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Satoh, Atsuhiro and Tanaka, Yasuhito

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Atsuhiro Satoh*
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.
and
Yasuhito Tanaka†
Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.

We study a symmetric free entry oligopoly in which firms produce differentiated goods so as to maximize their relative profits. The relative profit of each firm is the difference between its profit and the average of the profits of other firms. We show that whether firms determine their outputs or prices, the equilibrium price when firms maximize their relative profits is lower than the equilibrium price when firms maximize their absolute profits, but the equilibrium number of firms under relative profit maximization is smaller than the equilibrium number of firms under absolute profit maximization. This is because each firm is more aggressive and produces larger output under relative profit maximization than under absolute profit maximization.

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1 Introduction

In recent years, maximizing relative profit instead of absolute profit has aroused the interest of economists[1].

*atsato@mail.doshisha.ac.jp
†yasuhito@mail.doshisha.ac.jp
[1]For analyses of relative profit maximization see Gibbons and Murphy (1990), Lu (2011), Matsumura, Matsushima and Cato (2013), Satoh and Tanaka (2013), (2014), Schaffer (1989), Tanaka (2013a), (2013b) and Vega-Redondo

In Vega-Redondo (1997) it was shown that the equilibrium in an oligopoly with a homogeneous good under relative profit maximization is equivalent to the competitive equilibrium. Referring to Alchian (1950) and Friedman (1953) he argued that it is relative rather than absolute performance which should in the end prove decisive in the long run. With differentiated goods, however, the equilibrium in an oligopoly under relative profit maximization is not equivalent to the competitive equilibrium. In Tanaka (2013a) it was shown that under the assumption of linear demand and cost functions when firms in a duopoly with differentiated goods maximize their relative profits, the Cournot equilibrium and the Bertrand equilibrium are equivalent. Satoh and Tanaka (2014a) extended this result to an asymmetric duopoly in which firms have different cost functions. Satoh and Tanaka (2013) showed that in a Bertrand duopoly with a homogeneous good under relative profit maximization and quadratic cost functions there exists a range of the equilibrium price, and this range is narrower and lower than the range of the equilibrium price in duopolistic equilibria under absolute profit maximization shown by Dastidar (1995). Tanaka (2013b) showed that under relative profit maximization the choice of strategic variables, price or quantity, is irrelevant to the equilibrium of duopoly with differentiated goods[2].

We think that seeking for relative profit or utility is based on the nature of human. Even if a person earns a big money, if his brother/sister or close friend earns a bigger money than him, he is not sufficiently happy and may be disappointed. On the other hand, even if he is very poor, if his neighbor is more poor, he may be consoled by that fact. Also firms in an industry not only seek to improve their own performances but also want to outperform the rival firms. TV audience-rating race and market-share competition by breweries, automobile manufacturers, convenience store chains and mobile-phone carriers, especially in Japan, are examples of such behavior of firms.

In this paper we study a symmetric free entry oligopoly in which firms produce differentiated goods so as to maximize their relative profits. The relative profit of each firm is the difference between its profit and the average of the profits of other firms. We show that whether firms determine their outputs or prices, the equilibrium price when firms maximize their relative profits is lower than the equilibrium price when firms maximize their absolute profits, but the equilibrium number of firms under relative profit maximization is smaller than the equilibrium number of firms under absolute profit maximization. This is because each firm is more aggressive and produces larger output under relative profit maximization than under absolute profit maximization. Also we show that Cournot and Bertrand equilibria under relative profit maximization are equivalent.

An equilibrium of a free entry oligopoly is defined as a sub-game perfect equilibrium of the following two stage game.

1. There are many potential firms. In the first stage of the game each firm determines whether to enter or not to enter into the industry. If a firm does not enter, its absolute profit is zero.

(1997). In the analysis of delegation problem such as Miller and Pazgal (2001) the weight on the relative profit is treated as a means of the owner of a firm to control its firm, and the owner's objective itself is still the absolute profit of its firm. But in this paper we have an interest in the case where the owners of firms themselves seek to maximize the relative profits.

