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2005

Communications in Statistics—Simulation and Computation®, 34: 1069–1079, 2005

ISSN: 0361-0918 print/1532-4141 online
DOI: 10.1080/03610910500308719

Statistical Quality Control

An Examination of the Robustness to Non Normality of the EWMA Control Charts for the Dispersion

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The EWMA control chart is used to detect small shifts in a process. It has been shown that, for certain values of the smoothing parameter, the EWMA chart for the mean is robust to non normality. In this article, we examine the case of non normality in the EWMA charts for the dispersion. It is shown that we can have an EWMA chart for dispersion robust to non normality when non normality is not extreme.

Keywords Average run length; Control charts; Exponentially weighted moving average control chart; Median run length; Non normality; Statistical process control.

Mathematics Subject Classification 62N10.

1. Introduction

Control charts are a well-known tool in today's industry. The most known of them are the Shewhart control charts. However, they are unable to detect small shifts in a process quickly enough. For this reason, other charts have been implemented such as the Cumulative Sum (CUSUM) (Page, 1954) and the Exponentially Weighted Moving Average (EWMA) charts (Roberts, 1959).

The EWMA chart is popular because of another characteristic. As Montgomery (2001, p. 433) states: "It is almost a perfectly non parametric (distribution free) procedure". Borror et al. (1999), examined the Average Run Length (ARL) performance of the EWMA chart for the mean in non normal cases when the parameters of the process are known and concluded in the same result for certain

Received June 3, 2004; Accepted June 17, 2005
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values of the smoothing parameter. Recently, Stoumbos and Sullivan (2002) and Testik et al. (2003) extended the work of Borror et al. (1999) to the multivariate case of the EWMA chart. They concluded that a properly designed multivariate EWMA control chart is robust to the non normality assumption. The performance of the EWMA charts for the dispersion under non normality appears to have been studied so far only by Stoumbos and Reynolds (2000). In this article, we examine the performance under non normality of the charts discussed in Stoumbos and Reynolds (2000) along with a new suggestion and other EWMA charts that have been proposed (Domangue and Patch, 1991) for monitoring a process' dispersion.

The article outline is as follows. In Sec. 2, we present the EWMA charts for dispersion that have been implemented up to now together with a new proposal. Section 3 presents the methods for evaluating the performance of a control chart's behavior and ways to compute them. In Sec. 4, the results on the performance of the charts are given with some recommendations. The conclusions are summarized in Sec. 5.

2. The EWMA Control Charts for Monitoring the Process Dispersion

The first step in the setting up control charts is to gather data that are used to estimate the unknown parameters. These data are used to examine whether the process was in control at the time they were collected, since otherwise the parameter estimates computed would affect the ability of a chart to detect an out of control situation (see e.g., Sullivan and Woodall, 1996).

Let μ_0 and σ_0 denote the in-control values of the process parameters that are either known or estimated from a very large sample taken when the process is assumed to be in control. We want to detect any shifts of the dispersion in the process using EWMA charts that are known to be efficient for detecting small to moderate shifts in the parameters. For the remainder of this article we consider individual observations ($n = 1$) which are independent and identically distributed. We further assume that we are in the prospective setting (Phase II), where the estimates or the parameter values are used to monitor the process.

Several publications dealing with the subject of detecting shifts in the dispersion using an EWMA type chart have appeared in the literature (see, e.g., Acosta-Mejia and Pignatiello, 2000; Domangue and Patch, 1991; MacGregor and Harris, 1993). Our main concern is to detect increases in the process dispersion. It should be stressed that detecting decreases in the dispersion is equally important because they indicate an improvement in the process. Nevertheless, it is not probable that a reduction in the process standard deviation, or variance, will occur without a corrective action. Therefore, when an attempt to improve the quality of a process is taking place, the time that this possible change occurs is known. A control chart is one of the tools used to check for possible reduction in the variance before and after the corrective action. However, the main use of a control chart is to detect persistent or sudden shifts in a process at unknown times.

