Market versus Non-Market Assignment of Initial Ownership

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ABSTRACT: We study the assignment of initial ownership of a good when agents differ in their ability to pay. Selling the good at the market-clearing price favors the wealthy in the sense that they may acquire the good instead of poor buyers who value it more highly. Non-market assignment schemes, even simple random rationing, may yield a more efficient allocation than the competitive market would — if recipients of the good are allowed to resell. Schemes that favor the poor are even more desirable in that context. The ability to resell the good is critical to the results, but allowing resale also invites speculation, which undermines its effectiveness. If the level of speculation is sufficiently high, restricting resale may be beneficial.

KEYWORDS: efficiency, non-market assignment, merit-based assignment rules, need-based assignment rules, resale, speculation.

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When ye are passed over Jordan into the land of Canaan ... ye shall divide the land by lot for an inheritance among your families ... every man's inheritance shall be in the place where his lot falleth ... (Num. 33:51-54)

1 Introduction

Suppose that a government wishes to distribute a scarce good such as public land or radio spectrum. A natural way to assign ownership is to sell the good at the market-clearing price. Yet, the market is just one of many possible ways to assign ownership. Non-market assignment methods such as first-come-first-served or a lottery have a long history, as the passage above indicates, and their use remains widespread. Land was assigned on a first-come-first-served basis (i.e., by a race) during the 1889 *Oklahoma Land Rush*, and by lottery in 1901. Human organs are assigned to transplant patients by a priority rule that depends on factors including the recipient's age, the severity of the condition, and the distance between the donor and recipient. In Korea and Singapore, substantial numbers of new housing units are subject to price caps that are well below market prices, and the units are assigned by lottery.[1] In addition, lotteries have been used to assign immigration visas and jury duty, and to select conscripts for military service. School enrollment is assigned by non-price factors such as the location of a student's residence or test scores.

While the virtues of markets are well understood, the case for using markets to assign *initial ownership* of goods is less clear. After all, the Coase theorem is neutral about the use of markets in this regard: As long as frictionless trade is possible subsequently, it does not matter how initial ownership is assigned.[2] In fact, the market may not produce an efficient allocation when potential buyers have limited assets or face liquidity constraints. If a spectrum license is sold at a market-clearing price, it may go to a well-capitalized buyer instead of one who can generate higher value from the license but happens to be under-

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[1] In Singapore, most citizens live in units sold by the government at prices that are well below the market. Some 86 per cent of Singapore's citizens live in such units and 92 per cent of those residents own their units. (See "Building Homes, Shaping Communities," at http://www.mnd.gov.sg/, accessed on May 28, 2005.) The price cap is as low as half of the price on the resale market (Tu and Wong, 2002). The same multiple is given by Green, Malpezzi and Vandell (1994) for Korea. See also Kim (2002).

[2] The Coase Theorem is invoked frequently when new assignment schemes are proposed. A prominent example concerns the Federal Communications Commission (FCC) spectrum license auctions. Opponents of the FCC's favored design argued that the design would not affect the ultimate allocation, so revenue should be the only criterion. See the discussion in Milgrom (2004), which also contains several counterpoints.

capitalized or illiquid.[3] This concern is pertinent whenever the good being assigned has a large value relative to buyers' (liquid) assets. In such circumstances, the initial assignment of the good will affect the ultimate allocation.[4] We therefore ask how non-market methods of assigning initial ownership compare with the market.

Specifically, we develop a model in which the supply of an object is assigned to a mass of agents. The object could be a productive asset such as a license to operate a business, to exploit resources, or to export goods, in which case the willingness to pay reflects the monetary payoff that the asset will generate for the recipient. Alternatively, the object could be a consumption good such as housing or health care, in which case the willingness to pay reflects the utility from consuming the good. We then introduce binding wealth constraints, which capture liquidity constraints, capital market imperfections or consumers' income effects.[5] Our analysis of this model yields several results.

☐ EFFICIENCY OF NON-MARKET ASSIGNMENT:

Consistent with the discussion above, assigning initial ownership through a competitive market results in allocative *inefficiency*. A simple non-market scheme that assigns initial ownership randomly at a below-market price yields a more efficient allocation than the competitive market *if and only if resale is allowed*. Random assignment provides the good to some low-wealth buyers and low-valuation buyers who would not get it in the market. If resale is not permitted, the resulting allocation is less efficient than the market. If resale is permitted, low-valuation recipients will resell while those with low wealth and high valuations will not. As a consequence, more high-valuation buyers will get the good ultimately than in the competitive market, so efficiency is now greater.

☐ DESIRABILITY OF NEED-BASED ASSIGNMENT:

The benefits of non-market assignment depend in part on the information possessed by the government. If it has information about agents' preferences or wealth, the government can do better than simple random assignment. For housing or health care, an individual's

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[3]Salant (1997) describes the impact of binding liquidity constraints in FCC spectrum license auctions. The author, who participated in the bidding as a member of the GTE team, noted the importance of liquidity constraints: "We were very concerned about how budget constraints could affect bidding. ... In the [Major Trading Area] auction, budget constraints appeared to limit bids."

[4]In other words, the strong version of the Coase theorem will not hold in the sense that different assignments of initial ownership will result in different final allocations. The resulting allocation will still be Pareto efficient, so the weak version of the Coase theorem will continue to hold.

[5]The main results here hold with the weaker condition that the shadow value of wealth differs across buyers. Specifically, we show in Subsection 5.3 that the main results hold without quasi-linear utility.

valuation may be gleaned from her existing living arrangements or a doctor's assessment of her medical condition. Likewise, one's wealth may be inferred from earnings and asset holdings. While *merit-based* assignment (i.e., favoring those with the highest apparent valuation) is justifiable on efficiency grounds, it is not immediately clear that the same holds for *need-based* assignment (i.e., favoring the poor). In fact, the argument above supporting random assignment means that need-based schemes yield an even more efficient allocation, presuming resale is allowed, since more of the good is assigned initially to low-wealth-high-valuation individuals.

□ SPECULATION AND DESIRABILITY OF RESTRICTING RESALE:

Allowing resale enables agents to alleviate the misallocation resulting from non-market schemes. In fact, resale is crucial for such schemes to be beneficial. At the same time, unrestricted resale invites speculation, sometimes on a massive scale. For example, after the FCC decided to use a lottery to assign cellular telephone licenses, it received nearly 400,000 applications.[6] The volume of applications caused shelves to break at the FCC's processing center (Kwerel and Williams, 1993). The use of lotteries to assign housing at below-market prices in Korea engendered so much speculation that it has been blamed for volatility in housing prices.[7]

Speculation undermines non-market schemes by reducing the probability of assignment for those with low wealth but high valuations. Specifically, the benefit of non-market assignment vanishes as the number of would-be speculators grows without bound. We show that a broad class of non-market schemes yields the competitive market allocation in the limit. Although this result resembles the Coase theorem, the predicted allocation is inefficient, which points to a very different normative conclusion: Restricting or totally prohibiting resale may be socially desirable if the assignment technology is sufficiently effective at targeting high-valuation individuals.

Our results have broad applicability to the assignment of public resources and entitlements such as rights to exploit resources including minerals, forests, fish and wildlife; as well as immigration visas and exemptions from civic duty such as military service or jury duty. They apply to the assignment of private resources as well, presuming that the government can regulate the private market. (The housing markets in Korea and Singapore provide cases in point.) They also apply to government-led industrialization processes in many developing economies. For example, the Korean industrialization process was marked

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[6]The application fee was zero initially, and only $230 in 1993.
[7]See the references in Malpezzi and Wachter (2005), for example.

by licensing policies that targeted industries and firms for export quotas, trade protection and other privileges (Amsden, 1989). During a period dubbed the "licence raj," the Indian government controlled large areas of economic activity through the awarding of rights and "permissions" (Esteban and Ray, 2006).

Several specific conclusions can be drawn from the results. First, despite the widespread use of non-market assignment methods, their efficiency properties have not been well appreciated; we show that non-market schemes have efficiency benefits. Market-based schemes such as auctions are employed increasingly to assign government resources, replacing comparative hearings and lotteries. While a market-based approach is justified in many cases, it may warrant some modification in others, especially when liquidity constraints or wealth effects are important. In the same vein, intervention *in a competitive market* might be justified in some cases. Note that these observations do not detract from the fundamental value of markets: Well-functioning (resale) markets are crucial for non-market schemes to succeed.

Second, our findings provide an efficiency basis for need-based schemes that favor the poor. Such schemes are common in college admissions, license auctions, and subsidized housing programs. While these programs are often motivated by redistributive goals, our results suggest that they may be desirable from an efficiency standpoint as well. Note, however, that transferability of benefits, which is often prohibited by such programs, is necessary for the desirable efficiency performance.

Third, the previous point about transferability has implications for the design of non-market assignment schemes. Allowing transferability may undermine certain redistributive goals since, *ex ante*, it will invite speculators whose participation reduces the benefits that accrue to the target group. But, many programs are likely to experience an increase in efficiency if they allow transferability.

Last, our theory suggests that it may sometimes be desirable to restrict resale. The sale of human organs is outlawed in most countries, just as one cannot transfer the rights to enroll in a university or to immigrate. Restrictions on transferability are typically justified by paternalistic arguments (e.g., those who wish to sell an organ might not make rational decisions) or concerns about fairness (e.g., only the wealthiest patients will get a transplant). Our theory offers an efficiency rationale for such restrictions.

The remainder of this paper is organized as follows. Section 2 lays out the model, and it describes the efficient allocation and the competitive market allocation. Section 3 characterizes the outcome when the good is subject to a price cap and the available supply

is assigned randomly. Section 4 analyzes general assignment schemes and derives conditions under which they outperform the market. We discuss various generalizations in Section 5. The related literature is discussed in Section 6, with concluding remarks in Section 7.

2   The Model

2.1   Primitives

A good is available in fixed supply, $S \in (0, 1)$. The good is indivisible, and it is supplied at a constant marginal cost of c, up to S. The good may be owned by the government or supplied by private firms. It could a productive asset, a license to exploit resources, or a consumption good.

There is a mass $1 + m$ of buyers who each consume either zero or one unit of this good. They also consume a divisible numeraire called "money." Each buyer has two attributes: her endowment of money or *wealth*, w, and her *valuation* of the good, v; we refer to (w, v) as the buyer's *type*. If the good is a productive asset, v represents the profit that a firm can generate from the asset. If it is a consumption good, v represents a buyer's gross consumer surplus.

The attributes w and v are distributed independently over $[0, 1]^2$ and there is a non-zero density for almost every (w, v) in the support. Wealth is distributed according to the cumulative distribution function (cdf) $G(w)$. A unit mass of buyers have valuations $v \in [0, 1]$ distributed according to the cdf $F(v)$. The remaining m have a valuation of zero; these buyers have no real demand for the good but may still participate for speculative reasons. We therefore refer to them as *pure speculators*. The independence of w and v helps to isolate the role that each attribute plays and the effect of policy treatments based on each; most of our results are robust to the introduction of correlation.

Buyers are risk-neutral, with quasilinear utility. In particular, a type-(w, v) buyer gets utility $vx + w - p$ if she "consumes" the good with probability $x \in [0, 1]$ and pays $p \leq w$. Buyers cannot spend more than their wealth, however. A buyer with $w < v$ is *wealth-constrained* in the sense that she is unable to pay as much as she is willing to pay. The wealth constraint is a device for introducing "income effects," which are the fundamental source of our results. By contrast, the literature on mechanism design and auctions typically assumes quasilinear preferences without wealth constraints, so income effects never arise. Yet, such effects are often important in economic development and social programs, and in

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many auctions, for instance.

