Cross-border Banking, Spillover Effects and International Business Cycles

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CROSS-BORDER BANKING, SPILLOVER EFFECTS AND INTERNATIONAL BUSINESS CYCLES*

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Abstract

This paper studies the link between cross-border banking activities and the international propagation of real and financial shocks. We develop a two-country DSGE model with a bank capital channel and a financial accelerator, in which banks grant loans to domestic as well as to foreign firms. The model economy is calibrated to data from the U.S. and Canada. Our results suggest that following a positive technology shock and a tightening of home monetary policy, the existence of cross-border banking activities tends to amplify the transmission channel in both the domestic and the foreign country. However, cross-border banking activities tend to weaken the impact of shocks on foreign and home consumption because of the cross-border saving possibility between the two countries. Finally, our simulations suggest that under cross-border banking, correlations between macroeconomic variables of both countries become greater than in the absence of international banking activities. Overall, our results show sizable spillover effects of cross-border banking on macroeconomic dynamics and suggest cross border banking is an important source of the synchronization of business cycles between the U.S. and Canada.

J.E.L. Classification: E44, E52, G21
Keywords: Cross-border banking; bank capital, interest rate and exchange rate channels; business cycle synchronization.

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Contents

1 Introduction

The international transmission of shocks − the foreign spillover effects of shocks that originated at the national level − has drawn attention of many separate literatures, particularly in the aftermath of the recent global economic crisis. In macroeconomics, it's well established that the monetary policy implemented by a specific country affects policy in other countries via the bank capital, exchange rate and financial accelerator channels (see literature cited below). In forecasting, dynamic factor models have documented that international variables play a key role for the forecasting of national macroeconomic aggregates, especially at short horizon (Kopoin et al. (2013b)), suggesting that shocks originated at a national level have strong cross-border spillover effects. In finance, the cross-border transmission of financial shocks plays a central role in the globalization and integration of financial markets worldwide and has been identified as an important source of contagion episodes. As an example, the recent subprime crisis, while originating in the United States, also caused a drop in asset prices around the world and led to a synchronized downturn in most developed economies (see Figure 1 and Table 1).[1]

One of the key features that made the recent financial crisis so deep and widespread is the extent and nature of international banking integration, which led to unprecedented transmission of financial instability. Deteriorations in banks' national balance sheets represented adverse shocks to the home country's economy, but so did deteriorations in the quality of their loans in abroad countries.[2] In such a way, financial instability spread to countries in which domestic banks had signifiant asset holdings and foreign entrepreneurs found their access to credit restricted even if credit risk conditions in their country had remained unchanged. This suggests that financial shocks may increase countries' risk of experiencing financial crises and generate a synchronization of international business cycles.

The quantitative strength of this mechanism remains uncertain, however. To what extent does cross-border banking explain the transmission of national and international shocks? Does

[1] In the international framework, financial contagion occurs when international banks in the domestic country respond to deteriorations in their balance sheet by reducing cross-border loans to entrepreneurs living in a country not directly exposed to the initial shock.

[2] As an illustration, consider a negative shock to the banks' balance sheets. This shock, coupled with macro-prudential regulations such as maximum leverage ratio or minimum capital requirement, lead domestic banks to tighten cross-country loans.

it generate a strong synchronization of the international business cycles? This paper provides a framework with which these questions can be analyzed, by considering Canada as the domestic country and the U.S. as the foreign country. We develop a two-country DSGE model with a bank capital channel and a financial accelerator, in which banks receive deposits from home and foreign depositors, and grant loans to home and foreign firms.

Our simulations suggest that following a positive technology shock or a tightening of home monetary policy, the presence of cross-border banking tends to amplify the transmission channel in both domestic and foreign countries. However, cross-border banking activities tend to weaken the impact of shocks on consumption, because of the cross-border saving possibility between the two countries. These results are consistent with those arrived at in empirical studies using estimated vector autoregressive models (Markovic (2006), Beaton et al. (2014), Beaton and Desroches (2011), Teng-Xu (2012)). In addition, our results suggest that under cross-border banking, bilateral correlations between comparable macroeconomic aggregates become greater than those present without international banking activities in many cases. Overall, our results demonstrate sizable spillover effects of cross-border banking on the propagation of shocks and the synchronization of business cycles.

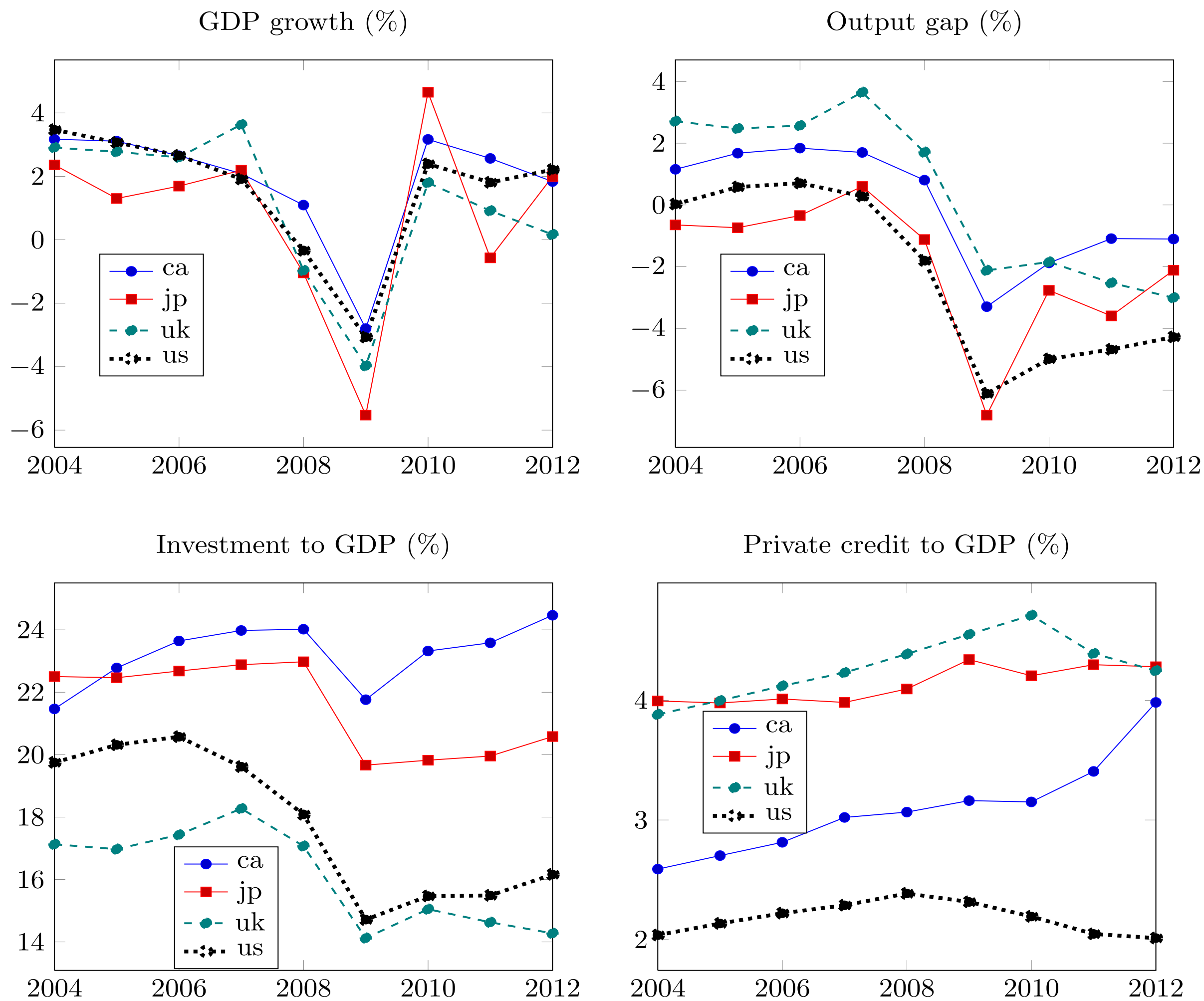
Table 1: Correlation of change in stock prices

Variables	S&P/TSX	NASDAQ	NIKKEI	FTSE100
TSX	1.000			
NASDAQ	0.759	1.000		
NIKKEI	0.659	0.641	1.000	
FTSE100	0.758	0.807	0.637	1.000

Note: This tables provides cross-correlations between four major financial indexes from 2001 to 2012. Data are from Bloomberg and the International Financial Statistics (IFS).

Previous work on the role of cross-border banking activities in the transmission of national and international shocks includes Mendoza and Quadrini (2010), who highlight the central role played by financial globalization in the recent world financial crisis. Although Mendoza and Quadrini (2010) also analyze financial globalization and spillovers of country-specific shocks, our setup differs from theirs in three important respects. The first difference is that our model considers an international credit contract between borrowers and lenders living in two integrated

Figure 1: International business cycles synchronization



Note: This figure illustrates the international business cycles synchronization in the U.S., Canada, UK and Japan. The top left chart displays the dynamic of GDP growth, whilst the bottom right chart presents the growth rate of the credit to the private sector in the aforementioned countries. Data are from Bloomberg, canadian socioeconomic database (Cansim) and International financial statistics.

financial markets with frictions in each market, rather than a global financial market as in Mendoza and Quadrini (2010). This specificity has the advantage of clearly identifying the role of financial openness in the transmission of international shocks. The second difference is that their analysis does not include international trade in the goods market.[3] As our model is calibrated

[3]This channel has also been identified as an important source of shocks transmission (Kopoin et al. (2013b)).

to US and Canadian data, this feature plays an important role since Canada is the largest single-country trading partner of the United States.[4] As a consequence, our model predicts consistent bilateral correlations and is more suited to analyze business cycles synchronization. Finally, in Mendoza and Quadrini (2010), financial intermediaries hold a fixed quantity of the productive capital (bank capital), in contrast to our model which allows (endogenous) bank capital accumulation. As consequence, negative shocks to bank capital lead to a reduction in the supply of loans, and thus to a fall in domestic and foreign investment and output.

Our paper is also close to Dedola and Lombardo (2012), who show that an unexpected increase in credit spreads in one country generates a similar rise in credit spreads in other financially integrated countries, independently of the exposure to foreign assets in the balance sheet of leveraged investors. However, as a model prediction difference, Dedola and Lombardo (2012) argue that business cycle synchronization is strong even if financial exposure is minimal. We find that higher financial exposure yields stronger business cycle synchronization. Next, Guerrieri et al. (2012) analyze cross-country spillover effects of sovereign default in a quantitative setting calibrated to the euro area and find sizable spillover effects of default from the periphery to the core through a drop in the volume of credit extended by the banking sector. Finally, Ueda (2012) uses data from the U.S. and Japan and shows that under globalization of financial intermediation, adverse shocks that hit one country affect the other, yielding business cycle synchronization on both the real and financial sides. In contrast to Ueda (2012) and Guerrieri et al. (2012), our model is calibrated to the two most integrated economies among OECD countries. In addition, our analysis employs a different financial contract to link investors and bankers, and includes prices and wages rigidities, which plays an important role in the transmission of shocks.

The rest of the paper is organized as follows. Section 2 describes the financial contract and the two-country model. Section 3 illustrates the parametrization of the model and presents the results of our quantitative simulation. Section 4 concludes.

[4]The United States and Canada conduct the world's largest bilateral trade relationship, with total merchandise trade (exports and imports) of about $429.7 billion in 2009.

2 Two-country DSGE Model with Financial Frictions

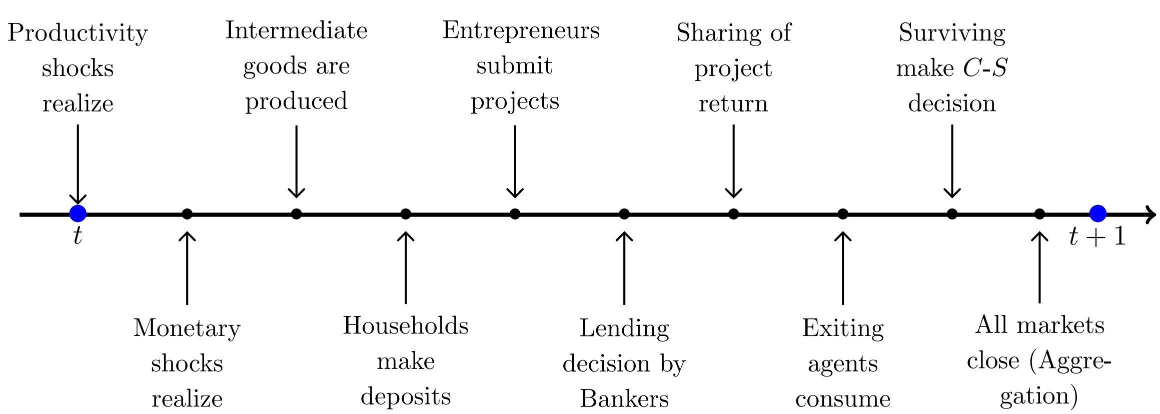
In this section, we describe a two-country model with financial frictions and cross-border banking. Within each country, there are households, entrepreneurs, bankers, intermediate goods producers, composite good producers, a monetary authority and a government. The population masses of workers, entrepreneurs and bankers is the same within each country and is: η^w for workers, η^e for entrepreneurs and η^b for bankers ($\eta^w = 1 - \eta^e - \eta^b$). Each period households supply differentiated labor, rent physical capital and receive lump-sum transfers from the monetary authority. An aggregate household labor input is assembled using a Dixit-Stiglitz form by a firm that sells it to intermediate goods producers. Households divide their high-powered money between currency, home deposits and foreign deposits. Bankers raise funds by issuing deposits liabilities to domestic and foreign households workers and make loans to home and foreign entrepreneurs.

Each country includes an intermediate goods production sector. In that sector, firms operating under monopolistic competition and facing Calvo-type pricing friction, use labor and capital to produce intermediate goods. Next, perfectly competitive firms produce both the domestic and foreign composite goods, which are then combined to form a final good. This final good is converted into consumption, investment and government spending. Each country is specialized in the production of a set of differentiated goods, but consumers in both countries consume domestic and foreign goods. This micro-founded economic structure has become standard in the new open economy macroeconomics literature (Obstfeld and Rogoff (2000), Kehoe and Perri (2002), Iacoviello and Minetti (2006), Perri and Quadrini (2011) and Kollmann et al. (2011)).

Entrepreneurs have access to external finance, but the maximum amount they can borrow depends on their net worth. As well, banks can issue loans to foreign and domestic entrepreneurs, and banking regulation, in the form of taxation of bank dividends, constrains banks' choices, so that lending cannot grow as freely as banks would want. Entrepreneurs use borrowing as well as their own fund to finance projects that result in the production of new capital goods. The model includes a financial accelerator mechanism as in Bernanke et al. (1999), because of an asymmetric information problem between borrower and lenders. Additionally, the model includes bank capital and exchange rate channels as described in Kopoin et al. (2013a), because of an asymmetric information problem between lenders and depositors, and the presence of an international goods market.[5] As the focus of our analysis is to investigate how cross-border banking activities and leverage constraints affect the international propagation of shocks across countries, we analyze separately whether leverage constraints are binding or not.

The two economies are similar in size and in the composition of agents, but are heterogeneous in their financial regulation systems. There is international trade in goods, whereas labor is assumed to be immobile across countries. The timing of events is summarized in Figure 2 and is as follows: at the beginning of each period, productivity and monetary policy shocks are realized in the home and foreign country. Intermediate goods are then produced, using capital and labor services and final goods are produced, using intermediates. Next, households make deposits in the foreign and domestic banks, entrepreneurs choose which project to undertake and banks combine households deposits and their own capital to finance project submitted by entrepreneurs and choose whether to monitor. In the last step, successful projects return of new capital is shared between the three agents according to the terms of the financial contract, and exiting agents transform their capital goods into consumption goods. Surviving agents make a consumption-savings decision.

Figure 2: Timing of events within each country

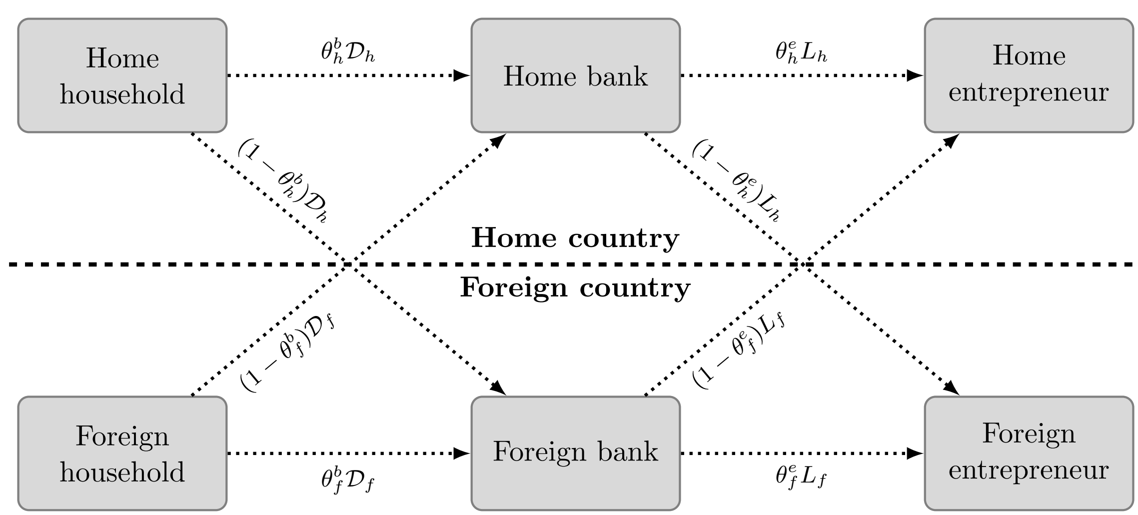


[5]The bank capital channel may be interpreted as a financial accelerator mechanism on the supply side (interaction between households savers and banks) and the exchange rate arises because of the impact of real exchange rate to the cost of commodity imports and to the foreign demand for composite goods production firms.