The EWMA chart of squared deviations from target (WR) was proposed by Wortham and Ringer (1971) for detecting a shift in the process standard deviation. The statistic of this chart is given by

$$S_i = \lambda(x_i - \mu_0)^2 + (1 - \lambda)\max(S_{i-1}, \sigma_0^2), \quad S_0 = \sigma_0^2$$

where λ is a smoothing parameter that takes values between 0 and 1 and S_0 is the initial value. The above statistic is defined in a way to detect only upward shifts and therefore we need only an upper control limit. This happens because, whenever S_i is less than σ_0^2, we set it equal to its starting value. The control limit of this chart is

$$UCL = \sigma_0^2 + h_S\sigma_0^2\sqrt{\left(\frac{2\lambda}{2-\lambda}\right)},$$

where h_S is a constant used to specify the width of the control limit. Note that σ_0^2 would be the mean and $\sigma_0^2\sqrt{2\lambda/(2-\lambda)}$ would be the asymptotic standard deviation of S_i if the reset were not used.

As Stoumbos and Reynolds (2000) point out, when the normality assumption is questionable for the observations, the WR statistic does not converge fast to normality because it is a weighted average of squared deviations. For this reason, they propose an EWMA chart of the absolute deviations from target (SR), adjusted for detecting only upward shifts. The statistic of this chart is

$$V_i = \lambda|x_i - \mu_0| + (1-\lambda)\max(V_{i-1}, \sigma_0\sqrt{2/\pi}), \quad V_0 = \sigma_0\sqrt{2/\pi}$$

where V_0 is the initial value. As in the case of the WR statistic, the above statistic can detect only upward shifts and therefore we need again only an upper control limit. The control limit of this chart is

$$UCL = \sigma_0\sqrt{2/\pi} + h_V\sigma_0\sqrt{1-(2/\pi)}\sqrt{\lambda/(2-\lambda)},$$

where h_V is a constant specifying the width of the control limit. Note that $\sigma_0\sqrt{2/\pi}$ would be the mean and $\sigma_0\sqrt{1-(2/\pi)}\sqrt{\lambda/(2-\lambda)}$ would be the asymptotic standard deviation of V_i if the reset were not used.

Hawkins and Olwell (1998, p. 82) suggested a different statistic for monitoring individual readings for scale changes. Specifically, they recommended the use of the differences $(X_n - \mu_0)$ CUSUMming the square root of their absolute values. Here, we introduce an EWMA type control chart using the suggestion of Hawkins and Olwell. Let $H = \sqrt{|x_i - \mu_0|}$, where x_i are our observations. It can be shown that if X is normally distributed $(\mathrm{N}(\mu_0, \sigma_0^2))$, then $E(H) = (2^{3/4})\Gamma(3/4)\sqrt{\sigma_0/2\pi}$ and $\mathrm{Var}(H) = \sigma_0\left(\sigma\sqrt{\frac{2}{\pi}} - \sqrt{2}\frac{\Gamma^2(3/4)}{\pi}\right)$, where the gamma function is defined as $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx, z > 0$. Then, the Hawkins-Olwell (HO) chart is based on the statistic

$$H_i = \lambda\sqrt{|x_i - \mu_0|} + (1-\lambda)\max\left(H_{i-1}, (2^{3/4})\Gamma(3/4)\sqrt{\sigma_0/2\pi}\right),$$

$$H_0 = (2^{3/4})\Gamma(3/4)\sqrt{\sigma_0/2\pi},$$

where H_0 is the initial value. The control limit of this chart is

$$UCL = (2^{3/4})\Gamma(3/4)\sqrt{\sigma_0/2\pi} + h_H\sqrt{\sigma_0\left((2/\sqrt{2\pi}) - \sqrt{2}\Gamma^2(3/4)/\pi\right)\lambda/(2-\lambda)},$$

where h_H is a constant specifying the width of the control limit. The mean of H_i is $(2^{3/4})\Gamma(3/4)\sqrt{\sigma_0/2\pi}$ and $\sqrt{\sigma_0((2/\sqrt{2\pi}) - \sqrt{2}\Gamma^2(3/4)/\pi)\lambda/(2-\lambda)}$ is the asymptotic standard deviation of H_i if the reset is not used.