The precise way in which income effects are modelled is not essential. As will be seen in Subsection 5.3, our main result holds in a more general setup in which buyers simply have different shadow values of wealth. Nonetheless, our simple way of introducing income effects — combining wealth constraints with quasilinear preferences — has an analytical advantage. The effect arises only when the wealth constraint binds here, so Utilitarian efficiency can be measured by the total value of the good consumed (less costs) as in the standard framework without income effects. This feature makes comparisons with the latter transparent, thereby isolating the role that income effects play. Consequently, we take as our welfare measure the sum of the valuations of those who consume the good, less the cost of production.[8] Since supply is inelastic, we will focus on *total value*, which is the sum of the valuations for those who consume.

The *efficient allocation* maximizes total value by providing the good to the S buyers with the highest valuations. Let $v^* > 0$ denote the critical valuation such that $1 - F(v^*) = S$. If all S buyers with valuations of v^* and above acquire the good, the allocation is efficient. The corresponding total value is

$$V^* := \int_0^1 \int_{v^*}^1 v dF(v) dG(w) = \int_{v^*}^1 v dF(v) = S\phi(v^*),$$

where

$$\phi(z) := \frac{\int_z^1 v dF(v)}{1 - F(z)}$$

is the expectation of a buyer's valuation, conditional on exceeding z.

2.2 Assignment Schemes

Throughout the paper we will compare the performance of three alternatives: (1) *a competitive market*, (2) *a non-market assignment scheme without transferability*, and (3) *a non-market assignment scheme with transferability*. A competitive market arises naturally when there are private firms and the government adopts a laissez-faire policy. If the good is

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[8]This Utilitarian criterion is widely used, and it is appealing since agents who do not yet know their types would seek to maximize aggregate utility. Suppose that consumers and producers are drawn from a large pool, with each individual equally likely to be selected for the consumer pool as the producer pool. Then, a second drawing selects active participants and their types. If the individuals were to select an assignment mechanism prior to realizing their types, they would vote for the one with the higher total value. Analogous justifications have been given by Vickrey (1945) and Harsanyi (1953, 1955).

supplied by the government, the outcome of a competitive market can be replicated by the government's acting as a Walrasian auctioneer or employing a multiunit auction.[9] A non-market assignment scheme can be implemented in a similar fashion in the two scenarios. That is, the government either charges a price below the market equilibrium and assigns the good according to its chosen priority rule, or it mandates that private firms follow the same assignment rule. The recipients of the good are allowed to resell it in a resale market in (3), but not in (2).

The three regimes are all observed in the housing market, for example, and they are employed or could be employed in various other settings:

- **Education:** Suppose that public school students are assigned to two schools based solely on place of residence. The number of students who actually *prefer* school A exceeds its capacity. The valuation now represents the premium that an individual is willing to pay for the right to attend A. Since the nominal price of attending the school is zero, the preference for A will be capitalized in housing prices. This corresponds to the market regime. Now suppose that slots in school A are awarded by lottery. This is a non-market assignment scheme without transferability. The final regime arises if a lottery awards *transferable* vouchers that confer the right to attend A.

- **Fugitive Property, Entitlements, and Government Resources:** Fugitive property — a good or resource whose ownership is not yet established — can be assigned to the individual who claims it first (*the rule of first possession*) or to the individual who owns property tied to it (*tied ownership*).[10] These methods correspond to assignment with transferability. The 1889 Oklahoma Land Rush and the 1901 Oklahoma Land Opening are examples of assignment with transferability. In 1906, government land in Oklahoma was sold by auction, which corresponds to a competitive market. Emissions permits are typically assigned based on historical emissions data and they are often transferable. A notable example is the Kyoto Protocol, which assigned emissions credits based partly on countries' levels of economic development, and allowed credits to be traded.[11]

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[9]The competitive market outcome can also be replicated when there is lobbying in which individuals offer bids to a government official, who makes inferences about their merits (as signaled by the bids) and then assigns the good. Esteban and Ray (2005) shows that a (refined) signaling equilibrium will generate an allocation that corresponds to the competitive market allocation we identify later.

[10]For instance, a landowner has the subsurface right to natural gas deposits underneath the land.

[11]See http://unfccc.int/resource/docs/convkp/kpeng.pdf, accessed October 30, 2006.

- **Health Care:** Health care is often provided via a non-market assignment scheme without transferability. A specific example involves organ transplants, where patients are placed in a queue and cannot switch places.[12] One could also employ a general assignment scheme with transferability in which patients waiting for a transplant may sell their places in the queue. Finally, a competitive market would assign organs to those willing and able to pay the market price.

- **Military Recruitment:** An all-volunteer army corresponds to the competitive market. A draft lottery is effectively a non-market assignment scheme without transferability. A draft with tradable deferments represents an assignment scheme with transferability. An example of this practice arose during the U.S. Civil War: conscripts could avoid service in the Union Army by paying non-draftees to take their places.

In the remainder of this section we focus on the competitive market.

2.3 A Competitive Market

A competitive market operates according to the standard textbook description. At any price, demand and supply are formed, and the price adjusts to clear the market. There is no supply at any price $p < c$, so the equilibrium must have $p \geq c$. At any price $p \geq c$, the entire supply, S, is available. On the demand side, the measure of buyers willing and able to pay p is

$$D(p) := [1 - G(p)][1 - F(p)].$$

We assume the good to be scarce in the sense of $D(c) > S$, so not every buyer who demands the good at the marginal cost can be accommodated. Hence, the market clears at the price $p^e > c$ such that

$$D(p^e) = [1 - G(p^e)][1 - F(p^e)] = S. \tag{1}$$

A couple of remarks are in order. First, $1 - F(v^*) = S$, so $[1 - G(v^*)][1 - F(v^*)] < S$, which implies $p^e < v^*$. This means that the equilibrium allocation is not efficient.

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[12]In the U.S., a patient awaiting a kidney transplant can effectively move to the front of the queue by locating a live donor. Roth, Sönmez, and Ünver (2004) discuss kidney exchanges wherein patients who have located incompatible live donors essentially trade donor kidneys.
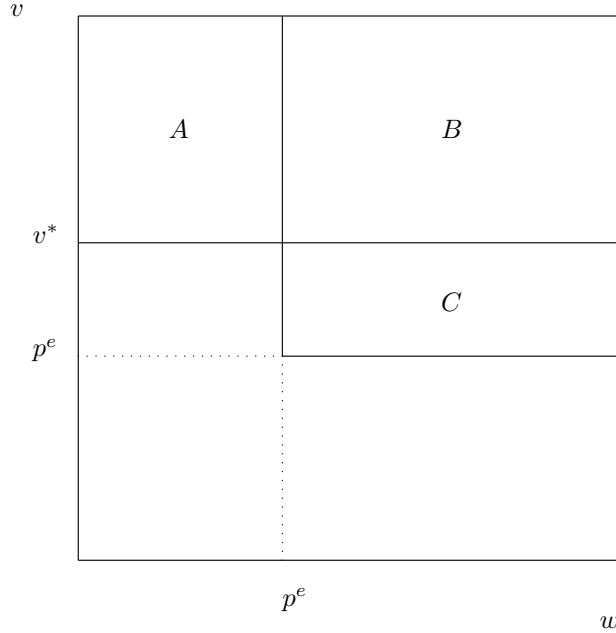
Figure 1: Benchmark Allocations

In Figure 1, the efficient allocation gives the good to all buyers in region $A + B$ while the market assigns it to those in $B + C$. Relative to the efficient allocation, the market favors high-wealth-low-valuation buyers (region C) over low-wealth-high-valuation buyers (region A).

The total value in the market equilibrium is:

$$V^e := \int_{p^e}^1 \int_{p^e}^1 v dF(v) dG(w) = [1 - G(p^e)] \int_{p^e}^1 v dF(v) = S\phi(p^e) < S\phi(v^*) = V^*.$$

The second equality holds since $[1 - G(p^e)][1 - F(p^e)] = S$, by (1), and the inequality holds since $p^e < v^*$ and ϕ is a strictly increasing function. The inefficiency is entirely attributable to the binding wealth constraints. If no buyers were constrained, the market-clearing price would satisfy $p = v^*$, yielding an efficient allocation.

Second, even though the market allocation is inefficient, it would not trigger any resale. Individuals who purchase the good have $v \geq p^e$, so they would only sell at prices exceeding p^e, but there would be no demand at such prices.[13] In other words, the inefficiency will not be mitigated by opening another market.

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[13]If there were an *active* resale market, the resale market price would equal the price in the original equilibrium; otherwise, buyers would switch from one market to the other. Hence, the allocation would be the same.

3 Analysis with Random Assignment

We begin the analysis of non-market assignment schemes with the simplest one: the price is capped at $\overline{p} \in [c, p^e)$, and the good is assigned randomly to those who demand it at that price. Random assignment is particularly easy to implement since it does not require any knowledge of buyers' preferences or wealth. We will analyze general assignment schemes in Section 4. For simplicity, we assume that each individual may participate in the assignment scheme only once.

3.1 Random Assignment without Transferability

We first consider the case in which resale is not permitted. Some of the goods we have discussed may not be transferable because the supplier mandates it or because there are legal restrictions. When the good cannot be transferred, only buyers whose valuation and wealth both exceed $\overline{p}$ will attempt to acquire it. In particular, pure speculators will not participate. The participants each receive the good with probability

$$\frac{S}{[1 - F(\overline{p})][1 - G(\overline{p})]}.$$

The expected valuation for such buyers is $\phi(\overline{p})$, and the aggregate quantity is S, so random assignment gives a total value of $S\phi(\overline{p})$. Since $\overline{p} < p^e$, we have $S\phi(\overline{p}) < S\phi(p^e)$, meaning that efficiency is lower than under the market.

The reason why a price cap is harmful here differs from the standard explanation, which is that quantity falls, resulting in a deadweight loss. There is no quantity effect here; rather, the fixed quantity is simply allocated less efficiently.

