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2014

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Abstract
This paper presents results concerning the nonlinear analysis of the mean annual value temperature time series corresponding to the Earth's global climate for the time period of 713 – 2004. The nonlinear analysis consists of the application of several filtering methods, the estimation of geometrical and dynamical characteristics in the reconstructed phase space, techniques of discrimination between nonlinear low dimensional and linear high dimensional (stochastic) dynamics and tests for serial dependence and nonlinear structure. All study results converge to the conclusion of nonlinear stochastic and complex nature of the global earth climate.

Keywords: Nonlinear dynamics; Correlation dimension; Lyapunov exponent; Mutual information function; Chaos.

JEL codes: C80; C88; Q50; Q52; Q54.

1. Introduction

The climate system of the Earth consists of natural spheres (atmosphere, biosphere, hydrosphere and geosphere), the humansphere (economy, society, culture) and their complex interactions (Schellnhuber, 1999; Halkos, 2013). Gedalin & Balikhin (2008) claim that these interactions as well as the multi-level structure of the system atmosphere-ocean-solid surface, ocean currents and winds, the differential absorption volume of solar radiation, the effects of chemical composition, etc lead to a three dimensional field of the distribution of temperatures presenting much smaller spatial and temporal scales compared to Earth radius and planet's rotation period respectively. At the same time, observations show episodes of abrupt change, starting with the sudden onset of large global warming (e.g. the end of the last ice age) to the active and rapid regional changes in hydroclimatic cycle, rainfall and drought (e.g. desert extension) (Rial et al., 2004).

In addition, there is a wide range of interactions found in the magnitudes of changes in temperatures that show multifractality (multiple self-similarity) in the Earth's climate on time scales of 1-100 years (Ashkenazy et al., 2003). These large-scale changes in temperatures mainly depend on external influences such as physical effects on air temperature near the Earth's surface, like for instance variations in volcanic eruptions, the El Niño Southern Oscillation phenomenon, active water cycle (Nordstrom et al., 2005), solar radiation cycles (Ozawa et al., 2003), changes in the Earth orbital motion and continents (Lin et al., 1991), while the state change occurs at a slow time scale.

Moreover, all human actions that affect the state of the climate system such as changes in the concentration of greenhouse gases and in small gas fractions controlling the content of stratospheric ozone, sulfur dioxide etc can be considered as small external shocks (Dymnikov & Gritsoun, 2001). All of the above non-linear and disproportionate inputs-outputs, the created chains of feedback and inner circles, multiple equilibria, astronomical effects etc create non-linear, stochastic and complex nature of global climate on Earth.

In this study we present tools as well as methodology of the modern nonlinear time series analysis used for tracing nonlinear and chaotic dynamics in the Earth's climate complex system. The nonlinear algorithm is applied in the annual temperature time series concerning the Global Earth Climate during the time period of 713-2004 (D'Arrigo et al. 2006a, b).[1] In order to extract useful information about the complex dynamics of the Earth's climate we use different filtering methods such as the AR(4) residuals[2] and the SVD analysis. In particular, we estimate geometrical and dynamical characteristics in the reconstructed phase space such as correlation dimension, mutual information and maximum Lyapunov exponent.

In addition, BDS test of independence and identical distribution and Brock's Residual test are applied to the AR(4) residuals of the Temperature time series, searching for more evidence of nonlinearity in the data. Finally, the method of stochastic surrogate data is employed for the exclusion of 'pseudo-chaos' caused by the nonlinear distortion of a purely stochastic process. Our results indicate the nonlinear stochastic profile of the Earth's complex climate dynamics.

[1] The data set is the Northern Hemisphere Tree-Ring-Based STD and RCS Temperature Reconstructions and the contributors are Rosanne D'Arrigo and Gordon Jacoby, Lamont-Doherty Earth Observatory, and Rob Wilson, University of Edinburgh.

[2] A number of autoregressive schemes to the data set at hand were fitted relying on the statistical significance of their components and selecting their complexity by the use of the AIC.

2. Results[3]

This section presents results concerning the nonlinear analysis of the annual temperature time series. The nonlinear analysis consists of the estimation of geometrical and dynamical characteristics in the reconstructed phase space and the techniques of discrimination between nonlinear low dimensional and linear high dimensional (stochastic) dynamics.[4]

2.1 Annual temperature time series

In Figure 1a the mean annual value temperature time series is presented corresponding to the Earth's global climate for the time period of 713 – 2004. As can be seen in this figure the time series is stationary apart from the last part where a significant increase takes place. This increase could be due to the so called "greenhouse" effect (for more details see Halkos, 2014).

Figure 1b presents the autocorrelation coefficient of the time series estimated for 100 lags. As can be seen the autocorrelation coefficient decays slowly indicating the presence of long range correlations of the underlying dynamics. The profile of the power spectrum, shown in Figure 1c indicates the presence of two distinct dynamics underlying the temperature time series, one corresponding to a power law scaling as seen in low frequencies indicating a long range correlation process and another one corresponding to a flat spectrum indicating an uncorrelated (white noise) process. Finally, in Figure 1d the slopes of correlation integral estimated for a fixed embedding dimension m=8 and for three different time reconstruction delays τ=2, 4, 6, is

[3] Part of the analysis was presented in the 6th International Conference *from Scientific Computing to Computational Engineering* (6th IC-SCCE), Athens, Greece, July 11 2014.

[4] A complete review concerning the methodology of nonlinear time series analysis and its application in various geophysical time series can be fount in Athanasiu & Pavlos (2001), Pavlos *et al.* (2004), Pavlos *et al.* (2007), Iliopoulos et al. (2008), Iliopoulos & Pavlos (2010) and references within.

presented. As can be observed there is not any saturation, $dlnc/dlnr = d_m \neq$ steady or scaling of the slopes for ln(r). This result indicates that the underlying dynamics do not correspond to a low dimensional attractor and is high dimensional (practically infinite degrees of freedom).

