhaps surprisingly, this restriction is not rejected: A likelihood ratio test produces a χ^2_{16} statistic of 12.3, significant only at the 72 percent level. The estimated pooled coefficient is 0.013 with a standard error of 0.002. Assuming a capital share of 0.33, the framework of the previous section tells us that this estimate is consistent with technological *regress* of 3.9 percent per year!

Columns 2 and 3 of Table 2 check whether these results are robust to the inclusion of a dummy variable for productivity slowdown and GDP growth. They show that, unlike the aggregate results which switched sign when DUM74 was added, the industry-level estimates are essentially unchanged by the addition of this variable. A number of the coefficients are estimated to be more positive than before and the pooled coefficient is now 0.016 (standard error 0.003), consistent with technological regress of almost 5 percent per year. Column 3 adds GDP growth and produces results closer to those in column 1 with a pooled coefficient of 0.012 (standard error 0.003). Again, the restriction of the coefficients on $\Delta a(t)$ being equal across all industries cannot be rejected for either of these regressions.

One interesting question about these results is whether they are stable over time. If the theoretical analysis of the previous section is correct, then the coefficient on the age of the capital stock should change over time if the amount of mis-measurement of capital input has varied. Given that the introduction of improved methodologies seems likely to have reduced mis-measurement more for recent observations than for older ones, such a pattern might be expected. However, Figure 2 shows that the main result is very stable across sub-samples. The figure shows that pooled coefficients on the age variable from the Column 3 regression (featuring the 1974 dummy and output growth) for a recursive set of samples starting with 1947-1975 and adding one observation for each period up to the full sample of 1947-1999. The figure shows very little variation in the estimated coefficient, with all samples producing a significantly positive value.

Investment Regressions: We next consider a slightly less restrictive approach to assessing the embodiment hypothesis by following equation (13) in regressing measured TFP growth on current and lagged values of investment growth. Table 3 reports results from a regression containing contemporaneous investment growth as well as four lagged values. As before, the second and third columns augment the base specification with the post-1973 dummy (the second column) and with this dummy and GDP growth (the third column). Given the difficulty in estimating and reporting each of the investment growth coefficients for all industries, only the pooled estimates are reported here.

The results of the previous section showed that, according to the usual derivations, we should expect the coefficients in this regression to be positive for recent values of investment growth and then to turn negative at longer lags. However, in line with the results from the average age regressions, almost all of the estimated coefficients are negative and many are statistically significant. Extending the regression to allow for longer lags produces a similar pattern for the near-term coefficients but turns up no noticeable pattern for the coefficients on the more distant lags.

Of course, if a pattern of unmeasured technological regress did exist, equations (17) and (18) imply that the coefficients in these regressions should turn positive at longer lags. However, in practice, it would be unlikely that such effects would be detected. Numerical calculations show that in the presence of low rates of unmeasured technological regress, the weights on the measured stock would be greater than the weights on the true stock for investments made over the previous decade, with this being made up for by a series of small differences between measured and actual weights at longer lags. Such longer-lagged effects would most likely be drowned out by the substantial amount of other statistical noise affecting measured TFP growth.

4 An Explanation for Our Findings?

Taken literally, the results of the previous section represent a puzzling pattern that appears incompatible with the embodiment hypothesis of under-measured quality improvements in capital goods. In this section, I present a possible explanation for these results that may still be compatible with embodiment.

Depreciation and Decay: In deriving the relationship between measured TFP growth and the age distribution of capital in Section 2, we noted that we were assuming the same "depreciation rate" δ when defining both the true productive capital stock in equation (3) and the measured capital stock in equation (6). The only source of mis-measurement related to the investment series.