3.2 Random Assignment with Transferability

Now consider random assignment with unrestricted resale. In other words, the buyers who obtained the good at $\overline{p}$ are allowed to sell it subsequently in a competitive resale market. We will see that the equilibrium resale price, $r_{\overline{p}}$, exceeds the cap so any buyer who receives the good can pocket $r_{\overline{p}} - \overline{p} > 0$ by reselling. Hence, all buyers who are able to pay $\overline{p}$ will participate in the random assignment, including the pure speculators. Since there are $[1 + m][1 - G(\overline{p})]$ such buyers, each participant will receive the good with probability

$$\rho(\overline{p}; m) := \frac{S}{[1 + m][1 - G(\overline{p})]}.$$

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Suppose that the resale price is $r > \bar{p}$. Resale demand at that price comprises the buyers who did not receive the good initially but who are willing and able to pay r:

$$RD(r) := [1 - F(r)][1 - G(r)][1 - \rho(\bar{p}; m)]. \tag{2}$$

Now consider resale supply. If a buyer with valuation v keeps the good, she will receive a net surplus of $v - \bar{p}$ since reselling gives $r - \bar{p}$. A successful buyer will resell if and only if $v < r$. Resale supply therefore equals the measure of the initial supply assigned to those with $v < r$:

$$RS(r) := S\left(\frac{F(r) + m}{1 + m}\right). \tag{3}$$

Equilibrium requires that resale demand equals resale supply: At $r = r_{\bar{p}}$,

$$[1 - F(r)][1 - G(r)][1 - \rho(\bar{p}; m)] = S\left(\frac{F(r) + m}{1 + m}\right)$$
$$\Rightarrow D(r) = S - \rho(\bar{p}; m)[1 - F(r)][G(r) - G(\bar{p})].$$

The product on the last line is the measure of buyers who are unable to purchase the good at the price $r_{\bar{p}}$ but would not resell the good if they had it. These buyers are depicted in Figure 2 as A'.
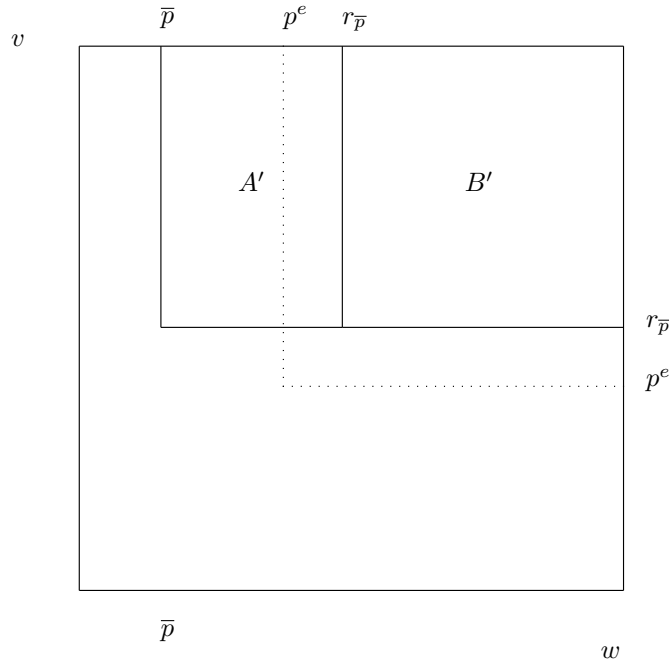

Figure 2: Random Assignment with Resale

Assigning the good to these buyers reduces the available supply in the resale market, causing the equilibrium resale price to be higher than p^e. To see this, suppose that the resale price were $r \leq p^e$. Then, by (1), we have $D(r) \geq S$, so the buyers who are willing and able to pay r (region B') would exhaust the entire S by themselves. (They are either assigned the good and keep it, or they buy it on the resale market.) In addition, some buyers in region A' are assigned the good, and do not resell it. Since there will be excess demand on the resale market when $r \leq p^e$, the equilibrium resale price, $r_{\bar{p}}$, must exceed p^e.

In sum, random assignment with resale yields total value $S\phi(r_{\bar{p}}) > S\phi(p^e) = V^e$.[14] That is, random assignment with transferability produces a strictly more efficient allocation than either random assignment without transferability or operating a competitive market.[15] This result rests on a simple insight: Shifting the initial assignment away from the wealthy to the poor improves the post-resale allocation because the former group has the ability to purchase the good on the open market, which the latter group lacks. The market does poorly in this regard since it tends to assign the good to buyers with purchasing power rather than those without. By contrast, random assignment does not screen buyers based on their purchasing power. Transferability of the good is also crucial to the beneficial performance of random assignment: Absent transferability, there is no efficiency rationale for assigning the good randomly.

A similar logic applies to reductions in the price cap. As $\bar{p}$ falls, more of the good is assigned to the poor. The ultimate allocation becomes more efficient since $r_{\bar{p}}$ rises as $\bar{p}$ falls, so efficiency is highest when the cap is at the lowest level at which supply is available, c. The formal results are now given.

PROPOSITION 1. *Random assignment without transferability is less efficient than the competitive market, and it becomes increasingly less efficient as the price cap, $\bar{p} > c$, falls. Random assignment with transferability is more efficient than the competitive market, and it becomes increasingly more so as the price cap falls.*

Although random assignment and resale are beneficial, speculation reduces the benefits by lowering the quantity assigned to low-wealth buyers. In particular, $\rho(\bar{p}; m) \to 0$ as $m \to \infty$, so speculators acquire essentially the entire supply. The resale market then mimics the original competitive market, and the equilibrium resale price approaches p^e. It follows that there is no benefit from random assignment and resale in the limit as $m \to \infty$.

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[14]Independence of v and w implies that, for any given $w \geq \bar{p}$, the expected value of v, conditional on exceeding $r_{\bar{p}}$, is $\phi(r_{\bar{p}})$. Since quantity equals S, the total value is $S\phi(r_{\bar{p}})$.

[15]High-valuation buyers with wealth $w < \bar{p}$ cannot get the good, so $r_{\bar{p}} < v^*$, which means that there is not full efficiency.

The general points from this section can be seen clearly in a discrete version of our model.

EXAMPLE 1. *(Discrete Types) Let the supply be $S = \frac{1}{2}$. The numbers (measures) of agents of different types are given by the following table:*

	$w = \hat{w}$	$w = 2$
$v = 2$	$\frac{1}{4}$	$\frac{1}{4}$
$v = 1$	$\frac{1}{4}$	$\frac{1}{4}$
$v = 0$	$\frac{m}{2}$	$\frac{m}{2}$

The low wealth is $\hat{w} \in (0, 1)$.

It is efficient for all agents with $v = 2$ to consume, but a competitive market cannot support that allocation. In order for all agents with $v = 2$ to obtain the good, the price must be no greater than $\hat{w} < 1$, but such a price would attract buyers with $v = 1$, leading to excess demand. Hence, $p > \hat{w}$ in a competitive market equilibrium, only agents with $w = 2$ obtain the good, and total value is

$$V^e = \frac{1}{4} \times 2 + \frac{1}{4} \times 1 = \frac{3}{4}.$$

Now suppose that the good is assigned randomly at price $\overline{p} = \hat{w}$, with no resale allowed. All agents with $v \geq 1$ will participate, so each has probability $\frac{1}{2}$ of obtaining the good. The resulting allocation yields total value $\frac{3}{4}$, just as the competitive market does.[16]

Finally, suppose that the good is assigned randomly at price $\overline{p} = \hat{w}$, with resale allowed. All agents will participate now, so each has probability $\frac{1}{2(m+1)}$ of obtaining the good. Unsuccessful agents with $v = 2$ will purchase on the resale market from successful ones with $v \leq 1$. Since there are more resellers than buyers at any $r > 1$, the resale market clears at $r = 1$. The resulting allocation yields total value[17]

$$W = \left(\frac{1}{4} + \frac{1}{8(m+1)} \right) \times 2 + \left(\frac{1}{2} - \frac{1}{4} - \frac{1}{8(m+1)} \right) \times 1 = \frac{3}{4} + \frac{1}{8(m+1)},$$

which exceeds $V^e = \frac{3}{4}$. At the same time, $W \to V^e$ as $m \to \infty$, so the benefit from non-market assignment disappears as the number of pure speculators increases without bound.[18]

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[16]This equivalence is an artifact of the feature that there are no buyers with valuation $v \in (\hat{w}, 1)$. The presence of such a buyer type would have favored the competitive market, just as Proposition 1 asserts.

[17]The measure $\frac{1}{4}$ with $(w, v) = (2, 2)$ get the good, via assignment or in the resale market, while the measure $\frac{1}{8(m+1)}$ with $(\hat{w}, 2)$ get it through the assignment scheme. The remaining supply goes to a subset of the buyers with $v = 1$.

[18]The speculation problem also explains why resale may be prohibited in social programs motivated

4 General Non-Market Assignment Schemes

Random assignment can be implemented with no information about agents' characteristics. When information is available, it may be used to produce a different initial assignment. For instance, a patient may get priority for an organ transplant based on his age and medical urgency, which likely reflect his valuation. The awarding of need-based scholarships provides an example of assignment based on wealth. Need-based assignment was seen in the preferences for small businesses in the FCC spectrum license auctions. Even a first-come-first-served scheme generates an allocation that is correlated with buyers' types. In this section, we study a general assignment scheme that depends on agents' characteristics.

Consider an *assignment technology*, labeled x, and suppose that a (measurable) set of agents, $\Omega \subset \Theta$, participates in the assignment. The assignment technology then induces an *assignment rule*, $x_\Omega : \Theta \mapsto [0,1]$, which determines the probability of assignment for each $(w,v) \in \Theta$. A *feasible assignment rule* satisfies $x_\Omega(w,v) = 0$ for each $(w,v) \in \Theta \backslash \Omega$, and

$$\int_\Theta x_\Omega(w,v)dF(v)dG(w) = S. \tag{4}$$

Let X denote the set of all feasible assignment rules. The assignment technology is then defined as the family, $\{x_\Omega\}_{\Omega \subset \Theta}$, of feasible assignment rules that it induces. Let $\mathcal{X}$ denote the set of all assignment technologies. This reduced-form approach enables us to treat a broad class of assignment schemes.

For much of our discussion, Ω will be fixed under a given assignment technology, so we drop the subscript, Ω, and use x to denote the induced assignment rule as well. We focus on assignment rules that are *separable* in that $x(w,v) = \alpha_x(w)\beta_x(v)$ for functions $\alpha_x : [0,1] \mapsto \Re_+$ and $\beta_x : [0,1] \mapsto \Re_+$. Let $X_S \subset X$ denote the set of feasible, separable assignment rules. Our interest is in the efficiency implications of assigning the good based on "merit" (i.e., valuations) and "need" (i.e., wealth). Separable assignment schemes allow us to evaluate policies targeting each attribute in the most transparent way.

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by redistributive goals. Allowing resale reduces a low-wealth buyer's probability of assignment from $\frac{1}{2}$ to $\frac{1}{2(1+m)}$. Although resale increases the value of the entitlement, it invites speculation, so the target group may be worse off. The beneficiaries of resale are the buyers with $(w,v) = (2,2)$ and the buyers with $v = 0$.

4.1 Characterization of Assignment Rules

Consider an assignment rule, $x \in X_S$, with $x(w,v) = \alpha_x(w)\beta_x(v)$. We can express the rule as

$$x(w,v) = S \cdot a_x(w)b_x(v),$$

where

$$a_x(w) := \frac{\alpha_x(w)}{\int_0^1 \alpha_x(\tilde{w})dG(\tilde{w})}$$

and

$$b_x(v) := \frac{\beta_x(v)}{\int_0^1 \beta_x(\tilde{v})dF(\tilde{v}) + m\beta_x(0)}.$$

This holds because

$$\left(\int_0^1 \alpha_x(\tilde{w})dG(\tilde{w})\right)\left(\int_0^1 \beta_x(\tilde{v})dF(\tilde{v}) + m\beta_x(0)\right)$$

$$= \int_0^1 \int_0^1 \alpha_x(\tilde{w})\beta_x(\tilde{v})dF(\tilde{v})dG(\tilde{w}) + m\int_0^1 \alpha_x(\tilde{w})\beta_x(0)dG(\tilde{w}) = S,$$

where the last equality follows from (4). With x expressed this way,

$$A_x(w) := \int_0^w a_x(\tilde{w})dG(\tilde{w})$$

represents the fraction of the supply assigned to agents with wealth less than w, and

$$B_x(v) := \int_0^v b_x(\tilde{v})dF(\tilde{v}) + mb_x(0)$$

represents the fraction assigned to agents with valuations less than v. That is, A_x and B_x are cumulative distribution functions of quantity across wealth levels and valuations, respectively.