2.1 The Financial Contract

Our setting of bank financing is based on the *costly state verification* (CSV) model developed by Bernanke et al. (1999) and Christiano et al. (2010), which we extend to allow cross-border lending and borrowing. The home country is denoted by the superscript h, and the foreign country by the superscript f. The basic idea of the financial contract is as follows: in period t, a representative entrepreneur in the home country borrows a fraction θ_h^e of its total financing needs from the home financial system and $1 - \theta_h^e$ from the foreign financial system. Conversely, a representative entrepreneur in the foreign country engages in a credit contract by borrowing a fraction θ_f^e from the foreign financial system and $1 - \theta_f^e$ from the home financial system, with $0 < \theta_h^e, \theta_f^e < 1$. In this context, θ_h^e and θ_f^e capture the degree of cross-country borrowing, which may be interpreted as the degree of financial globalization. On the supply side, domestic banks receive a fraction θ_h^b and $1 - \theta_f^b$ of deposits from home and foreign workers, respectively, and make loans to the home and foreign entrepreneurs. Correspondingly, foreign banks receive a fraction θ_f^b and $1 - \theta_h^b$ of deposits from foreign and domestic investors. Let $\mathcal{D}_{h,t}$ and $\mathcal{D}_{f,t}$ denote the total amount of deposits held by the home and foreign investors. Figure 3 describes the cross-border financial activities between borrowers and lenders.

Figure 3: Chained financial contract



2.1.1 The demand side of the credit contract

Entrepreneurs in the home country are assumed to be risk-neutral agents and each has a constant probability τ_h^e of surviving to the next period, which implies an expected lifetime of $1/(1-\tau_h^e)$. As well, a representative entrepreneur in the foreign country has an expected lifetime of $1/(1-\tau_f^e)$. This assumption precludes the possibility that the entrepreneurial sector will accumulate enough wealth to be fully self-financing. Entrepreneurs' net worth comes from supplying labor to intermediate goods producers and from profits accumulated from previous investments. Finally, following the CSV literature, we assume that the financial intermediary must pay a cost to observe the entrepreneur's realized return on capital.

A. Entrepreneurs and bankers in the home country

At the end of period t (going into period $t+1$), entrepreneur j in the home country combines net worth $\mathcal{N}_{h,t+1}^e(j)$ with bank loans to purchase new physical capital. The value of capital purchased in the home country is $\theta_h^e \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}$, where $\mathcal{Q}_{h,t}$ is the unit-price of capital at time t in the domestic country. Following Bernanke et al. (1999), we assume that the return to capital invested is subject to both aggregate and idiosyncratic risk: denote the ex-post aggregate return to capital by $\omega_h \mathcal{R}_{h,t+1}^k$ in period $t+1$, where ω_h is an idiosyncratic disturbance that affects project in the home country. This disturbance is constant across all entrepreneurs and is lognormally distributed with a cumulative distribution function demoted by $F_t(\omega_h)$.[6] The debt borrowed by this entrepreneur from the home financial intermediary is $\mathcal{B}_{h,t+1}(j) = \theta_h^e(\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}(j) - \mathcal{N}_{h,t+1}^e(j))$, while the complementary amount to run the entrepreneurial project is borrowed from the foreign financial intermediary.

Definition 1 : *A financial contract is characterized by:* ***(i)*** *The amount of debt that a representative entrepreneur borrows from the home financial intermediaries:* $\mathcal{B}_{h,t+1}(j) = \theta_h^e(\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}(j) - \mathcal{N}_{h,t+1}^e(j))$*;* ***(ii)*** *The threshold value of the idiosyncratic shock* ω_h*, defined as* $\overline{\omega}_h$*, such that entrepreneur repays its debt for* $\omega_h \geq \overline{\omega}_h$ *and declares default for* $\omega_h < \overline{\omega}_h$*;* ***(iii)***

[6]The mean and standard deviation of $\log(\omega_h)$ are 1 and σ_{ω_h}. In contrast to Christiano et al. (2010), we assume that the risk profile of entrepreneurs is constant over time.

The loan rate that the entrepreneur repays when he does not default, $Z_{h,t+1}$.

As defined above, when $\omega_h \geq \overline{\omega}_h$, the home entrepreneur pays the home financial intermediary an amount of $\theta_h^e(\overline{\omega}_h \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1})$ and keeps $\theta_h^e(\omega_h - \overline{\omega}_h)\mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}$. Otherwise, if $\omega_h < \overline{\omega}_h$, the entrepreneur receives nothing, and the financial intermediary audits the entrepreneur by paying a monitoring cost $\theta_h^e \mu \omega_h \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}$, with $0 < \mu < 1$, and thus receives net receipts of $(1-\mu)\theta_h^e \omega_h \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}$. If $Z_{h,t+1}$ is the loan rate, then, the home entrepreneur repays a promised debt amount of $Z_{h,t+1}B_{h,t+1}$ when $\omega_h \geq \overline{\omega}_h$ and keeps net receipts $\theta_h^e(\omega_h \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} - Z_{h,t+1}B_{h,t+1})$.

Formally, at the end of period t, expected payoffs of the entrepreneur and the banker are given by $V_{h,t+1}^e(\overline{\omega}_h, \mathcal{R}_{h,t+1}^k)$ and $V_{h,t+1}^b(\overline{\omega}_h, \mathcal{R}_{h,t+1}^k)$, where

$$V_{h,t+1}^e(\overline{\omega}_h, \mathcal{R}_{h,t+1}^k) = \begin{cases} \theta_h^e(\omega_h - \overline{\omega}_h)\mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} & \text{if } \omega_h \geq \overline{\omega}_h \\ 0 & \text{if } \omega_h < \overline{\omega}_h \end{cases}$$

and

$$V_{h,t+1}^b(\overline{\omega}_h, \mathcal{R}_{h,t+1}^k) = \begin{cases} \theta_h^e \overline{\omega}_h \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} & \text{if } \omega_h \geq \overline{\omega}_h \\ \theta_h^e(1-\mu)\omega_h \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} & \text{if } \omega_h < \overline{\omega}_h. \end{cases}$$

The entrepreneur is able to repay its loans at the contractual rate equal to the non-default loan rate $Z_{h,t+1}$ if the idiosyncratic shock ω_h is greater than $\overline{\omega}_h$. Consequently, the threshold value of the idiosyncratic shock ω_h and the non-default loan rate are jointly defined as follows:

$$Z_{h,t+1} = \frac{\overline{\omega}_h \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}}{\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} - \mathcal{N}_{t+1}^e}. \tag{2.1}$$

Bankers face an opportunity cost of funds in period t equal to the economy's riskless gross rate of return $R_{h,t+1}$. An incentive compatibility condition for a banker to participate in the financial contract is therefore $(\overline{\omega}_h Pr(\omega_h \geq \overline{\omega}_h) + (1-\mu)\mathbf{E}[\omega_h \mid \omega_h < \overline{\omega}_h]Pr(\omega_h < \overline{\omega}_h))\, \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} \geq R_{h,t+1}\mathcal{B}_{h,t+1}$, which may be expressed as: $R_{h,t+1}\mathcal{B}_{h,t+1} \leq (1-\mu)\int_0^{\overline{\omega}_h} \omega_h \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} dF(\omega_h) + (1-F(\overline{\omega}_h))Z_{h,t+1}\mathcal{B}_{h,t+1}$. Combining this equation with the threshold value of the idiosyncratic shock, this incentive constraint may be expressed as:

$$R_{h,t+1} \leq \frac{\left(\Gamma(\overline{\omega}_h) - \mu G(\overline{\omega}_h)\right) \mathbf{E}\left[\mathcal{R}^k_{h,t+1}\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}}{\left(\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} - \mathcal{N}^e_{h,t+1}\right)}, \tag{2.2}$$

where

$$\Gamma(\overline{\omega}_h) = \int_0^{\overline{\omega}_h} \omega_h dF(\overline{\omega}_h) + \overline{\omega}_h \int_{\overline{\omega}_h}^{\infty} dF(\overline{\omega}_h)$$
$$\mu G(\overline{\omega}_h) = \mu \int_0^{\overline{\omega}_h} \omega_h dF(\overline{\omega}_h).$$

This condition requires that the expected return on loan paid to the financial intermediary, net of auditing cost (in the case of default), should be greater than or equal to the financial intermediary's opportunity cost of lending. The home entrepreneurs could purchase capital goods, using their own net worth without entering a financial contract.

We impose a participation constraint to the entrepreneurs specified by $\int_{\overline{\omega}_h}^{\infty} \omega_h \mathbf{E}\left[\mathcal{R}^k_{h,t+1}\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} dF(\overline{\omega}_h) - Z_{h,t+1}\mathcal{B}_{h,t+1} \geq \mathbf{E}\left[\mathcal{R}^k_{h,t+1}\right] \mathcal{N}^e_{h,t+1}$, which states that it is more profitable for them to participate in the contract. Using the threshold value of the idiosyncratic shock, this participation constraint may be rewritten as: $[1 - \Gamma(\overline{\omega}_h)] \mathbf{E}\left[\mathcal{R}^k_{h,t+1}\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} \geq \mathbf{E}\left[\mathcal{R}^k_{h,t+1}\right] \mathcal{N}^e_{h,t+1}$.

B. Home bankers and foreign entrepreneurs

The foreign representative entrepreneur engages in a credit contract with the home financial intermediary to finance the difference between its expenditures on capital goods and its net worth. As in the home country, entrepreneurs in the foreign country are also subject to an idiosyncratic shock ω_f.[7] The participation constraint of the foreign entrepreneur may be specified as:

$$\underbrace{\left(1 - F(\overline{\omega}_f) - \overline{\omega}_f \int_0^{\overline{\omega}_f} dF(\overline{\omega}_f)\right)}_{1-\Gamma(\overline{\omega}_f)} \mathbf{E}\left[\mathcal{R}^k_{f,t+1}\right] \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1} \geq \mathbf{E}\left[\mathcal{R}^k_{f,t+1}\right] \mathcal{N}^e_{f,t+1} \tag{2.3}$$

Given the participation constraint, the amount of capital purchased by foreign entrepreneur using loans from the home financial intermediary is $(1 - \theta^e_f)\Upsilon_t \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}$, where $\mathcal{Q}_{f,t}$ is the

[7] As in the home country, we assume that ω_f follows a log-normal process.

price paid to capital in units of the household consumption index in the foreign country. Home financial intermediary faces to an incentive compatibility condition in the credit contract with foreign entrepreneur. This incentive condition may be expressed as:

$$R_{f,t+1} \leq \frac{\left(\Gamma(\overline{\omega}_f) - \mu G(\overline{\omega}_f)\right) \mathbf{E}\left[\mathcal{R}^k_{f,t+1}\right] \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}}{\left(\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1} - \mathcal{N}^e_{f,t+1}\right)}, \tag{2.4}$$

Proposition 1 : *Given the participation constraint of the entrepreneur and the incentive constraint of financial intermediary, the expected return received by the non-default entrepreneur in the home country may be expressed as:*

$$\begin{aligned} V^e_{h,t}(\theta^e_h, \theta^e_f) =& \theta^e_h \left(1 - \Gamma(\overline{\omega}_h)\right) \mathbf{E}\left[R^k_{h,t+1}\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} \\ &+ (1-\theta^e_h)\Upsilon_t \left(1 - \Gamma(\overline{\omega}_f)\right) \mathbf{E}\left[R^k_{f,t+1}\right] \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}, \end{aligned} \tag{2.5}$$

Proposition 2 : *Combining equation 2.3 and 2.4, the expected return on loan contract paid to the home financial intermediary when engages loan contract with the home and foreign entrepreneur is given by:*

$$\begin{aligned} V^b_h(\theta^e_f, \theta^e_f) \equiv& \theta^e_h \left(\Gamma(\overline{\omega}_h) - \mu G(\overline{\omega}_h)\right) \mathbf{E}\left[\mathcal{R}^k_{h,t+1}\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} \\ &+ (1-\theta^e_f)\Upsilon_t \left(\Gamma(\overline{\omega}_f) - \mu G(\overline{\omega}_f)\right) \mathbf{E}\left[\mathcal{R}^k_{f,t+1}\right] \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}, \\ V^b_f(\theta^e_f, \theta^e_f) \equiv& (1-\theta^e_h) \left(\Gamma(\overline{\omega}_h) - \mu G(\overline{\omega}_h)\right) \mathbf{E}\left[\mathcal{R}^k_{h,t+1}\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} \\ &+ \theta^e_f \Upsilon_t \left(\Gamma(\overline{\omega}_f) - \mu G(\overline{\omega}_f)\right) \mathbf{E}\left[\mathcal{R}^k_{f,t+1}\right] \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}, \end{aligned} \tag{2.6}$$

where Υ_t is the real exchange rate. The real exchange rate is used to express earnings in household consumption index in the foreign country to the units of the household consumption index in the home country.

Proposition 3 : *Denote by* $\nabla^e_h = \mathcal{N}^e_{h,t+1}/\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}$ *and* $\nabla^e_f = \mathcal{N}^e_{f,t+1}/\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}$*, the debt to equity ratio in the home and foreign country. Then, given the incentive constraints faced by* *the financial intermediary in the home and foreign credit contract:*

$\exists\ Z^b_{h,t+1}, Z^b_{f,t+1} \in [\min(R_{f,t+1}, R_{h,t+1}), \max(R_{h,t+1}, R_{f,t+1})]$ *defined as the expected return earned by the home and foreign financial intermediary on the loans to entrepreneur such that:*

$$\begin{aligned} Z^b_{h,t+1} &= \frac{1-\nabla^e_h}{\left(1-\nabla^e_h\right)+\nabla^e_{h,f/h}\left(1-\nabla^e_f\right)} R_{h,t+1} + \frac{1-\nabla^e_f}{\left(1-\nabla^e_f\right)+\nabla^e_{h,h/f}\left(1-\nabla^e_h\right)} R_{f,t+1}, \\ Z^b_{f,t+1} &= \frac{1-\nabla^e_h}{\left(1-\nabla^e_h\right)+\nabla^e_{f,f/h}\left(1-\nabla^e_f\right)} R_{h,t+1} + \frac{1-\nabla^e_f}{\left(1-\nabla^e_f\right)+\nabla^e_{f,h/f}\left(1-\nabla^e_h\right)} R_{f,t+1} \end{aligned} \tag{2.7}$$

where $\nabla^e_{h,f/h} = \left((1-\theta^e_f)\Upsilon_t \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}\right)/\theta^e_h \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}$, and $\nabla^e_{h,h/f} = \left(\nabla^e_{h,f/h}\right)^{-1}$. $\nabla^e_{f,f/h} = \theta^e_f \Upsilon_t \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}/\left((1-\theta^e_h)\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}\right)$, and $\nabla^e_{f,h/f} = \left(\nabla^e_{f,f/h}\right)^{-1}$. In this specification, $\nabla^e_{h,f/h}$ is defined as the ratio of capital borrowed by the foreign entrepreneur relative to the capital borrowed by the home entrepreneur from the home bank. Equation (2.7) shows that in absence of cross-border borrowing, the expected return earned by the each bank is the economy's riskless gross rate of return.

Proof. Appendix. □.

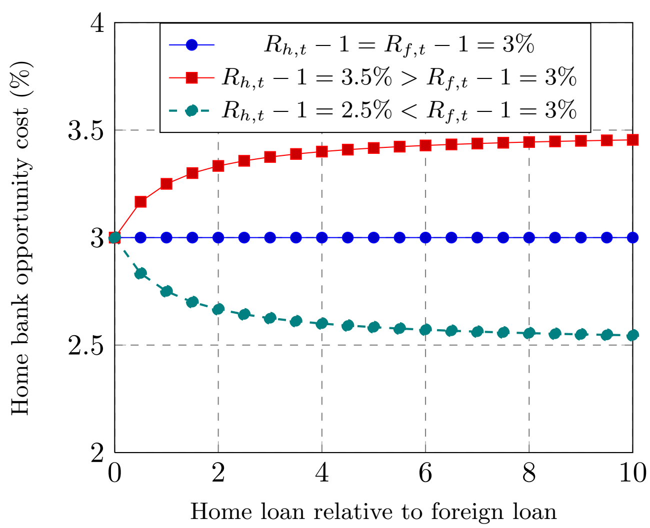


Figure 4: Effect of an increase in the home financing relative the foreign financing

Equation (2.7) shows that when the risk-free rate in the home country is equivalent to the that in the foreign country, $R_{h,t} = R_{f,t}$, then the opportunity cost face by the representative banker is independent to the the size of the cross-border banking, $Z^b_{h,t+1} = R_{h,t} = R_{f,t}, \ \forall \, \theta^e_h, \theta^e_f \in (0,1)$. As consequence, for given entrepreneurs with a same risk profile, bankers is indifferent in the financing of home and foreign projects. Figure 4 illustrates the expected return of the home bank for 3 cases: the blue line describes the case where the home and foreign risk-free rate are arbitrary set to 3% in annual basis. The red and green lines present the cases where the home risk-free rate is 3.5% and 2.5% respectively, while the foreign risk-free rate is set to 3%. Figure 4 shows that as the home loan relative to foreign loan increases, the expected return of the home bank tends to approximate the home risk-free rate.

