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The Short-Run Monetary Equilibrium with Liquidity Constraints

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Abstract

A theoretical framework is presented to characterise the money demand in deregulated markets. The main departure from the perfect capital markets setting is that, instead of assuming that investors can lend and borrow any amount of capital at a single (and exogenously determined) interest rate, a bounded money supply is considered. The problem of capital allocation can then be formulated in actuarial terms, in such a way that the optimal liquidity demand can be expressed as a Value-at-Risk. Within this framework, the monetary equilibrium determines the rate at which a unit of capital is exchanged by a unit of risk, or, in other words, it determines the market price of risk. In a Gaussian setting, such a price is expressed as a mean-to-volatility ratio and can then be regarded as an alternative measure to the Sharpe ratio. Finally, since the model depends on observable variables, it can be verified on the grounds of historical data. The Black Monday (October 1987) and the Dot-Com Bubble (April 2000) episodes can be described (if not explained) on this base.

Key words: Money demand; Monetary equilibrium; Economic capital; Distorted-probability principle; Value-at-Risk.

JEL-Classification: E41, E44, E52, G11.

1 Introduction

Financial institutions are bound to meet regulatory requirements and accordingly maintain reserves in the form of *equity* or *risk-capital* — which is typically provided by shareholders — and exchange cash-balances in *secondary* markets at the *inter-bank* rate of interest. The term *economic capital* is used as well, when the fact is wanted to be stressed, that demanded surpluses can differ from regulatory requirements due to discrepant expectations. Besides, frictions in capital markets are responsible for the appearance of premiums over the risk-free interest rate relying on *default* and *market* risk, as well as on transaction costs. Although the influence of *liquidity* is implicit in the three components just mentioned, there is no consensus about how to measure it or even about its *theoretical* role. After Modigliani and Miller (1958),

many scholars have neglected any liquidity effects, claiming that the only factors preventing investors from attracting all the funds required to carry out solvent projects, other than market imperfections, are unnecessary regulations and aversion to risk. Practitioners, on the other hand, have to deal with credit restrictions and have to face changing scenarios going from periods of high optimism and cheap money, to severe short-falls that sometimes occur all in a sudden and without apparent reasons in the middle of an expansionary cycle. Providing a theoretical framework for the characterisation of *liquidity*, capable of accounting for the behaviour of markets under *normal* circumstances as well as in times of crises, is the main purpose of this paper.

As it is well-known, one of the main assumptions upon which the perfect markets model is built, is that investors can borrow and lend any amount of capital at a single and exogenously determined rate of interest. Let us suppose instead, in the following, that the total money supply (containing any kind of instruments that can perform as *perfect substitutes* for money) cannot surpass a given stock M , which can be regarded as a guarantee that all standing liabilities can be honoured. At equilibrium, the supply and the demand for cash balances must be the same. In this context, *deregulation* — meaning the total freedom to allocate funds and cash-balances — will force investors to *compete* for the available funds when the total needs approaches (or surpasses) the aggregated supply. Moreover — as a general rule — increments and reductions in the stock of money must be respectively accompanied by reductions and increments in the rate of interest attained at equilibrium, for the price of money-substitutes is respectively increased and reduced in each case. The short-run monetary equilibrium can then be characterised by the following equation:

$$M = L \cdot k(r) \quad \text{with} \quad \frac{dk(r)}{dr} < 0 \quad (1)$$

where L and $k(r)$ respectively denote the cash-balance demanded to spend on the transactions of securities and the proportion of funds that is maintained as risk-capital. In a more general context, the function $k(r)$ represents the *preference for liquidity* of the market, while the equilibrium interest rate can be related to the cost of contracting *non-risky* debt (denoted by r_0) plus premiums and transaction costs.

Therefore, variations in the sock of money must be followed by *quantity* and *price* adjustments, respectively performed through changes in the variables L and r , while the predominant effect is determined by the magnitude of the *point-elasticity* of the money demand with respect to the interest rate, defined as $\epsilon_r(r) := \frac{r}{k(r)} \cdot \frac{\partial k(r)}{\partial r}$, so that $\Delta k(r) = \epsilon_r(r) \cdot \Delta r$, $\forall r$. Accordingly, the more *inelastic* the money demand, i.e. the more its elasticity approaches to zero, the more unwilling are investors in exchanging reserves for securities, so that adjustments are mostly performed through the interest rate. By contrast, the more *elastic* the money demand, i.e. the more its elasticity approaches to infinite, the more investors agree to modify their surpluses in order to keep unchanged the level of the interest rate. In the case of *perfect elasticity*, i.e. when $\epsilon_r = \pm\infty$, investors are satisfied at a single level of the interest-rate, and will modify their balances as much as it is required for the rate of interest to be fixed. Under such circumstances, liquidity-preference is *absolute*.

Let us now assume — as will be soon proposed, in *Section 2* — that every investor is the holder of a hedged portfolio whose series of (percentage) capital *profits and losses* ($P\&L$) is represented by a random variable X . Since M and L are respectively determined by lenders

and borrowers *outside* the market, they can be regarded as *control* variables, and hence the characterisation of the monetary equilibrium depends on obtaining a theoretical expression for the money demand function $k(r)$, the variable *of state*. Recall that investors can choose among three different strategies to suppress risk, namely, hedging, insurance and capital cushions, albeit decision-makers trying to avoid the costs to be afforded under a liquidity crisis prefer to avoid hedging. They should then demand equity until the level where they are indifferent between assuming the risk of default by themselves and transferring it to an insurance institution. From the actuarial point of view, such a contract minimises the sum of the costs afforded by the insured and the insurer, respectively represented by the cost of capital $r \cdot k$ and the excess of loss $E[(X + k)_-]$, with $(X + k)_- := -\min\{0, X + k\}$. On these grounds, the *optimal* level of economic capital is related to the optimal *deductible* or *retention* of a corresponding insurance policy. As long as the expectation operator is represented by the *distorted probability principle* (that will be soon defined in *Section 2*), the optimal level of capital, which minimises the cost of bankruptcy plus the cost of capital, is determined at the point where its opportunity cost equals the marginal return on risk, in such a way that the optimal or *rational* money demand is given by the quantile function of the underlying probability distribution.

Within this framework, the role of creditors in secondary markets can be corresponded to that of central managers in decentralised conglomerates, where an internal cost is established for exchanging capital inside the organisation and divisions are allowed to independently decide their cash-balances. Accordingly, in *Section 3*, the cash-balances demanded at different aggregation levels — from single investors, to companies, industries and economic sectors — are obtained by summing up the individual surpluses. Then the aggregated exposure corresponds to the sum of the individual exposures correlated according to the *comonotonic* dependence structure, in such a way that *precautionary* industries rely on the most pessimistic situation, when the failure in any single firm spreads all over the market (see Dhaene et al., 2002). On these grounds, a weaker version of the hypothesis of rational behaviour is proposed in *Section 3*, according to which the states of aggregated markets are not determined by a single rational decision-maker, but are rather described by the equilibrium attained when investors continuously and simultaneously put orders on equity and capital assets trying to build optimal portfolios. The optimal solution can then be approximated by comparing the daily realised profits and losses with the projected balances, while those firms that fail to follow such a strategy will be forced to eventually go on bankruptcy, for they will systematically demand too-much or too-less capital.

In a Gaussian setting, the monetary equilibrium determines a set of combinations of the supply and the demand for money, M and L , the rate of interest r , the expected return μ and the volatility σ . In this way, an alternative measure of risk to the *Sharpe ratio* is obtained in *Section 4*. As the Sharpe ratio, the new measure is expressed as a mean-to-volatility ratio, but instead of suppressing the value of the free-of-default interest rate from the expected return, the ratio of the stock of money to the cash flow spent in transactions is added to it. Such a difference is consistent with the fact that while the Sharpe ratio represents the equilibrium in markets where investors can modify their surpluses to any extent by exchanging balances at a single and constant rate of interest, in the alternative model presented in this paper, demand orders on short-term debt can raise the price of credit depending on the size of the involved funds, the credit quality of borrowers and the

restrictions put to liquidity. Besides, a rate of interest exists, equal to the probability that the capital loss is greater than the relative stock of money, or, in other words, that the cash-guarantee maintained at the aggregated level is insufficient to cover the realised loss. Such a rate corresponds to the return to be obtained when investing one monetary unit on the underlying risk and can accordingly be regarded as the *Internal Rate of Return on Risk (IRRR)*. Alternatively, the *IRRR* can be interpreted as the return to be paid for holding a given uncertain claim instead of investing on a non-risky zero-coupon bond with a predetermined maturity. Hence, for every fixed pair (M, L) , a risk-structure of interest-rates over the plane of expected returns and volatilities exists, providing the price to be asked for exchanging a given risk for equity.

In *Section 5*, the historical evolution during two decades of two representative indexes of the economy of United States, namely, the *DowJones* and the *Nasdaq*, is analysed on the grounds of the model for deregulated markets presented in this paper. Fluctuations in the flows of funds spent on the transactions of securities can then be corresponded to the observed paths of mean returns and volatilities, in accordance to the facts predicted by the model. In particular, the two episodes of liquidity crisis occurred in the period, the *Black Monday*, occurred in October 1987 and the *dot-com bubble*, from April 2000, are clearly distinguished by variations in the preference for liquidity and the *IRRR* — which are consistent with the underlying risk-parameters. Consequently, liquidity crises are perfectly possible in deregulated markets when these are confronted to restrictions on liquidity, as long as the variables involved in the monetary equilibrium follow certain patterns. Notably, such a result enters in no contradiction with the behaviour of *rational* decision-makers who minimise their losses on an individual base, and only the magnitude of the involved adjustments does depend on the attitude towards risk, a well as on the amount and kind of restrictions and regulations in financial markets. *Section 6* concludes the paper.

2 Preference for Liquidity as the Optimal Retention

Investors holding financial securities with random outcomes are exposed to unknown balances equal to the differences between the market values of outstanding assets and liabilities. When positive, such a surplus can be lent at the *overnight* interest rate in order to avoid keeping idle money, while, by contrast, short-term debt can be attracted when a net loss is suffered. Modigliani and Miller (1958) have noticed that, since in *perfect* capital markets such operations can be performed without restrictions, *rational* decision-makers — who are assumed to maximise value — should demand no equity, for it is costly and provides no benefit. However, frictions in capital markets can prevent this mechanism to fully operate in practice. As a matter of fact, premiums are normally charged on market prices depending on *credit-ratings* and the size of loans, evidencing that the cost of restructuring debt can be incremented when the magnitude of losses increases up to certain levels. Those firms that cannot afford such extra costs will be forced to go on bankruptcy. Under such circumstances, the financial position and so the performance of companies is affected by the preferred surpluses and consequently, *rational* decision-makers demand risk-capital (see Merton, 1997). Moreover, the prevailing level of the rate of interest provides a *measure* of the liquidity conditions, in such a way that the higher the level of the interest rate, the more difficult is

the access to debt and so the more *illiquid* is the market.