As discussed above, it is clear that the δ which defines the productive capital stock is a measure of the rate at which the productivity of an asset decays as it ages. In contrast, however, the empirical depreciation rates used to construct the BEA average age series and the BLS series on capital input are based on schedules for *economic* depreciation—meaning the rate at which assets lose value as they age—from a range of empirical studies, most notably the work of Charles Hulten and Frank Wykoff (1981). (Technically, the BLS uses a different schedule from the geometric pattern used by BEA, but the BLS bases its estimates on the BEA depreciation rates, and the two sets of stock estimates are very close in practice.) And it is well known that the economic depreciation rate, δ^e, obtained from asset-price studies is not conceptually the same as the productive decay rate δ that we used to define our productive capital stock in equation (3).[9]

In particular, when there are improvements in the quality of new investment goods that are not picked up by the measured price deflators, as assumed in equation (10), then a measured unit of capital will decline in value as it ages both because of its decline in productive capacity and because it is declining in quality relative to the newest units. Algebraically, then, we can write the economic depreciation rate used to construct measured capital stocks as

$$\delta^e = \delta + \gamma \tag{24}$$

i.e. the measured economic depreciation rate is the sum of the true productive decay rate and the rate of undetected improvement in the quality of capital goods. (Note that this argument implies that this is the same γ as in our earlier formulas for the measured stock.)

The relationship in equation (24) also underlies an important paper by Oliner (1989), which discussed the relationships between depreciation rates and quality-adjustments. Oliner pointed out that once statisticians have adjusted investment data for quality improvements, then the economic depreciation rate, δ^e, was not the correct rate to use to construct quality-adjusted capital stocks. Instead, the correct depreciation rate for quality-adjusted capital should be calculated by subtracting off the quality adjustment for investment, γ, with the failure to do so resulting in a double-counting of the effect of quality improvements on relative productivities of new and old units of capital.

These considerations tell us that, in contrast to equation (6) above, the measured capital stock should actually be written as

$$K^m(t) = I(t) + (1 - \delta - \gamma) I(t-1) + (1 - \delta - \gamma)^2 I(t-2) + ... \tag{25}$$

[9]See, for example, Jorgenson (1973) and Hulten and Wykoff (1996) for detailed discussions of this issue. Whelan (2002) discusses this issue in relation to measures of computer capital stocks.

And that its growth rate should be represented as

$$z_K(t) = w_0^m(t) z_I(t) + w_1^m(t) z_I(t-1) + w_2^m(t) z_I(t-2) + ... \tag{26}$$

where

$$w_n^m(t) = \frac{(1-\delta-\gamma)^n I(t-n)}{K(t-1)} \tag{27}$$

Revised Steady-State Calculations: We can now use the same method as before to derive the steady-state values of these weights. Again assuming that the true productive investment and capital stock are growing at rate g, and that the measured investment and capital stock are growing at rate $g-\gamma$, we have that

$$\frac{I^m(t)}{K^m(t-1)} = g - \gamma - \delta^e = g + \delta \tag{28}$$

which implies that

$$w_0^m = \frac{I^m(t-1)}{K^m(t-1)} = \frac{g+\delta}{1+g-\gamma} \tag{29}$$

Thus, the measured set of weights can now be written as

$$w_n^m = \frac{g+\delta}{1+g-\gamma}\left(\frac{1-\delta-\gamma}{1+g-\gamma}\right)^n \tag{30}$$

Note now that

$$w_0^m = \frac{g+\delta}{1+g-\gamma} > \frac{g+\delta}{1+g} = w_0 \tag{31}$$

In other words, the measured capital stock actually places a *higher* weight on current investment growth than the true capital stock; with each set of weights declining gradually, this inequality will also hold for other recent lags of investment growth. Thus, the coefficients on recent values of investment growth in the TFP regressions should in fact be negative, as we found, and increases in the average age of the capital stock should be associated with higher, not lower, TFP growth.

