



Munich Personal RePEc Archive

**Potential dividends and actual cash flows. Theoretical and empirical reasons for using ‘actual’ and dismissing ‘potential’, Or: How not to pull potential rabbits out of actual hats**

Vélez-Pareja, Ignacio and Magni, Carlo Alberto

Universidad Tecnológica de Bolívar, University of Modena and Reggio Emilia

19 February 2008

Online at <https://mpra.ub.uni-muenchen.de/7266/>  
MPRA Paper No. 7266, posted 19 Feb 2008 17:03 UTC

**Potential dividends and actual cash flows. Theoretical and empirical reasons for using ‘actual’ and dismissing ‘potential’,  
Or:  
How not to pull potential rabbits out of actual hats**

Ignacio Vélez–Pareja  
Universidad Tecnológica de Bolívar  
Cartagena, Colombia  
[ivelez@unitecnologica.edu.co](mailto:ivelez@unitecnologica.edu.co)  
[nachovelez@gmail.com](mailto:nachovelez@gmail.com)

Carlo Alberto Magni  
Department of Economics  
University of Modena and Reggio Emilia  
Modena, Italy  
[magni@unimo.it](mailto:magni@unimo.it)

First version: November 1, 2007

This version: February 19, 2008

---

**Acknowledgments.** We would like to thank Professor Edinson Caicedo from Universidad del Valle, Colombia and Professor David Andrés Londoño from Universidad Tecnológica de Bolívar, Colombia, for their comments. Any opinion and error are our sole responsibility.

**ABSTRACT**

Practitioners and academics in valuation include changes in liquid assets (potential dividends) in the cash flows. This widespread and wrong practice is inconsistent with basic finance theory. We present economic, theoretical, and empirical arguments to support the thesis. Economic arguments underline that only flows of cash should be considered for valuation; theoretical arguments show how potential dividends lead to contradiction and to arbitrage losses. Empirical arguments, from recent studies, suggest that investors discount potential dividends with high discount rates, which means that changes in liquid assets are not value drivers. Hence, when valuing cash flows, we should consider only actual payments.

**KEY WORDS.**

Cash flows, cash flow to equity, free cash flow, liquid assets, potential dividends, firm value, equity value, Modigliani and Miller, levered value, error in valuation.

**JEL CLASSIFICATION**

M21, M40, M41, G12, G31

## Introduction

In this paper, we give support to the idea that potential dividends that are not distributed (and are invested in liquid assets) should be neglected in firm valuation, because only distributed cash flows add value to shareholders. Hence, the definition of Cash Flow to Equity should include only the cash flow that is actually paid to shareholders (dividends paid plus share repurchases minus new equity investment). Although some authors warn against the use of potential dividends for valuing firms (Vélez-Pareja, 1999a, 1999b, 2004, 2005a, 2005b; Fernández, 2002, 2007; Tham and Vélez-Pareja, 2004; DeAngelo and DeAngelo, 2006, 2007), some respected authors (e.g. Copeland<sup>1</sup>, Koller and Murrin, 1994, 2000; Benninga and Sarig, 1997; Brealey and Myers, 2003; Damodaran, 1999, 2006, 2007) and many practitioners seem to support the idea that the Cash Flow to Equity has to include *undistributed* potential dividends.

To include undistributed potential dividends in valuation is admissible only if they are expected to be invested at the cost of equity capital,  $k_e$ , i.e. the net present value (NPV) of those investments is zero from the point of view of current shareholders. If the latter assumption held, then changes in liquid assets could be indeed included in the Cash Flow to Equity, because they would be value-neutral (see DeAngelo and DeAngelo, 2006; Magni, 2007). It should be noted that a *definition* of cash flow to equity is meant to be valid for all possible cases, and thus should not depend on a particular assumption about investment in liquid assets, otherwise the consequent definition of firm value would depend on a particular assumption about investment in liquid assets. Furthermore, this particular assumption violates Jensen's (1986) agency theory. Magni (2007) writes that "If NPV is zero, dividend irrelevance applies [...] if agency problems are present, managers tend to retain funds and invest them in *negative-NPV projects*, and therefore the zero-NPV assumption must be removed, so that dividend irrelevance does not apply any more" (p. 1). DeAngelo and DeAngelo (2006) claim that "When MM's assumptions are relaxed to allow retention, payout policy matters in exactly the same sense that investment policy does" (p. 293) and "irrelevance fails because some feasible payout policies do not distribute the full present value of FCF to currently outstanding shares" (p. 294). The zero-NPV assumption is not supported by the empirical data. Literature reports that holding liquid assets destroys value or at most does not create a significant amount of value. Schwetzler and Carsten (2003) report that in Germany "persistent excessive cash holdings lead to a *significant operating underperformance* [...] in line with expectations of the agency theory" (p. 25).<sup>2</sup> Harford (1997) finds that "cash-rich bidder *destroys seven cents of firm value* for every dollar of excess cash held" (p. 1983) and says that "the stock market appears to partially anticipate this behavior, as evidenced by the *negative stock market reaction to cash stockpiling*". (p. 1972). Finally, he says that "one might expect that stockpiling cash would be *greeted negatively by the market*" (p. 1992). Opler et al. (1999) say that "*holdings of liquid assets can make shareholders worse off* in some circumstances" (p. 2) and "*In a world of perfect capital markets*, holdings of liquid assets are *irrelevant*. If cash flow turns out to be unexpectedly low so that a firm has to raise funds to keep operating and invest, it can do so at zero cost. [...] Hence, if a firm borrows money and invests it in liquid assets, shareholders wealth is unchanged" (pp. 4-5). Finally, they write that "investing in cash can therefore have an adverse effect on firm value. To put it another way, *increasing firm's holdings of liquid assets by one dollar may increase firm value by less than one dollar*" (p. 11). Faulkender and Wang (2004) find "that the *marginal value of cash declines with larger cash holdings*, higher leverage, better access to capital markets, and as firms choose to

---

<sup>1</sup> Professor Tom Copeland in a private correspondence says: "If funds are kept within the firm you still own them -- hence 'potential dividends' are cash flow available to shareholders, whether or not they are paid out now or in the future."

<sup>2</sup> Italics ours from now on, unless otherwise specified.

distribute cash via dividends rather than repurchases” (p. 2) and they “estimate that for the mean firm-year in the sample, *the marginal value of cash is \$0.96*” (p. 24). On the other hand, Mikkelson and Partch (2003) conclude that “persistent large holdings of cash and equivalents have not hindered corporate performance”, (p. 2) and that “there is no evidence that large firms with lower insider stock ownership, higher inside board composition, or a controlling founder perform differently than other large cash firms” (p. 20). Pinkowitz, Stulz and Williamson (2003) write: “Strikingly, while *a dollar of cash is worth roughly a dollar of firm value* in the U.S., in countries with poor institutions, *a dollar of cash is consistently estimated to be worth less than 65 cents*” (p. 6). Pinkowitz, Williamson and Stulz (2007)’s result point out that one dollar of increase in dividends creates about ten dollars in value while one dollar increase in cash creates about 29 cents of value for countries with poor shareholders rights countries. In high corruption countries one dollar increase in dividends creates more than 6.5 dollars of value and one dollar of cash and cash equivalent create 33 cents of value. In the non corrupt countries and with good protection to the shareholders one dollar of extra dividends creates near four dollars in value and one dollar in liquid assets creates about one dollar in value. On the other hand, Pinkowitz and Williamson (2002) report that, on the average, one dollar in cash is 1.25 dollars worth in value.

This paper aims at reinforcing the arguments on the inconvenience of adding the change in liquid assets (undistributed potential dividends) as part of the cash flows to be used for firm valuation. The arguments used in this paper to support our thesis are of three types: economic, logical, and empirical.

As for the economic reasons, they highlight that to include the change in liquid assets in the definition of Cash Flow to Equity means (i) to confound stock with flows (ii) to adopt the very particular assumption that the full present value of liquid assets (computed at the cost of equity) will be distributed to shareholders, (iii) to break consistency between cash flow and financial statements, (iv) to distort taxes, (v) to refuse Jensen’s agency theory which implies that firms in real life tend to retain funds and invest them in low-return investments.

As for the logical arguments, we provide three formal proofs which show that a rational evaluator does not use undistributed potential dividends. The first proof relies on incompatibility of CAPM and the use of potential dividends for valuation. The second one shows that use of potential dividends is incompatible with the classical valuation theory (and therefore, with Modigliani and Miller’s approach). A third proof shows that investors including changes in liquid assets as value-creation factors fall prey to arbitrage losses.

As for the empirical argument, we analyze and work out some contributions in the recent literature, from which it can be inferred that investors value liquid assets much less than dividends. Also, the implicit discount rates for liquid assets are very high, which suggests that investors do not consider liquid assets as affecting a firm’s value.

The paper is organized as follows: section 1 shows that our definition of Cash Flow to Equity as dividends minus net capital contributions is consistent with basic finance, and, in particular, with Modigliani and Miller’s approach to valuation, whereas the definition widely used in many applied corporate finance textbooks and in real-life applications is not; this section also summarizes the different views advocated by several authors. Section 2 furnishes several economic reasons for supporting our thesis. Section 3 illustrates three logical arguments and section 4 makes use of findings and information collected and analyzed by Pinkowitz, Williamson and Stulz (2007) and other scholars to find indications of the irrelevance of liquid assets in value creation. Section 5 proposes a (theoretically correct) model to test the hypothesis that changes in liquid assets do not

add value: the model proposed will be used in future researches. Some remarks conclude the paper. (Main notational conventions are collected at the end of the paper).