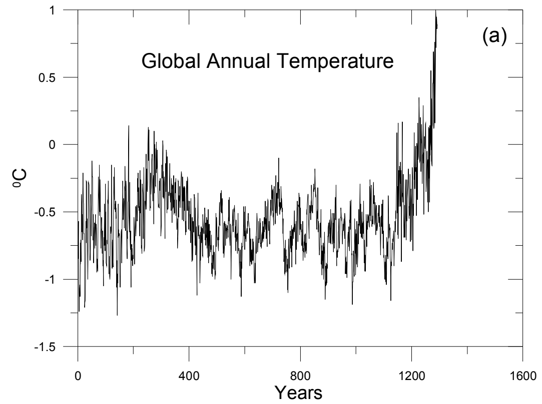

Figure 1a: Mean annual value temperature time series

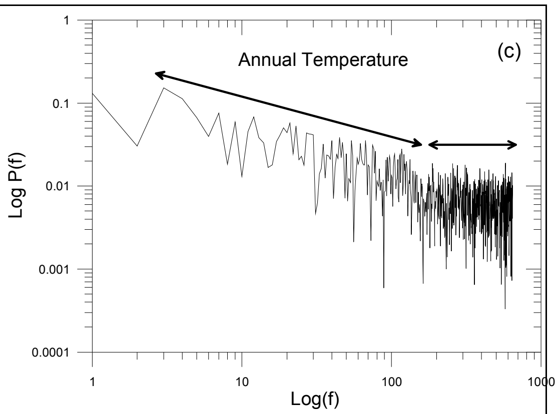

Figure 1c: Power spectrum

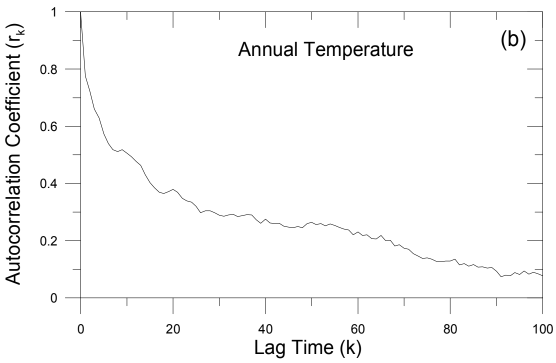

Figure 1b: Autocorrelation coefficient

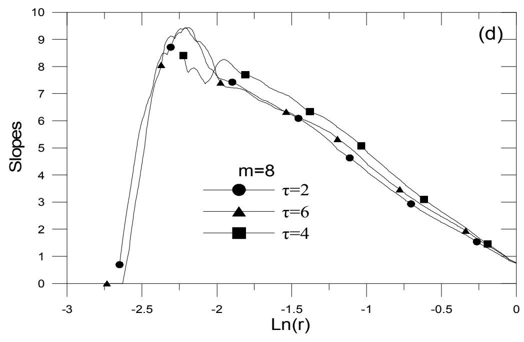

Figure 1d: Slopes of the correlation integrals as estimated for embedding dimension m=8 and time delay τ=2, 4, 6.

In Figure 2 we present results corresponding to the method of surrogate data. Usually, this method is used for the rejection of the null hypothesis of the "pseudo-chaos" existence and the results presented previously exclude the existence of low dimensional dynamics underlying the temperature time series. However, we can still use the surrogate data method as an indicator of nonlinearity or even high dimensional chaos (if the difference between the surrogate data and the original time series is significant).

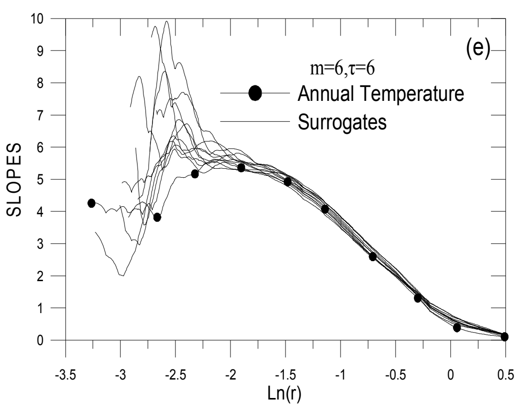

Figure 2a: Slopes of the correlation integrals of the original time series and 10 surrogate data as estimated for embedding dimension *m=6* and time delay τ=6.

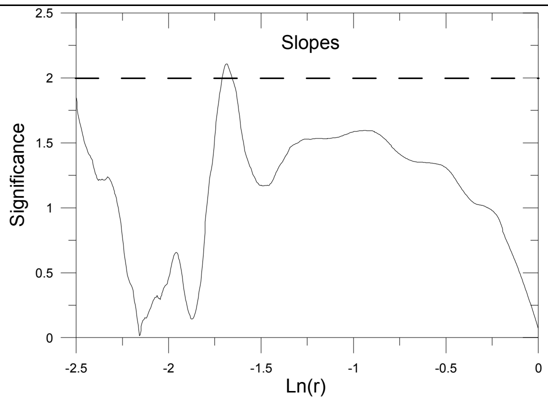

Figure 2d: Significance of the difference of the statistics between the original time series and 10 surrogate data concerning the slopes of the correlation integrals.

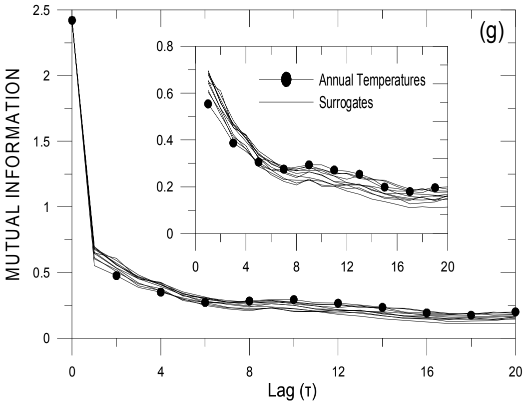

Figure 2b: Mutual Information estimated for the original time series and 10 surrogate data.

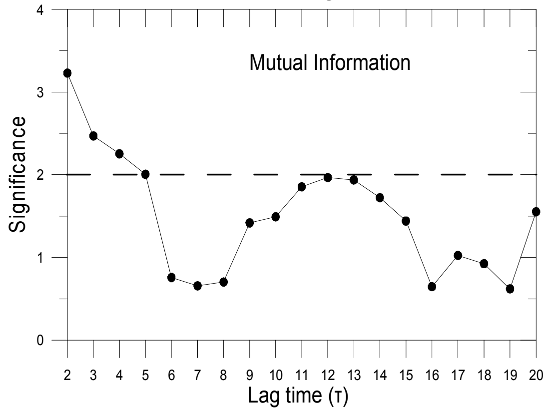

Figure 2e: Significance of the difference of the statistics between the original time series and 10 surrogate data concerning the mutual information.

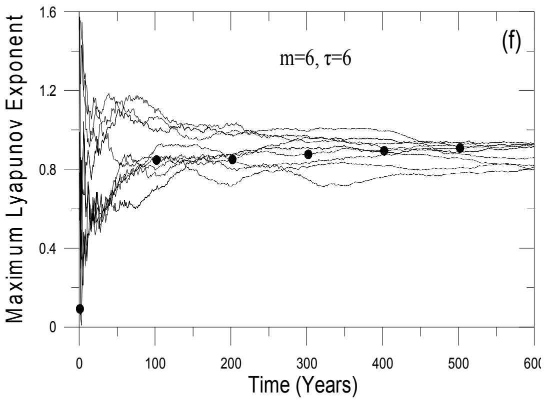

Figure 2c: Maximum Lyapunov exponent for the original time series and 10 surrogate data estimated for embedding dimension *m=6* and time delay τ=6.