Domangue and Patch (1991) introduced the omnibus EWMA control charts. The statistic used in these charts is $Z_i = (x_i - \mu_0)/\sigma_0$ and the proposed Domangue-Patch (DP) scheme is

$$A_i = \lambda|Z_i|^{\alpha} + (1-\lambda)A_{i-1},$$

where the starting value A_0 is set by the practitioner and it is usually equal to the asymptotic mean of A_i. Two different schemes were proposed by Domangue and Patch, one with $a = 0.5$ and the second with $a = 2$. In the case of independent samples from a normal process with mean μ_0 and standard deviation σ_0, Domangue and Patch (1991) showed that the asymptotic mean and variance of A_i for the DP1 scheme with $a = 1/2$ are $E(A_i) = (\sqrt{2}/\pi)^{1/2}\Gamma(3/4)$ and $\mathrm{Var}(A_i) = \frac{\sqrt{2}\lambda}{(2-\lambda)\pi}[\sqrt{\pi} - \Gamma^2(3/4)]$. In the case of the DP2 scheme, where $a = 2$, they proved that $E(A_i) = 1$ and $\mathrm{Var}(A_i) = \frac{2\lambda}{(2-\lambda)}$. Then, the control limit in each case is

$$UCL = E(A_i) + h_A \mathrm{Var}(A_i)^{1/2},$$

where h_A is a constant specifying the width of the control limit and either of the schemes signals whenever $A_i \geq UCL$. Note that these schemes can signal only upward because of the way they are constructed. Moreover, as Domangue and Patch point out these schemes are sensitive to increases in dispersion. Note that if we use in the computation of the scheme a reset as in the other charts already presented, then for $a = 1/2$ we end up with a chart that has exactly the same performance as the HO chart. However, since Domangue and Patch demonstrated that the charts they propose are able to detect an increase in the process dispersion, we choose to use them as they are.

For all the above schemes, we observe that they are vulnerable to shifts in the mean apart from the dispersion. Therefore, a signal of these charts might be the result of a change in the mean. This deficiency can be resolved by using the moving range (Hawkins and Olwell, 1998, p. 82) or by calculating at each point in time (observation) an estimate of the mean (MacGregor and Harris, 1993). However, the use of either of these techniques might lead to other problems such as dependence of the observations and since they involve cumbersome calculations they are not considered here.

3. Methods of Evaluating Control Charts Performance and Their Computation

In a control chart, we have two objectives. Firstly, when we are in control, we want the chart to signal as we have planned it to do (false alarm). Secondly, when the process is out of control, we want the control chart to signal as soon as possible. Different measures for evaluating the performance of a chart, concerning the previous two objectives, have been proposed. The most known measure is the average run length (ARL), which is based on the run length (RL) distribution. The number of observations (individual data), or samples (data in subgroups), needed for a control chart to signal is a run length or, equivalently, one observation of the RL distribution. The mean of the RL distribution is the ARL, which is actually the average number of observations needed for a control chart to signal. Usually, along with the ARL, the standard deviation of the run length (SDRL) is also computed. Alternatively, the ARL is expressed as the average number of observations to signal (ANOS). A measure similar to the ARL is the average time to signal (ATS), which is the average time needed for a control chart to signal and it is actually a product of the ARL and the sampling interval used in the case of fixed sampling.

From the preceding discussion, one can see that all these measures are related to the ARL. However, the sole use of the ARL has been criticized (see, e.g., Barnard, 1959; Bissell, 1969; Gan, 1993, 1994; Woodall, 1983). The disadvantage of the ARL is the skewness of the run length distribution in the in and out of control cases as well as in non normality cases. As a result, one can draw misleading conclusions based on the ARL. An alternative measure is the median run length (MRL), which is more credible since it is less affected by the skewness of the run length distribution (see, e.g., Gan, 1993, 1994).

In the context of EWMA charts, computing the previously stated measures of performance can be done by employing the integral equation method, the Markov chain method or via simulation (see e.g., Brook and Evans, 1972; Domangue and Patch, 1991; Lucas and Saccucci, 1990). The integral equation method is an exact method. However, estimates of the measures in question are not always obtainable. The Markov chain method can be implemented in the cases where the previous method cannot, but requires discretization of the continuity of the process using many steps. The simulation method is easy in its implementation and, when using a large number of iterations, the results are very close to the ones of the exact case. In this article, the simulation approach is used, with 200,000 iterations.

