SHOULD THE UTILITY FUNCTION BE DITCHED?

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ABSTRACT

This note takes a retrospective look at the Utility Construct still in use in economic science and compares it to a new approach based on recent findings in neuroscience. The results show that it is more natural and more compelling to go from the preference order to the price vector. Thus making the non-falsifiable utility apparatus superfluous.

I- INTRODUCTION

A C^2 function is one whose second derivatives in its arguments exist and are continuous. According to a group of economist-pioneers, a throw-off of some individual's C^2 utility function is a well-defined demand function, which is differentiable and homogeneous of degree zero in budget and prices, a corner stone of economic theory. The utility function itself is supposedly derived from a well-ordered individual preference relation, thus making the demand function dependent on two subjective concepts; namely, a C^2 utility function and a well-ordered preference relation. The economizing problem is essentially one of optimization, but the early pioneers were not sure as to how to proceed to optimization from a mere ranking, whether well-ordered or not. In addition, it was initially and erroneously believed that utility maximization would yield richness and robustness; hence the foray into utility, as preference was debased to a primitive concept.

However, the path from a well-ordered preference relation to a C^2 function is fraught with some dangerous anomalies. Initially, it was thought that a strictly convex and continuous preference relation suffices to yield a strictly quasi-concave utility function. Subsequently, however, it was realized that these properties might not have been strong enough to yield an everywhere differentiable demand function. Additional conditions would be required at the levels of both preference and utility. That realization spurred the vast research programme that gives substance to today's economists' discourse on utility. In retrospect, it is easily seen that economists had expanded a considerable amount of energy and time to figure out all conditions they thought were necessary to arrive at well-specified demand functions. The relevant and voluminous literature attests to that. See for example, Pareto (1896), Wold (1943, 1944), Samuelson (1947), Debreu (1959, 1960, 1964, 1976), Radner (1963), Sonnenschein (1965, 1971), Smeidler (1971), Koopmans (1972), Hildenbrand (1974), Shafer (1974), just to name a few. However, despite that effort, the preference order-utility construct remains to this day hazy and problematic. Worst still, its predictions are regularly falsified whether in simple observation, personal experience, or in experimental designs.

At first sight, one would think that economists should be uneasy when faced with such incongruities, but one should also bear in mind that when these concepts were being developed, they had no means of gazing into the workings of the human brain. This is no longer so, however. With more mature reflections and new tools[(1)] (developed by physicists), neuroscientists are now able to examine the brain's architecture in a non invasive manner. Their findings are compelling enough to encourage economists to revisit their construct; and this note is a first step in that direction.

I will first briefly review the existing construct mainly to see why the ensuing problems continue to defy satisfactory solutions. Next, I will examine an alternative approach based on the findings of neuroscience, and compare the two sets of results. As the results of the alternative approach are very compelling, hopefully, they might sway economists to finally view the utility function as an unnecessary appendage.

II- CONDITIONS FOR A WELL-ORDERED PREFERENCE RELATION

Until the late 1920s, the notion of preference relation was loosely equated with utility. At the same time though, many were still confusing preference with taste, which was thought to be fixed. Around the 1940s, however, economists began to differentiate the two. While recognizing preference as more basic, utility began to appear as a more convenient device to describe consumer behavior; although nobody knew how preferences were formed, and to this day it seems that whatever is said about preferences still leaves the associated utility function incompletely specified. To avoid pathologies, therefore, economists began to 'finick' both concepts so as to meet the requirements of maximizing behavior. At the same time, developments in set theory provide them with a language and rules to undertake the task with thoroughness. For tractability, I show in Table 1 a list of axioms and conditions that the preference relation is supposed to satisfy and to which I will refer throughout the discussion. To arrive at a C^2 utility function, still more are needed, but I will only mention them only in passing, because they will not be necessary for the present purpose. In fact, I believe that that additional panoply of conditions is outright superfluous. For in this case, mathematical formalism, though helpful in description and analysis, can not dictate consumer behavior. Further, to describe, mathematically or otherwise, one must first understand, although but in the present case, it seems that formalism had taken precedence over concreteness.

