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Innovation and imitation incentives in dynamic duopoly

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Abstract

We study entry in a growing market by ex-ante symmetric duopolists when sunk costs differ for the innovating and imitating firm. Strategic competition takes the form either of a preemption race or of a war of attrition, the latter being likelier when demand uncertainty is high. Industry value is maximized when firms seek neither to race nor to delay investment. Free imitation is socially costly, and if the consumer surplus resulting from imitation is not too large the socially optimal imitation cost, as may be induced by patent protection, involves preemption. Finally, we discuss endogenous entry barriers and contractual alternatives that increase the likelihood of preemption regimes, with differing implications for imitator entry. When the cost of imitation is low for instance, innovators are shown to rely more heavily on trade secrecy and patents. Welfare-enhancing takeovers and licensing are also shown to occur.

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Keywords: Dynamic oligopoly; Knowledge spillover; Real options

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1 Introduction

Profitable investment by a pioneering firm often breeds imitation by subsequent entrants. When developing an invention into a commercial product requires significant enough resources so that only a few firms may jockey to secure positions in an industry either as a first-mover or as a second entrant, product development takes the form of a noncooperative timing game. The central contribution of this paper is to develop a theoretical framework for strategic investment that integrates key features from existing research on innovation and the dynamics of firm entry in order to produce a useful and general formalization of first and second-mover advantage, and the consequences of the resulting investment incentives for industry dynamics.

The ease of imitation by a second entrant, as reflected by its irreversible cost of investment relative to that of the first mover, plays a central role in this framework. This central role in the theory is consistent with practice, as the ease of imitation depends on natural entry barriers but is also determined by the firms and regulators, through their choices of technology, licensing, and intellectual property protection levels.¹ Depending on whether imitation cost is high or low, the industry is characterized by a potential first- or second-mover advantage, and the strategic interaction between firms takes the form of a preemption race or of a war of attrition.

1.1 Results

The central proposition in this paper is the characterization of equilibrium for a class of games that ranges from attrition to preemption, from which we are able to derive a number of other results, in particular with respect to welfare.

First, as firms are taken to be ex-ante identical and their roles as innovator or imitator are endogenous, positional rents are dissipated. As one might expect, a higher imitation cost turns out to be associated with accelerated product introduction and a larger expected lead time for the innovating firm before the second entry occurs. We are therefore able to identify an optimal level of imitation cost for firms: industry value is maximized, all else equal, in those industries in which the cost of imitation is such that there is neither a race to preempt nor a war of attrition, *i.e.* in which firms do not compete for positional rents by rushing to enter or waiting unduly to innovate. The intuition behind this result runs as follows. When the imitation cost increases, a follower firm delays entry and has a lower expected value, so that a leader firm can expect to

¹Mansfield et al. [27] and Mansfield [26] are pioneering empirical studies that have fixed the perception of imitation upon which much subsequent theoretical work is based. See also Cohen et al. [8].

benefit from a longer monopoly phase, and therefore has a higher expected value ex ante. When firms engage in a preemption race, rent equalization pegs expected value to the follower value, and thus decreases with imitation cost. Conversely, in a war of attrition, the expected value of firms is that of the leader firm, which increases with imitation cost.

In addition to an optimal level of imitation cost for the industry, we identify the welfare trade-offs associated with raising imitation cost, as may arise with broader patent protection, for instance. We identify a positive lower bound for the optimal level of imitation cost, which must be sufficiently large so that firm entry into the industry is sequential, rather than simultaneous, with positive probability. Moreover, as raising the imitation cost above this lower bound both increases expected industry value and accelerates initial product development, broader patent protection results in a greater social surplus so long as the increased lag in imitation does not have too great an impact on consumer surplus. Socially optimal outcomes are thus more likely to involve a preemption regime than one of attrition under these circumstances, and it can be optimal to rule out imitator entry by means of a prohibitively high level of imitation cost.

Finally, we enlarge the set of alternatives that are available to firms to allow for the possibility that the innovator raises entry barriers by making its product more complex and hence difficult to reengineer, as well as contracting options like “pay for delay” and licensing agreements. In all of these cases, the expected value of innovating firms is shifted upward, so strategic interaction in the industry becomes more preemptive, and there are contrasting implications regarding firm values and the timing of the imitator’s entry.

Thus, in a model of the effect of imitation cost on entry behavior, we provide conditions under which a social optimum involves preemption, and argue that several alternative mechanisms exist that can substitute for regulatory measures in providing for dynamic efficiency.

1.2 An example: imitation cost in the biopharmaceutical industry

The questions we address were originally motivated by real-world situations in which the same firms can face contrasting technological conditions with respect to ease of imitation over the different business segments in which they operate. In the biopharmaceutical industry, typically, whereas medications are easily imitated thus justifying the industry’s systematic recourse to patent protection, in the vaccine segment technological conditions render imitation much more costly.²

²Another characteristic of the pharmaceutical industry is the uncertainty that is introduced by late-stage clinical trials regarding the outcome of an R&D project, most often after significant costs have already been sunk, but we

On the one hand, pharmaceutical firms typically rely on intellectual property rights in order to increase the costs of imitators for new drugs “which otherwise could be copied more easily than products whose production processes can be kept secret, or for which the time and relative expense needed to copy the invention are much higher” (Scherer and Watal [33], p. 4). If such patent protection is not available, a generic product can be introduced at a much lower fixed cost than incurred by the branded product supplier. In India, after the passage of the Patents Act 1970, and before the TRIPs (Trade Related aspects of Intellectual Property rights) agreements were enforced, pharmaceutical products became unpatentable, “allowing innovations patented elsewhere to be freely copied” (Lanjouw [24], p. 3). By reducing imitation costs, the absence of legal protection fostered the domestic production of generic formulations.

This ease of imitation is not found in the vaccine segment, as vaccines are made from living micro-organisms, and unlike drugs “are not easily reverse-engineered, as the greatest challenges often lie in details of production processes that cannot be inferred from the final product,” implying that “there is technically no such thing as a generic vaccine” (Wilson [37], p. 13). The regulatory implication is that a me-too vaccine supplier must pay for clinical trials to demonstrate the safety and efficacy of its product. There is no short-cut toward the bio-equivalence of a copied candidate vaccine, whose design and delivery require investments in technological capabilities and manufacturing facilities that comply with demanding regulatory standards. In the case of recent complex vaccines (*e.g.*, a tetravalent dengue virus vaccine), a follower must catch up with leading-edge R&D and manufacturing approaches (the technological challenges for the design a dengue virus vaccine are reviewed in Guey Chuen et al. [17]). The fixed cost that must be incurred by a new entrant for the delivery of a follow-on vaccine can thus be prohibitively high.³

1.3 Related literature

Our model of innovation and imitation builds upon an already rich literature dating back to Reinganum [31], who provides a foundation for dynamic game-theoretic models of duopoly investment that she construes as technology adoption. In a deterministic environment in which one of the firms can commit as a first investor, she identifies a diffusion equilibrium in which investments occur sequentially and result in a first-mover advantage. Fudenberg and Tirole [13] consider investment decisions when leader and follower roles are endogenous. With symmetric firms, there is a preemption race that accelerates the first investment, dissipating rents to the first investor so

do not seek to represent this specific feature in our model.

³We further discuss this example in light of the theoretical model in Section 4.1.

that firm values are equalized in equilibrium. In an otherwise similar framework but with asymmetric firms, Katz and Shapiro [23] allow either licensing or imitation to occur post-investment. They find that a second-mover advantage can arise, so that investment decisions take the form either of a preemption race or of a waiting game.

Some recent research on innovation dynamics has focused on informational spillovers, which are one of the important determinants of second-mover advantage, into models of duopoly investment. A key reference is Hoppe [20] which introduces uncertainty regarding the success of new technology adoption. The follower firm only invests if the new technology is profitable, so that when the likelihood of success is low, firms engage in a war of attrition. In a similar vein, Huisman and Kort [22] allow the follower to benefit from the subsequent arrival of a better technology, and Femminis and Martini [12] model a disclosure lag of random duration before the follower benefits from a spillover. The effect of informational spillovers on investment incentives has also been studied in models of learning by Décamps and Mariotti [9] and Thijssen et al. [35]. In these models, the first-mover's investment sends a profitability signal to the follower in addition to some ongoing background information that both firms receive. Depending on the relative importance of the preemption motive and the informational externality, firms engage in either preemption or attrition.

Through these different contributions runs a common thread, namely that to the extent an innovator's investment has positive spillovers for its rival, competition between otherwise symmetric firms takes the form either of a preemption race or of a war of attrition. We depart from prior work by deriving a symmetric Markov perfect equilibrium in a model of strategic investment that incorporates both potential first- and second-mover advantages, sparsely parametrized by the relative sunk costs of innovating and imitating firms. Thus, our paper integrates the characterizations of attrition by Hendricks et al. [18] and preemption in a stochastic setting by Thijssen et al. [36]. In so doing, we extend the so-called standard real option game framework⁴ by relaxing the assumption that leader and follower investment costs are identical and exogenous. The most closely related work that we have identified in this area are those of Pawlina and Kort [30] and Mason and Weeds [28], which introduce firm asymmetry into duopoly investment games in ways that are complementary to the approach we adopt here.

Several other strands of research provide broader context for our work. In particular, the ease

⁴Azevedo and Paxson [2] is a recent survey of this field, which draws from game theory and continuous time finance in order to incorporate strategic and payoff uncertainty into models of investment. Typical applications are to capacity investment, as in Boyer et al. [6], as well as investment in R&D, as in the present paper. For a thorough and pedagogical presentation, see Chevalier-Roignant and Trigeorgis [7].

of imitation is pertinent in determining optimum patents, as described by Gallini [15] from whom we follow the formal specification of the cost of “inventing around”. The dynamics of patent races can be studied with similar tools to the strategic investment research cited above, as in Weeds [38], although such applications more closely describe the invention stage of innovation whereas our focus is on the subsequent development or product introduction phase. Another stream of research dating back to Benoit [3] studies imitation incentives once an innovator has achieved incumbency. More recent papers such as Mukherjee and Pennings [29] and Henry and Ruiz-Aliseda [19] have identified the importance of the patenting, licensing, and reverse engineering decisions that then arise. Our analysis is also related to models of cumulative innovation, as exemplified by Green and Scotchmer [16] and Denicolò [10]. Lastly, the concepts of first and second mover advantage that we approach here from an economic standpoint are of broad interest to managers and strategists, and the survey of Lieberman and Montgomery [25] remains a useful reference in this area.

2 A model of new product development

This section describes a model of strategic investment in line with the characteristic features of innovation and imitation that we have identified. The assumptions regarding industry structure and firm conduct are presented in Sections 2.1 and 2.2, and equilibrium in Section 2.3.

2.1 Assumptions

Two otherwise identical firms may enter a new market by introducing their own version of a product. Product development involves uncertainty regarding future levels of final demand and irreversibility, as in the investment framework described by Dixit and Pindyck [11]. Organizational constraints preclude a firm from selling two variants of the product.

The introduction of the product generates a perpetual profit flow whose instantaneous value depends on the number of active firms: π_M when a single firm is active, and π_D when both are. These flow profit levels may reflect either standard duopoly competition ($0 < 2\pi_D \leq \pi_M$) or duopoly competition with significant complementary product differentiation ($\pi_M \leq 2\pi_D \leq 2\pi_M$). Flow profit is scaled by a multiplicative shock (Y_t) representing market size that follows a geometric Brownian motion ($dY_t = \alpha Y_t dt + \sigma Y_t dZ_t$ where $(Z_t)_{t \geq 0}$ is a standard Wiener process). Profit flows are assumed to begin instantaneously and with certainty once investment has occurred.⁵ Firms

⁵Thus, we do not purport to model lead times, and our approach contrasts with some of the related work on patent races or information spillovers cited above, in which the success of innovation is an additional stochastic

have a common and constant discount rate (r).