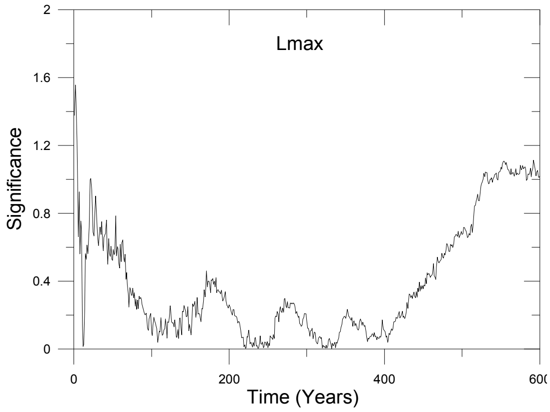

Figure 2f: Significance of the difference of the statistics between the original time series and 10 surrogate data concerning the maximum Lyapunov exponent.

In particular, Figure 2a presents the slopes of the correlation integral estimated for the original temperature time series and its 10 surrogate data, for embedding dimension m=6 and time delay τ=6. As can be seen there is no significant difference; a result depicted also in Figure 2d which presents the significance of the discrimination of the statistics of Figure 2a, attaining values below 2 for a wide range of ln(r). This result indicates high dimensional dynamics underlying the original time series.

In Figure 2b we present the mutual information estimated for the original temperature time series and for its 10 surrogate data, while in Figure 2e we present the significance of the statistics. The significance attains values above 2 for the three first values, however the mutual information value is lower than the corresponding of the surrogate data, a result that indicates linearity. Finally, Figure 2c presents the maximum Lyapunov exponent estimated for the original time series and its 10 surrogate data for parameters m=6 and τ= 6, while in Figure 2f the significance of the statistics is presented. Even though the maximum Lyapunov exponent is positive there is no difference from the corresponding exponents of the surrogate data. Overall, the results show that the original time series correspond to high dimensional dynamics.

3. Testing for Serial Dependence and Nonlinear Structure

The present study presents the results of two tests for nonlinear dependence in annual earth temperatures from 713 up to 2004. The first one is the BDS test of independence and identical distribution and the second one is Brock's Residual test. Similar studies have been carried out in Willey (1992), Frank and Stengos (1988a, b), Frank et al. (1988), Chavas and Holt (1993). These tests substitute the typical statistical tests using spectral analysis and autocorrelation function that fail to reveal statistically significant correlations on non-independent data and cannot discriminate between a stochastic explanation and a deterministically chaotic explanation of a time series.

The correlation coefficient diagram for the residuals of an AR(4) model fitted to temperature data in Figure 3a shows statistically insignificant correlations. This means that the AR(4) model succeeded in removing from the temperature series the linear structure. The correlation coefficient diagram and the flat profile of the log-log plot of the power spectrum (Figure 3b) indicate that AR(4) residuals are white noise residuals.

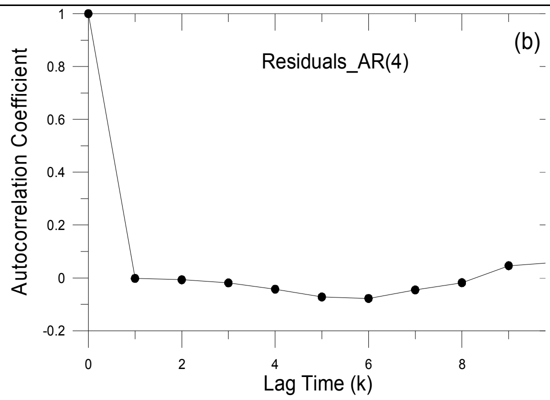

Figure 3a: Correlation coefficient of AR(4) residual series

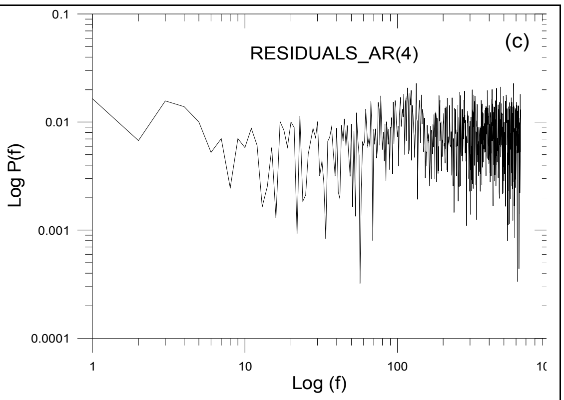

Figure 3b: Power spectrum of AR(4) residual series

3.1 BDS Test of Independence

Brock, Dechert and Scheinkman (BDS in Brock et al., 1996) created a test for time based dependence in a series. It is a test of the null hypothesis that the data are independent and identically distributed (i.i.d. data) against a variety of possible departures from independence including linear dependence, nonlinear dependence or chaos. This test can be applied to a series of residuals of estimated linear models in order to test for extra structure. For example, the residuals from an ARMA model can be tested to see if there is any nonlinear dependence in the series after the linear

ARMA model has been fitted. This way the BDS test of residuals performs a test of a new hypothesis of an underlying nonlinear process and thus can be employed as a test for nonlinearity.

Table 1: BDS Test for residuals generated by AR(4) process (e is a multiple of the standard deviation of the series)

e=0.5

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.003262	0.000790	4.127598	0.0000
3	0.002793	0.000524	5.326703	0.0000
4	0.001895	0.000261	7.258768	0.0000
5	0.001038	0.000114	9.121463	0.0000
6	0.000503	4.59E-05	10.94192	0.0000
7	0.000197	1.76E-05	11.15649	0.0000
8	6.81E-05	6.52E-06	10.44085	0.0000

e=1

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.009244	0.001920	4.814101	0.0000
3	0.014139	0.002353	6.009269	0.0000
4	0.016815	0.002161	7.779901	0.0000
5	0.016582	0.001738	9.540243	0.0000
6	0.014341	0.001294	11.08694	0.0000
7	0.011323	0.000915	12.37596	0.0000
8	0.008427	0.000624	13.50136	0.0000

e=1.5

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.010220	0.002044	4.998774	0.0000
3	0.020484	0.003348	6.117933	0.0000
4	0.031566	0.004108	7.684830	0.0000
5	0.041031	0.004410	9.303869	0.0000
6	0.046963	0.004381	10.72055	0.0000
7	0.048953	0.004135	11.84003	0.0000
8	0.047877	0.003763	12.72204	0.0000

e=2

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.007245	0.001440	5.029779	0.0000
3	0.016613	0.002746	6.050690	0.0000
4	0.029420	0.003918	7.508800	0.0000
5	0.044021	0.004892	8.997795	0.0000
6	0.057941	0.005651	10.25289	0.0000
7	0.069139	0.006202	11.14861	0.0000
8	0.075744	0.006563	11.54132	0.0000

To perform the test, a distance e is chosen. If a pair of points is considered, the probability of the distance between these points being less or equal to e will be constant in case the observations of the series are truly i.i.d. In Table 1, probabilities converge to 0 in all considered embedding dimensions and distances e.

