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EXTREMELY Rough Draft, Please Do Not Quote Without Permission. Comments Are Welcome

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ABSTRACT

This paper examines the effect of bubble-bursting policy in the case where the central bank sometimes tries to deflate an asset which is not, in fact, overpriced. We consider the case of a "semi-bubble," where some traders know that an asset is overpriced, but others do not. Unlike most previous papers on bubble policy, our framework assumes rational traders. We also assume a finite time horizon, to rule out infinite horizon type bubbles. The market's "fulfilled expectations" equilibria are derived, and standard tools of welfare economics are applied to evaluate the effect of anti-bubble policy.

Under the assumption that the announcements of the financial authority can help less informed traders to learn more about a risky asset, market equilibria are presented and compared. We show that, if sellers care relatively more about the states where the central bank makes a negative bubble-bursting announcement, an announcement policy interferes with the asset's ability to share risks. Conversely, if sellers care relatively less about the announcement states, an announcement policy improves risk sharing.

"Information leakage" plays an important role in our analysis. Because of this leakage, central bank announcements can initiate further information revelation between traders. That is, the leakage effect can reveal information that the central bank, itself, does not have. However, this information leakage may not be welfare improving. Also, this leakage effect makes it difficult to predict the effects of bubble-bursting policy. This may complicate both private investment strategies and public policy analysis.

1 Introduction

The theory of asset price bubbles has attracted a great deal of attention recently, due to the dramatic rise and bursting of asset prices in the late 1990s (Higgins and Osler 1997, Shiller 2005, Ofek and Richardson 2003) and real estate fluctuations (Case and Shiller 2003, McCarthy and Peach 2004). These fluctuations are of interest since, if they are due to bubbles, they may lead to a non-optimal allocation of resources in the economy. Thus, the possibility that movements in asset prices could be due to "self-fulfilling prophesies" instead of fundamentals puts traditional economic thinking in an awkward situation. As Stiglitz (1990) points out, "if asset prices do not reflect fundamentals well, and if these skewed asset prices have an important effect on resource allocations, then the confidence of economists in the efficiency of market allocations of investment resources is, to say the least, weakened."

Since bubbles can distort resource allocations, it is important to determine how monetary authorities should deal with any bubbles in the market (Bernanke and Gertler 1999, 2001, Cecchetti, Genberg, Lipsky and Wadhwani 2000, Cecchetti, Genberg and Wadhwani 2002, Bordo and Jeanne 2002). Economists' arguments generally concentrate on three aspects of anti-bubble policy. First, can anti-bubble policy actually deflate bubbles? Second, if it does, to what degree can this policy correct the misallocation of resources in the economy, that is, increase social welfare? Third, why is it that historical attempts to burst bubbles have had such unpredictable, and at times disastrous, results? This study addresses these questions in an environment where the monetary authority may have mistaken information about the economy.

In this paper, we use an explicit, fully endogenous model of a greater-fool "semi-bubble," with rational agents and finite horizons, to evaluate the effects of anti-bubble policy when the central bank has only limited information about the true state of the economy, and sometimes tries to burst a bubble when there is no bubble actually present.

We consider a model with several states of the world and two periods. Two representative traders, Bob (the buyer) and Susan (the seller), trade a risky asset in a perfectly competitive market. Initially, Susan is endowed with one unit of the risky asset in each of the possible states. The actual value of the asset will be revealed to traders in the second period. Traders have different initial information partitions and different marginal utilities of wealth in each state. These generate different willingnesses to pay, i.e., shadow prices, in each cell of each trader's information partition.[1]

Initially, in the absence of any intervention from the central bank, the market has an asset bubble problem. We specifically choose parameter values so that Bob's willingness to pay is greater than Susan's in all cells of their partitions. Bob will then bid up the asset price to his willingness to pay, and always buys from Susan. If, in some states, Susan knows the asset that she is selling is worthless, then Susan is a "bad seller." If Susan knows the asset that she is selling is potentially valuable, then Susan is a "good seller." Because good Susan pools together with bad Susan, Bob must risk buying from bad Susan, if he wants to buy from good Susan. These bad-Susan states are then the bubble states. We call this a "semi-bubble" because only one side of the market knows the asset is overpriced.

[1] We adapt the information modeling framework used in Milgrom and Stokey (1982) and Allen, Morris and Postlewaite (1993). Samuelson (2004) provides an extensive description of this modeling methodology.

Next, in addition to the two traders, assume there is a central bank with imperfect information about the states of the world. The central bank's information partition is different from the two traders' information partitions. Revelation of the central bank's information will further refine traders' information, and thus change their shadow price relationships.

To restrain the "bubble" problem, the central bank may make a negative announcement when it believes the asset is in a bubble, while it makes no announcement when it believes the asset is valuable. We call this an "announcement policy." If the central bank chooses never to make any announcement (similar to the initial situation where there is no central bank), we call this a "no-announcement policy."

Of course, since the central bank's bubble-bursting announcement is sometimes mistaken, agents will rationally discount any announcement to some degree. The major goals of this study are to determine the effects of an announcement policy, and to find conditions, if any, under which an announcement policy can improve social welfare.

Assume, then, that the central bank follows an announcement policy. After the central bank makes the anti-bubble announcement, both traders will refine their information partitions and have new shadow prices. This change in shadow price relationships may cause the new market prices to reveal more information to traders than does the initial announcement. We call this process "information leakage."

For example, recall that, in the no-announcement-policy situation, we assume that Bob will always buy from Susan, and Bob's willingness to pay is the market price. Then, in an announcement-policy situation, good Susan's shadow price may rise above Bob's after the anti-bubble announcement. Good Susan's willingness to pay will then become the market price in the corresponding announcement states. When Bob observes the market price rising above his own willingness to pay, he will conclude that he is dealing with good Susan, who has a non-zero willingness to pay. On the other hand, if Bob observes the market price remain unchanged at his own willingness to pay after an announcement, he will conclude that he is dealing with a bad Susan, and his willingness to pay in this state will drop to zero. These changing shadow price relationships can therefore cause some of Susan's private information to "leak" to Bob.

We concentrate on three cases, each allowing for the *ex ante* existence of a bubble.

- **Case 1**. In this case, Bob's shadow prices always remain larger than Susan's after an anti-bubble announcement, so no further information is revealed. Bob continues to buy in each state of the world, which makes the anti-bubble effort of the central bank in vain in the sense that it does not burst the asset bubble. But asset prices in announcement states may be lower than in no-announcement states. Consequently, asset prices in the bubble states may be lower than the prices without an anti-bubble announcement.
- **Case 2a**. In this case, owing to the post-announcement changes in shadow price relationships, further information is revealed to Bob. This is a "leakage" case. With more information, Bob can refine his information partition to be the same as Susan's, and is able to distinguish bad Susan from good Susan. That is, in certain announcement states the central bank warns of a bubble but information leakage tells Bob that he is dealing with good Susan. In Case 2a Bob's willingness-to-pay increases but stays below Susan's. Thus, Bob will not buy from Susan in these states and Susan's higher willingness to pay is

the market price. Also, in the states where the central bank warns of a bubble and Bob finds out that his is dealing with bad Susan, Bob's willingness-to-pay drops to zero. He therefore does not buy in this situation either, so trade does not happen in any of the announcement states in Case 2a. Trade only happens in the "no-announcement" states where the central bank believes that the risky asset is valuable.