Introducing the new product involves an irrecoverable fixed cost (I) for the the first firm that invests, *i.e.* for the *innovator*. A firm that observes its rival’s innovation can invest after, even immediately, as a second entrant, *i.e.* as an *imitator*. We assume that in addition to the various standard setup costs associated with bringing a product to market such as dedicated plant and equipment, marketing expenditures, and so forth, the follower incurs a cost of imitation of variable magnitude depending on technological, institutional, or other industry conditions. Introducing the alternative version thus involves an irrecoverable fixed cost (K), and we allow for the extreme case of costless imitation. The imitator’s fixed cost may be either higher or lower than the innovator’s, depending both on the difficulty of reengineering and on the degree of protection afforded to the intellectual property of innovators. If the second firm can develop the same product independently, for ex-ante identical firms, imitation is no more expensive than innovation ($K \leq I$) in the absence of intellectual property protection. When the complexity of the product is limited or legal protection is weak (when the breadth of patents is narrow) the imitation costs can still be low enough that the entry cost is lower for the follower than for the leader. On the other hand, greater technological complexity and stronger legal protection imply a higher cost for imitators who must invest in reverse engineering or invent around any patents held by the innovator, although this is mitigated by disclosure requirements that reduce unnecessary duplication of effort. Of course, in addition to natural barriers and technological impediments to imitation, legal rules may translate into more or less successful protection of innovators, depending on whether rights are transferred or not, whether there is a pool of rights involved or not, the breadth of patents and so forth. Conceivably, the product can be so complex or legal protection can be so strong as to make the second mover incur higher entry costs than the leader ($K > I$).⁶ Moreover, we later show that such a level of complexity legal protection can be socially optimal (in fact, that it can even be efficient to rule out the second firm’s entry altogether by setting an

element for firms. A lower cost of imitation in our model is consistent with an informational spillover as well, if innovation requires success on a large number of independent trials or a search process that an imitator can bypass.

⁶Our focus is the relation between innovation and imitation, but other circumstances can also lead to asymmetric fixed costs for ex-ante identical firms. If developing the new product involves *scarce assets*, such as prime location in real estate or natural resource extraction, then the imitator may face a higher cost ($K > I$). We do not pursue this interpretation actively. One area where doing so would make a difference is in the case of “ties”, where we will assume that if both firms seek to and effectively invest simultaneously, the investment cost is I , rather than $\max\{I, K\}$ as would be the case if firms had to compete for scarce assets in the input market. Also, *imperfect competition in input markets* may result in asymmetric investment costs. In Billette de Villemeur et al. [5], investment cost is determined endogenously by a strategic input supplier, resulting in a discounted input price for the first firm that invests ($I < K$).

arbitrarily high imitation cost $K^* = \infty$).

Finally, we suppose that information is complete and that $\alpha \geq 0$, $\sigma \geq 0$ (with one of these inequalities strict), $\alpha < r$, and $0 < Y_0 < (r - \alpha) I / \pi_M$,⁷ so that absent competition an innovating firm would initially prefer to delay investment and hold on to their growth option, either because of deterministic growth in demand, or volatility, or some combination of the two.

2.2 Threshold strategies and leader and follower payoffs

Firms decide independently when to introduce a new product. In order to focus on the economic aspects of the decision problems of the firms, we describe a reduced form market entry game that captures the relevant features of a more general dynamic entry game. This is done by taking firm strategies to consist of investment thresholds, which determine a stochastic time of investment, and by imposing a specific tie-breaking rule in the event that both firms face a known coordination problem in which they seek to invest simultaneously when it would only be optimal for one to do so (see Section A.1 for a description of the underlying dynamic game). This representation of the game gains in simplicity, and the equilibrium that is obtained is consistent with the equilibrium of the general entry game.

At any time $t \geq 0$ at which it has not yet invested, the strategy of a firm i , $i \in \{1, 2\}$ consists of an entry threshold $Y_i \in [Y_t, \infty)$ that, once reached for the first time and from below, triggers investment. Threshold choices result in (stochastic) investment times and endogenously determine the role of each firm as innovator or imitator. In the case of identical thresholds ($Y_i = Y_j$), a tie-breaking rule that subsumes the equilibrium of the underlying continuous time game randomly determines the roles of each firm. Under these conditions, industry dynamics in the product market may be viewed as resulting from a two stage interaction, where in the first stage (which determines the onset of the monopoly phase) the choices of initial entry thresholds (Y_i, Y_j) determine the roles of the firms, and in the second stage (the onset of the duopoly phase), the remaining firm enters at a threshold of its choice that we denote by Y_{-i}^e (with $Y_{-i}^e \geq Y_i$) for the moment.

The expected payoffs for innovators and imitators have the following specific forms:

$$L(Y_i, Y_{-i}^e) = \left(\frac{\pi_M}{r - \alpha} Y_i - I \right) \left(\frac{Y_t}{Y_i} \right)^\beta + \frac{\pi_D - \pi_M}{r - \alpha} Y_{-i}^e \left(\frac{Y_t}{Y_{-i}^e} \right)^\beta \quad (\text{innovator payoff}) \quad (1)$$

⁷This bound ensures that firms prefer to delay investment under preemption rather than invest immediately, even when imitation costs are large.

and

$$F(Y_i; K) = \left(\frac{\pi_D}{r - \alpha} Y_i - K \right) \left(\frac{Y_t}{Y_i} \right)^\beta \quad (\text{imitator payoff}) \quad (2)$$

where in both (1) and (2), β is shorthand for the function of parameters

$$\beta(\alpha, \sigma, r) := \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}}. \quad (3)$$

The function β given in (3) is a standard expression in real option models, satisfying $\beta > 1$ and $\lim_{\sigma \rightarrow 0} \beta = r/\alpha$. A lower value of β is associated with a greater incentive to wait (it is straightforward to check that $\partial\beta/\partial\alpha < 0$, $\partial\beta/\partial\sigma < 0$, and $\partial\beta/\partial r > 0$), so β may be interpreted as a measure of “impatience”. The $(Y_t/\bullet)^\beta$ terms in which β occurs reflect the expected discounting of the monetary units that are received when the stochastic process reaches the relevant thresholds $\{Y_i, Y_j\}$ for the first time.⁸

The leader (innovator) payoff is comprised of two terms, which correspond to the monopoly profit flow of the innovating firm and the negative impact on this profit flow of the second firm’s entry. The follower (imitator) payoff has the standard form of a growth option. Both payoff functions are quasiconcave over their domains (note that F is log-concave), and attain non-negative global maxima at $Y_L := (\beta(r - \alpha)I) / ((\beta - 1)\pi_M)$ and $Y_F := \max\{Y_t, (\beta(r - \alpha)K) / ((\beta - 1)\pi_D)\}$ respectively. We refer to these thresholds as the optimal standalone leader and follower thresholds, and they correspond to the open loop equilibrium identified by Reinganum [31].⁹ A key property of the payoff functions which is used throughout our analysis is that the leader payoff is nondecreasing in the imitation cost provided the follower invests at the optimal follower threshold ($\partial L(Y_i, Y_F) / \partial K \geq 0$), whereas the follower payoff is decreasing in the imitation cost ($\partial F(Y_i; K) / \partial K < 0$).

Lastly, both firms may introduce their respective products independently at the same moment, in which case we assume that they incur the same fixed cost. The corresponding payoff is denoted by $M(Y_i) := L(Y_i, Y_i) (= F(Y_i; I))$, which is maximized at $Y_S := (\beta(r - \alpha)I) / ((\beta - 1)\pi_D)$.

⁸In the deterministic case ($\sigma = 0$) and letting $t_i := \inf\{\tau, Y_\tau \geq Y_i\}$, $(Y_t/Y_i)^\beta = \exp(-r(t_i - t))$, which is the standard continuous time discounting term under certainty.

⁹For sufficiently low values of K ($K \in [0, K_l)$, $K_l := (\pi_D/\pi_M)I$), $Y_F < Y_L$. In this range, if roles were exogenously assigned, a follower would be willing to pay its rival to induce it to invest earlier. We mention this possibility for completeness, but the threshold K_l does not play a significant role in the rest of the analysis.

2.3 Equilibrium

We choose to focus on symmetric equilibrium. One reason for this is that as firms are taken to be symmetric ex-ante, it seems natural to suppose that they hold symmetric beliefs about each other's play at the beginning of the investment game. In so doing, the equilibrium described in Proposition 1 below is consistent with the earlier approaches of Fudenberg and Tirole [13], Hendricks et al. [18], and more recently, Thijssen et al. [36], but authors have occasionally proceeded differently, particularly with respect to attrition equilibrium.¹⁰ Another reason to focus on symmetric equilibrium is that it results in rent dissipation, a feature that is emphasized in the early timing game literature as surveyed by Fudenberg and Tirole [14], and which rationalizes a smooth dependence of equilibrium on imitation cost that is of compelling simplicity.

The reduced form game described in Section 2.2 occurs in two stages. First, firms compete in entry thresholds that endogenously determine their roles as innovators or imitators, and second, at once after initial entry has occurred, the remaining firm selects its own entry threshold. We therefore have

- Stage 1: both firms select initial entry thresholds (Y_i, Y_j) (or distribution thereof) that determine innovator and imitator roles;
- Stage 2: if a single firm (i) innovates, the remaining firm (j) then selects its imitator entry threshold.

To determine the equilibrium choices, notice that once the initial investment by at least one of the firms has occurred, any firm that remains out of the market holds a standard growth option. It prefers to wait if the first investment occurs early enough (before Y_F is reached), and otherwise it prefers to invest immediately. The optimal follower policy for a given first investment threshold Y_i is therefore $Y_F^* = \max\{Y_i, Y_F\}$, resulting in the follower value $F(Y_F^*; K)$. By backward induction, the first stage leader payoff is therefore $L(Y_i, Y_F^*)$.

In the first stage, a given firm i 's payoff is

$$V(Y_i, Y_j) = \begin{cases} L(Y_i, Y_F^*) & \text{if } Y_i < Y_j \\ p(Y_i; K) L(Y_i, Y_F^*) + p(Y_i; K) F(Y_F^*; K) + (1 - 2p(Y_i; K)) M(Y_i) & \text{if } Y_i = Y_j \\ F(Y_F^*; K) & \text{if } Y_i > Y_j \end{cases} \quad (4)$$

¹⁰A contrasting approach is that of Hoppe [20], who focuses attention on asymmetric equilibrium in pure strategies in a war of attrition. This approach applies if, for instance, the same entry game is played in several independent markets and pre-play communication allows the firms to coordinate, but we do not allow for this possibility here.

where the second line of (4) reflects the tie-breaking rule of the reduced form game, which runs as follows. If at a given moment both firms seek to invest, whereas it would only be optimal for one to do so (*i.e.* if $F(Y_F^*; K) > M(Y_i)$), then they face a known coordination problem (Fudenberg and Tirole [13]). If coordination is successful, either firm is equally likely to invest as a leader or as a follower, with probability

$$p(Y_i; K) = \begin{cases} \frac{F(Y_F^*; K) - M(Y_i)}{L(Y_i, Y_F^*) + F(Y_F^*; K) - 2M(Y_i)} & \text{if } L(Y_i, Y_F^*) \geq F(Y_F^*; K) \\ 0 & \text{if } L(Y_i, Y_F^*) < F(Y_F^*; K) \end{cases}. \quad (5)$$

The expression $(1 - 2p(Y_i; K))$ thus corresponds to the probability that a “mistaken” simultaneous investment outcome occurs.¹¹

Note that there are two kinds of simultaneous investment outcomes that can arise in the model. If one firm invests first and thereby takes the role of innovator, but does so at a sufficiently high threshold ($Y_i \geq Y_F$), the remaining firm then chooses to invest immediately after, although it takes the follower role so its payoff is $F(Y_i; K)$. On the other hand, if both firms attempt to invest simultaneously without coordinating their investments, they both receive the same payoff $M(Y_i)$.

In the proposition that follows, we establish that there exists a critical imitation cost, which we denote by \widehat{K} ,¹² that determines the nature of the duopoly investment game. This imitation cost is defined implicitly by the condition that firms in equilibrium are indifferent between the innovator and imitator payoffs when these are evaluated at the optimal standalone thresholds, that is $L(Y_L, Y_F^*) = F(Y_F^*; \widehat{K})$ (note that Y_F^* is a function of K). Provided that $K \geq \widehat{K}$ so that this is well-defined in $[Y_t, \infty)$, it is also useful to define a critical level of the multiplicative shock Y_t , which we denote by Y_P , and which is usually referred to as the preemption point. This is the first threshold at which firms are indifferent between innovating and imitating, *i.e.* $L(Y_P, Y_F^*) = F(Y_F^*; K)$. Then,

Proposition 1 *The duopoly investment game has a unique symmetric equilibrium and there exists a threshold imitation cost $\widehat{K} \leq I$ such that:*

(i) *if the imitation cost is low ($K < \widehat{K}$), firms play a game of attrition. The innovator investment threshold ($\min \{ \widetilde{Y}_i, \widetilde{Y}_j \}$) is bounded below by the standalone leader threshold (Y_L) and distributed*

¹¹A noteworthy contrast between our model and the standard real option game is that the values of the leader and follower payoffs generally differ at Y_F because of the asymmetry in investment costs. For some values of K , the behavior of the mistake probability $1 - 2p(Y_i; K)$ is non-monotonic (See Section A.1).