## 1. Definition(s) of Cash Flow to Equity

This section proposes a definition of Cash Flow to Equity which is consistent with Modigliani and Miller's (1958, 1963) and Miller and Modigliani's (1961) approach to valuation. Let EBV be the equity book value and  $CS_{t+1}$  be the capital stock contributed by shareholders up to time  $t+1$ . As known, change in equity book value is equal to change in capital stock plus net income (NI) minus dividends paid to shareholders (Div):

$$EBV_{t+1} - EBV_t \equiv CS_{t+1} - CS_t + NI_{t+1} - Div_{t+1}$$

Using the variation symbol "d",<sup>3</sup>  $dEBV_{t+1} = EBV_{t+1} - EBV_t$  is the change in equity book value and  $dCS_{t+1} = CS_{t+1} - CS_t$  represents the net capital contributions made by shareholders in the year (i.e.  $dCS_{t+1} = \text{new equity investment} - \text{shares repurchases}$ ). Therefore, we may rewrite the above equation as

$$dEBV_{t+1} = NI_{t+1} - (Div_{t+1} - dCS_{t+1}). \quad (1)$$

Note that increase in book value is split into two terms:  $NI_t$  is an accounting item, whereas  $(Div_{t+1} - dCS_{t+1})$  is a cash flow; in particular, the latter represents the cash flow that equity holders actually receive (net of capital contributions made during the year). It is just this cash flow that adds value to the firm. We therefore define Cash Flow to Equity as

$$CFE_{t+1} := Div_{t+1} - dCS_{t+1}. \quad (2)$$

It is worth noting that our eq. (1) is equivalent to Miller and Modigliani's (1961) eq. (4) at p. 414, according to which

$$\text{Issuance of New Equity} = \text{Increase in book value} - (\text{Net Income} - \text{Dividends})$$

(we remind that there are no shares repurchases in Miller and Modigliani's approach, so that  $\text{Issuance of New Equity} = dCS_{t+1}$ ). Our notion of CFE in eq. (2) is exactly what Miller and Modigliani (1961) use to compute the firm value: their eq. (17) at p. 419 highlights the difference between dividends paid and net capital contributions: using our symbols, their formula is

$$E_t = \frac{1}{(1 + \rho)} [(Div_{t+1} - dCS_{t+1}) + E_{t+1}]$$

where  $E_t$  is the equity market value at time  $t$  (and  $\rho$  is Miller and Modigliani's symbol for the cost of equity capital). It is worth noting that Miller and Modigliani (henceforth MM) also propose the (equivalent) *stream of earnings approach* to valuation. Their eq. (9) is as follows:

$$E_0 = \sum_{t=0}^{\infty} \frac{NI_{t+1} - dEBV_{t+1}}{(1 + \rho)^{t+1}}$$

---

<sup>3</sup> Henceforth, change in a variable  $y$  is defined as:  $dy_{t+1} := y_{t+1} - y_t$ .

Inspecting the numerator, the reader may note that our eqs. (1) and (2) just imply

$$CFE_{t+1} = NI_{t+1} - dEBV_{t+1}, \text{ so that } E_0 = \sum_{t=0}^{\infty} [CFE_{t+1} / (1 + \rho)^{t+1}].$$

As a result, our approach is consistent with MM's approach. By contrast, a large part of practitioners and some corporate finance scholars seem to be willing to propose a different definition. For example, Damodaran (1998, 2006a, 2006b) proposes the use of cash flow available for distribution for valuing a firm's equity, even if it will be not paid to the equity holder. The cash available for distribution is usually called "potential dividends"; the part of it which is not actually distributed is invested in liquid assets, i.e. cash and short-term investments such as securities, bonds etc. Damodaran's definition of cash flow to equity, which is widely adopted in applied corporate finance, may be formalized as follows:

$$CFE^*_{t+1} = CFE_{t+1} + Div_{pot(t+1)} \quad (3)$$

where  $Div_{pot(t+1)}$  denote *undistributed* potential dividends. The latter are defined as the increase in liquid assets:

$$Div_{pot(t+1)} = dLA_{t+1}$$

with  $LA_{t+1} = C_{t+1} + STI_{t+1}$  ( $C$ =cash,  $STI$ =short-term investments).

To put it in different terms, let us divide the assets of the firm into two categories: Fixed Assets net of cumulated depreciation (NFA), and Working Capital (WC), defined as difference between current assets (cash+short-term investments+accounts receivable+inventories) and current liabilities (accounts payable):

$$WC_{t+1} = C_{t+1} + STI_{t+1} + AR_{t+1} + Inv_{t+1} - AP_{t+1} .$$

Then,

$$EBV_{t+1} = NFA_{t+1} + WC_{t+1} - D_{t+1} \quad (4)$$

where  $D$  is the book value of the debt. From eq. (4),  $dEBV_{t+1} = dNFA_{t+1} + dWC_{t+1} - dD_{t+1}$ . Hence, making use of eqs. (1)-(2), CFE may also be computed with the so-called indirect method:

$$CFE_{t+1} = NI_{t+1} - (dNFA_{t+1} + dWC_{t+1} - dD_{t+1}). \quad (5)$$

where  $dNFA_{t+1} = NFA_{t+1} - NFA_t = Investment\ in\ Fixed\ Assets_{t+1} - Depreciation_{t+1}$  represents the so-called *net capital expenditure*.<sup>4</sup>

By contrast, a frequent definition in textbooks turns out to be

$$CFE^*_{t+1} = NI_{t+1} - (dNFA_{t+1} + dWC^{nc}_{t+1} - dD_{t+1}) \quad (6)$$

with  $WC^{nc}$  being noncash (operating) working capital:

---

<sup>4</sup> Eqs. (5)-(6) may obviously be written as

$$\begin{aligned} CFE_{t+1} &= NI_{t+1} + Depreciation_{t+1} - Investment\ in\ fixed\ assets_{t+1} - dWC_{t+1} + dD_{t+1}. \\ CFE^*_{t+1} &= NI_{t+1} + Depreciation_{t+1} - Investment\ in\ fixed\ assets_{t+1} - dWC^{nc}_{t+1} + dD_{t+1}. \end{aligned}$$

$$WC_{t+1}^{nc} = WC_{t+1} - LA_{t+1}, \quad (7)$$

so that  $dAR_{t+1} + dInv_{t+1} - dAP_{t+1} = dWC_{t+1}^{nc}$  (see, for example, Damodaran, 1999, p. 128; Damodaran, 2006a, p. 79). In eq. (5) working capital is inclusive of undistributed potential dividends  $dLA_{t+1}$ , whereas in eq. (6) working capital excludes  $dLA_{t+1}$ . That is why the term in parenthesis in eq. (5) is greater than the term in parenthesis in eq. (6). As a result, CFE is smaller than CFE\*.

Damodaran (2006b) acknowledges the valuation divergences derived from using potential dividends rather than actual cash flows. After presenting a notion of cash flow to equity equal to dividends paid to shareholders, he extends the definition to include share repurchases: “we extend our definition of cash returned to stockholders to include stock buybacks, thus implicitly assuming that firms that accumulate cash by not paying dividends return use them to buy back stock.” (Damodaran, 2006b, p. 19). This definition (which is strikingly similar to ours) is then dismissed in favor of a notion of cash flow to equity that includes change in liquid assets and that is named “free cash flow to equity”: he writes that “the free cash flow to equity model does not represent a radical departure from the traditional dividend discount model” (p. 20) even though he is aware that this model implies a well-determined assumption: “When we replace the dividends with FCFE to value equity, we are doing more than substituting one cash flow for another. We are implicitly assuming that the FCFE will be paid out to stockholders” (p. 21). He is perfectly aware that this assumption is harmless *only* if the excess cash “is invested in fairly priced assets (i.e. assets that earn a fair rate of return and thus have zero net present value)” (p. 24). He correctly observes that “when the FCFE is greater than the dividend and the excess cash either earns below-market interest rates or is invested in negative net present value assets, the value from the FCFE model will be greater than the value from the dividend discount model” (p. 24). And he himself admits that “there is reason to believe that this is not as unusual as it would seem at the outset” (p. 24). Nevertheless, in his textbooks and papers he seems to contravene his very arguments, given that he favors the potential-dividends model over the other ones:

Actual dividends ... may be much lower than the potential dividends (that could have been paid out) ... When actual dividends are less than potential dividends, using a model that focuses only on dividends will *understate the true value* of the equity in a firm. (Damodaran, 2008, slide 106)

firms do not always pay out what they can afford to in dividends. A more realistic estimate of equity value may require us to estimate the potential dividend—the cash flow that could have been paid out as a dividend (Damodaran, 2006a, p. 111)

Obviously, the fact that firms do not pay out what they can afford is not a good reason for favoring potential dividends as opposed to actual cash flow. Quite the contrary, if firms do not pay out cash flows that is available, then value is affected; in principle, whatever the magnitude of potential dividends, if a firm pays out no dividend over the life of the enterprise, the equity’s value is zero, as MM correctly recognize (MM, p. 419, footnote 12). And this is true even if the firm has invested in positive NPV projects (see also DeAngelo and DeAngelo, 2006).

Benninga and Sarig (1997, p. 36) share Damodaran’s view: “Free Cash Flow (FCF) [is] a concept that defines the amount of cash that the firm *can* distribute to security holders ... Cash and marketable securities are the best example of working capital items that we exclude from our definition of [change in net working capital], as they are the firm’s stock of excess liquidity” adjustment. When calculating free cash flow (and cash flow to equity) these authors do not subtract



$dLA_{t+1}$ , which entails that they abide by eq. (6) above (they make use of the term “net working capital” while meaning “operating net working capital”, inclusive of change in liquid assets). If changes in liquid assets are not included in the working capital, then any increase in liquid assets is not listed as a reduction in the cash available (as is correctly done in eq. (5)), even if they are deposited in the bank or in an investment fund. Using the funds (a cash excess) for increasing liquid assets reduces the cash available for distribution; if that is not listed in the calculation of FCF, then the increase appears to be an amount available for distribution even if it is not, and hence the cash flows to investors (FCF or CFE) are overstated, as well as the consequent firm value.