The intuition for these results is fairly simple. Relative to the true productive capital stock, the measured stock underestimates the impact of a unit of new investment relative to that of a unit of old investment; *ceteris paribus*, this effect causes empirical growth accounting to underestimate the effect on productivity growth of recent investment growth. However, all else is not equal in this case, because the mis-measurement of technological improvements in capital goods leads empirical researchers to weight past units of investment by $(1-\delta^e)^n$ instead of $(1-\delta)^n$ and this causes an *underestimation* of the effect of past investment growth on current productivity. Our calculations show that this latter effect actually dominates the first, more traditional effect.

In theory, one could combine our newly derived weights with the regression of Table 2 to estimate exactly which positive rate of embodiment is most consistent with the estimated relationship between TFP and investment growth. However, the pattern of the coefficients in Table 2 is not consistent with the predictions of equations (17) and (30) that the coefficients on investment should become less negative in a monotonic fashion as the lags get longer, so empirical estimates based on this pattern would not be of much use.

Figure 3 may provide part of the explanation for the failure of equations (17) and (30) to exactly explain the estimated relationship between TFP and investment. It shows the weights w_n of capital of various ages in the capital stock for a specific set of parameter values ($\delta = 0.07$, $\gamma = 0.10$, $g = 0.13$). It also shows the weights for the measured stock under the traditional approach of equation (18) and the alternative approach of equation (30). While the traditional analysis suggests that the gap between the weights for the actual stock and those for the measured stock are often large, our alternative formula for the measured weights produces a series that is much closer to the weights for the actual stock. (This pattern turns out to be robust to the choice of a wide range of realistic parameter values.)

Once one takes into account the relatively small gaps between the actual and measured weights suggested by our alternative calculations, and the likely level of statistical noise affecting this regression, one would be surprised if the tight restrictions imposed by equations (13), (17) and (30) were actually satisfied in the data. However, allowing for error in the measurement of the productive decay rate can reconcile the hypothesis of unmeasured technological improvements in capital goods with the otherwise puzzling pattern of a negative relationship between measured TFP growth and changes in the age of the capital stock.

Adjustment Costs: It is possible, of course, that there are other explanations for the positive relationship between measures of TFP and the age of the capital stock. One other possibility is that adjustment costs associated with the installation of new capital could be playing some role instead of, or in addition to, the explanation just provided. That said, the pattern of the results in Table 3 do not seem very consistent with this explanation. Adjustment costs due to high levels of investment should be expected to have a negative effect on the level of TFP for one or perhaps two years after which the level of TFP should revert to its baseline level. So, if adjustment costs were driving the results, one would expect to coefficients on contemporaneous and lagged investment in Table 3 to be negative, but also expect that the coefficients at longer lags would then turn positive with similarly magnitudes. However, this is not the pattern found. Instead, the findings of long-lagged negative effects of investment growth are consistent with the "economic versus productive depreciation" argument put forward in the paper.

5 Conclusions

This paper has presented evidence that, in contrast to the usual intuition underlying tests of the embodiment hypothesis, increases in the age of the capital stock appear to be associated with higher levels of measured TFP. In relation to the embodiment hypothesis of under-measured quality improvements in capital, there appear to be two possible explanations for this result.

The first explanation is that the traditional embodiment hypothesis is simply wrong, and that statistical agencies are not underestimating the rate of quality improvement in new capital. In light of the significant body of evidence suggesting the official price indexes understate quality improvements, this explanation may not find many takers. Thus, this paper advances an alternative explanation, which is that the embodiment hypothesis may be correct but that the motivating assumption underlying the "direct method" to testing the hypothesis—that embodiment implies growth accounting underestimates the effect on current productivity of new investments relative to old—is incorrect. The traditional analysis is correct that embodiment implies that empirical growth accounting undermeasures the productive effect of a unit of new investment relative to a unit of old investment. However, embodiment also implies that the empirical analysis based on capital stocks constructed from economic depreciation rates will place too low a weight on these old units of capital, and this latter effect will tend to dominate.