Let us then assume that every investor holds a portfolio combining equity and a residual exposure X , represented by the series of (random) capital *Profit and Losses* ($P\&L$) of the value of the firm V (equal to assets minus liabilities), with $X_t = \frac{V_t - V_{t-1}}{V_{t-1}}$. Let the magnitude of the guarantee be expressed as a proportion k of the volume of transactions, equal to the nominal demand for money L , such that $K = L \cdot k$. In other words, the cash-balance demanded for *precautionary motives* is expressed as a fraction of the funds demanded for the *transactions motive*, while the factor k represents the *relative* money demand for precautionary motives (see Howells and Bain, 2005). Moreover, since each monetary unit invested on risk produces the percentage return X , the total $P\&L$ accrued from the fund is equal to $L \cdot X = L \cdot \Delta V$. According to Dhaene et al. (2003) and Goovaerts et al. (2005), within a class of firms facing the same cost of capital r , the optimal level of economic capital can be determined by minimising the total bankruptcy costs:

$$\min_k E_\theta [(X + k)_-] + r \cdot k \quad (2)$$

where the parameter θ accounts for the state of expectations of the decision-maker — dependent on information and the risk-aversion profile. From the actuarial point of view, the term $E_\theta [(X + k)_-]$ represents the *excess of loss* suffered by the issuer of an insuring policy (or *layer*, see Dhane et al., 2006) covering the loss of the mutual fund X , where the *retention* or *deductible* is equal to k . The *optimal* level of surplus, as determined by *Equation 2*, is then corresponded to the *optimal* retention-level, in the sense that it minimises the total cost to be paid by the insurer. Therefore, the optimal deductible characterises the surplus at which decision-makers are indifferent between establishing an insurance contract and assuming the cost of insolvency (which can be estimated by the term $E_\theta [(X + k)_-]$) by themselves. Choosing a different cash-balance can only reduce the price of the policy at the time that the cost of the guarantee is augmented at a higher rate or vice-versa. Allowing different estimations of θ due to market imperfections provides a justification to the appearance of a demand for insurance (see Venter, 1991, and Wang et al., 1997).

In order to obtain a solution for the optimisation problem of *Equation 2*, an expression for the expectation operator E_θ is required. In lines with Wang (1995), the following risk-principle will be used:

$$E_\theta[X] = \int x dF_{\theta,X}(x) = \int T_{\theta,X}(x) dx = \int T_X(x)^{\frac{1}{\theta}} dx \quad (3)$$

where $F_{\theta,X}$ and $T_{\theta,X}$ respectively denote the *distorted cumulative* and the *distorted tail* probability functions representing the underlying risk, which are defined as $T_{\theta,-X}(k) = P_\theta \{-X > k\} = P_\theta \{X \leq -k\} = F_{\theta,X}(-k) = \int_{-\infty}^{-k} dF_{\theta,X}(x)$. Consequently, the more the aversion-to-risk of decision-makers (i.e. the higher the magnitude of θ), the higher the price of risk, while the more risk-lover the decision-maker, the lower the price of risk. *Risk-neutral* decision-makers are characterised by $\theta = 1$, in which case the distorted probability principle is equal to the traditional expectation operator. In the general case, the risk-principle can be defined based on a given *distortion* function $\varphi : [0, 1] \rightarrow [0, 1]$ (see Wang et al., 1997):

$$E_\varphi[X] = \int x dF_{\varphi,X}(x) = \int T_{\varphi,X}(x) dx = \int \varphi(T_X(x)) dx$$

Risk-loving and aversion-to-risk attitudes are then respectively characterised by convex and concave distortion functions, while risk-neutrality is represented by the identity function $\varphi(x) = x \forall x$. This is, in fact, the approach followed by Dhaene et al. (2003) and Goovaerts et al. (2005) when defining their mechanism for capital allocation. The risk-principle of *Equation 3* corresponds to the case when the *proportional-hazards* distortion $\varphi_\theta(p) = p^{\frac{1}{\theta}}$ is introduced. This choice is made for the sake of simplicity. However, all the analysis that follows could be equivalently established in terms of distortion functions.

As long as the expectation operator is defined according to *Equation 3*, the excess of loss can be written as:

$$E_\theta [(X + k)_-] = - \int_{-\infty}^{-k} (x + k) \cdot dF_{\theta,X}(x)$$

Then by applying the Leibniz integral rule for the differentiation of a definite integral whose limits are functions of the differential variable,¹ the following expression for the optimal cash-balance is obtained by applying Lagrange optimisation:

$$\frac{\partial}{\partial k} E_\theta [(X + k)_-] + r = -T_{\theta,-X}(k^*) + r = 0$$

Hence decision-makers demand reserves up to the point where its opportunity cost equals the marginal return of risk and the *optimal* money demand is given by the inverse function of the tail-probability:

$$k_{\theta,-X}(r) = T_{\theta,-X}^{-1}(r) = F_{\theta,-X}^{-1}(1 - r) \quad (4)$$

or equivalently:

$$k_{\theta,-X}(r) = T_{-X}^{-1}(r^\theta) = F_{-X}^{-1}(1 - r^\theta)$$

According to this specification, the money-demand follows a non-increasing and continuous path — as long as the probability function describing risk is continuous. Besides, the minimum and maximum levels of surplus are respectively demanded when $r \geq 1$ and $r \leq 0$. Discrepancies relative to preferred cash-balances can thus be explained on the basis of the underlying risks, expectations and the opportunity cost of capital.

Notice that although in *Equation 4* the expectations parameter θ eventually affects the cost of capital, from the mathematical point of view, it is not this variable but rather the underlying probability distribution what is distorted — a fact that has been stressed by choosing the notation $F_{\theta,X}$ and $T_{\theta,X}$. The reader should then not confuse this setting, where expectations distort the *risk perceptions* of decision-makers, with that of heterogeneous estimations of the cost of capital. In fact, a main hypothesis that will be established soon in *Section 3*, when characterising the aggregate money demand and the monetary equilibrium, is that investors can lend and borrow capital at a single interest rate r , which does not

¹According to this rule:

$$\frac{\partial}{\partial z} \int_{u(z)}^{v(z)} f(x, z) dx = \int_{u(z)}^{v(z)} \frac{\partial f(x, z)}{\partial z} dx + f(v(z), z) \cdot \frac{\partial v(z)}{\partial z} - f(u(z), z) \cdot \frac{\partial u(z)}{\partial z}$$

necessarily coincide, however, with the risk-free interest rate r_0 . Thus, the model describes the situation of markets where lenders cannot fully observe the composition of the portfolios held by borrowers (due to the *opacity* of financial decisions, as suggested by Merton, 1997) and cannot efficiently discriminate through the price of liabilities. Under such circumstances, creditors will prefer to behave as *price-takers* and to assign the cost of capital to the different markets — or *classes* of securities — in accordance to the recommendations established by regulators, researchers, specialised organisations (like credit rating agencies), the media and the orders putted by their colleagues.

3 The Short-Run Monetary Equilibrium

Let us now characterise the total *P&L* obtained when adding the returns at different levels of aggregation — going from small investors and firms, to big companies, conglomerates, industries and economic sectors — when the individual capital *P&L* and expectations are respectively represented by the series of random variables X_1, \dots, X_n and informational parameters $\theta_1, \dots, \theta_n$. It is common knowledge that a benefit due to *diversification* arises when individual exposures show dependence to some extent, for losses in some components can be compensated by gains obtained in others, in such a way that the amount of capital required for *diversified* portfolios is supposed to be lower than the sum of the surpluses of individual funds when treated as stand-alone entities. However, central managers can take advantage of diversification only if a *centralised* mechanism is established — i.e. only if they can impose the surpluses to be maintained by subsidiaries. Allowed to decide the levels of risk-capital according to their will, as it is the case in *decentralised* organisations, the total cash-balance raised by the conglomerate will be equal to the sum of divisional contributions. Then the question is under what circumstances the level of reserves demanded at the aggregate level is equal to the sum of the cash-balances preferred by individuals. Given that such surpluses are expressed in terms of the *quantiles* of the underlying probability distributions (as in *Equation 4*), we would specifically like to know whether the inverse probability function of the sum $X = X_1 + \dots + X_n$ is equal to the sum of the quantiles of the marginal distributions.

As proved by Dhaene et al. (2002), the property of the sum of the quantiles mathematically characterises the *comonotonic* dependence structure, where *comonotonicity* represents a case of extreme dependence, when no diversification effect can be attained by pooling risks together, for all of them move in the same direction. In fact, given a random vector (X_1, \dots, X_n) with marginal cumulative distribution functions $(F_{X_1}, \dots, F_{X_n})$, the *comonotonic* random vector (X_1^c, \dots, X_n^c) is mathematically defined in such a way that if U denotes the random variable *uniformly* distributed in the interval $[0, 1]$, such that $F_U(u) = u \forall u \in [0, 1]$, $F_U(u) = 0 \forall u < 0$ and $F_U(u) = 1 \forall u > 1$, the following identity holds in distributions:

$$(X_1^c, \dots, X_n^c) = (F_{X_1}^{-1}(U), \dots, F_{X_n}^{-1}(U))$$

Equivalently, the vector (X_1^c, \dots, X_n^c) is said to be *comonotonic* if a random variable Z exists, as well as a set of *non-decreasing* functions h_1, \dots, h_n , such that:

$$(X_1^c, \dots, X_n^c) = (h_1(Z), \dots, h_n(Z))$$

In this way, the realisation of a single event (which can be either related to the uniform random variable U or the risk Z) simultaneously determines all the components of any *comonotonic* random vector. Moreover, since both the series of functions $(F_{X_1}^{-1}, \dots, F_{X_n}^{-1})$ and (h_1, \dots, h_n) are *non-decreasing*, all the components of the vector (X_1^c, \dots, X_n^c) *move in the same direction*. Hence, as already stated, the quantile function of the sum of the components of any comonotonic random vector is equal to the sum of the component quantile functions, a fact that supports the use of the *comonotonic dependence structure* to characterise the aggregate money demand in deregulated markets — according to the terms established in the previous paragraph.

In this context, as long as capital is supplied by financial intermediaries at a single interest rate r , the following expression is obtained for the aggregate money demand:

$$k_{\theta, -X}(r) = T_{\theta, -X}^{-1}(r) = \sum_{i=1}^n T_{\theta_i, -X_i}^{-1}(r) \quad (5)$$

where the process of capital P&L of the *market* portfolio and the aggregated effect of the distortions introduced by investors are respectively described by the *comonotonic sum* $X = X_1^c + \dots + X_n^c$ and the informational parameter θ , and where $T_{\theta, -X} = \left(\sum_{i=1}^n T_{\theta_i, -X_i}^{-1}\right)^{-1}$ denotes the distribution function of the comonotonic sum when the marginal returns are described either by the cumulative or by the tail probability functions, respectively denoted by $(F_{\theta_1, -X_1}, \dots, F_{\theta_n, -X_n})$ and $(T_{\theta_1, -X_1}, \dots, T_{\theta_n, -X_n})$. Thus *precautionary* industries rely on the most pessimistic case, when the failure in any single firm spreads all over the market. Such a result can be regarded as the price to be paid for decentralisation — for central managers can take advantage of diversification only if a centralised mechanism is implemented.