Copeland, Koller and Murrin’s (1990, 1994, 2000) (henceforth CKM) definition of free cash flow is consistent with Damodaran’s and Benninga and Sarig’s: they define it as “gross cash flow (NOPLAT plus depreciation) minus gross investment (increases in working capital plus capital expenditures)” (CKM, 2000, p. 138). Like with Benninga and Sarig, they employ the expression “working capital” but refer to “operating working capital”, as they explicitly state at p. 168. As a consequence, their definition of cash flow to equity does not exclude undistributed potential dividends:

$$\text{Cash flow to equity} = \text{Dividends paid} + \text{Potential Dividends} + \text{Equity repurchases} - \text{Equity Issues}$$

(see, for example, CKM, 1994, p. 480, Exhibit 16.3; CKM, 2000, Exhibit 21.2 at p. 430 and Exhibit 21.10 at p. 438). Admittedly, in the first edition of their book (CKM, 1990) they seemed to be inclined to accept a strict definition of cash flow to equity as cash flow paid to shareholders: in their Exhibit 13.2 at p. 379 one finds, referred to equity,

$$\text{Free Cash Flow} = \text{Dividends to equity}$$

and in the same page they explicitly refer to “free cash flow to shareholders, which is mathematically identical to dividends”. Yet, from the second edition of their book a radical shift toward potential dividends is consummated, albeit with no justification.

Brealey and Myers (2003, p. 75) write that “free cash flow is the amount of cash that a firm *can pay out* to investors after paying for all investments necessary for growth.” Therefore, while not being explicit, they seem to share the above mentioned authors’ stance. Their notion of working capital is consistent with the above mentioned authors, who do not subtract the change in liquid assets: “Working capital summarizes the net investment in short-term assets associated with a firm, business or project ... Working capital = inventory + accounts receivable – accounts payable.” (p. 126).<sup>5</sup>

While the practice of using potential dividends rather for valuing firms is a widespread one, there are some authors who consider it an error and correctly use only actual payments to shareholders for defining cash flow to equity. For example, Fernández (2002, p. 171) clearly states that “the forecast equity cash flow in a period must be equal to forecast dividends plus share repurchases in that period” and “the ECF in a period is the increase in cash (above the minimum cash, whose

---

<sup>5</sup> It is worth underlining that these authors use their (operating) free cash flow to value the whole firm, inclusive of all assets: “Discounting the FCFs at the WACC or at the RADR reflecting the risk of assets give us *the value of the firm as a whole*—the value of the firm’s assets. This value equals the sum of the values of *all* the securities that the firm has issued [...] In finance jargon this is usually called the **value of the firm**” (Benninga and Sarig, 1997, p. 83. Italics and boldface in original).

increase is included in the increase in WCR) during that period, before dividend payments, share repurchases and capital increases” (Fernández, 2002, p. 172); “considering the cash in the company as an equity cash flow when the company has no plans to distribute it” (Fernández, 2007, p. 26) is a frequent error in real-life applications:

In several valuation reports, the valuer computes the present value of positive equity cash flows in years when the company will not distribute anything to shareholders. Also, Stowe, Robinson, Pinto, and McLeavey (2002) say that “Generally, Equity Cash Flow and dividends will differ. Equity Cash Flow recognizes value as the cash flow available to stockholders even if it is not paid out.” Obviously, that is not correct, unless we assume that the amounts not paid out are reinvested and obtain a return equal to  $K_e$  (the required return to equity). (Fernández, 2003, p. 10)

His notion of working capital explicitly includes liquid assets:

$$\text{WCR} = \text{Cash} + \text{Accounts receivable} + \text{Inventories} - \text{Accounts payable}$$

(e.g. Fernández, 2002, p. 39), where “Cash” is Fernández’s term for “liquid assets”. Accordingly, his definition of equity cash flow is equivalent to our eq. (5) (Fernández, 2002, p. 178 and Tables 9.6-9.8 at pp. 179-181). Cash flows to equity must be necessarily be computed with eq. (2) above; “otherwise, we will be making hypotheses about what use is given to the part of the equity cash flow that is not used for dividends (cash, investments, repaying debt, etc.) and it will be necessary to subtract it beforehand from the equity cash flow” (Fernández, 2002, p. 179):

Shrieves and Wachowicz (2001, p. 35) commendably stress that working capital is inclusive of cash and marketable securities. Their eq. (2) defining cash flow paid to shareholders is equivalent to our eq. (5) above (they use the symbol *Div* to mean payments to shareholders). Admittedly, they seem to misinterpret CKM’s (1994) definition, because they write that the latter is “equivalent to our expression for dividends” (p. 36). However, despite of their slip of the pen, they correctly notice that by omitting liquid assets from the notion of working capital cash flows are overstated and observe that such an omission is harmless only *if* change in liquid assets have zero NPV.

DeAngelo and DeAngelo (2007) argue that a firm’s equity is not given by potential dividends but by cash flow paid to shareholders. The former (they label them “free cash flow”) determines the *investment value*, the latter leads to the *distribution value*: “Investment value is the discounted value of the FCF *to the firm* generated by its investment policy, which determines the firm’s *capacity* to make payouts. Distribution value is the discounted value of the cash payouts to currently outstanding shares, i.e., the cash flow paid *to stockholders*, which determines the market value of equity” (p. 16, italics in original). They underline that “value is generated for investors *only to the extent that this capacity is transformed into actual payouts*” (DeAngelo and DeAngelo, 2006, p. 309). Only if “the full PV of FCF is distributed to investors, variation in the timing of the stream of payouts and in their form (e.g. dividends versus stock repurchase) has no effect on stockholder wealth” (DeAngelo and DeAngelo, 2007, p. 25).

Penman (2007, p. 39) underlines that “Owner’s equity increases from value added in business activities (income) and decreases if there is a net payout to owners. Net payout is amounts paid to shareholders less amount received from share issues. As cash can be paid out in dividends or share repurchases, *net payout is stock repurchases plus dividends minus proceeds from share issues*”. He also writes that “it is noncontroversial that the price of a security is expressed as the ‘present value’ of the expected future payoffs to holding the security” (Penman, 1992, p. 466), where ‘payoffs’ unambiguously refers to “the payoffs for equity securities” (p. 466). His notions of “net cash flow

to shareholders” (p. 239) or “net dividend” (p. 241) are consistent with our notion of cash flow to equity:

$$\text{Net dividend} = \text{Cash dividend} + \text{Share repurchases} - \text{Share Issues}$$

(p. 241, Microsoft Corporation example). However, while he is perfectly aware that “The theory of finance describes equity valuation in terms of expected future dividends” (Penman and Sougiannis, 1998, p. 348), in his textbook he explicitly adopts the convention of assuming that dividend irrelevance holds (Penman, 2007, p. 96), so that using either net dividend or cash flow available for distribution is immaterial to the final result.

Our stance is that *nomina sunt omina*:<sup>6</sup> cash flow is a flow of cash and therefore (i) stock items in the Balance Sheet should not be considered flows, (ii) changes in stock items should be considered flow of cash only if they are paid to (or received from) shareholders. Hence, our definition of CFE as dividends minus net capital contributions (see eq. (2) above). If management keeps cash holdings instead of distributing it, shareholder wealth is affected unless the unpaid cash is invested in zero-NPV projects and distributed to shareholders over the life of the enterprise. But, usually in real life, as empirical evidence suggests, keeping cash holdings destroys value: the NPV of those invested funds is not zero (below zero).

## 2. Economic and financial arguments

This section summarizes economic and financial reasons for including in the cash flow only what indeed is a flow of cash.

**Cash flows and stocks.** To consider as cash flow items that are listed in the Balance Sheet is to deny the basic concept in valuation: cash flows. And they are just that, flows of money, whereas items in the balance sheet are stocks. It is a contradiction to say that an item is at the same time a line (or part of it) in the balance sheet and a line in the cash flow. Furthermore, if one considered cash flow what it is such only potentially, then one should consistently consider any asset in the firm as potential dividends, because assets may be sold and the cash may be distributed to investors: but then one should consider assets in the firm as representing both stocks and flows, which is a contradiction.

**Modigliani and Miller’s approach.** Modigliani and Miller’s (1958, 1963) approach to firm valuation only takes account of cash flows paid to investors. There are no “potential dividends” in their articles. The same is true even in MM (1961) where the irrelevance of dividends is proved. As DeAngelo and DeAngelo (2006) underline, in MM’s 1961 paper there are no retained funds, and the assumption is “to mandate 100% free cash flow payout in every period” (p. 293). MM do not deal with potential dividends retained in the firms and invested in liquid assets. There are no investments in liquid assets in MM (1961). MM’s thesis may be extended to the case of retention of FCF only if that investment is made at the opportunity cost of equity and “provided that managers distribute the full present value of FCF” (DeAngelo and DeAngelo, 2006, p. 303. See also Magni, 2007). Therefore, it is true that the unpaid dividends will be distributed at the terminal date, so that their value is captured in the terminal or continuing value. But it is never sufficiently stressed that irrelevance holds if and only if a perfect market exists where cash excess not paid is invested at the opportunity cost of equity. In real life, excess cash is invested in liquid assets at some available rate

---

<sup>6</sup> This Latin sentence means “names indicate what they mean”.

that might be greater, equal or lower than the cost of equity. This means that the NPV of those undistributed funds can be greater, equal or less than zero.

**Consistency between cash flows and financial statements.** There should be a complete consistency between cash flows and financial statements. If one says that every dollar available belongs to the equity holders, then that fact should be reflected in the financial statement. That is, those funds should appear as effectively distributed. In a valuation where a finite planning horizon is considered, decisions on what to do with excess cash are reflected in the financial model. This implies that if management foresees to invest in marketable securities, that decision should appear in the financial statements. On the other hand, if cash holdings are invested in additional operating assets, that decision should be included in the analysis; if they are devoted to acquisitions or buyouts, again, that decision should be reflected in the cash flows with all the financial implications it has. After this finite planning horizon, one makes the assumption that all available cash is distributed to equity and debt holders; the finite planning horizon should cover the largest possible period and at the end, a continuing or terminal value should measure the value generated for perpetuity.

