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Sleep is an important part of life, with an individual spending an estimated 32 years of her life asleep. Despite this importance, little is known about life satisfaction and sleep duration. Using German panel data, it is shown that sleep is an important factor for life satisfaction and that maximal life satisfaction is associated with about eight hours of sleep on a typical weekday. This figure represents, on average, an hour more than people currently sleep suggesting that more sleep would lead to a higher reported satisfaction with life.

Keywords: Sleep, Life Satisfaction, GSOEP, fixed effects

JEL codes: C23, D10, I31

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Sleep duration and life satisfaction

"Sleep is for wimps!"

Reputedly said by Margaret Thatcher, who apparently existed on four hours of sleep a day.

"It's not an indulgence, it's not a luxury, and actually a good night's sleep can have a huge impact on your ability to come up with novel solutions to complex problems."

Russell Foster CBE, Professor of Circadian Neuroscience

Abstract: Sleep is an important part of life, with an individual spending an estimated 32 years of her life asleep. Despite this importance, little is known about life satisfaction and sleep duration. Using German panel data, it is shown that sleep is an important factor for life satisfaction and that maximal life satisfaction is associated with about eight hours of sleep on a typical weekday. This figure represents, on average, an hour more than people currently sleep suggesting that more sleep would lead to a higher reported satisfaction with life.

1. Introduction and motivation

Many news articles, and some academic ones, about sleep start with the cliché about how it is important because it is such a large part of our day. This article, which empirically investigates the amount of sleep associated with maximum life satisfaction, is no exception. It has been estimated that 36 percent of our lives are spent asleep, approximately 32 years in total if we live until 90 (Tufnell 2014). The sample of German individuals investigated here sleep for an average of 7 hours per weekday, and nearly 8 hours per day at the weekend. Given this importance, it is somewhat surprising that it is little researched from a well-being point of view. There is a large medical literature about sleep and its associations with health generally, and a very small 'economics of sleep' literature despite its economic importance. As just one example of this importance, the market for sleep aids is estimated to be a multi-billion dollar industry (Walsh and Engelhardt 1999). The current study is one of the first within economics that investigates life satisfaction and the amount of sleep an individual has. Overall, the results indicate that the duration of sleep that is associated with optimal life satisfaction is, on average, substantially higher than that which individuals actually have.

The medical literature is briefly discussed in the next section and, broadly, finds negative health consequences from both short and long sleep durations. These health consequences include increased weight gain and obesity, and the effects of such weight gain including, for example, diabetes and hypertension. Also discussed in Section 2 is the brief 'economics of sleep' literature. The 'economics of sleep' treats the amount of sleep an individual has as a choice: individuals can choose how to split their day up between work, leisure and sleep in an attempt to maximise utility.

The third section describes the data which come from the well-known German Socio Economic Panel (SOEP) which has, since 2008, included questions about the amount of sleep a respondent has. That it is panel data is particularly advantageous because as well as the evidence that sleep patterns differ substantially across different subgroups (in, for example, Biddle and Hamermesh 1990; Szalontai 2006), there is also evidence that individuals have specific time-invariant differences regarding their need for sleep (Aeschbach et al. 2003; Van Donegan et al. 2005). The use of fixed effects (FE), in section 3, usefully allows these individual time-invariant needs to be controlled for. This section provides descriptive statistics, the overall averages given above, and the most notable trend is with health: healthy individuals sleep longer than less healthy individuals. As well as employing fixed effects (FE) and pooled ordinary least squares (OLS), the methodological approach to attempt to find maximal life satisfaction is briefly explained. The remainder of the paper presents, in Section 4, the result. Section 5 contains a discussion of these results as well as suggestions for future research. Finally Section 6 concludes.

2. Literature review

The medical literature is rather large, and the discussion here mainly follows literature reviews and meta-analyses that investigate the association of sleep duration with health problems. Following this there is a brief discussion of the economics of sleep literature, after which potential links are made between happiness and sleep.

Patel and Hu (2008) review the medical literature between 1966 and 2007 which investigated a possible link between sleep duration and weight gain. A motivation for this review was the combined trends of falling sleep duration and increased weight gain over approximately this period in the USA. However, as Lucassen et al. (2012) explain, almost all of these studies can only look for correlations and tendencies regarding sleep duration and weight gain (and other health factors). This is because experiments which may give insight towards causality are difficult to undertake: it is difficult for participant to behave in the way they should according to their allocation group. Patel and Hu (2008) split their analysis between studies that investigated children and those that investigated adults. The eleven childhood cross-section studies they review all reported a positive association between short sleep duration and increased obesity, with changes in how obesity and short sleep duration are defined at different ages to reflect different 'norms' at different ages. These studies use samples from different and diverse countries, and control for different potential confounders. In adulthood, this broad outcome is also found in the majority of cross-section studies (though, unlike the childhood studies, not all of them). These studies also vary in terms of sample size, age range, potential confounders, and country investigated. Some of these studies, however, use BMI as the sole measure of weight and obesity which is occasionally troublesome because BMI can classify some sportsmen, for example athletic and healthy rugby players and American footballers, as obese. Many of these adult-based studies also investigated the impact of long sleep duration. Overall, they found a U-shape relationship with respect to sleep duration and weight gain (and obesity). Sleeping for a long time – over 11 hours in many of these studies – is also associated with weight gain. As Patel and Hu (2008) report, the handful of longitudinal studies investigating this issue support the link between short sleep duration and weight gain.