Imaging a typical consumer, denoted by i, facing consumption set X, which a collection of consumption bundles as its elements. In general, X is the non negative orthant in R^k_+ (k =1, 2, …,n), where R is the reals. For ease of exposition, however, I posit X = {x, y, z}, where x, y, and z are thing-like entities over which consumer i expresses a value judgement at a particular instant of time. A binary relation P on X is a subset of the Cartesian product on X with itself. Also P stands for a weak preference. Thus writing (xPy) means that x is preferred to y, because consumer i considers x 'as good as' y. The binary relation becomes an equivalence relation, I, under certain conditions, in the sense that writing (xIy) means that i is 'indifferent' between x and y; in other words, (xIy) if and only if (xPy) and (yPx), implying the symmetry of I.

Still later on, economists realized that a weak preference relation could not ensure continuity in P, hence, nor in u(.), the utility function, because the upper and lower contour sets of P are 'closed' relative to X. They then posited P' as a strict preference relation (a stronger assumption). Thus (xP'y) means that x is 'strictly preferred to' y, because i believes that x is 'as good as' y, but not conversely. The supposed advantage here is that under P', the upper and lower contour sets are 'open' relative to X.

Referring to Table 1, a P' that meets A1-A4 and C1 is said to be linear or complete. The equivalence relation satisfying B1-B3 follows. Monotonicity (C2) too follows naturally for most goods. Thus, the difference between P and P' is reduced to the following: P satisfies A3, A4 and B1, while the indifference relation given by,

$$\mathbf{I(x) = \{y \in X \mid (yP'x)\} \cap \{y \in X \mid (xP'y)\},}$$

TABLE 1: Axioms and conditions* for a Well-Ordered Preference Relation

A- The binary Relation P'

∀(x, y, z) ε X:

1- *Irreflexivity* ………………………………………. ¬(xP'x)
2- *Asymmetry* ………………………………….. (xP'y) → ¬ (yP'x)
3- *Connexity* …………………………………….(xP'y) → (xP'z) ∨ (zP′y)
4- *Transitivity* ………………………………….. (xP'y) ∧ (yP'z) → (xP'z) .

B- The Equivalence Relation I

1- *Reflexivity* ……………………… ……………….. (xP'x), (yP'y) ∧ (zP'z)
2- *Symmetry* …………………… ……………….…. (xP'y) → (yP'x)
3- *Transitivity* ……………………….. ……………….. as in A-4

C- Additional Conditions

1- *Continuity* ……….. ……. If (xP'y) and there exists a sequence z^t such that lim z^t →x* close to x → (zP'y) → open upper ∧ lower contour sets relative to X.
t →n
2- *Monotonicity* ……………………………… ∀(x, y, z) ε X, more is preferred to less.

i., e., the intersection of the contour sets are closed under P and open under P'. It can immediately be seen why P' does not admit B1, for, x, say, cannot be strictly preferred to itself. However, this difference is a consequence of C1 that ensures continuity and which at once seems to introduce surreptitiously time in the analysis. In words, C1 says that if (xP'y), say, and if there exits a sequence z^t that converges to some x* close to the bundle x, then (zP'y). Then, it is not farfetched to assume that the consumer can order X in time, as the sequence converges in finite time. If z_1, z_2, z_3, … are subsequent selections, then there is a time dimension, which somehow had gotten lost in the analysis; I will later return the question of time and to other questions relative to the empirical content of A4. For the time being, let us suppose that if P' satisfies A1- A4 and C1, the relation I will satisfy B1-B3. Then P' is said to be *linear or complete*. We may also call it an 'empirical relational system', represented by: ⟨X, P', I⟩. To justify that claim and to say more about it, Theorem 1 may be invoked:

Theorem 1 If $\langle X, P', I\rangle$ is linear or complete, then it has a countable base and it can be represented by an ordinal one-dimensional scale.

Next, Zorn's Lemma (1935) can be also be invoked to justify the following consequential statements (CT):

CT-1 An ordered set, X, is said to be *well-ordered* if X and each of its non-empty subsets have first elements in the prescribed ordering of X.

CT-2 Every well-ordered set X is *totally ordered*.

CT-3 The converse of *CT-2* holds if X is finite.