Having characterised the money demand of the market, the determination of the monetary equilibrium is straightforward. Indeed, since the demanded and the supplied cash-balances must be the same at equilibrium, letting M denote the total supply of money and letting L (as usually) denote the total amount of funds invested on securities, the following equation must hold:

$$M = K_{\theta, -X}(r) = L \cdot k_{\theta, -X}(r)$$

Replacing the liquidity-preference function $k_{\theta, -X}(r)$ according to *Equation 5*, we obtain that:

$$M = T_{\theta, -X}^{-1}(r) \cdot L \quad \Leftrightarrow \quad m := \frac{M}{L} = T_{\theta, -X}^{-1}(r) \quad \Leftrightarrow \quad r = T_{\theta, -X}(m) \quad (6)$$

The discount factor $T_{\theta, -X}^{-1}(r)$ can then be regarded as the *market-price of risk*, for it corresponds to the rate at which a unit of investment on risk can be exchanged for a unit of money in the market. The condition $T_{\theta, -X}^{-1}(r) < 1$ is consistent with the fact that holders of risky assets rely on *still unrealised* payments — whose magnitude overcomes the volume of transactions guaranteed by the stock M . Within this context, the rate $r = T_{\theta, -X}(m)$ represents the return obtained when investing one monetary unit on the underlying risk and can be accordingly regarded as the *Internal Rate of Return on Risk (IRRR)*, while the ratio

$m = \frac{M}{L}$ represents the *supply-to-demand for transactions ratio*, which can be alternatively regarded as a *relative* or *effective* money-supply.

Thus, while the *Internal Rate of Return (IRR)* of a zero-coupon bond represents the opportunity cost of receiving a cash-flow at some future date instead of today (see, for example, Hull, 2000), the IRRR can be interpreted as the return to be paid for holding a given uncertain claim instead of investing on a *non-risky* zero-coupon bond with a predetermined maturity. Both coefficients can then be considered as measures of *liquidity*, and we can thus explicitly define the *premium for liquidity* over a benchmark (risk-free) rate of interest r_0 :

$$\tau_{\theta,X} = IRRR - r_0 = T_{\theta,-X}(m) - r_0 = T_{\theta,-X}\left(\frac{M}{L}\right) - r_0 \quad (7)$$

Alternatively, the IRRR corresponds to the probability that the current stock of money will suffice to cover the expected imbalance of the market portfolio. In fact, the lower the IRRR, the lower the tail-probability that the excess of loss of the market portfolio was greater than the real money-supply. Such a situation is consistent with (overnight) markets where investors face less difficulties to adjust their end-of-day balances, or equivalently, more *liquid* markets. By contrast, the tail-probability of the market portfolio increases with the IRRR, and so markets are more *illiquid* the greater the IRRR. Moreover, the higher the money spent in transactions with respect to the stock of capital in a given market, the higher its corresponding IRRR, since in this case the expansion of output is, to a greater extent, *nominal* in nature or, equivalently, more *inflationary*. Conversely, less inflated funds, characterised by greater supply-to-demand ratios, are assigned lower internal returns. Within this framework, the IRRR determines a discount factor for inflationary trends.

We can now try to assess whether the traditional belief that markets can be stimulated or *heated up* by diminishing the level of the interest rate — i.e. by raising the stock of money M — and contracted or *cooled down* by augmenting the risk-free interest-rate — i.e. by decreasing the stock of money M — can be replicated in the new model. In fact, given that a supply-to-demand ratio m_0 can be found, with $r_0 = T_{\theta,-X_0}(m_0)$, where X_0 represents the series of *P&L* in primary markets, the rate of interest r_0 can be also interpreted as an IRRR, while the liquidity-premiums can be expressed in terms of the corresponding exposures and relative supplies:

$$\tau_i = T_{\theta,-X_i}(m_i) - r_0 = T_{\theta,-X_i}(m_i) - T_{\theta,-X_0}(m_0)$$

Therefore, only those portfolios with positive premiums provide space for financial intermediation and add value to their firms, and the more value is added, the greater the magnitude of the liquidity-premium. Consequently, more credit will be provided by intermediaries to projects offering higher liquidity-premiums and more transactions will be established for that kind of securities, or in other words, such markets can be regarded as more *excited* or *heated up*. By contrast, markets offering lower liquidity-premiums are *cooled down*. In fact, credit becomes scarcer as the IRRR decreases (for the space for financial intermediation is reduced in this way) and eventually disappear at all when $IRRR < r_0$. We can properly say that the market is *frozen* under such circumstances. On these grounds, the *IRRR* can be regarded as a measure of the *temperature* of markets.

The relationship above can be also established in general terms, for as long as m_1, \dots, m_n denote the relative money-demands for the available securities, the *relative* premiums for liquidity are given by:

$$\tau_{i,j} = T_{\theta, -X_i}(m_i) - T_{\theta, -X_j}(m_j)$$

When markets are found in a situation apart from equilibrium, flows of capital must be produced until the internal returns offered by traded securities are equalised. This is the only situation where intermediaries are indifferent about which securities to include in their portfolios, for in this case the market prices $T_{\theta, -X_i}^{-1}(IRRR)$ are the same for every risk — and no capital profits can be obtained by modifying the composition of portfolios. In other words, every relative premium should be the same at equilibrium — premium differentials are only possible in transient periods. The persistence of imbalances is a signal of frictions and market imperfections.

Back to *Equation 6*, notice that while the supply-to-demand ratio is affected by creditors and investors, who respectively control the supply and the demand for money, the discount factor $T_{\theta, -X}^{-1}(r)$ provides a measure of the market's response, or in other words, while the variables M and L are *exogenously* determined, the informational parameter θ and the statistical description of the aggregated portfolio X are intrinsic characteristics of the market and so they are *endogenously* determined. The only variable involved in the monetary equilibrium which is not directly observable is the IRRR, though its value is uniquely determined once the rest of the parameters are fixed. Changes in the stock of money must then be followed by adjustments in the transacted volumes as well as in the IRRR and the market conditions. There is ample evidence suggesting that probability distributions and market beliefs are indeed expected to change in response to monetary flows. When parametric risk-descriptions are introduced, such a fact is confirmed by the existence of variable parameters in practice.² As a consequence, *repricing* is constantly required — at least while the price of risk is required to depend on the underlying exposure — and sometimes violent adjustments are needed.

Within this framework, money is continuously circulating or *flowing* to securities, mutual funds and investment projects in general — according to the will of investors trying to maximise capital profits — in such a way that the market determinants are continuously modified, affecting in turn the preferences of decision-makers and so the volumes of transactions. Moreover, every equilibrium can be attained by a set of combinations of transacted volumes and risk-parameters. The monetary equilibrium is thus the result of the simultaneous and constant interaction of central banks, investors and markets, and multiple equilibria are allowed. Accordingly, the natural state of markets is to remain in constant evolution and followed paths are not reversible in general. Finally, the existence of a *long-term equilibrium* is only possible if the expectations of decision-makers and the probability distributions describing risks are *stationary*, for in this case alone disturbances in credit conditions can be interpreted as natural adjustments occurring in *transient* periods.

²As a matter of fact, models of variable volatility, like ARCH and GARCH, are very popular in banking and insuring industries. See Hull, 2000.

4 The Market-Price of Risk and the Risk-Structure of Interest-Rates in Gaussian Markets

Let us assume in the following that individual exposures are distributed as Gaussians with means μ_1, \dots, μ_n and volatilities $\sigma_1, \dots, \sigma_n$, while the contributions of economic sectors to the aggregated money demand for transactions L are given by the coefficients $\lambda_1, \dots, \lambda_n$, with $0 \leq \lambda_i \leq 1 \forall i$, such that $L_i = \lambda_i \cdot L$ and $L = L_1 + \dots + L_n$, or equivalently, $\lambda_i = \frac{L_i}{L_1 + \dots + L_n}$. Volatilities are expressed as proportions of the levels of income and can be interpreted as the volatilities of different funds as well as the distorted volatilities of the same Gaussian exposure, or some intermediate case. Under such conditions, the series of capital $P\&L$ of the *market* portfolio, equal to the comonotonic sum of individual exposures, is also a Gaussian random variable, whose mean and volatility are given by their corresponding weighted averages (as demonstrated by Dhaene et al., 2002). Consequently, since the individual returns $-X_1, \dots, -X_n$ are also distributed as Gaussian, with the same volatilities but mean returns with opposite sign, i.e. $-\mu_1, \dots, -\mu_n$, we obtain from *Equation 4* that the money-demand of the economy is equal to:

$$k_{\mu, \sigma}(r) = \sigma \cdot \Phi^{-1}(1 - r) - \mu \quad (8)$$

with:

$$\mu = \sum_{i=1}^n \lambda_i \cdot \mu_i \quad \& \quad \sigma = \sum_{i=1}^n \lambda_i \cdot \sigma_i$$

where Φ denotes the cumulative distribution function of the *standard Gaussian* distribution, whose mean and volatility are respectively equal to zero and one.

From *Equations 6* and *8*, the following identity determines the levels of the internal return r , the relative money supply m and the risk-parameters μ and σ compatible with the short-run monetary equilibrium in Gaussian markets:

$$m = \sigma \cdot \Phi^{-1}(1 - r) - \mu$$

or equivalently:

$$R := \frac{\mu + m}{\sigma} = \Phi^{-1}(1 - r) = \Psi^{-1}(r) \quad \Leftrightarrow \quad r = \Psi\left(\frac{\mu + m}{\sigma}\right) = \Psi(R) \quad (9)$$

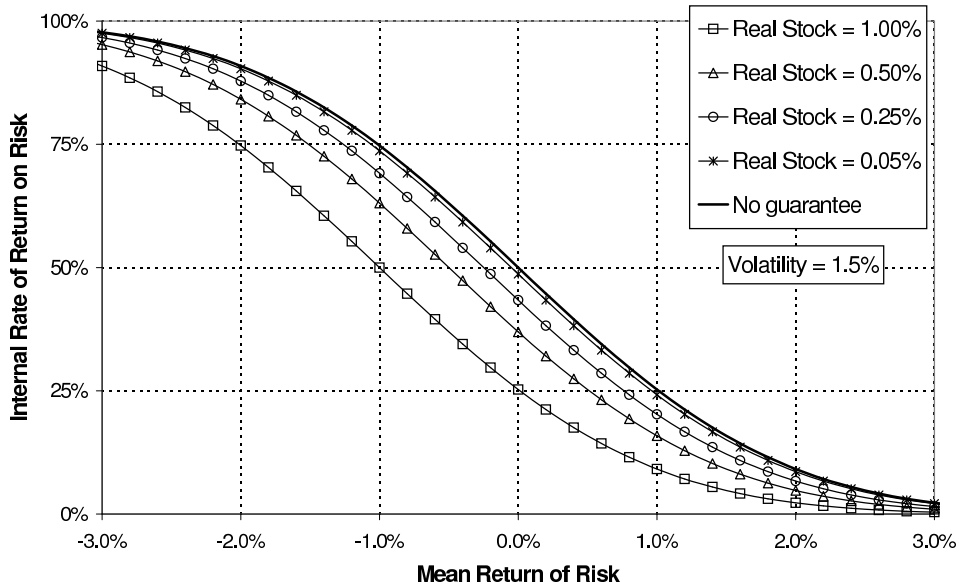
The *mean-to-volatility* ratio R represents the rate at which investors agree to exchange the sure flow $(\mu + m)$ for the uncertain return $(X + m)$, and can accordingly be regarded as an alternative measure of risk to the *Sharpe* ratio:

$$\text{Sharpe Ratio} := \frac{\mu - r_0}{\sigma}$$

where r_0 represents the return offered by the benchmark portfolio. As it is well known, the Sharpe ratio is equal to the slope of a straight line relating the expected return of the fund with its volatility, such that $\mu = r_0 + S \cdot \sigma$ (see Sharpe, 1964). Such a result depends on the assumption that the current stock of money suffices to honour all contracted liabilities,

so that investors can borrow or lend capital without restrictions at the interest-rate r_0 . By contrast, inflationary and deflationary trends (respectively characterised by increasing and diminishing m) induce the market-price of risk R to rise and to decrease respectively. Within a class of risks with the same market-price, different real stocks of money are necessarily corresponded to different risk-parameters. Thus, for the prices of risks to remain stable with varying m , adjustments in the mean returns and volatilities of the traded securities are required.