¹ There is no collinearity problem with sleep and health in all of the regressions presented below.

Another review, Lucassen et al. (2012), investigates sleep curtailment and obesity, together with insulin resistance. The results are consistent with those of Patel and Hu (2008), and the authors argue that the relationship may well be bidirectional in nature, linked by decreased energy expenditure, increased appetite, and immunological changes. Lucassen et al. (2012) also report on the studies that find a link between long sleep duration and weight gain. As well as sleep duration, poor sleep quality is also argued to be a factor. A further review and meta-analysis looks at sleep quality (measured by difficulty in initiating sleep, and difficulty in maintaining sleep) as well as quantity and the incidence of type 2 diabetes (Cappuccio et al. 2010). In brief, they find that both the quality and quantity of sleep significantly predict the risk of an individual developing type 2 diabetes. The causal mechanisms are argued to be metabolic – including changes in the hormones ghrelin ('the hunger hormone') and leptin ('the satiety hormone') – and their consequences on appetite and energy expenditure. They also link long sleep duration to depressive symptoms, low socio-economic status, unemployment, a low level of physical activity, and poor general health. Like the studies investigated by Patel and Hu (2008) and Lucassen et al. (2012), the studies analysed by Cappuccio et al. come from different countries, and consider a wide range of potential confounders. From a medical point of view it appears clear that too much sleep or too little is undesirable.

There are interesting, tentative links analysed between sleep and wellbeing in the medical literature. For example, from studies of narcoleptics who have a deficit of hypocretin, a neurotransmitter associated with wakefulness and which has recently been found to be associated with positive emotion. This research is summarised in a New York Times article which suggests that this potentially explains the finding that narcoleptics are six times more likely than average to suffer from depression (O' Connor 2013). Recent medical research outlines links between poor sleep (and circadian rhythm disruption) with schizophrenia and other mental illnesses (Foster et al. 2013; Jagannath et al. 2013). The Foster et al. (2013) study states that the association is not well understood however, though the generation of sleep and mental health share overlapping neural mechanisms. Similarly, in a presentation Russell Foster stated that "the really exciting news is that mental illness and sleep are not simply associated but they are physically linked within the brain. The neural networks that predispose you to normal sleep, give you normal sleep, and those that give you normal mental health are overlapping" (Foster 2013). Sleep, it appears from the medical literature, is very important for well-being.

The 'economics of sleep' literature adds sleep to the more common work and leisure trade off, noting that studies which ignore sleep ignore the possibility that individuals may be able to (for example) increase both work and leisure at the same time.³ This literature views sleep as an investment in energy and alertness, but also in terms of its potential opportunity cost: less leisure and /or income. As Asgeirsdottir and Zoega (2011) assert 'sleep and resting make us alert and can enhance the experiences of both work and leisure' (p.150). In this study, the authors assume that utility can be maximised and depends on (the log of) daytime alertness. From this starting point they develop a theoretical model dependent upon the trade-offs involved regarding the benefits and opportunity costs of sleep, as well as the trade-off between work and leisure. Their model indicates (among other results) that sleep will be reduced when an individual's wage is higher (and vice versa),

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² In the New York Times article, the journalist explicitly uses the word happiness specifically when talking about hypocretin but the study referred to discusses positive emotions rather than happiness.

³ This could be achieved by decreasing the amount of sleep one has.

and demonstrate this empirically with Icelandic data. They show, using a method exploiting 'within' change, that between 2007 and 2009, when the average real wage dropped, there was a tendency for sleep duration to increase.⁴

Other studies focus on this sleep and work trade off, particularly with respect to students and studying (Gillen-O'Neel et al. 2013; Baert et al. 2015). Students are particularly apposite for studying this trade-off because, the studies argue, students are unusually flexible (compared to other adults) in being able to choose the amount of sleep they have. More generally, whether sleep duration is a matter of personal choice or not is discussed in Biddle and Hamermesh (1990). They argue that it is, to a large extent, and hence suitable for an economic investigation.

The theoretical model developed by Asgeirsdottir and Zoega (2011) was an attempt to derive and find a solution to an inter-temporal utility-maximization problem regarding sleep duration. Their subsequent empirical work found, as mentioned above, some results consistent with some of their model's predictions. However, their empirical work does not attempt to calculate the optimal duration of sleep. With utility being neither measurable nor operational as a concept finding the amount of sleep consistent with maximum utility is impossible to do. An alternative is to employ life satisfaction as a proxy for utility. There is an extensive debate around whether utility and life satisfaction are the same or not, and whether the existence of surveys that have life satisfaction data means that utility can be measured or not. Prominent examples are as follows: Van Praag Ferrer-i-Carbonell and 2007; Vendrik and Woltjer 2007; Clark et al. 2008; Frey 2008. The empirical work below sidesteps this, and attempts instead to find the sleep duration associated with maximal life satisfaction making no comments on utility.

3. Data and Methodology

Since 2008, the SOEP has included questions on the amount of sleep an individual has, both on a normal weekday and at the weekend. The answers are given in complete hours. On average individuals in the sample sleep for 7 hours on weekdays and nearly 8 hours on the weekend. Importantly, the question that asks about the amount of sleep an individual has on a weekday specifically refers to a normal workday. Discussions of weekday results are thus assumed to reflect a typical workday. Table 1 shows this along with the averages for other categories of individuals in the sample.

⁴ Interestingly, for this investigation, the authors state that they are going to explain how their model is going to help explain self-assessed happiness though they do not do so.