In order to study the effect of non normality in the performance of the EWMA charts for dispersion we use the same types of distributions as in Borror et al. (1999) and Stoumbos and Reynolds (2000); symmetric and skewed ones. Specifically, we simulated observations in the skewed case from the Gamma(a, b) distribution with probability density function

$$f(x; \alpha, b) = \left\{ \begin{matrix} \dfrac{b^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp(-bx) & x > 0 \\ 0, & x \leq 0 \end{matrix} \right\},$$

where the mean is α/b and the variance is α/b^2. In the sequel, we set b equal to unity without loss of generality. Under this condition, as α increases, the gamma distribution approaches the normal. In the symmetric case, we simulated observations from the $t(k)$ distribution with probability density function

$$f(x; k) = \frac{\Gamma((k+1)/2)}{\sqrt{\pi}\,\Gamma(k/2)} \frac{1}{((x^2/k)+1)^{(k+1)/2}}, \quad -\infty < x < \infty,$$

where k are the degrees of freedom, the mean is 0, and the variance is $k/(k-2)$. The t distribution is symmetric about 0, but it has more probability in the tails than the normal. Moreover, as the degrees of freedom increase, the t distribution approaches the normal.

In the simulation algorithm, the parameter values we simulated from, are $\alpha = 0.5$, 2, 4 and $b = 1$ in the gamma case, and $k = 4$, 10, 30 in the t distribution case. The in-control mean, when we are in the gamma case, is equal to α/b and the variance is α/b^2. When we have a t distribution, the in-control mean is 0 and the variance is $k/(k-2)$. The values under the normal distribution are calculated also in each case for studying the non normality effect. The values of λ chosen are 0.05, 0.1, and 0.2, which are usually the chosen values for studying the non normality effect (see e.g., Borror et al., 1999; Reynolds and Stoumbos, 2001; Stoumbos and Reynolds, 2000). The values of (h_S, h_V, h_H, h_A) are chosen so that, under normality, they give the same in-control value for ARL approximately 370.4. The simulation study was conducted using Fortran and the random deviates where generated using the Microsoft IMSL Library routines RNNOR, for the normal, RNGAM, for the gamma and RNSTT, for the t distribution. Also, in all the cases, results are displayed for asymptotic control limits. Finally, all the out-of-control computations performed in this paper are made under the assumption of immediate occurrence of the shift at the beginning of the process.

4. Results

Tables 1 and 2 contain the results for the five EWMA charts for dispersion (WR, SR, HO, DP1, and DP2). The results are displayed for three combinations of λ and the corresponding h_S, h_V, h_H, and h_A values. The second row of Table 1 gives the five different h_S, h_V, h_H, and h_A values, which are calculated so as to give, under normality, an in-control value of ARL equal to 370.4. The same values for these h constants are used in Table 2 and, for this reason, are not displayed.

Table 1 summarizes the results for the in-control case for the gamma and the t distributions (ARL(0)), while Table 2 contains the results in the out-of-control case for the Gamma and the t distributions (ARL(1)). In each table, the ARL, MRL, and SDRL values for the normal distribution have also been included, in order to identify the non normality effect. In Table 2, the out-of-control process variance is computed by multiplying the in-control process variance by 1.2 for the first 5 columns and by 1.4 for the remaining 5 ones.

The conclusions drawn from these tables are the following. When the process is in control, the HO chart, for $\lambda = 0.1$, has a satisfactory non normality performance. Additionally, the DP1 chart for, $\lambda = 0.2$, gives also results comparable to the normal ones when we are in control. One also may conclude that the other charts are much less robust regarding non normality for every combination of the smoothing parameter and the process parameters presented. Most of the time, they lead to a larger number of false alarms than the nominal. However, the HO and DP1 can give for certain parameters, very large ARL values. As the value of α in the gamma case and k in the t, become larger so does the ARL and MRL for WR, SR, and DP2. On the other hand, the ARL and MRL values for the HO and DP1 decrease when $\lambda = 0.05$, $\lambda = 0.1$, and increase for $\lambda = 0.2$.