2.1.2 The supply side of the credit contract

The supply side of the credit contract specifies the deposits contract between financial intermediaries and depositors. In period t, the home bank receives a fraction of home deposits, $\theta^b_h \mathcal{D}_{h,t+1}$, and a fraction of foreign deposits, $(1-\theta^b_f)\mathcal{D}_{f,t+1}$, from the home and foreign workers, respectively. In our specification, we consider a continuum number of financial intermediaries indexed by $j \in (0, \eta^b)$. A representative financial intermediary in the home country combines deposits received and its own net worth $\mathcal{A}_{h,t+1}(j)$ to satisfy loans demand $\theta^e_h \mathcal{L}^d_{h,t+1}(j) \equiv \theta^e_h \left(\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}(j) - \mathcal{N}^e_{h,t+1}(j)\right)$, and $(1-\theta^e_f)\mathcal{L}^d_{f,t+1}(j) \equiv (1-\theta^e_f)\left(\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}(j) - \mathcal{N}^e_{f,t+1}(j)\right)$ to the home and foreign entrepreneurs, respectively. In this specification, $\mathcal{L}^d_{h,t+1}$ and $\mathcal{L}^d_{f,t+1}$ denote the total demand of loans in the home and foreign country. The home bank and the foreign bank face a minimum capital requirement constraint: this constraint specifies that their date t capital should not be smaller than a fraction ι_h of the bank's assets $\mathcal{A}_{h,t+1}$. Formally, the loans demand in the home country and its loanable fund function may be expressed as:

$$\begin{cases} \mathcal{L}^d_{h,t} &= \theta^e_h \left(\mathcal{Q}_{h,t-1}\mathcal{K}_{h,t} - \mathcal{N}^e_{h,t}\right) + (1-\theta^e_f)\Upsilon_t \left(\mathcal{Q}_{f,t-1}\mathcal{K}_{f,t} - \mathcal{N}^e_{f,t}\right) \\ \mathcal{L}^s_{h,t} &= (1-\iota_h)\mathcal{A}_{h,t} + \theta^b_h \mathcal{D}_{h,t} + (1-\theta^b_f)\Upsilon_t \mathcal{D}_{f,t} \end{cases}$$

Similarly, the loans demand in the foreign country and its loanable fund function may be expressed as:

$$\begin{cases} \mathcal{L}^d_{f,t} & = (1-\theta^e_h)\left(\mathcal{Q}_{h,t-1}\mathcal{K}_{h,t} - \mathcal{N}^e_{h,t}\right) + \theta^e_f \Upsilon_t \left(\mathcal{Q}_{f,t-1}\mathcal{K}_{f,t} - \mathcal{N}^e_{f,t}\right) \\ \mathcal{L}^s_{f,t} & = (1-\iota_f)\Upsilon_t \mathcal{A}_{f,t} + (1-\theta^b_h)\mathcal{D}_{h,t} + \theta^b_f \Upsilon_t \mathcal{D}_{f,t} \end{cases}$$

where $\mathcal{A}_{f,t+1}$ denotes the foreign financial intermediaries equity and ι_h and ι_f are the capital requirement ratios in the home and foreign country, respectively. Given the total loans demand in each country, the home and foreign banks' budget constraints are:

In the home country:

$$(1-\iota_h)\mathcal{A}_{h,t+1} + \theta^b_h R^D_{h,t}\mathcal{D}_{h,t+1} + (1-\theta^b_f)\Upsilon_t R^D_{f,t}\mathcal{D}_{f,t+1} \leq \\ + \theta^e_h \underbrace{[\Gamma(\overline{\omega}_h) - \mu G(\overline{\omega}_h)]}_{\Psi(\overline{\omega}_h)} \mathbf{E}\left[\mathcal{R}^k_{h,t+1}\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} + (1-\theta^e_f)\Upsilon_t \underbrace{[\Gamma(\overline{\omega}_f) - \mu G(\overline{\omega}_f)]}_{\Psi(\overline{\omega}_f)} \mathbf{E}\left[\mathcal{R}^k_{f,t+1}\right] \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1} \tag{2.8}$$

In the foreign country:

$$(1-\iota_f)\Upsilon_t\mathcal{A}_{f,t+1} + (1-\theta^b_h) R^D_{h,t}\mathcal{D}_{h,t+1} + \theta^b_f\Upsilon_t R^D_{f,t}\mathcal{D}_{f,t+1} \leq \\ + (1-\theta^e_h) \underbrace{[\Gamma(\overline{\omega}_h) - \mu G(\overline{\omega}_h)]}_{\Psi(\overline{\omega}_h)} \mathbf{E}\left[\mathcal{R}^k_{h,t+1}\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} + \theta^e_f\Upsilon_t \underbrace{[\Gamma(\overline{\omega}_f) - \mu G(\overline{\omega}_f)]}_{\Psi(\overline{\omega}_f)} \mathbf{E}\left[\mathcal{R}^k_{f,t+1}\right] \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1} \tag{2.9}$$

2.1.3 The optimal financial contract

In the home country, the optimal financial contract consists of a maximization of the home entrepreneur expected return, $V^e_{h,t+1}(\overline{\omega}_h, \mathcal{K}_{h,t+1})$, given to incentive compatibility constraint of the home and foreign financial intermediary. In the foreign country, the optimal financial contract also consists of a maximization of the foreign entrepreneur expected return, $V^e_{f,t+1}(\overline{\omega}_f, \mathcal{K}_{f,t+1})$, given to incentive compatibility constraint of the home and foreign financial intermediary.

Formally, the optimal financial contract between entrepreneur and bank consists to choose the size of new capital (investment) and the threshold value of the idiosyncratic shock that maximizes the end-of-contract level of net worth for entrepreneur subject to the bank's zero profit condition.

$$\max_{\mathcal{K}_{h,t+1},\omega_h} \left(\theta_h^e \left(1-\Gamma(\overline{\omega}_h)\right) \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} + (1-\theta_h^e)\Upsilon_t \left(1-\Gamma(\overline{\omega}_f)\right) \mathbf{E}\left[\mathcal{R}_{f,t+1}^k\right] \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}\right)$$
$$\text{s.t.} \begin{cases} V_{h,t+1}^b(\overline{\omega}_h, \mathcal{K}_{h,t+1}) & \geq Z_{h,t+1}^b \left[\theta_h^e \left(\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} - \mathcal{N}_{h,t+1}^e\right) + (1-\theta_f^e)\Upsilon_t \left(\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1} - \mathcal{N}_{f,t+1}^e\right)\right] \\ V_{f,t+1}^b(\overline{\omega}_h, \mathcal{K}_{f,t+1}) & \geq Z_{f,t+1}^b \left[\theta_f^e \Upsilon_t \left(\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1} - \mathcal{N}_{f,t+1}^e\right) + (1-\theta_h^e) \left(\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} - \mathcal{N}_{h,t+1}^e\right)\right] \end{cases} \tag{2.10}$$

$$\max_{\mathcal{K}_{f,t+1},\omega_f} \left(\Upsilon_t\theta_f^e \left(1-\Gamma(\overline{\omega}_f)\right) \mathbf{E}\left[\mathcal{R}_{f,t+1}^k\right] \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1} + (1-\theta_f^e) \left(1-\Gamma(\overline{\omega}_h)\right) \mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right] \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}\right)$$
$$\text{s.t.} \begin{cases} V_{h,t+1}^b(\overline{\omega}_h, \mathcal{K}_{h,t+1}) & \geq Z_{h,t+1}^b \left[\theta_h^e \left(\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} - \mathcal{N}_{h,t+1}^e\right) + (1-\theta_f^e)\Upsilon_t \left(\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1} - \mathcal{N}_{f,t+1}^e\right)\right] \\ V_{f,t+1}^b(\overline{\omega}_h, \mathcal{K}_{f,t+1}) & \geq Z_{f,t+1}^b \left[\theta_f^e \Upsilon_t \left(\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1} - \mathcal{N}_{f,t+1}^e\right) + (1-\theta_h^e) \left(\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1} - \mathcal{N}_{h,t+1}^e\right)\right] \end{cases} \tag{2.11}$$

The first order conditions of the problem are the first order necessary conditions associated with the optimization problem, and the zero profit condition in each country:

$$\begin{aligned} \frac{\mathbf{E}\left[\mathcal{R}_{h,t+1}^k\right]}{Z_{h,t+1}^b} &= \frac{\chi_1\theta_h^e + \chi_2(1-\theta_h^e)Z_{f,t+1}^b/Z_{h,t+1}^b}{\theta_h^e\left[1-\Gamma(\overline{\omega}_h)\right] + \left[\Gamma(\overline{\omega}_h) - \mu G(\overline{\omega}_h)\right]\left[\chi_1\theta_h^e + \chi_2(1-\theta_h^e)\right]}, \quad //\mathcal{K}_{h,t+1} \\ F(\overline{\omega}_h) &= 1 - \left[1 - F(\overline{\omega}_h) - \mu\overline{\omega}_h f(\overline{\omega}_h)\right] \frac{\chi_1\theta_h^e + \chi_2(1-\theta_h^e)}{\theta_h^e}. \quad //\overline{\omega}_h \end{aligned} \tag{2.12}$$

$$\begin{aligned} \frac{\mathbf{E}\left[\mathcal{R}_{f,t+1}^k\right]}{Z_{ft+1}^b} &= \frac{\chi_1(1-\theta_f^e)Z_{h,t+1}^b/Z_{f,t+1}^b + \chi_2\theta_f^e}{\theta_f^e\left[1-\Gamma(\overline{\omega}_f)\right] + \left[\Gamma(\overline{\omega}_f) - \mu G(\overline{\omega}_f)\right]\left[\chi_1(1-\theta_f^e) + \chi_2\theta_f^e\right]}, \quad //\mathcal{K}_{f,t+1} \\ F(\overline{\omega}_f) &= 1 - \left[1 - F(\overline{\omega}_f) - \mu\overline{\omega}_f f(\overline{\omega}_f)\right] \frac{\chi_1(1-\theta_f^e) + \chi_2\theta_f^e}{\theta_f^e}. \quad //\overline{\omega}_f \end{aligned} \tag{2.13}$$

where χ_1 and χ_2 represent the Lagrange multipliers associated with the bank's zero profit condition in the home and foreign country, respectively. Denote by:

$$\Delta_m = \frac{\left[1-F(\overline{\omega}_m)\right]}{\left[1-F(\overline{\omega}_m) - \mu\overline{\omega}_f f(\overline{\omega}_m)\right]}, \quad m \in (h, f) \tag{2.14}$$

then, the Lagrange multipliers are :

$$\chi_1 = \frac{\left[\theta^e_f(1-\theta^e_h)\Delta_f - \theta^e_h\theta^e_f\Delta_h\right]}{\left[1-\theta^e_h-\theta^e_h\right]}, \quad \chi_2 = \frac{\left[\theta^e_h(1-\theta^e_f)\Delta_h - \theta^e_h\theta^e_f\Delta_f\right]}{\left[1-\theta^e_h-\theta^e_h\right]}. \tag{2.15}$$

Proposition 4: *After substituting out the expression for Lagrange multipliers and rearranging, the expected discounted return to capital for entrepreneur is given by:*

$$\mathbf{E}\left[\mathcal{R}^k_{h,t+1}\right]/Z^b_{h,t+1} \equiv \psi_h\left(\Delta_h, \nabla^e_h, \nabla^e_f, \nabla^e_{h,f/h}, \theta^e_h, \theta^e_f, R_{h,t+1}, R_{f,t+1}\right),$$
$$\mathbf{E}\left[\mathcal{R}^k_{f,t+1}\right]/Z^b_{f,t+1} \equiv \psi_f\left(\Delta_f, \nabla^e_h, \nabla^e_f, \nabla^e_{f,h/f}, \theta^e_h, \theta^e_f, R_{h,t+1}, R_{f,t+1}\right).$$

When $\theta^e_h = 1$ and $\theta^e_f = 1$, we obtain the no cross-border borrowing results in the Bernanke et al. (1999), in which the return to capital will be equated to the marginal cost of external finance in equilibrium for each country.

2.1.4 Dynamic behaviour of net worth

Following the timing of events described in Figure 2, once the representative entrepreneur has settled its international debt contract, the accumulated capital is sold back to capital producers and the entrepreneur's net worth in period $t+1$ is then determined. At this step, a fixed fraction of entrepreneurs and bankers exit the economy and a fraction of their total accumulated net worth is consumed, surviving agents save all their accumulated net worth. As mentioned in Christiano et al. (2010), net worth transfers are relatively small, which helps to ensure that entrepreneurs and bankers do not accumulate enough net worth to escape the financial frictions. FInally, if $V^e_{h,t}$ and $V^e_{f,t}$ are the accumulated wealth by home and foreign entrepreneurs, then the aggregate entrepreneurial net worth of the home entrepreneur, $\mathcal{N}^e_{h,t}$, and the foreign entrepreneur, $\mathcal{N}^e_{f,t}$ at the end of period t are given by: $\mathcal{N}^e_{h,t} = \tau^e_h V^e_{h,t} + \eta^e \mathcal{W}^e_{h,t}$, $\mathcal{N}^e_{f,t} = \tau^e_f V^e_{f,t} + \eta^e \mathcal{W}^e_{f,t}$. Likewise, the bank capital accumulated by the home and foreign banker are: $\mathcal{A}_{h,t+1} = \tau^b_h V^b_{h,t} + \eta^b \mathcal{W}^b_{h,t}$, $\mathcal{A}_{f,t+1} = \tau^b_f V^b_{f,t} + \eta^b \mathcal{W}^b_{f,t}$, where $V^e_{h,t}$, $V^b_{h,t}$, $V^e_{f,t}$ and $V^b_{f,t}$ are defined above.

2.2 The General Macroeconomic Environment

We now embed contracting problem between bankers and entrepreneurs within a two-country dynamic stochastic general equilibrium model. This will allow to endogeinize the risk-free interest rate in both countries, as well as the return to capital and the relative price of capital, which were taken as given in the previous pages. To do so, we add to the model important agents such as households, retailers, a government sector that conducts fiscal policy and a central bank that formulates and implements monetary policy.

2.2.1 Households

Preferences and endowments: The economy of each country is composed of a continuum of infinitely-lived households of mass η^w indexed by $i \in (0, \eta^w)$. In each country, households obtain utility from consumption and the liquidity services of holding money, while they receive disutility from contributing work effort to the production of intermediate goods. The preferences of the representative household i in the home country are given by the following lifetime utility function, separable into consumption, real money balances and hours worked:

$$\mathbf{E}_t \left\{ \sum_{l=0}^{\infty} \beta^{t+l} U \left(\left\{ \mathcal{C}^w_{t+l}(i) - \gamma \mathcal{C}^w_{t+l-1}(i) \right\}, \frac{\mathcal{M}_{t+l}}{\mathcal{P}_{t+l}}(i), 1 - \mathcal{H}^w_{t+l}(i) \right) \right\}, \tag{2.16}$$

where the period utility function $U : \mathbb{R}^3_+ \to \mathbb{R}$ is strictly increasing, strictly concave, and satisfies the Inada conditions. There is habit persistence in consumption, captured by γ, to account for the stylized facts about consumption. In addition, $U(.)$ is assumed to have a constant-relative-risk aversion (CRRA) form.

The consumption basket and the labour supply of the representative home household are represented by $\mathcal{C}^w_t(i)$ and $\mathcal{H}^w_t(i)$ respectively. $\mathcal{P}_t$ is the aggregate level of domestic country price and $\mathcal{M}_t(i)/\mathcal{P}_t$ therefore denotes real money balances at the end of period t. In equation (2.16), $\beta \in (0, 1)$ represents the household's discount factor, which is related to the inverse of the risk-free interest rate in credit markets and $\mathbf{E}_t$ is the conditional expectation operator evaluated at time t.

Household consumption is a CES function of domestic and foreign final goods, assembled as

$$\mathcal{C}_t^w = \left[\omega_c^{1/\eta_c}\mathcal{C}_{h,t}^{w\ (\eta_c-1)/\eta_c} + (1-\omega_c)^{1/\eta_c}\mathcal{C}_{f,t}^{w\ (\eta_c-1)/\eta_c}\right]^{\eta_c/(1-\eta_c)}, \quad \eta_c > 0. \tag{2.17}$$

where $C_{h,t}^w$ and $C_{f,t}^w$ denote consumption of home-produced goods and foreign-produced goods, respectively. Further, η_c is the elasticity of substitution between home-produced goods and foreign-produced goods and ω_c represents the weight of home-produced goods in the aggregate domestic consumption. For instance, if $\omega_c > 1/2$, then the representative household in the home country has a home bias in the aggregate consumption.