In order to enhance the evidence of nonlinearity, the BDS test is applied to a series of shuffled residuals (Scheinkman and LeBaron 1989) and to a series of surrogate residuals (Schreiber and Schmitz 1996). The shuffled residuals is a new series of the same length as the original AR(4) residual series, created by random sampling from it with replacement. The surrogate residual series has the same length, the same autocorrelation function and probability density with the series of AR(4) residuals. Here, the surrogate residuals are constructed using the improved algorithm of Schreiber and Schmitz.

3.1.1 BDS Test for shuffled AR(4) residuals

The shuffled AR(4) residuals are used to check the reliability of the BDS test. According to Scheinkman and LeBaron (1989) we get the original time series and sampling randomly with replacement from it. The shuffled series will have the same length as the original. As shuffling destroys the alleged non-linear structure of the data, the statistical BDS (z-statistic) should detect the difference between the shuffled and initial time series of AR(4) residuals.

3.1.2 BDS Test for surrogate AR(4) residuals

The surrogate data are random numbers with the same probability density function and the same autocorrelation function as the original time series. They are generated using the improved algorithm Schreiber-Schmitz (Schreiber and Schmitz 1996). The BDS statistic should detect in the case of surrogate data as well, the difference between the surrogate and original AR(4) residual time series.

Table 2: BDS Test for the shuffled series of AR(4) residuals (e is a multiple of the standard deviation of the series)

e=0.5

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	-2.01E-05	0.000703	-0.028636	0.9772
3	-0.000409	0.000459	-0.891495	0.3727
4	-0.000162	0.000225	-0.720191	0.4714
5	-7.70E-05	9.63E-05	-0.800314	0.4235
6	-4.33E-05	3.82E-05	-1.133358	0.2571
7	-1.47E-05	1.44E-05	-1.022919	0.3063
8	-9.45E-06	5.24E-06	-1.803039	0.0714

e=1

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	-0.000268	0.001790	-0.149962	0.8808
3	-0.000940	0.002174	-0.432576	0.6653
4	-0.000463	0.001979	-0.233736	0.8152
5	-0.000303	0.001577	-0.192206	0.8476
6	6.40E-05	0.001163	0.054976	0.9562
7	0.000222	0.000815	0.272031	0.7856
8	0.000274	0.000551	0.497182	0.6191

e=1.5

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	-0.000902	0.001961	-0.459705	0.6457
3	-0.001740	0.003199	-0.543934	0.5865
4	-0.001142	0.003909	-0.292209	0.7701
5	3.50E-05	0.004180	0.008361	0.9933
6	0.001631	0.004136	0.394342	0.6933
7	0.002416	0.003888	0.621390	0.5343
8	0.003353	0.003525	0.951351	0.3414

e=2

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	-0.001187	0.001392	-0.852656	0.3939
3	-0.001676	0.002655	-0.631513	0.5277
4	-0.001730	0.003790	-0.456534	0.6480
5	-0.000597	0.004733	-0.126043	0.8997
6	0.001837	0.005469	0.335914	0.7369
7	0.003755	0.006004	0.625474	0.5317
8	0.006207	0.006356	0.976601	0.3288

Table 3: BDS Test for a randomly selected series of surrogate AR(4) residuals (e is a multiple of the standard deviation of the series)

e=0.5

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.001355	0.000790	1.713933	0.0865
3	0.000931	0.000524	1.775276	0.0759
4	0.000390	0.000261	1.493032	0.1354
5	0.000166	0.000114	1.461026	0.1440
6	6.20E-05	4.59E-05	1.349033	0.1773
7	2.40E-05	1.76E-05	1.363166	0.1728
8	9.89E-06	6.52E-06	1.516046	0.1295
9	3.52E-06	2.35E-06	1.495197	0.1349
10	5.71E-07	8.31E-07	0.687196	0.4920

e=1

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.003955	0.001920	2.059653	0.0394
3	0.004974	0.002353	2.114055	0.0345
4	0.004333	0.002161	2.004720	0.0450
5	0.003042	0.001738	1.750358	0.0801
6	0.001844	0.001294	1.425599	0.1540
7	0.001094	0.000915	1.195582	0.2319
8	0.000486	0.000624	0.779270	0.4358
9	0.000162	0.000415	0.391184	0.6957
10	6.36E-05	0.000270	0.235989	0.8134

e=1.5

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.005025	0.002044	2.457751	0.0140
3	0.008404	0.003348	2.510072	0.0121
4	0.009652	0.004108	2.349746	0.0188
5	0.008760	0.004410	1.986365	0.0470
6	0.007402	0.004381	1.689747	0.0911
7	0.006157	0.004135	1.489052	0.1365
8	0.004442	0.003763	1.180364	0.2379
9	0.003034	0.003334	0.909864	0.3629
10	0.002102	0.002893	0.726560	0.4675

e=2

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.003870	0.001440	2.686398	0.0072
3	0.007293	0.002746	2.656265	0.0079
4	0.009613	0.003918	2.453417	0.0142
5	0.009802	0.004892	2.003532	0.0451
6	0.009653	0.005651	1.708092	0.0876
7	0.009292	0.006202	1.498362	0.1340
8	0.007954	0.006563	1.211922	0.2255
9	0.006512	0.006759	0.963461	0.3353
10	0.005344	0.006817	0.783967	0.4331

The results clearly show that in both series of shuffled and surrogate AR(4) residuals the null hypothesis for i.i.d data cannot be rejected since z statistic takes statistically insignificant values in the vast majority of embedding dimensions and e.

The BDS statistic (z-statistic in Table 4) is expected to diagnose the difference between the original AR(4) residuals and the shuffled and surrogate residuals, given that their generating process destroys the (possible) nonlinear structure of the original residuals. If the residuals are i.i.d. data then the BDS test statistic is normally distributed [z-statistic~N(0,1)].

The values of z-statistic for AR(4) residuals exceed the critical value for every significant level, in all embedding dimensions and distances e. The conclusion is to reject the null hypothesis of independence of AR(4) residuals (Halkos, 2006). This is an indication that the series is generated by a nonlinear process, given that all the linear structure has been removed. The results also show that in both series of shuffled and surrogate residuals the i.i.d. null hypothesis is retained, since z-statistic values are less than critical values in the vast majority of embedding dimensions and distances e.

Aiming to check the reliability of BDS statistic in our data, we apply the following methodology:

Step 1: We generate 30 surrogate residual series using the Schreiber and Schmitz algorithm,

Step 2: We perform BDS test in each one of them and

Step 3: We compute mean value μ_{surro} and standard deviation σ_{surro} of the distribution of the z-statistics of step 2.