Table 1: Average sleep for different groups of individuals

	Average s	leep (hours)	Person-year	observations
	Weekday	Weekend	Weekday	Weekend
Everyone	7.01	7.88	80,206	79,887
Male	7.00	7.93	38,127	37,970
Female	7.01	7.83	42,079	41,917
Income: 0-37.8%*	7.16	7.56	29,593	29,462
Income: 37.8-40%*	7.13	8.20	1,793	1,789
Income: 40-60%	7.03	8.11	16,280	16,210
Income: 60-80%	6.88	8.06	16,352	16,301
Income: 80-100%	6.82	8.01	16,188	16,125
Married	7.01	7.75	47,550	47,364
Single	7.06	8.49	19,998	18,946
Divorced	6.85	7.69	6,547	6,512
Separated	6.76	7.61	1,419	1,413
Widowed	7.02	7.27	5,667	5,627
Employed	6.88	8.08	34,936	34,807
Self-employed	6.94	7.83	4,895	4,875
Retired	7.20	7.37	21,851	21,732
Unemployed	7.04	7.65	3,918	3,908
Govern. employed	6.80	8.03	3,312	3,301
Apprentice	7.07	8.96	1,893	1885
Not in lab mkt	7.05	7.59	4,912	4,901
Health: v good	7.20	8.38	7,150	7,123
Health: good	7.09	8.11	31,427	31,315
Health: satisf.	6.99	7.78	26,897	26,790
Health: poor	6.75	7.33	14,669	14,599
Educ: high	7.01	7.88	18,002	17,941
Educ: medium	7.00	7.85	47,750	47,569
Educ: low	7.00	7.78	11,242	11,179
No child in HH	7.05	7.84	58,932	58,705
One child in HH	6.92	8.06	11,120	11,060
Two child. in HH	6.87	7.93	7,819	7,789
3+ child. in HH	6.85	7.87	2,335	2,333
Age: 15-20	7.26	9.20	3,359	3,354
Age: 21-30	7.10	8.56	10,137	10,111
Age: 31-40	6.92	8.01	11,149	11,114
Age: 41-50	6.83	7.93	15,985	15,912
Age: 51-60	6.84	7.72	14,713	14,665
Age: over 60	7.10	7.61	45,591	45,298

Note: SOEP data used: Socio-Economic Panel (SOEP), data for years 2008-2012, version 36, SOEP, 2013, doi:10.5684/soep.v36. *The first income category is everyone with a recorded income of zero (37.8% of individuals), the second category has an income of greater than zero up until the upper limit of the second quartile (i.e. 37.8% - 40%).

These averages support somewhat the argument of Biddle and Hamermesh (1990) and Asgeirsdottir and Zoega (2011): people with a higher income sleep for a (slightly) shorter duration on a normal weekday than those with less income. This may reflect the opportunity cost of sleep, as the authors just mentioned suggest, and perhaps a related necessity to spend more time at work. Difficult to

assess with this dataset but an individual's higher income and/or wealth may increase the utility they get from extra leisure. The health categories are interesting too, and suggest that, on average, healthier individuals sleep for longer than unhealthy individuals. In every case, on average people sleep more on the weekends than in the week, though for the retired and the widowed (categories which may overlap somewhat) the difference is approximately ten minutes and a quarter of an hour respectively.

The sample contains five waves of the SOEP from 2008 to 2012. The dependent variable comes from individual responses to the following question 'we would like to ask you about your satisfaction with your life in general", which is coded on a scale from 0 (completely dissatisfed) to 10 (completely satisfied). The medical literature, some of which was briefly discussed above, finds that both too much and too little sleep are associated with health problems, thus it is likely to be more appropriate to model the relationship between happiness and amount of sleep as a curvilinear relationship rather than a linear one. This seems more appropriate on a priori grounds too: it seems unreasonable to expect a constant impact on well-being of an extra hour of sleep with maximum well-being associated with either zero or twenty-four hours. Thus coefficients for sleep duration and sleep duration squared are used to find the turning point for the amount of sleep associated with maximum life satisfaction. This is akin to those studies investigating the relationship between age and well-being, which use the coefficients for age and age squared to find the turning point for the age associated with minimum life satisfaction (for example, Blanchflower and Oswald 2008).

Multivariate regressions are run, starting with pooled OLS before moving on to fixed effects. The latter is preferred because of its well-known ability to control for individual heterogeneity. This can help to control for unusual shift patterns somewhat. If an individual works shifts, and does not change his job, this shiftwork can be said to be controlled for with FE estimates. The SOEP does have data about shiftwork and other unusual working patterns, however it is not in the same waves as those that contain the questions about sleep duration. As the introduction discusses, individuals can exhibit quite a bit of heterogeneity in sleep patterns and sleep needs. Pooled OLS is included too for three main reasons. Firstly, the calculation of coefficients does not (unlike fixed effects) just rely on changes 'within' individuals, so coefficients can be obtained for variables which change for individuals rarely or not at all like, for example, gender. Secondly, fixed effects results, being calculated solely from the data of the individuals in the sample, should ideally not be generalised to the wider population (unlike OLS). And thirdly, to act as a robustness check for the preferred fixed effects results. In each regression, socio-demographic controls are included; almost everything in table 1 is included in the regression either explicitly or implicitly in the role of the reference category. Additionally controls are included for the region an individual lives in (one of the sixteen German länder) and the year of the interview to capture region and year specific variation. A difference is with respect to income which does not enter the equation to be estimated as five dummy variables representing quintiles. Instead income is included as real income, deflated by the CPI, is included in thousands of euros. Two further alternatives for income are used in additional

⁵ A negative (and significant) coefficient on sleep would represent the former situation, and a positive (and significant) coefficient would represent the latter.