These functions will prove useful in characterizing how an assignment rule treats an agent based on each attribute. We say that x *merit-dominates* $y \in X_S$ if B_x first-order stochastically dominates (FOSD) B_y: $\forall v$, $B_x(v) \leq B_y(v)$.[19] In words, x assigns greater quantities to high-valuation buyers than y does. Rules x and y are *merit-equivalent* if $B_x(\cdot) = B_y(\cdot)$. The assignment rule is *merit-blind* if $b_x(v)$ is constant for all $v \geq \overline{p}$. Random assignment is an obvious example of a merit-blind rule as it awards the good with equal probability to all buyers whose valuations exceed the price cap.

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[19]The merit-dominance is *strict* if the inequality is strict for a positive measure of valuations. The analogous condition makes subsequent dominance definitions strict as well.

There are analogous conditions for cases in which *lower wealth* is favored. We say that *x need-dominates y* if A_y FOSD A_x: $\forall w$, $A_x(w) \geq A_y(w)$. Then, x is more likely to assign the good to low-wealth buyers than y is. Rules x and y are said to be *need-equivalent* if $a_x(\cdot) = a_y(\cdot)$. Finally, x is *need-blind* if $a_x(w)$ is constant for all $w \geq \bar{p}$, in which case the probability of receipt is independent of wealth for $w \geq \bar{p}$.

4.2   Non-Market Assignment without Transferability

We first consider a general non-market assignment rule, $x \in X_S$, without resale. There is a price cap, $\bar{p} \in [c, p^e)$, so only agents with $(w, v) \geq (\bar{p}, \bar{p})$ will participate. The assignment rule will therefore satisfy $x(w, v) = 0$ if $w < \bar{p}$ or $v < \bar{p}$. Total value is

$$V(x_{\bar{p}}) \quad := \quad \int_0^1 \int_0^1 vx(w, v)dF(v)dG(w).$$

The following proposition provides a ranking of schemes based on merit-dominance.

PROPOSITION 2. *If $x \in X$ [strictly] merit-dominates $y \in X$, then $V(x) \geq [>]V(y)$.*

PROOF: Rewrite the total value as

$$
\begin{aligned}
V(x) \quad &= \quad S \int_0^1 \int_0^1 va_x(w)b_x(v)dF(v)dG(w) \\
&= \quad S \int_0^1 vb_x(v)dF(v) \\
&= \quad S \int_0^1 vdB_x(v).
\end{aligned}
$$

Since x [strictly] merit-dominates $y \in X$, B_x [strictly] FOSD B_y, so the result follows. ∎

An assignment rule that puts relatively more weight on high valuations yields greater efficiency. A couple of implications immediately follow. We first note that it does not matter how an assignment rule treats buyers with different wealth levels here: All that matters for efficiency is how it screens based on valuations.

COROLLARY 1. (IRRELEVANCE OF NEED-BASED SCREENING) *If $x \in X_S$ and $y \in X_S$ are merit-equivalent, then $V(x) = V(y)$.*

Any assignment rule that is merit-dominated by a merit-blind rule is less efficient than the latter rule, which is itself welfare-equivalent to random assignment, by Corollary 1. The previous section established that random assignment is strictly less efficient than the competitive market, so the following result is also immediate.

COROLLARY 2. (DRAWBACK OF NON-MARKET ASSIGNMENT WITHOUT TRANSFERABIL-
ITY) *Any assignment rule in X_S that is merit-dominated by the random assignment rule
(given the same binding price cap) yields a strictly less efficient allocation than the market
does.*

It follows that a merit-blind assignment rule is strictly less efficient than the competitive
market. That is, any assignment rule associated with purely need-based screening is less
efficient than the market. This implies that favoring the poor cannot be justified from an
efficiency perspective *if the good is not transferable.*

The *strict* dominance of the market over merit-blind rules implies that any rule that is
modestly merit-superior to random assignment will also do worse than the market efficiency-
wise. In order for non-market assignment without transferability to improve efficiency,
substantial merit-based screening must be feasible.

4.3   Non-Market Assignment with Transferability

The good is again assigned by $x \in X_S$, with a price cap of $\bar{p} < p^e$, but now the recipients
are allowed to resell the good. The subsequent resale price must exceed $\bar{p}$ so all buyers with
$w \geq \bar{p}$ will participate. At the same time, $x(w,v) = 0$ for any $w < \bar{p}$.

Suppose that the resale price is r. An agent who fails to get the good initially will
demand a unit on the resale market if she is willing and able to pay r, so resale demand is

$$RD(r) := \int_r^1 \int_r^1 [1 - x(w,v)]dF(v)dG(w).$$

Buyers who get the good initially will keep it if $v \geq r$, so the quantity supplied on the
resale market is

$$RS(r) := S - \int_{\bar{p}}^1 \int_r^1 x(w,v)dF(v)dG(w).$$

$RD(\cdot)$ is nonincreasing, $RS(\cdot)$ is nondecreasing, and both are continuous functions. Further,
$RD(1) = 0 < RS(1)$, and $RD(0) > 0 = RS(0)$. Hence, there exists an equilibrium resale
price, $r(x)$, that clears the market: $RD(r(x)) = RS(r(x))$.

We can rewrite the market-clearing condition as:

$$RD(r) - RS(r) = D(r) - S + K_x(r) = 0, \tag{5}$$

where

$$K_x(r) := \int_{\bar{p}}^r \int_r^1 x(w,v)dF(v)dG(w) = S[A_x(r) - A_x(\bar{p})][1 - B_x(r)]$$

is the measure of buyers with wealth $w \in [\bar{p}, r]$ who get the good initially and keep it.[20] Since $K_x(r) \geq 0$, $RD(r) - RS(r) \geq D(r) - S > 0$ for any $r < p^e$, so the equilibrium resale price cannot be less than p^e. In fact, $r(x) > p^e$ if $K_x(p^e) > 0$.

Total value is now

$$W(x) := \int_{\bar{p}}^{r(x)} \int_{r(x)}^{1} v x(w, v) dF(v) dG(w) + \int_{r(x)}^{1} \int_{r(x)}^{1} v dF(v) dG(w).$$

The first term represents the value realized by high-valuation-low-wealth buyers who are assigned the good, while the second represents the value realized by those with high valuations and high wealth, all of whom buy the good on the resale market if not assigned it.

The subsequent characterization refers to a new property. We say that the assignment rule x *relatively merit-dominates* the rule y if $B_x(0) \leq B_y(0)$ and

$$\frac{b_x(v')}{b_x(v)} \geq \frac{b_y(v')}{b_y(v)},$$

for every $v' \in (v, 1)$. This concept implies merit-dominance as it requires the latter to hold for all subsets of valuations. We say that x is *meritorious* if it relatively merit-dominates a merit-blind rule, in which case $b_x(v') \geq b_x(v)$ for all $v' > v$. Likewise, x is *demeritorious* if it is relatively merit-dominated by a merit-blind rule. Meritorious (demeritorious) assignment rules dominate (are dominated by) merit-blind rules in a stronger sense than merit-dominance.

Let X_S^+ and X_S^- denote, respectively, the sets of all meritorious and demeritorious rules in X_S. Note that both sets include merit-blind rules.

PROPOSITION 3. *If a meritorious rule, $x \in X_S^+$, relatively merit-dominates $y \in X_S$, and if $r(x) \geq [>] r(y)$, then $W(x) \geq [>] W(y)$.*

PROOF: See the Appendix.

As was the case with random assignment, Proposition 3 shows that a higher resale price indicates a more efficient allocation. Several important conclusions can then be drawn. The first concerns a sufficient condition for assignment schemes to improve upon the competitive market equilibrium.

COROLLARY 3. (SUPERIORITY OF MERITORIOUS ASSIGNMENT WITH TRANSFERABILITY) *Fix a meritorious assignment rule, $x \in X_S^+$, with $\bar{p}_x < p^e$ and $A_x(p^e) > 0$. The rule produces a strictly more efficient allocation than the competitive market does.*

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[20]Note that $A_x(\bar{p}) = 0$ since $x(w, v) = 0$ for $w < \bar{p}$.

PROOF: Fix $x \in X_S^+$ with $\bar{p}_x < p^e$ and $A_x(p^e) > 0$. Since x is meritorious, we have $B_x(p^e) \le p^e < 1$; together with $A_x(p^e) > 0$, this means $K_x(p^e) > 0$. It follows that $r(x) > p^e$.

Now consider $y \in X_S$ with $\bar{p}_y = p^e$ and $b_y(v) = b_y(v')$ for every $v \ne v'$. Then, $A_y(p^e) - A_y(\bar{p}_y) = 0$, so $K_y(p_e) = 0$. This means that $r(y) = p^e$, which in turn implies that y generates the same allocation as the competitive market. Since $r(x) > p^e = r(y)$, and since x relatively merit-dominates y (given $x \in X_S^+$), Proposition 3 gives the result. ∎

This result generalizes the main point of Proposition 1. The requirement that $A_x(p^e) > 0$ simply means that the scheme awards the good to some buyers who are willing but unable to pay p^e. These recipients would not resell the good if $r = p^e$, which pushes the resale price above the competitive market price. Except for this condition, the result does not require much in terms of how the assignment depends on wealth levels. In other words, a weakly meritorious assignment rule does strictly better than the competitive market, largely independent of how it treats different levels of wealth. Even merit-blind assignment rules *strictly* dominate the market. This means that some demeritorious assignment schemes could do better than the market, given transferability. We next use the proposition to demonstrate the benefit of need-based assignment schemes.

COROLLARY 4. (BENEFIT OF NEED-BASED ASSIGNMENT SCHEMES) *If $x \in X$ relatively merit-dominates and need-dominates $y \in X$, then $W(x) \ge W(y)$. If either dominance is strict, then $W(x) > W(y)$.*

PROOF: Given Proposition 3, it suffices to show that $r(x) \ge r(y)$, with a strict inequality for a strict ranking. Relative merit-dominance by x over y implies $1 - B_x(r) \ge 1 - B_y(r)$ for all r, whereas need-dominance implies $A_x(r) \ge A_y(r)$. Hence, $K_x(r) \ge K_y(r)$, for all r, which implies $r(x) \ge r(y)$. If either dominance is strict, then $K_x(r) > K_y(r)$ when $r = r(y)$, so $r(x) > r(y)$. ∎

While it is unsurprising that merit-based rules can improve efficiency, it is noteworthy that need-based rules can have the same effect. If x and y are merit-equivalent, but the former need-dominates the latter, x produces a more efficient ultimate allocation. Wealthy buyers can purchase the good from a reseller if they do not get it initially; the poor lack the means to do so. As a consequence, an initial assignment that gives the good dispro-portionately to the poor produces a more efficient allocation than the market, if resale is allowed. Combined with Corollary 1, this illustrates the importance of resale for need-based assignment to have an efficiency benefit.