Next, the home and foreign goods bundles are themselves assumed to be CES aggregates of differentiated home and foreign-produced goods such as

$$\begin{aligned} \mathcal{C}_{h,t}^w &= \left[\left(\frac{1}{\eta^w}\right)^{1/\xi_c} \int_0^{\eta^w} \mathcal{C}_{h,t}^w(i)^{1-\frac{1}{\xi_c}} di\right]^{\frac{\xi_c}{\xi_c-1}}, \\ \mathcal{C}_{f,t}^w &= \left[\left(\frac{1}{\eta^w}\right)^{1/\xi_c} \int_0^{\eta^w} \mathcal{C}_{f,t}^w(i)^{1-\frac{1}{\xi_c}} di\right]^{\frac{\xi_c}{\xi_c-1}}, \end{aligned} \tag{2.18}$$

where ξ_c represents the intra-temporal elasticity of substitution between goods produced within each country.[8] Consequently, the aggregate price of the final good is a CES function of the home currency price of goods produced in the home and foreign country

$$\mathcal{P}_t = \left[\omega_c\mathcal{P}_{h,t}^{(1-\eta_c)} + (1-\omega_c)\mathcal{P}_{f,t}^{(1-\eta_c)}\right]^{1/(1-\eta_c)}, \tag{2.19}$$

whereas the foreign aggregate price is defined analogously (throughout, asterisks denote foreign variables)

$$\mathcal{P}_t^* = \left[\omega_c^*\mathcal{P}^{*\,(1-\eta_c)}_{h,t} + (1-\omega_c^*)\mathcal{P}^{*\,(1-\eta_c)}_{f,t}\right]^{1/(1-\eta_c)}. \tag{2.20}$$

where $\mathcal{P}^*{}_{h,t}$ and $\mathcal{P}^*{}_{f,t}$ are, respectively, the foreign currency price of home and foreign-produced goods. Finally, using the first order conditions, the domestic and foreign composite good are

[8] Consistent with the assumption of specialization in production, the elasticity of substitution is higher among brands produced within a country, than across types of national goods, that is, $\xi_c > \eta_c$.

optimally determined as

$$\mathcal{C}^w_{h,t} = \omega_c \left(\frac{\mathcal{P}_{h,t}}{\mathcal{P}_t}\right)^{-\eta_c} \mathcal{C}^w_t \qquad \text{and} \qquad \mathcal{C}^w_{f,t} = (1-\omega_c)\left(\frac{\mathcal{P}_{f,t}}{\mathcal{P}_t}\right)^{-\eta_c} \mathcal{C}^w_t . \tag{2.21}$$

The representative household in the home country holds $\theta^b_h \mathcal{D}_{h,t}$ units of real deposits in the home bank and $(1-\theta^b_h)\mathcal{D}_{h,t}$ units of real deposits in the foreign bank. Real deposits pay the gross interest rate $R^D_{h,t}$ in the home country and $R^D_{f,t}$ in the foreign country.

In period t, domestic workers supply a differentiated labour to home entrepreneurs for which they receive a nominal wage of $\mathcal{W}^w_t(i)$. Workers pay taxes on their wage and the tax rates imposed by the home and foreign country are, respectively, given by τ_w and τ^*_w. In contrast to capital, which is perfectly mobile internationally, labour is assumed to be immobile between countries. As a result, the typical household's budget constraint is given by:

$$\begin{aligned} &\mathcal{C}^w_{t+l}(i) + \frac{\mathcal{M}_{t+l}(i)}{\mathcal{P}_{t+l}} + \theta^b_h \mathcal{D}_{h,t+l}(i) + (1-\theta^b_h)\Upsilon_{t+l}\mathcal{D}_{f,t+l}(i) = \ \theta^b_h \frac{R^D_{h,t}\mathcal{D}_{h,t+l-1}(i)}{\pi_{t+l}} \\ &\frac{\mathcal{M}_{t+l-1}(i)}{\mathcal{P}_{t+l}} + \Upsilon_{t+l}(1-\theta^b_h)\kappa_{t+l}\frac{R^D_{f,t}\mathcal{D}_{f,t+l-1}(i)}{\pi^*_{t+l}} + (1-\tau_w)\mathcal{W}^w_{t+l}\mathcal{H}^w_{t+l}(i) + Tr_{t+l}(i)\,, \end{aligned} \tag{2.22}$$

where $\mathcal{Q}_t$ is the nominal exchange rate and $\Upsilon_t = \mathcal{Q}_t\mathcal{P}^*_t/\mathcal{P}_t$ is the real exchange rate, defined as the relative price of final goods and $Tr_{t+l}(i)$ denotes a lump-sum money transfer from the home monetary authority to the home domestic household. Further, π_{t+l} and π^*_{t+l} are the domestic and foreign inflation rate, respectively, and $\kappa(.)$ represents the premium associated with foreign deposits. This function captures the costs (or benefits) for home workers of undertaking positions in international financial markets.[9] Finally, a quadratic adjustment cost function is introduced to penalize swift changes in the stock of physical capital. Thus, with δ representing the depreciation rate of capital, the law of motion for capita is $\mathcal{I}_{h,t} = \mathcal{K}_{h,t} - (1-\delta)\mathcal{K}_{h,t-1} + \Psi(\mathcal{K}_{h,t}, \mathcal{K}_{h,t-1})$, where $\Psi(.,.)$ represents adjustment costs.[10]

[9] Formally, $\log(\kappa_{t+l}) = \varpi \left[\exp\left(\frac{\Upsilon_{t+l}\mathcal{D}_{f,t+l}}{\mathcal{Y}_{t+l}}\right) - 1\right] \varepsilon^e_{t+l}$, where ϖ is a parameter that captures the risk of foreign deposits. Computationally, a premium on the foreign interest rate function is introduced to help the system have a well-defined steady state. In addition, we add a time-varying shock, ε^e_t, to the risk premium function to capture the changes in the foreign debt. We assume that this shock follows a random walk with drift given by: $\log(\varepsilon^e_t) = \rho_e \log(\varepsilon^e_{t-1}) + \epsilon^e_t, \quad \text{where} \ \ \epsilon^e_t \sim N(0,1)$.

[10] $\Psi(\mathcal{K}_{h,t}, \mathcal{K}_{h,t-1}) = \frac{\phi}{2}\left(\frac{\mathcal{K}_{h,t}}{\mathcal{K}_{h,t-1}} - 1\right)^2 \mathcal{K}_{h,t-1}, \quad \phi > 0.$

Wage setting:

The decision related to wage setting is made following the New Keynesian framework as in Smets and Wouters (2007) or Christiano et al. (2010). Each household supplies a differentiated labour type used by intermediate good producers. The household has monopoly power over its own-type labour input and sets the wage using Calvo's partial indexation mechanism. The labour input supplier aggregates different labour types using the constant-elasticity-of-substitution aggregator $\mathcal{H}_t^w = \left(\int_0^{\eta^w} \mathcal{H}_t^w(i)^{\frac{\xi_w-1}{\xi_w}} di\right)^{\frac{\xi_w}{\xi_w-1}}$, where $0 \leq \xi_w \leq +\infty$ is the elasticity of substitution among different types. This aggregator maximizes profits subject to the production function and given the price, $\mathcal{W}_t^w(i)$ of each labour type, and the price of the composite labour input, $\mathcal{W}_t^w$. The first order conditions imply that $\mathcal{H}_t^w(i) = \left(\frac{\mathcal{W}_t^w(i)}{\mathcal{W}_t^w}\right)^{-\xi_w} \mathcal{H}_t^w$ and $\mathcal{W}_t^w = \left(\int_0^{\eta^w} \mathcal{W}_t^w(i)^{1-\xi_w} di\right)^{\frac{1}{1-\xi_w}}$.

Following Calvo (1983), households' wage setting is as follows: each period, a fraction $1-\phi_w$ of households reoptimize their wages, while the remaining ϕ_w households cannot change their wages optimally, but can index it wages to past inflation, with a degree of indexation captured by $\chi_w \in (0,1)$. This nominal rigidity implies that a household that has reoptimizer for k periods has, by period $t+k$, a wage equal to $\prod_{s=1}^{k} \frac{\pi_{t+s-1}^{\chi_w}}{\pi_{t+s}} \frac{\mathcal{W}_t^w(i)}{\mathcal{P}_t}$. Under the labour demand for each type derived above, demand for the labour of this household is $\mathcal{H}_{t+k}^w(i) = \left(\prod_{l=1}^{k} \frac{\pi_{t+l-1}^{\chi_w}}{\pi_{t+l}} \frac{\mathcal{W}_t^w(i)}{\mathcal{W}_{t+k}^w}\right)^{-\xi_w} L_{t+k}^w$, and the overall price for the composite labour input is a geometric average of past and newly reoptimized wages: $\pi_t^{1-\xi_w} \mathcal{W}_t^{w\,1-\xi_w} = \eta^w(1-\phi_w)\pi_t^{1-\xi_w} \widetilde{\mathcal{W}^w}_t^{1-\xi_w} + \phi_w \mathcal{W}_{t-1}^{w\;1-\xi_w} \pi_{t-1}^{1-\xi_w}$.

Given preferences and the budget constraint, the home household's optimization problem consists of choosing $(\mathcal{C}_t^w(i), M_{t+1}(i), \mathcal{D}_{h,t+1}(i), \mathcal{D}_{f,t+1}(i))$ for all $t \in [0,\infty)$ to maximize lifetime utility function, $\mathbf{U}_t(\dots)$, as in:

$$\max \left\{ \mathbf{U}\left(\left(\mathcal{C}_t^w(i) - \gamma \mathcal{C}_{t-1}^w(i)\right), \mathcal{M}_t/\mathcal{P}_t(i), 1 - L_t(i)\right)\right\}$$

$$\text{s.t.} \quad \begin{cases} \mathcal{C}_t^w(i) + \mathcal{M}_t(i)/\mathcal{P}_t + \theta_h^b \mathcal{D}_{h,t}(i) + (1-\theta_h^b)\Upsilon_t \mathcal{D}_{f,t}(i) = \theta_h^b R_{h,t}^D \mathcal{D}_{h,t-1}(i)/\pi_t \\ + \mathcal{M}_{t-1}(i)/\mathcal{P}_t + \Upsilon_t(1-\theta_h^b)\kappa_t R_{f,t}^D \mathcal{D}_{f,t-1}(i)/\pi_t^* + (1-\tau_w)\mathcal{W}_t^w L_t(i) + Tr_t(i), \end{cases}$$

where $\mathbf{U}(.) = log\left(\mathcal{C}_t^w(i) - \gamma \mathcal{C}_{t-1}^w(i)\right) + \psi log\left(1 - L_t(i)\right) + \zeta_M log\left(\mathcal{M}_t(i)/\mathcal{P}_t\right)$ and $\Lambda_t(i)$ is therefore the Lagrange multiplier associated with the budget constraint.

Household's first-order conditions (except for labor and wages) are given by:

$$\frac{1}{\mathcal{C}_t^w(i) - \gamma \mathcal{C}_{t-1}^w(i)} - \beta\gamma \mathbf{E}_t \left[\frac{1}{\mathcal{C}_{t+1}^w(i) - \gamma \mathcal{C}_t^w(i)} \right] = \Lambda_t(i); \tag{2.23}$$

$$\frac{\zeta_M}{\mathcal{M}_t(i)/\mathcal{P}_t} + \beta \mathbf{E}_t \left[\frac{\Lambda_{t+1}(i)}{\pi_{t+1}} \right] = \Lambda_t(i); \tag{2.24}$$

$$\beta \mathbf{E}_t \left[\Lambda_{t+1}(i) \frac{R_{h,t+1}^D}{\pi_{t+1}} \right] = \Lambda_t(i); \tag{2.25}$$

$$\beta \mathbf{E}_t \left[\Lambda_{t+1}(i) \Upsilon_{t+1} \frac{R_{f,t+1}^D}{\pi_{t+1}^*} \right] = \Upsilon_t \Lambda_t(i); \tag{2.26}$$

Capital good producers:

As in Bernanke et al. (1999) or Christiano et al. (2010), we assume that there are competitive physical capital producers in each country. They purchase the newly produced physical capital as well as the undepreciated physical capital to put new capital in place for period $t+1$. The first order conditions from these producers' maximization problem lead to:[11]

$$\begin{aligned} &\beta \mathbf{E}_t \left[\frac{\Lambda_{t+1}(i) \mathcal{Q}_{h,t+1}}{\Lambda_t(i)} \left(1 - \delta + \phi \left(\frac{\mathcal{K}_{h,t+1}(i)}{\mathcal{K}_{h,t}(i)} - 1 \right) \frac{\mathcal{K}_{h,t+1}(i)}{\mathcal{K}_{h,t}(i)} \right) \right] \\ =&\mathcal{Q}_{h,t} \left(1 + \phi \left(\frac{\mathcal{K}_{h,t}(i)}{\mathcal{K}_{h,t-1}(i)} - 1 \right) \right); \end{aligned} \tag{2.27}$$

2.2.2 Distribution and Good Production

The distribution sector is composed of intermediate and final good producers. Domestic intermediate goods producers consist of domestic and foreign firms operating under monopolistic competition. Output produced by these producers is then assembled into a composite domestic good and a composite foreign good by perfectly competitive firms. Finally, this domestic composite good is combined with imported foreign goods to produce the final good, which is

[11] Derivations of this equation is documented in Christiano et al. (2010)

allocated to consumption and investment.

2.2.3 Intermediate Good Production

Domestic Intermediate Good Production:

Intermediate goods are produced by monopolistically competitive firms under nominal rigidities *à la Calvo (1983)*. The producer of good j combines rented capital $\widetilde{\mathcal{K}}_t(j)$ with the labour supply of households, bankers and entrepreneurs to produce the differentiated intermediate goods $\mathcal{Y}_t$. A fraction of this intermediate good is used to produce the composite domestic good and the remaining part, $\mathcal{Y}_t^x$, is exported. Each intermediate good producer j has access to a production function represented by

$$\mathcal{Y}_t(j) = a_t^{\mathcal{Y}} f\left(\widetilde{\mathcal{K}}_t, \mathcal{H}_t^w, \mathcal{H}_t^e, \mathcal{H}_t^b\right) - \Theta, \tag{2.28}$$

where the non-negative parameter Θ represents fixed costs of production and $a_t^{\mathcal{Y}}$ is a serially correlated technology shock that follows the stochastic process $\log(a_t^{\mathcal{Y}}) = (1-\rho_a)\log(a^{\mathcal{Y}}) + \rho_a log(a_{t-1}^{\mathcal{Y}}) + \epsilon_t^{\mathcal{Y}}$.[12] The cost minimization problem of these producers is given by:

$$\begin{aligned}
\min_{\mathcal{K}_t(j),\mathcal{H}_t^w(j),\mathcal{H}_t^e(j),\mathcal{H}_t^b(j)} & \left\{ r_t^k \widetilde{\mathcal{K}}_t(j) + \mathcal{W}_t^w \mathcal{H}_t^w(j) + \mathcal{W}_t^e \mathcal{H}_t^e(j) + \mathcal{W}_t^b \mathcal{H}_t^b(j) \right\} \\
s.c. \quad \mathcal{Y}_t(j) &= a_t^{\mathcal{Y}} \widetilde{\mathcal{K}}_t(j)^{\alpha_k} \mathcal{H}_t^w(j)^{\alpha_w} \mathcal{H}_t^e(j)^{\alpha_e} \mathcal{H}_t^b(j)^{\alpha_b} - \Theta \\
\widetilde{\mathcal{K}}_t(j) &= \widetilde{\mathcal{K}}_{h,t} + \widetilde{\mathcal{K}}_{f,t} \\
\widetilde{\mathcal{K}}_{h,t} &= \theta_h^e \mathcal{K}_{h,t} \\
\widetilde{\mathcal{K}}_{f,t} &= (1-\theta_h^e)\mathcal{K}_{f,t},
\end{aligned} \tag{2.29}$$

where r_t^k denotes the rental rate on capital services. The nominal wage on the labour inputs from households, entrepreneurs and bankers are denoted by $\mathcal{W}_t^h$, $\mathcal{W}_t^e$ and $\mathcal{W}_t^b$, respectively. Let mc_t be the Lagrange multiplier associated with the production function constraint, which can be interpreted as the real marginal cost of producing one additional unit of output. The first order conditions from 2.29 are:

$$r_t^k(j) = mc_t \frac{\alpha_k \mathcal{Y}_t(j)}{\mathcal{K}_t(j)}, \tag{2.30}$$

[12] Θ is calibrated to ensure that economic profits of capital producer is roughly equal to zero in the equilibrium

$$\mathcal{W}_t^w = mc_t \frac{\alpha_w \mathcal{Y}_t(j)}{\mathcal{H}_t^w(j)}, \quad \mathcal{W}_t^e = mc_t \frac{\alpha_e \mathcal{Y}_t(j)}{\mathcal{H}_t^e(j)}, \quad \mathcal{W}_t^b = mc_t \frac{\alpha_b \mathcal{Y}_t(j)}{\mathcal{H}_t^b(j)}. \tag{2.31}$$

After producing, the home entrepreneur sells the undepreciated capital stock at price $\mathcal{Q}_{h,t+1}$ to the home capital producer, so that the average rate of return of capital across entrepreneurs is:

$$\mathcal{R}_{h,t}^k = \frac{mc_t \frac{\alpha_k \mathcal{Y}_t(j)}{\mathcal{K}_{h,t}(j)} + \mathcal{Q}_{h,t+1}(1-\delta)}{\mathcal{Q}_{h,t}}. \tag{2.32}$$

Price-setting is as follows. Each period, a fraction $1-\phi_h$ of domestic firms can reoptimize their prices. When allowed to do so, firm chooses an output price $\widetilde{\mathcal{P}}_{h,t}(j)$ to maximize discounted real profits. The remaining firms can only index their prices to the previous periods' inflation rate, with the degree of indexation controlled by $\chi_h \in (0,1)$. An intermediate good producer j allowed to reoptimize at time t realizes that the chosen price today, $\widetilde{\mathcal{P}}_{h,t}(j)$, will be after l periods with no reoptimizing

$$\mathcal{P}_{h,t+l}(j) = (\pi_{h,t+1})^{\chi_h} \times (\pi_{h,t+2})^{\chi_h} \times \cdots \times (\pi_{h,t+l-1})^{\chi_h} \times \mathcal{P}_{h,t}(j) = \prod_{s=1}^{l-1} (\pi_{h,t+s})^{\chi_h} \mathcal{P}_{h,t}(j), \tag{2.33}$$

where $\pi_{h,t+l} = \mathcal{P}_{h,t+l}/\mathcal{P}_{h,t+l-1}$. The problem of the reoptimizing firm is then:

$$\begin{aligned} &\max_{\widetilde{\mathcal{P}}_{h,t}(j)} \mathbf{E}_t \sum_{l=0}^{\infty} (\beta\phi_h)^l \Lambda_{t+l} \left\{ \left(\prod_{s=1}^{l-1} (\pi_{h,t+s})^{\chi_h} \frac{\widetilde{\mathcal{P}}_{h,t}(j)}{\mathcal{P}_{h,t+l}} - mc_{t+l} \right) \mathcal{Y}_{t+l}(j) \right\} \\ &s.c. \quad \mathcal{Y}_{t+l}(j) = \left(\prod_{s=1}^{l-1} (\pi_{h,t+s})^{\chi_h} \frac{\widetilde{\mathcal{P}}_{h,t}(j)}{\mathcal{P}_{h,t+l}} \right)^{-\xi_h} \mathcal{Y}_{t+l}, \end{aligned} \tag{2.34}$$

where Λ_{t+l} is the marginal utility of wealth for a firm j after $t+l$ periods.