- **Case 2b**. In this case, as in Case 2a, the anti-bubble announcement causes further information leakage. With additional information, Bob again refines his information partition to be the same as Susan's. However, unlike Case 2a, after Bob learns that Susan is good, he will buy from her because his willingness to pay rises above good Susan's in the announcement states. Bob will not buy from bad Susan because he knows the asset in that situation to be worthless. Bob continues to buy from Susan in the "no-announcement" states. In Case 2b, Bob avoids trade with bad Susan and only trades with good Susan.

In addition to these three cases, there are three parallel cases, Case 3, 4a and 4b, where Bob does not buy from Susan in the no-announcement states. However, in this paper we focus on Cases 1, 2a and 2b, for brevity.

For each of these cases, we derive the welfare effects of the announcement policy. In Case 1, we prove that, when Susan puts less weight than does Bob on the states where the central bank makes a "bubble bursting" announcement, it is possible for the anti-bubble announcement to improve social welfare. In Cases 2a and 2b, where the policy leads to additional information leakage in these cases, we identify endogenously generated factors that make it more or less difficult to improve social welfare.

We can view these results in terms of policy's effect on an asset's risk-sharing attributes, as in Hirshleifer (1971). If Susan puts more weight on the announcement states than on the no-announcement states, an announcement policy from the central bank reduces the asset's ability to share risk. Here, Susan bears more risk than she otherwise would because she gets less in the announcement states, which she weighs more, and gets more in the no-announcement states, which she weighs less, so her welfare is reduced as a result of the announcement. Conversely, if Susan cares more about the no-announcement states than about the announcement states, then an announcement policy from the central bank improves the asset's ability to share risk. Bob, on the other hand, acts like a risk neutral market maker. His expected welfare is unchanged in our model.

Information leakage plays an important role in our analysis. Because of this leakage, central bank announcements can initiate further information revelation between traders. In general, one would expect this additional information revelation to improve social welfare. However, we will show that this additional information revelation can reduce welfare, depending on its effect on the asset's ability to share risk. Also, the leakage effect is unpredictable. This significantly complicates both public policy and the efforts of private investors to anticipate the price effects of this policy.

The remainder of the paper proceeds as follows. Section 2 compares our bubble framework to others in the literature. Section 3 develops market equilibria under the announcement policy versus the no-announcement policy. Section 4 presents the welfare analysis and our propositions, and Section 5 concludes.

2 Previous Literature

In his famous "beauty contest" analogy, Keynes (1936) describes an equity market as an environment in which speculators anticipate "what average opinion expects average opinion to be" rather than trying to find the underlying fundamentals of an asset. Since Keynes, many bubble models have been presented in the literature. Standard models of rational bubbles use an infinite-horizon framework, where agents buy overpriced assets because they believe those assets' prices will inflate forever (Tirole 1985). However, in these models, bubbles actually tend to improve welfare, since the bubble assets facilitate saving for future consumption. It seems unlikely that actual historical bubbles serve this function. Also, as argued by Kent and Lowe (1997), "the size of the bubble is indeterminate" in these models, so "there is no way of tying down the size" of any response of bubbles to policy actions.

For these reasons, previous studies of anti-bubble policy normally assume exogenously generated bubbles for risky assets. However, this method of modeling bubbles makes it difficult to relate the welfare effects of bubble policy to the underlying market failures which generate the bubble, because these market failures are not explicitly derived. Few previous studies therefore examine the welfare implications of anti-bubble policies.

Allen et al. (1993) provide a different approach to bubble research by modeling bubbles in a finite-horizon framework with rational agents. In their model any bubble must burst eventually. They use asymmetric information and short-sales constraints to model a "strong bubble," where everyone knows that the asset is overpriced. However, as a result of the information asymmetry, no one knows whether anyone else also knows that the asset is overpriced, so agents hope to sell their assets to the next "greater fool" before prices collapse. Their paper proves that bubbles are possible even if everyone is rational and knows that the overpriced asset will soon collapse.

Allen et al. (1993) present a very interesting approach to modeling asset bubbles. However, in its original form their model is too complicated to work with. Conlon (2004) simplified the Allen et al. approach by reducing the required number of agents from three to two. This simplification makes the comparison of equilibria easier. The present study simplifies further by focusing on a "semi-bubble," where only the seller knows that the asset is overpirced.

Other theoretical papers also model the intuition of "greater fool" bubbles from different perspectives. Abreu and Brunnermeier (2003) show that synchronization risk can prevent arbitrageurs from taking short positions to correct mispricing. A stock bubble grows until a significant number of arbitrageurs attack together. As a result, when arbitrageurs conclude that large scale synchronization is not likely, they will choose to "ride" the bubble for the time being. Similarly, in de Long, Shleifer, Summers and Waldmann (1990), rational arbitrageurs make profits by exploiting "positive feedback traders." When good news is confirmed, arbitrageurs buy into an asset and inflate its price. They then expect to sell the equity to the positive feedback traders in the next period since they know the positive feedback traders will follow the good news and drive prices up even more.

Harrison and Kreps (1978) and Scheinkman and Xiong (2003) model bubbles as a result of overconfidence. In their models, heterogeneous beliefs arise because of the existence of overconfident traders. Asset owners have the option to sell the asset to other traders with more optimistic priors. Traders trade at a price higher than their valuation of expected dividends because they believe that they will be able to find more optimistic buyers willing to pay even higher prices. The overconfidence framework enables researchers to characterize properties of asset bubbles, such as trading frequency and asset price volatility. In particular, Scheinkman and Xiong (2003) show that their model is consistent with the characteristics of actual historical bubbles. They also suggest that a small trading tax could reduce speculative trading, but not overpricing.

Greater-fool type bubbles may not just be a theoretical possibility. Recent empirical studies also provide some support for the theory. For example, citing the theoretical framework of Abreu and Brunnermeier (2003), Temin and Voth (2004) show that during the South Sea bubble, Hoare's Bank, a leading player in the event, knew that a bubble was in progress and "nonetheless invested in the stock: it was profitable to ride the bubble." They also show that its informational advantage did not prevent Hoare's Bank from inflating the bubble. Thus, informed investor behavior does not necessarily lead to attacks on a bubble. Brunnermeier and Nagel (2004) provide more recent evidence of investors riding a bubble. They investigate the activities of hedge funds during the growth of the "dot-com bubble." They find that hedge funds, which are normally considered to represent "rational arbitrageurs," did not correct the price misalignments in the market. In their sample period, hedge fund portfolios were deeply long in high-tech stocks. Further, hedge fund managers were able to predict the price peak of individual stocks and reduced their positions before prices collapsed. Brunnermeier and Nagel (2004) show that hedge fund portfolios earned excess returns of about 4.5 percent per quarter compared with well-matched portfolios.