¹²See Section A.2 for a characterization of $\widehat{K} := \left((1 + \beta((\pi_M/\pi_D) - 1)) / (\pi_M/\pi_D) \right)^{1/(\beta-1)} I$.

continuously over a possibly disconnected support, and imitator investment occurs either at the optimal standalone follower threshold (Y_F) or immediately after the innovator's entry.

(ii) if the imitation cost is intermediate ($K = \widehat{K}$), firms invest at the optimal standalone leader and follower thresholds (Y_L, Y_F), with either firm equally likely to be the innovator or the imitator.

(iii) if the imitation cost is high ($K > \widehat{K}$), firms play a game of preemption. The innovator and imitator investment thresholds are (Y_P, Y_F), with either firm equally likely to be the innovator or the imitator.

In order to illustrate the cases described in Proposition 1, Figures 1 – 5 depict leader and follower payoffs in the first stage of the game, for different typical values of the imitation cost. Throughout these figures, as the imitation cost increases, the follower payoff shifts down and towards the right, and the optimal standalone follower threshold Y_F increases. Because of the longer monopoly phase, the first stage leader payoff $L(Y_i, Y_F^*)$ accordingly shifts upward over the sequential investment range (Y_0, Y_F). Note that the optimal standalone leader threshold Y_L is independent of K , and the leader payoff function has a kink at Y_F when imitator entry becomes immediate. In Figures 1 and 2, there is a *second-mover advantage* (in the sense that $L(Y_L, Y_F) < F(Y_F; K)$) and the game is one of attrition. Figure 3 represents the intermediate case in which the imitation cost attains its critical value, $K = \widehat{K}$, and there is neither a first-mover advantage nor a second-mover advantage. In Figures 4 and 5, there is a *first-mover advantage* (in the sense that $L(Y_L, Y_F) > F(Y_F; K)$), and the game is one of preemption, with the first investment occurring at Y_P .

With respect to the expected values that firms attain in equilibrium, in the symmetric equilibrium described in Proposition 1, rent dissipation occurs whenever the firms play a game of attrition ($K < \widehat{K}$) or of preemption ($K > \widehat{K}$). To state the following corollary, some further notation is necessary. Since $L(Y_i, Y_F^*)$ can have two local maxima, in some attrition cases in which the imitation cost is sufficiently low such as that illustrated in Figure 1, the global maximum of this function may be attained at $Y_S := \arg \max M(Y_i)$, which then corresponds to the lower bound of the support of innovator entry thresholds. Thus, define $Y_L^* := \arg \max L(Y_i, Y_F^*)$, which may be either Y_L or Y_S , to refer to the lower bound of all the threshold distributions in attrition equilibrium.

Corollary 1 *In a symmetric equilibrium, the expected payoffs of firms are identical and equal to $\min\{L(Y_L^*, Y_F^*), F(Y_F; K)\}$, that is to the lowest of the diffusion equilibrium payoffs.*

The dependence of the threshold imitation cost \hat{K} on model parameters is straightforward to characterize. The next corollary gives sensitivity results with respect to the intensity of competition in the product market (π_M/π_D) and discounting (β).

Corollary 2 *The more intense product market competition is (high π_M/π_D) and the more firms discount the future (high β), the more likely it is that preemption occurs, and conversely for attrition.*

To interpret this corollary, recall that the process Y_t is stochastic, and there is an option value for firms to wait before investing that is positively related to volatility. Provided that there is an inherent advantage to imitation ($K < I$), for some levels of the parameters this option value can outweigh any preemption motive to secure monopoly rents. Thus, as $\partial\beta/\partial\sigma < 0$, by Corollary 2 *an attrition regime is more likely in industries with greater demand volatility*. This comparative static effect is of particular importance because it stands in opposition to several of the mechanisms that are discussed in the rest of the paper. As the next sections show, institutional conditions such as intellectual property protection and firm choices regarding both technology and licensing generally serve to make market entry regimes more preemptive and attrition relatively rare, absent significant demand uncertainty.

3 Normative economics of imitation cost

The main result of the previous section highlights the key role that is played by the sunk cost of imitation in determining the nature of strategic competition and the properties of market entry in equilibrium. This imitation cost is likely to be determined by several different factors including technological development and the level of intellectual property protection. It thus varies from industry to industry, and can be influenced ex-ante, most commonly upward, by regulators. Such considerations raise the question of determining what may be desirable levels of imitation cost. At first glance, a higher imitation cost is socially wasteful in and of itself but also serves to hasten innovator entry, however different effects arise in the preemption and attrition regimes that need to be examined carefully.

In fact, it turns out that social welfare is not a quasiconcave function of imitation cost and therefore generally has local maxima in the attrition and preemption regimes.¹³ Nevertheless, we

¹³Thus, models that focus attention exclusively on either attrition (second-mover advantage) or preemption (first-mover advantage) *run the risk of identifying only local maxima of welfare*.

are able to provide a partial characterization of the social optimum, as well as the underlying welfare trade-offs associated with raising imitation cost. As a preliminary step to conducting this more thorough welfare analysis we first consider industry performance only, which allows us to derive a useful intermediate result regarding industry value.

3.1 Industry performance

Understanding the determinants of first and second-mover advantages is a question of long-running interest to scholars of industry dynamics, both in economics and in strategic management, even if the research agendas of the two disciplines differ (strategic management focuses on asymmetric and heterogeneous firms). In the terminology of Lieberman and Montgomery [25], the framework we have developed sheds light on the role of *technological leadership* and *free-riding* incentives (and to a lesser extent, on the *preemption of scarce assets*, see footnote 5 above) insofar as these are accurately represented by variations in imitation cost. The effect of changes in imitation cost on equilibrium choices and profit can then be described as follows.

A first and seemingly obvious consideration that emerges from our framework is that lower imitation cost is a necessary, but not a sufficient condition for second mover advantage. Since $\hat{K} \leq I$, if firms have identical fixed costs, there is an inherent first-mover advantage that results from the monopoly phase of the game ($L(Y_L, Y_F^*) \geq F(Y_F^*; I)$). The degree of first-mover advantage in this case is determined by the relative importance of monopoly profit in the product market (π_M/π_D). A second-mover advantage arises because of conditions on inputs, and requires that the relative advantage of imitation (I/K) be sufficient to compensate for foregone monopoly profit. Thus the empirical presence of lower costs for imitators, as has been observed by different authors (Mansfield et al. [27], Samuelson and Scotchmer [32]), does not by itself ensure that firms will find it desirable to pursue so-called imitation strategies in a dynamic setting.

As imitation always occurs at some point (provided duopoly profits are positive) a key question regarding firm investment decisions in our model is not whether imitation will occur, but rather when, and what consequences imitator entry may have for the incentive to innovate. To begin, note that the higher is the imitation cost, the higher is the standalone threshold for the follower firm (Y_F), although actual follower entry may occur either at this threshold or later if the investment game is one of attrition. The effect of higher imitation cost on the first firm's entry threshold is indirect. In the attrition regime, it is the distribution of innovator entry thresholds (of $\min\{\tilde{Y}_i, \tilde{Y}_j\}$) and therefore entry times that is shifted forward by higher imitation costs, with a higher value of K hastening innovator entry in a stochastic sense (the hazard rate of en-

try increases). In the preemption regime, rent equalization results directly in a lower preemption threshold (Y_P) when imitation cost increases. The link between imitation cost and earlier product introduction is thus similar to that identified by Katz and Shapiro [23]. Finally, the distribution of follower investment thresholds (Y_F^*) does not exhibit a simple relationship to K . As imitation cost increases, imitator entry occurs later if the innovator enters early (if $\min\{\tilde{Y}_i, \tilde{Y}_j\} < Y_F$) and earlier if the innovator enters late (if $\min\{\tilde{Y}_i, \tilde{Y}_j\} > Y_F$). However, the gap (and therefore the expected time lag) between leader and follower entry increases stochastically with imitation cost. Thus, higher imitation costs, as may result from higher technical or regulatory entry barriers, may be said to accelerate innovative investment and delay the onset of imitative investment once innovation has occurred.

Lastly, the equilibrium described in Proposition 1 leads to a simple result regarding industry performance. Because in the different regimes of attrition and preemption, competition between firms to secure either second or first mover advantages results in the dissipation of any potential rents, it is only when the level of the imitation cost is such that neither of these regimes occurs (case (ii), $K = \hat{K}$) that investment thresholds are set optimally from the standpoint of industry profit. Thus, all else equal, it is in those industries in which imitation costs approach this level so that firms do not have a strong incentive to seek positional advantages of either sort that industry value is most likely to be maximized. Note also that there is a threshold $\tilde{K} < \hat{K}$ (see Appendix 1) such that if $K \leq \tilde{K}$, the expected firm value $M(Y_S)$ is independent of imitation cost. There is also therefore a range of imitation cost levels, $[\tilde{K}, \hat{K}]$, over which greater resource costs are not detrimental but instead strictly beneficial to the industry. By shielding an innovator from excessively rapid imitation, the incentive of innovators to unduly delay product introduction is thereby reduced, and more timely product introduction benefits imitators as well.

Proposition 2 *Viewed as a function of imitation cost, expected industry value is initially constant, single-peaked, and attains its maximum when neither attrition nor preemption occur ($K = \hat{K}$).*

3.2 Optimal intellectual property protection

We take it that regulators can broadly influence the relative cost of imitation, at least upward, through their choice of intellectual property protection levels. Given this unique instrument, the imitation cost K can be viewed as a decision variable of the regulator in which case we find it natural to consider a second-best welfare benchmark in which firms are free to select their entry thresholds and product market output or prices.

Before deriving the formal result, it is possible to discuss welfare intuitively in light of the intermediate results described above. Roughly speaking, since individual entry decisions do not account for the positive effect of entry on consumer surplus, one would expect that firms tend to enter too late from a social standpoint. Since industry value is maximized when the imitation cost is at the critical value \widehat{K} , a constrained social optimum would therefore involve setting a higher imitation cost so as to induce some degree of preemption. But although this reasoning is suggestive of the answer, a more detailed analysis is necessary in order to accurately depict the effect of imitation cost on welfare.

To begin with, expected welfare can be broken down into three parts: expected industry value, consumer surplus from innovator entry, and consumer surplus from imitator entry. The first of these is maximized at the critical imitation cost \widehat{K} , whereas the other two depend on K through the innovator and imitator entry thresholds. A higher imitation cost unambiguously accelerates innovator entry which in turn increases consumer surplus, so the second of these components is increasing in K . The last component is more difficult to characterize, in particular in an attrition regime, since an increase in K may either delay (through its effect on the optimal threshold Y_F at which imitator entry occurs with positive probability) or hasten imitator entry (if innovator entry occurs after Y_F so imitator entry follows immediately). So long then as the consumer surplus from imitator entry is not too large, the effect of imitation cost on the first two components of welfare dominates, and the social optimum involves some degree of preemption.