Letting Let $\widetilde{p}_{h,t} = \widetilde{\mathcal{P}}_{h,t}/\mathcal{P}_t$ and assuming that all firms of type j adopt the same pricing strategy, the first order condition relating to optimal pricing of the domestic intermediate good j leads to

$$\widetilde{p}_{h,t} = \frac{\xi_h}{\xi_h - 1} \frac{\mathbf{E}_t \sum_{l=0}^{\infty} (\beta\phi_h)^l \Lambda_{t+l} mc_{t+l} \left(\prod_{s=1}^{l-1} \frac{(\pi_{h,t+s})^{\chi_h}}{\pi_{h,t+s+1}} \right)^{-\xi_h} \mathcal{Y}_{t+l}(j)}{\mathbf{E}_t \sum_{l=0}^{\infty} (\beta\phi_h)^l \Lambda_{t+l} \left(\prod_{s=1}^{l-1} \frac{(\pi_{t+s}^h)^{\chi_h}}{\pi_{t+s+1}^h} \right)^{1-\xi_h} \mathcal{Y}_{t+l}(j)}. \tag{2.35}$$

Foreign Intermediate Goods

The domestic economy imports foreign intermediate goods via a continuum of importing firms indexed by $j \in (0,1)$. There is a monopolistic competition in this market and each good is an imperfect substitutes for the other in the production of the composite imported good, $\mathcal{Y}_{f,t}$, which is assembled by a representative competitive firm. Foreign price-setting follows Calvo's assumption, as in the home country.[13] Each period, a fraction $1-\phi_f$ of firms can reoptimize their prices. When allowed to do so, firm chooses the price of its output, $\widetilde{\mathcal{P}}_{f,t}(j)$, in order to maximize discounted real profits. Similar to what was the case in the home country price setting, the first order condition yields

$$\widetilde{\mathcal{P}}_{f,t}(j) = \frac{\xi_f}{1-\xi_f} \frac{\mathbf{E}_t \sum_{k=0}^{\infty} (\beta\phi_f)^k \Lambda_{t+k} \Upsilon_t \mathcal{Y}_{f,t+k}(j)}{\mathbf{E}_t \sum_{k=0}^{\infty} (\beta\phi_f)^k \Lambda_{t+k} \mathcal{Y}_{f,t+k}(j)/\mathcal{P}_{f,t+k}}, \tag{2.36}$$

where ξ_f represents the elasticity of substitution between differentiated imported goods and s_{t+l} is the real exchange rate, the equivalent to marginal cost of production in the domestic country.

2.2.4 Rest of the economy

Monetary policy is conducted by the home central bank, which manages the short-term nominal interest rate $R_t^D = (1+r_t^D)$ in response to fluctuations in the domestic GDP, $\mathcal{Y}_t$, and in consumer price inflation π_t. This Taylor rule allows the central bank to set the nominal interest rate through open market operations and is specified as[14]:

$$\log\left(R_{h,t}^D/\overline{R}_h^D\right) = \lambda_r \log\left(R_{h,t-1}^D/\overline{R}_h^D\right) + (1-\lambda_r)\left(\lambda_\pi log\left(\pi_t/\overline{\pi}\right) + \lambda_y \log\left(\mathcal{Y}_t/\overline{\mathcal{Y}}\right)\right) + \log\left(\vartheta_{h,t}\right), \tag{2.37}$$

with $\lambda_r \in (0,1)$. In equation (2.37), $\overline{\pi}$ and $\overline{\mathcal{Y}}$ represent the target level of inflation and the target level of output of the domestic economy, respectively. The term $\vartheta_{h,t}$ denotes a random shock to monetary policy, that follows a first-order autoregressive process given by:

[13] This Calvo-type staggered price setting in the imported goods market allows to capture incomplete exchange rate pass-through from import to domestic prices

[14] The use of the previous period interest rate allows to match the smooth profile in observed interest rate

$\log(\vartheta_{h,t}) = \rho_{h,mp}\log(\vartheta_{h,t-1}) + \epsilon_t^{h,mp}$, with $\epsilon_t^{h,mp} \sim N(0,1)$. The foreign monetary policy variables (the interest rate paid for holding foreign bonds, $R_{f,t}^D$, and the foreign inflation, π_t^*) are both exogenous.[15] Formally, we use two stochastic processes to capture the quantitative impact of the foreign monetary policy shock. Thus,

$$\log\left(R_{f,t}^D/\overline{R}_f^D\right) = \lambda_r \log\left(R_{f,t-1}^D/\overline{R}_f^D\right) + (1-\lambda_r)\left(\lambda_\pi log\left(\pi_t^*/\overline{\pi}^*\right) + \lambda_y \log\left(\mathcal{Y}_t^*/\overline{\mathcal{Y}}^*\right)\right) + \log\left(\vartheta_{f,t}\right), \tag{2.38}$$

and $\log(\vartheta_{f,t}) = \rho_{f,mp}\log(\vartheta_{f,t-1}) + \epsilon_t^{f,mp}$ where $\rho_{f,mp} \in (0,1)$ denotes the persistence of the foreign monetary policy shock and $\overline{R}_f^D$ denotes the targeted steady-state value of the foreign interest rate, $R_{f,t}^D$. The government budget constraint is given by: $\mathcal{G}_{h,t} + \mathcal{M}_{t-1}/\mathcal{P}_t + Tr_t = \tau_w \mathcal{W}_t^w \mathcal{H}_t^w + \mathcal{M}_t/\mathcal{P}_t$. Government spending is exogenous and determined using the following stochastic equation: $\log(\mathcal{G}_{h,t}) = (1-\rho_G)\log(\overline{\mathcal{G}}_h) + \rho_G \log(\mathcal{G}_{h,t-1}) + \epsilon_{G,t}$, where $\overline{G}_h$ denotes the targeted steady-state value of government spendings.

2.3 Market clearing and competitive equilibrium

The market clearing conditions require that the private and government demand be equal to the supply. In the aggregate, it follows that

$$\mathcal{H}_t^w = \int_0^{\eta^h} \mathcal{H}_t^w(i)di, \quad \mathcal{H}_t^e = \int_0^{\eta^e} \mathcal{H}_t^e(j)dj, \quad \mathcal{H}_t^b = \int_0^{\eta^b} \mathcal{H}_t^b(j)dj; \tag{2.39}$$

and

$$\mathcal{H}_t = \mathcal{H}_t^w + \mathcal{H}_t^b + \mathcal{H}_t^e. \tag{2.40}$$

Exiting banks and entrepreneurs consume the value of their available wealth. This implies the following for aggregate consumption of entrepreneurs and bankers

$$\begin{aligned}\mathcal{C}_t^e =& (1-\tau^e)\theta_h^e \int_0^{\eta^e} \left((1-\Gamma(\overline{\omega}_h))\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}(i)\mathcal{R}_{h,t}^k(i)\right) di + \\ & (1-\theta_h^e)(1-\tau^e)\int_0^{\eta^e} \left((1-\Gamma(\overline{\omega}_f))\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}(i)\mathcal{R}_{f,t}^k(i)\right) di;\end{aligned} \tag{2.41}$$

[15] This assumption is consistent with our small open economy setting in which the foreign monetary variables are exogenously determined.

$$\begin{aligned}\mathcal{C}_t^b =&(1-\tau^b)\int_0^{\eta^b} \theta_h^e \left((\Gamma(\overline{\omega}_h) - \mu_h G(\overline{\omega}_h)\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}(i)\mathcal{R}_{h,t}^k(i)\right) di + \\ &(1-\tau^b)(1-\theta_f^e)\int_0^{\eta^b} \left((\Gamma(\overline{\omega}_f) - \mu_f G(\overline{\omega}_f))\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}(i)\mathcal{R}_{f,t}^k(i)\right) di.\end{aligned} \tag{2.42}$$

Equilibrium in the goods markets requires that production be equal to aggregate demand:

$$\begin{aligned}\mathcal{Y}_t &= \mathcal{C}_t^w + \mathcal{C}_t^e + \mathcal{C}_t^b + \mathcal{I}_t + \mu_h \theta_h^e G(\overline{\omega}_h)\mathcal{R}_{h,t}^k \mathcal{Q}_{h,t}\mathcal{K}_{h,t} \\ &+ \mu_f(1-\theta_f^e)\Upsilon_t G(\overline{\omega}_f)\mathcal{R}_{f,t}^k \mathcal{Q}_{f,t} K_{f,t}.\end{aligned} \tag{2.43}$$

Definition (Competitive equilibrium) *A competitive equilibrium is defined as a set of functions for (i) households' policies $\mathcal{C}_t^w(i)$, $\mathcal{I}_t(i)$, $\mathcal{K}_{h,t}(i)$ and $\mathcal{K}_{f,t}(i)$ that solve the maximization problem of the household; (ii) firms' policies $\mathcal{K}_t(j)$, $\mathcal{H}_t^w(j)$, $\mathcal{H}_t^e(j)$, $\mathcal{H}^b(j)$, $\mathcal{W}_t^e$, $\mathcal{W}^b$ and $\mathcal{W}_t^w(i)$ that solves firms' maximization problems; (iii) optimal financial contract that solve the maximization problem associated with the financial contract; (iv) aggregate prices $\mathcal{P}_{h,t}$, $\mathcal{P}_{f,t}$ and P_t and (v) saving and consumption decision rules for bankers and entrepreneurs.*

3 Model Simulations

In this section we present the results of some quantitative experiments that assess the extent to which cross-border banking activities affect the transmission of real and financial shocks and international business cycles.

3.1 Model parametrization

To evaluate how financial frictions in credit markets combined with cross-border banking activities affect the propagation of shocks, we set the parameters of our baseline model to reflect the key features of the Canadian and the U.S. economy. Most of these parameters are widely used and their values have become standard in the financial frictions literature (Christiano et al. (2010), Dib (2010) and Meh and Moran (2010)). The home country is calibrated from Canadian data whereas the foreign country is parametrized from the US data.

In the financial market, parameters that are related to the capital production and the optimal financial contract between bankers and entrepreneurs are calibrated following Carlstrom and

Fuerst (1997), Bernanke et al. (1999), and Christiano et al. (2010). Accordingly, the steady state value of the bank's minimum capital requirement is set to $\iota_h = 20\%$ in Canada and $\iota_f = 10\%$ in the United States. The monitoring cost is set to $\mu = 2.5\%$ in both countries and cross-border banking parameters in the home and foreign country are fixed to $\theta_h^e = 0.75$ and $\theta_f^e = 0.85$. Recall that $1 - \theta_h^e$ and $1 - \theta_f^e$ represent the size of foreign demand of fund, and therefore, $\theta_h^e = \theta_h^e = 1$ refers to the case of an absence of cross-border borrowing activities. In the supply side, it is difficult to measure the actual degree of banking globalization that matches our model. Thus, the parameters θ_h^b and θ_f^b are set to $\theta_h^b = \theta_f^b = 0.85$, which are consistent to estimations in Christiano et al. (2010).

The remaining parameters governing the financial contract are set following Bernanke et al. (1999). Accordingly, the steady state risk spreads, $\mathcal{R}_h^k - R_h$ and $\mathcal{R}_f^k - R_f$ are set to 200 basis points, approximately the historical average spread between the prime lending rate and the six-month Treasury bill rate. The annualized business failure rate, $F(\overline{\omega}_h)$ in the home country and $F(\overline{\omega}_f)$ in the foreign country, are set to 3%, the approximate rate in the Canadian and U.S. historical data. The ratio of capital to net worth, ∇_h^e and ∇_f^e is fixed to 50% in each country, the approximate value in the Canadian and U.S. data.

In the representative household's utility function, the weight on labour supply ψ is set to 9.05, which leads to a steady-state value of household work effort equal to 30% of available time. Following results in Christiano et al. (2010) and Meh and Moran (2010), the parameter indexing habit formation, γ, is fixed to 0.65.

The household's discount factor, β, is set to 0.99, implying a long-run nominal interest rate of 4% annually in both countries. The share of capital in the production function of intermediate good producers in the home and foreign country, $\alpha_k = \alpha_{kh} = \alpha_{kf}$, is set to 0.36 and the depreciation rate of capital is fixed to 0.025, standard values in the New Keynesian literature. As we want to reserve a small role in the production for the hours worked by bankers and entrepreneurs, we set the share of the labour input of households, α_w, to 0.63. Then we choose $\alpha_b = \alpha_e = 0.005$, reflecting an equal contribution of bankers and entrepreneurs in the intermediate good production process which allows entrepreneurs and bankers to always have non-zero net worth.

The parameter capturing the fixed costs in the production function, Θ, is set in order to ensure that the steady state value of profits equals zero. The persistence of the technology shock, ρ^a, is set to 0.95 and its standard deviation is 0.0015, which ensure that the model's simulated output volatility equal that of observed aggregate data. The nominal price rigidity parameter as well as the nominal wage-setting parameter are set following Calvo's model of staggered price and wage adjustment. As in Christiano et al. (2005), the probability of not reoptimizing for price and wage setters in the domestic country, ϕ_h and ϕ_w, are fixed to 0.75 and 0.64, respectively. The elasticity of of substitution between domestic intermediate goods, ξ_h, and the elasticity of substitution between domestic labour types, ξ_w, are set to 8 and 21, respectively. The price and wage indexation parameters, χ_h and χ_w, are calibrated to 0.15. These values are estimated in Christensen and Dib (2008) for the U.S. economy and are commonly used in the literature. Correspondingly, the probability of not reoptimizing for foreign price setters, ϕ_f, is set to 0.5, while the elasticity of substitution between foreign intermediate goods production, ξ_f, is calibrated to 8.

Domestic and foreign monetary policy parameters λ_r, λ_π et λ_y are set of 0.8, 1.5 and 0.1/4, respectively. These values satisfy the Taylor principle and are consistent with those estimated in Clarida et al. (2000). The standard deviation of both domestic and foreign monetary policy shocks is fixed to 0.0016, $\rho_{h,mp} = \rho_{f,mp} = 0.0016$, which ensures that a one-standard deviation shock moves the interest rate by 0.6 percentage points. This value is consistent with the empirical estimates reported in Christiano et al. (2005).

The remaining parameters and steady-state ratios of the baseline model are set in order to ensure that the model's steady state match to standard New-Keynesian models. Thus, household consumption to GDP is calibrated to be 73%. Also, the investment-to-GDP ratio and Capital-to-GDP ratio are set 18% and 12. Finally, the domestic good to final good ratio and imported good to final ratio are set to 70% and 30%. Tables 2 and 3 report the calibration and the steady-state values of some key variables.