Many researches have addressed the issue of asset bubbles and monetary policy. Bernanke and Gertler (1999) and (2001) argue that unstable asset prices should not be the central bank's major concern unless these price movements influence the central bank's expectation of inflation. That is, central banks should not respond to asset price fluctuations in general. They argue that an inflation-targeting monetary policy will stabilize the economy well, even during an asset market boom or collapse.

Several other studies are more positive about central bank policies towards asset price misalignments. Cecchetti et al. (2002) and Kent and Lowe (1997) present their arguments in an inflation-targeting framework. Cecchetti et al. (2002) argue that the central bank can improve macroeconomic stability by reacting differently towards different types of asset price misalignments. Kent and Lowe (1997) suggest that an anti-bubble reaction from the central bank today helps it to realize long-term inflation targets by reducing future asset price instabilities.

Bordo and Jeanne (2002) discuss the difference between a proactive monetary policy and a reactive monetary policy. They suggest that, in some circumstances, a reactive strategy by a central bank, focusing only on deviations from inflation targets, may lead to more loss of output than a proactive policy which incorporates asset prices into the central bank's monetary objectives. However, they also suggest that such an optimal monetary policy cannot be generalized into a simple policy rule since it depends on the economic variables in a "complex, non-linear" way.

Conlon (2005), finally, uses a "greater fool" bubble model to evaluate the effect of anti-bubble policy. However, unlike our analysis, he assumes that the central bank only tries to deflate prices when assets are, in fact, overpriced. Conlon shows that central bank announcements which are accurate in this sense can improve social welfare, if they reduce the lemons problem (Akerlof 1970) caused by bad sellers who know their assets are overpriced. However, he also shows that bubble bursting can actually make the lemons problem worse.

3 Model and Market Equilibria

In this section, we analyze anti-bubble policy in a revised version of the Allen et al. (1993) type greater fool model. For simplicity, we use the Conlon (2004) two-agent version of the model, and simplify further by considering a "semi-bubble" model. That is, in period one, only half of the agents know that the asset is overpriced. In addition, a central bank may make an announcement about what it believes to be the true state of the market. Traders will then refine their knowledge about the market based on the announcement. Like other agents in the market, the central bank possesses only partly correct knowledge about the true state. As is standard, we assume a short-selling constraint in this model.[2]

We adopt the "fulfilled expectations" equilibrium notion of Kreps (1977) to analyze equilibrium in the market. Different traders have different information partitions and marginal utilities. Traders thus have different shadow prices in different states of the

[2] Many studies have investigated empirically different types of short-sales constraints and their effects. For example, Ofek and Richardson (2003) discuss the role of short-sales constraints in the DotCom bubble. They argue that there were substantial short-sales restrictions due to stock lock-up agreements among late 90s hi-tech stocks. D'Avolio (2002) analyzes the security lending market and shows that the supply of loanable stocks can be very scarce, which causes a significant short-sales constraint. Alexander and Peterson (1999) and Alexander and Peterson (2007) investigate the role of short-sales regulations, e.g., the price test (or "up-tick rule), in constraining short sales. Lamont (2004) discusses legal and short-squeeze pressures from short sales targeted firms, which increase short-sales costs for short sellers and yield another source of short-sales constraints.

world. Also, we assume that their elasticities of demand are infinite at these shadow prices. With short-sales constraints, the actual market price in each state is then bid up to the highest of the traders' shadow prices in that state.

In our model, there is a risky asset, whose market is competitive, with two representative traders, Susan (the seller) and Bob (the buyer), and a central bank. There are five states and two time periods. The discount factor is one. There is uncertainty in the first period, but the true state of the world is revealed in the second period.

The state space Ω is:

$$\Omega = \{ e^B ,\ e^G_{1a} ,\ e^G_{2a} ,\ e^G_{1n} ,\ e^G_{2n} \}.$$

Here e^B is the state where Susan is "bad" and knows that the asset is worthless, and e^B, e^G_{1a} and e^G_{2a} are the states where the central bank believes that Susan could be bad and makes an anti-bubble announcement. In the states e^G_{1n} and e^G_{2n} the central bank believes that Susan is good and makes no announcement. Therefore subscript *n* means "no announcement" from the central bank, and subscript a means "announcement." Assume that all five states are equally likely, so each state has a probability of one-fifth. The risky asset only pays a dividend, of 10, in two states, e^G_{2a}, e^G_{2n}. Subscript *2* means "dividend of ten pays in the second period, subscript *1* means "no dividend will be paid in both periods." The traders' initial information partitions are:

$$P_S = \{ e^B \},\ \{ e^G_{1a} ,\ e^G_{2a} ,\ e^G_{1n} ,\ e^G_{2n} \}$$

$$P_B = \{ e^B ,\ e^G_{1a} ,\ e^G_{2a} ,\ e^G_{1n} ,\ e^G_{2n} \}$$

That is, Susan knows whether or not the true state is e^B, but Bob has no information about which state has occurred. The central bank's information partition is:

$$P_{CB} = \{ e^B,\ e_{1a}^G,\ e_{2a}^G \},\ \{ e_{1n}^G,\ e_{2n}^G \}$$

Thus, if the state is e_{1n}^G, say, then the central bank knows the state is either e_{1n}^G or e_{2n}^G, but it does not know which one it is.

Suppose that Susan initially owns one unit of the risky asset in all five states. This risky asset only pays a dividend, of 10, in two states, e_{2a}^G and e_{2n}^G. Because of her initial information partition, if the true state of the world is e^B, Susan is a "bad seller" since she knows that the asset she is planning to sell actually is worthless. If the true state of the world is any one of e_{1n}^G, e_{1a}^G, e_{2n}^G, or e_{2a}^G, Susan is a "good seller," as indicated by the superscript *G,* since she believes that the asset has a positive expected value. Bob, on the other hand, cannot distinguish between the different states of the world. Bob can be seen as an uninformed trader in the asset market, since he depends on public information.

Marginal utilities of wealth for Susan and Bob are given in the following table:

Table 1: Marginal Utilities

State	e^B	e_{1a}^G	e_{2a}^G	e_{1n}^G	e_{2n}^G
MU_E	y_B	y_{1a}	y_{2a}	y_{1n}	y_{2n}
MU_F	z_B	z_{1a}	z_{2a}	z_{1n}	z_{2n}

Like the traders, the central bank has only limited information about the true state of the world. In states e^B, e_{1a}^G, e_{2a}^G, the central bank knows that Susan could be a bad seller. Therefore, if it is following an announcement policy, it will make a "bubble-bursting" announcement. In states e_{1n}^G and e_{2n}^G, the central bank knows that Susan is a good seller, so it will not make a bubble-bursting announcement in these two states.