This result is upheld as significant and it authorizes its authors to claim that if $\langle X, P', I\rangle$ is well-ordered, then there exists a continuous utility function u: $R^k_+ \rightarrow R_+$. Furthermore, if u and u' are both maps of the empirical relational system $\langle X, P', I\rangle$ into a real number R, then u' is a monotone transformation of u.

At this juncture, it might be of interest to stress once more that P' is selected over P, because the latter, although more basic, was thought to be unable to meet all the associated topological conditions to generate a well-defined u. Can P' do so? Not quite. The consumption sets of the poor are difficult to well-order due to product differentiation (i., e., lack of information). The consumption sets of non-poor (most consumers) consist of necessities and discretionary goods. That is, those two types of goods are not really comparable. In such a case, the continuity assumption (C1) is violated. Only a lexicographic order is possible, as trade-offs between necessities and discretionary goods do not exist. Indeed, the very definition of the binary relation is not satisfied. The monotonicity assumption is satisfied in most cases; that is, when the entities are thing-likes, but not generally as in the case of leisure, for example, where less may be preferred to more. More telling though is that consumers continue to make intransitive choices, and sometime they are observed even acting in a manner that seems contrary to the dictates of utility maximization. Following that another debate ensued. Sonnenschein (1965) argued that continuity (C1) and connexity (A3) suffice to ensure transitivity (A4). Shafer (1971) followed, arguing that continuity and transitivity just imply connexity. But again, many known relations simply do not display continuity as in the case of lexicographic ordering. Faced with this dilemma, economists became overtly finicky with the derived u, adding conditions such as convexity, upper-semi-continuity, indirectness, separability, Cobb-Douglas-ness, etc. in an attempt to bloc eventual anomalies. As a result, instead of the ordinal scale allowed by Theorem 1, they have inadvertently produced a theoretically safe construct, also one that has been proven not to have any empirical content.

III- AN ALTERNATIVE

I have given the reader a succinct account of why these early pioneers were so eager to construct a C^2 utility function. One can easily understand that, knowing preciously little about how preference was formed, they had to rely on Boolean reasoning and operations to grapple with the slipperiness of such a concept. More recently, however, new tools have been made available. Now neuroscientists can readily observe brains' workings easily and non-intrusively[(2)]. As a result, the brain comes to be viewed as essentially a dynamic input/output structure. Mathematically and succinctly put, there exist a set of external input stimuli, satellite structures (such as the visual cortex), two state spaces, (L) and (χ), and a number of maps. L is the limbic system or the brain stem, χ is the frontal cortex, some of the maps are 'onto' while others are 'one-to-one'. Moreover, L is the site of memory (factual and emotive experience), and χ is where computation, deduction and overall evaluation take place.

To clarify, consider an example of how external stimuli are mapped onto L, and χ. Seeing a bundle x, say, means that light patterns are received in the photoreceptor cells in the retina, where they are immediately converted into neuronal signals, and the neurotransmitter acetylcholine is released throughout the prefrontal cortex and to all its satellite structures (Crick and Koch, 2003; Koch, 2007). This gives rise to an emergent property, e, which focuses and directs attention to a task, implying also that the seer is in the state of wakefulness (Greenfield, 2007). Next, neuronal signals, made out of specialized neurons, traveled through the optic nerve to the lateral geniculate nucleus in the thalamus, and from there to the visual cortex (where light patterns are identified) via the neural cords (called optic radiation, leading to the occipital lobe) and on to the hippocampus (Martinez-Conde, 2006; Martinez-Conde and Macknik,2007). The later harbors what neuroscientists call 'neuronal cliques', which are specialized neurons invariantly organized hierarchically from the general to the specific (Tsien, 2007). Different cliques or circuits respond to perceptual, factual and emotional aspects of the inputs. Beside, the hippocampus receives information from photoreceptors, from the geniculate nucleus, from the vestibular system, from the locus coerulus, from the amygdala, etc. and from other neurons encoded with general and abstract information to be used in future situations. Thus, the identification of x fed to χ by L and other satellite structures for a final evaluation is a mixture of factual and emotional assessments of the image of x or something likes it. This complex flows may be conveniently divided into two categories, namely objective information, I_0, and the emotive mixture, e_L. Therefore, state space χ may be represented by a dynamical system $\langle I_0, e_L, e\rangle$ that brings these two flows of neurotransmitters into an equilibrium; that equilibrium is of course, an informational equilibrium, but it is similar to a thermodynamic one.