To formalize this reasoning, consumer surplus is taken to be scaled by the market size parameter Y_t , as is the case for firm profits. Let CS_M and CS_D then denote the unit flow of consumer surplus under monopoly and under duopoly respectively. The social discount rate is assumed be identical to that of firms for simplicity. Under these conditions, for given innovator and imitator thresholds social welfare at time t can be written as

$$\begin{aligned}
 W(K) = & \\
 & \underbrace{\left(\frac{CS_M + \pi_M}{r - \alpha} \min\{Y_i, Y_j\} - I \right) \left(\frac{Y_t}{\min\{Y_i, Y_j\}} \right)^\beta}_{\text{welfare from innovation}} + \underbrace{\left(\frac{(CS_D + 2\pi_D) - (CS_M + \pi_M)Y_F^* - K}{r - \alpha} \right) \left(\frac{Y_t}{Y_F^*} \right)^\beta}_{\text{welfare from imitation}}
 \end{aligned} \tag{6}$$

so as to reflect the successive monopoly and duopoly phases of the industry. In (6), recall that the equilibrium thresholds $\min\{Y_i, Y_j\}$ and Y_F^* are themselves functions of K , and that the time at which these thresholds are reached is stochastic. In addition, in an attrition regime the equilibrium investment thresholds $\{Y_i, Y_j\}$ chosen by firms are themselves stochastic as well. In order to

identify the trade-offs involved in setting an optimal level of the imitation cost, it is useful to re-express the social welfare function (6) and take expectations over the equilibrium distribution of entry thresholds so as to get

$$\mathbb{E}_{\tilde{Y}_i, \tilde{Y}_j} W(K) = \mathbb{E}_{\tilde{Y}_i, \tilde{Y}_j} \left[\underbrace{2V(\tilde{Y}_i, \tilde{Y}_j)}_{\text{industry value}} + \underbrace{\frac{\text{CS}_M}{r - \alpha} \left(\min \{ \tilde{Y}_i, \tilde{Y}_j \} \right)^{-(\beta-1)} Y_t^\beta}_{\text{consumer surplus from innovation}} + \underbrace{\frac{(\text{CS}_D - \text{CS}_M)}{r - \alpha} Y_F^{*-(\beta-1)} Y_t^\beta}_{\text{consumer surplus from imitation}} \right]. \quad (7)$$

The first summand in (7) is the industry value. It is independent of the distribution of $\min \{ \tilde{Y}_i, \tilde{Y}_j \}$ and equal to $2 \min \{ L(Y_L^*, Y_F^*), F(Y_F^*; K) \}$, which is single-peaked with respect to K with a maximum at \hat{K} by Proposition 2. The second term is the consumer surplus that results from innovative investment. The expected value of this term increases with K , since a higher imitation cost shifts the distribution of innovator entry thresholds (which may be degenerate, *e.g.* under preemption) leftward. The third term is the consumer surplus that results from the imitator's entry into the market. Since a greater imitation cost raises the standalone imitation threshold Y_F , the follower's entry can occur later when K increases, delaying the second wave of surplus that accrues to consumers. In the attrition regime, there is therefore a single possibly ambiguous effect of higher imitation cost on welfare, which involves this third term. So long as the effect of imitator entry on consumer surplus is therefore not too large, the first two effects dominate the last one, in which case social welfare is increasing until \hat{K} . The precise argument regarding the relationship between imitation cost and welfare is given in Section A.5, and we summarize some of the main points below.

First, within the range of preemption regimes ($K > \hat{K}$), the innovator and imitator entry thresholds are respectively Y_P and Y_F , so it is sufficient and straightforward to identify the local maximum of (6). For some parameter values, the optimum is a corner solution ($K^* = \infty$), signifying that the social optimum involves a single active firm. This corner solution arises if the contribution of the second firm's entry to welfare (which consists both of the direct welfare effect of the imitator's investment and its strategic effect on the timing of innovator entry) is relatively small, as occurs for instance if there is collusion in the product market (if $2\pi_D \approx \pi_M$).

Second, within the range of attrition regimes, there is a local maximum of welfare as well. To establish this, it is sufficient to show that social welfare is decreasing at the critical value \hat{K} . For simplicity, set $\text{CS}_M = 0$ (*e.g.* assume that a monopoly innovator practices perfect price discrimination) so that the middle term in the welfare expression (7) drops out. Also, note that the expected industry value term $2V(\tilde{Y}_i, \tilde{Y}_j)$ reaches a maximum at \hat{K} also so $\partial V(\tilde{Y}_i, \tilde{Y}_j) / \partial K \Big|_{\hat{K}} =$

0. The behavior of social welfare at this critical point is therefore determined by the remaining term, the consumer surplus from imitation. In an attrition regime, near \hat{K} (for $K \in (\tilde{K}, \hat{K})$), this term has two parts, depending on whether the innovator invests before Y_S (in which case imitator investment occurs at Y_F) or after (in which case imitator investment occurs immediately afterward). Accounting for the equilibrium distribution of Y_F^* therefore gives this term as

$$\frac{(\text{CS}_D - \text{CS}_M)}{r - \alpha} Y_F^{-(\beta-1)} Y_t^\beta \left(G_{\min}(Y_S; K) + \int_{Y_S}^{\infty} (Y_F/s)^{\beta-1} dG_{\min}(s; K) \right) \quad (8)$$

where $G_{\min}(\bullet)$ is the distribution of innovator entry thresholds (see Section A.2), which depends notably on K . However, it can be shown that $G_{\min}(Y_S; \hat{K}) = 1$ and $\partial G_{\min}(Y_S; \hat{K}) / \partial K = 0$. Around the critical value \hat{K} therefore, changes in K have a second-order effect on the distribution of entry thresholds compared with Y_F . Thus, an envelope argument on the welfare expression (7) establishes that $\partial \mathbb{E}_{\tilde{Y}_i, \tilde{Y}_j} W(\hat{K}) / \partial K < 0$.

Finally, either of the local maxima (under attrition or preemption) can be a global maximum, depending on the relative magnitude of the consumer surplus resulting from innovation and from imitation.

To summarize,

Proposition 3 *The social optimum involves a limited level of second mover advantage ($K^* > \tilde{K}$). Provided that a monopoly innovator does not perfectly price discriminate ($\text{CS}_M > 0$) and that the consumer surplus generated by the imitator's entry ($\text{CS}_D - \text{CS}_M$) is not too large, the social optimum involves preemption rather than attrition ($K^* > \hat{K}$). Moreover, the optimal imitation cost can be so high as to preclude imitator entry ($K^* = \infty$).*

4 Patenting intensity, reverse engineering, and licensing

The framework we have developed is readily extended to incorporate other aspects that are usually linked with innovation and imitation. One is the option that an innovating firm has to raise the entry barrier of the imitator, either by technological choices in product development that render reverse engineering more costly, or by strengthening the patentability of its product. Another is to incorporate the possibility of contracting, which typically takes the form of technology transfer that reduces the follower's imitation cost in a context similar to a licensing agreement, but can also involve a "pay for delay" agreement or a takeover. Both of these features add an intermediate stage to the investment game, once the innovator's entry has occurred and before the imitator

invests, and have as a common feature that, by raising the standalone value of the innovating firm, they tend to favor first-mover advantage and the emergence of preemption regimes.

4.1 Endogenous entry barrier

As a protective measure for its new product, in a first extension of the framework, the innovating firm may rely on a varying degree of either legal or technical protection in order to influence the imitation cost of a subsequent entrant. In case of legal protection, the imitation cost level reflects the breadth of patents, with wider patents implying higher costs for inventing around to develop a non-infringing imitation. Moreover, firms may decide to pursue patent protection more or less aggressively, as is the case for pharmaceutical firms as discussed in the introduction. In case of technical protection, the imitation costs are imparted by reverse engineering, and increase with the complexity of the copied product. For instance, an innovating firm can render its product more difficult to disassemble, or even add misleading complexity.¹⁴

Such choices may be incorporated into the framework of this paper by introducing a decision by the innovating firm at the time of its investment to expend an additional irrecoverable cost, which we denote by ρ , that raises the imitating firm's sunk cost by an amount $f(\rho)$, where f is taken to be an increasing and concave function with $f(0) = 0$ for simplicity. The cost ρ is deducted from the innovator payoff $L(Y_i, Y_{-i}^e)$ defined in (1). The investment costs of the innovator and imitator are then redefined as $I := I_0 + \rho$ and $K := K_0 + f(\rho)$, where I_0 and K_0 represent the baseline values where no effort is exerted on raising rival cost. With respect to the sequence of decisions, the choice of ρ arises once the roles of firms are determined, and before the entry of the second firm. This corresponds to adding an intermediate stage to the game, so that we have:

- Stage 1': both firms select initial entry thresholds (Y_i, Y_j) that determine innovator and imitator roles;
- Stage 2': if a single firm (i) innovates, it selects a degree of patenting effort and product complexity (ρ);
- Stage 3': the remaining firm (j) then selects its imitator entry threshold.

Proceeding by backward induction, with an endogenous entry barrier for imitation, the imitator payoff is a nonincreasing function of K and therefore of the innovator's effort, ρ , whereas its

¹⁴Samuelson and Scotchmer [32] provide an extensive review of the economic and legal aspects of reverse engineering practices.

entry threshold $Y_F^*(\rho) = \max\{Y_i, Y_F(\rho)\}$ is nondecreasing in ρ . In stage 2', the innovator now faces a post-entry decision problem, $\max_\rho (L(Y_i, Y_F^*(\rho)) - \rho)$. Provided that Y_i is not too large, the optimum solution is interior, and the optimal cost-raising effort satisfies¹⁵

$$\beta \left(\frac{\pi_M}{\pi_D} - 1 \right) \left(\frac{Y_i}{Y_F(\rho^*)} \right)^\beta f'(\rho^*) = 1. \quad (9)$$

The reasoning for stage 1' proceeds as in Section 2, save that the innovation and imitation payoffs now take the respective forms $L(Y_i, Y_F^*(\rho^*))$ and $F(Y_F^*(\rho^*); K_0 + f(\rho^*))$, and the equilibrium is as characterized in Proposition 1.

Figures 1–5 are useful to get insight into the effect of endogenous entry barriers. The optimal choice of effort is positive over a range of investment thresholds, resulting in a lower follower payoff, whereas the leader payoff is higher. We can therefore conclude that incorporating patenting and technological choice into the analysis results in a more preemptive strategic investment game, lowering the critical imitation cost threshold \widehat{K}_0 that separates the two regimes. Moreover, when the imitation cost is at this critical level, so that firms do not compete for positional rents, the standalone innovator entry threshold is greater than Y_L since an innovator faces an additional sunk cost ρ^* . Innovative firms generally find it optimal to exert some effort to increase the level of entry barriers, except in the case where the innovator enters at a very high threshold in a waiting game. When this effort is positive, it decreases with the baseline imitation cost, which it supplements ($\partial\rho^*/\partial K_0 < 0$). *This latter result is in line with the biopharmaceutical industry case discussed in Section 1.2: firms typically place greater reliance on patenting in the medications segment, in which natural entry barriers are low, than in the vaccines segment.*

Finally, some comparative statics results follow directly from the first-order condition (9). For instance, it is straightforward to show that greater monopoly profits or a more productive cost-raising technology, such as greater patent protection (higher f'), are associated with a higher endogenous entry barrier. Ambiguous effects can also arise, for instance with respect to volatility. Greater volatility results in a higher entry barrier if the innovator invests at a relatively low threshold (as is more likely to occur in the preemption regime where the follower's option value is important), and conversely in a lower entry barrier in industries with attrition and where follower entry is often instantaneous, so $\partial\rho^*/\partial\sigma \geq 0$.

¹⁵If ρ is expenditure on patenting, the corner solution is of economic interest in that it reflects the choice of a firm *not* to patent and rely on lead times instead. This outcome is more likely to arise if a fixed cost of patenting is introduced, or if the innovator invests sufficiently late in an attrition regime so that follower investment is immediate, and attaining an effective degree of patent protection ($Y_F^*(\rho^*) > Y_i$) is therefore prohibitively costly.

4.2 Contractual alternatives

Having established the baseline investment incentives of innovators and imitators, it is possible next to broaden the range of contracting possibilities offered to firms. An often-discussed alternative in the context of innovation and imitation is licensing. Suppose therefore that some of the knowledge developed by the innovator can be transferred to the second firm. Other types of contracts can also arise. If an imitator can commit not to enter the market over a certain period, an innovator might consider a pay-for-delay agreement,¹⁶ if such are allowed, and an extreme case of a pay-for-delay agreement is a takeover, in which case imitator entry never occurs. We do not propose in this section to study the optimal form of contract in full generality, in particular in the case of the licensing agreement. Rather, we make the simplifying assumption that firms have one type of contractual alternative, which consists in making a lump-sum payment contingent upon investment time or threshold, and which may incorporate a technology transfer.¹⁷ This simple form of contract suffices to illustrate a diversity of outcomes. Also, we assume that the contract is written by the innovator, who holds all the bargaining power.

The formal effect of introducing this contractual alternative is to add an intermediate stage to the game, which has the form of a dynamic agency problem in which the innovator seeks to guide the imitating firm's investment behavior. Let K_0 denote an incompressible level of imitation cost, reflecting such items as distribution and marketing expenses, and K_I denote that part of the imitator's product development cost that can be eliminated by a knowledge transfer from the innovator, so the fixed cost of the imitator is $K := K_0 + K_I$. The sequence of events is:

- Stage 1": both firms select initial entry thresholds (Y_i, Y_j) that determine innovator and imitator roles;
- Stage 2": if a single firm (i) innovates, it proposes a contract (license, pay for delay) involving lump sum transfer fee (φ) ;
- Stage 3": the remaining firm (j) decides whether or not to accept the contract and selects its entry threshold.