3.1 The Central Bank with a No-Announcement Policy

First, let us consider the market equilibria when the central bank does not reveal its information to the market, so the central bank is practicing a no-announcement policy. The shadow prices for Susan and Bob in period one can be calculated by taking marginal-utility-weighted values of the dividends, for each cell in their partitions. These shadow prices are given in Table 2, with sp_I^E and sp_I^F given by:

$$sp_I^S = \frac{10(y_{2a} + y_{2n})}{y_{1n} + y_{1a} + y_{2a} + y_{2n}}, \; sp_I^B = \frac{10(z_{2a} + z_{2n})}{z_B + z_{1n} + z_{1a} + z_{2a} + z_{2n}} \tag{1}$$

Table 2: Shadow Prices in Case I (with no-announcement policy)

	e^B	e_{1a}^G	e_{2a}^G	e_{1n}^G	e_{2n}^G
Susan	0	sp_I^S	sp_I^S	sp_I^S	sp_I^S
Bob	sp_I^B	sp_I^B	sp_I^B	sp_I^B	sp_I^B

With these shadow prices, we obtain the following market equilibria.

Case I: If $sp_I^B \geq sp_I^S$, then there are two equilibria. In one equilibrium, trade will happen in all five states at the price sp_I^B. Bob will be hurt eventually if the true state of the world is e^B, e_{1a}^G, or e_{1n}^G. However, a semi-bubble exists only in state e^B because Bob pays for a worthless asset which Susan knows is worthless.

There is also a second, non-bubble equilibrium, with price equal to zero in state e^B, and price equal to sp_{II}^B in the other four states, (with the formula of sp_{II}^B given in Case II below). Intuitively, this is possible because bad Susan is more eager to sell than good Susan, which may lead to different selling efforts by bad Susan and good Susan, and leak further information to the other side of the trade. Suppose Susan tries harder to sell the asset if the true state is e^B (because she knows the asset really is worthless). Bob knows the information partition of Susan, so he might conclude that the true state of the world is e^B. In this case, the market price drops to zero in state e^B, yielding the second market equilibrium. However, since we are interested in bubble-bursting announcements, we focus on the first equilibrium, which has a semi-bubble in state e^B.

Also, because an asset bubble exists only in Case I, we will be concentrating on this case in the welfare analysis in Section 4. However, we also derive the market equilibria in the other two cases, Case II.1 and Case II.2 below, to complete the analysis of the no-announcement policy situation.

Case II: If $sp_I^B < sp_I^S$, then good Susan will tend to bid the market price up to her shadow price, so sp_I^S will be the market price in states e_{1a}^G, e_{2a}^G, e_{1n}^G, and e_{2n}^G. However, since Susan's shadow price in state e^B is zero, Susan will not bid price above sp_I^B in state e^B. Bob would then observe this price difference and learn that the true state of the world is e^B. That is, Susan's information will be revealed to Bob through the market price (this is similar to the information leakage effect we will discuss in Subsection 3.2 below). The shadow prices for both types of trader will then be as in Table 3, with:

$$sp_{II}^S = sp_I^S,\ sp_{II}^B = \frac{10(z_{2a} + z_{2n})}{z_{1n} + z_{1a} + z_{2a} + z_{2n}}. \tag{2}$$

Table 3: Shadow Prices in Case II

	e^B	e^G_{1a}	e^G_{2a}	e^G_{1n}	e^G_{2n}
Susan	0	sp^S_{II}	sp^S_{II}	sp^S_{II}	sp^S_{II}
Bob	0	sp^B_{II}	sp^B_{II}	sp^B_{II}	sp^B_{II}

Case II leads to two sub-cases.

Case II.1: In this case, even after the bubble state e^B is revealed to Bob, Bob's confidence in the market improves only a little. Thus, Bob won't buy even when the true state is not the bubble state e^B. Intuitively, the fear of getting a worthless asset in state e^B does not concern Bob very much, so the information revealed, that he is not in the bubble state, has little positive influence on Bob's shadow price. This happens if Bob's marginal utility of consumption in state e^B is very small. Therefore his new shadow price, sp^B_{II}, is roughly the same as sp^B_I and, so, will still be smaller than sp^S_I. Thus sp^S_I will still be the market price in states e^G_{1a}, e^G_{2a}, e^G_{1n}, and e^G_{2n}. There will be no trade in these states. Also, no bubble exists in state e^B, since the price is zero at e^B.

Case II.2: In this case, after Bob learns that he is not in the bubble state e^B, his confidence in the market improves significantly. Here, the fear of getting a worthless asset in state e^B concerns Bob greatly. So, when Bob learns that the state is not e^B, Bob's shadow price in other states rises above Susan's. This case happens when Bob's marginal utility of consumption in state e^B is very large, so sp^B_{II} rises a great deal above sp^B_I. Thus, even though $sp^B_I < sp^S_I$, sp^B_{II} becomes larger than sp^S_I, since $sp^B_{II} \gg sp^B_I$. Trade will then happen in states e^G_{1a}, e^G_{2a}, e^G_{1n}, and e^G_{2n} at price sp^B_{II}.

3.2 The Central Bank with An Announcement Policy

If the central bank reveals its information, Susan's and Bob's partitions become

$$P_S = \{e^B\}, \{e^G_{1a}, e^G_{2a}\}, \{e^G_{1n}, e^G_{2n}\}$$

$$P_B = \{e^B, e^G_{1a}, e^G_{2a}\}, \{e^G_{1n}, e^G_{2n}\}$$

Here Susan forms P_S by incorporating new information from the central bank's announcement about whether or not states e^B, e^G_{1a}, or e^G_{2a} occurred. Bob knew nothing before the announcement, so his information partition, P_F, after the announcement, is identical to the information partition of the central bank. That is, when the central bank makes an announcement, Bob knows that the true state is one of e^B, e^G_{1a}, and e^G_{2a}, but he does not exactly know which one is the true state. On the other hand, when the central bank makes no announcement, Bob know that the true state is one of e^G_{1n}, e^G_{2n}, he does not know the exact state either.

The new shadow prices in period one are shown in Table 4:

Table 4: Shadow Prices with Announcement Policy—Case 1

	e^B	e^G_{1a}	e^G_{2a}	e^G_{1n}	e^G_{2n}
Susan	0	$\frac{10y_{2a}}{y_{1a}+y_{2a}}$	$\frac{10y_{2a}}{y_{1a}+y_{2a}}$	$\frac{10y_{2n}}{y_{1n}+y_{2n}}$	$\frac{10y_{2n}}{y_{1n}+y_{2n}}$
Bob	$\frac{10z_{2a}}{z_B+z_{1a}+z_{2a}}$	$\frac{10z_{2a}}{z_B+z_{1a}+z_{2a}}$	$\frac{10z_{2a}}{z_B+z_{1a}+z_{2a}}$	$\frac{10z_{2n}}{z_{1n}+z_{2n}}$	$\frac{10z_{2n}}{z_{1n}+z_{2n}}$

We next consider the various possible market equilibria in the presence of an announcement policy. In order for the central bank's announcement to always have a negative influence on the market price, we assume that the following shadow price relationships hold throughout our analysis.

$$\text{For Susan:} \quad \frac{10y_{2a}}{y_{1a}+y_{2a}} = sp_{1A}^{S} < \frac{10y_{2n}}{y_{1n}+y_{2n}} = sp_{1NA}^{S}, \tag{3}$$

$$\text{For Bob:} \quad \frac{10z_{2a}}{z_B+z_{1a}+z_{2a}} = sp_{1A}^{B} < \frac{10z_{2n}}{z_{1n}+z_{2n}} = sp_{1NA}^{B}. \tag{4}$$

Here the subscript *A* represents "announcement," and *NA* represents "no announcement."