In all of this, the role of the emergent factor, e ε [-1 1] can not be overemphasized. It exists only in the state of wakefulness (compared to the state of dormancy); it is the maestro in charge of the traffic of neurotransmitters from various parts of the brain; it may be associated with judgement and its intensity varies with the intensity of the input stimulus (Greenfield, 2007). If the informational assessment realized in state space χ is a 'scalar field', then e gives the field a direction.

I have shown elsewhere (Dominique, 2006), that the assessment of the state space χ is none other than a "potential" that must be minimized using a differential map. It suffices for the present purpose to say that the equilibrium value, P, is some sort of a linear combination of I_0 and e_L. The stronger are the intensities of these flows the higher is P. With an appropriate additional transformation, I was able to identify the topology of the brain as a bi-modal cusp catastrophe given by:

(i) $\Psi(\langle I_0, e_L, e\rangle)\ \varepsilon\ C^{\infty}(R^{v+c}, R),$

where $v \leq 2$ is the corank and c is the codim of Ψ. To put it differently, the two ordinal scales in χ are mapped onto two equilibrium manifolds, M_+ and M_-. M_+ maps equilibrium points onto R_+ and, analogously, M_- maps equilibria onto R_-.

In the present case, appealing to the Thom's (1975) Classification Theorem proved very useful in identifying and analyzing the nature of $\mathscr{P}$ in equation (i) above. Since neuroscientists affirm that signals travel at speed up to 525 feet a second, I then ignored slow dynamics to concentrate on points mapped onto R_+ under very fast dynamics. When they are projected onto R_+, such points are not equidistant near the maximal element (the highest point in the direction in which preference runs) as the equilibrium manifold is a curved surface. Hence, preference indices near but inferior to the maximal element are more compact, while those near but superior to the minimal element are more unclenched. This means that the *continuity property of* P *is not guaranteed*. Finally, it can immediately be seen that the manner in which all additional or new information are appraised in χ may shift an equilibrium point on M_+ or may even cause it to jump onto M_- ; implying that it is possible to go from preference to dis-preference in a natural way.

If equilibria are given by P, and the well-ordered preference is denoted R_{xyz}, solving the economizing problem implies applying the maps Đ, h_1, h_2 and M. That is, Đ: $\langle I_0, e_L, e\rangle \rightarrow \mathcal{P}$, h_2: $\mathcal{P} \rightarrow R_{xyz}$, such that:

(ii) $$M = R_{xyz} \rightarrow P,$$

where M, is a composite map or a market (simplified to a pure exchange) that takes us to the price vector P directly from the ordering.

The way M is constructed is as follows: First, it ignores points on M_-, as goods eliciting such equilibria are not in the consumption set X. Suppose consumers are limited to 3 (i =1,2, 3), then P_x , P_y , P_z, mapped onto R_+ yield the ordering $(xP'y) \wedge (yP'z)$, call it $\mathfrak{R}^i$. In equilibrium, that ordering is preserved when it is mapped onto market shares, $\propto$, and onto total equilibrium expenditures ($\propto$B). Since in equilibrium, expenditures are equal to market values (V), then, we have:

$$\mathfrak{R}^i_{xyz} \rightarrow V_x > V_y > V_z = (\propto^i_x B^i) > (\propto^i_y B^i) > (\propto^i_z B^i). \text{ If } V_x = (\propto^i_x B^i) \rightarrow \Delta Px / \Delta x < 0, \forall (x,y,z) \, \varepsilon \, X.$$

If now $\mathfrak{R}^i_{xyz}$ is mapped onto the budget share set $\{\propto^i\}$ such that $\Sigma_i \propto_{xyz} = 1$, and if supply is fixed, then in equilibrium:

$$\Sigma_i \propto_{xi} > \Sigma_i \propto_{yi} > \Sigma_i \propto_{zi} \rightarrow P_x > P_y > P_z .$$

Thus, the utility construct is unnecessary, and there is no explicit effort on the part of the consumer to maximize anything.