¹⁶The emblematic practice of "pay for delay" contracts, as occurs between a pharmaceutical firm and a generic manufacturer, is an application of our framework.

¹⁷If royalty payments associated with a licensing agreement are a flow, the imitator retains a valuable option to invest in product development on its own later on. Other equilibria then arise which are analogous to those described by Hori and Mizuno [21]'s model of access charges.

The reservation value of the follower if it rejects any contract with the innovator is the value which results from the equilibrium described in Section 2, $F(Y_F^*; K_0 + K_I)$. Because this reservation value is not constant over time (until it is eventually realized at Stage 3") it is useful to denote it as $F_0(Y_t)$. Any contract must therefore satisfy the participation constraint $F(Y_F^*(\varphi); K_0 + \varphi) \geq F_0(Y_t)$. The transfer φ is allowed to be time dependent, but we omit the time subscript for simplicity. There are then two subcases to consider, which depend on the optimal number of firms for the industry. The first corresponds to a standard industrial organization framework, whereas the second allows for a sufficient degree of product complementarity, so that duopoly profits exceed the monopoly level. This latter case is of interest in that it illustrates a richer interaction between the two firms.

Case *i.* ($2\pi_D \leq \pi_M$) If profits are maximized when there is a single firm in the industry, an innovator may seek to pay the imitator its reservation value in order to delay entry indefinitely (a takeover), provided that entry barriers are sufficient to preclude any further entry. If product market competition is severe enough and the innovator does not enter excessively late in an attrition regime, the innovator indeed finds it optimal to buy out and shut down the imitator in stage 2". The expected payoff function of the leader in stage 1" is therefore shifted up, accelerating innovation in a preemption regime and raising industry value in an attrition regime. As this contracting option eliminates the imitator entry in the market, the resulting consequences for economic welfare are ambiguous. Although it is generally preferable from a static welfare standpoint to have competition in the product market, recall that as detailed Section 3 a monopoly can be more dynamically efficient (when $K^* = \infty$). *In such cases, a takeover can raise total welfare.* As an example, suppose that the product market functions as a cartel ($2\pi_D = \pi_M$) and that the imitation cost is such that firms are in a preemption regime. Then, leaving the level of industry profit unchanged, product introduction occurs earlier when the first firm can conduct a takeover than under standard preemption resulting in a higher level of consumer surplus.

If a takeover is not feasible, the best option for the innovator is to allow follower entry at the usual threshold Y_F^* , but set a maximum license fee $\varphi^* = K_I$. As licensing reduces the innovator's irreversible cost of investment by the expected licensing revenue $K_I(Y_i/Y_F^*)^\beta$, the leader payoff is higher, the standalone innovator threshold is lower, and the absence of duplication of effort reduces the industry's resource cost.

Case *ii.* ($2\pi_D > \pi_M$) If the second firm's entry increases industry profit, as may occur if there is product differentiation and sufficient product market complementarity, then there is a finite optimal imitator entry threshold for the industry, given by $Y_F^{**} := \beta(r - \alpha)K_0/(\beta - 1)(2\pi_D - \pi_M)$. This threshold is greater than the standalone imitator threshold if the amount of transfer-

able technology is small or if product market complementarity is not too strong (formally, if $K_0/(K_0 + K_I) > 2 - \pi_M/\pi_D$), but can be smaller otherwise, in which case an innovator seeks to accelerate imitator entry. In all cases, the innovator induces the industry optimum by setting a license fee policy that consists of a forcing contract, *i.e.*

$$\varphi^* = \frac{\pi_D}{r - \alpha} Y_F^{**} - K_0 - F_0(Y_F^{**}) \quad (10)$$

if the follower invests at $Y_{-i} = Y_F^{**}$ and no license otherwise.

Although this description of contractual alternatives is clearly simplified, it is sufficient to shed further light on innovation incentives. In comparison with the baseline model, the expected value of the imitator in the first stage of the game is unchanged. Therefore, the main effect of introducing a supplemental instrument is generally to raise leader profit, thereby accelerating innovative investment in preemptive regimes, and raising industry profit in attrition regimes. The consequences for imitative investment can be quite different, and range from accelerated imitation, to no imitation at all.

4.3 Synthesis

As shown in this section, allowing a richer set of interactions between innovating and imitating firms heightens potential first-mover advantages, raising the likelihood that strategic investment dynamics take the form of a preemption race. Where technological choices and contractual alternatives can have contrasting effects is with respect to the timing of imitator entry, which is naturally delayed when entry barriers are endogenous, but which may be accelerated or eliminated by contractual measures, so that the latter may be thought of as inducing a greater variance in imitation outcomes. Naturally, real-world firms may also concurrently face the different decisions described in this section, and a structured examination of the trade-offs that may then arise requires additional assumptions and is left for further work.

Proposition 4 *In an extended framework for strategic investment:*

- (i) *a firm that innovates early enough increases its expected value by raising the imitating firm's entry cost, resulting in a more preemptive investment regime with delayed imitation, whereas*
- (ii) *if an innovating firm can contract with the imitating firm, the optimal contract involves either a takeover (which can be welfare-increasing) or a licensing agreement. These contracts result in a more preemptive investment regime and earlier innovation, whereas imitation may be either accelerated or delayed.*

5 Conclusion

We have sought to develop an integrative framework so as to study some long-standing questions regarding the allocation of resources to innovation and to imitation under imperfect competition, in line with both established research on innovation and more recent theory on strategic investment. In the classic work in this field, intellectual property protection, and notably patent policy, is motivated with reference to a trade-off between static and dynamic inefficiencies, as described by Arrow [1] in a setting with monopoly and competition. The analysis of this trade-off under imperfect competition and in a dynamic setting highlights an altogether different channel through which changes in the cost of imitation influence firm choices, by altering the nature of strategic competition (attrition *vs.* preemption). The broad message that emerges from the study of the duopoly case remains consistent with this seminal work – notably, that some degree of protection should be afforded to innovators when the cost of imitation is relatively small. At the same time, alternative mechanisms such as technological choice and contracting alternatives exist that can, to an extent, substitute for the regulatory protection of innovators so that a natural dynamic allocation need not be less efficient than a regulated one.

Among the extensions of the framework that we have identified but have not pursued here, a possible next step in the analysis is to study incremental innovation (or “versioning”) among existing firms in a market. In this setting, it is more likely that simultaneous investment equilibrium solutions arise, suggesting that firms might coordinate on investment timing. It is not much further to go to examine the possibility of cooperation in product development with these tools as well.

A Proofs

A.1 Tie-breaking rule

The market entry game described in Section 2.2 is a reduced form of a timing game in which firms choose (stochastic) investment times rather than thresholds. A complete specification of the stochastic payoff case can be found in Thijssen et al. [36], of which we only give the most summary description here.¹⁸

¹⁸Note that these authors focus their analysis on the preemption case, but the specification of the timing game also applies to attrition.

In the general timing game, firms choose so-called simple strategies that consist at any moment t_0 of a pair of functions $(G_i^{t_0}, \alpha_i^{t_0})$, the first of which is the cumulative distribution function of a firm's future investment times over the sample space, and the second of which is an investment intensity or "atom function" that allows for coordination between firms at times when simultaneous investment by both firms is not optimal that satisfies a number of consistency conditions, with $\alpha_i^{t_0}(\cdot) \in [0, 1]$. With simple strategies, at any time t at which both firms simultaneously attempt to invest, dropping time subscripts and the argument K for simplicity, firm i 's probability of investing before its rival is

$$p(i) = \frac{\alpha_i(1 - \alpha_j)}{\alpha_i + \alpha_j - \alpha_i\alpha_j} \quad (11)$$

and its expected payoff is

$$\tilde{V}_i(\alpha_i, \alpha_j) = p(i) L(Y; Y_F^*) + p(j) F(Y_F^*; K) + (1 - p(i) - p(j)) M(Y). \quad (12)$$

In a symmetric equilibrium in mixed strategies and positive investment intensity $((\alpha_i, \alpha_j) \neq (0, 0))$, α_j should be such that firm i is indifferent between different intensities, *i.e.* $\partial \tilde{V}_i / \partial \alpha_i = 0$ where

$$\frac{\partial \tilde{V}_i}{\partial \alpha_i} = \alpha_j \frac{-\alpha_j (L(Y; Y_F^*) - M(Y)) + (L(Y; Y_F^*) - F(Y_F^*; K))}{(\alpha_i + \alpha_j - \alpha_i\alpha_j)^2}. \quad (13)$$

The sign of (13) depends on the comparison of $L(Y; Y_F^*)$ with $M(Y)$ and $F(Y_F^*; K)$. For all $Y < (=) Y_F^*$, we have $L(Y; Y_F^*) > (=) M(Y; Y_F^*)$. Therefore, a symmetric equilibrium in mixed strategies with positive investment intensity exists only if $K > \hat{K}$ so that $L(Y_L; Y_F^*) > F(Y_F^*; K)$. Its support is then $\{Y \leq Y_F | L(Y; Y_F^*) > F(Y_F^*; K)\}$, with

$$\alpha_i^* = \alpha_j^* = \frac{L(Y; Y_F^*) - F(Y_F^*; K)}{L(Y; Y_F^*) - M(Y)} \quad (14)$$

which, given the investment probabilities (11) yields the tie-breaking rule (the investment probabilities) and the reduced form payoff $V(Y_i, Y_j)$ in (4).

A noteworthy feature of our model is that the behavior of the investment probabilities differs qualitatively from the case studied by Thijssen et al. [36].¹⁹ To begin with, α_i^* is not monotone in Y over its support. Moreover, for $K \in (\hat{K}, I)$, the condition $L(Y; Y_F^*) = F(Y_F^*; K)$ has two distinct roots $Y_P < Y_{P'}$ (*see Figure 4*). This means that, for some intermediate values of K , the preemption range is a strict subset of $[Y_P, Y_F]$ so that, even though firms are in a preemption regime, off the equilibrium path (or, if the game were to begin at a large enough level Y_0), the

¹⁹In another extension of the standard real option game model, Billette de Villemeur et al. [4] provide a case where the simultaneous investment probability reaches 1 in the interior of (Y_P, Y_F) .

probability of simultaneous investment converges to zero and firms then face a subgame that is a war of attrition. \square

A.2 Proof of Proposition 1

In this section we first identify and characterize the threshold \widehat{K} . We then characterize the innovator value function $L(Y_i, Y_F^*)$. Finally, we describe the equilibrium strategies in the attrition ($K < \widehat{K}$) and preemption regimes ($K \geq \widehat{K}$).

Characterization of \widehat{K}

We first verify that \widehat{K} is well defined. Recalling that $Y_F^* := \arg \max F(Y_i; K)$, if $K = 0$ we have $Y_F^* = Y_i$, and the follower investment in stage 2 of the game occurs immediately after the leader has invested. In that case, from (1) and (2) we have

$$L(Y_i, Y_F^*) = \left(\frac{\pi_D}{r - \alpha} Y_i - I \right) \left(\frac{Y_t}{Y_i} \right)^\beta < \frac{\pi_D}{r - \alpha} Y_i \left(\frac{Y_t}{Y_i} \right)^\beta = F(Y_F^*; 0), \quad (15)$$

for all $Y_i \geq Y_t$. Next, as K increases, $L(Y_i, Y_F^*)$ shifts upwards since Y_F^* is nondecreasing in K and $\partial L(Y_i, Y_j) / \partial Y_j \geq 0$, all Y_i, Y_j . Also, $F(Y_F^*; K)$ shifts downward since Y_F^* is a maximizer of $F(Y_i; K)$ and $\partial F(Y_i; K) / \partial K < 0$, all Y_i . The maximum values at Y_L and Y_F necessarily satisfy $\partial L(Y_L, Y_F^*) / \partial K \geq 0$ and $\partial F(Y_F^*; K) / \partial K < 0$, with $\lim_{K \rightarrow \infty} F(Y_F^*; K) = 0$. Therefore, there exists a unique level of the imitation cost \widehat{K} such that $L(Y_L, Y_F^*) = F(Y_F^*; \widehat{K})$.