**Tax distortion.** When one includes in the CFE the excess cash invested in liquid assets, one does distort taxes. Instead of recording an explicit return (usually a low return) that is taxed and listed in the Income Statement, one “creates” a virtual return equivalent to the cost of equity,  $k_e$ , which is not taxed in reality. Here, by virtual return we mean the return obtained implicitly when we distribute the cash flow to the owners (the debt and equity holders). This virtual and “automatic” return is one of the assumptions when we discount cash flows, as was recognized decades ago (see Lorie and Savage, 1955). When we assume that funds that are invested at a rate usually lower than the discount rate are part of the cash flow, we are assuming implicitly that those flows are reinvested at the same discount rate, but those returns are not listed in the Income Statement and not taxed, and go directly to the pocket of the owners. Here, it is important to take account of this: cash flows for valuation are what goes out from the firm and is paid to the capital owners (debt and equity holders). When that cash flow is received by them, it is assumed that they invest those funds at their respective rates of return (cost of equity,  $k_e$ ). The problem arises when the firm does not pay out the cash flow and invests it in liquid assets. If the amounts appear in the cash flows, then it is “as if” the equity holder had received them, but in fact she has not, and when one discounts the cash flows at the cost of equity or at WACC, one is assuming that the investor is reinvesting it at the opportunity cost of equity rate, but it does not happen in reality because she has not received them (the funds are invested in cash and/or in marketable securities). It might sound paradoxical<sup>7</sup>, but, actually, the greater the cash flow drawn out by debt and equity holders, the greater the firm’s value. The value of the firm is not in the funds that remain inside the firm, but in the funds that go out. Firm value increases if and only if cash is actually pulled out from the firm and distributed to the owners of debt or equity. Therefore, to consider potential dividends as actually pulled out by the firm is like trying to pull potential rabbits out of actual hats.

**Zero-NPV assumption in real-life applications.** One argument often used to justify inclusion the cash holdings as a cash flow is just that: the net present value of those investments is zero. We might agree on this assumption as a conservative approach to avoid excessive optimism as well as the determination (in a forecast) of a very high return on those invested funds. However, in

---

<sup>7</sup> Intuitively, people think of cash flows as net cash after adding inflows and subtracting all outflows. But inflows for capital providers are outflows for the firm.

constructing pro forma financial statements (forecasting) one should look the history of the firm and estimate the historical returns on those funds and forecast them accordingly to the historical average return. If the forecasted return is lower than the discount rate of the cash flows, the effect is that there is a destruction of value. If higher, a creation of value occurs. The idea of assuming without question that the NPV of the investment in market securities is zero implies that whatever the analyst or the management will do with those funds are of no concern to investors, because they funds do not contribute to value creation (they are value-neutral). Then, a simple conclusion could be drawn if one accepted the idea that the investment of cash excess does not affect the firm value: whether one keeps it in the safe box, in the bank, invested in an investment fund or whatever else, it will make no difference because the NPV is zero. This should lead many people and institutions to refrain from offering solutions for cash management purposes, such as the following.

“No financial discipline is more important and yet more overlooked and misunderstood than the essential principles and practices of corporate cash and liquidity management. Any business' success – from its long-term prospects to its short-term stability – depends on smart, efficient, and creative cash management solutions for handling cash inflows and outflows. The lesson is simple: Those companies that handle cash best thrive most.” (Northwestern University, <<http://www.scs.northwestern.edu/pdp/npdp/cm/>>, accessed November 21, 2007).

A stockholder would not accept to be virtually rewarded with “potential” dividends that never go to her pocket; likewise, banks or, in general, debt holders would not accept that interest or principal payments should be paid with “potential” interest and principal payments. Will practitioners and teachers accept an invitation to tell their customers and students to disregard the importance of cash management? Will they spread the idea that keeping the excess cash in the bank account (with no interest) would mean the same in terms of value than investing it wisely? Obviously not. If managers do not or are not expected to fully distribute the cash available, one should not “create” cash flow where it is not. Certainly, there should be ways to avoid that managers waste the excess cash in bad investments. Jensen (1986) says it in a rather graphical manner: “The positive market response to debt in oil industry takeovers ... is consistent with that additional debt increases efficiency by forcing organizations with large cash flows but few high-return investment projects to disgorge cash to investors. The debt helps prevent such firms from wasting resources on low-return projects” (pp. 10-11) Further, he writes that “Free cash flow is cash flow in excess of that required to fund all projects that have positive net present values when discounted at the relevant cost of capital. Conflicts of interest between shareholders and managers over payout policies are especially severe when the organization generates substantial free cash flow. The problem is how to motivate managers to disgorge the cash rather than investing it at below the cost of capital or wasting it on organization inefficiencies” (p. 2) and “Managers with substantial free cash flow can increase dividends or repurchase stock and thereby pay out current cash that would otherwise be invested in low-return projects or wasted” (p. 3). Therefore, although corporate financial theory may conveniently employ the assumption of full distribution of cash generated, in practice this does not happen, as Jensen underlines, so there is a need for distinguishing between potential cash flow available for distribution and actual cash flow effectively paid out to equity and debt holders.<sup>8</sup> The excess cash that managers refuse to pay to equity holders should not be included in the effective (actual) distributed cash flow. The fact that *in theory* firms should distribute the available cash does not imply that the analyst should assume that in the future all available cash will be distributed, if historically the firm has not distributed it. To assume that cash holdings are a cash flow that creates

---

<sup>8</sup> Admittedly, “potential cash flow” is a linguistic contradiction, because if distribution does not occur, then the undistributed potential cash flow is not a cash flow at all, as previously seen.

value is counter to evidence: “the theory is empirically refutable, predicting that firms will distribute the full PV of FCF, an implication that differentiates it from Jensen’s (1986) agency theory” (DeAngelo and DeAngelo, 2006, p. 295).

### 3. Logical arguments

We formalize in this section three arguments that aim at logically supporting the thesis according to which undistributed potential dividends do not add value to shareholders (and therefore must not be included in the definition of Cash Flow to Equity). In particular, they show that the use of potential dividends for valuation: (a) does not comply with the CAPM, (b) does not comply with the basic tenet of valuation theory, (c) does not comply with the no-arbitrage principle.

**Potential dividends and CAPM.** The use of undistributed potential dividends is in clear contradiction with the Capital Asset Pricing Model. When the CAPM is used to estimate the cost of equity,  $k_e$ , one uses dividends paid out to calculate the historical stock returns and historical beta; one never uses potential dividends.

[1] Suppose (i) an investor uses the CAPM for computing the cost of equity and (ii) uses undistributed potential dividends for valuation. Then, the firm lies on and above the SML.

Assume, with no loss of generality, that  $dLA_{t+1} > 0$ . Due to (i), the following relation holds:

$$k_e = r_f + \beta_e (r_m - r_f) \quad (8)$$

with  $\beta_e = \text{cov}(\tilde{r}_e, \tilde{r}_m) / \sigma_m^2$ , where  $\tilde{r}_e = \tilde{F}_{t+1} / E_t - 1$  is the random rate of return and  $\tilde{F}_{t+1}$  denotes the cum-dividend equity value at time  $t+1$ . This implies

$$E_t = \frac{F_{t+1}}{1 + k_e} \quad (9a)$$

with  $F_{t+1}$  being the expected value of  $\tilde{F}_{t+1}$ . However, due to (ii),

$$E_t = \frac{F_{t+1} + dLA_{t+1}}{1 + k_e}. \quad (9b)$$

Eqs. (8) and (9a) tell us that the firm lies *on* the SML, whereas (9b) tells us that the firm lies *above* the SML, given that it implies  $k_e = r_f + \beta_e (r_m - r_f) + dLA_{t+1} / E_t$ .

**The basic tenet of valuation theory.** Section 1 has shown that our definition of CFE is consistent with MM’s approach to valuation. MM, in turn, strictly abide by a basic tenet of valuation theory: value depends on cash flow received by the investor. This tenet may be formalized as:

$$E_t = \frac{E_{t+1} + \text{Cash Flow paid to shareholders}_{t+1}}{1 + k_e} \quad (10)$$

(see, for example, Miller and Modigliani, 1961, eq. (2)).

[2] Suppose (i) an investor uses undistributed potential dividends for valuation and (ii) accepts the basic tenet of valuation theory. Then, she incurs contradictions.

Note that one must have  $dLA_{t+1} \neq 0$ , otherwise the use of undistributed potential dividends is meaningless. If undistributed potential dividends enters valuation, then  $E_{t+1} = E_t(1+k_e) - CFE^*_{t+1}$ . But

$$\text{Cash Flow paid to shareholders}_{t+1} = CFE^*_{t+1} - dLA_{t+1},$$

because  $dLA_{t+1}$  are, by assumption, not distributed to shareholders. Therefore

$$dLA_{t+1} = E_t(1+k_e) - E_{t+1} - \text{Cash flow paid to shareholders}_{t+1}. \quad (11)$$

Eqs. (10)-(11) imply  $dLA_{t+1} = 0$ . But this contradicts the assumption  $dLA_{t+1} \neq 0$ .

**The arbitrage argument.** As is well-known, the no-arbitrage principle is a cornerstone in financial theory (Varian, 1987) and decision theory (Smith and Nau, 1994), and, more generally, represents a norm of rationality in economics (Nau and McCardle, 1991).

[3] Suppose an investor uses undistributed potential dividends for valuation. Then, she is open to arbitrage losses.