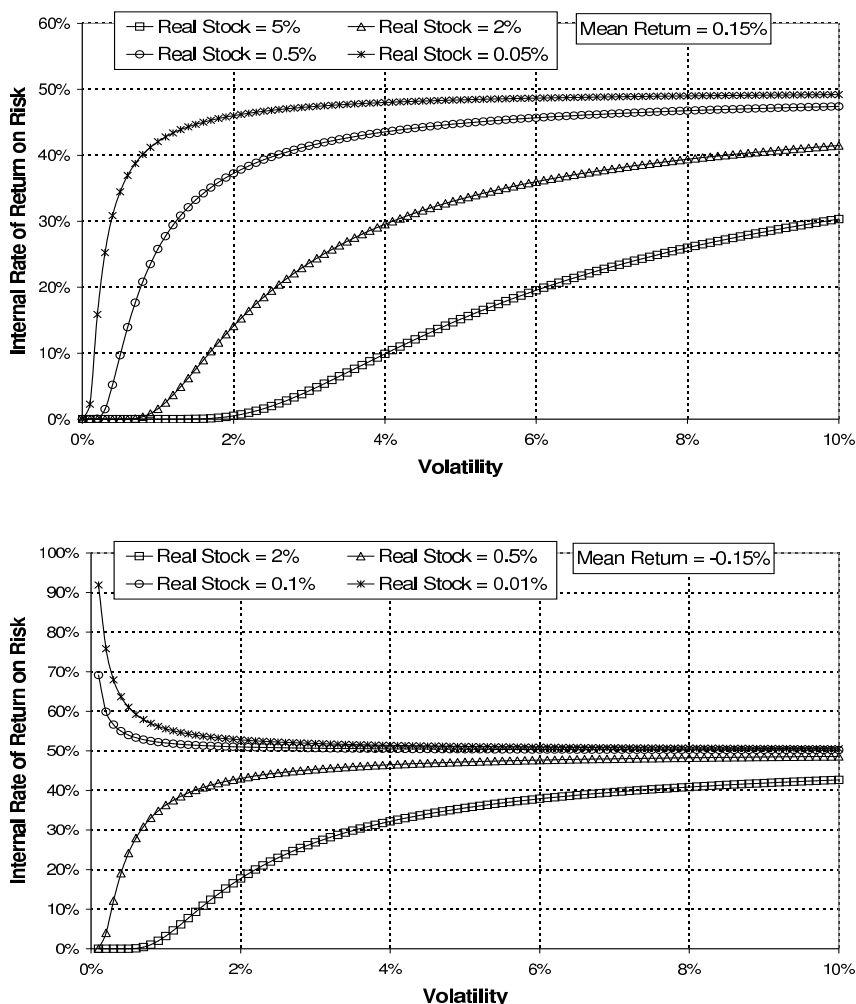
Figure 1: Risk-Structure of Interest Rates as a Function of the Expected Return.



Given a fixed stock of money m , a *risk-structure* of interest rates on the plane of mean returns and volatilities is determined by *Equation 9*, where each rate represents the return to be asked for receiving a sure cash-flow instead of an uncertain payment — while, as stated in the previous section, each rate in the *term-structure* of interest rates represents the return to be asked for receiving a cash-flow today instead of at a future date and the corresponding *discount factor* gives the price of a *zero-coupon bond*. As depicted in *Figure 1*, given a stock of money and a level of volatility, the expected return and the internal rate of return on risk are inversely related. Recall that the market price of risk is incremented when the IRRR decreases and vice-versa. Therefore, since the curve is moved to the left when the surplus is increased, within a class of securities showing the same variability, the market price of risk (or equivalently, the level of the IRRR) can be maintained while the mean return is falling only if the relative supply of reserves is incremented. This result provides a theoretical explanation of why investors show liquidity preference, namely, the same market price (or the same internal return) can be assigned to risks with different expected returns only if a higher cash-balance is provided to the portfolio containing the lower expected return.

The dependence of the risk-structure on volatility is shown in *Figure 2*. As depicted in the upper graph of *Figure 2*, within a class of securities offering the same *positive* expected return, the IRRR rises with variability, at the time that for every level of volatility, the

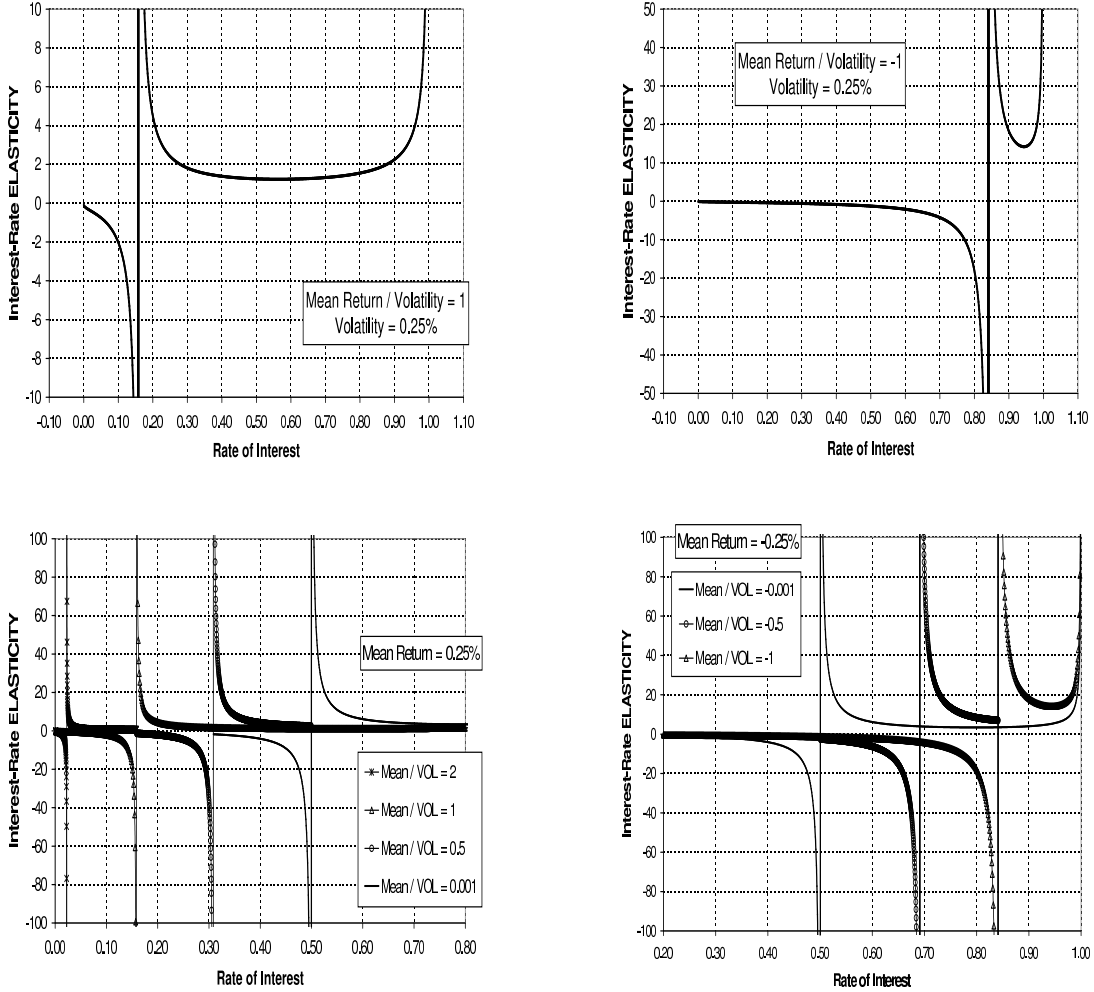
Figure 2: Risk-Structure of Interest Rates as a Function of Volatility.



IRR decreases with the real stock of money. Consequently, the market price of risk can be inflated both by incrementing liquidity and by controlling the variability of income — relations that are compatible with economic intuition. In the lower graph of *Figure 2*, the risk-structure is shown for a class of funds offering a *negative* expected return equal in magnitude to the return used in the upper graph. Notice that the curve presents the same pattern in both situations, as long as the real stock of money is greater in magnitude than the expected loss — although the IRR is lower for negative expected returns. However, the relationship between IRRs and volatilities is reverted when the surplus is lower than the expected loss, i.e. when the money-supply *does not suffice* to honour all outstanding liabilities, in such a way that higher IRRs (or lower market-prices) are observed for lower volatilities. In other words, the higher the volatility, the more investors prefer to invest in cash instead of risk, and so the more expensive is risk with respect to money, in such a way that lower IRRs are obtained for higher volatilities and vice-versa. The IRR is *infinite* (or the market price is equal to zero) when instability is vanished. Moreover, the asymptote

separating both trends at the axis $r = 0.5$ confirms that volatility tends to infinite when the IRRR tends to 0.5, no matter from where we approach to this value, i.e. $\sigma \uparrow +\infty$ both when $r \uparrow 0.5$ and $r \downarrow 0.5$. Hence, high variability can be predicted around this value.

Figure 3: Interest-Rate Elasticity of the Money-Demand.



In order to assess the instability of the monetary equilibrium, let us analyse the point-elasticity of the money demand with respect to the rate of interest. An explicit expression can be given for such elasticity in the model. In fact, let ϕ denote the *density* probability function of the standard Gaussian distribution, with $\phi(x) = (2\pi)^{-\frac{1}{2}} \cdot \exp\left(-\frac{x^2}{2}\right)$. Then, from *Equation 8*, the following expression is obtained for the *point interest-rate-elasticity* of the money demand — representing the percentage change of demanded cash balances in response to a percentage movement of the rate of interest:

$$\epsilon_r\left(r, \frac{\mu}{\sigma}\right) = \frac{r}{k_{\mu,\sigma}(r)} \cdot \frac{dk_{\mu,\sigma}(r)}{dr} = -(2\pi)^{\frac{1}{2}} r \exp\left(\frac{[\Phi^{-1}(1-r)]^2}{2}\right) \cdot \left[\Phi^{-1}(1-r) - \frac{\mu}{\sigma}\right]^{-1} \quad (10)$$

since for any level of probability $p = \Phi(x)$:

$$\phi(x) = \frac{\partial\Phi(x)}{\partial x} \implies \frac{\partial\Phi^{-1}(p)}{\partial p} = \frac{1}{\phi(\Phi^{-1}(p))}$$

The interest-rate-elasticity is then determined by the rate of interest and the *mean-to-volatility* ratio $\frac{\mu}{\sigma}$. The sign of the variation of the money demand with respect to the interest rate can then be explicitly determined:

$$\epsilon_r \left(r, \frac{\mu}{\sigma} \right) < 0 \Leftrightarrow \Phi^{-1}(1-r) - \frac{\mu}{\sigma} > 0 \Leftrightarrow r < 1 - \Phi \left(\frac{\mu}{\sigma} \right) = \Psi \left(\frac{\mu}{\sigma} \right) =: r_{\mu,\sigma} \quad (11)$$

where Ψ denotes the tail-probability function of a standard Gaussian distribution. Thus, as depicted in *Figure 3*, the interest rate elasticity of the money demand is negative and decreasing in r when $r < r_{\mu,\sigma}$, and it is positive with an U-shape when $r > r_{\mu,\sigma}$. Since the interest rate elasticity converges to $-\infty$ and $+\infty$ when the internal return converges to $r_{\mu,\sigma}$ from the left and from the right respectively, i.e. $\epsilon_r \downarrow -\infty$ when $r \uparrow r_{\mu,\sigma}$ and $\epsilon_r \uparrow +\infty$ when $r \downarrow r_{\mu,\sigma}$, an asymptote is produced at the point $r = r_{\mu,\sigma}$, which is located to the left of the axis $r = 0.5$ when the mean-to-volatility ratio is greater than zero, and to the right of the axis $r = 0.5$ when this ratio is less than zero. Moreover, the greater the magnitude of the mean-to-volatility ratio, the more the asymptote $r = r_{\mu,\sigma}$ approaches to axis $r = 0$ and $r = 1$ when the ratio is greater and lower than zero respectively. Within this framework, the level $r = r_{\mu,\sigma}$ represents a *turning-point* point in capital markets, since the demand for money is respectively restrained and stimulated with r when $r < r_{\mu,\sigma}$ and $r > r_{\mu,\sigma}$.