[2] Usually the relative profit of a firm in a duopoly or an oligopoly is defined as the difference between the absolute profit of this firm and the absolute profit of the rival firm (or the average of the absolute profits of the rival firms). Alternatively we can define the relative profit as the ratio of the profit of a firm to the total profit in the industry. In Satoh and Tanaka (2014b) we compare these two definitions in a duopoly.

2. In the second stage each firm, which has entered in the first stage, determines the output or the price of its good.

In the next section we present the model, in Section 3 we analyze Cournot and Bertrand equilibria under relative profit maximization, and in Section 4 we compare relative and absolute profit maximization.

2 The model

There are n firms ($n \geqq 2$). The firms produce differentiated substitutable goods. The output and the price of the good of Firm $i, i \in \{1, 2, \dots, n\}$ are denoted by x_i and p_i. The marginal cost $c > 0$ is common. There is a fixed cost $f > 0$, which is also common to all firms.

The inverse demand functions of the goods produced by the firms are

$$p_i = a - x_i - b \sum_{j=1, j \neq i}^{n} x_j,\ i \in \{1, 2, \dots, n\}. \tag{1}$$

where $a > c$ and $0 < b < 1$. b is a substitution parameter. The larger the value of b is, the more substitutable the goods are. These inverse demand functions are symmetric.

By symmetry we can assume that all x_j for all $j \neq i$ are equal at any equilibrium. Differentiating (1) with respect to p_i yields

$$1 = -\frac{\partial x_i}{\partial p_i} - (n-1)b\frac{\partial x_j}{\partial p_i},$$

and

$$0 = -b\frac{\partial x_i}{\partial p_i} - [1 + (n-2)b]\frac{\partial x_j}{\partial p_i}.$$

Then, we obtain

$$\frac{\partial x_i}{\partial p_i} = -\frac{1 + (n-2)b}{1 + (n-2)b - (n-1)b^2}, \tag{2}$$

and

$$\frac{\partial x_j}{\partial p_i} = \frac{b}{1 + (n-2)b - (n-1)b^2},\ j \neq i.$$

Thus,

$$\frac{\partial x_i}{\partial p_i} - \frac{\partial x_j}{\partial p_i} = -\frac{1 + (n-1)b}{(1-b)[1 + (n-1)b]} = -\frac{1}{1-b} \tag{3}$$

3 Relative profit maximization

3.1 Cournot equilibrium

The absolute profit of Firm i is written as

$$\pi_i = (a - x_i - b \sum_{j=1, j \neq i}^{n} x_j)x_i - cx_i - f,\ i \in \{1, 2, \dots, n\}.$$

We denote the relative profit of Firm i by Π_i. It is written as follows,

$$\Pi_i = \pi_i - \frac{1}{n-1}\sum_{j=1,j\neq i}^{n}\pi_j = [a - x_i - b\sum_{j=1,j\neq i}^{n} x_j - c]x_i - f$$
$$-\frac{1}{n-1}\sum_{j=1,j\neq i}^{n}\{[a - x_j - b\sum_{k=1,k\neq j}^{n} x_k - c]x_j - f\}.$$

The condition for maximization of Π_i with respect to x_i is

$$a - 2x_i - b\sum_{j=1,j\neq i}^{n} x_j - c + \frac{b}{n-1}\sum_{j=1,j\neq i} x_j = 0.$$

By symmetry, we can assume that all x_i's are equal. Then, this equation is rewritten as

$$a - [2 + (n-1)b]x_i - c + \frac{b}{n-1}(n-1)x_i = a - [2 + (n-2)b]x_i - c = 0.$$

The equilibrium outputs and prices are

$$\tilde{x}_i^C = \frac{a-c}{2+(n-2)b},$$

and

$$\tilde{p}_i^C = \frac{(1-b)a + [1 + (n-1)b]c}{2+(n-2)b}.$$

C indicates *Cournot*.