Case 1 (Bob's shadow price always above Susan's and he always buys): If $sp_{1NA}^{S} \le sp_{1NA}^{B}$ and $sp_{1A}^{S} \le sp_{1A}^{B}$, then there is still a bubble equilibrium in which Bob always buys from Susan. The market prices in period 1 will be Bob's shadow prices, i.e., sp_{1A}^{B} in states e^B, e_{1a}^{G}, e_{2a}^{G}, where an announcement is made, and sp_{1NA}^{B} in the states e_{1n}^{G}, e_{2n}^{G}, where no announcement is made. Trade will always occur, even in the bubble state e^B.[3]

Case 2 (Bob's shadow price falls below Susan's if the central bank makes an announcement): If $sp_{1NA}^{S} \le sp_{1NA}^{B}$ but $sp_{1A}^{S} > sp_{1A}^{B}$, the anti-bubble announcement decreases Bob's confidence a great deal, so his willingness-to-pay is lower than good Susan's. Good Susan will tend to bid the market price up to her shadow price in the announcement states e_{1a}^{G} and e_{2a}^{G}. Since Susan's shadow price in state e^B is zero, bad Susan in the competitive market will not bid price above Bob's shadow price, sp_{1A}^{B}, in state e^B. If the true state is e^B, Bob observes that the market price does not change and thus identifies the state as e^B, since he knows Susan's information partition. Conversely, if Susan is good, Bob will realize that too. Bob will then have the same information

[3] Of course, as in Subsection 3.1 above, there is also a no-bubble equilibrium. For simplicity, we focus on the bubble equilibrium.

partition as Susan. That is, the information revealed by the central bank induces additional "information leakage." The market price will drop to zero in state e^B, and rise to $P_A' = 10z_{2a}/(z_{1a}+z_{2a})$ in states e_{1a}^G and e_{2a}^G. Here, P_A' is Bob's shadow price after the central bank announcement and information leakage, so the new shadow price schedule will be as in Table 5. The new shadow prices lead to two sub-cases.

Table 5: Shadow Prices with Announcement Policy—Case 2

	e^B	e_{1a}^G	e_{2a}^G	e_{1n}^G	e_{2n}^G
Susan	0	$\frac{10y_{2a}}{y_{1a}+y_{2a}}$	$\frac{10y_{2a}}{y_{1a}+y_{2a}}$	$\frac{10y_{2n}}{y_{1n}+y_{2n}}$	$\frac{10y_{2n}}{y_{1n}+y_{2n}}$
Bob	0	P_A'	P_A'	$\frac{10z_{2n}}{z_{1n}+z_{2n}}$	$\frac{10z_{2n}}{z_{1n}+z_{2n}}$

Case 2a (No trade if the central bank makes announcement): In this sub-case, the anti-bubble announcement depresses Bob's willingness-to-pay very much, and the information leakage does not cause his confidence to recover enough in states e_{1a}^G and e_{2a}^G, so Bob does not buy after an announcement (though he always buys if the central bank turns out not to make an announcement). That is, Bob's confidence does not rise much if he learns that he is not facing bad Susan. This happens if z_B, his marginal utility of consumption in the bubble state e^B, is very small. In this case, P_A' will be roughly the same as sp_{1A}^B, so, since $sp_{1A}^B < sp_{1A}^S$, we have $P_A' < sp_{1A}^S$ still, so Bob does not buy from Susan in states e_{1a}^G and e_{2a}^G. Trade will not happen in state e^B either, since both traders

know the asset is worthless. However, trade will happen in states e_{1n}^{G} and e_{2n}^{G} at market price sp_{1NA}^{B}.

Case 2b (trade happens when Susan is good): In this sub-case, the anti-bubble announcement depresses Bob's willingness-to-pay, but information leakage causes his confidence to recover more significantly. After the bad Susan is identified, Bob always buys from good Susan. Bob is more concerned about the danger of buying from bad Susan and, so, is greatly relieved when he learns that Susan is good. This happens when z_B is very large, so $P_A' > sp_{1A}^{S}$ since $P_A' \gg sp_{1A}^{B}$. Trade happens in states e_{1a}^{G} and e_{2a}^{G}, and the market price is P_A'. However, the market price is zero in state e^{B} since both traders know it is the bubble state. Trade also happens in the no-announcement states e_{1n}^{G} and e_{2n}^{G}, and the price is sp_{1NA}^{B}.

Now we have enough equilibria to do the welfare analysis (Case I without the announcement policy, and Cases 1, 2a, and 2b with an announcement policy). But for completeness we derive the remaining equilibria in Cases 3, 4a, and 4b below. These are similar to Cases 1, 2a and 2b, but trade does not happen if the central bank turns out not to make an announcement.

Case 3: If $sp_{1A}^{S} \le sp_{1A}^{B}$ but $sp_{1NA}^{S} > sp_{1NA}^{B}$, the market price in states e^{B}, e_{1a}^{G} and e_{2a}^{G} will be sp_{1A}^{B}, and the market price in state e_{1n}^{G} and e_{2n}^{G} will be sp_{1NA}^{S}. In states e^{B}, e_{1a}^{G} and e_{2a}^{G}, trade will happen. In states e_{1n}^{G} and e_{2n}^{G}, trade will not happen. Trade happens only after the announcement. The semi-bubble still exists after the announcement.

Case 4: If $sp_{1A}^{B} < sp_{1A}^{S}$ and $sp_{1NA}^{B} < sp_{1NA}^{S}$, then good Susan will always bid the market price up to her shadow price, so sp_{1A}^{S} will be the market price in states e_{1a}^{G} and e_{2a}^{G}, and sp_{1NA}^{S} will be the market price in states e_{1n}^{G} and e_{2n}^{G}.

However, in state e^{B} Susan's shadow price is zero, so Susan in the competitive market will not bid price above Bob's shadow price sp_{1A}^{B} in the announcement states. Bob then observes the difference in market prices and learns that the true state is e^{B}. That is, we again have "information leakage" as the result of the central bank announcement. Bob's information partition will be refined, yielding the same shadow price schedule as in Table 5.

Case 4a: In this case, even after the bubble state is revealed to Bob, he still does not trade, because his confidence in states e_{1a}^{G}, e_{2a}^{G} changes very little. Bob is not concerned about the bubble state e^{B} as an important factor influencing his trading decision. This case happens if Bob's marginal utility in the bubble state, z_B, is very small, so his new shadow price, P_A', will be roughly the same as sp_{1A}^{B}. Since $sp_{1A}^{B} < sp_{1A}^{S}$, P_A' will then still be smaller than sp_{1A}^{S} also. In this equilibrium, the market prices are sp_{1A}^{S} in states e_{1a}^{G}, e_{2a}^{G}, and sp_{1NA}^{S} in states e_{1n}^{G}, e_{2n}^{G}. Trade does not happen in state e^{B} because both traders know the asset is worthless.