IV-FINAL REMARKS

I have tried to show that Theorem 1 allows for a simple ordinal scale:

(iii) $u(\langle X, P', I\rangle): R^{k} \rightarrow R_{+}$,

as equation (iii) does not completely specify u(.). But to satisfy the imperative of utility maximization, economists construct:

(iv) $u(\langle X, P', I\rangle)\ \varepsilon\ C^{2}(R^{k}, R_{+})$;

this produces uncomfortable anomalies. By integrating the findings of neuroscience, we arrive at equation (ii) above, which is more concrete and presents a number of advantages over the conventional one. To wit:

Firstly, it dispenses with the need of continuous ordinal scales in conformity with nature itself, which is not continuous. Secondly, it is robust to a violation of the monotonicity condition, because it accounts for the context; thing-like objects are special cases. Thirdly, it distinguishes between transitivity and ambivalence. In this alternative, transitivity occurs when two imperfect substitute bundles receive the same ranking; the information set of the consumer is considered (by him or her) complete and, therefore, there is no reason for the consumer to appear inconsistent. Ambivalence, on the other hand, may occur when the substitutes have different attractive characteristics, but the information set is incomplete; in that case, the consumer can not decide on the ranking. Fourthly, it gives substance to what Simon (1985) has termed ‘objective’ and ‘subjective’ rationalities; here, they appear as a linear combination. Fifthly, it resolves the dilemma of the so-called called Ultimatum Game[(3)]. That is, the proportion offered to player B might be deemed unfair and, therefore, B turns it down to punish greed; in such a case, the flow e_L dominates I_0. Sixthly, it resolves a phenomenon observed by Tversky and Kaneman[(4)] (1981). That is, by letting x stands for money, and y for theater ticket, we have: (yP’x) in state 1, and (xP’y) in state 2; this is explained by what philosophers call ‘change in preference horizon’ or ‘different states of nature’ by economists. Seventhly, the conventional presentation can not explain changes over time in the utility function; here the time element is ever present. Eighthly, the conventional approach can not accommodate the impacts of emotions on choices, as alluded to by theorists such as John Elster, Herbert Simon, John M. Keynes, etc... Frank Knight, for example, believed that "human activity is largely impulsive response to stimuli and suggestions."

Finally, it should be observed that an equilibrium point mapped onto R_+ is a valuation, V. The higher is the position of x in the ranking, the higher is the V of x, commanding a corresponding proportion $\propto$ of consumer i's budget B. Hence, if both quantity and price could be made perfectly divisible, individual consumer demand could be a rectangular hyperbola. Indeed, in Dominique (1999, ch.3), it is shown how the invertible map M is constructed for a simple 3 goods-3 consumer economy to minimize a potential market gain, and how it easily maps $\Re$ onto P (the equilibrium price vector of a pure exchange model), but without the hypothesis of utility maximization. And variations in the price vector are perfectly correlated with changes in the ranking order.

NOTES

1 For example, Computed tomography scans (CT), Positron Emission tomography scans (PET), Magnetic Resonance Imaging (MRI), Functional Magnetic Imaging (fMRI), Magneto-encephalography (MEG), Trans-cranial Stimulation (TMS), etc.

2 Many of their findings are discussed in the popular press. See, for example, *The New York Times*, June 17, 2006, and November 22, 2006, *US& World Report*, Oct. 18, 2006, and *Scientific American*, July, August and October 2007.

3 This experiment involves two players, A and B. A, say, is given a certain sum, s, and A must give a proportion k of his or her choosing to B. If B accepts k s, then B may keep (1-k) s. However, if A refuses k s, as she deems k too low, both players end up with nothing. The experiment shows that if k is less than 20 percent of s, the offer is rejected.

4 Tversky and Kaneman have observed that 88 percent of subjects in their sample say if they arrived at a theater to buy a $ 10 ticket for a play and then discovered that they had lost $10 on the way, they would buy the ticket anyway. But if the ticket was bought in advance and discovered when they arrived in the theater that they had lost the ticket, 54 percent say they would not buy another ticket. In the conventional approach, the 54 percent would be labeled 'irrational'.

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