Full efficiency may even be attainable if wealth is observable. Consider an assignment rule with $x^*(w, v) = 1$ if $w \leq w^*$, and $x^*(w, v) = 0$ otherwise, where $(1 + m)G(w^*) = S$. This rule assigns the good to all buyers with wealth w^* or below (region $A + B$ in Figure 3).
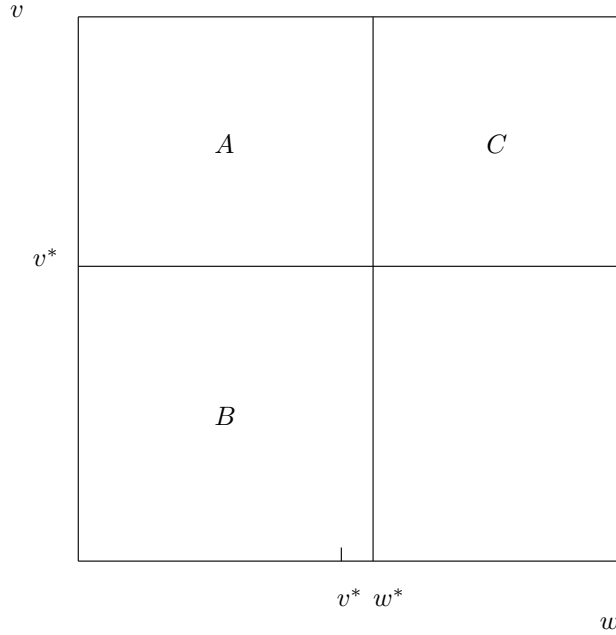

Figure 3: Efficiency of Need-Based Assignment

If $v^* \leq w^*$, this rule achieves full efficiency.[21] The buyers with $v \geq v^*$ (region $A + C$ in Figure 3) will end up with the good, which is efficient. The requirement that $v^* \leq w^*$ is satisfied when S is sufficiently large. In that case, x^* assigns the good to so many buyers that those not assigned the good are all financially capable of purchasing on the resale market. Note that this benefit from need-based screening does not depend on the independence of v and w, although it does depend on observability of w.

We next examine the effect of a change in the price cap. When the cap changes, it alters the set of buyers who participate, so the assignment rule itself changes. We make two natural assumptions about how the rule then changes.

CONDITION (R): Suppose that x and x' are rules induced by a given assignment technology,

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[21]When the resale price is r, supply will be $RS(r) = S\frac{F(r)+m}{1+m}$. Meanwhile, those not assigned the good are willing to buy on the resale market if $v \geq r$, so resale demand is $RD(r) = [1 - F(r)][1 - G(\max\{r, w^*\})]$. Since $RD(r) \gtreqless RS(r)$ if $r \lesseqgtr v^*$, the equilibrium resale price is v^*.

with price caps $\bar{p} < p^e$ and $\bar{p}' < \bar{p}$, respectively. Then, (i) x and x' are merit-equivalent (i.e., $b_{x'}(\cdot) = b_x(\cdot)$), and (ii) $a_{x'}(w) < a_x(w)$ for $w \in [\bar{p}, 1]$.

Property (i) says that the assignment across valuations is unchanged, which is sensible since all buyers with wealth exceeding the price cap participate, so the cap affects participation along the wealth dimension only. Property (ii) reflects the equally plausible condition that adding buyers with lower wealth reduces the assignment probability for each of the existing participants. This would hold, for example, if the relative probabilities among the existing participants are unchanged: $x'(w, v) = \lambda x(w, v)$ for $w \in [\bar{p}, 1]$, for some $\lambda < 1$. The following result shows that the lowest feasible cap is then optimal.

COROLLARY 5. (BENEFIT OF LOWERING PRICE CAPS) *Lowering the price cap, $\bar{p} > c$, increases efficiency when resale is permitted, given a meritorious assignment technology satisfying Condition (R).*

PROOF: Let x and x' be meritorious rules induced by a given assignment technology, with price caps $\bar{p} < p^e$ and $\bar{p}' < \bar{p}$, respectively. Then, Condition (R) means that x and x' are also merit-equivalent. Hence, x' relatively merit-dominates x, and $B_{x'}(\cdot) = B_x(\cdot)$.

We next prove that $A_{x'}(w) > A_x(w)$ for all $w \in [\bar{p}, 1)$. Fixing w, we have

$$A_x(w) = \int_{\bar{p}}^{w} a_x(\tilde{w}) dG(\tilde{w}),$$

and

$$A_{x'}(w) = \int_{\bar{p}'}^{w} a_{x'}(\tilde{w}) dG(\tilde{w}).$$

Hence, for $w \geq \bar{p}$,

$$\frac{dA_{x'}(w)}{dw} = a_{x'}(w)g(w) < a_x(w)g(w) = \frac{dA_x(w)}{dw},$$

where the inequality follows from Condition (R). Together with $A_{x'}(1) = A_x(1)$, this implies that $A_{x'}(w) > A_x(w)$ for all $w \in [\bar{p}, 1)$. Combining these facts, we have

$$K_{x'}(r) = A_{x'}(r)[1 - B_{x'}(r)] > A_x(r)[1 - B_x(r)] = K_x(r)$$

for any $r > \bar{p}$, so $r(x') > r(x)$. Proposition 3 then implies that $W(x') > W(x)$. ∎

4.4   Speculation and Regulation of Resale

Although resale mitigates the inefficiency of non-market assignment schemes, it also engenders speculation. Speculators — low-valuation buyers who participate in the assignment

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solely for the purpose of reselling — reduce the probability that the good is assigned to those with high valuations but low wealth. This diminishes the benefits of employing a non-market assignment scheme, possibly to the point that regulation of resale becomes desirable.[22]

Regulation of resale could take the form of a blanket prohibition, or it could entail not allowing buyers to profit from resale. For instance, in the 3G spectrum auctions in the United Kingdom, resale of the licenses was not permitted.[23] Owners of subsidized housing units may not be allowed to resell at a profit for a certain period of time.[24] Similarly, if a designated entity sells a spectrum license during the first five years, it must reimburse the FCC for the entire bidding credit plus interest.[25]

In order to assess the impact of speculation, we examine the effect of varying the number of pure speculators. Consider a general class of assignment technologies, $\mathcal{X}$, with price cap $\overline{p} \leq p^e$. An assignment technology, $x \in \mathcal{X}$, is called *non-concentrating* if there exists $N > 0$ such that, for any set $\Omega \subset \Theta$ of participants and $\forall (w, v), (w', v') \in \Omega$, we have

$$\frac{x_\Omega(w, v)}{x_\Omega(w', v')} \leq N,$$

irrespective of the measure, m, of pure speculators.

Let $\mathcal{X}_{NC} \subset \mathcal{X}$ be the set of non-concentrating assignment technologies. Members of this set cannot perfectly screen buyers. This condition is sensible in many environments in which agents' types are not accurately observed, making complete exclusion of certain types of buyers impossible. Random assignment is an obvious example; another has constant (and bounded) relative assignment probabilities. Assignment of the good using any assignment technology within this class leads to the same allocation, as $m \to \infty$.

PROPOSITION 4. *Any $x \in \mathcal{X}_{NC}$ followed by resale yields the competitive market allocation in the limit as $m \to \infty$.*

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[22]Speculation may also entail a direct welfare cost. The price caps on new housing in Korea have been criticized for encouraging speculation and diverting resources away from other productive investment activities.

[23]See Klemperer (2004). This did not stop the acquisition of licenses through a corporate takeover, however.

[24]See 42 U.S.C. §12875 for a discussion of restrictions in the "Housing Opportunities for People Everywhere" (HOPE) program in the U.S. When a housing unit is sold within six years, the seller may not receive any "undue profit." That is, the seller must disgorge proceeds exceeding the original price, adjusted for inflation.

[25]See Federal Communications Commission 47 CFR Part 1 [WT Docket No. 05-211; FCC 06-52].

PROOF: Fix any $x \in \mathcal{X}_{NC}$. Suppose that $r > p^e$ is the resale price that follows assignment by x. Index the market-clearing condition in (5) by the measure of pure speculators:

$$RD^m(r) - RS^m(r) = D(r) - S + K_x^m(r), \qquad (6)$$

where $K_x^m(r)$ is the measure of buyers with $(w, v) \in [\bar{p}, r] \times [r, 1]$ who are assigned the good. (Feasibility of x requires $\bar{p} \leq p^e$.) Since $r > \bar{p}$, all buyers with $w \geq \bar{p}$ participate in the assignment scheme.

For each $(w, v) \in [\bar{p}, r] \times [r, 1]$ and for $(w', v') \in \Omega = [\bar{p}, 1] \times [0, 1]$, we have

$$x(w, v) \leq Nx(w', v'),$$

from which it follows that

$$x(w, v) \leq \frac{\int_{\bar{p}}^1 \int_0^1 Nx(w', v')dF(v')dG(w') + m \int_{\bar{p}}^1 Nx(w', 0)dG(w')}{(1+m)[1 - G(\bar{p})]} = \frac{NS}{(1+m)[1 - G(\bar{p})]}.$$

Hence,

$$K_x^m(r) = \int_{\bar{p}}^r \int_r^1 x(w, v)dF(v)dG(w) \leq \frac{\int_{\bar{p}}^r \int_r^1 NSdF(v)dG(w)}{[1+m][1 - G(\bar{p})]} \leq \frac{NS}{1+m}.$$

Thus, $K_x^m(r)$ must converge to zero as m rises without bound, implying $RD^m(r) - RS^m(r) < 0$ for all $m > M$, for some $M > 0$. This means that the equilibrium resale price, r_x^m, must converge to p^e as $m \to \infty$. ∎

This is reminiscent of the Coase theorem, as the initial assignment does not matter much, but the ultimate outcome is *inefficient* here. When there is substantial participation by pure speculators, most of the supply will be resold, thereby mimicking the competitive market. Since that outcome is inefficient, it may be desirable to discourage speculation. One common approach is to prohibit resale for a period of time. For example, as noted above, owners of subsidized housing may effectively be prohibited from reselling for a certain period.

To examine the effects of such a regulation, we reinterpret our model so that the good has a lifespan normalized to one, and it generates a flow surplus of v to a type-(w, v) buyer. The recipient is prohibited from reselling the good until time $z \in [0, 1]$. This approach encompasses no restriction on resale (i.e., $z = 0$) and total prohibition of resale (i.e., $z = 1$) as special cases. For simplicity, we assume that there is no discounting.[26]

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[26]Suppose that the agents discount the future at the rate $r > 0$. Then, the subsequent results will hold if the total period is T such that $\int_0^T e^{-r\tau}d\tau = 1$, and $z = \int_0^t e^{-r\tau}d\tau$ for $t \in [0, T]$.