⁶ Future research can better control for shift work, and other unusual working hour patterns, and see if there is a modifying effect on the sleep duration on maximum life satisfaction. Though this cannot be undertaken with this dataset, the weekday sleep question asks about a normal workday potentially lessening the impact of not being able to control for unusual working hours.

estimates. In these further regressions the log of income is used, as is common in life satisfaction work and, separately, the inverse hyperbolic sine of income. This latter transformation is well explained by Burbidge et al. (1988), and increasingly used by economists when investigating wealth (Woolley 2011). This is one of the first to use it in life satisfaction work though arguably this should be more commonly used. A major advantage in comparison to the log transformation is that it can handle zero values and in this sample. As the note under table 1 states, there are a substantial number of individuals with zero income. Using the log transformation drops all of these individuals from the sample, and is thus far from ideal. A 'technical' solution keeping to keep them in the sample would be to give all of these individuals an income of one euro, which is also far from ideal. A better alternative is to use the inverse hyperbolic sine of income.

4. Results

Table 2 presents the results of the pooled OLS regressions for the full sample, with the columns differing only by their treatment of income. The headline result is that the normal weekday sleep duration associated with maximum life satisfaction is approximately 1 hour and 20 minutes longer (1.33 hours) than the average sleep that the individuals in the sample actually have. This is the minimum found in all three columns. The results for the weekend differ by the choice of income measurement, and help to highlight the advantage of the inverse hyperbolic sine of income when compared to the log transformation of income. The amount of hours slept at the weekend is only statistically significantly associated with life satisfaction when the log transformation is employed for income. This is different from the other two columns because this transformation excludes individuals with no income from the estimation. This explains the considerably lower sample size (over 20,000 fewer observations) in the middle estimate, also explains why the preferred results are in the first and third columns, and illustrates why an alternative transformation than the log transformation should be considered for income. Thus, from the OLS estimates, the correct interpretation that the amount of hours slept at the weekend has no effect on well-being and that only if people in Germany slept longer during weekdays would well-being increase.

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⁷ The wealth literature favours this transformation over the log transformation because it can handle negative values (not so important for income) as well as zero values unlike the log transformation.

⁸ In all cases, here and below, the sleep duration with respect to maximum life satisfaction has been calculated with the 7 decimal place results for hours of sleep and hours of sleep squared, rather than the two decimal places presented in the table. Furthermore these results (OLS and FE) are robust to the inclusion of a variable to control for children in the household, with the sleep coefficients being the same to at least three decimal places.

Table 2: Sleep duration and life satisfaction, pooled OLS, SOEP 2008-2012

VARIABLES	(1) Life Satisfaction	(2) Life Satisfaction	(3) Life Satisfaction
Hours sleep: weekday	0.55***	0.53***	0.55***
riours sieep. weekday	(0.041)	(0.056)	(0.041)
Hours sleep squared: weekday	-0.03***	-0.03***	-0.03***
riours sieep squarea. weekday	(0.003)	(0.004)	(0.003)
Hours sleep: weekend	-0.01	0.09**	-0.01
riours sieep. weekend	(0.033)	(0.040)	(0.033)
Hours sleep squared: weekend	0.00	-0.00**	0.00
riours sieep squareu. Weekenu	(0.002)	(0.002)	(0.002)
Real Income (thousands)	4.38***	(0.002)	(0.002)
near meeme (measanas)	(0.260)		
Log real income	(0.200)	0.13***	
20g rear meanic		(0.007)	
Real Income (IHS)		(5.55.)	0.08***
			(0.006)
Male	-0.14***	-0.15***	-0.13***
	(0.012)	(0.015)	(0.012)
Married	0.32***	0.33***	0.32***
	(0.019)	(0.020)	(0.019)
Divorced	0.00	0.04	0.00
	(0.027)	(0.029)	(0.027)
Separated	-0.20***	-0.18***	-0.20***
·	(0.046)	(0.049)	(0.046)
Widowed	0.13***	0.06	0.14***
	(0.030)	(0.056)	(0.030)
Self-employed	-0.06***	-0.02	-0.03
	(0.024)	(0.023)	(0.024)
Government employed	0.16***	0.16***	0.14***
	(0.030)	(0.029)	(0.030)
Apprentice	0.10**	0.12**	0.07*
	(0.045)	(0.048)	(0.045)
Unemployed	-0.78***	-0.79***	-0.68***
	(0.028)	(0.038)	(0.030)
Retired	0.13***	0.24***	0.28***
	(0.027)	(0.048)	(0.032)
Very good health	2.53***	2.42***	2.53***
	(0.026)	(0.030)	(0.026)
Good health	1.88***	1.76***	1.88***
	(0.018)	(0.023)	(0.018)
Satisfactory health	1.14***	0.99***	1.14***
	(0.017)	(0.023)	(0.017)
More than High School education	0.26***	0.14***	0.27***
	(0.022)	(0.028)	(0.022)
High School Education	0.14***	0.04*	0.13***
	(0.018)	(0.025)	(0.018)
Age: 21-30	-0.24***	-0.26***	-0.30***
	(0.046)	(0.055)	(0.047)
Age: 31-40	-0.41***	-0.46***	-0.50***
	(0.049)	(0.059)	(0.050)
Age: 41-50	-0.52***	-0.55***	-0.59***
Age. 41-30	(0.050)	(0.060)	(0.051)