In fact, the interest-rate $r = r_{\mu,\sigma}$ determines a change in the sign of the liquidity-preference function, or in other words, a transition from a state of the market where cash-balances are mostly maintained for precautionary purposes, to a state where cash is actually used to cover current imbalances:

$$k_{\mu,\sigma}(r) > 0 \Leftrightarrow r < r_{\mu,\sigma} \quad \text{and} \quad k_{\mu,\sigma}(r) \leq 0 \Leftrightarrow r \geq r_{\mu,\sigma}$$

When $r > r_{\mu,\sigma}$, the opportunity cost of capital is high enough to make (free of default) lending more attractive than risk taking — more risk is substituted for equity — or in other words, risk has become so expensive (in relative terms to money) that investors *cannot afford it* and accordingly prefer to demand more money even when its price is raising, as it is the case with *Giffen's goods*. On these grounds, the point $r = r_{\mu,\sigma}$ determines a transition from a *liquid state*, where credit is available and positive balances are attained, to an environment where negative balances are mostly obtained and most of the portfolios become *insolvent*, so that the credit-supply is restricted. It is important to realise that first negative cash-balances are obtained, which force investors to use their own reserves, so that it is more difficult to find short-run funds by exchanging balances with other intermediaries. Up to this moment, the total money supply provided by (pure) creditors has not been altered, but it will once signals appear of massive default and the demand for cash-balances raises due to the fail in traditional means of attracting liquidity by overnight operations. Hence, liquidity literally *evaporates* when $r > r_{\mu,\sigma}$, so that $r = r_{\mu,\sigma}$ can be regarded as a *boiling-point*.

In conclusion, the *IRRR*, μ and σ can be regarded as *physical* properties of markets, which at equilibrium are connected to each other according to *Equation 9* for any given

level of the relative stock of money $m = \frac{M}{L}$, a variable that is simultaneously controlled by lenders and borrowers. In practice, markets are most likely to be found in the form of *mixed-phase* regimes, in such a way that while some investors holding portfolios satisfying $r > r_{\mu,\sigma}$ are willing to lend their surpluses overnight at the interest-rate r , others holding portfolios satisfying $r < r_{\mu,\sigma}$ are obliged to borrow in order to avoid insolvency. This fact sustains the appearance of capital markets for the exchange of cash-balances among investors and financial institutions. Besides, the market price of risk (as defined in *Equation 9*) respectively attains positive and negative levels when $r < 0.5$ and when $r > 0.5$, and accordingly, inflows and outflows of capital are respectively expected in the former and the later case. Therefore, the intersections of the IRRR with the axe $r = 0.5$ are corresponded to changes in the sign of the capital flows, thus giving a theoretical explanation for economic cycles.

5 Twenty Years in the History of the *DowJones* and the *Nasdaq* Composite Indexes

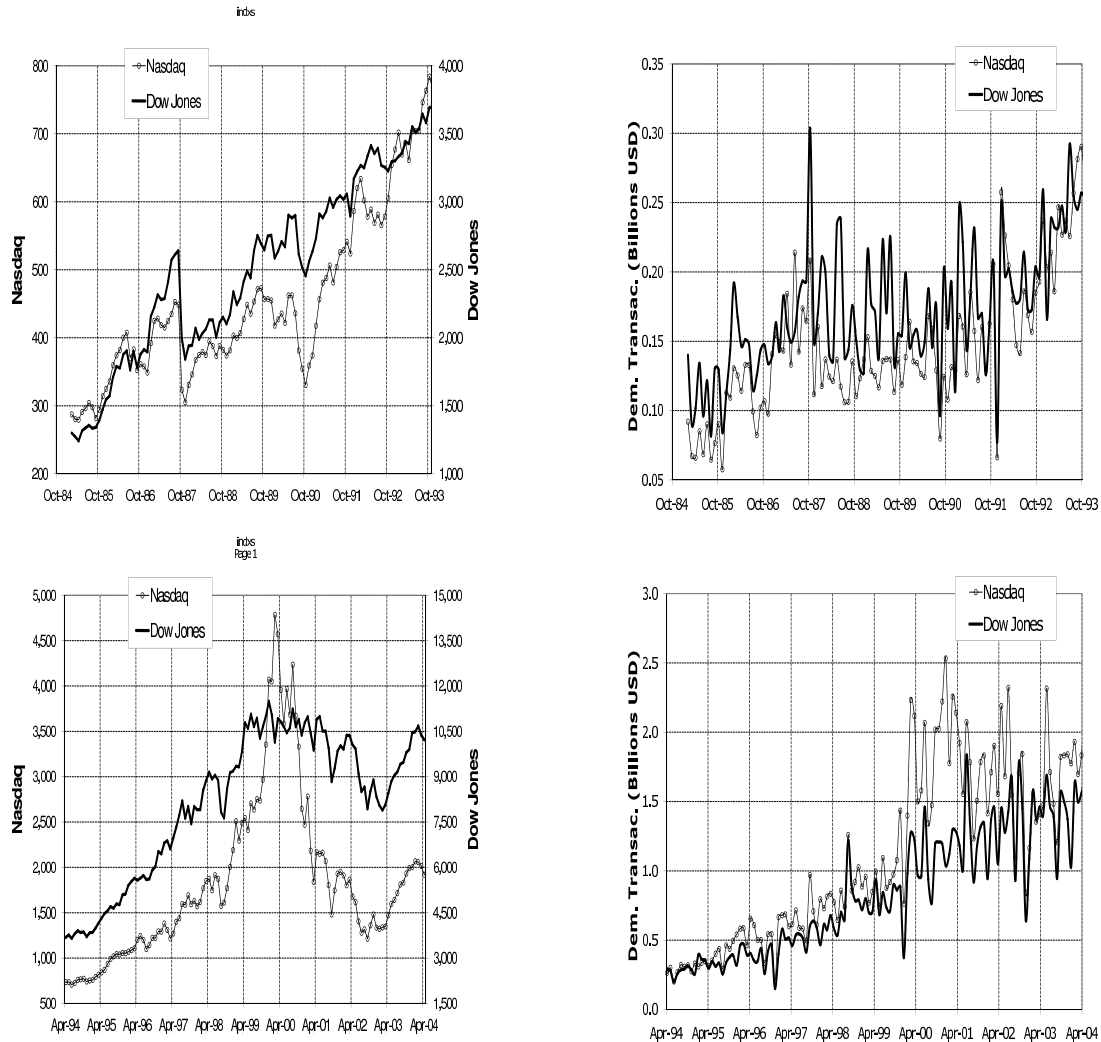
As established in *Equation 9*, except for the IRRR, the monetary equilibrium only involves *observable* variables under Gaussian parametrisations. Let us then investigate how the statistical evidence provided by the series of *P&L* of two representative indexes of the economy of United States, namely, the *DowJones* and the *Nasdaq*, can be interpreted on the grounds of the theoretical framework proposed in this paper. As it is well-known, the *DowJones* is computed as the average of the stock prices of 30 of the largest and most held companies in the United States and so it is regarded as a measure of the performance of the industrial sector. Instead, the *Nasdaq* is specialised on firms belonging to the technological sector. The daily values of both indexes and the amounts of funds daily spent on transactions in the corresponding sectors are respectively depicted in the left and the right columns of *Figure 4*. The data have been obtained from $\langle \text{http} : // \text{www.yahoo.com/finance} \rangle$. Two periods will be separately analysed, the first one going from *October 1984* to *October 1993* and the second one going from *April 1994* to *April 2004*, respectively depicted in the upper and the lower rows of *Figure 4*.

Two episodes of crisis can be clearly noticed in *Figure 4*: one, occurred in October 1987, that affected both indexes, and the other, occurred in April 2000, that primarily affected the *Nasdaq*.³ In fact, on Monday October 19th, 1987, the *DowJones* lost a 29.2% of its value to recover only a 5.9% the day after, thus accumulating in one week, in the two consecutive days, losses of 42.1% and 36.2% respectively. The *Nasdaq* lost a 12.8% of its value the same day and 9.9% the day after, thus accounting for weekly losses of 20.2% and 32.6% on October 19th and 20th respectively. The international nature of the crisis can be illustrated by noticing that towards the end of October 1987, the stock markets in Hong-Kong, Australia, the United Kingdom, the United States and Canada had respectively fallen 45.8%, 41.8%, 26.4%, 22.7% and 22.5%, and so in other economies. These are among the most severe declines observed during the whole past century. A striking feature of the

³A summary on *financial bubbles* and, in particular, on the *October 87* and *April 2000* stock crashes, as well as recommended references on the subject, can be found at $\text{http} : // \text{en.wikipedia.org/}$ looking for "stock market bubble", "Black Monday (1987)" and "dot-com bubble".

episode is the short time that took the bubble to explode and its lack of fundamentals, for no major news or events appeared during the prior days to the crash. Several alternative explanations have since then been given, based on liquidity restrictions, technical aspects of trading, rationality and psychological behaviour in general, among others.

Figure 4: Twenty Years of Evolution of the *Dow Jones* and the *Nasdaq* Composite Indexes and their Corresponding Daily Transacted Funds.



Unlike the stock crash of October 1987, the episode of April 2000 (depicted in the lower row of *Figure 4*) only affected companies related to the *Information & Technology* (IT) sector, and so it is referred to as the *dot-com bubble*. As shown in *Figure 4*, and in accordance with the policy of low interest rates that the Federal Reserve Bank was engaged in after 1990 (see the graphs in the left column of *Figure 7*), both the *Nasdaq* and the *Dow Jones* composite indexes rose steadily during the nineties and reached a maximum during the first quarter of 2000. However, while the *Dow Jones* attains a maximum at 11,722.98 points on January 14th, 2000, and then falls until reaching 9,811.24 points on March 14th, 2000, the *Nasdaq* attains a maximum at 5,048.62 points on March 10th, 2000, and then falls until reaching 3,400.91 points on May 31st, 2000. Thus, although both indexes adjusted in more

or less the same period of time (roughly two months), the magnitude of the adjustment is greater in the case of the *Nasdaq* index, which lost 36.64% of its value, while the *DowJones* only lost a 16.31% of its value. In both cases, the onset of the boom can be found in the second semester of 1998, while the end of the adjustment can be found towards April 2003, when the indexes attained the levels they had towards the end of 1998 (around 7,500 points for the *DowJones* and 1,500 points for the *Nasdaq*) and a new bullish trend seems to start.