Case 4b: In this case, after the bubble state e^{B} is revealed to Bob, his confidence in the other announcement states improves significantly. That is, the fear of buying from bad Susan concerns Bob greatly. So after Bob identifies the bubble state e^{B}, his shadow prices in other announcement states increase a great deal and he buys in these states. This case happens if z_B is very large, so $sp_{1A}^{B} \ll P_A'$, and it is possible that $sp_{1A}^{S} < P_A'$. The market prices are zero in state e^B, P_A' in states e_{1a}^{G}, e_{2a}^{G}, and sp_{1NA}^{S} in states e_{1n}^{G}, e_{2n}^{G}. Trade will happen only in states e_{1a}^{G} and e_{2a}^{G} after the announcement. The revelation by the central bank increases Bob's confidence in trading.

4 Welfare Analysis of the Anti-Bubble Policy

We now have several equilibria under different market situations. To examine the welfare effects of central bank announcements, it is first necessary to explain how we measure social welfare.

In this paper, social welfare is measured by Bob's consumer surplus and Susan's producer surplus. However, Bob's consumer surplus is zero. This is because both Susan's and Bob's marginal utilities in each state of the world both are constant, but Bob is different from Susan in the sense that his demand curve in this model is horizontal, i.e., his elasticity of demand is infinite, so his consumer surplus in this model is zero. On the other side, Susan can sell only her limited endowment of the asset, since she is short-sales constrained, so she cannot sell an infinite amount of the asset. Thus, her seller surplus, or producer surplus, can be non zero. Social welfare from the asset transaction is therefore equal to Susan's producer surplus, i.e., her benefit from any sales of her endowed asset.

Before we discuss the social welfare effects of anti-bubble announcements, let us limit the set of equilibria that we consider.

In Section 3.1 above, the central bank never makes an announcement with respect to an asset bubble in the economy. Traders behave as we have analyzed in Case I, Case

II.1 and II.2. We will focus on Case I from Section 3.1, where there is a semi-bubble in the asset market, so the "no announcement" asset price, P_{NAP}, refers to the market price in Case I.

In Section 3.2 above, we considered the case where the central bank follows an announcement policy. It makes an announcement if it believes that there is an asset bubble. Here we use P_A to denote the market price when the central bank believes there is an asset bubble and makes an announcement. Similarly, we use P_{NA} to denote the market price when the central bank is following the announcement policy, but it makes no actual announcement, since it believes there is no danger of an asset bubble in the market. In the asset market, if traders hear an announcement, they know that the danger of an asset bubble is more serious, so they pay less in those states, though price does not necessarily fall to zero, since they know the central bank also has imperfect information. If traders do not hear any announcement, they will conclude that the danger of a bubble in the market is less likely, so they will trade with more confidence and pay higher prices. We therefore always choose parameter values such that

$$P_A \leq P_{NAP} \leq P_{NA}. \tag{5}$$

We also adopt another assumption to simplify the analysis. In Case 1, Case 2a and 2b, under the announcement policy, we assume that Bob continues to buy from Susan if the central bank turns out not to make an announcement. For brevity, we focus on these cases, and ignore the parallel cases, Case 3, 4a and 4b, where trade ceases when no announcement is made. Thus, we will compare the equilibrium in Case I with the equilibria in Cases 1, 2a and 2b.

4.1 Conditions under which Announcements Increase Social Welfare

This section analyzes the welfare effects of an anti-bubble announcement policy. We start with the non-announcement policy Case I, where the central bank does not reveal any information, and Bob always buys from Susan, whether she is good or bad. That is, we require $sp_I^S \leq sp_I^B$ as in Case I from Section 3.1, and compare welfare in this case to welfare in Cases 1, 2a and 2b from Section 3.2. Again, since Bob's elasticity of demand is infinite, his consumer surplus in this model is zero, so we focus on Susan's producer surplus in the welfare analysis.

In Case I, the central bank is practicing a no-announcement policy. Susan's expected utility from the trade is equal to the (constant) market price times her expected marginal utility in those states. This gives:

$$\frac{10(z_{2a}+z_{2n})}{z_B+z_{1n}+z_{1a}+z_{2a}+z_{2n}}\times\left(\frac{y_B+y_{1n}+y_{1a}+y_{2a}+y_{2n}}{5}\right). \tag{6}$$

In the three announcement-policy cases (Case 1, Case 2a and Case 2b), the central bank announces its information to the market, so Susan's expected utilities can be written as follows.

In Case 1, Susan always sells the asset at Frank's shadow price, so her expected utility from the market is:

$$\frac{10z_{2a}}{z_B+z_{1a}+z_{2a}}\times\frac{y_B+y_{1a}+y_{2a}}{5}+\frac{10z_{2n}}{z_{1n}+z_{2n}}\times\frac{y_{1n}+y_{2n}}{5}. \tag{7}$$

In this case, the central bank announcement does not prevent Bob from buying from bad Susan. That is, The central bank announcement does not burst the semi-bubble in state e^B. However, since we assume $P_A \leq P_{NAP} \leq P_{NA}$, the announcement will in some degree decrease the market price in state e^B and also in the other two announcement states,

e_{1a}^G and e_{2a}^G. The announcement policy makes Susan worse off in the announcement states, but makes her better off in the no-announcement states.

In Case 2a, Susan keeps the asset in the announcement states, including state e_{2a}^G, where the dividend is 10. On the other hand, she sells it in the no-announcement states at Bob's shadow price. Thus, her expected utility from the market is:

$$10 \times \frac{y_{2a}}{5} + \frac{10 z_{2n}}{z_{1n} + z_{2n}} \times \frac{y_{1n} + y_{2n}}{5}. \tag{8}$$

The central bank announcement prevents Bob from buying from Susan in the announcement states. "Bad Susan" is worse off, and trading volume falls to zero if the central bank makes a bubble announcement.

In Case 2b, Susan sells in all the non-bubble states, at Frank's shadow price, so her expected utility from the market is:

$$\frac{10 z_{2a}}{z_{1a} + z_{2a}} \times \frac{y_{1a} + y_{2a}}{5} + \frac{10 z_{2n}}{z_{1n} + z_{2n}} \times \frac{y_{1n} + y_{2n}}{5}. \tag{9}$$

The central bank announcement prevents Bob from buying the risky asset from bad Susan, but trade happens in all of the other four states between Bob and good Susan. The central bank announcement does burst the asset bubble while retaining the trading volume in the non-bubble states. Both Cases 2a and 2b involve information leakage, because bad Susan's knowledge of state e^B ends up being revealed to the market.