Age: 51-60	-0.42***	-0.45***	-0.49***
	(0.051)	(0.061)	(0.052)
Age: Over 60	-0.07	-0.15**	-0.13**
	(0.055)	(0.065)	(0.055)
Regional controls	Yes	Yes	Yes
Wave controls	Yes	Yes	Yes
Constant	3.75***	3.37***	3.67***
	(0.133)	(0.197)	(0.133)
Sleep duration for maximal LS:			_
Weekday	8.31	8.53	8.33
Weekend	n.a.	8.78	n.a.
Observations	68,782	45,689	68,782
R-squared	0.255	0.236	0.254

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1. Reference categories: single, employed, poor health, less than high school education, age 15-20. SOEP data used: Socio-Economic Panel (SOEP), data for years 2008-2012, version 36, SOEP, 2013, doi:10.5684/soep.v36.

The coefficients obtained for the socio-demographic controls, while not of direct interest, are as expected given previous results in the literature. Good health is positive, significant and substantial with respect to life satisfaction whereas unemployment is negative, significant and substantial. Income and marriage are also positively related to life satisfaction; being male and being separated are both negatively related with life satisfaction. Finally, the age dummy variables are in line with the oft-found U-shape between age and life satisfaction (Blanchflower and Oswald 2008; Cheng et al. 2014).

Table 3 presents results from fixed effects estimates, recognising (and controlling for) the individual heterogeneity regarding people's sleep patterns and needs (Aeschbach et al. 2003; Van Donegan et al. 2005).

Table 3: Sleep duration and life satisfaction, fixed effects, SOEP 2008-2012

	(1)	(2)	(3)
VARIABLES	Life Satisfaction	Life Satisfaction	Life Satisfaction
Hours sleep: weekday	0.28***	0.31***	0.28***
	(0.045)	(0.064)	(0.045)
Hours sleep squared: weekday	-0.02***	-0.02***	-0.02***
	(0.003)	(0.005)	(0.003)
Hours sleep: weekend	0.12***	0.13***	0.12***
	(0.037)	(0.043)	(0.037)
Hours sleep squared: weekend	-0.01**	-0.01*	-0.01**
	(0.002)	(0.003)	(0.002)
Real Income (thousands)	1.06**	, ,	, ,
,	(0.429)		
Log real income	,	0.02	
		(0.014)	
Real Income (IHS)		()	0.02*
()			(0.009)
Married	0.15**	0.21***	0.15**
	(0.059)	(0.060)	(0.059)
Divorced	0.18**	0.27***	0.18**
Divorceu	(0.083)	(0.088)	(0.083)
Sonaratod	-0.10	-0.04	-0.10
Separated			
Midamad	(0.086)	(0.090)	(0.086)
Widowed	-0.40***	-0.30*	-0.40*** (0.400)
0.16	(0.099)	(0.176)	(0.100)
Self-employed	-0.05	-0.04	-0.05
	(0.053)	(0.054)	(0.053)
Government employed	-0.14	-0.13	-0.14
	(0.091)	(0.093)	(0.091)
Apprentice	-0.00	0.04	-0.01
	(0.056)	(0.064)	(0.056)
Unemployed	-0.51***	-0.60***	-0.51***
	(0.035)	(0.041)	(0.035)
Retired	-0.01	0.04	-0.00
	(0.048)	(0.062)	(0.050)
Very good health	1.15***	1.16***	1.15***
	(0.033)	(0.038)	(0.033)
Good health	0.90***	0.89***	0.90***
	(0.023)	(0.028)	(0.023)
Satisfactory health	0.56***	0.54***	0.56***
	(0.019)	(0.025)	(0.019)
More than High School education	-0.15	-0.10	-0.16
J	(0.112)	(0.134)	(0.113)
High School Education	-0.10	-0.00	-0.10
	(0.086)	(0.102)	(0.086)
Age: 21-30	0.06	0.05	0.06
	(0.065)	(0.075)	(0.065)
Age: 31-40	0.14*	0.11	0.14
78C. JI-40	(0.084)	(0.094)	(0.085)
Ago: 41 E0			
Age: 41-50	0.15	0.10	0.14
A F1 CO	(0.096)	(0.105)	(0.097)
Age: 51-60	0.15	0.09	0.15
	(0.107)	(0.116)	(0.107)

Age: Over 60	0.21*	0.20	0.21*
	(0.119)	(0.130)	(0.119)
Regional controls	Yes	Yes	Yes
Wave controls	Yes	Yes	Yes
Constant	0.99	4.30***	4.68***
	(0.912)	(0.476)	(0.385)
Sleep duration for maximal LS:			
Weekday	7.90	7.87	7.89
Weekend	11.75	12.54	11.79
Observations (person-year)	68,782	45,689	68,782
R-squared	0.056	0.061	0.056
Individuals	23,987	16,703	23,987

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1. Reference categories: single, employed, poor health, less than high school education, age 15-20. SOEP data used: Socio-Economic Panel (SOEP), data for years 2008-2012, version 36, SOEP, 2013, doi:10.5684/soep.v36.