3.2 Findings

In this section we present the simulated path of the main real and financial variables of the two-country economy in response to real and financial shocks. In this stochastic simulation,

Table 2: Parameter Calibration: Baseline model

	Parameters	Description	Values
Preferences	β	Discount factor	0.99
	γ	Habit formation	0.65
	ψ	Weight of leisure in utility	9.05
	ζ_M	Elasticity of money demand	0.00183
Technologies and final good production	α_k	Capital share	0.36
	α_w	Workers labor share	0.6399
	α_e	Entrepreneur labor share	0.00005
	α_b	Bankers labor share	0.00005
	ω_c	Share of domestic good in final good	0.7
	η_c	Elasticity of substitution (home and foreign goods)	0.59
	δ	Depreciation rate of capital	0.02
	τ_w	Labor income tax rate	0.25
	ρ_a	Autocorrelation of home technology shock	0.95
	σ_ϵ	Standard deviation of home technology shock	0.01
Financial sector	$F(\overline{\omega}_h)$, $F(\overline{\omega}_f)$	Annualized business failure rate	3%
	∇^e_h, ∇^e_f	Ratio of capital to net worth	50%
	μ	Monitoring cost	0.025
	τ_e	Entrepreneur's death probability	0.78
	τ_b	Banker's death probability	0.72
Nominal rigidities	ξ_w	Elasticity of labor supply	21
	ξ_h	Elasticity of substitution for domestic goods	8
	ξ_f	Elasticity of substitution for foreign goods	8
	ϕ_w	Wage reoptimization probability	0.64
	ϕ_h	Domestic price reoptimization probability	0.75
	ϕ_f	Foreign price reoptimization probability	0.5
	χ_w	Degree of wage indexation	0.15
	χ_h	Degree of price indexation	0.15
Monetary policy	λ_r	Taylor rule: Interest smoothing	0.8
	λ_π	Taylor rule: inflation coefficient	1.5
	λ_y	Taylor rule: GDP coefficient	0.025
	$\rho_{h,mp}$	Autocorrelation of home monetary policy shock	0.95
	$\rho_{f,mp}$	Autocorrelation of foreign monetary policy shock	0.95
	ρ_G	Autocorrelation of government spending shock	0.98
	$\sigma_{h,mp}$	Standard deviation of home monetary policy shock	0.0016
	$\sigma_{h,mp}$	Standard deviation of foreign monetary policy shock	0.0016
	σ_G	Standard deviation government spending shock	0.0016

Table 3: Steady-state values and ratios: Baseline model

Parameters	Description	Values
	Steady-state values	
$\mathcal{R}_h^k = \mathcal{R}_f^k$	Gross real interest rate of investment projects	1.2118
$\overline{\pi}_h = \overline{\pi}_f$	Inflation rate	$1.02^{1/4}$
$\overline{R}_h^D = \overline{R}_f^D$	Gross real interest rate of deposits	1.0101
	Steady-state ratios	
$\widetilde{\mathcal{C}}_h/\mathcal{Y}_h =$	Households consumption to GDP ratio	61%
$\widetilde{\mathcal{C}}_f/\mathcal{Y}_f =$	Households consumption to GDP ratio	56%
$\eta^b\widetilde{\mathcal{N}}_h^e/\mathcal{Y}_h = \eta^b\widetilde{\mathcal{N}}_f^e/\mathcal{Y}_f$	Entrepreneurs net worth to GDP ratio	0.18%
$\eta^e\widetilde{\mathcal{A}}_h/\mathcal{Y}_h = \eta^e\widetilde{\mathcal{A}}_f/\mathcal{Y}_f$	Bankers net worth to GDP ratio	0.24%
$\widetilde{\mathcal{I}}_h/\mathcal{Y}_h = \widetilde{\mathcal{I}}_f/\mathcal{Y}_f$	Investment to GDP ratio	22.4%
$\widetilde{\mathcal{K}}_h/\mathcal{Y}_h = \widetilde{\mathcal{K}}_f/\mathcal{Y}_f$	Capital to GDP ratio	12

we focus on three main shocks: *(i)* a positive productivity shock in the home country; *(ii)* a monetary policy tightening in the home country; and *(iii)* a monetary policy tightening in the foreign country. These three shocks will allow us to investigate the role of cross-border banking activities in the transmission of international shocks transmission.

Figures 5, 7 and 9 illustrate the responses of macroeconomic variables (output, consumption, investment, inflation and the real exchange rate) and Figures 6, 8 and 10 show responses of financial variables, such as entrepreneurial net worth, banking net worth, the risk premium and the home-foreign debt ratio defined as $\nabla^e_{h,h/f} = \theta^e_h \mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}/\left[(1-\theta^e_f)\Upsilon_t \mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}\right]$.

The simulations θ^e_h, θ^e_f, θ^b_h and θ^b_f, we compare two models characterized by their size of cross-border financial activities. The first model is our baseline two-country economy calibrated from the U.S. and Canadian data in which we allow cross-border lending and borrowing activities between bankers and entrepreneurs. In this benchmark model, the size of cross-border financial activities between entrepreneurs and bankers is kept at 25% for the home country ($\theta^e_h = 0.75$) and 15% for the foreign country ($\theta^e_f = 0.85$) and the international deposit parameters are set to 20% ($\theta^b_h = \theta^b_f = 0.8$). These parameters are consistent with those found by Devereux and Yetman (2010), Ueda (2012) and Guerrieri et al. (2012) and reflect the size of financial transactions between the U.S. and Canada (BIS, consolidated banking statistics). In our setting, the Canadian economy is denoted by the home economy whereas the U.S. economy is designated to be the foreign economy. The second model considers very limited cross-border banking activity, by setting $\theta_h^e = \theta_f^e = \theta_h^b = \theta_f^b = 0.99$. This latter economy is a standard two-country New Keynesian model without international banking activities that we refer to BGG model with only an international goods market (*standard two-country BGG model*). Finally, we analyze bilateral correlations between real and financial variables to assess the role cross-border banking activities play in the synchronization of international business cycles.

3.2.1 Productivity shock in the home country

In this simulation, we consider a productivity shock that affects the production frontier (recall that $\mathcal{Y}_t(j) = a_t^{\mathcal{Y}} f\left(\widetilde{\mathcal{K}}_t, \mathcal{H}_t^w, \mathcal{H}_t^e, \mathcal{H}_t^b\right) - \Theta$) and where the stochastic level of productivity $a_t^{\mathcal{Y}}$ evolves following a first order Markov process:

$$\begin{pmatrix} \log(a_{h,t}^{\mathcal{Y}}) - \log(a_h^{\mathcal{Y}}) \\ \log(a_{f,t}^{\mathcal{Y}}) - \log(a_f^{\mathcal{Y}}) \end{pmatrix} = \begin{bmatrix} \rho_a & 0 \\ 0 & \rho_a \end{bmatrix} \begin{pmatrix} \log(a_{h,t-1}^{\mathcal{Y}}) - \log(a_h^{\mathcal{Y}}) \\ \log(a_{f,t-1}^{\mathcal{Y}}) - \log(a_f^{\mathcal{Y}}) \end{pmatrix} + \begin{pmatrix} \epsilon_{h,t}^{\mathcal{Y}} \\ \epsilon_{f,t}^{\mathcal{Y}} \end{pmatrix}, \tag{3.1}$$

where $\epsilon_{h,t}^{\mathcal{Y}}$ and $\epsilon_{f,t}^{\mathcal{Y}}$ are country-specific innovations with mean 0 and equal standard deviation σ_ϵ. The serial correlation between these two innovations is captured by ρ_ϵ, set to 0.3 in the baseline model. As calibrated, this shock increases the productivity of wholesale goods-producing sector by one percent ($\sigma_\epsilon = 1\%$) at the impact and returns to the steady-state level with an autoregressive parameter of 0.95 ($\rho_a = 0.95$).

A positive technology shock expands the production frontier in the home country and lowers the marginal cost of production. As the supply of home-produced goods increases, prices of home-produced goods relative to those of foreign-produced goods decline, but given the nominal stickiness in prices this can only happen gradually, over time. As a consequence, real wages and real income increase and consumption follows with a slight delay given the habit persistence in preferences. Employment temporally decreases and higher expected returns on capital stimulate investment (Figure 5). Then, demand of home-produced goods increases, leading to an important depreciation of the real exchange rate and a deterioration in the terms-of-trade of the domestic economy. The initial depreciation in the exchange rate raises retail import prices only gradually given the price stickiness *à la Calvo* in the importing goods sector. As the relative price adjustments take time, a strong deficit in the current account emerges mainly because of the positive income effect on imports. The initial deficit in the current account and the term-of-trade gain in the foreign country leads to a sharing of the wealth effects between the two economies and as a result, output rises in the foreign country after a slight fall. Foreign investment increases in the case of no cross-border financial activities and decreases in the baseline model. In the case of no cross-border banking activities, the positive spillover effect of the home productivity shock results in a rise of foreign output and investment. However, with a higher cross-border banking activity, foreign investors will increase their investment in the home country and bankers will increase their financing toward the home country. As result, home investment increases more (by 1% additional) and foreign investment decreases.

Figure 6 presents the responses of home and foreign financial variables following the same positive shock in to home productivity. As the production frontier in the home country expands, expected returns on capital in the home country increase, pushing up both entrepreneurial and baking net worth. Foreign entrepreneurial and banking net wealth also increase, because of positive spillover effects of the home technology shock. However, foreign banking net worth increases more in the baseline model because it receives the positive spillover effects of the return on capital from the home country. The total effect on the two countries is greater in the presence of cross-border banking activities than that in the standard two-country BGG model. Our results are consistent with those in Ueda (2012) and Dedola and Lombardo (2012), who find that the financial accelerator, bank capital and exchange rate channels are enhanced in the presence of cross-border banking activities. Overall, with a positive technology shock in the home country, we synchronization in the responses of GDP, investment, consumption, entrepreneurial and banking net worth.

3.2.2 Home and foreign monetary policy shock

Monetary policy in our model affects real activity and relative prices because prices and wages are not fully flexible in the short run. Monetary policy impacts works by affecting directly the nominal interest rate, in turn real variables are affected by changes to the real interest rate. For illustration, Figures 7 and 8 report the responses to a tightening of monetary policy in the home country. The rise in the nominal interest rate increases the real interest rate, which leads to a rise in the cost of funding for home entrepreneurs. As a consequence, net worth decreases and home output and investment fall. This generates a negative effect on domestic demand, following the uncovered interest rate parity (UIP) mechanism, the exchange rate appreciates in the case of no cross-border banking. The exchange rate reaction implies an additional transmission mechanism to the monetary policy shock and generates a further negative contribution to the home GDP via net exports. As our monetary policy shocks are uncorrelated, a rise in the home nominal interest rate leads to an increase of home prices relative to foreign prices and this will create a higher demand of foreign goods. Higher demand of foreign-produced goods leads to a positive terms-of-trade and a wealth effect that boost foreign consumption and investment in the long run. As consequence, a tightening of home monetary policy leads to a fall in the foreign output in the short run because of the expenditure switching effect. An increase in foreign output provides a positive wealth effect to households so they consume more in the absence of cross-border banking activities. With cross-border banking activities, foreign households will increase their deposits and foreign consumption go back to the steady states a little faster than those in the model without cross-border banking activities. Compared to the standard BGG model with an international goods market, cross-border banking activities tend to amplify the transmission channel of the monetary policy shock on output and investment in both the domestic and the foreign country. However, cross-border banking activities tend to weaken the impact of shocks on foreign and home consumption because of the cross-border saving possibility between the two countries. Following a rise in the opportunity cost of funding in home country, home entrepreneurs and bankers' net worth decrease. However, without cross-border banking activities, foreign entrepreneurs' net worth decreases less because of the relatively weak opportunity cost of funding in the foreign country.

Foreign monetary policy shock affects the foreign and the home economy by generating opposite reactions comparing to the home monetary shock, nearly of a same magnitude. This result is consistent to our prediction since financial frictions are fully represented in the two countries and because of the symmetry effect in this two-country DSGE model. Thus following a monetary policy shock in the foreign country, foreign output and investment decrease, whereas home investment and output increase.

3.2.3 International business cycle synchronization

Finally, Tables 4 and 5 report bilateral correlations for key macro-financial variables (GDP, consumption, investment, bankers and entrepreneurs net worth). For this purpose, we consider two types of shocks: *(i)* productivity shocks and *(ii)* monetary policy shocks. For the monetary policy shock, we set the bilateral correlations of shocks to zero. However, we consider correlated productivity shocks with a correlation parameter set to (0.3).[16] For comparison purposes, the tables also report bilateral correlations predicted by the so called *standard two-country BGG model* and our two-country baseline model. In response to the productivity shock, a bilateral correlation between home and foreign GDP is 0.28 and the bilateral correlation for investment is 12% without cross-border baking and 42% in the baseline model. The bilateral correlation between home and foreign investment is -24% in the standard two-country model and 32% in the baseline model. Entrepreneurs and Bankers net worth are less sensitive to the international banking activities. However, the bilateral correlation between entrepreneurs and bankers net worth is higher under cross-border banking. In response to the monetary policy shock, a bilateral correlation for GDP is -5% and the bilateral correlation for investment is -91%. Under cross-border banking, a bilateral correlation for GDP is 8% and the bilateral correlation for investment is -77%. The cross-correlation between consumption is 10% under cross-border banking, whereas is negative in the standard two-country BGG model. In the financial market, the cross-correlation of entrepreneur and bankers net worth moves from -19% without cross-banking activities to 93% under cross-border banking. Standard open economy macro models without banks (Backus et al. (1995), Baxter (1995), Kehoe and Perri (2002)) too generate cross-country correlations of output and investment that are lower than the empirical cross-country correlations. Overall, under cross-border banking, predicted bilateral correlations become greater than those without international banking activities in many cases.

[16]This parameter is consistent with the empirical correlations found in Ueda (2012).

4 Concluding Remarks

This paper has developed a two-country DSGE model with real-financial linkages to address the role played by cross-border banking activities on the dynamics of shock propagation and in the synchronization of international business cycles. The model includes an international credit contract between bankers and investors, on the one hand, and between bankers and entrepreneurs, on the other. This introduces cross-border lending and borrowing and enhances integration of financial markets between the two countries. The model economy is calibrated to U.S. and Canadian data.

Our simulations suggest that following a positive technology shock and a tightening of home monetary policy, cross-border banking activities tend to amplify the transmission channel on output and investment in both the domestic and the foreign country. However, cross-border banking activities tend to weaken the impact of shocks on foreign and home consumption, because of the cross-border saving possibility between the countries.

Furthermore, our simulations suggest that in the presence of cross-border banking, bilateral correlations between macroeconomic aggregates become greater. Overall, our results show sizable spillover effects of cross-border banking in the propagation of shocks and suggest cross-border banking is an important source of the synchronization of business cycles between the U.S. and Canada.

Going forward, more research is needed to provide better macro and microfoundations of Canada-U.S. interdependence coming from international banking activities. In addition, estimating the two-country DSGE would allow the introduction of more shocks than observed variables and therefore properly test and identify common factors in the structural shocks.

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5 Appendix

5.1 Proof of the proposition 3

With our specification, the expected return received by the bank at the end of the financial contract in bot countries are defined as:

$$Z^b_{h,t+1} = \frac{V^b_h(\theta^e_h, \theta^e_f)}{\left(\theta^e_h\left(Q_{h,t}K_{h,t+1} - N^e_{h,t+1}\right) + (1-\theta^e_f)\Upsilon_t\left(Q_{f,t}K_{f,t+1} - N^e_{f,t+1}\right)\right)},$$

and

$$Z^b_{f,t+1} = \frac{V^b_f(\theta^e_h, \theta^e_f)}{\left((1-\theta^e_h)\left(Q_{h,t}K_{h,t+1} - N^e_{h,t+1}\right) + \theta^e_f\Upsilon_t\left(Q_{f,t}K_{f,t+1} - N^e_{f,t+1}\right)\right)},$$

Using equations (2.3) and (2.4), the expected return may be rewritten as:

$$Z^b_{h,t+1} = \frac{\theta^e_h\left(Q_{h,t}K_{h,t+1} - N^e_{h,t+1}\right)R_{h,t+1} + (1-\theta^e_f)\Upsilon_t\left(Q_{f,t}K_{f,t+1} - N^e_{f,t+1}\right)R_{f,t+1}}{\theta^e_h\left(Q_{h,t}K_{h,t+1} - N^e_{h,t+1}\right) + (1-\theta^e_f)\Upsilon_t\left(Q_{f,t}K_{f,t+1} - N^e_{f,t+1}\right)},$$

$$Z^b_{f,t+1} = \frac{(1-\theta^e_h)\left(Q_{h,t}K_{h,t+1} - N^e_{h,t+1}\right)R_{h,t+1} + \theta^e_f\Upsilon_t\left(Q_{f,t}K_{f,t+1} - N^e_{f,t+1}\right)R_{f,t+1}}{(1-\theta^e_h)\left(Q_{h,t}K_{h,t+1} - N^e_{h,t+1}\right) + \theta^e_f\Upsilon_t\left(Q_{f,t}K_{f,t+1} - N^e_{f,t+1}\right)},$$

Denote by $\nabla^e_{h,f/h} = \left((1-\theta^e_f)\Upsilon_t\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}\right)/\theta^e_h\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}$, and $\nabla^e_{h,h/f} = \left(\nabla^e_{h,f/h}\right)^{-1}$. $\nabla^e_{f,f/h} = \theta^e_f\Upsilon_t\mathcal{Q}_{f,t}\mathcal{K}_{f,t+1}/\left((1-\theta^e_h)\mathcal{Q}_{h,t}\mathcal{K}_{h,t+1}\right)$, and $\nabla^e_{f,h/f} = \left(\nabla^e_{f,f/h}\right)^{-1}$:

Hence:

$$Z^b_{h,t+1} = \frac{1-\nabla^e_h}{\left(1-\nabla^e_h\right) + \nabla^e_{h,f/h}\left(1-\nabla^e_f\right)}R_{h,t+1} + \frac{1-\nabla^e_f}{\left(1-\nabla^e_f\right) + \nabla^e_{h,h/f}\left(1-\nabla^e_h\right)}R_{f,t+1},$$

$$Z^b_{f,t+1} = \frac{1-\nabla^e_h}{\left(1-\nabla^e_h\right) + \nabla^e_{f,f/h}\left(1-\nabla^e_f\right)}R_{h,t+1} + \frac{1-\nabla^e_f}{\left(1-\nabla^e_f\right) + \nabla^e_{f,h/f}\left(1-\nabla^e_h\right)}R_{f,t+1}.$$

5.2 Analytical Expressions for the Variables Appearing in the Credit Contracts

In this section we lay out the analytical expression to compute $G_t(\overline{\omega}_h)$, $G_t(\overline{\omega}_f)$, $\Gamma_t(\overline{\omega}_h)$ and $\Gamma_t(\overline{\omega}_f)$. Following Bernanke et al. (1999) and Christiano et al. (2010), we assume that ω_h and ω_f follow log-normal distributions, with $E_t[\omega_h] = E_t[\omega_f] = 1$. We denote the cumulative distribution function of the two random variables by $F_t(\omega_h)$ and $F_t(\omega_f)$, and we assume that the variance of $\log(\omega_h)$ and $\log(\omega_f)$ are $\sigma^2_{\omega_h}$ and $\sigma^2_{\omega_f}$, respectively.