By assumption, investors in the market use eq. (3) (or, equivalently, eq. (6)) to value assets. Let us consider a firm traded in the market: let  $\tilde{a}$  be the periodic (random) payment to shareholders, and let  $\tilde{b}$  represent (random)  $\text{Div}_{\text{pot}}$ , which, by definition, are invested in liquid assets and not distributed.<sup>9</sup> If the investors positively evaluate  $\tilde{b}$ , then the market price of  $\tilde{b}$  is  $\frac{b}{i}$  where  $b$  is the expected value of  $\tilde{b}$  and  $i$  represents some (positive) expected rate of return. However, according to eq. (3), both payments to shareholders and changes in liquid assets are discounted to compute the equity market value. The latter is then priced at  $\frac{a}{r} + \frac{b}{i}$  where  $r$  is the appropriate discount rate (possibly equal to  $i$ ) for  $a$  (expected value of  $\tilde{a}$ ). An arbitrageur fixes two amounts  $h_1, h_2 > 0$ , so that  $h_1 + h_2 < \frac{b}{i}$  and proposes an investor a contract whereby the investor pays the arbitrageur an immediate sum equal to  $(\frac{a}{r} + \frac{b}{i} - h_1)$  and the arbitrageur guarantees periodic payments equal to those distributed by the firm to its shareholders. To the investor, this contract is equivalent to directly owning the firm's equity. The market value of the firm's equity is  $\frac{a}{r} + \frac{b}{i}$ , but she only spends  $(\frac{a}{r} + \frac{b}{i} - h_1)$  to receive that value. Therefore, NPV is positive to the investor:

$$\text{NPV} = h_1 > 0$$

so she accepts the contract.

---

<sup>9</sup> For the sake of simplicity (to avoid notational pedantry), we assume  $\tilde{a}$  and  $\tilde{b}$  do not vary over time, but this assumption is by no means restrictive. To avoid pedantry as well we assume infinite-horizon so that summation symbols are not needed.

Then, the arbitrageur proposes a second contract according to which the investor immediately receives from the arbitrageur an amount equal to  $(\frac{a}{r} + h_2)$  and periodically pays off the arbitrageur an amount equal to the cash flow that will be distributed by the firm to its shareholders. The investor accepts again, because she will pay  $\tilde{a}$ , whose present value is  $\frac{a}{r}$ , but immediately receives a greater amount, so that

$$NPV = h_2 > 0.$$

This strategy results in an arbitrage loss for the investor and an arbitrage profit for the arbitrageur (see Table 1). (To avoid arbitrage, one needs to value change in liquid assets  $b$  at zero).

| <b>Table 1. Arbitrage loss for an investor<br/>who uses eq. (3) for valuation</b> |                                    |                           |
|---|------------------------------------|---------------------------|
|   | Cash flows to the investor*        |                           |
|   | time 0                             | time $t = 1, 2, 3, \dots$ |
| First contract  | $-\frac{a}{r} - \frac{b}{i} + h_1$ | $\tilde{a}$               |
| Second contract   | $\frac{a}{r} + h_2$                | $-\tilde{a}$              |
| Total   | $h_1 + h_2 - \frac{b}{i} < 0$      | 0                         |

\*The cash flows to the arbitrageur are the same cash flows changed in sign

To sum up:

From [1]: if one uses the CAPM for computing the cost of equity, then one *may not* use undistributed potential dividends for valuation

From [2]: if one uses undistributed potential dividends for valuation, then one does not accept the basic tenet of valuation theory

From [3]: if one does not accept the basic tenet of valuation theory, one is open to arbitrage losses.

#### 4. Empirical Evidence: is the glass half full or half empty?

There is empirical evidence that dividends and not liquid assets are what increases firm value. Pinkowitz, Williamson and Stulz (2007) (henceforth PWS 2007) have examined publicly traded firms for 35 countries and divided the sample between corrupt and non corrupt countries and between countries with good and poor protection to minority equity holders according to two Investor Protection Indices. Using our symbols, the model tested by PWS 2007 is described as



$$V_t = \alpha + \beta_1 \text{Earn}_t + \beta_2 d\text{Earn}_t + \beta_3 d\text{Earn}_{t+1} + \beta_4 d\text{NA}_t + \beta_5 d\text{NA}_{t+1} + \beta_6 \text{RD}_t + \beta_7 d\text{RD}_t + \beta_8 d\text{RD}_{t+1} + \beta_9 \text{I}_t + \beta_{10} d\text{I}_t + \beta_{11} d\text{I}_{t+1} + \beta_{12} \text{Div}_t + \beta_{13} d\text{Div}_t + \beta_{14} d\text{Div}_{t+1} + \beta_{15} dV_{t+1} + \beta_{16} d\text{LA}_t + \beta_{17} d\text{LA}_{t+1} + \varepsilon_t$$

where  $V$  is the market value of the firm calculated at fiscal year end as the sum of the market value of equity and the book values of short-term debt and long-term debt; NA denotes net assets (defined as total assets minus liquid assets); Earn is earnings before extraordinary items plus interest, deferred tax credits, and investment tax credits; RD is research and development expense; I is interest expense; and Div is common dividends paid. The results by PWS 2007 are shown in Table 2.

**Table 2. Estimates of the value of \$1 increase in dividends and in cash holdings**

| Value Estimate of One Dollar More of | High corruption countries | Low corruption countries | Poor shareholders rights countries | Good shareholders rights countries |
|--------------------------------------|---------------------------|--------------------------|------------------------------------|------------------------------------|
| Dividends/Total Assets               | 6.56                      | 4.03                     | 9.8                                | 4.07                               |
| Cash/Total Assets                    | 0.33                      | 0.91                     | 0.29                               | 0.95                               |

Source: Pinkowitz, Williamson and Stulz (2007), Table 2

They allow us to estimate the implicit discount rate that makes an extra dollar in each of those components equal to the amount announced by the authors. In Table 3a we assume a non growing perpetuity; hence the discount rate is  $1/PV$ . The discount rate the market applies to one dollar in cash is extremely high, while the discount rate for the dividends is “reasonable”. Even if we assume that the \$1 is for ten years (length of the study) the order of magnitude for the cost of equity  $k_e$  in the case of cash is the same. For dividends, it ranges from 0.37% to 21.2%; for cash, it ranges from 105.2% to 343.8%.

**Table 3a. Implicit discount rate in PWS 2007, Table 2, assuming a Perpetuity**

|                      | High Corruption Countries | Low Corruption Countries | Poor shareholders Rights countries | Good shareholders Rights countries |
|----------------------|---------------------------|--------------------------|------------------------------------|------------------------------------|
| PV (div = \$1)       | 6.56                      | 4.03                     | 9.8                                | 4.07                               |
| $k_e$ for perpetuity | 15.24%                    | 24.81%                   | 10.20%                             | 24.57%                             |
| PV(cash = \$1)       | 0.33                      | 0.91                     | 0.29                               | 0.95                               |
| $k_e$ for perpetuity | 303%                      | 110%                     | 345%                               | 105%                               |

**Table 3b. Implicit discount rate in PWS 2007, Table 2, assuming single period**

|                      | High Corruption Countries | Low Corruption Countries | Poor shareholders Rights countries | Good shareholders Rights countries |
|----------------------|---------------------------|--------------------------|------------------------------------|------------------------------------|
| PV (div = \$1)       | 6.56                      | 4.03                     | 9.8                                | 4.07                               |
| $k_e$ for perpetuity | -84.8%                    | -75.2%                   | -89.8%                             | -75.4%                             |
| PV(cash = \$1)       | 0.33                      | 0.91                     | 0.29                               | 0.95                               |
| $k_e$ for perpetuity | 203.0%                    | 9.9%                     | 244.8%                             | 5.3%                               |

If one assumes that those coefficients are related to one period and not to a perpetuity one obtains the results shown in Table 3b. In these cases we can see that the “implicit discount rate” for one

period has huge differences between cash and dividends. We can say in general that the market punishes cash (compared with dividends) as seen by the discount rate, which is much greater than the discount rate for dividends. In Table 4a some findings are shown, assuming a perpetuity: Pinkowitz and Williamson (2002) (henceforth PW 2002) imply an average discount rate of 80% (1/1.25); according to Pinkowitz, Stulz and Williamson (2003) (henceforth PSW 2003), one dollar is worth 0.65, so that the discount rate would be 153.85%; according to Faulkender and Wang (2004, p. 23) (henceforth FW 2004), one dollar is worth 0.97 (discount rate: 103.1%); according to Harford (1997), the value of one dollar in cash is 0.956 (discount rate: 104.6%). The discount rates shown are very high and unusual and might be interpreted as something undesirable to the market. In any case, one can only infer that dividends create much more value than cash, not that cash is positively evaluated. (The results for a single period are shown in Table 4b).

Furthermore, Table 5 shows that dividends are between 20 and 34 times more relevant than cash in value creation in “bad” countries and about 4 times more relevant than cash in “good” countries.

**Table 4a. Implicit discount rate in other findings, assuming a perpetuity**

|                      | Harford 1997 | PW 2002 | PSW 2003 | FW 2004 |
|----------------------|--------------|---------|----------|---------|
| PV(cash = \$1)       | 0.956        | 1.25    | 0.65     | 0.97    |
| $k_e$ for perpetuity | 104.6%       | 80.0%   | 153.9%   | 103.1%  |

**Table 4b. Implicit discount rate in other findings, assuming a single period**

|                      | Harford 1997 | PW 2002 | PSW 2003 | FW 2004 |
|----------------------|--------------|---------|----------|---------|
| PV(cash = \$1)       | 0.956        | 1.25    | 0.65     | 0.97    |
| $k_e$ for perpetuity | 4.60%        | -20.00% | 53.85%   | 3.09%   |

As the saying goes: whether a glass is half full or half empty depends on the attitude of the person looking at it. Those who see the glass half full might say: this is a proof that cash creates value and hence should be included in the cash flows. Others (and we are in this number) see the glass half empty and say: the market is adverse to “potential” dividends because it discounts them with a huge rate, so trying to keep their value down to zero. Hence, they should not be included as cash flow because this is counter to evidence and overstates the value of the firm.

The findings presented in PW 2002 and PWS 2007 just suggest that (changes in) liquid assets should not be included in the FCF or CFE because they inflate the value of the firm. The meaning of the above mentioned results is that one dollar in liquid assets creates less than one dollar in value and that liquid assets or “potential” dividends are something not desirable by investors.