When accounting for individual heterogeneity, the normal weekday sleep duration associated with maximal happiness falls by approximately 20 minutes. However, the broad message is the same as it was for the pooled OLS estimates: individuals living in Germany sleep, on average, around an hour less than that commensurate with optimal life satisfaction on a normal weekday. With the fixed effects regressions, in contrast to the OLS ones of table 2, the amount of sleep at the weekend is significantly associated with life satisfaction too (albeit at a 90% level of significance for the least preferred estimation employing the log of real income). The maximums that these weekend figures generate seem overly high: half of the day in bed at the weekend provides peak life satisfaction? This will be investigated below via robustness checks, which will also provide a check on the interesting (and more plausible) week-day results.

A short note on the controls which are again, broadly, as expected: higher income (in two the preferred measures) is associated with higher life satisfaction; married individuals are happier than single people, divorced individuals are too (which is not an unusual result in the literature); health again is associated with increased life satisfaction and unemployment with lower life satisfaction. With fixed effects education is now insignificantly related to life satisfaction, a result highly likely to do with a lack of 'within' variation: individuals, over the lifecycle, do not, on average, change their level of education; a simple check reveals that variation within individuals is six times less than variation between individuals. The age coefficients no longer follow the U-shape, being insignificant. This is not an unusual result for fixed effects analysis with wave dummy variables (where they play a similar role with respect to within-a-person variation: getting older by one year is the same as moving to the subsequent wave). More curious is the constant term for the left hand column (i.e. real income), which is insignificant. A likely explanation is found in the regional dummies which are all (highly significant) and have a value of at least three.

Are the results for maximum sleep similar when the sample is restricted by gender? And what about when young people are excluded, given common perception that asserts that teenagers need more sleep than adults (National Sleep Foundation, 2015)? The next subsection, robustness tests, answers these questions.

4.1 Subsample results

The main result from Section 4 was that Germans appear, on average, to sleep for approximately an hour less than the amount consistent with maximum life satisfaction on a normal weekday. A curious result came from the coefficients obtained via fixed effects for sleep duration on the weekend, which lead to a conclusion that maximal life satisfaction was achieved by spending half the weekend in bed. This subsection presents results from subsamples (male, female, young adults, less young adults) and discusses them with respect to those two results just mentioned. Apart from the sample having a restriction by gender or age, the equations estimated are the same (though, for brevity, the results of the controls are not shown). As well as investigating the full sample results in more detail, investigating subsamples to more systematically find differences between key demographic groups reflects the advice of the evolutionary psychologist Buss (2000) who argued that happiness is best investigated in small subsamples. The gender subsample results are presented and discussed before two age subsamples.

Table 4a: Sleep duration for maximal life satisfaction, pooled OLS, males, SOEP 2008-2012

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	(1)	(2)	(3)
VARIABLES	Life Satisfaction	Life Satisfaction	Life Satisfaction
	Income	Log income	IHS income
Sleep duration for maximal LS:	_	_	
Weekday	8.74	8.74	8.81
Weekend	8.09	8.30	8.30
Observations	34,324	23,739	34,324
R-squared	0.273	0.253	0.271

Full controls used (as in tables 2 and 3). Income is real, having been deflated by the CPI.

Table 4b: Sleep duration for maximal life satisfaction, pooled OLS, females, SOEP 2008-2012

	(1)	(2)	(3)
VARIABLES	Life Satisfaction	Life Satisfaction	Life Satisfaction
	Income	Log income	IHS income
Sleep duration for maximal LS:			
Weekday	8.09	8.38	8.08
Weekend	8.51	n.a.	8.47
Observations	34,458	23,739	34,324
R-squared	0.241	0.253	0.271

Full controls used (as in tables 2 and 3). Income is real, having been deflated by the CPI. In both tables SOEP data used: Socio-Economic Panel (SOEP), data for years 2008-2012, version 36, SOEP, 2013, doi:10.5684/soep.v36.

Splitting the sample by gender supports the main result that individuals living in Germany (on average) sleep approximately an hour or more less than that associated with maximal life satisfaction on a normal week day. Males seem to require approximately an extra half-an-hour for peak life satisfaction when compared to females. The weekend sleep duration results, when significant (and thus maximums are appropriate to calculate), are similar to those obtained for the whole sample via pooled OLS. The long weekend sleep duration for the full sample obtained by fixed effects estimation, a result no longer found in the FE results for the separate genders of Tables 5a

⁹ This latter result raises questions about opportunity cost and the preciseness of the data for sleep, issues that are returned to in the discussion section below.

and 5b, which present the maximums obtained by fixed effects estimation for males and females respectively.

Table 5a: Sleep duration for maximal life satisfaction, fixed effects, males, SOEP 2008-2012

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	(1)	(2)	(3)		
VARIABLES	Life Satisfaction	Life Satisfaction	Life Satisfaction		
	Income	Log income	IHS income		
Sleep duration for maximal LS:					
Weekday	8.57	8.20	8.57		
Weekend	9.06	9.80	9.07		
Observations (person-year)	34,324	23,739	34,324		
R-squared	0.063	0.071	0.062		
Individuals	11,821	8,530	11,821		

Full controls used (as in tables 2 and 3). Income is real, having been deflated by the CPI.

Table 5b: Sleep duration for maximal life satisfaction, fixed effects, females, SOEP 2008-2012

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	(1)	(2)	(3)
VARIABLES	Life Satisfaction	Life Satisfaction	Life Satisfaction
	Income	Log income	IHS income
Sleep duration for maximal LS:			
Weekday	7.37	7.55	7.38
Weekend	n.a.	n.a.	8.47
Observations (person-year)	34,458	21,950	34,458
R-squared	0.053	0.056	0.053
Individuals	12,166	8,173	12,166

Full controls used (as in tables 2 and 3). Income is real, having been deflated by the CPI. SOEP data used: Socio-Economic Panel (SOEP), data for years 2008-2012, version 36, SOEP, 2013, doi:10.5684/soep.v36.