$G_t(\overline{\omega}_h)$, $G_t(\overline{\omega}_f)$ are the expected return from the default entrepreneurs in the home and foreign country, respectively. Using the log-normal assumption, they may be expressed as:

$$G_t(\overline{\omega}_h) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\omega_h)-\frac{1}{2}\sigma^2_{\omega_h}}{\sigma_{\omega_h}}} . \exp\left(-\frac{\vartheta_h^2}{2}\right) d\vartheta_h, \tag{5.1}$$

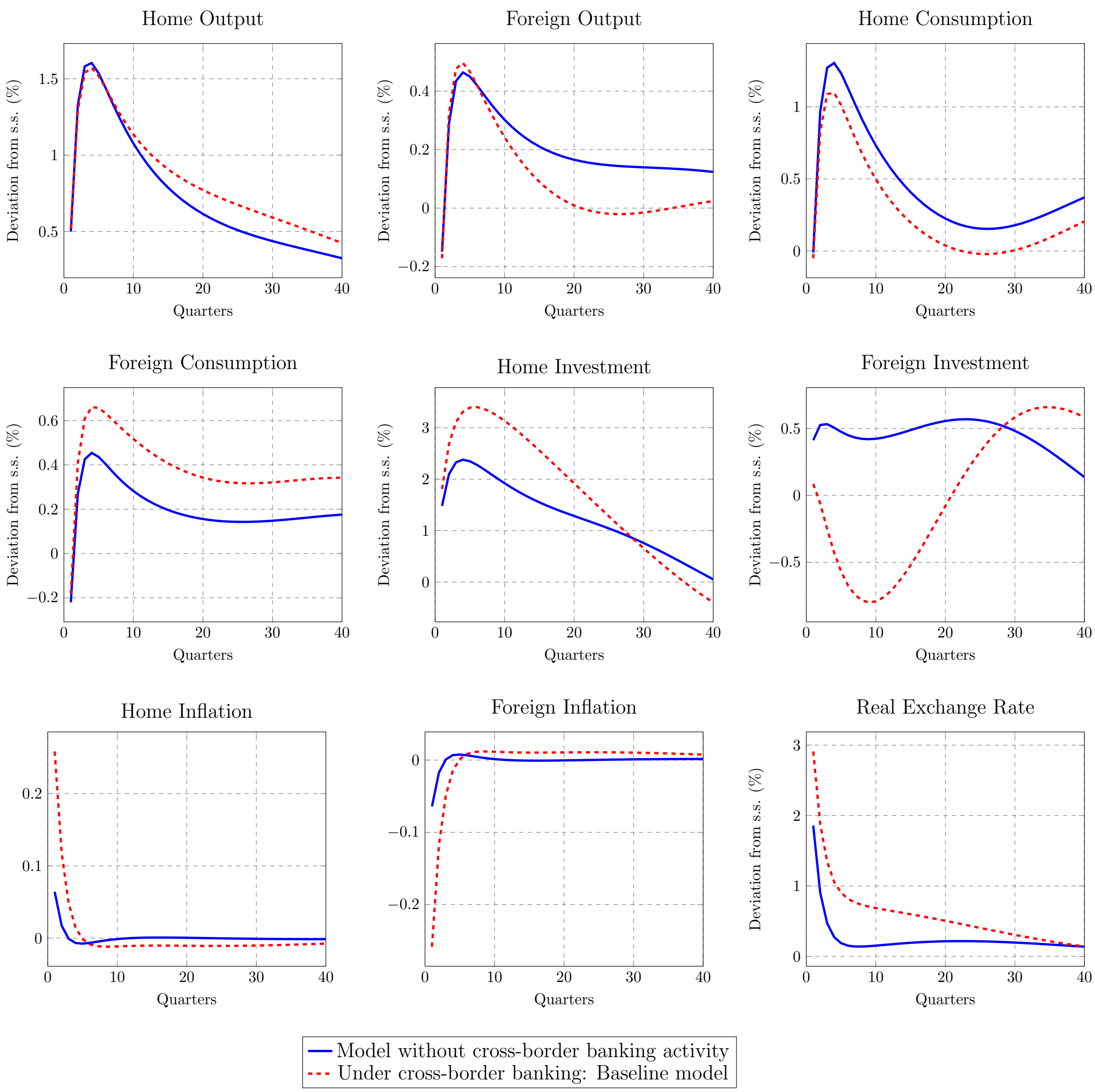
$$G_t(\overline{\omega}_f) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log\left(\omega_f\right)-\frac{1}{2}\sigma^2_{\omega_f}}{\sigma_{\omega_f}}} . \exp\left(-\frac{\vartheta_f^2}{2}\right) d\vartheta_f. \tag{5.2}$$

$\Gamma_t(\overline{\omega}_h)$ and $\Gamma_t(\overline{\omega}_f)$ are the net share profit going to bankers in the setting of the financial contract. Again, using the log-normal assumption, they may expressed as:

$$\Gamma_t(\overline{\omega}_h) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\omega_h)-\frac{1}{2}\sigma^2_{\omega_h}}{\sigma_{\omega_h}}} . \exp\left(-\frac{\vartheta_h^2}{2}\right) d\vartheta_h + \frac{\overline{\omega}_h}{\sqrt{2\pi}} \int_{\frac{\log(\omega_h)+\frac{1}{2}\sigma^2_{\omega_h}}{\sigma_{\omega_h}}}^{+\infty} . \exp\left(-\frac{\vartheta_h^2}{2}\right) d\vartheta_h, \tag{5.3}$$

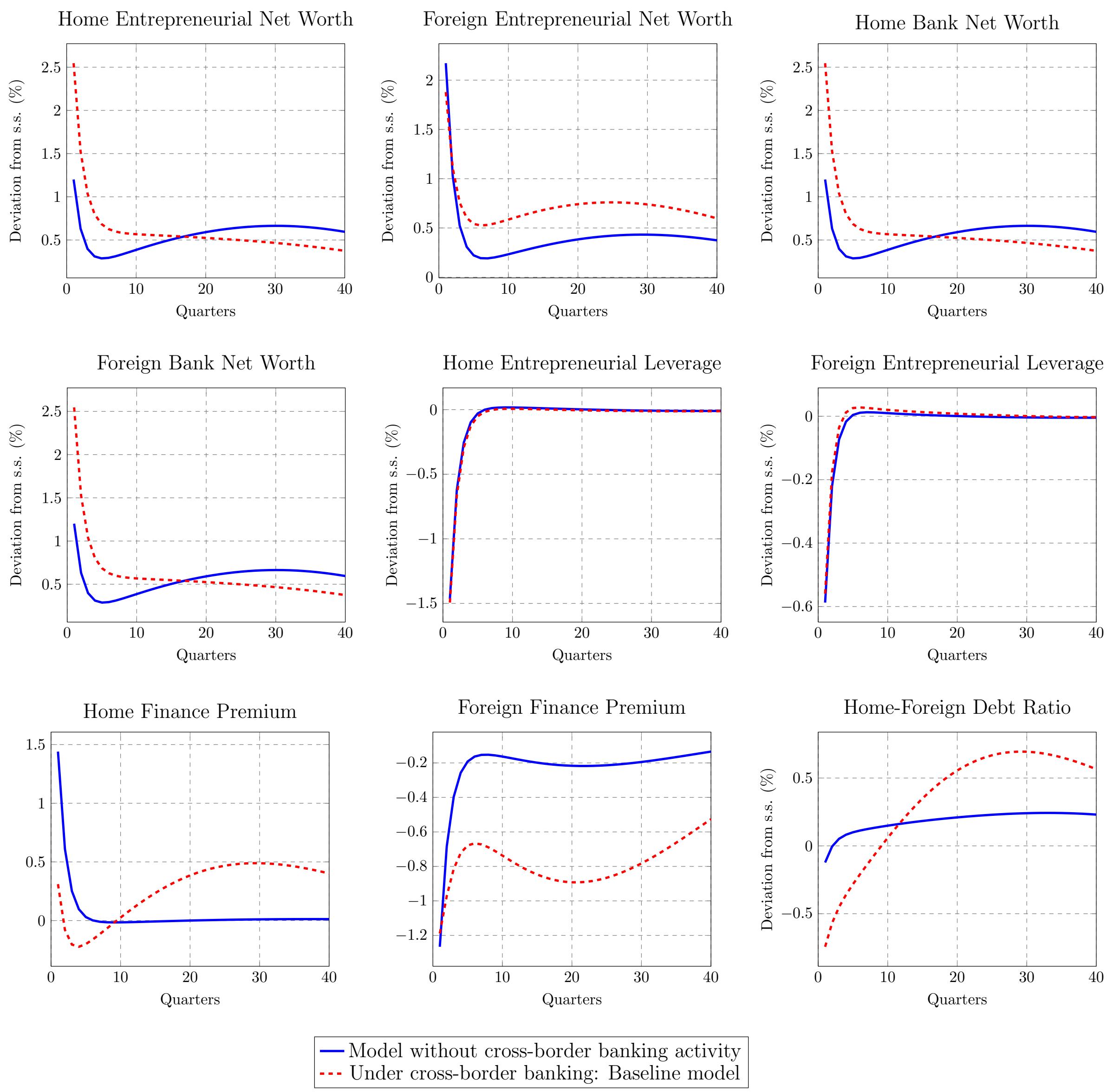
$$\Gamma_t(\overline{\omega}_f) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log\left(\omega_f\right)-\frac{1}{2}\sigma^2_{\omega_f}}{\sigma_{\omega_f}}} . \exp\left(-\frac{\vartheta_f^2}{2}\right) d\vartheta_f + \frac{\overline{\omega}_f}{\sqrt{2\pi}} \int_{\frac{\log\left(\omega_f\right)+\frac{1}{2}\sigma^2_{\omega_f}}{\sigma_{\omega_f}}}^{+\infty} . \exp\left(-\frac{\vartheta_f^2}{2}\right) d\vartheta_f. \tag{5.4}$$

Figure 5: IRF of real variables from a home productivity shock



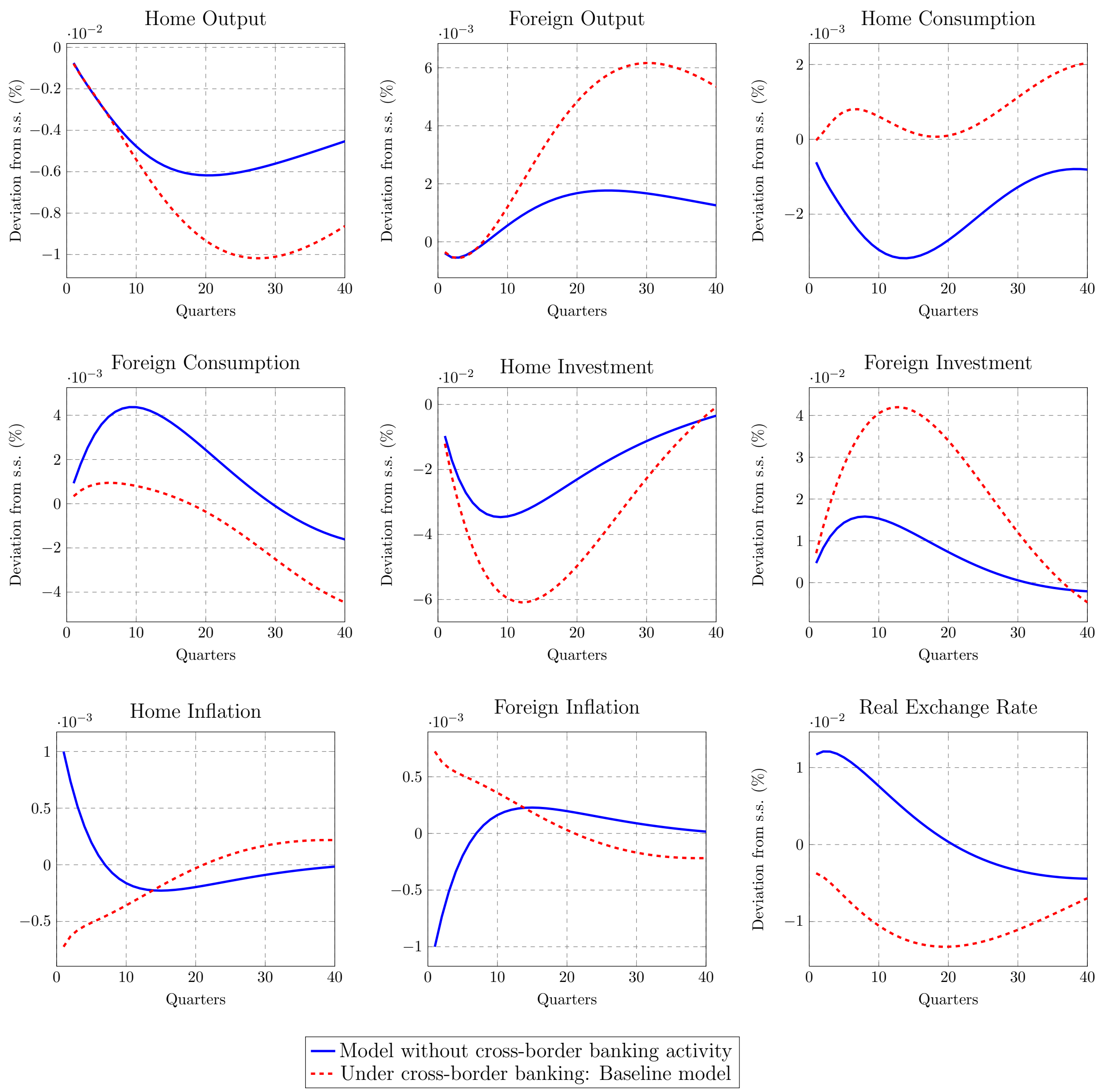
Notes: This figure presents impulse response functions from the simulated DSGE model to illustrate the effect of cross-border banking activities on the transmission of a shock to home productivity. Responses are expressed in percentage deviation except inflation and interest rate variables, which are expressed in basis points.

Figure 6: IRF of financial variables from a home productivity shock



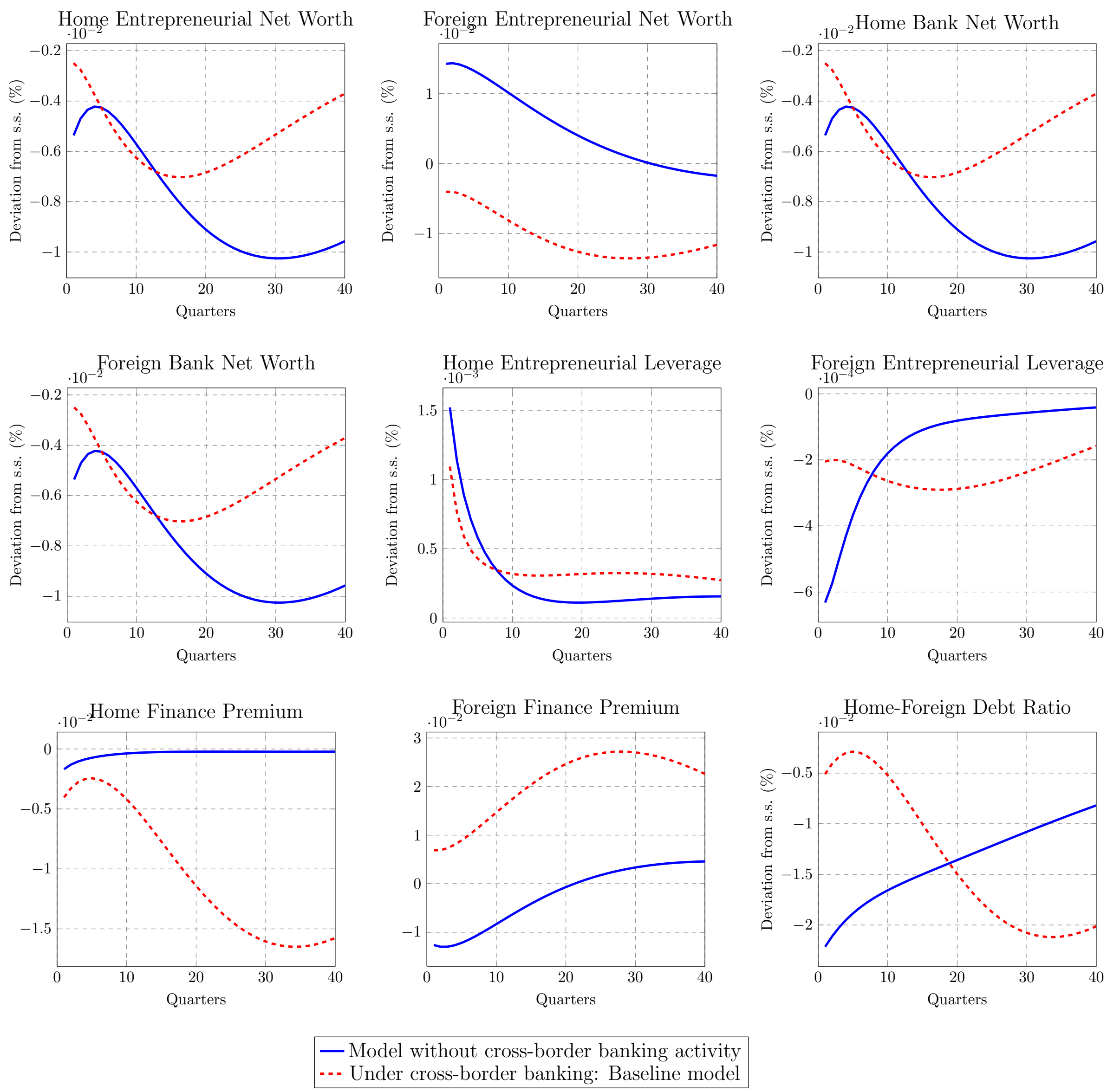
Notes: This figure presents impulse response functions from the simulated DSGE model to illustrate the effect of cross-border banking activities on the transmission of a shock to home productivity. Responses are expressed in percentage deviation except inflation and interest rate variables, which are expressed in basis points.