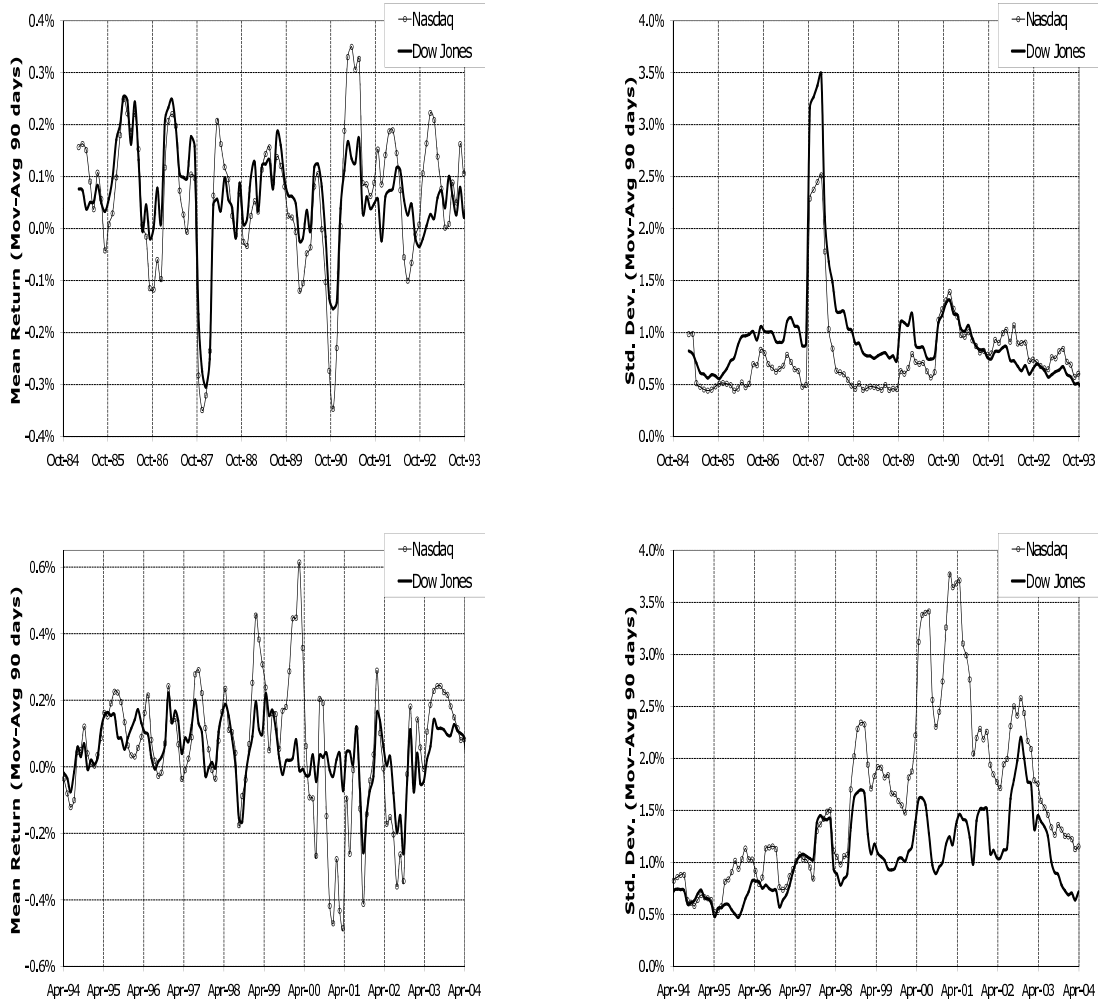
Inspecting the graph at the lower right corner of *Figure 4*, it can be noticed, in the first place, that the funds demanded for transactions follow a growing path starting in the mid-nineties, a fact that is consistent with the liberalisation of capital markets and the expansionary monetary policy followed by the Federal Reserve Bank in that period. Moreover, although the total inflows of capital are shared in roughly equal parts most of the time, such a tendency is abandoned in the period starting shortly after April 1999 and extending until shortly before April 2003. It is well-known, indeed, that towards the mid-nineties, a sentiment of growing confidence about the future of the dot-com firms started to predominate. As a consequence, the credit conditions were facilitated to this industry — both through the reduction of premiums and controls, as through the publishing of new stock — thus pushing the market values of these firms to increase and raising still more, in turn, the demand for cash balances. As already stated, such variations in the rate of growth of the money supply must be necessarily followed by adjustments in the relative prices of securities, consistent with the corresponding variations in the demands of funds for transactions and the market responses. In a Gaussian setting, risks are completely characterised by their expected returns and volatilities and accordingly, the effects just mentioned should be observed on these variables.

5.1 Market Responses to the Observed Monetary Flows

Recall that the monetary equilibrium has been established in *Equation 6*, in such a way that the market price of risk, determined by the optimal proportion of cash to be maintained as risk-capital (or the inverse function of the tail-probability of the underlying exposure, as specified in *Equation 4*) is equal to the supply-to-demand ratio. Consequently, the equilibrium depends on the interactions between borrowers and lenders on the one hand, who respectively control the demand and the supply of money, and the response of the market on the other, which can be regarded as the *involuntary* output of the decisions simultaneously and independently made by investors, intermediaries and monetary authorities, and which is corresponded to the probability distributions describing the processes of percentage returns. In a Gaussian setting, every risk is uniquely determined by its expected return and volatility. Then any level of the interest rate determines a set of compatible combinations of the supply-to-demand ratio, the mean return and the volatility, as established in *Equation 9*. On these grounds, while M and L can be regarded as *control* variables, μ , σ and the *IRRR* must be rather considered as variables *of state*.

Within this framework, the peak of trading activity occurred in October 1987, which can be distinguished by the sudden increment in the volumes of funds spent on transactions — which moved from levels between 15 and 20 millions USD, to about 30 millions USD, as depicted in the graph located to the upper right corner of *Figure 4* — must be followed by a response of the market through the expected return and volatility. Looking at the

Figure 5: Expected Returns and Volatilities of the Series of Capital $P&L$ of the *Dow Jones* and the *Nasdaq* Composite Indexes.



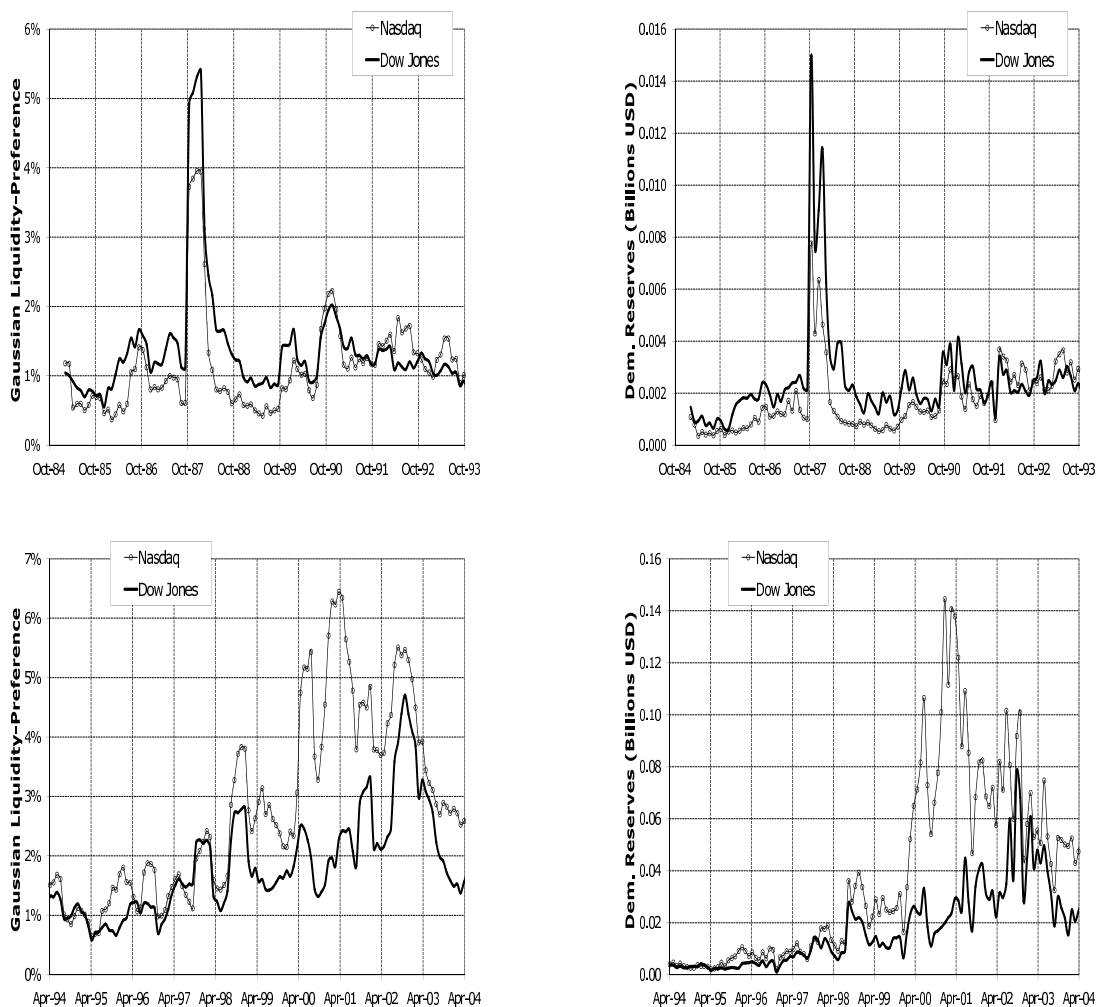
first row of *Figure 5*, we see that this is indeed the case, since drastic adjustments in the expected returns and the volatilities of both composite indexes are observed. Thus, while their expected returns decreased following paths very close to each other, moving from levels around 0.15% to levels around -0.4% , the volatility of the *Dow Jones* jumped from a level below 1.0% to a level over 3.5%, at the time that the volatility of the *Nasdaq* raised from a level around 0.5% to a level close to 2.5%. The fact that the mean returns decreased until reaching negative values suggests that the orders putted in the security markets were mostly *selling* orders. It additionally implies, still from *Equation 9*, that in order to peg the market price of risk R to the level determined by the market price of the free-of-risk security $\Phi^{-1}(1-r_0) = \Psi^{-1}(r_0)$, investors were next obliged to reduce the cash-balances demanded for transactions, for in this way the supply-to-demand for transactions ratio can be inflated to compensate for the negative contribution of the mean return. This explains why (as shown in *Figure 4*) the sudden increment of the volume for transactions is turned into a drastic reduction of even greater magnitude.