These FE results for males maintain the result for the whole sample of individuals sleeping for at least an hour less than that commensurate with maximal life satisfaction. However, for the female sample the difference between the amount slept and the sleep duration commensurate with maximum life satisfaction is between just over twenty minutes to just over half an hour. Further estimates (not shown) attempted to see if the male and female weekday difference was due to labour market participation. Restricting the sample to just the employed the differences between males and females are maintained, though the difference is half the size in shown in table 5. The weekend results, while high (for males), are not nearly as high as those obtained for the full sample. A striking difference between the OLS and FE results by gender is that females have a higher amount of sleep for maximal life satisfaction than males in the OLS estimates, a result reversed when estimated by FE.

One possibility is that the peak long weekend sleep duration for the full sample is driven by young people. Studies often find that young people have need of a longer sleep duration (Graham 2000; Carskadon 2011); the descriptive statistics above are suggestive of young Germans sleeping for longer than older Germans, especially at weekends. The next tables show the results for the young and the no-longer young. The cut-off point creating the subsamples is 30 years of age. This is somewhat arbitrary but a choice has to be made at some age. Moreover Piper (2014a), in a study of the life satisfaction of young British individuals (also aged up to 30), gives general reasons for studying particular age ranges separately, and reasons for investigating young people specifically.

Table 6a: Sleep duration for maximal life satisfaction, pooled OLS, 30 years old or younger, SOEP 2008-2012

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	(1)	(2)	(3)
VARIABLES	Life Satisfaction	Life Satisfaction	Life Satisfaction
	Income	Log income	IHS income
Sleep duration for maximal LS:			
Weekday	8.28	8.76	8.24
Weekend	n.a.	n.a.	n.a.
Observations	9,854	8,059	9,854
R-squared	0.184	0.172	0.183

Full controls used (as in tables 2 and 3). Income is real, having been deflated by the CPI.

Table 6b: Sleep duration for maximal life satisfaction, pooled OLS, over 30 years old, SOEP 2008-2012

	(1)	(2)	(3)
VARIABLES	Life Satisfaction	Life Satisfaction	Life Satisfaction
	Income	Log income	IHS income
Sleep duration for maximal LS:			
Weekday	8.35	8.50	8.41
Weekend	n.a.	8.51	n.a.
Observations	58,928	37,630	58,928
R-squared	0.264	0.250	0.264

Full controls used (as in tables 2 and 3). Income is real, having been deflated by the CPI. SOEP data used: Socio-Economic Panel (SOEP), data for years 2008-2012, version 36, SOEP, 2013, doi:10.5684/soep.v36.

Based on the pooled OLS results, the young and old people subsample results are unremarkable in comparison with the full sample results in table 2. The weekend results are almost always statistically insignificant, hence the maximums being not applicable.

Table 7a: Sleep duration for maximal life satisfaction, fixed effects, 30 years old or less, SOEP 2008-2012

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	(1)	(2)	(3)
VARIABLES	Life Satisfaction	Life Satisfaction	Life Satisfaction
	Income	Log income	IHS income
Sleep duration for maximal LS:			
Weekday	7.78	7.39	7.78
Weekend	n.a.	n.a.	n.a.
Observations (person-year)	9,854	8,059	9,854
R-squared	0.066	0.063	0.066
Individuals	4,255	3,725	4,255

Full controls used (as in tables 2 and 3). Income is real, having been deflated by the CPI.

Table 7b: Sleep duration for maximal life satisfaction, fixed effects, over 30 years old, SOEP 2008-2012

	(1)	(2)	(3)
VARIABLES	Life Satisfaction	Life Satisfaction	Life Satisfaction
	Income	Log income	IHS income
Sleep duration for maximal LS:			
Weekday	8.02	8.13	8.13
Weekend	11.41	n.a.	11.42
Observations (person-year)	58,928	37,630	58,928
R-squared	0.055	0.063	0.055
Individuals	20,326	13,509	20,326

Full controls used (as in tables 2 and 3). Income is real, having been deflated by the CPI. SOEP data used: Socio-Economic Panel (SOEP), data for years 2008-2012, version 36, SOEP, 2013, doi:10.5684/soep.v36.

With fixed effects analysis the weekday results are unremarkable and generally supportive of the finding that individuals sleep less than that associated with maximum life satisfaction.¹⁰ The fixed effects results for the weekend indicate that, in contrast to the speculation offered above, older people are driving the long sleep duration value obtained in table 3.

5. Discussion and limitations

The key result is that, on a normal week day, people living in Germany sleep for approximately an hour less than that associated with maximum life satisfaction. In other words, they are likely to be more satisfied with their life if they sleep for longer though the effect is not substantial. For example, with the first column of table two the increase in life satisfaction of is approximately 0.06 on the 0 to 10 life satisfaction scale. However, for greater changes, the effect is more noteworthy: an individual sleeping eight hours rather than her usual six is just over half of the life satisfaction benefit of being married compared to being single. The economics of sleep literature states that the main cost of more sleep is its opportunity cost, however this analysis suggests that there are still net benefits to be had by sleeping longer (when measured in terms of life satisfaction). There may be wider 'external' costs though which individuals do not consider when thinking about their life satisfaction. If every German was to sleep for an extra hour, there may well be an impact on the economy which (in terms of production and consumption), subsequently, might have knock-on effects in terms of an individual's future life satisfaction. This is not easily modelled, and so remains speculation.