Consider an assignment technology, $x \in \mathcal{X}$, with price cap $\overline{p} < p^e$, and suppose that buyers with $(w, v) \geq (\overline{p}, \hat{v})$ participate. Then, the resulting assignment rule is $x_{[\overline{p},1] \times [\hat{v},1]} =: \xi(\overline{p}, \hat{v})$. When the resale market opens at z, it operates just as before, with the surplus rescaled by the remaining time, $1 - z$. The resale equilibrium price is then $(1-z)r(\xi(\overline{p}, \hat{v}))$, where $r(\cdot)$ solves (5). If an agent with valuation $v \in [\hat{v}, 1]$ is assigned the good and then resells it at time z, he receives a payoff of

$$zv + (1-z)r(\xi(\overline{p}, \hat{v})) - \overline{p}.$$

The restriction on resale means that the payoff from assignment is strictly increasing in a reseller's valuation. This feature may discourage agents with low valuations from participating. In particular, given $\overline{p} > 0$, one can select $z \in (0, 1)$ so that pure speculators have no incentive to participate:

$$(1-z)r(\xi(\overline{p}, 0^+)) < \overline{p}, \tag{7}$$

where $\xi(\overline{p}, 0^+)$ is the assignment rule when all buyers with wealth above $\overline{p}$ and valuations *strictly* above zero participate. Let $\hat{z} := \inf\{z | (1-z)r(\xi(\overline{p}, 0^+)) < \overline{p}\}$ be the smallest z that would discourage them. (Assume that they do not participate when indifferent.) Since $\overline{p} < p^e \leq r(\xi(\overline{p}, 0^+))$, we have $\hat{z} > 0$. With resale prohibited until $z = \hat{z}$, the assignment rule yields total value of

$$\hat{W} := \hat{z}V(\xi(\overline{p}, 0^+)) + (1 - \hat{z})W(\xi(\overline{p}, 0^+)).$$

Total value does not depend on m since pure speculators do not participate. Since $V(\cdot) \leq W(\cdot)$, the following is then true.

COROLLARY 6. *Consider a non-concentrating assignment technology, x, with price cap $\overline{p} < p^e$. Suppose that x dominates the competitive market when the good is not transferable. Then, prohibiting resale until $\hat{z} \in (0, 1)$ yields a more efficient allocation than unrestricted resale $(z = 0)$ if $m > M_1$, for some $M_1 > 0$. Prohibiting resale altogether $(z = 1)$ produces a more efficient allocation than unrestricted resale if $m > M_2$, for some $M_2 > M_1$.*

With transferability, the allocation approaches the original competitive market allocation as m rises without bound. If the corresponding assignment rule is sufficiently meritorious, restricting resale may produce a more efficient allocation than both the market and the assignment scheme with transferability. As such, there is an efficiency rationale for restricting resale.[27]

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[27]Suen (1989) rationalizes restrictions on transferability when a queue is used to ration a good. Allowing resale raises the benefit from acquiring the good, which raises the incentive to incur socially wasteful time costs.

EXAMPLE 2. *Return to the discrete case from Example 1. It showed that random assignment with resale yields total value $W = \frac{3}{4} + \frac{1}{8(m+1)}$ when $\bar{p} = \hat{w}$. The market and random assignment without resale would both yield total value of $\frac{3}{4}$.*

Now suppose that resale is prohibited until $z \in [0,1]$. The equilibrium resale price will be $r = 1 - z$.[28] Pure speculators will not participate if

$$1 - z \le \bar{p} = \hat{w},$$

or $z \ge 1 - \hat{w}$. Random assignment, with $\bar{p} = \hat{w}$ and a resale ban until $z = 1 - \hat{w}$, yields total value

$$\hat{W} = (1 - \hat{w}) \times \frac{3}{4} + \hat{w} \times \frac{7}{8} = \frac{3}{4} + \frac{1}{8}\hat{w}.$$

This is clearly higher than the value generated by the market or random assignment with total prohibition of resale. (As was shown in Proposition 1, random assignment with total prohibition of resale cannot dominate the market.) Comparing W and $\hat{W}$, we see that restricting resale is desirable if and only if $m \ge \frac{1 - \hat{w}}{\hat{w}}$.

EXAMPLE 3. *Consider the same example but with a merit-based assignment technology: a buyer with $v = 2$ is twice as likely to get the good as one with $v = 1$, who is in turn twice as likely to get the good as a buyer with $v = 0$, if they all participate. That is, a buyer with valuation v gets the good with probability $\frac{2^{v-1}}{m+3}$. If resale is prohibited, pure speculators do not participate, so the buyers with $v = 2$ get the good with probability $\frac{2}{3}$, while those with $v = 1$ get it with probability $\frac{1}{3}$. This yields total value $V(x) = \frac{5}{6}$, which exceeds $\frac{3}{4}$, the value under the market.*

Now suppose that unrestricted resale is allowed. Then, all buyers participate. The resale market clears at $r = 1$ and yields total value of $\frac{11+3m}{4(3+m)}$.

Suppose, next, that resale is prohibited just long enough to keep the pure speculators out. That is, $z = 1 - \hat{w}$ again. With this restriction on resale, the assignment scheme yields total value of

$$\hat{W}(x) = (1 - \hat{w})\frac{5}{6} + \hat{w} \times \frac{11}{12} = \frac{5}{6} + \frac{1}{12}\hat{w}.$$

This value is higher than that attainable by the market and by total prohibition of resale. It also dominates unrestricted resale if and only if

$$\frac{5}{6} + \frac{1}{12}\hat{w} \ge \frac{11 + 3m}{4(3 + m)} \Rightarrow m \ge \frac{3(1 - \hat{w})}{1 + \hat{w}} =: M_1.$$

---

[28]For $r \in (1 - z, 2(1 - z)]$, unsuccessful buyers with $(2,2)$ demand the good and successful ones with $v \le 1$ wish to sell; since there are more of the latter than the former, there is excess supply. For $r < 1 - z$, all unsuccessful buyers with $v \ge 1$ wish to buy, which would require $S = 1$, so there is excess demand. Hence, the equilibrium resale price must be $1 - z$.

We also see that, unlike the case of random assignment, total prohibition of resale may dominate unrestricted resale. Specifically, the former dominates the latter here if and only if

$$\frac{5}{6} \geq \frac{11 + 3m}{4(3 + m)} \Leftrightarrow m \geq 3 =: M_2.$$

Note that $M_1 < M_2$ so the number of speculators needed for partial prohibition to outperform unrestricted resale is smaller than is needed for complete prohibition to do so.

5 Discussion

In this section we examine several additional issues and discuss the robustness of our results.

5.1 In-kind versus Cash Subsidies

The type of assignment scheme employed here improves efficiency by subsidizing the poor. In particular, it is an in-kind subsidy as a good is awarded at a below-market price. This raises the question of whether the same efficiency benefits would obtain with a direct cash subsidy. Providing each agent with a sufficiently large cash subsidy would eliminate the impact of wealth constraints, but such a policy may not be feasible or socially desirable since financing of the subsidy could have its own welfare costs. By contrast, our in-kind subsidy mechanism is budget balanced and does not require financing.

There does exist a budget-balanced cash subsidy scheme that can replicate a non-market assignment rule with *unrestricted resale.* Fix a feasible assignment rule, x, with price cap, $\overline{p}$, and suppose that it entails a resale price of $r(x)$. Now let a cash subsidy scheme be employed in place of this in-kind scheme; specifically, $r(x) - \overline{p}$ is awarded to S buyers, using the same assignment rule, x.[29] This policy endows each successful type-(w, v) buyer with the ability to pay at least $r(x)$. The ensuing competitive market clears at the price $r(x)$, and it implements the same allocation as the original non-market assignment rule with resale.

Despite this observation, the results of the current paper are still relevant because a cash subsidy cannot replicate an in-kind subsidy in one important respect: controlling speculation.[30] We have seen that speculators can be deterred by imposing restrictions on

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[29]The rule stipulates that only buyers with $w \geq \overline{p}$ may receive the subsidy; this can be enforced by requiring buyers to post a bond of $\overline{p}$.

[30]It is also possible that cash subsidies are simply not feasible. For example, the relevant government

resale, which can result in a strictly more efficient allocation than a competitive market if there are sufficiently many speculators. By contrast, a cash subsidy cannot be made unattractive to speculators in the same way that an in-kind subsidy can be. If a cash subsidy is awarded according to a non-concentrating assignment technology, it will only mimic the allocation from a competitive market as the number of pure speculators grows without bound. Hence, as in Corollary 6, non-market assignment of the good with restricted resale dominates a cash subsidy scheme (using the same assignment technology) for a sufficiently large m. This point is made more concrete in the next example.

EXAMPLE 4. *Revisit Example 2. Random assignment with $\bar{p} = \hat{w}$ and a ban on resale until $z = 1 - \hat{w}$ yields total value $\hat{W} = \frac{3}{4} + \frac{1}{8}\hat{w}$. We now show that this scheme can dominate a cash subsidy.*

Suppose that the cash subsidy is assigned randomly, consistent with the in-kind subsidy scheme. For a cash subsidy to have an effect, the recipient of the subsidy must be able to pay at least \$1 since that is what high-wealth-low-valuation buyers are willing (and able) to pay. That is, the subsidy must be at least $1 - \hat{w}$. Suppose that a subsidy $s \geq 1 - \hat{w}$ is given to a measure ℓ of randomly chosen buyers. Budget balancing means that the subsidy cannot exceed the proceeds from the sale, which implies

$$(1 - \hat{w})\ell \leq \frac{1}{2}(p - c) \leq \frac{2 - c}{2} \Leftrightarrow \ell \leq \frac{2 - c}{2(1 - \hat{w})}$$

(since the price cannot exceed \$2). All buyers will apply, so the probability that a given buyer receives the cash subsidy is at most

$$\frac{\ell}{1 + m} \leq \frac{2 - c}{2(1 - \hat{w})(1 + m)}.$$

This means that at most $\frac{2-c}{8(1-\hat{w})(1+m)}$ high-valuation-low-wealth buyers get the good. Hence, the total value resulting from any budget-balanced cash subsidy program is bounded above by

$$2 \times \left(\frac{1}{4} + \frac{2 - c}{8(1 - \hat{w})(1 + m)} \right) + 1 \times \left(\frac{1}{4} - \frac{2 - c}{8(1 - \hat{w})(1 + m)} \right) = \frac{3}{4} + \frac{2 - c}{8(1 - \hat{w})(1 + m)}.$$

If $m > \frac{2-c}{(1-\hat{w})\hat{w}} - 1$, this is less than $\hat{W} = \frac{3}{4} + \frac{1}{8}\hat{w}$, the value attainable under random assignment with restricted resale.

agency could be a regulatory body whose authority is limited to regulating price in an industry. Then, the decision of how to assign the good is separate from any subsidy or taxation decision.

This example shows that random assignment of a good, with a restriction on resale, can do strictly better than any budget-balanced cash-subsidy scheme, for sufficiently large m. We conclude that the ability to control speculation distinguishes an in-kind subsidy from a cash subsidy.[31]

5.2 Elastic Supply

Our analysis has assumed that supply is perfectly inelastic, which is an appropriate assumption when a government itself determines supply, as with licenses to produce or immigration visas. It may also be a reasonable assumption for health care, school choice, and even housing in the short run. Yet, supply may be rather responsive to a price cap in other situations.

Suppose that the good is supplied competitively according to a twice-differentiable, strictly convex, aggregate cost function, $c(\cdot)$. The supply at price p is $S(p) \in \arg\max_{q \geq 0} pq - c(q)$, which implies $c'(S(p)) = p$. Note that $S(\cdot)$ is increasing and differentiable. The competitive equilibrium is characterized by p^e satisfying $D(p^e) = S(p^e)$.