Using the aggregate data from PWS 2007 and without splitting the sample between good and bad countries, one finds that one extra dollar of cash is valued at zero by the market. This is an average, and it compares with the above mentioned findings. The data from PWS 2007 represent evidence of this fact. Data cover ten years (1988-1998) for 35 countries with some exceptions such as India, Philippines, Turkey and Peru. This means that they had near 69,000 observations. The values in Table 6 are the mean of the medians for each variable for each country. Market Value to Book Value (V/B) is the sum of market equity value plus book value of debt divided by book value of assets. Dividends and liquid assets are the percent of those items in the Balance Sheet on Total

assets. The liquid assets are not calculated individually but they are added (PWS 2007 use the term “Cash”).

| Value Estimate of One Dollar More of | High Corruption Countries | Low Corruption Countries | Poor Shareholders Rights countries | Good Shareholders Rights countries |
|--------------------------------------|---------------------------|--------------------------|------------------------------------|------------------------------------|
| Dividends/Total Assets               | 6.56                      | 4.03                     | 9.8                                | 4.07                               |
| Cash/Total Assets                    | 0.33                      | 0.91                     | 0.29                               | 0.95                               |
| Dividends/Cash                       | 19.88                     | 4.43                     | 33.79                              | 4.28                               |

Using the aggregated data shown in PWS 2007 we have run several regressions between V/B, Cash and Dividends. V/B is the dependent variable and the other two are independent variables. The aggregated data we have used are shown in Table 6. We depict the scattered data for each pair of independent and dependent variables: in Exhibit 1 we can observe a trend: the higher the dividend, the higher the V/B. In Exhibit 2 the reader can observe a slight downward trend: the higher the cash level, the lower the ratio V/B. Notice that the three variables are scaled or normalized by book value of total assets. This means that one may compare firms of different size and from different countries and years. We have analyzed eighteen regression models with the following structure:

1. Lin Lin model with the following structure:

$$V/B = \beta_0 + \beta_1 \times \text{Cash} + \beta_2 \times \text{Dividends}$$

2. Log Log model with the following structure:

$$\ln(V/B) = \beta_0 + \beta_1 \times \ln(\text{Cash}) + \beta_2 \times \ln(\text{Dividends})$$

3. Lin Log model with the following structure:

$$V/B = \beta_0 + \beta_1 \times \ln(\text{Cash}) + \beta_2 \times \ln(\text{Dividends})$$

4. Lin Lin Log model with the following structure:

$$V/B = \beta_0 + \beta_1 \times \text{Dividends} + \beta_2 \times \text{Cash} + \beta_3 \times \ln(\text{Cash}) + \beta_4 \times \ln(\text{Dividends})$$

5. Log Lin Log model with the following structure:

$$\ln(V/B) = \beta_0 + \beta_1 \times \text{Dividends} + \beta_2 \times \text{Cash} + \beta_3 \times \ln(\text{Cash}) + \beta_4 \times \ln(\text{Dividends})$$

We are aware that the model lacks good specification because there are other variables that have to be included, it is based on aggregated data and it does not represent the true universe studied by PWS 2007. This exploration only gives hints and trends.

After the regressions have been run, significant models are only those with the independent variable Dividends in linear form. This means linear (Lin Lin) and semi logarithmic (Log Lin) models. In particular, the only significant variable is Dividends. In Table 7 we summarize the statistical analysis. We expected to find that the relationship between V/B and CFE (dividends) be linear, because MM’s valuation model is linear (see eq. (16) in section 5). And as is suggested by Table 7, the results in terms of statistical significance corroborate the linearity between value and CFE. In particular, one dollar in dividends creates about 8 dollars in value (see Exhibit 1). We are aware that this analysis is rather restricted because we are dealing with aggregated data and not with the raw data PWS 2007 worked with. Another restriction is related to the interpretation of the results. Admittedly, we do not have full information regarding the specific model used in each case of the

report. For instance, we do not know whether cash refers to change in cash or level of cash, or if it is located at a different (future) period compared with the firm's market value. This is actually critical if we are interested in defining the coefficient of the independent variable as the increase in value for each extra dollar in the variable.

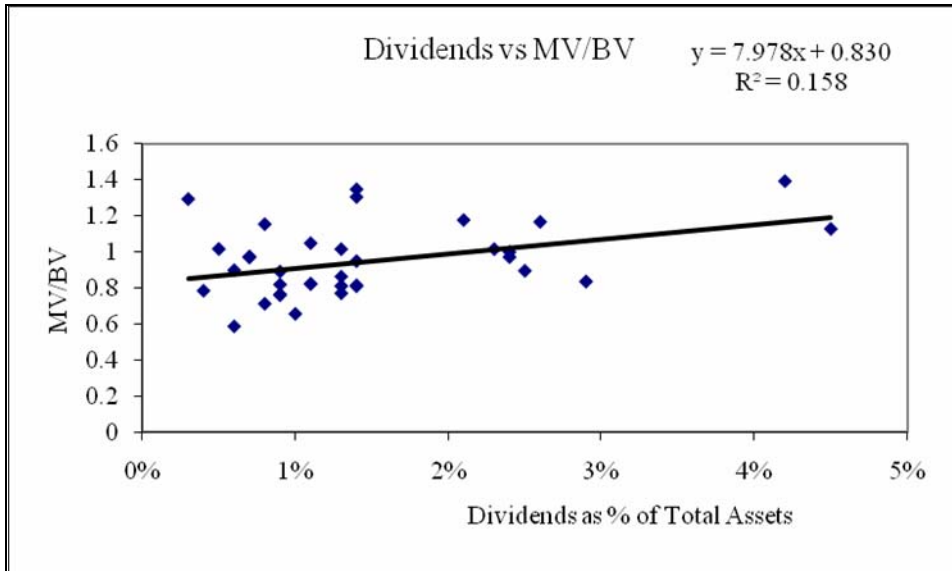


Exhibit 1. Dividends and V/B ratio.

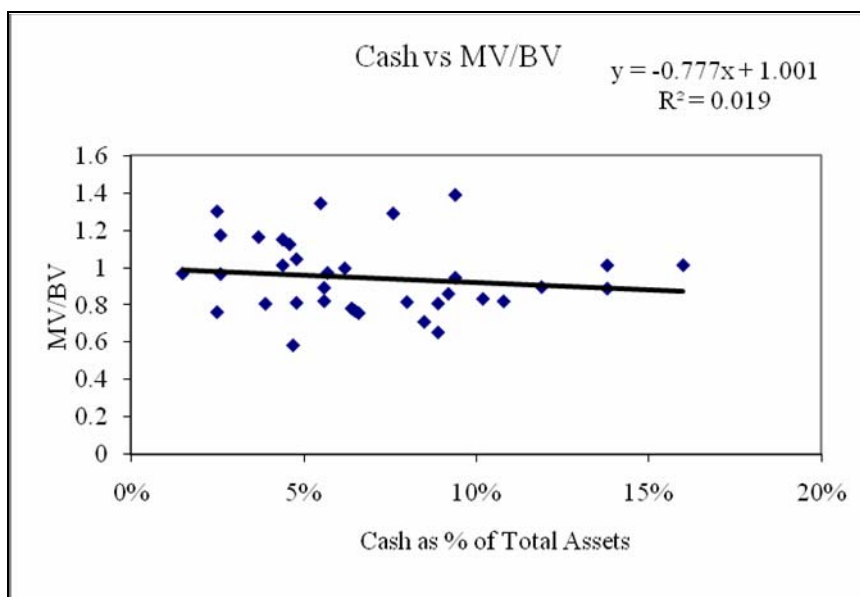


Exhibit 2. Cash and V/B ratio.

**Table 6. Dividends, Liquid Assets and V/B**

| Country       | Dividends | Cash  | V/B   | Mean # of firms per year |
|---------------|-----------|-------|-------|--------------------------|
| Argentina     | 1.3%      | 6.5%  | 0.769 | 14.2                     |
| Australia     | 2.3%      | 4.4%  | 1.013 | 137.8                    |
| Austria       | 0.9%      | 6.6%  | 0.758 | 34.3                     |
| Belgium       | 1.3%      | 8.9%  | 0.809 | 57.6                     |
| Brazil        | 0.6%      | 4.7%  | 0.586 | 46.1                     |
| Canada        | 0.7%      | 2.6%  | 0.967 | 292.8                    |
| Chile         | 4.5%      | 4.6%  | 1.125 | 32.7                     |
| Denmark       | 0.9%      | 13.8% | 0.889 | 88.6                     |
| Finland       | 0.9%      | 8.0%  | 0.817 | 58.8                     |
| France        | 0.8%      | 8.5%  | 0.711 | 309.8                    |
| Germany       | 1.1%      | 5.6%  | 0.822 | 272.1                    |
| Greece        | 2.6%      | 3.7%  | 1.164 | 40.4                     |
| Hong Kong     | 2.9%      | 10.2% | 0.834 | 100.1                    |
| India         | 1.4%      | 2.5%  | 1.301 | 57.3                     |
| Ireland       | 1.4%      | 9.4%  | 0.947 | 37                       |
| Italy         | 1.0%      | 8.9%  | 0.655 | 104.8                    |
| Japan         | 0.5%      | 16.0% | 1.014 | 1,442.7                  |
| Korea (South) | 0.4%      | 6.4%  | 0.783 | 101.3                    |
| Malaysia      | 1.4%      | 5.5%  | 1.344 | 140.3                    |
| Mexico        | 0.7%      | 5.7%  | 0.972 | 33.1                     |
| Netherlands   | 1.4%      | 4.8%  | 0.813 | 113.6                    |
| New Zealand   | 2.4%      | 1.5%  | 0.969 | 25.4                     |
| Norway        | 0.6%      | 11.9% | 0.897 | 64.8                     |
| Peru          | 1.1%      | 4.8%  | 1.046 | 10.6                     |
| Philippines   | 0.3%      | 7.6%  | 1.29  | 22                       |
| Portugal      | 0.9%      | 2.5%  | 0.763 | 25.9                     |
| Singapore     | 1.3%      | 13.8% | 1.013 | 75.9                     |
| South Africa  | 2.5%      | 5.6%  | 0.893 | 102.6                    |
| Spain         | 1.4%      | 3.9%  | 0.808 | 77.8                     |
| Sweden        | 1.3%      | 9.2%  | 0.861 | 89.6                     |
| Switzerland   | 1.1%      | 10.8% | 0.821 | 88.5                     |
| Thailand      | 2.1%      | 2.6%  | 1.174 | 86.5                     |
| Turkey        | 4.2%      | 9.4%  | 1.389 | 13.2                     |
| UK            | 2.4%      | 6.2%  | 0.997 | 962.4                    |
| USA           | 0.8%      | 4.4%  | 1.151 | 1,751.2                  |