The comparison of the evolution of the capital inflows and the risk-parameters depicted in *Figures 4* and *6* suggests that the crisis episodes of October 1987 and April 2000 are actually of a different nature. Thus, while the former event can be regarded as a *singularity*, for it involves two adjustments of high magnitude occurred within a short period of time — in fact, the volatility levels of October 1987 had been reestablished towards October 1988, one year later — in the later case, the volatility of the *Nasdaq* surpasses the volatility of the *DowJones* during roughly five years — from April 1998 until April 2003 — and several jumps are observed, notably the one produced around April 2000, when the bubble bursted. It is also worth noticing that the volatility of the *Nasdaq* was steadily decreasing on the months previous to the crash, falling from a level of 2% in April 1999 to 1.5% in December 1999, while the volatility of the *DowJones*, although still decreasing, was maintained more or less pegged to 1% in the same period. According to *Equation 9*, such volatility declines are needed to compensate for the reduction experienced by the supply-to-demand for transactions ratios $m = \frac{M}{L}$, due to the increments in the flows of funds spent in transactions. However, although the expected return of the *Dow Jones* maintained a decreasing path during the whole year of 1999 and started to oscillate around zero since the first quarter of 2000, the mean return of the *Nasdaq* started to raise from October 1999 and eventually passed the barrier of 0.4% by the end of 1999. These paths are consistent with the increment observed in the proportion of funds spent on securities belonging to the *Nasdaq* (see *Figure 4*), for as long as capital can be borrowed at the same rate of interest in both industries, the market price of risk (as determined in *Equation 9*) have to be the same in the both of them.

The end of the story is well-known: the *Nasdaq* felt dramatically in April 2000, pushing the mean return to zero towards the end of the month and leading this coefficient to negative values from the next month on. The bearish mood only stopped towards August 2000, when expected returns around -0.4% were observed. In the meantime, volatilities were sharply incremented, specially that of the *Nasdaq*, which by May 2000 attained a level more than three times its value in December 1999. Investors responded by drastically reducing the funds demanded for transactions, but negative expected returns and high volatilities persisted during the following quarters. Only after April 2003, the volatility of the *Nasdaq* attained the levels observed at the onset of the crisis (towards the end of 1998) and its mean return entered a new positive cycle.

We can then conclude that episodes of liquidity crises merely represent *physical* adjustments in liberalised capital markets — where investors fit their cash-balances in order to profit from capital gains — which can be precisely described on the grounds of the historical data. Besides, notice that the result enters in no contradiction with the *rationality* of decision makers, nor with the *efficiency* of the clearing mechanism. In fact, since multiple combinations of the involved parameters are compatible with the monetary equilibrium, or in other words, since multiple states of the markets are possible, additional conditions are needed — corresponding to a certain *preferred* equilibrium — to define *efficiency*. More precisely, a main conclusion of the model built in *Sections 2, 3* and *4*, is that capital inflows must necessarily produce changes in the mean returns and the volatilities of the series of capital *P&L* of the alternative securities, in such a way that *re-pricing* is constantly required, as well as adjustments in the cost of capital and the preference for liquidity. In the next paragraphs, we will investigate whether the magnitude of these adjustments can be measured by the *IRR* and the liquidity-preference coefficients.

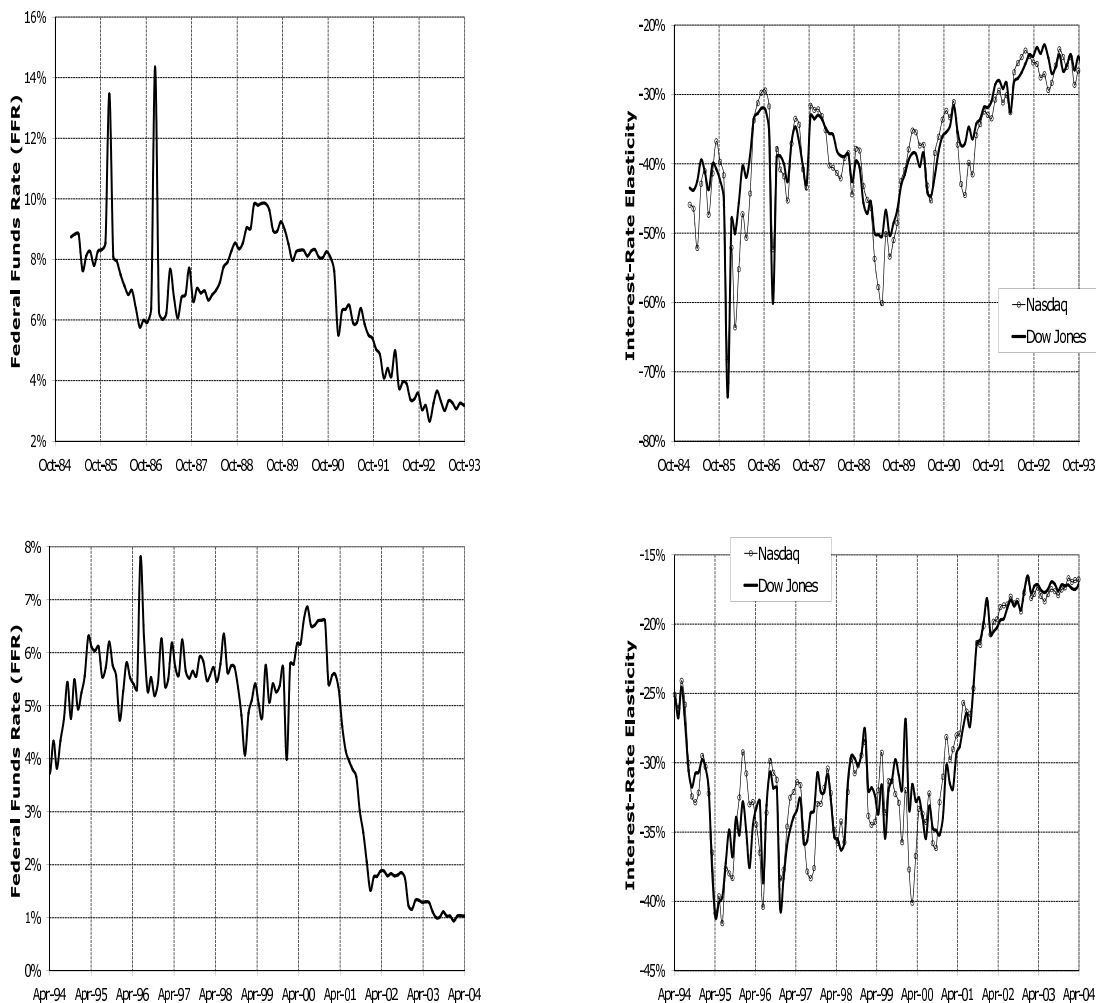
Figure 6: Preference for Liquidity and Demanded Reserves.



5.2 Adjustments to Liquidity-Preference without Liquidity Constraints

Let us suppose in the following that investors are allowed to exchange overnight balances without restrictions at the free-of-risk interest rate r_0 , which will be represented by the *Federal Funds Rate (FFR)* and whose evolution is depicted in the left column of *Figure 7*. This is equivalent to assume that lenders are *price-takers* and that no transaction costs nor credit premiums are charged to the price of debt. As already stated in *Section 2*, such a framework has been proposed by Modigliani and Miller (1958) to characterise the capital structure of financial institutions, leading them to conclude that the value of firms does not depend on the amount of debt and hence any level of capital maintained as a cash balance represents a burden that provides no benefit. It has been also established in *Section 2* that concerns about the level of the cost of capital in the future lead investors to demand equity. Under such conditions, the *optimal* cash balance is determined through an *optimal* insurance contract, in such a way that the excess of loss to be paid by the insurer is minimised.

Figure 7: Federal Funds Rate and Interest-Rate Elasticity without Liquidity Restrictions.



The optimal demand for money is then given by the inverse of the quantile function of the underlying exposure, as in *Equations 4 and 8*, and the level of reserves $K_{\mu,\sigma}$ demanded by a Gaussian market is given by:

$$K_{\mu,\sigma}(r_0) = L \cdot k_{\mu,\sigma}(r_0) = L \cdot [\sigma \cdot \Phi^{-1}(1 - r_0) - \mu] \quad (12)$$

Let us then analyse the empirical evidence provided by the series of capital *P&L* of the *Dow Jones* and the *Nasdaq* during the nineteen eighties and the nineteen nineties — as in the previous subsections. We would like to know whether the money demand remains more or less stable — as assumed in standard macroeconomic analysis — and whether the episodes of liquidity crises already described can be corresponded to adjustments in the liquidity-preference and the demanded reserves functions.

As depicted in the upper row of *Figure 6*, both the preference for liquidity and the demand for cash-balances are uniformly bounded during nearly the whole period going from October 1984 to October 1993, the only exception being the sharp increments shown in

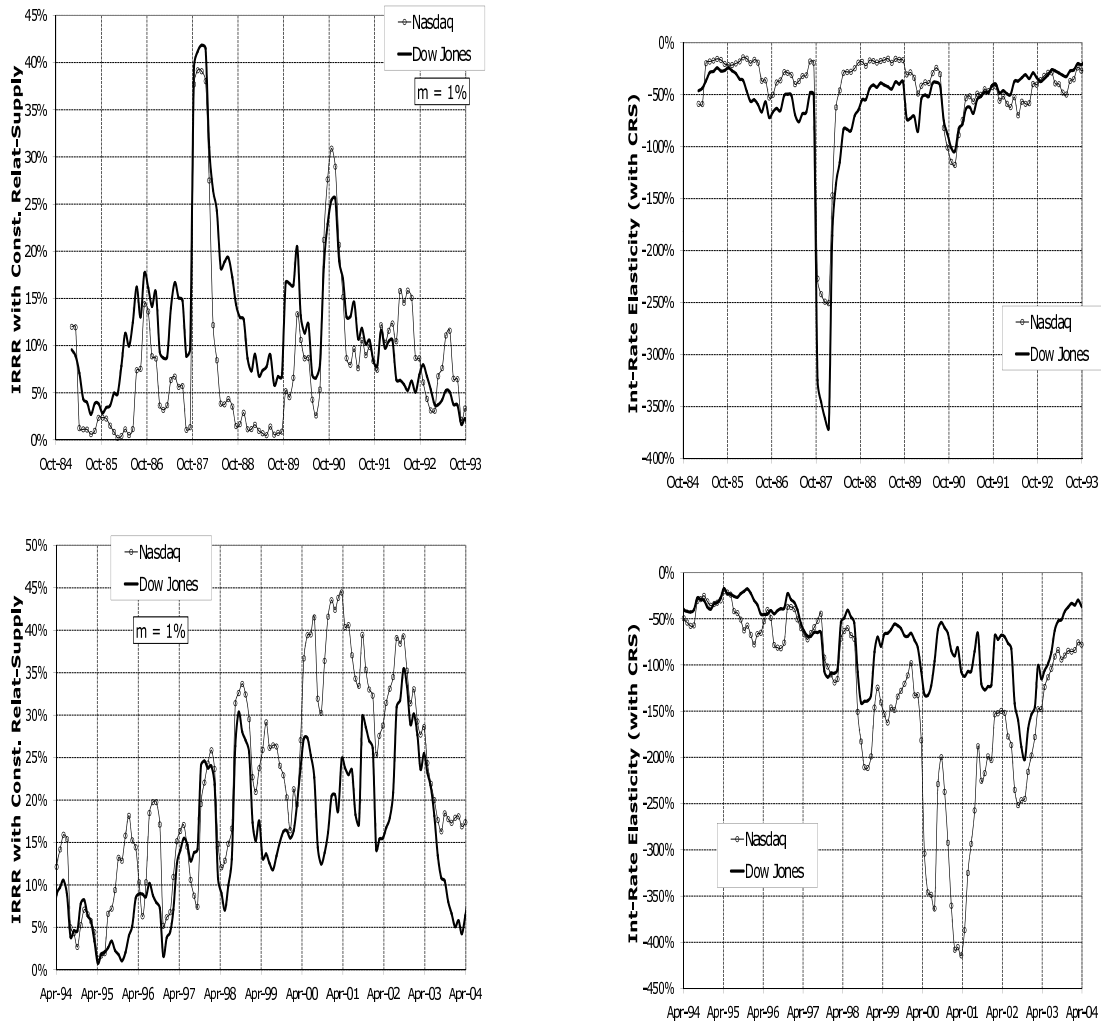
October 1987. It can be also noticed that the times of turbulence only lasted for a short time — for the levels that predominated before the crisis had already been attained towards October 1988 — and that its effects were suffered both by the *DowJones* and the *Nasdaq*, although the magnitude of the required adjustment was greater for the former index. In fact, in the case of the *DowJones*, the level of demanded reserves was multiplied by eight around October 1987 (going from about USD 2 MM to about USD 16 MM), a result that is compatible with the fact that the liquidity demand was multiplied by five around October 87 (rising from 1% to 5%), while the level of demanded cash-balances L was multiplied by 1.5 (rising from less than USD 200 MM to about USD 300 MM). Regarding the *Nasdaq*, the level of demanded reserves and the preference for liquidity were respectively amplified about 8 and 6 times, while the demand for cash balances was multiplied by 1.3 approximately. Therefore, the episode of crisis of October 1987 is precisely described by the demand functions defined in *Equations* 8 and 12, because not only the moment when it occurred is clearly identified, also its short-lasting nature is captured.