Next, the threshold \widehat{K} is given by the solution in K to $L(Y_L, Y_F) = F(Y_F; K)$ (necessarily $Y_L \leq Y_F$) which after simplification gives

$$\widehat{K} = \left(\frac{1 + \beta((\pi_M/\pi_D) - 1)}{(\pi_M/\pi_D)^\beta} \right)^{1/(\beta-1)} I, \quad (16)$$

and it is direct to check that $\widehat{K} < (=)I$ for $\pi_D < (=)\pi_M$.

Characterization of $L(Y_i, Y_F^)$*

We characterize the maximum of the function $L(Y_i, Y_F^*)$ over $[Y_t, \infty)$. There can be two local maxima, at $Y_L = \arg \max L(Y_i, Y_F)$ and $Y_S = \arg \max M(Y_i)$, with $Y_L \leq Y_S$. Let \widetilde{K} denote the imitation cost such that $L(Y_L, Y_F^*) = M(Y_S)$ which, for $Y_F^* = Y_F$ yields

$$\widetilde{K} = \left(\frac{\beta((\pi_M/\pi_D) - 1)}{(\pi_M/\pi_D)^\beta - 1} \right)^{1/(\beta-1)} I. \quad (17)$$

Then, $Y_L(Y_S)$ is a unique global maximum of $L(Y_i, Y_F^*)$ if $K > \tilde{K}$ ($K < \tilde{K}$).

Let $K_l := (\pi_D/\pi_M)I$ denotes the imitation cost for which that $Y_L = Y_F$. Then, it is useful to note the following ranking of the imitation costs in (16) and (17):

$$K_l \leq \tilde{K} \leq \hat{K}. \quad (18)$$

(the inequalities in (18) are strict if $\pi_M > \pi_D$). Indeed, straightforward calculations show that $\tilde{K} \geq K_l$ if and only if

$$(\beta - 1)(\pi_M/\pi_D)^\beta - \beta(\pi_M/\pi_D)^{\beta-1} + 1 \geq 0, \quad (19)$$

and that $\hat{K} \geq \tilde{K}$ if and only if

$$(\pi_M/\pi_D)^\beta - \beta((\pi_M/\pi_D) - 1) - 1 \geq 0. \quad (20)$$

Both of these conditions hold for all $\beta, \pi_M/\pi_D \geq 1$.

Attrition equilibrium

For $K < \hat{K}$ we have $L(Y_i, Y_F^*) < F(Y_F^*; K)$, all Y_i , so firms play a waiting game. Note that in this case there is no “mistaken” simultaneous investment and the simultaneous move payoff is $M(Y_i)$. There are two subcases to consider, $K < \tilde{K} (\leq \hat{K})$ and $\tilde{K} \leq K < \hat{K}$.

attrition subcase a.

If $K < \tilde{K}$, we know from the characterization of L above that $L(Y_i, Y_F^*)$ has a unique global maximum at Y_S and decreases over (Y_S, ∞) . Any play in $[Y_L, Y_S]$ is dominated by investing at Y_S (see Figure 1). Firms therefore play a standard war of attrition with complete information over $[Y_S, \infty)$. By Theorem 3 of Hendricks et al. [18] and a continuity argument at Y_S (since $(\partial L/\partial Y_i)(Y_S, Y_F^*) = 0$), there is a unique nondegenerate symmetric equilibrium in which firms randomize their entry triggers over $[Y_S, \infty)$ according to the cumulative distribution

$$G_a(Y_i; K) = 1 - \exp \int_{Y_S}^{Y_i} \frac{M'(s)}{F(s; K) - M(s)} ds \quad (21)$$

that results in an expected payoff of $M(Y_S)$. Substituting for the functions F and M and integrating gives the expression

$$G_a(Y_i; K) = 1 - \left(\frac{Y_i}{Y_S}\right)^{\frac{\beta I}{I-K}} \exp - \left\{ \frac{\beta I}{I-K} \left(\frac{Y_i}{Y_S} - 1\right) \right\}. \quad (22)$$

Note that as $Y_F \leq Y_S$, follower entry always occurs immediately after the first investment.

attrition subcase b.

If $\tilde{K} \leq K < \hat{K}$, we know from the characterization of L above that $L(Y_i, Y_F^*)$ has a global maximum at Y_L and a local maximum at Y_S . Because the leader payoff $L(Y_i, Y_F^*)$ is not monotonic over $[Y_L, Y_S]$, the attrition game is nonstandard. Let $Y_{S'}$ denote the unique solution in $[Y_L, Y_F]$ to the condition $L(Y_{S'}, Y_F) = M(Y_S)$. To verify that this threshold is well-defined, note that $Y_L \leq Y_F \leq Y_S$ since $K_l \leq \tilde{K} \leq K$ and $\hat{K} \leq I$, and that $L(Y_i, Y_F^*)$ is continuous and monotone decreasing on $[Y_L, Y_F]$ (see *Figure 2*). To derive the equilibrium, we proceed by backward induction.

First, the subgame starting at Y_S is a standard war of attrition, so conditionally on Y_S being reached, firms randomize their entry times according to the distribution G_a given in (21) above.

Next, any play in $(Y_{S'}, Y_S)$ is weakly dominated by investing at Y_S . The expected payoff in the subgame starting at $Y_{S'}$ is therefore $M(Y_S)$.

Lastly, consider a truncation of the game at $Y_{S'}$ so that firms are constrained to effectively invest over the interval of thresholds $[Y_L, Y_{S'}]$ with a terminal payoff $M(Y_S)$. This is a standard war of attrition with complete information and by Hendricks et al. [18] there is a unique nondegenerate symmetric equilibrium in which firms randomize their entry triggers over $[Y_L, Y_{S'}]$ with a possible terminal mass point $q := \Pr\{Y_i = Y_{S'}\}$. Because expected payoffs are constant over the support of mixed strategies, q satisfies the condition that $qM(Y_S) + (1 - q)F(Y_F^*; K) = L(Y_L, Y_F^*)$. The equilibrium of the truncated game therefore has firms randomizing their entry thresholds over $[Y_L, Y_{S'})$ according to the cumulative distribution

$$G_0(Y_i; K) = 1 - \exp \int_{Y_L}^{Y_i} \frac{\partial L(s, Y_F^*) / \partial Y_i}{F(Y_F^*; K) - L(s, Y_F^*)} ds \quad (23)$$

with an atom $q(Y_{S'}) = (F(Y_F^*; K) - L(Y_L, Y_F^*)) / (F(Y_F^*; K) - M(Y_S))$. Evaluating the integral in (23) yields the expression

$$G_0(Y_i; K) = \frac{L(Y_L, Y_F) - L(Y_i, Y_F)}{F(Y_F; K) - L(Y_i, Y_F)} \quad (24)$$

and in particular,

$$G_0(Y_{S'}; K) = \frac{\left((\pi_M / \pi_D)^\beta - 1 \right) I^{1-\beta} - \beta \left((\pi_M / \pi_D) - 1 \right) K^{1-\beta}}{K^{1-\beta} - I^{1-\beta}}. \quad (25)$$

With these elements, the symmetric equilibrium of the full attrition game over $[Y_L, \infty)$ can be described. Note first that if a firm plays an atom at $Y_{S'}$, the other firm strictly prefers delaying entry until Y_S . In a symmetric equilibrium first entry thresholds are therefore continuously

distributed over the disconnected support $[Y_L, Y_{S'}] \cup [Y_S, \infty)$. On this support, the cumulative distribution function

$$G_b(Y_i; K) = \begin{cases} G_0(Y_i; K) & \text{if } Y_L \leq Y_i \leq Y_{S'} \\ G_0(Y_{S'}; K) & \text{if } Y_{S'} < Y_i < Y_S \\ G_0(Y_{S'}; K) + (1 - G_0(Y_{S'}; K)) G_a(Y_i; K) & \text{if } Y_S \leq Y_i \end{cases} \quad (26)$$

is the symmetric subgame perfect Nash equilibrium, and results in an expected payoff $L(Y_L, Y_F^*)$. For $Y_i < Y_F$, follower entry occurs later than the first entry, and immediately after otherwise.²⁰

Preemption equilibrium

For $K > (=) \widehat{K}$ we have $L(Y_L, Y_F^*) > (=) F(Y_F^*; K)$ and so there exists a unique $Y_P \in (Y_t, Y_L]$ such that $L(Y_P, Y_F^*) = F(Y_F^*; K)$. We refer to “preemption” when the inequality is strict so that $Y_P < Y_L$. Both firms seek to invest at Y_P , with equal probability of being an innovator or of effectively entering as an imitator at Y_F . The structure of the game is very similar to a standard preemption game and the arguments establishing equilibrium are similar, although two additional points need to be made.

If $K < I$, the equilibrium condition $L(\cdot, Y_F^*) = F(Y_F^*; K)$ has another root $Y_{P'} \in (Y_L, Y_F)$ (see Section A.1). In this case, and in contrast with standard preemption games, in a subgame where $Y_t > Y_{P'}$ and that is never reached on the equilibrium path, firms play a war of attrition resulting in an expected payoff $L(Y_t, Y_F^*)$. As $L(Y_t - \varepsilon, Y_F^*) > L(Y_t, Y_F^*)$, a firm prefers to enter earlier rather than to reach this subgame.

Second, although simultaneous investment is generally not an equilibrium in the standard new market model of strategic investment, the suboptimality of simultaneous investment needs to be verified here because of the difference between leader and follower investment costs. Investment at the optimal simultaneous investment threshold Y_S results in a payoff $M(Y_S)$ and evaluating,

$$\frac{L(Y_L, Y_F^*)}{M(Y_S)} = \left(\frac{\pi_M}{\pi_D} \right)^\beta - \beta \left(\frac{\pi_M}{\pi_D} - 1 \right) \left(\frac{I}{K} \right)^{\beta-1}. \quad (27)$$

This ratio is increasing in K and therefore over the preemption range for which simultaneous equilibrium might arise, it is minimized at \widehat{K} . Substituting \widehat{K} for K and simplifying gives $L(Y_L, Y_F^*)/M(Y_S) = \left(I/\widehat{K} \right)^{(\beta-1)} \geq 1$, with strict inequality if $\pi_M > \pi_D$. The best response to $Y_{-i} = Y_S$ is Y_L if $K = \widehat{K}$. Therefore firms seek to preempt one another before the simultaneous investment threshold is reached, for all $K \geq \widehat{K}$. \square

²⁰Note that the equilibrium solution satisfies certain continuity properties in K . For $K = \widetilde{K}$, $Y_L = Y_{S'}$ and $q(Y_{S'}) = 1$ in the truncated game, yielding $G_0(Y_{S'}) = 0$, whereas for $K = \widehat{K}$, $G_0(Y_{S'; \widehat{K}}) = 1$.