Source: Pinkowitz, Williamson and Stulz, (2007), Table 1

**Table 7. Significant models with dividends as independent variables**

| Regression Model with linear Dividends        | Coefficient<br>(p-value)    |
|---|-----------------------------|
| 1. Lin Lin one independent variable Dividends | 7.97813497<br>(0.018068378) |
| 2. Log Lin one independent variable Dividends | 8.30632596<br>(0.01771795)  |

### 5. Moving forward: a proposal for future tests

The previous sections have shown that change in liquid assets must not be included in the notion of CFE. However, future research should bring corroboration to the idea that the market do not positively value change in liquid assets. Armed with the theoretical toolbox we have developed in section 1, this last section proposes a model for tests, which, we expect, will add empirical evidence. The model is based on standard results of corporate financial theory and, in particular, on Modigliani and Miller (1963). We use the following equation

$$V = V_U + V_{TS} = D + E \quad (12)$$

where  $V$  is the value of the levered firm,  $V_U$  is the value of the unlevered value,  $V_{TS}$  is the value of tax savings,  $D$  is the market value of debt (assumed to be equal to its book value). The cash flow paid to the investors is related to CFE as follows:

$$FCF + TS = CFE + CFD = CCF \quad (13)$$

where  $TS$  is Tax Savings (or Tax Shield),  $CFD$  is the Cash Flow to Debt and  $CCF$  is the Capital Cash Flow (see Ruback, 2002). From eqs. (12)-(13), and using the fact that  $D$  is the present value of  $CFD$ ,

$$V = PV(FCF \text{ at } WACC^{FCF}) = PV(CFE \text{ at } k_e) + D = PV(CCF \text{ at } WACC^{CCF}) \quad (14)$$

where  $PV$  is present value,  $WACC^{FCF}$  is the Weighted Average Cost of Capital for the FCF,  $k_e$  is the levered cost of equity,  $D$  is the market value of debt (we assume nominal debt equal to market value of debt), and  $WACC^{CCF}$  is the Weighted Average Cost of Capital for the CCF (i.e. pre-tax WACC). Hence,

$$V_t = \frac{V_{t+1} + CCF_{t+1}}{1 + WACC_{t+1}^{CCF}} \quad (15)$$

Using eqs. (2) and (13) and the relation  $CFD_{t+1} = I_{t+1} - dD_{t+1}$ , eq. (15) becomes

$$V_t = \frac{V_{t+1} + \text{Div}_{t+1} + I_{t+1} - (dCS_{t+1} + dD_{t+1})}{1 + \text{WACC}_{t+1}^{\text{CCF}}}, \quad (16)$$

where, as seen, the variation in capital stock  $dCS_{t+1}$  is obtained as difference between new equity investment and shares repurchases occurred in the year. If markets positively valued changes in liquid assets, then the levered value of the firm, as empirically evidenced by the market, would not be the theoretically correct eq. (16), but would be the following

$$V_t = \frac{V_{t+1} + \text{Div}_{t+1} + \text{Div}_{\text{pot}(t+1)} + I_{t+1} - (dCS_{t+1} + dD_{t+1})}{1 + \text{WACC}_{t+1}^{\text{CCF}}} \quad (17)$$

which differs from eq. (16) for the fact that change in liquid assets  $\text{Div}_{\text{pot}(t+1)}$  are included in the valuation formula. We expect to find that in eq. (17)  $\text{Div}_{\text{pot}}$  are of no value, or, as a weaker thesis, that the value of  $\text{Div}_{\text{pot}}$  is much lower than dividends actually paid to shareholders.

For the implementation of the test, we will need to collect information which is usually publicly available:

1. D = Financial debt (book value as a proxy of market value)
2. NS = # shares in the market
3. Pr = Market price per share
4. V = Firm value = # shares  $\times$  Market Price + debt (book value) = NS  $\times$  Pr + debt
5. I = Interest payments
6. CS = (cumulated) capital stock contributed by shareholders
7. Div = Dividends paid in cash
8. C = Cash
9. STI = Short-term investments (marketable securities)
10. B = Book value of total assets

In order to “normalize” the data and avoid problems of size, currency, time, etc. we will divide each variable by the book value of total assets in  $t$ , in the same way PWS 2007 do. As all variables will be divided by book value of total assets, the ratio  $V/B$  represents Tobin’s Q. These independent variables are the proxies for components of equation (17). While PWS 2007 do not include payments of debt, we will include them, and, tentatively, we will not include variables that they include, such as R&D expense. Hence, our econometric model will be (for each firm)

$$V_t = \beta_0 + \beta_1 V_{t+1} + \beta_2 dD_{t+1} + \beta_3 I_{t+1} + \beta_4 dCS_{t+1} + \beta_5 \text{Div}_{t+1} + \beta_6 \text{Div}_{\text{pot}(t+1)} + \varepsilon_t \quad (18)$$

where all variables are now meant to be divided by book value of assets. With this model, the value of the firm depends on the cash flows the owners of equity and debt expect to receive in the future and on potential dividends as well. Each of the elements of this model attempts to capture investors’ expectations on future cash flows and value. As all variables are divided by total assets we have Tobin’s Q as the dependent variable, while the independent variables will be a percent of total assets in  $t$ . With this model we attempt to measure how much value is created for a given value and cash flows in the following period. An alternate model is

$$E_t = \beta_0 + \beta_1 E_{t+1} + \beta_4 dCS_{t+1} + \beta_5 \text{Div}_{t+1} + \beta_6 \text{Div}_{\text{pot}(t+1)} + \varepsilon_t \quad (19)$$

where a proprietary approach is followed, and where  $E_t$  is measured as number of shares times market price per share (we will use data from Latin American countries). Given that eq. (19) requires less information, we intend to use the latter when validating data.

While the two models in eqs. (18)-(19) are consistent with standard finance theory, we do not intend to claim that they are fully explanatory nor to make use of them for forecasting purposes. The models are meant to provide information on the relevance of the independent variables, in particular the relevance/irrelevance of  $Div_{pot(t+1)}$  to value creation. To this end, the various betas are to be interpreted as the discount factors for the independent variables. In particular,  $\beta_6$  is the discount factor for change in liquid assets, i.e. it represents the value at time  $t$  of one dollar of  $Div_{pot(t+1)}$  available at time  $t+1$ .

Our hypothesis may be phrased in a strong or in a soft version:

**Strong version:** We expect  $\beta_6$  to be statistically zero (or close to zero). This means that investors try to set down the value of  $Div_{pot(t+1)}$  by discounting it with an infinite or at least very high a very high discount rate, because they do not consider (undistributed) potential dividends relevant for valuation.

**Soft version:** We expect  $Div_{pot(t+1)}$  to be evaluated much less than actual dividends. In econometric terms, we expect  $\beta_6$  to be much smaller than  $\beta_5$ .<sup>10</sup>

If our hypothesis will be corroborated (as already implied by both modern finance theory and the reported empirical findings), the practice of assuming that liquid assets are part of the cash flows will be (not only theoretically but also) empirically refuted.

## Conclusions

Economics, and in particular, financial economics provide rigorous theoretical tools for valuing assets. The theory is unambiguous in stating that the value of an asset depends on the cash flow actually received by investors, not on the cash flows that *could* be received, unless undistributed potential dividends are invested in zero-NPV activities and their full present value is distributed to shareholders (DeAngelo and DeAngelo, 2006, Magni, 2007). But if, historically, the firm has not distributed all the available cash, there is no reason to assume that in the future it will be different. While some authors correctly recognize that only cash flow paid to shareholders should be used for equity valuation (Vélez-Pareja, 1999a, 1999b, 2004, 2005a, 2005b; Shrieves and Wachowicz, 2001; Fernández, 2002, 2007; Tham and Vélez-Pareja, 2004; DeAngelo and DeAngelo, 2007), several authors in applied corporate finance and a large part of practitioners include use potential dividends for computing a firm's equity value (e.g. Benninga and Sarig, 1997; Damodaran, 1998, 2006; Copeland, Koller and Murrin, 1994, 2000) This paper aims at showing that the practice of adding liquid assets to cash flows actually paid is at odds with finance theory and seems to be inconsistent with empirical findings. Cash Flow to Equity should be defined as dividends paid minus net capital contributions, i.e. dividends plus shares repurchases minus new equity investment.

The issue is tackled in the paper in three ways: economic, logical, empirical. Economically, several reasons are given which confirm that only actual cash flows are relevant in valuation; logically, three formal proofs are provided that make use of the CAPM, of the basic tenet in valuation theory, and of arbitrage theory; empirically, recent findings in the literature are analyzed

---

<sup>10</sup> See Table 3a and 3b in section 4 and the difference between discount rates.



(among which Pinkowitz, Williamson and Stulz, 2007) from which evidence is drawn that market values cash holdings much less than dividends paid. An interpretation of these findings is that the market does not consider *potential* dividends a value driver. Further empirical work will be done with a model that is rigorously deduced from the theory (in particular, from Modigliani and Miller's approach to valuation).