Table 4: BDS statistics for annual earth temperature data, AR(4) residuals, shuffled and surrogate AR(4) residuals (e is a multiple of the standard deviation of the series)

dE	e	z-original temperature data	z-AR(4) residuals	z-shuffled AR(4) residuals	z- surrogate AR(4) residuals
2	0.5	45.24913	4.127598	-0.028636	-2.217687
3	0.5	57.37534	5.326703	-0.891495	-1.074243
4	0.5	72.22246	7.258768	-0.720191	-0.547121
5	0.5	93.62769	9.121463	-0.800314	0.314762
6	0.5	126.4490	10.94192	-1.133358	0.692186
7	0.5	177.9179	11.15649	-1.022919	0.841111
8	0.5	261.0943	10.44085	-1.803039	0.177648
2	1	39.84653	4.814101	-0.149962	-2.346665
3	1	45.91428	6.009269	-0.432576	-1.248528
4	1	50.96965	7.779901	-0.233736	-0.298179
5	1	57.74778	9.540243	-0.192206	0.094204
6	1	66.57659	11.08694	0.054976	0.138372
7	1	78.26005	12.37596	0.272031	0.209266
8	1	94.22354	13.50136	0.497182	0.233027
2	1.5	36.70460	4.998774	-0.459705	-2.372047
3	1.5	39.50391	6.117933	-0.543934	-1.424346
4	1.5	40.56742	7.684830	-0.292209	-0.455554
5	1.5	41.99484	9.303869	0.008361	-0.047905
6	1.5	43.73042	10.72055	0.394342	0.007883
7	1.5	45.99300	11.84003	0.621390	0.188030
8	1.5	48.80326	12.72204	0.951351	0.271578
2	2	34.58071	5.029779	-0.852656	-2.236987
3	2	35.68591	6.050690	-0.631513	-1.233640
4	2	35.05630	7.508800	-0.456534	-0.279808
5	2	34.70645	8.997795	-0.126043	0.098948
6	2	34.47040	10.25289	0.335914	0.267017
7	2	34.41821	11.14861	0.625474	0.544960
8	2	34.52596	11.54132	0.976601	0.659747

In Table 5 the difference between the BDS statistic of 30 surrogate residual series and the original residual series is evaluated, computing significance $S = \frac{|\mu_{original} - \mu_{surro}|}{\sigma_{surro}}$, a quantity without units of measure (Papaioannou, 2000). $\mu_{original}$ is the z-statistic of the original AR(4) residuals. When the value of significance S is higher than 2-3, then, the probability that the original AR(4) residuals does not belong in the same family of the surrogate data is higher than 0.95-0.99.

Table 5: Evaluation of the Significance S

dE	e	Mean z-surro	z-AR(4)	std z-surro	S
2	0.5	0.046493	4.1276	1.20897689	3.3756704
3	0.5	-0.00535	5.3267	1.04810525	5.0873204
4	0.5	-0.01428	7.2588	1.04883585	6.93442741
5	0.5	0.024699	9.1215	1.06811817	8.51666211
6	0.5	0.123513	10.9419	1.13927845	9.49582332
7	0.5	0.231672	11.1565	1.3338781	8.19027475
8	0.5	0.283587	10.4409	1.65918973	6.12185161
2	1	0.079205	4.8141	1.12199284	4.22007595
3	1	0.075377	6.0093	0.96038479	6.17869293
4	1	0.066815	7.7799	0.98676735	7.81651824
5	1	0.068257	9.5402	0.96808426	9.78421362
6	1	0.142875	11.0869	0.98222931	11.1420262
7	1	0.164796	12.376	1.0405768	11.7350334
8	1	0.126813	13.5014	1.08324927	12.3467311
2	1.5	0.110494	4.9988	1.16931166	4.18049912
3	1.5	0.105937	6.1179	0.99625702	6.03455028
4	1.5	0.096003	7.6848	0.99167558	7.65250008
5	1.5	0.071912	9.3039	0.98810418	9.34313228
6	1.5	0.126033	10.7206	0.98864506	10.7162498
7	1.5	0.164304	11.84	1.03462214	11.2849856
8	1.5	0.171682	12.722	1.0558221	11.886773
2	2	0.122381	5.0298	1.21619584	4.0350567
3	2	0.115367	6.0507	1.02923922	5.76671855
4	2	0.101784	7.5088	0.98702881	7.50435657
5	2	0.056389	8.9978	0.99608486	8.97655523
6	2	0.088089	10.2529	0.98540762	10.3153365
7	2	0.105492	11.1486	1.04405218	10.577161
8	2	0.127608	11.5413	1.06849091	10.6820679

S values presented in Table 5 are considerably high, meaning that the BDS statistic results differently for surrogate and original data. This is another evidence to reject the hypothesis that AR(4) residuals are linearly dependent noisy data.

3.2 Brock's Residual Test

Brock (1986) has proposed a test based on the invariance to linear transformations (like an AR process) that holds for chaotic data: if one transforms chaotic data linearly, both the original and the transformed data may have the same correlation dimension and Lyapunov exponents.

The process followed here (proposed by Brock and Sayers, 1988) permits a nonlinear test for the existence of a deterministic system and the refusal of a linear generating process if accepted. The dimension and the maximum Lyapunov exponent of the residuals is estimated and compared with the dimension and the maximum Lyapunov exponent of the original data. If any nonlinear structure exists, these values will be untouched.

Figure 4a depicts the estimated correlation dimension for the original and the AR(4) residuals series, for embedding dimensions ranging from 2 to 7. Figure 4b depicts the maximum Lyapunov exponent of the original temperature series and the AR(4) residuals series.

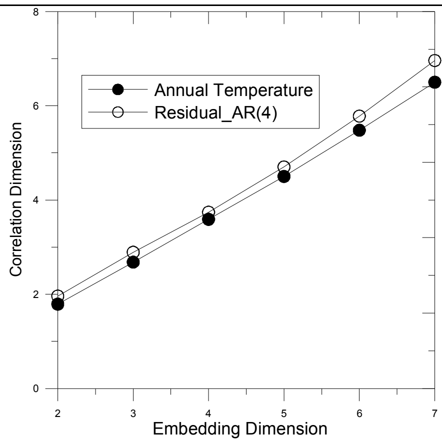

Figure 4a: Estimation of the correlation dimension for the annual earth temperatures and the AR(4) residuals for embedding dimensions ranging from 2 to 7.

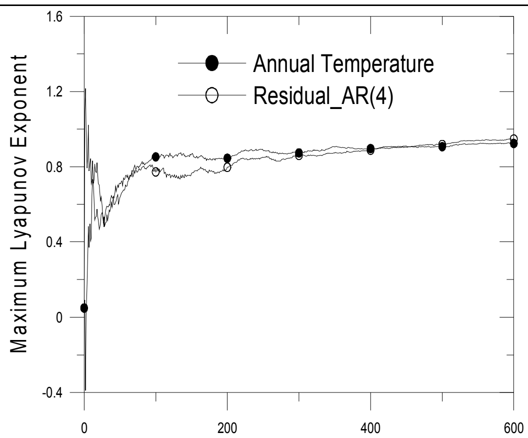

Figure 4b: Maximum Lyapunov exponent of the original temperature series and the AR(4) residuals series.