3.2 Bertrand equilibrium

The absolute profit of Firm i is written as

$$\pi_i = (p_i - c)x_i - f,\ i \in \{1, 2, \dots, n\}.$$

The relative profit of Firm i, Π_i, is written as follows,

$$\Pi_i = \pi_i - \frac{1}{n-1}\sum_{j=1,j\neq i}^{n}\pi_j$$
$$= (p_i - c)x_i - f - \frac{1}{n-1}\sum_{j=1,j\neq i}[(p_j - c)x_j - f],$$

The condition for maximization of Π_i with respect to p_i is

$$x_i + (p_i - c)\frac{\partial x_i}{\partial p_i} - \frac{1}{n-1}\sum_{j=1,j\neq i}(p_j - c)\frac{\partial x_j}{\partial p_i} = 0.$$

By symmetry, we can assume that all $\frac{\partial x_j}{\partial p_i}$'s for $j \neq i$ are equal, and all p_i's are equal. Then, this equation is rewritten as

$$x_i + (p_i - c)\left(\frac{\partial x_i}{\partial p_i} - \frac{\partial x_j}{\partial p_i}\right) = 0. \tag{4}$$

Substituting (3) into (4), we get

$$\begin{aligned} & x_i - \{a - [1 + (n-1)b]x_i - c\}\frac{1 + (n-1)b}{(1-b)[1 + (n-1)b]} \\ = & \; x_i - \frac{a - [1 + (n-1)b]x_i - c}{1-b} = 0. \end{aligned} \tag{5}$$

The equilibrium outputs and prices are obtained as follows,

$$\tilde{x}_i^B = \frac{a - c}{2 + (n-2)b},\ i \in \{1, 2, \dots, n\}.$$

and

$$\tilde{p}_i^B = \frac{(1-b)a + [1 + (n-1)b]c}{2 + (n-2)b},\ i \in \{1, 2, \dots, n\}. \tag{6}$$

B indicates *Bertrand*. We have $\tilde{x}_i^B = \tilde{x}_i^C$ and $\tilde{p}_i^B = \tilde{p}_i^C$. Thus, when firms maximize their relative profits, Cournot and Bertrand equilibria are equivalent.

We denote the equilibrium output and price of the good of each firm under relative profit maximization by $\tilde{x}_i$ and $\tilde{p}_i$.

The equilibrium profit of each firm is expressed by

$$\Pi_i = \frac{(\tilde{p}_i - c)^2}{1-b} - f = (1-b)\tilde{x}^2 = \frac{(1-b)(a-c)^2}{[2 + (n-2)b]^2} - f.$$

The condition for free entry of firms, ignoring integerness of the number of firms, is

$$\frac{(1-b)(a-c)^2}{[2 + (n-2)b]^2} = f, \tag{7}$$

or

$$\frac{(\tilde{p}_i - c)^2}{1-b} = f, \tag{8}$$

or

$$(1-b)\tilde{x}_i^2 = f.$$

Therefore, the equilibrium output and price of the good of each firm are as follows

$$\tilde{x}_i = \sqrt{\frac{f}{1-b}},$$

and

$$\tilde{p}_i = \sqrt{(1-b)f} + c.$$

Solving (7) for n, we get

$$\tilde{n} = \frac{(a-c)\sqrt{(1-b)f} - 2(1-b)f}{bf}.$$

$\tilde{n}$ denotes the equilibrium number of firms in the case of relative profit maximization.

Some discussions Comparing the first order conditions for relative profit maximization in the Cournot oligopoly and those in the Bertrand oligopoly, we can provide the reason why these results hold. At the Cournot equilibrium all x_i's are equal, and then the first order conditions are reduced to

$$a - [2 + (n-2)b]x_i - c = 0. \tag{9}$$

The first order condition at the Bertrand equilibrium, (5), is rewritten as

$$a - [2 + (n-2)b]x_i - c + (n-1)b[a - [2 + (n-2)b]x_i - c] = 0. \tag{10}$$

Since $(n-1)b + 1 \neq 0$, (10) implies (9). This is because

$$\frac{\partial \Pi_j}{\partial x_i} = -\frac{\partial \Pi_i}{\partial x_i} = -\{a - [2 + (n-2)b]x_i - c_i]\}$$

at the equilibrium of a symmetric oligopoly.