Now suppose that price is capped at $\bar{p} < p^e$ and the good is assigned randomly. For simplicity, assume that $m = 0$. Also assume that the good is transferable, so all individuals with $w \geq \bar{p}$ will participate. A resale equilibrium exists, and it is characterized by the price $r_{\bar{p}}$ satisfying

$$D(r_{\bar{p}}) = S(\bar{p}) - \rho(\bar{p})[1 - F(r_{\bar{p}})][G(r_{\bar{p}}) - G(\bar{p})]. \tag{8}$$

Welfare is now

$$\hat{W}(\bar{p}) := S(\bar{p})\phi(r_{\bar{p}}) - c(S(\bar{p})). \tag{9}$$

Capping the price and randomly assigning the good presents a tradeoff. It has the same benefit seen previously, but the quantity supplied falls. Moreover, the resulting deadweight loss is not negligible here, even if the cap is just below the market price, since the buyers losing access may be wealth constrained but have valuations well above p^e. Nonetheless, our result may still hold: If supply is sufficiently inelastic, random assignment continues to outperform the competitive market.

PROPOSITION 5. *If $\frac{S'(p^e)}{S(p^e)} < \frac{f(p^e)}{F(p^e)}$, there exists a price cap, $\bar{p} < p^e$, such that random assignment with transferability yields greater welfare than the competitive market does.*

PROOF: See the Appendix.

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[31]Other explanations for why an in-kind subsidy may dominate a cash subsidy can be found in Bruce and Waldman (1991), Coate (1995), and Blackorby and Donaldson (1988).

5.3 More General Preferences

We previously considered preferences with the property that the marginal utility of money was constant, and some buyers had binding wealth constraints. We now show that our main result is robust to more general preferences.

There are again two goods: the good being assigned and money. The former is indivisible, with buyers demanding either zero or one unit, and the latter is divisible. As before, an agent has a valuation $v \in [0, 1]$ for the good, which is distributed according to F. Each agent has a zero endowment of the good. An agent also has a wealth, w, which is now distributed over $[1, \infty)$ according to G. The good is supplied by the government or by competitive risk-neutral suppliers at zero cost, up to $S \in (0, 1)$.

A buyer of type (w, v) receives utility of $vx + \eta(w - \pi)$, where x is the probability of obtaining the good and π is the (net) payment. The function $\eta : \Re_+ \mapsto \Re_+$ measures the utility of money; it satisfies $\eta'(\cdot) \geq 1$ and $\eta''(\cdot) \leq 0$, with $\eta''(y) < 0$ for some $y > 0$. The assumption that $\eta'(\cdot) \geq 1$ ensures that no buyer is willing to spend more than v; along with $w \geq 1$, this means that buyers are not wealth constrained.[32] Even though no buyers are wealth constrained, initial wealth still matters because of the concavity of η: A buyer with high wealth has a lower marginal utility of money than does a buyer with low wealth. This feature is the fundamental driver of our main result, as we now show.

Two regimes will be compared: *the competitive market* and *random assignment with transferability*. Utilitarian efficiency maximizes aggregate utility, i.e., the aggregate valuation of the good in addition to the utility from consumption of money. This means that efficiency again requires the good to be assigned according to valuations alone, but it also requires that the payment burden (which may be necessary to finance supply of the good, for example) be allocated based on shadow values of money, with more of the burden going to buyers with higher wealth.

The competitive market is inefficient on both accounts. Given any price p, a buyer with (w, v) would buy the good if and only if

$$v + \eta(w - p) \geq \eta(w). \tag{10}$$

Let $\Omega^+(p)$ be the associated set of buyers, and let $\delta(p) := \Pr\{(w, v) \in \Omega^+(p)\}$ be the measure. Since δ is strictly decreasing, there is a unique equilibrium price, $p^e < 1$, with $\delta(p^e) = S$. High-wealth buyers have a lower shadow value of money, so they are more likely

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[32]The assumption also avoids the uninteresting case in which one can improve Utilitarian efficiency simply by transferring money from buyers to sellers.

to obtain the good than low-wealth buyers, all else equal. Thus, the good is inefficiently allocated. In addition, the payment burden is not efficiently allocated, as purchasers all pay the same price.

Random assignment with transferability can improve upon the competitive market on both accounts. Suppose that the price is capped at $\bar{p} < p^e$ and the good is randomly assigned to demanders at that price. As before, the resale price will exceed $\bar{p}$, so all buyers participate. Hence, each buyer receives the good with probability $\rho = \frac{S}{1+m}$.

Resale demand is $(1-\rho)\delta(r)$ if the resale price is $r \geq p^e$, while a recipient of the good would resell it if and only if

$$v + \eta(w - \bar{p}) < \eta(w + r - \bar{p}). \tag{11}$$

Let $\Omega^-(r, \bar{p})$ denote the set of *potential* resellers, with $\sigma(r, \bar{p})$ its measure. The latter function has partial derivatives satisfying $\sigma_1 > 0$ and $\sigma_2 \geq 0$, respectively. The resale supply at r is given by $\rho\sigma(r, \bar{p})$. Let $r(\bar{p})$ denote the equilibrium resale price, which satisfies

$$(1-\rho)\delta(r) = \rho\sigma(r, \bar{p}). \tag{12}$$

It is not difficult to see that $r(\bar{p}) > p^e$ if and only if $\bar{p} < p^e$.[33]

A binding price cap dominates the competitive market on both accounts noted above. The lower price improves the chances that low-wealth buyers obtain the good. At the same time, the low price reduces their payment burden, compared to the competitive market. This burden is now absorbed partly by the lower price paid to the suppliers or the government, and by the unsuccessful buyers with high wealth who pay more than the competitive market price to purchase on the resale market. These changes render random assignment with transferability superior to the market.

PROPOSITION 6. *There exists $\bar{p} < p^e$ such that random assignment, with a price cap of $\bar{p}$ and unrestricted resale, yields a more (Utilitarian) efficient allocation than the competitive market does.*

Proof: See the Appendix.

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[33]If $\bar{p} = p^e$ and $r = p^e$, the marginal types identified by (10) and (11) are precisely the same, so $\sigma(p^e, p^e) = 1 + m - \delta(p^e)$, and (12) is satisfied at $r = p^e$. If $\bar{p}$ is lowered, σ falls, so we must have $r(\bar{p}) > p^e$.

6 Related Literature

The current paper fits in a line of research showing that wealth constraints may lead to inefficient allocations. Che and Gale (1998) show this in the context of standard auctions when bidders differ in their valuations and wealth. Gali and Fernandez (1999) study the matching of workers to inputs when the workers differ in ability and wealth. They compare the market and a tournament, and find that both regimes provide efficient matching given perfect capital markets. Inefficiencies arise with imperfect capital markets, in which case the tournament does relatively better. Finally, Esteban and Ray (2006) consider a government awarding licenses to produce. A government concerned about efficiency assigns licenses based on lobbying expenditures since lobbying is a signal of productivity; however, wealthier sectors find it *less costly* to lobby, which jams the productivity signal. The resulting allocation corresponds to the market regime in our context. Esteban and Ray focus on how allocative efficiency varies as the underlying wealth distribution changes. The current paper finds the same signal-jamming effect of binding wealth constraints, but it focuses on a different issue; namely, how alternative assignment mechanisms compare in this environment. In fact, our results yield an interesting implication of their setup: If the government is unconcerned about efficiency, lobbying will not arise. The resulting situation would correspond to random assignment here, which dominates the allocation generated by an efficiency-minded government.

A second, related literature considers assignment of goods when markets are not permitted to operate. Abdulkadiroğlu and Sönmez (1999) and (2003), along with Roth, Sönmez, and Ünver (2004) have proposed ingenious algorithms for improving allocative efficiency—without using transfers—when wealth constraints are important. The algorithms may not deliver full efficiency, however. The current paper suggests that introducing transfers in the form of resale may enhance efficiency, so our approach is complementary.

A final, relevant literature rationalizes market intervention based on criteria that differ from Utilitarian efficiency. Weitzman (1977) took as a benchmark the allocation of goods that would prevail if all consumers had the mean income. He then showed that an equal allocation of goods may be closer to the benchmark than the market allocation is.[34] Sah (1987) compared different regimes from the perspective of the (homogeneous) members of the poorest group. They preferred quantity controls (maximum purchases) with resale permitted to quantity controls without resale, which they in turn preferred to the competitive

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[34]Closeness to the benchmark was determined using a mean-square-error criterion. This criterion does not give an exact measure of the welfare cost of misallocation, however.

market. Wijkander (1988) showed that capping price and allocating the good randomly favors certain income groups. If the welfare function puts unequal weights on different consumers' utility, capping price may raise welfare. The current paper differs from this literature by maintaining a focus on efficiency throughout. Also, none of these papers deal with resale and its regulation, which are crucial aspects of our analysis.

7 Concluding Remarks

This paper has asked how the initial ownership of a good should be assigned when some buyers have binding wealth constraints. We have shown that non-market assignment schemes, even simple random assignment, may outperform an unregulated market. Schemes that place goods directly in the hands of high-valuation buyers obviously work well, but so do schemes that target low-wealth buyers. In fact, full efficiency may be realized by assigning the entire stock to the poor. The ability to resell the good is critical to these results.

The results provide a basic efficiency rationale for using non-market methods to assign public resources and for regulating competitive markets when valuations of the good in question are significant relative to individuals' liquid assets. Our analysis has assumed away transaction costs, however. When these costs are substantial, one must weigh the benefits of non-market assignment against the transaction costs that may be incurred. Their presence will likely favor schemes that minimize post-assignment reallocation, and thus will favor market-oriented methods. Even in these circumstances, it may still be desirable to embed some dependence on wealth into these schemes. The preferences for small firms in the FCC auctions may be an appropriate implementation of such an embedding.

It is likely that a variety of non-market methods will continue to be used, and our results have implications for how these methods can be improved. Many current non-market schemes do not permit transferability of benefits, but the present work has identified a tradeoff associated with transferability. On the one hand, we have shown that allowing transferability will improve efficiency when speculators are not numerous. On the other hand, restricting transferability is desirable when there are many potential speculators and the assignment schemes are sufficiently good at targeting recipients with high valuations.

It is not difficult to imagine resale in many assignment programs that do not currently permit it. Moreover, the resale rules can be adapted to control speculation and to accommodate other institutional constraints. We sketch how resale can be introduced in several contexts.

The U.S. government assigns 50,000 permanent resident visas per year by lottery under the *Diversity Immigrant Visa Program*.[35] Becker (1987) proposed selling visas to a pool of qualified applicants. A simple alternative is to retain the lottery system but permit recipients to resell their visas to other "qualified" applicants (e.g., others within the original pool). Our results suggest that this change would yield greater efficiency than both the current system and the Becker proposal.[36]

A lottery could be used to assign transferable educational vouchers. With the pool of recipients and the transfer process appropriately regulated to discourage speculation, such a system may assign school enrollment more efficiently than would a system of local attendance zones or random assignment of non-transferable vouchers. In the same vein, allowing patients to swap places in the queue for human organs or other health care procedures via a (possibly supervised) resale market may be desirable. One could likewise imagine employing a draft with tradable deferments for military recruitment.[37] In each of these cases other objectives or institutional details loom large, but the results here argue for consideration of non-market assignment schemes and transferability.

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[35]Eligibility is restricted to individuals from countries with low rates of immigration to the U.S. See http://www.travel.state.gov/pdf/dv07FinalBulletin.pdf, accessed July 8, 2006.

[36]Other factors may enter a policymaker's welfare function. For instance, visa sales would generate revenue for the Treasury.

[37]Tobin (1970) noted that the same conclusion can be reached on the basis of egalitarian concerns. He first pointed out that the all-volunteer army was "just a more civilized and less obvious way of ... allocating military service to those eligible young men who put the least monetary value on their safety and on alternative uses of their time." He added that the difference between the two schemes is who pays — general taxpayers or the individuals who wish to avoid military service.