Although the ARL values in the cases of WR and SR in Stoumbos and Reynolds (2000) are the same as the ones in this work, we propose the HO chart as a better alternative to the SR that they propose. They also check the performance of combined charts which are able to detect both increases in the mean and variance. However, in this article we deal only with the EWMA charts for dispersion. The fact that these charts can also detect an increase in the process

Table 1
In-control ARL, MRL, and SDRL values for upward shifts

		WR	SR	HO	DP1	DP2
		when $\lambda = 0.05$				
	h	2.876	2.604	2.436	2.1492	2.495
N(μ, σ^2)	ARL	370.4	370.4	370.4	370.4	370.4
	MRL	260	260	264	259	257
	SDRL	361.3	358.1	353.6	361.8	368.3
G(4, 1)	ARL	151.3	304.2	444.1	490.5	181.2
	MRL	106	213	312	340	124
	SDRL	148.0	296.3	431.7	486.5	183.0
G(2, 1)	ARL	112.4	267.5	522.5	641.5	140.3
	MRL	79	187	365	444	95
	SDRL	110.0	262.1	511.3	640.5	144.1
G(0.5, 1)	ARL	67.8	185.8	840.3	2449.9	94.8
	MRL	47	130	583	1679	63
	SDRL	66.8	184.3	837.1	2489.1	99.9
t_4	ARL	112.6	271.0	792.9	2208	147.4
	MRL	79	189	549	1515	100
	SDRL	110.8	267.4	787.3	2251	151.4
t_{10}	ARL	186.0	329.9	476.8	591.8	216.2
	MRL	131	231	336	411	149
	SDRL	180.9	321.1	462.4	588.2	218.0
t_{30}	ARL	258.8	358.8	401.2	424.1	303.1
	MRL	200	252	282	296	211
	SDRL	279.8	346.4	389.3	417.6	301.4
		when $\lambda = 0.1$				
	h	3.432	2.916	2.628	2.409	3.094
N(μ, σ^2)	ARL	370.4	370.4	370.4	370.4	370.4
	MRL	259	257	260	259	258
	SDRL	365.9	360.8	359.2	363.6	367.4
G(4, 1)	ARL	129.7	237.0	380.8	421.1	147.2
	MRL	91	166	265	293	102
	SDRL	127.7	231.7	374.1	418.8	147.3
G(2, 1)	ARL	95.6	191.6	388.3	472.1	111.8
	MRL	66	133	271	328	77
	SDRL	94.8	188.9	382.1	469.3	112.7
G(0.5, 1)	ARL	59.2	120.2	399.4	816.4	73.1
	MRL	41	83	278	564	50
	SDRL	58.6	119.1	395.1	822.3	74.7
t_4	ARL	97.7	187.4	441.5	882.4	116.4
	MRL	68	131	307	609	80
	SDRL	96.1	185.5	438.1	890.9	117.5
t_{10}	ARL	167.7	269.7	394.1	470.4	185.0
	MRL	117	189	275	326	128
	SDRL	165.0	264.5	388.0	467.3	184.9
t_{30}	ARL	270.6	334.3	378.3	397.1	283.2
	MRL	190	234	264	277	196
	SDRL	264.6	326.7	371.1	392.2	283.8

(continued)

Table 1
Continued

		WR	SR	HO	DP1	DP2
		when $\lambda = 0.2$				
	h	4.112	3.215	2.742	2.584	3.821
$N(\mu, \sigma^2)$	ARL	370.4	370.4	370.4	370.4	370.4
	MRL	256	257	257	259	258
	SDRL	368.9	363.4	363.3	366.4	368.8
G(4, 1)	ARL	113.4	171.8	281.2	319.7	121.9
	MRL	79	120	196	221	84
	SDRL	112.6	169.0	277.9	318.3	121.4
G(2, 1)	ARL	83.5	131.4	240.8	296.4	91.3
	MRL	58	92	167	205	63
	SDRL	82.7	129.5	238.0	294.4	91.3
G(0.5, 1)	ARL	52.5	81.0	179.6	291.4	59.4
	MRL	36	57	125	201	41
	SDRL	51.8	80.3	178.3	293.2	59.4
t_4	ARL	86.7	130.1	238.3	383.0	96.0
	MRL	60	91	166	266	67
	SDRL	86.4	128.7	235.7	383.1	95.8
t_{10}	ARL	152.1	212.3	302.6	353.4	162.0
	MRL	106	148	211	246	112
	SDRL	150.3	210.0	298.0	352.3	162.5
t_{30}	ARL	257.1	303.7	347.4	365.5	264.7
	MRL	178	212	243	255	184
	SDRL	255.4	299.4	343.2	362.4	262.3

mean, as already pointed out in Sec. 3, is of no concern to us here. Another reason for not considering combined schemes is the extra complexity in these charts for practitioners.