There is no apparent difference between the correlation dimensions of the original series and the AR(4) residuals. However convergence of the dimension estimates does not occur, which indicates high dimensional dynamics. These results imply that the two series are stochastic, not chaotic.

Caution is required in the interpretation of this diagnostic. A bias has been shown in the case of relatively small data sets (100 to 2000 observations) as the series studied here of 1292 observations. This bias corresponds to estimation errors in the dimension estimates leading to rejection of deterministic chaos even if it exists (Brock 1988; Hsieh 1989; Ramsey at al. 1990).

4. Nonlinear Analysis of AR(4) Residuals

In Figure 5 we present results concerning the method of surrogate data. In particular, in Figure 5a the slopes of correlation integral estimated for the AR(4) residuals time series and 30 surrogate data are shown. For the estimation we used embedding dimension m=7 and time delay τ =3. As it can be observed there is no difference, a result clearly depicted in Figure 5d which shows the significance of discrimination statistics between the AR(4) time series and its surrogates.

The value of significance is $S < 2$ for all Ln(r). This result reveals the high dimensional character of the underlying dynamics corresponding to the residuals. In addition, in Figure 5b we present the comparison of the mutual information of the AR(4) time series and its surrogates, while in Figure 5e the significance of the discriminating statistics is shown. The significance attains large values above 2 for some τ, indicating long range nonlinear interactions.

Finally in Figures 5(c, f) the difference between the AR(4) time series and its surrogate data is obvious and significant, a result indicating that the AR(4) time series is more deterministic and less stochastic, since the Lmax of the AR(4) is much smaller from the corresponding of surrogates. Thus, the filtering used in order to generate the AR(4) residuals, revealed a hidden nonlinear dynamics of lesser complexity, in contrast to the analysis of the original annual temperature.

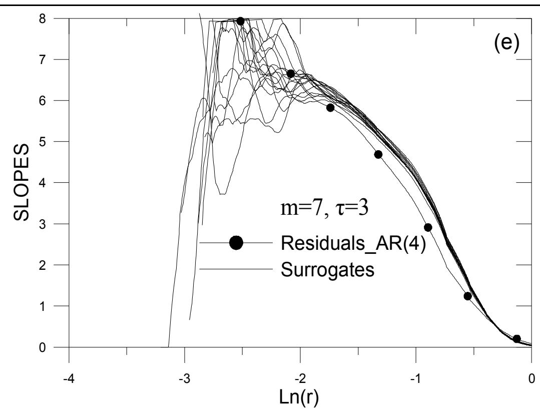

Figure 5a: Slopes of the correlation integrals of the AR(4) residuals and 30 surrogate data as estimated for embedding dimension *m=7* and time delay τ=3.

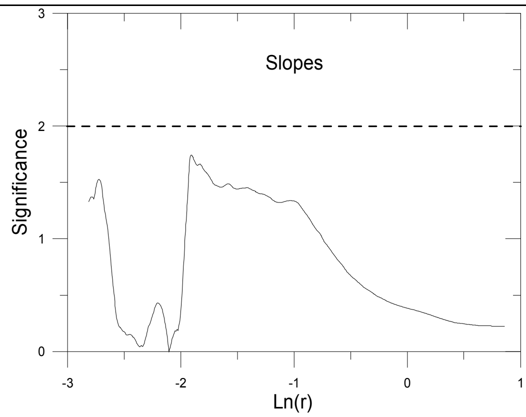

Figure 5d: Significance of the difference of the statistics between the AR(4) residuals and 30 surrogate data concerning the slopes of the correlation integrals.

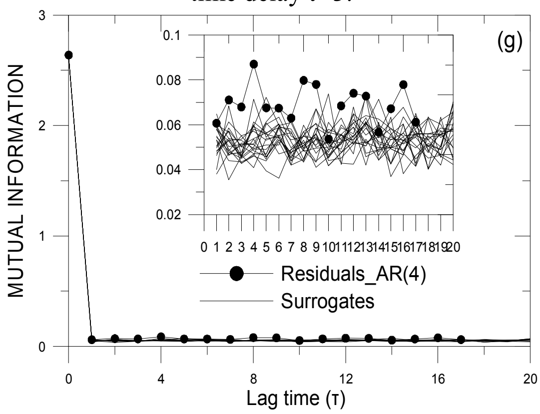

Figure 5b: Mutual Information estimated for the AR(4) residuals and 30 surrogate data.

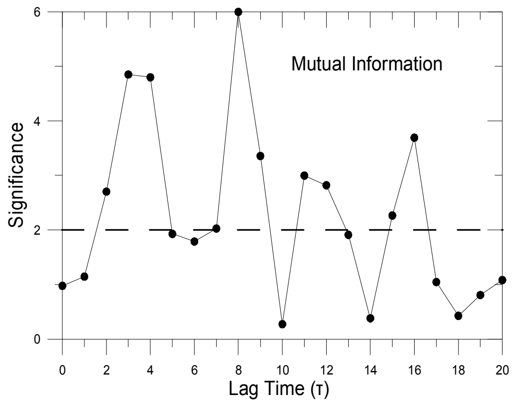

Figure 5e: Significance of the difference of the statistics between the AR(4) residuals and 30 surrogate data concerning the mutual information.

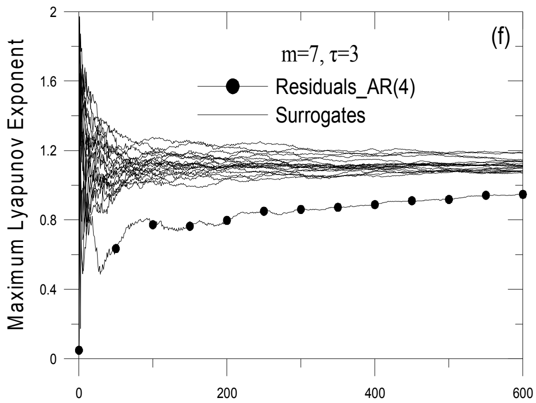

Figure 5c: Maximum Lyapunov exponent for the AR(4) residuals and 30 surrogate data estimated for embedding dimension *m=7* and time delay τ=3.