A.3 Proof of Corollary 2

To establish the corollary we characterize the effect of β and π_M/π_D on $\widehat{K}(\pi_M/\pi_D, \beta)$, defined in (16) above. Provided that $\pi_D < \pi_M$, we have $\partial\widehat{K}/\partial(\pi_D/\pi_M) < 0$, and $\partial\widehat{K}/\partial\beta < 0$: evaluating the partial derivatives and rearranging yields

$$\frac{\partial\widehat{K}}{\partial(\pi_M/\pi_D)} = -\beta((\pi_M/\pi_D) - 1)(1 + \beta((\pi_M/\pi_D) - 1))^{\frac{2-\beta}{\beta-1}}(\pi_M/\pi_D)^{\frac{1-2\beta}{\beta-1}}I \quad (28)$$

and

$$\frac{\partial\widehat{K}}{\partial\beta} = \frac{-1}{(\beta-1)^2} \left[\ln \frac{1 + \beta((\pi_M/\pi_D) - 1)}{\pi_M/\pi_D} - \frac{(\beta-1)((\pi_M/\pi_D) - 1)}{1 + \beta((\pi_M/\pi_D) - 1)} \right] \widehat{K}. \quad (29)$$

The sign $\partial\widehat{K}/\partial(\pi_M/\pi_D) < 0$ follows directly from (28). Together with $\widehat{K}(1, \beta) = I$ and $\lim_{\pi_M/\pi_D \rightarrow \infty} \widehat{K}(\pi_M/\pi_D, \beta) = 0$, we thus also obtain that $0 \leq \widehat{K} \leq I$. The sign $\partial\widehat{K}/\partial\beta < 0$ is determined by the middle (bracketed) term in (29). More specifically, since $\ln x > (x-1)/x$ for $x > 0$ and $x \neq 1$, we have $\ln[(1 + \beta((\pi_M/\pi_D) - 1))/(\pi_M/\pi_D)] > (\beta-1)[(\pi_M/\pi_D) - 1]/[1 + \beta((\pi_M/\pi_D) - 1)]$, which together with $\widehat{K}(\pi_M/\pi_D, \beta) > 0$ is sufficient to conclude. \square

A.4 Section 3.1 arguments and industry optimum (Proposition 2)

Sensitivity analysis of investment thresholds

Consider first the innovator threshold \widetilde{Y}_I . If $K < \widetilde{K}$ (or $K = \widetilde{K}$), the hazard rate of the distribution of first entry thresholds implied by (23) is

$$h_a(Y_i; K) = \frac{\beta I}{I - K} \left(\frac{1}{Y_i} - \frac{1}{Y_S} \right), \quad (30)$$

so $\partial h/\partial K \geq 0$. For $\widetilde{K} < K < \widehat{K}$, the hazard rate corresponding to (26) is defined by parts. Over $[Y_L, Y_{S'})$ the hazard rate is

$$h_0(Y_i; K) = \frac{-\partial L(Y_i, Y_F^*)/\partial Y_i}{F(Y_F^*; K) - L(Y_i, Y_F^*)} \quad (31)$$

which has a numerator that is independent of K , so $\partial h_0/\partial K = -(\partial(F-L)/\partial K)(\partial L/\partial Y_i)/(F-L)^2 \geq 0$. Similarly over $[Y_{S'}, \infty)$ we have $\partial h/\partial K \geq 0$. Note that the hazard rate is discontinuous at $Y_{S'}$ and Y_S , but as $\partial Y_{S'}/\partial K \geq 0$ and $\partial Y_S/\partial K = 0$, it increases over the entire range $[Y_S, \infty)$. Finally, for $K > \widehat{K}$, Y_P decreases with K . Therefore the first entry threshold of each firm decreases with K (stochastically in the attrition regime and deterministically in the preemption regime), as does the minimum of the two, which is the threshold for both firms.

With respect to imitator investment, the second entry threshold Y_F^* is a random variable. In the attrition regime, it decreases in a stochastic sense over (Y_F, ∞) , that is when follower entry is immediate, but increases otherwise. However, the difference between the first and second entry thresholds is simple to characterize. For $K_l \leq K < \hat{K}$, $Y_F^* - \tilde{Y}_I = \max\{0, Y_F - \tilde{Y}_I\}$ is distributed over $[0, Y_F - Y_L]$ as $1 - G_\bullet(Y_F - s)$. So the difference between the second and the first entry threshold increases with K (stochastically in the attrition range and deterministically in the preemption range).

Industry optimum

The proposition follows directly from the equilibrium value resulting from rent equalization, $\mathbb{E}_{\tilde{Y}_i, \tilde{Y}_j} V(\tilde{Y}_i, \tilde{Y}_j) = \min\{L(Y_L^*, Y_F^*), F(Y_F; K)\}$ and the sensitivity of L and F to K . Note that for $K \leq \tilde{K}$, $\min\{L(Y_L^*, Y_F^*), F(Y_F; K)\} = M(Y_S)$ which is independent of K and that at $K = \hat{K}$, $M(Y_S) < L(Y_L, Y_F^*) = F(Y_F; \hat{K})$. Therefore, $\mathbb{E}_{\tilde{Y}_i, \tilde{Y}_j} V(\tilde{Y}_i, \tilde{Y}_j)$ is constant over $[0, \tilde{K}]$, increasing over (\tilde{K}, \hat{K}) , and decreasing over (\hat{K}, ∞) . \square

A.5 Imitation cost and consumer surplus (Proposition 3)

The argument is divided into four parts. We first determine the optimal imitation cost level, K_P , in the closure of the preemption regime ($K \geq \hat{K}$). Second, we establish a lower bound for the optimal imitation cost ($K^* \geq \tilde{K}$). Third, we establish the existence of a local optimum of welfare under attrition ($\tilde{K} \leq K < \hat{K}$). Finally, we conclude by comparing the optimum under preemption with the optimal welfare that is attained in the attrition regime.

Socially optimal imitation cost in preemption regime

Suppose that $K \geq \hat{K}$, so entry thresholds are Y_P and Y_F . The social welfare function (7) then has the form

$$W(K) = \left(\frac{\pi_M + \text{CS}_M}{r - \alpha} Y_P - I \right) \left(\frac{Y_t}{Y_P} \right)^\beta + \left(\frac{(2\pi_D + \text{CS}_D) - (\pi_M + \text{CS}_M)}{r - \alpha} Y_F - K \right) \left(\frac{Y_t}{Y_F} \right)^\beta. \quad (32)$$

Noting that Y_P and Y_F are functions of K , with $Y_P \in [Y_{\text{NPV}}, Y_L]$ differentiable and strictly decreasing (where $Y_{\text{NPV}} = (r - \alpha)I/\pi_M$ is the Marshallian investment threshold for a monopoly firm). Using the preemption equilibrium condition $L(Y_P, Y_F) = F(Y_F; K)$ which implicitly defines the ratio $(Y_F/Y_P)^\beta$, the derivative of (32) can be expressed as

$$\frac{dW}{dK} = \left(\frac{Y_t}{Y_F} \right)^\beta \left(\left(\beta \frac{\pi_M}{\pi_D} - (\beta - 1) \right) \frac{\left(1 + \frac{\text{CS}_M}{\pi_M} \right) Y_P - Y_L}{Y_L - Y_P} - \beta \frac{(2\pi_D + \text{CS}_D) - (\pi_M + \text{CS}_M)}{\pi_D} + (\beta - 1) \right). \quad (33)$$

If $\text{CS}_M = 0$ (perfect price discrimination under monopoly), the $Y_L - Y_P$ terms in (33) cancel out and it is straightforward to verify that $dW/dK < 0$, so that \widehat{K} is a maximum. For $\text{CS}_M > 0$, (33) satisfies $\lim_{\widehat{K}} dW/dK = +\infty$ and is strictly decreasing in K . Then if $\lim_{\infty} dW/dK < 0$, there is a unique root $K_P > \widehat{K}$ that constitutes an interior optimum, which occurs if

$$\left(\beta^2 \frac{\pi^M}{\pi^D} - (\beta - 1)^2 \right) \frac{\text{CS}_M}{\pi_M} - \beta \frac{\text{CS}_D}{\pi^D} - 2 > 0. \quad (34)$$

For notational simplicity, in what follows we allow $K_P = \infty$.

Lower bound on socially optimal imitation cost

If $K < \widetilde{K}$ (*attrition subcase a.* in Section A.2 above) so firms randomize investment triggers over $[Y_S, \infty)$ according to the distribution $G_a(Y_i; K)$ and imitator entry is immediate, then $W(K) < W(\widehat{K})$. To see this, note first that by Proposition 2, industry value is lower at K than at \widehat{K} , so it suffices to show that expected consumer surplus is lower also. But at \widehat{K} , innovator and imitator entry occur at the standalone thresholds Y_L and $Y_F = (\beta(r - \alpha)\widehat{K}) / ((\beta - 1)\pi_D)$, whereas the lower bound of the entry threshold distribution under attrition is $Y_S = (\beta(r - \alpha)I) / ((\beta - 1)\pi_D) \geq Y_F$ with $G_a(Y_S; K) < 1$. Therefore, both investments occur later if $K < \widetilde{K}$ than they do at the critical imitation cost \widehat{K} , resulting in lower consumer surplus and welfare.

Existence of local maximum in attrition regime

Consider the value of $\mathbb{E}_{\widetilde{Y}_i, \widetilde{Y}_j} W(\widehat{K})$ as K approaches \widehat{K} from below. Since $V(\widetilde{Y}_i, \widetilde{Y}_j)$ is maximized at \widehat{K} , at this critical value the sign of $d\mathbb{E}_{\widetilde{Y}_i, \widetilde{Y}_j} W(\widehat{K})/dK$ depends only on the behavior of the consumer surplus term. For simplicity, consider the third term, consumer surplus from imitation (the argument for the other term is similar). As noted in the text, the consumer surplus from imitation is given by

$$\frac{\text{CS}_D - \text{CS}_M}{r - \alpha} Y_F^{-(\beta-1)} Y_t^\beta \left(\underbrace{G_{\min}(Y_S'; K)}_{\text{lagged imitator entry}} + \underbrace{\int_{Y_S}^{\infty} (Y_F/s)^{\beta-1} dG_{\min}(s; K)}_{\text{immediate imitator entry}} \right). \quad (35)$$

To determine the value of the derivative of this expression with respect to K at \widehat{K} , recall that the distribution of entry thresholds is given by $G_{\min}(Y_i; K) = 1 - (1 - G_{\bullet}(Y_i; K))^2$. Consider

the first summand in (35). Since $G_0(Y_{S'}; \widehat{K}) = 1$, $G_{\min}(Y_{S'}; \widehat{K}) = 1$. Moreover $\partial G_{\min}/\partial K = 2(1 - G_{\bullet})\partial G_{\bullet}/\partial K$ so $\partial G_{\min}(Y_{S'}; \widehat{K})/\partial K = 0$. Therefore, in (35) only the direct effect of K on Y_F matters for welfare at \widehat{K} . A similar argument applies to the consumer surplus from innovation term in (7), with no direct effect since Y_L is independent of K .

Therefore,

$$\lim_{\widehat{K}_-} \frac{d\mathbb{E}_{\widetilde{Y}_i, \widetilde{Y}_j} W(\widehat{K})}{dK} = -(\beta - 1) \frac{\text{CS}_D - \text{CS}_M}{r - \alpha} Y_F^{-\beta} Y_t^\beta \frac{\partial Y_F}{\partial K} \leq 0. \quad (36)$$

Since $W(K)$ is continuous, we conclude that if $\text{CS}_D > \text{CS}_M$, there exists a (local) optimum imitation cost level K_A in the attrition range that satisfies $\widetilde{K} < K_A < \widehat{K}$.

Global welfare optimum

From above, we therefore know that $\lim_{\widehat{K}_-} dW(K)/dK < 0$ and that, for $\text{CS}_M > 0$, $\lim_{\widehat{K}_+} dW(K)/dK > 0$ so that welfare has local maxima in both the attrition (specifically, for $K \in (\widetilde{K}, \widehat{K})$) and preemption ranges (whereas the local maximum under preemption is $K_P = \widehat{K}$ if $\text{CS}_M = 0$). Either type of local maximum can be a global maximum, depending on the relative magnitude of the consumer surplus resulting from innovation or from imitation. Since $W(K)$ is continuous in the model's parameters, we can consider successively the cases where one of these surpluses becomes arbitrarily small to show this.

First, if $\text{CS}_M = 0$, $K_P = \widehat{K}$, and since $\lim_{\widehat{K}_-} dW(K)/dK < 0$, social welfare is maximized for a value of K that lies in the attrition range.

Alternatively, suppose that $\text{CS}_M > 0$ (so $K_P > \widehat{K}$) and take $\text{CS}_D - \text{CS}_M = 0$. Then, for $K \leq \widehat{K}$, $W(K) \leq W(\widehat{K}) < W(K_P)$ where the first inequality results from Proposition 2 and because innovator entry thresholds satisfy $\widetilde{Y}_i \geq Y_L$. Therefore, the local optimum in the preemption range K_P is a global welfare maximum in this case. \square

A.6 Endogenous entry barrier

In stage 3', the imitator payoff depends on the cost-raising effort ρ :

$$F(Y_i; K) = \left(\frac{\pi_D}{r - \alpha} Y_i - K_0 - f(\rho) \right) \left(\frac{Y_t}{Y_i} \right)^\beta. \quad (37)$$

The optimal standalone imitator threshold is $Y_F(\rho) = (\beta(r - \alpha)(K_0 + f(\rho))) / ((\beta - 1)\pi_D)$, yielding an optimal choice $Y_F^*(\rho) = \max\{Y_i, Y_F\}$. In stage 2', an innovator having entered at the threshold Y_i chooses a level of effort that maximizes

$$L(\rho) = \left(\frac{\pi_M}{r - \alpha} Y_i - I_0 - \rho \right) + \frac{\pi_D - \pi_M}{r - \alpha} Y_F^*(\rho) \left(\frac{Y_i}{Y_F^*(\rho)} \right)^\beta. \quad (38)$$

If $Y_i \leq Y_F(0)$, the solution is the interior solution ρ^* given by (9), whereas if Y_i is arbitrarily large, $L(\rho^*) < L(0)$, so both types of solutions arise. Note that in contrast with the payoff functions shown in Figures 1 – 3, since the optimal effort ρ^* depends on the innovator's entry threshold Y_i , with $\partial\rho^*/\partial Y_i \geq 0$, the follower payoff in the first stage of the game, $F(Y_F^*(\rho^*); K_0 + \rho^*) (\leq F(Y_F^*(0); K_0))$ is decreasing rather than constant. In stage 1' therefore, the payoff $V(Y_i, Y_j)$ given by (4) is defined for the underlying innovator and imitator payoffs $L(Y_i, Y_F^*(\rho^*))$ and $F(Y_F^*(\rho^*); K_0 + f(\rho^*))$. At the critical imitation cost \widehat{K}_0 that separates the attrition and preemption regimes and because $F(Y_F^*(\rho^*); K_0 + \rho^*)$ is decreasing in Y_i , at the standalone optimum Y_L , $L(Y_L, Y_F^*(\rho^*)) > F(Y_F^*(\rho^*); \widehat{K}_0 + f(\rho^*))$ so the equilibrium condition is satisfied above Y_L , and innovator entry occurs “too late” from the standpoint of industry value. Finally, since $L(\rho^*) \geq L(0)$, the same arguments apply as with Proposition 1 to rule out a simultaneous investment equilibrium in the preemption regime.