## References

- Benninga, S.Z. and Sarig, O.H. (1997) *Corporate Finance. A Valuation Approach*. McGraw-Hill.
- Brealey, R. and Myers, S.C. (2003) *Principles of Corporate Finance*, 7th edition, New York: McGraw Hill-Irwin.
- Copeland, T.E., Koller, T. and Murrin, J. (1990) *Valuation: Measuring and Managing the Value of Companies*. John Wiley & Sons.
- Copeland, T.E., Koller, T. and Murrin, J. (1994) *Valuation: Measuring and Managing the Value of Companies*, 2<sup>nd</sup> Edition. John Wiley & Sons.
- Copeland, T.E., Koller, T. and Murrin, J. (2000) *Valuation: Measuring and Managing the Value of Companies*, 3<sup>rd</sup> Edition. John Wiley & Sons.
- Damodaran, A. (1999) *Applied Corporate Finance. A User's Manual*. New York: John Wiley & Sons.
- Damodaran, A. (2006a) *Damodaran on Valuation*, second edition. Hoboken, NJ: John Wiley & Sons.
- Damodaran, A. (2006b) Valuation approaches and metrics: a survey of the theory and evidence. Available at SSRN: <<http://www.stern.nyu.edu/~adamodar/pdfiles/papers/valuesurvey.pdf>> (November). Originally published in *Foundations and Trends® in Finance*, 1(8), 693-784, 2005.
- Damodaran, A., (2008) Equity instruments: Part I. Discounted cash flow valuation, <[www.stern.nyu.edu/~adamodar/pdfiles/eqnotes/packet1.pdf](http://www.stern.nyu.edu/~adamodar/pdfiles/eqnotes/packet1.pdf)>, (updated January 14, 2008). Accessed February 16, 2008.
- DeAngelo, H. and DeAngelo, L. (2006) The Irrelevance of the MM dividend irrelevance theorem. *Journal of Financial Economics*, 79, 293–315.
- DeAngelo, H. and DeAngelo, L. (2007) Payout policy pedagogy: what matters and why. *European Financial Management*, 13(1), 11-27.
- Faulkender, M.W. and Wang, R. (2004) Corporate financial policy and the value of cash. *Working Paper* (July 23). Available at SSRN: <<http://ssrn.com/abstract=563595>>.
- Fernández, P. (2002) *Valuation Methods and Shareholder Value Creation*. San Diego: Academic Press.

- Fernández, P. (2007) Company valuation methods. The most common errors in valuation. *Working paper* (February). Available at SSRN: <<http://ssrn.com/abstract=274973>>.
- Harford, J. (1999) Corporate cash reserves and acquisitions. *Journal of Finance*, 54(6), 1969-1997. Available at SSRN: <<http://ssrn.com/abstract=2109>> or DOI: 10.2139/ssrn.2109, (January 30, 1997).
- Jensen, M.C. (1986) Agency cost of free cash flow, corporate finance, and takeovers. *American Economic Review*, 76(2), 323-329. Available at SSRN: <<http://ssrn.com/abstract=99580>> or DOI: 10.2139/ssrn.99580.
- Lorie, J.H. and Savage, L.J. (1955) Three problems in rationing capital. *Journal of Business*, 28, October. Reprinted in E. Solomon (Ed.) 1959. *The Management of Corporate Capital*, Glenco, Illinois: The Free Press.
- Magni, C.A. (2007) Relevance or irrelevance of retention for dividend policy irrelevance. *Working Paper* (November). Available at SSRN: <<http://ssrn.com/abstract=1027401>>.
- Mikkelson, W.H. and Partch, M. (2003) Do persistent large cash reserves lead to poor performance?. *Journal of Financial and Quantitative Analysis*, 38(2), 275-294, June. *Working Paper*. Available at SSRN: <<http://ssrn.com/abstract=186950>> or DOI: 10.2139/ssrn.186950.
- Miller, M.H. and Modigliani, F. (1961) Dividend policy, growth and the valuation of shares. *The Journal of Business*, 34(4), 411-433, October.
- Modigliani, F. and Miller, M.H. (1958) The cost of capital, corporation taxes and the theory of investment. *American Economic Review*, 47, 261-297.
- Modigliani, F. and Miller, M.H. (1963) Corporate income taxes and the cost of capital: a correction. *American Economic Review*, 53, 433-443.
- Nau, R.F. and McCardle, K.F. (1991) Arbitrage, rationality, and equilibrium. *Theory and Decision*, 31(2-3), 199-240, September.
- Opler, T.C., Pinkowitz, L., Stulz, R.M. and Williamson, R. (1999) The determinants and implications of corporate cash holdings. *Journal of Financial Economics*, 52(1), 3-46. NBER Working Paper No. W6234. Available at SSRN (October 1997): <<http://ssrn.com/abstract=225992>>
- Penman, S. (1992) Return to fundamentals. *Journal of Accounting, Auditing and Finance*, 7, 465-483 (Fall). Reprinted in R. Brief and K.V. Peasnell (Eds.), *Clean Surplus: A Link Between Accounting and Finance*, New York and London: Garland Publishing.
- Penman, S. and Sougiannis, T. (1998) A comparison of dividends, cash flow, and earnings approaches to equity valuation. *Contemporary Accounting Research*, 15(3), 343-383, Fall.
- Penman, S. (2007) *Financial Statement Analysis and Security Valuation*, third edition. New York: McGraw Hill.

- Pinkowitz, L., Williamson, R. and Stulz, R.M. (2007) Cash holdings, dividend policy, and corporate governance: a cross-country analysis. *Journal of Applied Corporate Finance*, 19(1),81-87, Winter.
- Pinkowitz, L., Stulz, R.M. and Williamson, R. (2003) Do firms in countries with poor protection of investor rights hold more cash?. *Dice Center Working Paper* No. 2003-29, (November). Available at SSRN: <<http://ssrn.com/abstract=476442>> or DOI: 10.2139/ssrn.476442.
- Pinkowitz, L. and Williamson, R.G. (2002) What is a dollar worth? The market value of cash holdings. *Working Paper* (October). Available at SSRN: <<http://ssrn.com/abstract=355840>> or DOI: 10.2139/ssrn.355840.
- Ruback, R. S. (2002). Capital cash flows: a simple approach to valuing cash flows. *Financial Management*, Summer, 85-103.
- Schwetzler, Bernhard and Reimund Carsten (2003), Valuation effects of corporate cash holdings: Evidence from Germany. *HHL Working Paper*. Available at SSRN: <<http://ssrn.com/abstract=490262>>.
- Shrieves, R.E. and Wachowicz, J.M. (2001) Free Cash Flow (FCF), Economic Value Added (EVA ), and Net Present Value (NPV): A Reconciliation of Variations of Discounted-Cash-Flow (DCF) Valuation. *The Engineering Economist*, 46(1), 33-52.
- Smith, J.E. and Nau, R. (1995) Valuing risky projects: option pricing theory and decision analysis. *Management Science*, 41(5) 795-816, May.
- Tham, J. and Véléz-Pareja, I. (2004) *Principles of Cash Flow Valuation*. Academic Press.
- Varian, H.R. (1987) The arbitrage principle in financial economics. *The Journal of Economic Perspectives*, 1(2), 55-72, Autumn.
- Véléz-Pareja, I. (1999a) Construction of free cash flows: a pedagogical note. Part I (December). *Working Paper*. Available at SSRN: <<http://ssrn.com/abstract=196588>>
- Véléz-Pareja, I. (1999b) Construction of free cash flows: a pedagogical note. Part II (December). *Working Paper*. Available at SSRN: <[http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=199752](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=199752)>.
- Véléz-Pareja, I. (2004) The correct definition for the cash flows to value a firm (free cash flow and cash flow to equity) (September 30), *Working Paper*. Available at SSRN: <<http://ssrn.com/abstract=597681>>.
- Véléz-Pareja, I. (2005a) Once more, the correct definition for the cash flows to value a firm (free cash flow and cash flow to equity). *Working Paper* (February 3). Available at SSRN: <<http://ssrn.com/abstract=642763>>.
- Véléz-Pareja, I. (2005b) Construction of cash flows revisited. *Working Paper* (August 16). Available at SSRN: <[http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=784486](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=784486)>.

---

### Main notational conventions

---

|                     |  |
|---------------------|--|
| AP                  | Accounts Payable   |
| AR                  | Accounts Receivable  |
| B                   | Book value of total assets                                     |
| C                   | Cash   |
| CCF                 | Capital Cash Flow  |
| CFD                 | Cash Flow to Debt  |
| CFE                 | Cash Flow to Equity  |
| CFE*                | Potential dividends (CFE + change in liquid assets)            |
| CS                  | Capital stock  |
| d                   | Variation symbol   |
| D                   | Debt (market value=book value)                                 |
| Div                 | Dividends  |
| Div <sub>pot</sub>  | Undistributed potential dividends                              |
| E                   | Equity market value  |
| EBV                 | Equity book value  |
| $\tilde{F}, F$      | Cum-dividend equity market value (random and expected)         |
| FCF                 | Free Cash Flow   |
| I                   | Interest on debt   |
| Inv                 | Inventories  |
| $k_e$               | Cost of equity   |
| LA                  | Liquid assets  |
| NFA                 | Net Fixed Assets (fixed assets minus accumulated depreciation) |
| NI                  | Net income   |
| PV                  | Present Value  |
| STI                 | Short-term investment  |
| TS                  | Tax Shield (Tax Savings)                                       |
| V                   | Market value of the firm                                       |
| V <sub>TS</sub>     | Market value of the tax shield                                 |
| V <sub>U</sub>      | Market value of the unlevered firm                             |
| WACC                | Weighted Average Cost of Capital                               |
| WACC <sup>CCF</sup> | Pre-tax Weighted Average Cost of Capital                       |
| WC                  | Working capital  |
| WC <sup>nc</sup>    | noncash Working Capital  |
| $\beta_e$           | Beta of equity   |
| $r_f, r_m$          | Risk-free rate, expected market rate of return                 |
| $\sigma_m^2$        | Variance of market rate of return                              |

---