We next determine the conditions under which the anti-bubble policy increases social welfare, i.e., Susan's expected utilities from the market. That is, we compare Susan's expected welfare from Case I above to her expected welfare in Cases 1, 2a, and 2b.

We first consider Case 1, where the anti-bubble announcement does not actually burst the asset bubble.

Proposition 1: In Case 1 (where buyers buy regardless of the anti-bubble announcement), the announcement policy increases social welfare if and only if

$$\frac{z_B + z_{1a} + z_{2a}}{z_{1n} + z_{2n}} > \frac{y_B + y_{1a} + y_{2a}}{y_{1n} + y_{2n}}. \tag{10}$$

Proof: Because Bob always buys in Case 1, and his welfare is unaffected by the anti-bubble policy, his welfare gain in the announcement states (paying less) should be equal to his welfare loss in the no-announcement states (paying more), i.e.,

$$(P_{NAP} - P_A)(z_B + z_{1a} + z_{2a}) = (P_{NA} - P_{NAP})(z_{1n} + z_{2n}). \tag{11}$$

This can also be verified by plugging in the formulas for $P_{NAP} = sp_I^B$, $P_A = sp_{1A}^B$, and $P_{NA} = sp_{1NA}^B$. For Susan, her welfare decreases in the announcement states (receiving less) and increases in the no-announcement states (receiving more). When her loss is smaller than her gain, then her welfare (social welfare) increases. This happens when

$$(P_{NAP} - P_A)(y_B + y_{1a} + y_{2a}) < (P_{NA} - P_{NAP})(y_{1n} + y_{2n}). \tag{12}$$

Rearranging equation (11) and inequality (12) slightly gives:

$$\frac{P_{NA} - P_{NAP}}{P_{NAP} - P_A} = \frac{z_B + z_{1a} + z_{2a}}{z_{1n} + z_{2n}}, \tag{13}$$

$$\frac{P_{NA} - P_{NAP}}{P_{NAP} - P_A} > \frac{y_B + y_{1a} + y_{2a}}{y_{1n} + y_{2n}}. \tag{14}$$

Combining equation (13) and inequality (14) gives Condition (10). **QED**

Proposition 1 says that, in Case 1, social welfare (i.e., Susan's welfare) will increase as a result of the anti-bubble policy if Susan puts less weight on the announcement states relative to the no announcement states than does Bob.

This proposition can be understood as follows. We assume that $P_A \leq P_{NAP} \leq P_{NA}$ always holds. Thus, the announcement shifts wealth from Susan to Bob in the announcement states, and from Bob to Susan in the no-announcement states. The overall expected change in Bob's welfare is zero. Using this as a benchmark, we can look at the change in Susan's welfare. If Susan puts relatively less weight on the announcement states than does Bob, she will care less about the lower P_A after the announcement. On the other hand, if Susan puts more weight on the no-announcement states than does Bob, the increased P_{NA} will dominate her welfare change, so Susan's welfare, i.e., social welfare, increases.

If condition (10) holds, so Susan cares more about the no-announcement states than about the announcement states, then the announcement policy improves the asset's ability to share risk. Susan bears less risk since she will get more in the no-announcement states which she weighs more, and get less in the announcement states which she weighs less. Thus, her welfare, and so the social welfare, increases as a result of the announcement policy. Similarly, if Condition (10) is violated, then the announcement interferes with the asset's ability to share risk, as in Hirshleifer (1971).

Proposition 1 shows the effect of the bubble policy in the case where a semi-bubble exists both with and without the announcement policy. We now look at the two "information leakage" cases, where the central bank's announcement causes additional information leakage, which bursts the asset semi-bubble. We first look at Case 2b.

Proposition 2: In Case 2b (where trade happens after the announcement if Susan is good), the announcement policy increases social welfare if and only if

$$\frac{z_B + \alpha(z_{1a} + z_{2a})}{z_{1n} + z_{2n}} > \frac{y_B + \alpha(y_{1a} + y_{2a})}{y_{1n} + y_{2n}}, \tag{15}$$

where $\alpha = (P_{NAP} - P_A')/P_{NAP}$. Here α is positive but less than one, because we assume $P_A' \leq P_{NAP} \leq P_{NA}$

Proof: Because Bob only buys from good Ellen in Case 2b, and his welfare is unaffected by the anti-bubble policy, his welfare gain in the announcement states (paying less) should be equal to his welfare loss in the no-announcement states (paying more), i.e.,

$$P_{NAP}\ z_B + (P_{NAP} - P_A')(z_{1a} + z_{2a}) = (P_{NA} - P_{NAP})(z_{1n} + z_{2n}). \tag{16}$$

This can also be verified by plugging in the formulas for $P_{NAP} = sp_I^B$, $P_{NA} = sp_{1NA}^B$, and $P_A = 10z_{2a}/(z_{1a} + z_{2a})$. For Susan, her welfare decreases in the announcement states (receiving less) and increases in the no-announcement states (receiving more). When her loss is smaller than her gain, then her welfare (social welfare) increases. This happens when

$$P_{NAP}\ y_B + (P_{NAP} - P_A')(y_{1a} + y_{2a}) < (P_{NA} - P_{NAP})(y_{1n} + y_{2n}). \tag{17}$$

Dividing Equation (16) by P_{NAP} gives

$$z_B + (\frac{P_{NAP} - P_A'}{P_{NAP}})(z_{1a} + z_{2a}) = (\frac{P_{NA} - P_{NAP}}{P_{NAP}})(z_{1n} + z_{2n}),$$

or

$$z_B + \alpha(z_{1a} + z_{2a}) = (\frac{P_{NA} - P_{NAP}}{P_{NAP}})(z_{1n} + z_{2n}), \tag{18}$$

where $\alpha = (P_{NAP} - P_A')/P_{NAP}$. Similarly, (17) is equivalent to:

$$y_B + \alpha(y_{1a} + y_{2a}) < (\frac{P_{NA} - P_{NAP}}{P_{NAP}})(y_{1n} + y_{2n}). \tag{19}$$

Rearranging equation (18) and inequality (19) slightly gives:

$$\frac{P_{NA} - P_{NAP}}{P_{NAP}} = \frac{z_B + \alpha(z_{1a} + z_{2a})}{z_{1n} + z_{2n}}, \tag{20}$$

$$\frac{P_{NA}-P_{NAP}}{P_{NAP}} > \frac{y_B+\alpha(y_{1a}+y_{2a})}{y_{1n}+y_{2n}}. \tag{21}$$

Combining equation (20) and inequality (21) gives Condition (15). **QED**

The intuition behind Condition (15) is similar to that in Condition (10). A difference is that both agents' marginal utilities are weighted by an endogenous factor α. The factor α may be understood as follows.

In Case 2b the central bank announcement reduces Bob's shadow price. If information leakage between Susan and Bob informs Bob that Susan is good, then this gives Bob more confidence and raises the price he is willing to pay, so his shadow price P_A' rises back up and becomes the market price, i.e., P_A' rises above Susan's shadow price and gets closer to P_{NAP}. We can view α as a measure of the information leakage effect, that is, when α approaches zero, the leakage effect is more prominent.