Beyond the embodiment hypothesis, the empirical results presented here also show that the observed pro-cyclicality of measured TFP does not appear to be related to pro-cyclical mis-measurement of capital input. If anything, because TFP growth is positively correlated with changes in the age of the stock it appears that the opposite is the case: Empirical growth accounting is more likely to understate the growth rate of capital input in a recession than in a boom.

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Table 1: TFP Growth and the Age of the Capital Stock, Aggregate Analysis

Constant	0.010 (.003)	0.024 (.006)	0.004 (.004)
Δ Average Age	-0.019 (.016)	0.022 (.021)	0.015 (.011)
DUM74		-0.019 (.007)	-0.013 (.004)
GDP Growth			0.462 (.044)
R^2	0.026	0.153	0.744

Standard errors in parentheses. Sample is annual data from 1950-1999. DUM74 equals one before 1974 and zero afterwards. GDP growth refers to the nonfarm business sector.

Table 2: Coefficients on the Change in the Average Age, Industry-Level Analysis

		With DUM74	
	Base Equation	No GDP Growth	With GDP Growth
Food Products	0.029 (.019)	0.029 (.020)	0.030 (.019)
Textiles	0.007 (.008)	0.007 (.008)	0.007 (.008)
Apparel	0.003 (.006)	0.000 (.007)	0.002 (.007)
Paper	0.051 (.014)	0.062 (.014)	0.054 (.014)
Printing	-0.007 (.012)	0.011 (.012)	0.010 (.012)
Chemicals	0.038 (.015)	0.053 (.015)	0.048 (.015)
Petroleum Products	0.010 (.004)	0.013 (.004)	0.008 (.004)
Rubber and Plastics	0.031 (.015)	0.039 (.017)	0.038 (.017)
Lumber	0.015 (.014)	0.028 (.016)	0.022 (.016)
Furniture	0.013 (.008)	0.018 (.009)	0.016 (.009)
Stone Products	0.015 (.006)	0.017 (.007)	0.012 (.007)
Primary Metals	0.016 (.010)	0.029 (.012)	0.017 (.010)
Fabricated Metals	0.002 (.008)	0.003 (.010)	0.001 (.009)
Industrial Machinery	0.027 (.016)	0.011 (.015)	-0.005 (.015)
Electrical Machinery	0.016 (.016)	0.005 (.015)	-0.001 (.015)
Instruments	-0.012 (.013)	-0.005 (.016)	-0.013 (.018)
Miscellaneous	0.011 (.014)	0.019 (.016)	0.018 (.016)
POOLED	0.013 (.002)	0.016 (.003)	0.012 (.003)

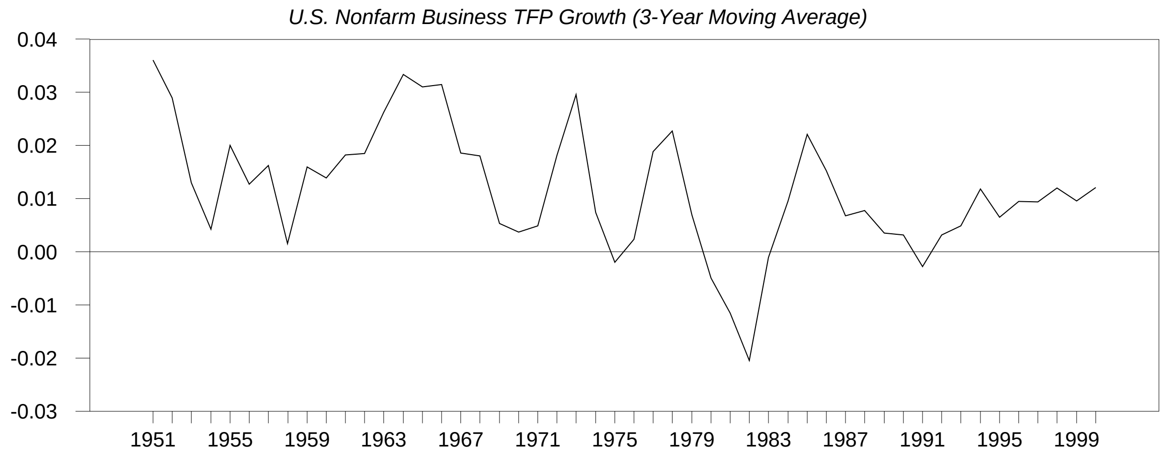
Dependent variable is TFP growth. Estimated using SUR. Standard errors in parentheses. Sample is annual data from 1950-1999. DUM74 is a dummy variable equalling one after 1974 and zero before.