With regard to the signs of the comparative statics mentioned in the text, since f is concave, it is sufficient to determine those of $\partial^2 L / \partial \rho \partial \pi_M$ and $\partial^2 L / \partial \rho \partial \sigma$. The sign of the former is immediate, and for the second,

$$\partial^2 L(Y_i; Y_F(\rho^*)) / \partial \rho \partial \sigma = \beta \left(\frac{\pi_M}{\pi_D} - 1 \right) \left(\frac{2\beta - 1}{\beta - 1} + \beta \ln \left(\frac{Y_i}{Y_F(\rho^*)} \right) \right) \left(\frac{Y_i}{Y_F(\rho^*)} \right)^\beta f'(\rho^*) \frac{\partial \beta}{\partial \sigma}. \quad (39)$$

The sign of the second bracketed term depends in the investment threshold of the innovator. It is negative if $Y_i < e^{-\beta(2\beta-1)/(\beta-1)} Y_F(\rho^*)$ and zero or positive otherwise. \square

A.7 Contractual alternatives

Note that an alternative interpretation, viewing the rights to profit streams as real assets, runs as follows. After the initial investment, the imitator holds a natural call option on duopoly profit flow π_D at a strike price of K . Until that option is exercised, the innovator perceives a flow of differential monopoly rent $\pi_M - \pi_D$, but it can alternatively sell what constitutes a call option on

the duopoly profit flow to the imitator, at a strike price φ , and with a transaction cost K_0 that is paid by the imitator. When $\pi_M < 2\pi_D$, this is an efficient trade.

Formally, in a typical licensing setup, the innovator's decision in stage 2" takes the form:

$$\max_{\varphi} V_{\text{lic.}}(\varphi) = \left(\varphi - \frac{\pi_M - \pi_D}{r - \alpha} Y_F^*(\varphi) \right) \left(\frac{Y_t}{Y_F^*(\varphi)} \right)^{\beta} \quad (40)$$

where $Y_F^*(\varphi)$ is the follower's investment threshold depends on φ_t , and the time subscript is dropped for simplicity. We consider the cases discussed in the text successively.

case i. ($2\pi_D \leq \pi_M$)

For simplicity, we assume that the firm's environment is such that the different decision problems (takeover, technology licensing) are exclusive, so that they can be treated separately. First, suppose that $2\pi_D < \pi_M$ and a takeover is feasible. The innovator's payoff in this case is

$$L_{\text{tak.}}(Y_i) = \left(\frac{\pi_M}{r - \alpha} Y_i - I - \varphi \right) \left(\frac{Y_t}{Y_i} \right)^{\beta} \quad (41)$$

and the optimal transfer (to the imitator) is $\varphi = F_0(Y_t)$. As imitator entry reduces industry flow profit, a takeover is always efficient for the firms and is straightforward to verify that $L_{\text{tak.}}(Y_i) > L(Y_i, Y_F^*)$. The leader's stage 1" payoff is thus $\max L_{\text{tak.}}(Y_i)$. Note that the optimal standalone threshold Y_L is higher when takeovers are allowed, so the distribution of innovator entry thresholds is shifted right in an attrition regime. The case of a license fee as described in the text is direct.

To establish that a takeover increases welfare, consider the case where there is a cartel on the product market, $2\pi_D = \pi_M$, so imitator entry leaves consumer surplus unchanged. If $K > \widehat{K}$, preemption occurs and industry value is set at $F_0(Y_t)$ by rent equalization regardless of whether takeovers are allowed or not. A takeover is efficient in this case if the first firm enters earlier when it can make a takeover offer to its rival, *i.e.* if the lower root of $L_{\text{tak.}}(Y_i) = F(Y_F; K)$ is lower than Y_P , which holds since $L_{\text{tak.}}(Y_i) > L(Y_i, Y_F^*)$.

case ii. ($2\pi_D > \pi_M$)

In stage 2", the innovator chooses φ^* so as to optimize the payoff (40). For $Y_i \leq \min\{Y_F, Y_F^{**}\}$, the stage 1" leader payoff is correspondingly

$$L_{\text{lic.}}(Y_i, Y_F^{**}) = \left(\frac{\pi_M}{r - \alpha} Y_i - I \right) \left(\frac{Y_t}{Y_i} \right)^{\beta} + \left(\varphi - \frac{\pi_M - \pi_D}{r - \alpha} Y_F^{**} \right) \left(\frac{Y_t}{Y_F^{**}} \right)^{\beta} \quad (42)$$

and the reasoning proceeds similarly to above. \square

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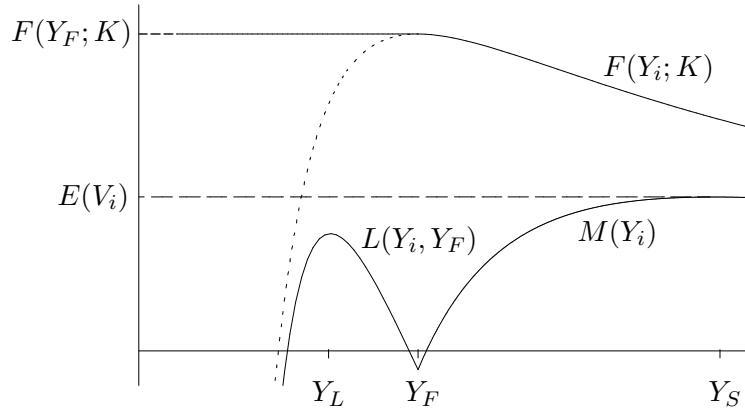


Figure 1: Attrition regime, $K \in [0, \tilde{K})$. Y_S is a global maximum of the leader payoff, innovator entry thresholds are distributed over $[Y_S, \infty)$, and imitator entry occurs immediately after. Note that if $K < K_l$, then $Y_F < Y_L$.

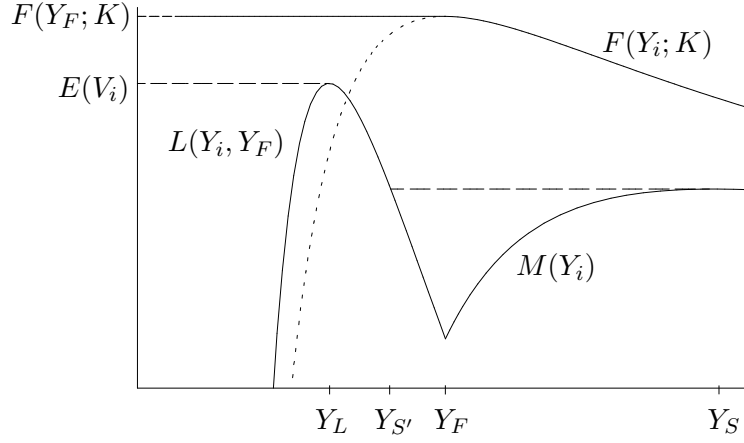


Figure 2: Attrition regime, $K \in [\tilde{K}, \hat{K})$. The leader payoff has two local maxima (Y_L, Y_S), innovator entry thresholds are distributed over $[Y_L, Y_{S'}] \cup [Y_S, \infty)$, and imitator entry occurs either at Y_F (if $\min\{\tilde{Y}_i, \tilde{Y}_j\} \in [Y_L, Y_{S'}]$) or immediately otherwise (if $\min\{\tilde{Y}_i, \tilde{Y}_j\} \in [Y_S, \infty)$).

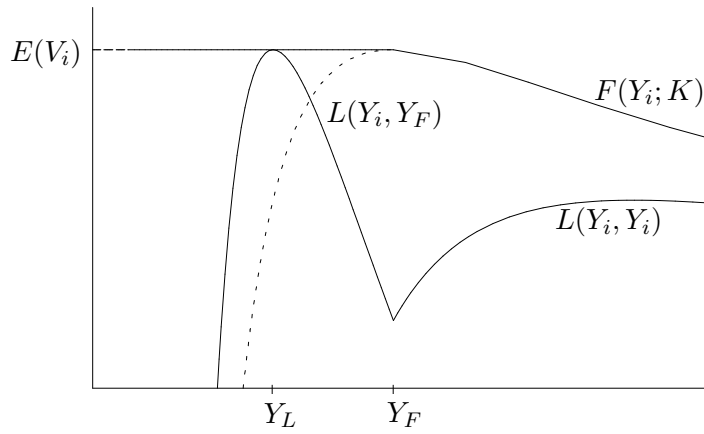


Figure 3: Critical case, $K = \hat{K}$. The innovator and imitator enter at Y_L and Y_F respectively.

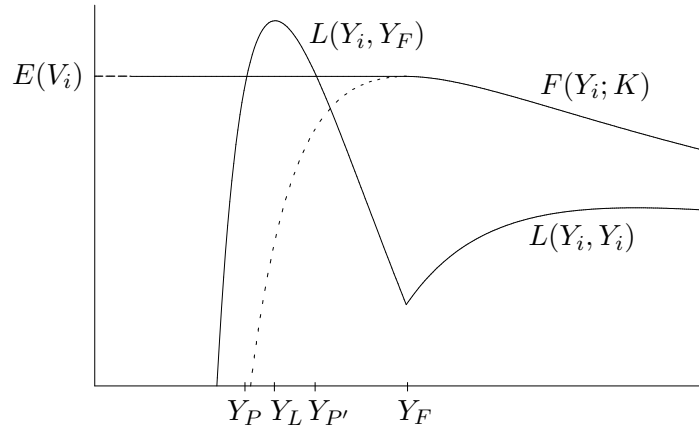


Figure 4: Preemption regime, $K \in (\widehat{K}, I)$. The innovator enters at Y_P and the imitator at Y_F . There is war of attrition off the equilibrium path (over $(Y_{P'}, \infty)$).

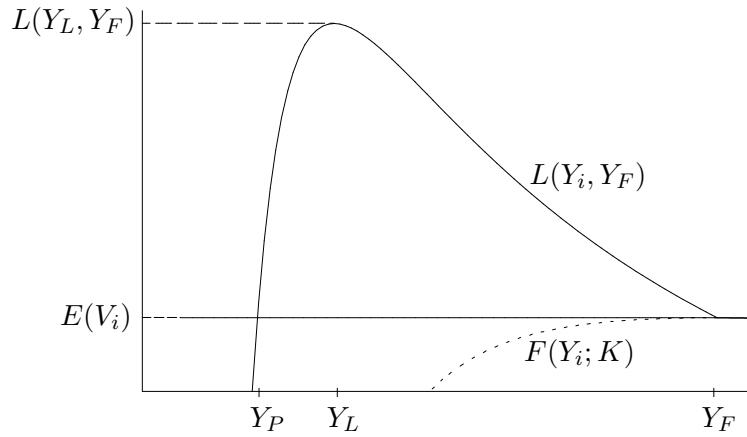


Figure 5: Preemption regime, $K \in [I, \infty)$. The innovator enters at Y_P and the imitator at Y_F . Note that the dotted curve represents $F(Y_i; K)$ whereas the corresponding solid curve is the concentrated follower payoff $F(Y_F^*; K)$.