Figure 7: IRF of real variables from a home monetary policy shock



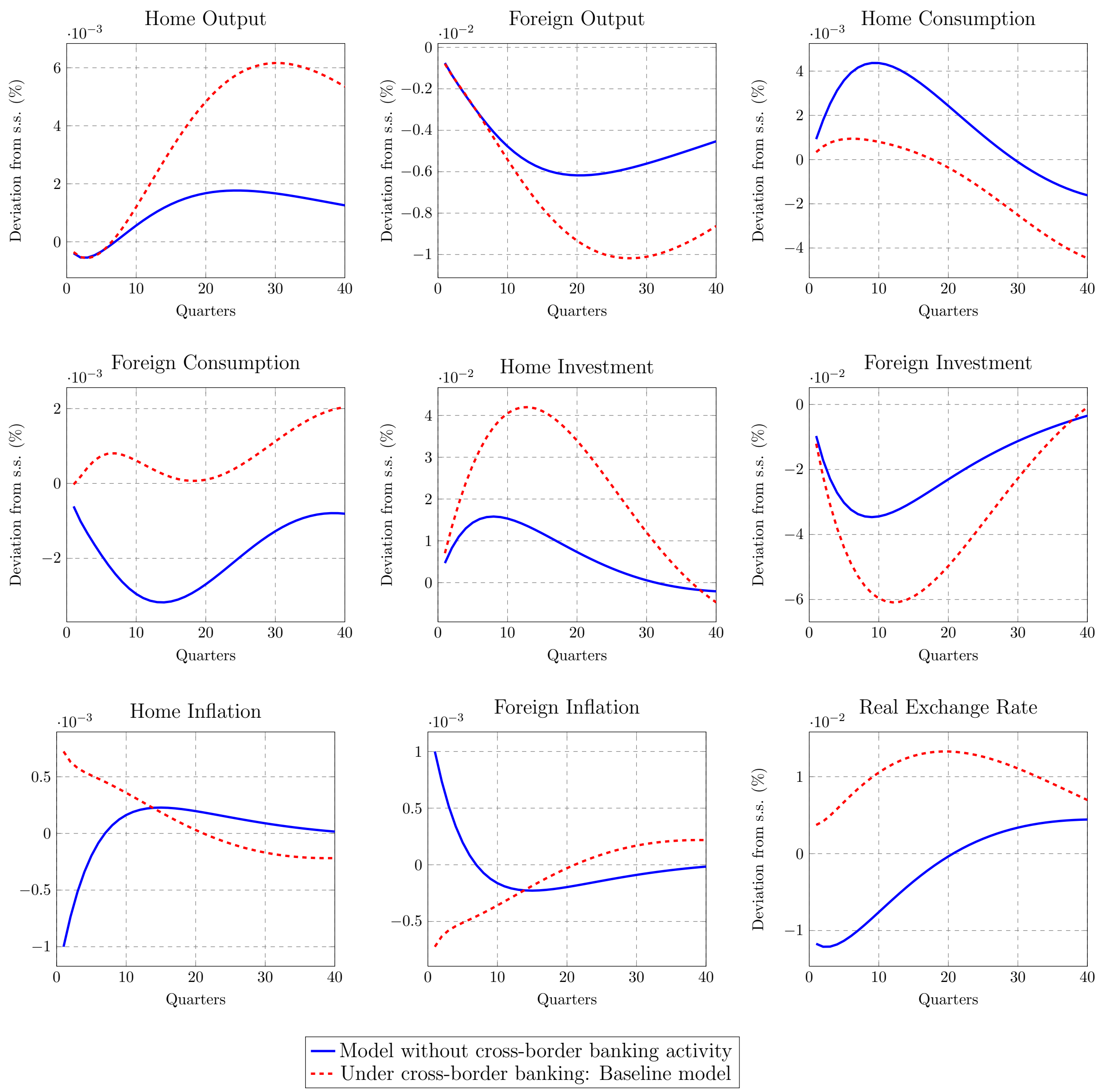
Notes: This figure presents impulse response functions from the simulated DSGE model to illustrate the effect of cross-border banking activities on the transmission of a home monetary policy shock. Responses are expressed in percentage deviation except inflation and interest rate variables, which are expressed in basis points.

Figure 8: IRF of financial variables from a home monetary policy shock



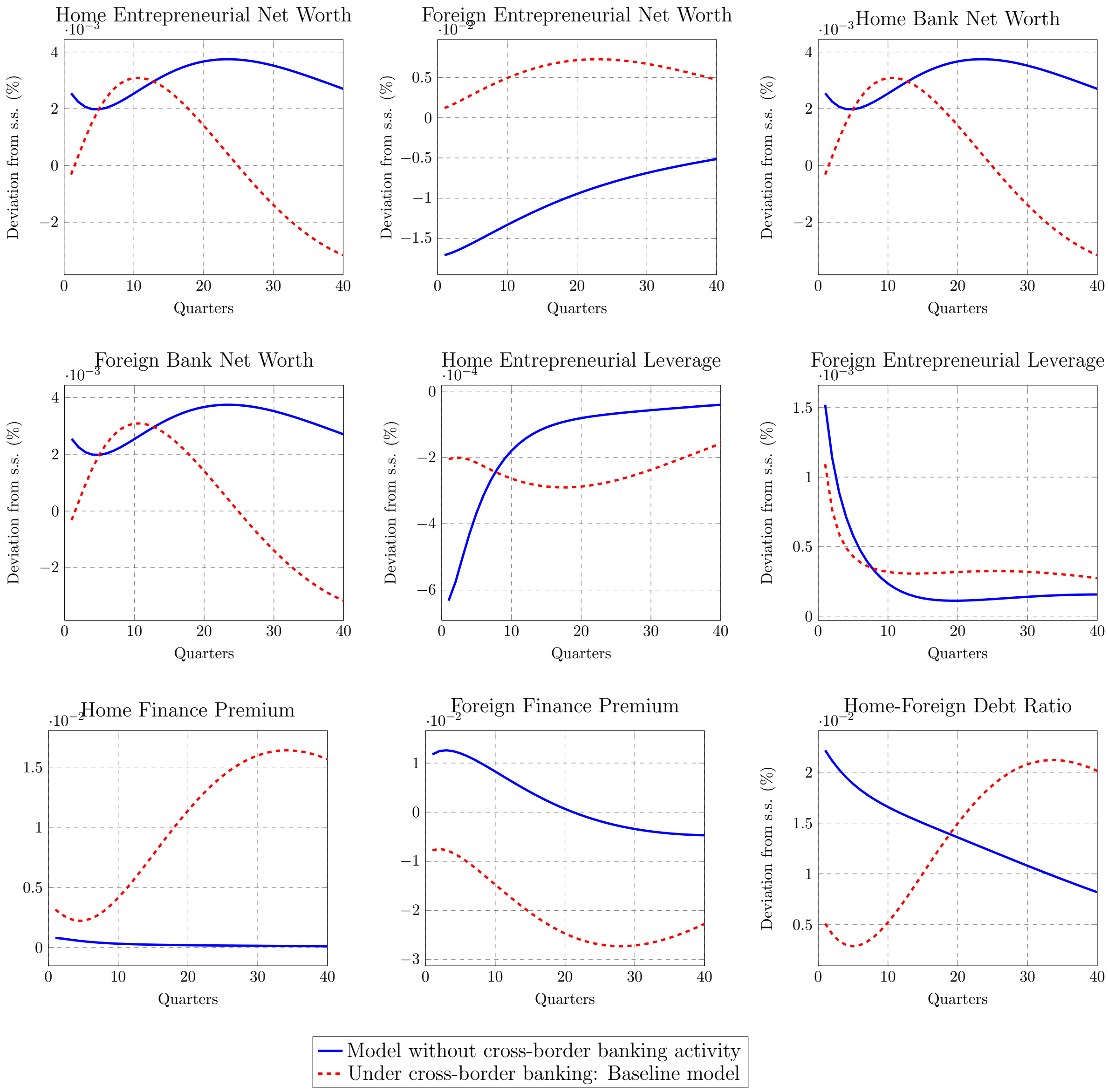
Notes: This figure presents impulse response functions from the simulated DSGE model to illustrate the effect of cross-border banking activities on the transmission of a home monetary policy shock. Responses are expressed in percentage deviation except inflation and interest rate variables, which are expressed in basis points.

Figure 9: IRF of real variables from a foreign monetary shock



Notes: This figure presents impulse response functions from the simulated DSGE model to illustrate the effect of cross-border banking activities on the transmission of a foreign monetary policy shock. Responses are expressed in percentage deviation except inflation and interest rate variables, which are expressed in basis points.

Figure 10: IRF of financial variables from a foreign monetary shock



Notes: This figure presents impulse response functions from the simulated DSGE model to illustrate the effect of cross-border banking activities on the transmission of home monetary policy shock. Responses are expressed in percentage deviation except inflation and interest rate variables, which are expressed in basis points.

Table 4: Predicted Bilateral Correlations (home productivity shock)

Variables	Model	$\mathcal{Y}_h$	$\mathcal{Y}_f$	$\mathcal{C}_h$	$\mathcal{C}_f$	$\mathcal{I}_h$	$\mathcal{I}_f$	$\mathcal{N}_h^e$	$\mathcal{N}_f^e$	$\mathcal{A}_h$	$\mathcal{A}_f$
$\mathcal{Y}_h$	BGG	1.00	0.12	0.78	0.53	0.79	0.03	0.76	0.79	0.76	0.79
	Baseline	1.00	0.42	0.80	0.52	0.85	0.30	0.75	0.68	0.75	0.68
$\mathcal{Y}_f$	BGG	0.12	1.00	0.53	0.81	0.07	0.79	0.24	0.19	0.24	0.19
	Baseline	0.42	1.00	0.52	0.84	0.27	0.87	0.35	0.44	0.35	0.44
$\mathcal{C}_h$	BGG	0.78	0.53	1.00	0.58	0.40	0.17	0.54	0.55	0.54	0.55
	Baseline	0.80	0.52	1.00	0.51	0.47	0.27	0.57	0.50	0.57	0.50
$\mathcal{C}_f$	BGG	0.53	0.81	0.58	1.00	0.17	0.47	0.48	0.48	0.48	0.48
	Baseline	0.52	0.84	0.51	1.00	0.24	0.57	0.53	0.60	0.53	0.60
$\mathcal{I}_h$	BGG	0.79	0.07	0.40	0.17	1.00	-0.24	0.63	0.61	0.63	0.61
	Baseline	0.85	0.27	0.47	0.24	1.00	0.32	0.55	0.54	0.55	0.54
$\mathcal{I}_f$	BGG	0.03	0.79	0.17	0.47	-0.24	1.00	-0.06	-0.06	-0.06	-0.06
	Baseline	0.30	0.87	0.27	0.57	0.32	1.00	0.25	0.30	0.25	0.30
$\mathcal{N}_h^e$	BGG	0.75	0.35	0.57	0.53	0.55	0.25	1.00	0.87	1.00	0.87
	Baseline	0.76	0.24	0.54	0.48	0.63	-0.06	1.00	0.98	1.00	0.98
$\mathcal{N}_f^e$	BGG	0.68	0.44	0.50	0.60	0.54	0.30	0.87	1.00	0.87	1.00
	Baseline	0.79	0.19	0.55	0.48	0.61	-0.06	0.98	1.00	0.98	1.00
$\mathcal{A}_h$	BGG	0.75	0.35	0.57	0.53	0.55	0.25	1.00	0.87	1.00	0.87
	Baseline	0.76	0.24	0.54	0.48	0.63	-0.06	1.00	0.98	1.00	0.98
$\mathcal{A}_f$	BGG	0.68	0.44	0.50	0.60	0.54	0.30	0.87	1.00	0.87	1.00
	Baseline	0.79	0.19	0.55	0.48	0.61	-0.06	0.98	1.00	0.98	1.00

Notes: This table shows bilateral correlations between the home and the foreign country for a positive home productivity shock. For illustration, we consider two kinds of models: *(i)* A standard BGG model with an international goods market without cross-border banking activities; and *(ii)* our baseline model with cross-border banking parameters calibrated from the U.S. and Canadian Economy.

Table 5: Predicted Bilateral Correlations (home monetary shock)

Variables	Model	$\mathcal{Y}_h$	$\mathcal{Y}_f$	$\mathcal{C}_h$	$\mathcal{C}_f$	$\mathcal{I}_h$	$\mathcal{I}_f$	$\mathcal{N}_h^e$	$\mathcal{N}_f^e$	$\mathcal{A}_h$	$\mathcal{A}_f$
$\mathcal{Y}_h$	BGG	1.00	-0.05	0.52	0.55	0.57	-0.34	0.72	0.85	0.72	0.85
	Baseline	1.00	0.08	0.86	0.13	0.64	-0.37	0.76	-0.14	0.76	-0.14
$\mathcal{Y}_f$	BGG	-0.05	1.00	0.31	0.31	-0.19	0.35	-0.46	-0.43	-0.46	-0.43
	Baseline	0.08	1.00	0.11	0.76	-0.19	0.47	-0.15	-0.21	-0.15	-0.21
$\mathcal{C}_h$	BGG	0.52	0.31	1.00	-0.06	0.46	-0.30	0.25	0.21	0.25	0.21
	Baseline	0.86	0.11	1.00	0.10	0.68	-0.58	0.52	-0.30	0.52	-0.30
$\mathcal{C}_f$	BGG	0.55	0.31	-0.06	1.00	0.07	-0.03	0.11	0.36	0.11	0.36
	Baseline	0.13	0.76	0.10	1.00	-0.37	0.30	0.04	-0.27	0.04	-0.27
$\mathcal{I}_h$	BGG	0.57	-0.19	0.46	0.07	1.00	-0.91	0.46	0.54	0.46	0.54
	Baseline	0.64	-0.19	0.68	-0.37	1.00	-0.77	0.51	-0.46	0.51	-0.46
$\mathcal{I}_f$	BGG	-0.34	0.35	-0.30	-0.03	-0.91	1.00	-0.30	-0.38	-0.30	-0.38
	Baseline	-0.37	0.47	-0.58	0.30	-0.77	1.00	-0.35	0.56	-0.35	0.56
$\mathcal{N}_h^e$	BGG	0.76	-0.15	0.52	0.04	0.51	-0.35	1.00	-0.19	1.00	-0.19
	Baseline	0.72	-0.46	0.25	0.11	0.46	-0.30	1.00	0.93	1.00	0.93
$\mathcal{N}_f^e$	BGG	-0.14	-0.21	-0.30	-0.27	-0.46	0.56	-0.19	1.00	-0.19	1.00
	Baseline	0.85	-0.43	0.21	0.36	0.54	-0.38	0.93	1.00	0.93	1.00
$\mathcal{A}_h$	BGG	0.76	-0.15	0.52	0.04	0.51	-0.35	1.00	-0.19	1.00	-0.19
	Baseline	0.72	-0.46	0.25	0.11	0.46	-0.30	1.00	0.93	1.00	0.93
$\mathcal{A}_f$	BGG	-0.14	-0.21	-0.30	-0.27	-0.46	0.56	-0.19	1.00	-0.19	1.00
	Baseline	0.85	-0.43	0.21	0.36	0.54	-0.38	0.93	1.00	0.93	1.00

Notes: This table shows bilateral correlations between the home and the foreign country for a tightening home monetary policy. For illustration, we consider two kinds of models: *(i)* A standard BGG model with an international goods market without cross-border banking activities; and *(ii)* our baseline model with cross-border banking parameters calibrated from the U.S. and Canadian Economy.