In the condition, when α approaches zero, i.e., the leakage effect is very significant, then

$$\frac{z_B+\alpha(z_{1a}+z_{2a})}{z_{1n}+z_{2n}} \approx \frac{z_B}{z_{1n}+z_{2n}} \text{ and } \frac{y_B+\alpha(y_{1a}+y_{2a})}{y_{1n}+y_{2n}} \approx \frac{y_B}{y_{1n}+y_{2n}}, \tag{22}$$

so Condition (15) becomes

$$\frac{z_B}{z_{1n}+z_{2n}} > \frac{y_B}{y_{1n}+y_{2n}}. \tag{23}$$

This means that, when Susan puts less weight than Bob does on the bubble state relative to the no-announcement states, social welfare increases.

We can therefore interpret the information leakage effect as follows. With a significant information leakage effect, holding everything else the same, α tends to be a small value, i.e., P_A' is close to P_{NAP}. This implies that the central bank's bubble-bursting announcement has less net impact on the asset price in announcement states e_{1a}^{G} and e_{2a}^{G}, so the negative effect of the announcement only reduces P_{NAP} slightly to P_A'. Thus, since $P_{NAP} \approx P_A'$, Susan's welfare changes very little in states e_{1a}^{G} and e_{2a}^{G}, so the change in social welfare is decided by the trade-off between Susan's welfare loss in state e^{B} and her welfare gain in states e_{1n}^{G} and e_{2n}^{G}, as in (23). If Susan cares relatively less about the bubble state than Bob, as in (23), then welfare is improved by bubble bursting, as one would expect.

We now develop the condition in the Case 2a where anti-bubble announcements eliminate trade entirely. Bob only buys when the central bank makes no announcement.

Proposition 3: In Case 2a (where trades only happen when the central bank makes no announcement), the announcement policy increases social welfare if and only if

$$\frac{z_B + z_{1a} - \beta \cdot z_{2a}}{z_{1n} + z_{2n}} > \frac{y_B + y_{1a} - \beta \cdot y_{2a}}{y_{1n} + y_{2n}}, \tag{24}$$

where $\beta = (10 - P_{NAP}) / P_{NAP}$. Here, β is positive since P_{NAP} is smaller than 10, the maximum possible dividend.

Proof: Because in Case 2a, Bob only buys when the central bank makes no announcement, and his welfare is unaffected by the anti-bubble policy, his welfare gain in the announcement states (paying less) should be equal to his welfare loss in the no-announcement states (paying more). Also he suffers utility loss for not getting dividend in state e_{2a}^{G}. This all gives

$$P_{NAP}\,(z_B + z_{1a} + z_{2a}) = (P_{NA} - P_{NAP})(z_{1n} + z_{2n}) + 10 z_{2a}. \tag{25}$$

For Susan, her welfare decreases in the announcement states (receiving less) and increases in the no-announcement states (receiving more), and she also gains utility in state e_{2a}^{G} from getting the dividend. When her loss is smaller than her gain, her welfare (social welfare) increases, so

$$P_{NAP}(y_B + y_{1a} + y_{2a}) < (P_{NA} - P_{NAP})(y_{1n} + y_{2n}) + 10\, y_{2a}\,. \tag{26}$$

Equation (25) is equivalent to:

$$(z_B + z_{1a}) + \frac{(P_{NAP} - 10)}{P_{NAP}} z_{2a} = \frac{(P_{NA} - P_{NAP})}{P_{NAP}} (z_{1n} + z_{2n}). \tag{27}$$

Inequality (26) is equivalent to:

$$(y_B + y_{1a}) + \frac{(P_{NAP} - 10)}{P_{NAP}} y_{2a} < \frac{(P_{NA} - P_{NAP})}{P_{NAP}} (y_{1n} + y_{2n}). \tag{28}$$

Rearranging equation (27) and inequality (28) gives

$$\frac{(P_{NA} - P_{NAP})}{P_{NAP}} = \frac{z_B + z_{1a} - \beta z_{2a}}{z_{1n} + z_{2n}}, \tag{29}$$

$$\frac{(P_{NA} - P_{NAP})}{P_{NAP}} > \frac{y_B + y_{1a} - \beta y_{2a}}{y_{1n} + y_{2n}}, \tag{30}$$

where $\beta = (10 - P_{NAP}) / P_{NAP}$. Combining equation (29) and inequality (30) yields Condition (24). **QED**

The intuition behind the condition for Proposition 3 is the same as that in Propositions 1 or 2. A difference is that both agents' marginal utilities are weighted by an endogenous and negative factor β.

The condition can be rewritten as:

$$\frac{P_{NAP}(z_B + z_{1a} + z_{2a}) - 10 z_{2a}}{(P_{NA} - P_{NAP})(z_{1n} + z_{2n})} > \frac{P_{NAP}(y_B + y_{1a} + y_{2a}) - 10 y_{2a}}{(P_{NA} - P_{NAP})(y_{1n} + y_{2n})}. \tag{31}$$

On the left hand side of the condition, $P_{NAP}(z_B + z_{1a} + z_{2a})$ is Bob's welfare gain from paying nothing to Susan, but because trade does not happen in state e_{2a}^{G}, Bob suffers a loss of $10 z_{2a}$ units of utility from the lost dividend. Also $(P_{NA} - P_{NAP})(z_{1n} + z_{2n})$ is Bob's welfare loss from paying more to Susan in the announcement states, We know Bob's welfare loss is compensated by his gain, so the left hand side of inequality (28) is one.

On the right hand side of the condition, unlike Bob, the numerator is Susan's net welfare loss and the denominator is her net gain, since she is the seller. The condition suggests that if Susan puts more weights on the no-announcement states relative to the announcement states than Bob does, then her expected welfare gain will be larger than her expected loss, and social welfare increases.

5 Conclusion

In this paper, we use a revised Allen et al. (1993) type of greater fool bubble framework to evaluate the effect of anti-bubble policy. Unlike most other bubble models, this framework includes rational agents and a finite horizon, which rules out infinite-horizon bubbles as in Tirole (1985). An endogenously generated asset bubble in our model provides us with insights about the development and bursting of an asset bubble. In this framework, it is not necessary to assume that the central bank is smarter than other agents. As long as the central bank's announcement contains some useful information, we show that it is possible, though by no means necessary, that an announcement policy benefits the economy.

Specifically, we show that anti-bubble policy affects an asset's ability to facilitate risk sharing. When sellers care relatively more about the states where the central bank makes negative announcement, an announcement policy reduces the asset's ability to share risks. Conversely, when sellers care relatively more about the no-announcement states, the announcement policy improves the asset's ability to share risks.

We also identify an information leakage effect initiated by an imperfect anti-bubble announcement from the central bank. That is, traders may be able to get additional information about the asset by observing price changes after the announcement. Because of the leakage effect, more information is revealed to traders than the central bank initially possesses. This leakage effect may explain why the effects of anti-bubble policy have often been extremely unpredictable during historical bubble episodes.

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