Table 3: Regression of TFP Growth on Current and Lagged Investment Growth

		With DUM74	
	Base Equation	No GDP Growth	With GDP Growth
$z_I(t)$	-0.001 (.003)	-0.000 (.003)	-0.005 (.003)
$z_I(t-1)$	-0.014 (.003)	-0.015 (.003)	-0.009 (.003)
$z_I(t-2)$	-0.011 (.003)	-0.011 (.003)	-0.011 (.003)
$z_I(t-3)$	0.000 (.003)	0.001 (.003)	0.000 (.003)
$z_I(t-4)$	-0.005 (.003)	-0.006 (.003)	-0.008 (.003)

Estimated using SUR. Coefficients restricted to be equal across all industries. Standard errors in parentheses. Sample is annual data from 1950-1999. DUM74 is a dummy variable equalling one after 1974 and zero before.

Figure 1


U.S. Nonfarm Business TFP Growth (3-Year Moving Average)
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
1951
1955
1959
1963
1967
1971
1975
1979
1983
1987
1991
1995
1999

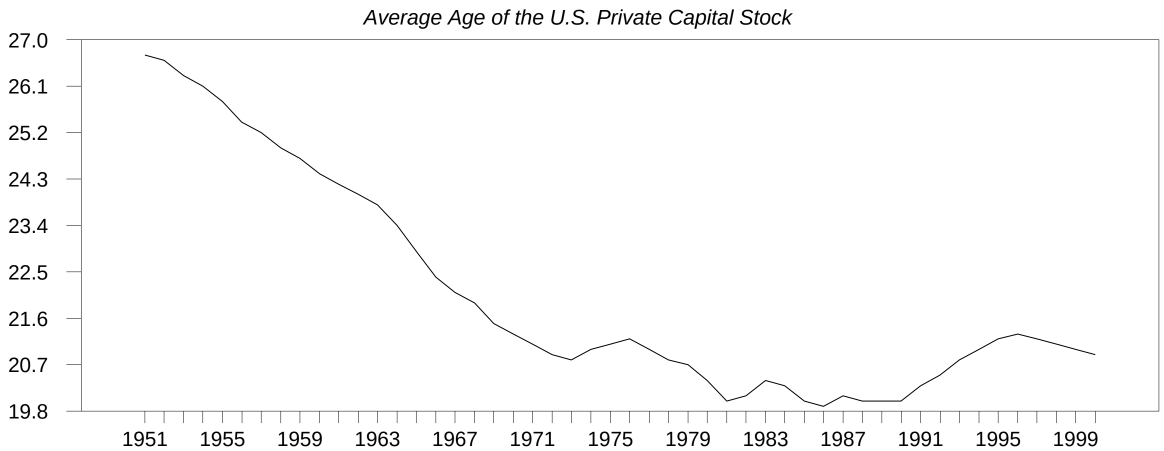

Average Age of the U.S. Private Capital Stock
27.0
26.1
25.2
24.3
23.4
22.5
21.6
20.7
19.8
1951
1955
1959
1963
1967
1971
1975
1979
1983
1987
1991
1995
1999