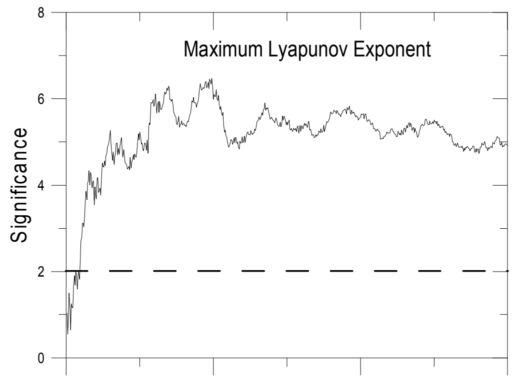

Figure 5f: Significance of the difference of the statistics between the AR(4) residuals and 30 surrogate data concerning the maximum Lyapunov exponent.

5. Analysis SVD Components

In Figure 6a we present the first SVD component, V1, of the temperature time series, which captures the general trend of the original time series. The profile of the log-log power spectrum shown in Figure 6b reveals a power law scaling indicating scale invariance and long range correlations of the underlying dynamics. However, as we can see in Figure 6c the slopes of the correlations integrals, estimated for m=7 and τ=2, did not reveal the needed saturation or scaling, indicating the high dimensional nature of the underlying dynamics.

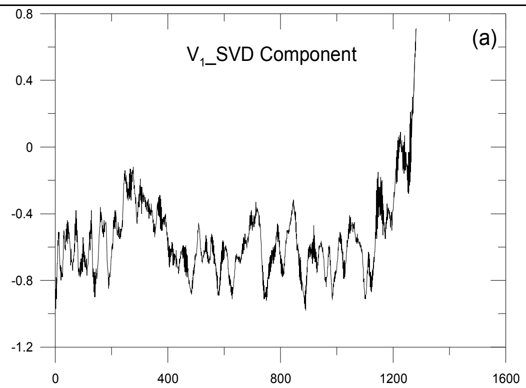

Figure 6a: SVD component time series, V1.

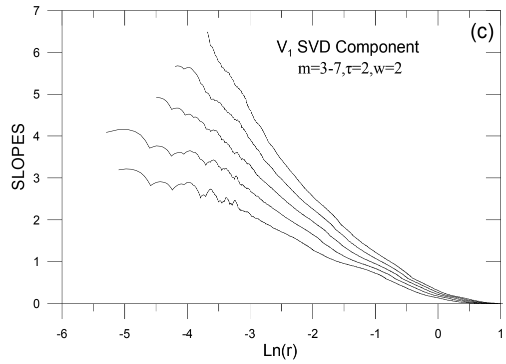

Figure 6c: Slopes of the correlation integrals for embedding dimension $m=7$ and delay time τ=2.

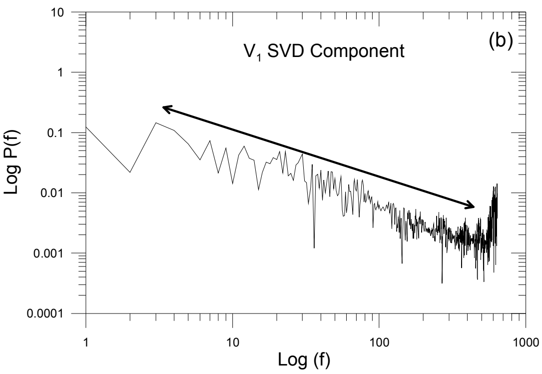

Figure 6b: Power spectrum of SVD V1.

Figures 7(a-c) show the slopes of the correlation integrals, the mutual information and the maximum Lyapunov exponent in comparison with the corresponding 30 surrogates, while Figures 7(d-f) show the related significances of the discrimination of statistics. The results show the first SVD component V1, reveals a hidden nonlinearity and nonlinear long range correlations, while there is a significant difference of low dimensionality for radius ln(r) = -3 till -2. These results are in agreement and further strengthen the results of the nonlinear analysis concerning AR(4) residuals.

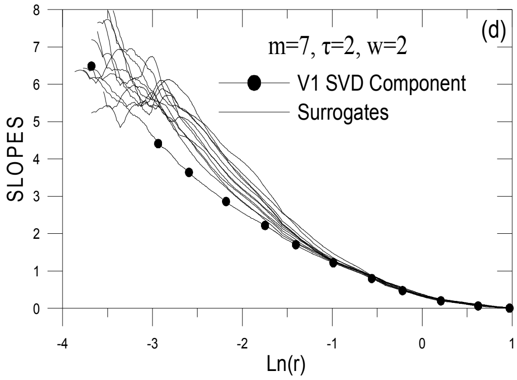

Figure 7a: Slopes of the correlation integrals of the SVD V1 component and 20 surrogate data as estimated for embedding dimension *m=7* and time delay *τ*=2.

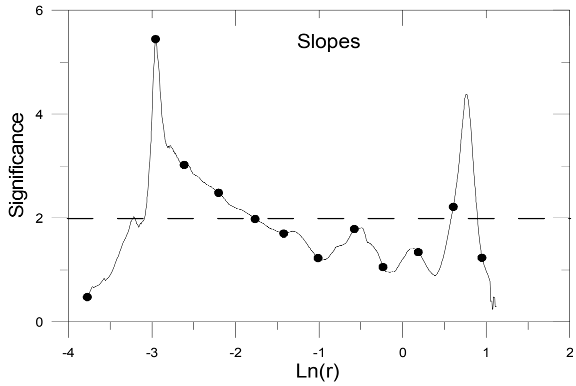

Figure 7d: Significance of the difference of the statistics between the SVD V1 component and 20 surrogate data concerning the slopes of the correlation integrals.

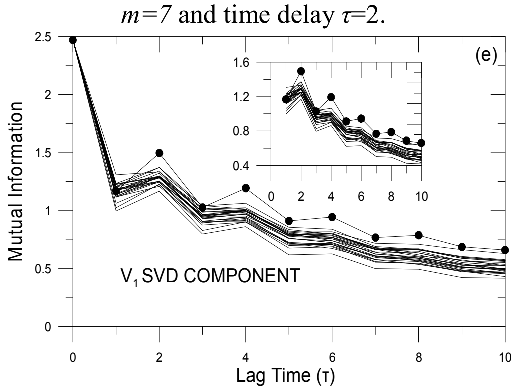

Figure 7b: Mutual Information estimated for the SVD V1 component and 20 surrogate data.

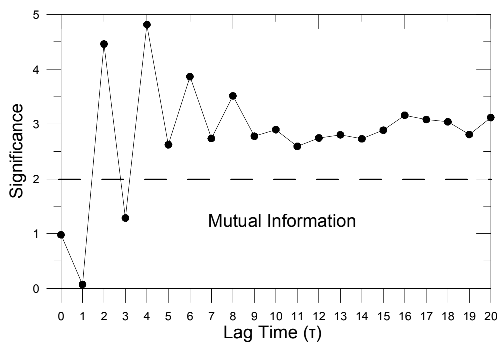

Figure 7e: Significance of the difference of the statistics between the SVD V1 component and 20 surrogate data concerning the mutual information.