In the out of control cases, as the shift increases, the non normality effect decreases. Note that a direct comparison of the different schemes is not possible because they do not have the same in control ARL or MRL values. We observe that the out of control ARL performance of the HO when $\lambda = 0.1$ and DP1 when $\lambda = 0.2$, is not that close to the normal in the case of extreme non normal situations like the gamma(0.5, 1) and $t(4)$. For observations coming from less extreme distributions the performance of these two charts is acceptable especially if we take into consideration that the other charts seem to have a great sensitivity to the normality assumption in the in control case.

Consequently, the HO and DP1 charts may be recommended when normality is questionable for specific values of the smoothing parameter λ. The WR and DP2 charts are not recommended since their performance in both in-control and out-of-control situations is far from the results under normality. The SR chart appears not to be performing well for skewed distributions but leads to better results for small values of λ in the symmetric case.

Table 2
Out-of-control ARL, MRL, and SDRL values for control limits

		1.2					1.4				
	Shift	WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
					$\lambda = 0.05$						
$N(\mu, \sigma^2)$	ARL	113.3	116.2	126.1	114.4	100.8	55.6	58.4	65.9	58.6	48.8
	MRL	81	84	92	82	7	41	44	50	44	37
	SDRL	105.5	105.1	113.1	104.5	94.2	48.2	48.5	54.2	48.7	42.5
G(4, 1)	ARL	66.0	111.0	167.3	171.9	68.8	36.2	54.5	80.2	77.6	35.5
	MRL	47	80	120	121	48	27	40	59	56	26
	SDRL	62.6	103.0	155.7	164.8	67.5	32.9	47.8	70.2	70.0	33.3
G(2, 1)	ARL	61.1	119.3	214.1	237.9	67.6	39.2	67.0	111.9	116.8	40.6
	MRL	43	85	152	166	46	28	48	81	83	28
	SDRL	58.6	113.4	203.1	233.4	68.0	36.8	61.2	101.9	110.5	39.5
G(0.5, 1)	ARL	49.4	117.1	420.7	910.3	62.7	38.5	82.9	252.4	444.3	46.0
	MRL	35	82	293	623	41	27	58	177	304	31
	SDRL	48.2	114.3	413.2	933.2	65.9	37.3	80.2	245.3	454.7	47.8
t_4	ARL	75.5	159.5	417.6	930.3	91.0	59.5	116.2	283.6	556.6	68.4
	MRL	53	112	291	638	62	42	82	199	381	46
	SDRL	73.6	155.8	410.6	946.7	92.9	57.7	113.1	277.1	569.0	69.6
t_{10}	ARL	78.7	131.2	212.2	244.2	82.5	51.6	81.4	133.4	147.1	52.4
	MRL	56	93	151	170	57	37	59	96	103	37
	SDRL	75.2	124.4	201.6	240.1	81.3	48.5	75.3	123.4	141.9	50.9
t_{30}	ARL	86.1	125.1	171.6	175.4	85.8	50.7	73.6	106.6	107.1	49.6
	MRL	61	90	123	123	60	37	54	77	77	35
	SDRL	81.2	116.9	159.2	168.4	83.2	46.7	66.1	96.1	99.9	47.0
					$\lambda = 0.05$						
$N(\mu, \sigma^2)$	ARL	124.1	123.3	131.8	123.2	113.0	60.7	61.7	68.2	62.2	54.6
	MRL	88	88	94	88	80	44	45	50	45	39
	SDRL	119.8	116.6	123.5	116.2	109.1	56.3	55.6	60.7	55.6	50.5
G(4, 1)	ARL	60.5	94.9	147.9	156.7	62.8	34.3	48.7	71.9	73.6	34.2
	MRL	43	67	105	110	44	25	35	52	53	24
	SDRL	58.9	91.0	140.8	152.0	61.8	32.4	45.0	66.2	68.6	32.8
G(2, 1)	ARL	54.8	94.4	171.7	195.0	59.8	36.0	55.9	94.3	102.0	37.4
	MRL	38	66	120	136	42	25	40	67	72	26
	SDRL	53.5	91.5	165.8	191.4	59.5	34.5	52.7	88.5	98.1	36.6
G(0.5, 1)	ARL	43.5	81.7	230.3	400.3	51.6	34.6	60.6	151.6	237.8	39.3
	MRL	30	57	161	276	35	24	42	106	164	27
	SDRL	42.8	80.3	226.4	404.6	52.5	33.7	59.4	147.7	240.1	39.9
t_4	ARL	67.4	116.8	254.6	449.8	76.3	53.8	88.6	182.3	300.3	59.6
	MRL	47	82	178	310	53	38	62	127	207	41
	SDRL	66.0	114.7	250.4	454.0	76.7	52.5	86.9	179.2	302.4	59.6
t_{10}	ARL	73.0	111.1	175.9	203.6	76.4	48.5	70.1	111.2	126.4	49.5
	MRL	51	79	124	142	53	34	50	79	89	35
	SDRL	70.9	107.2	169.2	200.5	75.6	46.7	66.6	105.4	122.7	48.3
t_{30}	ARL	82.9	114.0	156.5	164.2	83.5	48.7	66.6	95.6	100.3	48.6
	MRL	58	81	111	116	59	35	48	68	71	34
	SDRL	80.5	109.1	149.4	159.0	81.4	46.1	62.4	89.3	95.7	47.0