Figure 2

Recursive Regression Estimates of Average Age Coefficient

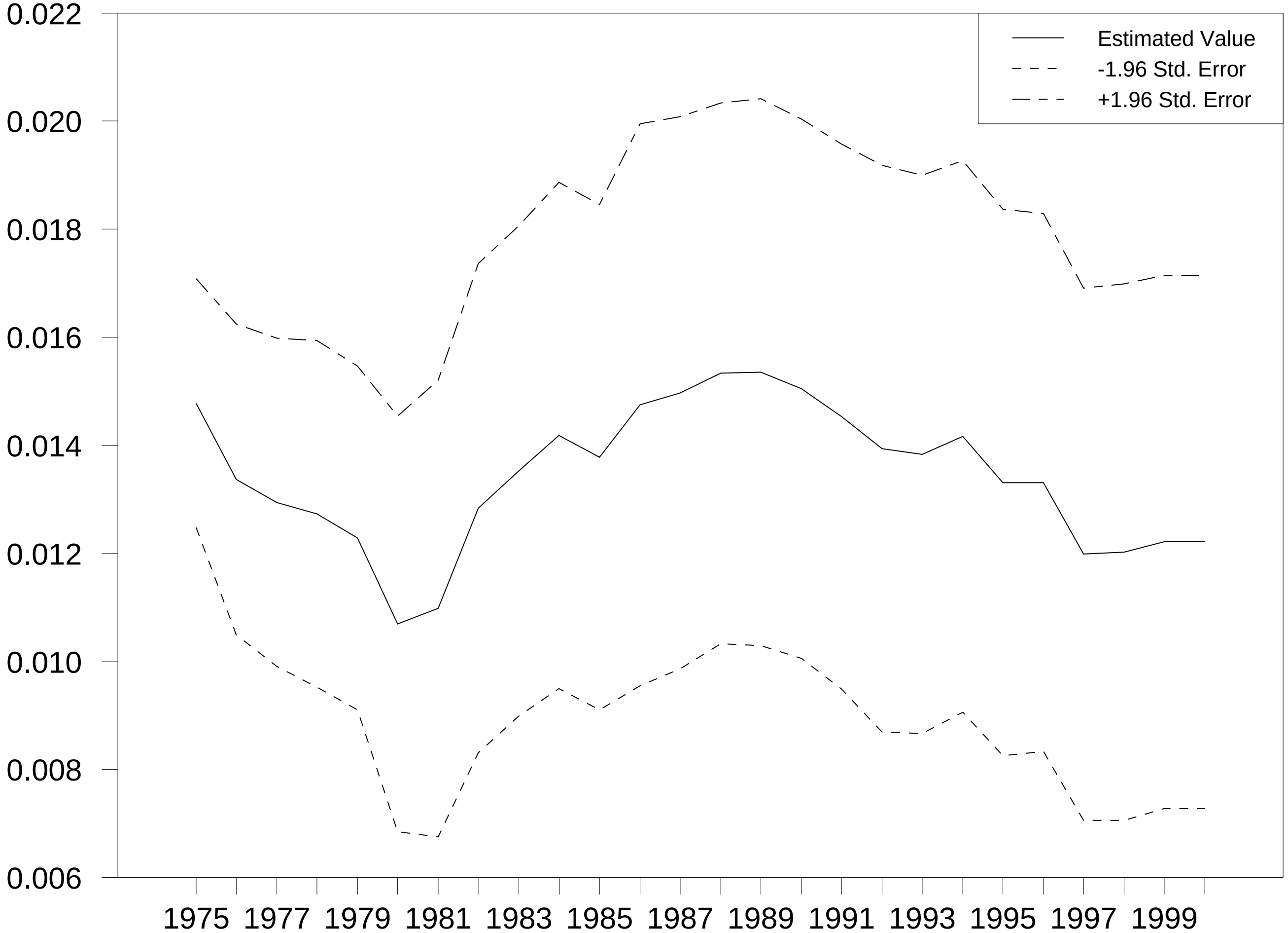
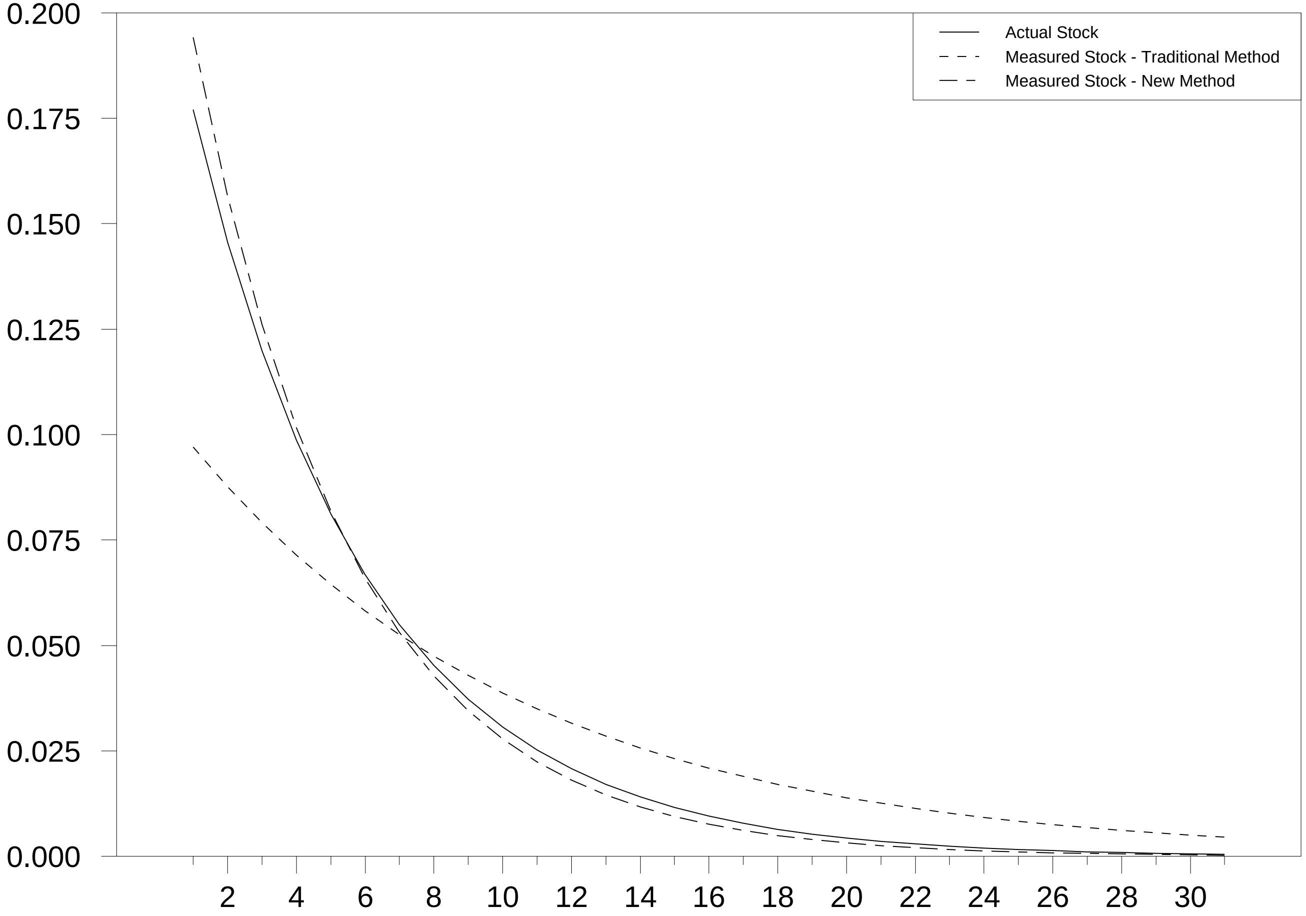


Figure 3

Weights in the Capital Stock for Current and Past Investment



Based on gamma=0.1, g=0.13, delta=0.07