4 Comparison between relative profit maximization and absolute profit maximization

4.1 Cournot equilibrium under absolute profit maximization

The condition of absolute profit maximization for Firm i x_i is

$$a - c - 2x_i - b\sum_{j=1, j\neq i}^{n} x_j = 0, \ i \in \{1, 2, \dots, n\}.$$

The equilibrium outputs, prices and profits of the firms are

$$x_i^C = \frac{a-c}{2 + (n-1)b},$$

$$p_i^C = \frac{a-c}{2 + (n-1)b} + c,$$

and

$$\pi_i^C = \left[\frac{a-c}{2 + (n-1)b}\right]^2 - f.$$

C indicates *Cournot*. The condition for free entry of firms, ignoring integerness of the number of firms, is

$$\left[\frac{a-c}{2 + (n-1)b}\right]^2 = f, \tag{11}$$

or

$$(x_i^C)^2 = f,$$

or

$$(p_i^C - c)^2 = f.$$

Solving (11) for n, we get

$$n^C = \frac{(a-c)\sqrt{f} - (2-b)f}{bf}.$$

n^C denotes the equilibrium number of firms at the Cournot equilibrium under absolute profit maximization. Also we have

$$p_i^C = \sqrt{f} + c,$$

and

$$x_i^C = \sqrt{f}.$$

4.2 Bertrand equilibrium under absolute profit maximization

(2) holds also in the case of relative profit maximization. Then, the condition of absolute profit maximization for Firm i with respect to p_i is

$$x_i + (p_i - c)\frac{\partial x_i}{\partial p_i} = x_i - (p_i - c)\frac{1 + (n-2)b}{1 + (n-2)b - (n-1)b^2} = 0.$$

The equilibrium outputs, prices and profits of the goods of the firms are

$$x_i^B = \frac{[1 + (n-2)b](a-c)}{[1 + (n-1)b][2 + (n-3)b]},$$

$$p_i^B = \frac{(1-b)(a-c)}{2 + (n-3)b} + c,$$

and

$$\pi_i^B = \left[\frac{1 + (n-2)b}{1 + (n-1)b}\right]\frac{(p_i^B - c)^2}{1-b} - f.$$

B indicates *Bertrand*. The condition for free entry of firms, ignoring integerness of the number of firms, is

$$\frac{(1-b)[1 + (n-2)b](a-c)^2}{[1 + (n-1)b][2 + (n-3)b]^2} = f, \tag{12}$$

or

$$(x_i^B)^2 = \frac{[1 + (n-2)b]}{(1-b)[1 + (n-1)b]}f,$$

or

$$(p_i^B - c)^2 = \frac{(1-b)[1 + (n-1)b]}{[1 + (n-2)b]}f.$$

Denote the number of firms which satisfies (12) by n^B. It is the equilibrium number of firms at the Bertrand equilibrium under absolute profit maximization. Also we have

$$p_i^B = \sqrt{\frac{[1 + (n^B - 1)b]}{[1 + (n^B - 2)b]}(1-b)f} + c,$$

and

$$x_i^B = \sqrt{\frac{[1 + (n^B - 2)b]}{[1 + (n^B - 1)b]}\left(\frac{f}{1-b}\right)}.$$

4.3 Comparison of the equilibrium prices

Comparing $\tilde{p}_i$ with p_i^B

$$\tilde{p}_i - p_i^B = \sqrt{(1-b)f} - \sqrt{\left[\frac{1+(n^B-1)b}{1+(n^B-2)b}\right](1-b)f},$$

Since $\frac{1+(n^B-1)b}{1+(n^B-2)b} > 1$, we have $\tilde{p}_i - p_i^B < 0$.