(continued)

Table 2
Continued

	Shift	1.2 WR	SR	HO	DP1	DP2	1.4 WR	SR	HO	DP1	DP2
					$\lambda = 0.05$						
$N(\mu, \sigma^2)$	ARL	136.7	133.4	137.7	132.0	128.8	69.1	67.0	71.1	67.6	63.5
	MRL	95	94	97	93	90	49	48	51	48	45
	SDRL	134.5	129.8	133.1	128.0	127.1	67.1	63.6	66.6	63.4	61.4
G(4, 1)	ARL	56.0	76.9	116.3	128.7	57.6	33.1	42.1	59.8	63.8	32.9
	MRL	39	54	82	90	40	23	30	42	45	23
	SDRL	54.9	74.7	113.1	126.5	56.8	32.0	40.1	56.7	61.2	32.0
G(2, 1)	ARL	49.6	71.4	119.6	140.2	52.3	33.5	45.4	70.2	79.4	34.5
	MRL	35	50	84	97	36	24	32	49	56	24
	SDRL	48.7	69.9	116.4	138.7	51.9	32.6	44.0	67.5	77.1	34.0
G(0.5, 1)	ARL	39.5	57.8	117.1	174.9	43.4	31.5	44.6	83.7	118.3	34.3
	MRL	28	40	81	121	30	22	31	59	82	24
	SDRL	38.8	57.2	115.6	175.8	43.4	30.9	43.6	82.1	118.5	34.2
t_4	ARL	61.2	87.0	149.0	223.9	65.8	49.4	68.1	112.3	162.1	52.7
	MRL	43	61	104	155	46	35	48	79	113	37
	SDRL	60.3	85.8	146.8	224.1	65.6	48.6	66.7	110.2	161.8	52.4
t_{10}	ARL	68.8	91.0	133.8	156.3	71.5	46.2	59.5	85.7	98.4	47.1
	MRL	48	64	94	109	50	32	42	61	69	33
	SDRL	67.5	89.0	130.2	154.6	70.5	45.1	57.7	82.3	96.4	46.2
t_{30}	ARL	80.6	100.7	134.1	144.6	81.0	47.9	59.0	80.1	86.6	47.9
	MRL	56	70	94	101	56	34	41	57	61	34
	SDRL	79.2	98.7	130.4	141.2	80.0	46.7	56.8	76.8	84.3	46.9

5. Discussion

In this article, the effect of non normality on the EWMA charts for the process dispersion was examined in the case of individual observations ($n = 1$). A detailed simulation study based on the ARL, MRL, and SDRL was presented and conclusions were drawn. Two of the presented EWMA charts for the dispersion have an acceptable performance when the non normality effect is not extreme for certain values of the smoothing parameter λ and they are recommended for use in such a cases.

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