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Appendix: Proofs

PROOF OF PROPOSITION 3: Let $\psi_x(v) := \frac{\int_v^1 \tilde{v} dB_x(\tilde{v})}{1 - B_x(v)}$ denote the expected value conditional on exceeding $v > 0$, given the distribution B_x. Total value can be expressed as:

$$
\begin{aligned}
W(x) &= \int_{\underline{p}}^{r(x)} \int_{r(x)}^1 vx(w,v)dF(v)dG(w) + \int_{r(x)}^1 \int_{r(x)}^1 vdF(v)dG(w) \\
&= S \cdot \int_{\underline{p}}^{r(x)} \int_{r(x)}^1 va_x(w)b_x(v)dF(v)dG(w) + (1 - G(r(x))) \int_{r(x)}^1 vdF(v) \\
&= S \cdot A_x(r(x))[1 - B_x(r(x))] \left(\frac{\int_{r(x)}^1 vdB_x(v)}{1 - B_x(r(x))} \right) + D(r(x)) \left(\frac{\int_{r(x)}^1 vdF(v)}{1 - F(r(x))} \right) \\
&= K_x(r(x))\psi_x(r(x)) + D(r(x))\phi(r(x)) \\
&= S \left[\left(1 - \frac{D(r(x))}{S} \right) \psi_x(r(x)) + \left(\frac{D(r(x))}{S} \right) \phi(r(x)) \right];
\end{aligned}
$$

where the first equality follows by definition; the second and third follow by substituting for $x(w,v)$ and integrating; and the last one follows from (5). If x is meritorious, then $\psi_x(r(x)) \geq \phi(r(x))$.

Suppose that x merit-dominates $y \in X$ and $r(x) \geq [>]r(y)$. Then,

$$
\begin{aligned}
W_x &= S \left[\left(1 - \frac{D(r(x))}{S} \right) \psi_x(r(x)) + \left(\frac{D(r(x))}{S} \right) \phi(r(x)) \right] \\
&\geq S \left[\left(1 - \frac{D(r(y))}{S} \right) \psi_x(r(x)) + \left(\frac{D(r(y))}{S} \right) \phi(r(x)) \right] \\
&\geq S \left[\left(1 - \frac{D(r(y))}{S} \right) \psi_y(r(x)) + \left(\frac{D(r(y))}{S} \right) \phi(r(x)) \right] \\
&\geq [>] \; S \left[\left(1 - \frac{D(r(y))}{S} \right) \psi_y(r(y)) + \left(\frac{D(r(y))}{S} \right) \phi(r(y)) \right] \\
&= W_y;
\end{aligned}
$$

where the first inequality follows from $r(x) \geq r(y)$ (which implies $D(r(x)) \leq D(r(y))$) and from $\psi_x(r(x)) \geq \phi(r(x))$; the second follows from the relative merit-dominance of x over y; and the third one follows from the fact that the conditional expectations, $\psi_y(\cdot)$ and $\phi(\cdot)$, are strictly increasing in the relevant region. ∎

PROOF OF PROPOSITION 5: It suffices to show that $\hat{W}'(p^e) < 0$ here, which means that lowering the cap increases total value. To that end, for $\overline{p} \leq p^e$, rewrite total value as:

$$
\hat{W}(\overline{p}) = \{\rho(\overline{p})[G(r_{\overline{p}}) - G(\overline{p})] + [1 - G(r_{\overline{p}})]\} \int_{r_{\overline{p}}}^1 vdF(v) - c(S(\overline{p})).
$$

37

Since $r(p^e) = p^e$, we have

$$\hat{W}'(p^e) = -(1 - G(p^e))f(p^e)r'(p^e)p^e - (1 - \rho(p^e))g(p^e)r'(p^e)\int_{p^e}^1 vdF(v)$$

$$-\rho(p^e)g(p^e)\int_{p^e}^1 vdF(v) - c'(S(p^e))S'(p^e)$$

$$= -(1 - G(p^e))f(p^e)r'(p^e)p^e - F(p^e)[1 - F(p^e)]g(p^e)r'(p^e)\phi(p^e)$$

$$-[1 - F(p^e)]^2 g(p^e)\phi(p^e) - S'(p^e)p^e, \qquad (13)$$

where the second equality holds since $\rho(p^e) = S(p^e)/[1 - G(p^e)] = D(p^e)/[1 - G(p^e)] = 1 - F(p^e)$, $\phi(z) = \int_z^1 vdF(v)/[1 - F(z)]$, and $c'(S(p^e)) = p^e$.

Totally differentiating both sides of (8) and using $\rho(p^e) = 1 - F(p^e)$ yields

$$r'(p^e) = -\frac{S'(p^e) + g(p^e)[1 - F(p^e))]^2}{g(p^e)(1 - F(p^e))F(p^e) + f(p^e)(1 - G(p^e))}.$$

Substituting this into (13) and collecting terms, we get

$$\hat{W}'(p^e) = -\frac{[\phi(p^e) - p^e](1 - F(p^e))g(p^e)[D(p^e)f(p^e) - S'(p^e)F(p^e)]}{g(p^e)(1 - F(p^e))F(p^e) + f(p^e)(1 - G(p^e))}$$

$$= -\frac{[\phi(p^e) - p^e](1 - F(p^e))g(p^e)[S(p^e)f(p^e) - S'(p^e)F(p^e)]}{g(p^e)(1 - F(p^e))F(p^e) + f(p^e)(1 - G(p^e))}.$$

Hence, $\hat{W}'(p^e) < 0$ if and only if $\frac{S'(p^e)}{S(p^e)} < \frac{f(p^e)}{F(p^e)}$. ∎

PROOF OF PROPOSITION 6: Fix any $\bar{p} < p^e$, and suppose that the equilibrium resale price is $r(\bar{p}) \geq p^e$. We prove the result using a bound on aggregate utility. The first step is to ask what aggregate buyer utility would equal if the resale price is $r(\bar{p})$ but there is a constraint on buyer behavior; specifically, the buyers are constrained to behave so that the competitive allocation arises. Unsuccessful agents with $(w, v) \in \Omega^+(p^e)$ purchase on the resale market at the price $r(\bar{p})$ and successful agents with $(w, v) \notin \Omega^+(p^e)$ resell the good at $r(\bar{p})$. Then, the competitive allocation arises, and the buyers enjoy aggregate utility of

$$\Gamma(\bar{p}) = \rho \int_{\Omega^+(p^e)} (v + \eta(w - \bar{p}))dF(v)dG(w) + (1 - \rho)\int_{\Omega^+(p^e)} (v + \eta(w - r(\bar{p}))dF(v)dG(w)$$

$$+ (1 - \rho)\int_{\Omega^-(p^e, p^e)} \eta(w)dF(v)dG(w) + \rho \int_{\Omega^-(p^e, p^e)} \eta(w + r(\bar{p}) - \bar{p})dF(v)dG(w).$$

The first product pertains to high-valuation buyers who get the good through the initial assignment; the second represents the ones who purchase on the resale market. The third product covers the low-valuation buyers who never get the good; the fourth represents the

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ones who are assigned the good but resell it. Note that when $\bar{p} = p^e$, we have $r(\bar{p}) = p^e$ and $\Gamma(\bar{p})$ is precisely the same aggregate utility as is generated by a competitive market.

The derivative of aggregate utility with respect to $\bar{p} < p^e$ is

$$
\begin{aligned}
\Gamma'(\bar{p}) &= -\rho\delta(p^e)\mathbb{E}[\eta'(w-\bar{p})|\Omega^+(p^e)] - r'(\bar{p})(1-\rho)\delta(p^e)\mathbb{E}[\eta'(w-r(\bar{p}))|\Omega^+(p^e)] \\
&\quad - (1 - r'(\bar{p}))\rho\sigma(p^e, p^e)\mathbb{E}[\eta'(w+r(\bar{p})-\bar{p})|\Omega^-(p^e, p^e)].
\end{aligned}
$$

Totally differentiating (12) with respect to $\bar{p}$ and invoking the implicit function theorem, we obtain

$$
-r'(\bar{p}) = \frac{\rho\sigma_2(r(\bar{p}), \bar{p})}{\rho\sigma_1(r(\bar{p}), \bar{p}) - (1-\rho)\delta'(r(\bar{p}))} \geq 0.
$$

It is not difficult to see that $\sigma_2(p^e, p^e) \leq -\delta'(p^e)$.[38] Hence,

$$
0 \leq -r'(p^e) \leq \frac{\rho\sigma_2(p^e, p^e)}{-(1-\rho)\delta'(p^e)} \leq \frac{\rho}{(1-\rho)},
$$

with one of the inequalities being strict. Substituting in, we obtain

$$
\begin{aligned}
\Gamma'(p^e) &= -\rho\delta(p^e)\mathbb{E}[\eta'(w-p^e)|\Omega^+(p^e)] - r'(p^e)(1-\rho)\delta(p^e)\mathbb{E}[\eta'(w-p^e)|\Omega^+(p^e)] \\
&\quad - (1 - r'(p^e))\rho\sigma(p^e, p^e)\mathbb{E}[\eta'(w)|\Omega^-(p^e, p^e)] \\
&= -(\rho + r'(p^e)(1-\rho))\delta(p^e)\mathbb{E}[\eta'(w-p^e)|\Omega^+(p^e)] \\
&\quad - (1 - r'(p^e))\rho\sigma(p^e, p^e)\mathbb{E}[\eta'(w)|\Omega^-(p^e, p^e)] \\
&< -(\rho + r'(p^e)(1-\rho))\delta(p^e) - (1 - r'(p^e))\rho\sigma(p^e, p^e) \\
&= -(\rho + r'(p^e)(1-\rho))S - (1 - r'(p^e))\rho(1 + m - S) \\
&= -\rho(1 + m) = -S.
\end{aligned}
$$

The lone inequality follows because $-r'(p^e) \leq \frac{\rho}{(1-\rho)}$, so $r'(p^e)(1-\rho) \geq -\rho$, $\eta'(w) \geq [>] 1$ [for a positive measure of w], and $\eta(\cdot)$ is strictly concave. The second-to-last equality follows from the fact that $\sigma(p^e, p^e) = 1 + m - \delta(p^e)$ and $\delta(p^e) = S$.

The string of inequalities implies that $\Gamma(\bar{p}) + S \cdot \bar{p} > \Gamma(p^e) + S \cdot p^e$, for some $\bar{p} < p^e$, whereas the competitive market yields total utility of $\Gamma(p^e) + S \cdot p^e$. At the same time, random assignment with a price cap of $\bar{p} < p^e$ will give strictly higher utility to the buyers than $\Gamma(\bar{p})$, since all buyers are weakly better off and some are strictly better off at the resale equilibrium with $r(\bar{p})$ than with the restricted behavior. Hence, the *overall* aggregate utility is strictly higher with random assignment and the price cap, $\bar{p}$. ∎

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[38]Note, first, that $\Omega^+(p^e) = \Theta\backslash\Omega^-(p^e, p^e)$, where $\Theta := [1, \infty) \times [0, 1]$. Next, observe that for any $\epsilon > 0$,

$$
\Omega^+(p^e - \epsilon) \supset \Theta\backslash\Omega^-(p^e, p^e - \epsilon).
$$

These two facts imply $-\delta'(p^e) \geq \sigma_2(p^e, p^e)$.