The economic literature presents neat theoretical models aimed at maximising utility. However, the brief literature discussion suggests and the related references argue this cannot be measured and operationalised: such models can never offer a quantifiable sleep duration which equates to maximum utility. Instead, the use of life satisfaction data offers an alternative, and enables an empirical assessment of various factors (like sleep duration). Given the medical literature, the equations in the economic literature that link sleep, alertness, wages, and utility are rather simplistic and do not – perhaps necessarily cannot – reflect some of the nuances involved in these relationships.

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¹⁰ The one occasion when the maximum is closest to the actual average is the least preferred model for reasons discussed above.

¹¹ These results are ceteris paribus results, found by substituting the different hours for weekday sleep duration and weekday sleep duration squared in the real income (i.e. first column) results of table 2 (OLS) and table 3 (FE).

This study does not address the quality of someone's sleep, only its quantity. This is sometimes measured by questionnaires which ask if an individual has trouble getting to sleep, and trouble maintaining sleep, (though trouble getting to sleep is likely to be linked with sleep duration). A dataset dedicated to sleep may include variables capturing these factors, however the SOEP, though excellent, does not ask these questions. There is, however, a question which asks about an individual's satisfaction with their sleep. Satisfaction with sleep has a positive and strongly significant association with life satisfaction: the more satisfied someone is with their sleep the more satisfied they are with their life. This is unsurprising, and the relationship between the two satisfactions is (highly) likely to be endogenous. This means that it is not really appropriate to include both variables in an OLS or fixed effects estimate. One possibility is to use GMM techniques however the necessary diagnostic tests do not support such use. A possible limitation is that the answers that individuals give to the questions about sleep duration are always in exact hours and thus not precise. However, as a broad average on a normal day this seems acceptable. One study (find reference), not using the SOEP, note that in their dataset "sleep time" is only a measure of time set aside to sleep and thus it is "possible that other activities that take place in bed are reported as sleep." 13

As well as the amount of sleep an individual has being important for life satisfaction, a further possibility is that having the ability (or freedom) to choose how much sleep to have could be similarly important for life satisfaction. Many socio-economic factors may prevent individuals from being able to enjoy the freedom to choose how much sleep they have. Discretionary time has been proposed as a new measure of freedom, in a discussion of overwork and subsequent time-crunch concerns (Goodin et al. 2008). The economics of sleep literature is predicated on people having a choice regarding how much they sleep; this is 'willed behaviour' in Asgeirsdottir and Zoega (2011). Discussions of a time crunch resulting from overwork are suggestive that people may face constraints in terms of their choice of sleep duration. Parents may also face time crunch issues, though in this investigation, as a footnote above states, having a child or children in the house makes almost no difference to the results presented above.

Finally, it would be interesting to learn if this result of Germans sleeping less than the amount commensurate with maximum life satisfaction is found in other countries: an interesting avenue for future research. Perhaps sleep needs are physiological, with requirements shared by humans across cultures rather than differing by cultures? Other recent research has looked for a physiological basis for common findings in the life satisfaction literature. An example is Weiss et al. (2012) which investigated the well-being of apes and found a U-shape, like that often found for humans, in the relationship between age and well-being. A further example of a likely physiological explanation for life satisfaction is found by Blanchflower and Oswald (2007) who find that high blood pressure is negatively correlated with happiness. As the medical literature shows, sleep may well be a link between the body and the mind with good (poor) sleep promoting good (poor) physical and mental health outcomes and life satisfaction.

6. Concluding remarks

Sleep is a very important part of our lives. It provides restoration, renewing energy levels, and as burgeoning medical research investigates helps the brain support and promote mental health. Recent medical research suggests that sleeping shares many similar neural networks as good mental health, and that sleep and life satisfaction may be more important for each other than we yet understand. More research will determine this, but some of the neuroscience literature indicates

¹² A discussion regarding GMM techniques, and particularly dynamic panel data estimation, and life satisfaction is found in Piper (2014b).

 $^{^{13}}$ It is conceivable that individuals are reading academic articles in bed rather than sleeping.

that adequate sleep seems likely to be intimately connected with good mental health. Supporting this, this research has shown that there is a strong association between sleep and life satisfaction.

In the nationally representative sample employed in this investigation, the SOEP, Individuals living in Germany sleep, on average, for 7 hours on a normal week day. Slight differences exist within different socio-economic categories. For example, and perhaps most noticeably, an interesting difference is due to health with very healthy people sleeping for longer than people with poor health. However, this difference is less than 30 minutes. As table 1 shows, averages for different categories within the sample do not vary by much. The striking result found above is that, on average and in every case, the sleep duration associated with optimum life satisfaction is substantially higher than that actually enjoyed. This difference is most commonly an hour, though in some cases even greater. Reported well-being in Germany would increase if people slept for longer. A burgeoning medical literature is finding reasons why, including the neural networks in the brain which are shared between good sleep and good mental health. The precise mechanisms linking sleep and well-being are not yet well understood, though it is clear there are links.

To benefit from sufficient sleep and life satisfaction, the research above indicates that, on average, individuals should sleep for about eight hours on a typical weekday, which represents an hour more a day than currently taken. As one of the two opening epigraphs states sleep is neither an indulgence nor a luxury; it has health benefits and (even controlling for health) more of it should, in general, improve life satisfaction.

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