Comparing p_i^B with p_i^C,

$$p_i^B - p_i^C = \sqrt{\left[\frac{1+(n^B-1)b}{1+(n^B-2)b}\right](1-b)f} - \sqrt{f}.$$

Since

$$\left[\frac{1+(n^B-1)b}{1+(n^B-2)b}\right](1-b) - 1 = \frac{-(n^B-1)b^2}{1+(n^B-2)b} < 0,$$

we have $p_i^B - p_i^C < 0$.

Therefore, we have shown

$$\tilde{p}_i < p_i^B < p_i^C.$$

4.4 Comparison of the equilibrium numbers of firms

Compare $\tilde{n}$ with n^B. The equation for the equilibrium number of firms in the Bertrand oligopoly under absolute profit maximization is a cubic equation, and its closed-form solution is very complicated. So, implicit comparison is appropriate. Assume that the number of firms at the Bertrand equilibrium under absolute profit maximization and that at the Bertrand equilibrium under absolute profit maximization are equal. Then, from (7) and (12), we have

$$\frac{(1-b)[1+(n-2)b]}{[1+(n-1)b][2+(n-3)b]^2} - \frac{1-b}{[2+(n-2)b]^2} > 0. \tag{13}$$

Since $\frac{1-b}{[2+(n-2)b]^2}$ is a decreasing function of n, (13) means that the equilibrium number of firms under relative profit maximization is smaller than that at the Bertrand equilibrium under absolute profit maximization.

Compare n^C and n^B. Assume that the number of firms at the Cournot equilibrium and that at the Bertrand equilibrium under absolute profit maximization are equal. Then, from (11) and (12) we have

$$\left[\frac{1}{2+(n-1)b}\right]^2 - \frac{(1-b)[1+(n-2)b]}{[1+(n-1)b][2+(n-3)b]^2} > 0. \tag{14}$$

Since $\frac{1-b}{[2+(n-1)b]^2}$ is a decreasing function of n, (14) means that the equilibrium number of firms at the Cournot equilibrium under absolute profit maximization is larger than that at the Bertrand equilibrium under absolute profit maximization. Therefore, we have shown

$$\tilde{n} < n^B < n^C.$$

4.5 Comparison of the equilibrium outputs per firm

Comparing $\tilde{x}_i$ with x_i^B

$$\tilde{x}_i - x_i^B = \sqrt{\frac{f}{1-b}} - \sqrt{\frac{[1+(n^B-2)b]}{[1+(n^B-1)b]}\left(\frac{f}{1-b}\right)}.$$

Since $\frac{1+(n^B-2)b}{1+(n^B-1)b} < 1$, we have $\tilde{x}_i - x_i^B > 0$.

Comparing x_i^B with x_i^C,

$$x_i^B - x_i^C = \sqrt{\frac{[1+(n^B-2)b]}{[1+(n^B-1)b]}\left(\frac{f}{1-b}\right)} - \sqrt{f}.$$

Since

$$\left[\frac{1+(n^B-2)b}{1+(n^B-1)b}\right]\frac{1}{(1-b)} - 1 = \frac{(n^B-2)b^2}{1+(n^B-1)b} > 0,$$

we have $x_i^B - x_i^C > 0$.

Therefore, we have shown

$$\tilde{x}_i > x_i^B > x_i^C.$$

Summarizing the results,

Proposition 1. *Whether firms determine their outputs or prices,*

1. *the equilibrium price when firms maximize their relative profits is lower than the equilibrium price when firms maximize their absolute profits;*
2. *The equilibrium number of firms under relative profit maximization is smaller than the equilibrium number of firms under absolute profit maximization;*
3. *The equilibrium output per firm under relative profit maximization larger than the the equilibrium output per firm under absolute profit maximization.*

The reason why the equilibrium number of firms under relative profit maximization is smaller than that under absolute profit maximization is to be that each firm is more aggressive and produces larger output under relative profit maximization than under absolute profit maximization.

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