Let us now analyse the episode of April 2000. As it is shown in the lower row of *Figure* 6, although the liquidity-preference of both indexes rose steadily from roughly the first quarter of 1997 until the last quarter of 2002, the magnitude of this coefficient is greater for the *Nasdaq*, specially after April 2000. The crisis signaled at this point can then be regarded as one particular adjustment occurred at the middle of a more extended period of turbulence primarily affecting the *IT* industry. Inspecting the graph located to the lower right corner of *Figure* 6, we can additionally notice that the increment in the preference for liquidity is partly compensated by a reduction in the funds demanded for transactions L (see also *Figure* 4), in such a way that only from the beginnings of the year 2000 until the ends of 2003 the balances demanded for precautionary motives surpassed the levels considered as *normal* up to that moment. However, since the lift of the liquidity-preference of the *Nasdaq* is sharpest on April 2000, the burst of the bubble can be established at this point, in accordance to the analysis of the previous subsection. In conclusion, as it was the case with the episode of October 1987, the crisis of April 2000 can be fully described by means of the money demand functions. Thus, in the first place, the time of the crash is clearly distinguished; the signal appears only in the *Nasdaq* index in the second place, in accordance to the fact that the *DowJones* did not actually collapse at that moment, and finally, the adjustment is produced within a broad period of time — and it is not a *singularity*, as in October 1987.

In order to asses the instability of the money demand, the point-interest-rate elasticity (defined in *Equation* 10) is depicted in the right column of *Figure* 7. It is firstly worth noticing that the coefficient is always negative, while its magnitude is always greater than 15% and sometimes surpasses the levels of 40% or even 70%. Besides, as a general rule, the demand for reserves is more elastic for higher levels of the interest rate (compare with the graphs at the left column of *Figure* 7, where the evolution of the Federal Funds Rate is depicted). However, the major peaks are not observed around any of the crisis episodes. This fact is actually not surprising, since in the situation that is currently under analysis, investors can borrow and lend any amount of capital at the Federal Funds Rate r_0 . Under such circumstances, any movement in the rate of interest (i.e. every *price* adjustment) is *exogenous*, and it is then this variable what mainly determines the *propensity* of decision-makers to modify their cash-balances. But in practice investors have to pay a premium over

the risk-free interest rate to borrow capital in secondary markets, which is dependent on the market conditions and it is accordingly corresponded to the current stock of money. Another particular case will then be considered in the next subsection, according to which the total cash balance M will be determined so as to peg the relative money supply $m = \frac{M}{L}$ to a given constant, and the interest rate of equilibrium will be defined so as to satisfy *Equation 9*.

Figure 8: Internal Rate of Return on Risk (IRRR) and Interest-Rate Elasticity of the Money-Demand with Constant Relative-Supply.



5.3 Adjustments to the Cost of Capital with Constant Relative Money Supply

Let us now assume that decision-makers are obliged to fit their reserves in order to peg the ratio of equity to the funds spent on transactions, K/L , to a fixed proportion α . Such could be a restriction established by a regulatory authority, but it could be also imposed as a tacit requirement in markets with informational asymmetries. In fact, when intermediaries cannot

fully observe the portfolios held by their customers, the leverage ratios provide a signal of the capacity to pay liabilities back and thus explicitly affect the price of debt. Under such circumstances, investors choose the capital structure corresponded to the cost of capital they are willing to pay, and lenders must be compensated by a return that cannot surpass the *IRRR* — for this coefficient represents the average capital profit accrued by the underlying portfolio and thus the maximum price that borrowers can afford — which is obtained by replacing $m = \alpha$ in *Equation 9* (see also *Equation 6*):

$$r_\alpha = 1 - \Phi\left(\frac{\mu + \alpha}{\sigma}\right) \quad (13)$$

Besides, from *Equations 10* and *13*, the following expression is obtained for the point-interest-rate elasticity:

$$\epsilon_r(r_\alpha) = -(2\pi)^{\frac{1}{2}} \cdot \exp\left[\frac{(\mu + \alpha)^2}{2\sigma^2}\right] \cdot \frac{\alpha r_\alpha}{\sigma} \quad (14)$$

The market cost of capital is then equal to the probability that the normalised capital return surpasses the market price of risk. *Ceteris paribus*, both greater expected returns and lower volatilities (representing *better* prospects for risks) lead to lower liquidity premiums, although they can also lead to elasticities of greater magnitude. Similarly, lower expected returns and greater volatilities (both characterising risks with *worse* prospects) lead to greater liquidity premiums, but can also lead to lower elasticities. Therefore, the interest-rate elasticity — and thus the *instability* of the market — increases with the premium for liquidity.

The internal returns on risk and the elasticities of the money demand, computed according to *Equations 13* and *14*, are respectively depicted in the left and the right columns of *Figure 8*, when the relative money supply is maintained unchanged at the level $\alpha = 0.1\%$. It can then be observed that, in the same way the episodes of crisis were distinguished by sudden changes in the preference for liquidity when the cost of capital was peg to the *Federal Funds Rate* in the previous sub-section, they are corresponded to adjustments in the *IRRR* and the interest-rate elasticity when the leverage ratios are fixed. In fact, notice (in the upper row of *Figure 8*) how, around October 1987, the *IRRR* jumps to a historical maximum, while the interest-rate elasticity falls to a historical minimum. Besides, the signal appears in both indexes and its influence is short-lasting — in accordance to the empirical facts characterising this crisis. Regarding the episode of April 2000, the times of turbulence are located on a broader interval, with a clear predominance of the signals related to the *Nasdaq* — in consistence with the fact that only this index crashed on that date — and the most unexpected movements are produced precisely in April 2000.

6 Conclusions

According to standard macroeconomic analysis, in *perfect* capital markets decision-makers can lend and borrow any amount of cash at a single and exogenously determined interest rate. Moreover, at equilibrium, the total supply must be equal to the total demand of money, which can be expressed as a proportion of the total liabilities, as in *Equation 1*. Such a factor determines the *liquidity-preference* of the market and depends on the level of

the prevailing interest rate. Under the assumption that the preference for liquidity remains stable and it is distributed more or less homogeneously across economic sectors, the effect of the transactions between economic agents can be neglected. Trends at the macroscopic level are then explained on the basis of economic fundamentals and *psychological* phenomena, in such a way that, in particular, *bullish* and *bearish* trends are possible if risk-loving and aversion-to-risk attitudes respectively predominate in the markets, and transitions between states are understood as changes in the mood of investors from optimism to pessimism and vice-versa, or alternatively, as the result of the sudden release of up to the moment privately held information.

In real markets, however, premiums and controls reduce the access to credit, thus encouraging investors to seek protection against bankruptcy through insurance and risk-capital cushions. The *optimal* level of surplus must then be determined at the level where decision-makers appear indifferent to which strategy to implement — for in this way alone capital and insurance markets are *at equilibrium*. The problem can be established in actuarial terms by minimising the cost of capital plus the excess of loss of the residual exposure — in such a way that the sum of the costs to be afforded by insureds and insurers is minimised — whose solution is determined by the *quantile* function of the probability distribution describing risk (see *Equations 2 and 4*). Allowed to decide their levels of reserves at will, investors act as subsidiaries in *decentralised* organisations, where the internal price for exchanging balances is given and the total surplus is equal to the sum of the divisional contributions (as in *Equations 5 and 8*). Besides, as long as different expectations about risks are maintained, different configurations are compatible with a same aggregated stock of money.

Within this theoretical framework, the monetary equilibrium determines the combinations of supplied and demanded cash balances that are compatible with the underlying exposures and expectations, and the market level of the rate of interest is expressed as the tail probability distribution (see *Equations 6 and 9*). Alternatively, the market interest rate represents the return obtained when investing one monetary unit on risk and can be accordingly regarded as the Internal Rate of Return on Risk (IRRR), while the quantile function can be regarded as the market price of risk — as an extended measure to the Sharpe ratio — for it corresponds to the rate at which a unit of investment on risk is exchanged for a unit of money at equilibrium. In a Gaussian setting, given fixed stocks of money and aggregated reserves, a risk-structure of interest rates is determined in the plane of expected returns and volatilities (depicted in *Figures 1 and 2*). Consequently, given any fixed level of expected return, the level of the IRRR respectively increases and decreases with volatility when $IRRR < 0.5$ and $IRRR > 0.5$ (see *Figure 2*), in consistence with the fact that solvent and insolvent portfolios respectively predominate in the former and the later case. Additionally, given any fixed level of volatility, the IRRR decreases with the mean return and the curve is moved to the left when the real stock of money $m = \frac{M}{L}$ is incremented (see *Figure 1*), in such way that the same internal return can be assigned to risks with different expected returns only if higher cash balances are provided to the exposures providing lower expected returns. This result provides a theoretical explanation of why investors show liquidity-preference.

In conclusion, variations in the rate of growth of the funds spent on transactions or in the stock of money must be necessarily followed by market adjustments reflected in the relative prices of securities. In Gaussian settings, such effects should be observed on the mean returns

and volatilities, for in this situation these parameters completely characterise risks. Thus different scenarios are corresponded to different combinations of the involved parameters, some of which can be related to *bullish* (in markets with positives mean returns and low volatilities) and *bearish* trends (in markets with negative mean returns and high volatilities). In this way, both *normal* times, when credit is plentiful, and times of liquidity crises are possible in the model, depending on the flows of capital and the paths followed by the risk parameters. Although such factors as the aversion-to-risk and the states of information of decision-makers might trigger the initial flows of money or might affect its final effects, the clearing mechanism of capital markets (determined by the established transactions) is *physical* in nature — and not necessarily *psychological*, as claimed in classic macroeconomics. The predictions of the model can be verified by analysing the historical data, as proved in *Section 5*.

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