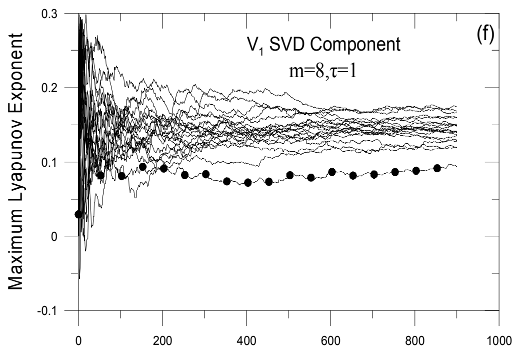

Figure 7c: Maximum Lyapunov exponent for the SVD V1 component and 20 surrogate data estimated for embedding dimension *m=8* and time delay *τ*=1.

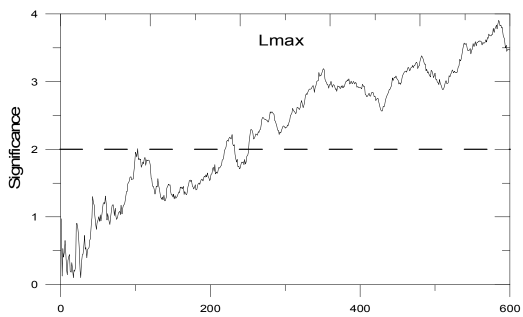

Figure 7f: Significance of the difference of the statistics between the SVD V1 component and 20 surrogate data concerning the maximum Lyapunov exponent

The next SVD components (Figures. 8(a-d)) capture the noise component that affects the original data. As it can be seen in these Figures the V2 SVD component corresponds to a high dimensional and linear dynamic process. In particular, the power spectrum has a flat profile (Figure 8b), the mutual information of the V_2 SVD component cannot be discriminated from its surrogates (Figure 8d) and the slopes of the correlation integral do not reveal a saturation or a scaling profile (Figure 8c).

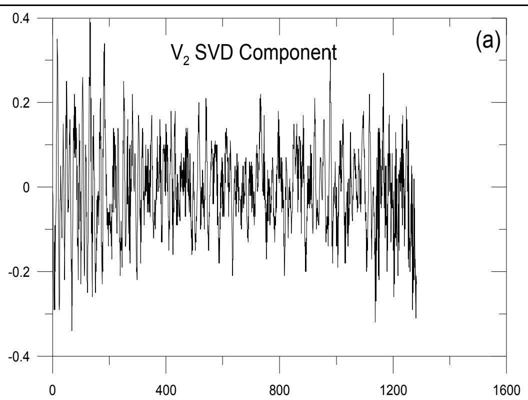

Figure 8a: SVD component time series, V2.

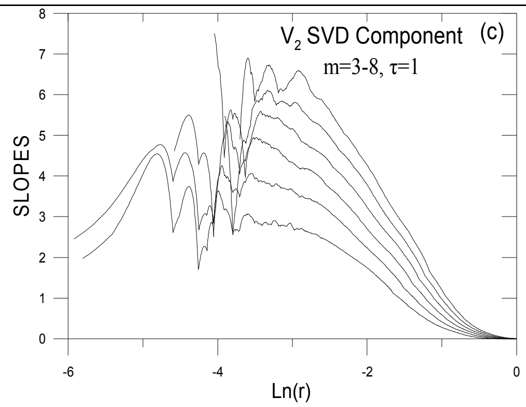

Figure 8c: Slopes of the correlation integrals for embedding dimension $m=8$ and delay time $\tau=1$.

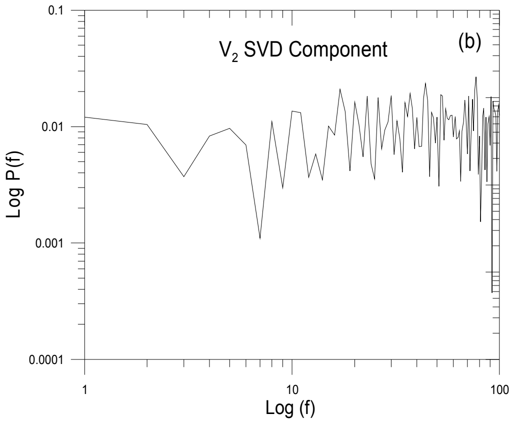

Figure 8b: Power spectrum of SVD V2.

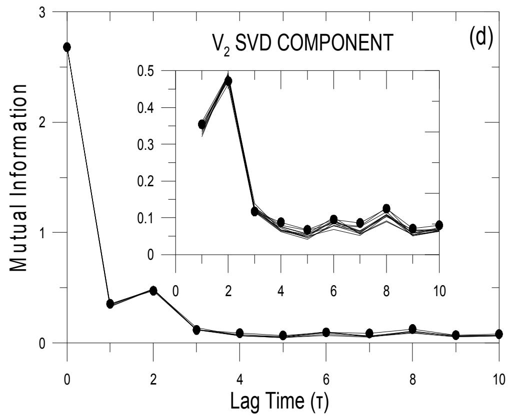

Figure 8d: Mutual Information estimated for the SVD V2 component and 20 surrogate data.

The analysis of the next SVD component, V2, which corresponds to the rapid fluctuations of the original signal, revealed the high dimensionality and the linearity of the component, properties reminiscent of white noise.

6. Conclusions

In the analysis performed, an AR(4) model was fitted to temperature data to remove the linear structure and the AR(4) residual series was tested using tools and methodology of the modern nonlinear time series. After applying the BDS test of independence and identical distribution in AR(4) residuals, the null hypothesis of independent and identically distributed data was rejected and the underlying nonlinear data structure was revealed. In addition, Brock's Residual test results for both the original and the AR(4) residual series led to the rejection of a linear generating process for temperatures.

Next, geometrical and dynamical characteristics in the reconstructed phase space such as correlation dimension, mutual information and maximum Lyapunov exponent were estimated for the temperature data and the AR(4) residuals. In order to prove the difference between the (potential) nonlinear process and a nonlinear distortion of a purely stochastic process, we generated 30 series of surrogate data, with discriminating statistics the BDS statistic, the maximum Lyapunov exponent and the mutual information. The nonlinear algorithm for computing geometrical and dynamical characteristics of the climate system was used once again, after filtering original temperature data with SVD Analysis. The comparison with stochastic surrogate data analysis provided more evidence for nonlinearity.

The results indicated the nonlinear stochastic profile of the Earth's complex climate dynamics, finding no evidence of deterministic chaos. Analysis of the (original) temperature time series revealed a linear stochastic component (white noise), being dominant in climate dynamics. Applying nonlinear techniques in whitened series gave strong evidence of nonlinearity and resulted in an underlying dynamics with many degrees of freedom. This conclusion is in accordance with the widely accepted opinion that earth's climate is highly nonlinear and stochastic.

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