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# **Decomposition of a Certain Cash Flow Stream: Differential Systemic Value and Net Final Value**

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May 2000

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MPRA Paper No. 7308, posted 22 Feb 2008 18:14 UTC

# Decomposition of a Certain Cash Flow Stream: Differential Systemic Value and Net Final Value

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published in: *Proceedings of the XXIV annual AMASES Conference*, Association for Mathematics Applied to Economics and Social Sciences, Padenghe, Italy, 6-9 September 2000.

**Abstract.** This paper proposes a new way of decomposing net present values and net final values in periodic shares. Such a decomposition generates a new notion of residual income, radically different from the classical one available in the financial and accounting literature. While the standard residual income is formally computed as profit minus cost of capital times actual capital invested, the new paradigm introduces a fourth element: the capital invested in the so-called *shadow project*. Such a capital is the counterfactual capital that the investor would own if, at time 0, he invested his funds at the cost of capital, rather than in the project. Two important features are found: in primis, the new residual income is obtained as the sum of the standard residual incomes and the interest earned on past standard residual incomes; in secundis, the new paradigm is shown to be additive: the net final value of the project is computed as the sum of all periodic shares (residual incomes) with no capitalization process (abnormal earnings aggregation). A generalization is provided for a levered portfolio of projects, and a fourthfold decomposition is reached: (i) periodic decomposition, (ii) opportunity account decomposition, (iii) project decomposition, (iv) financing decomposition.

## Note to the reader:

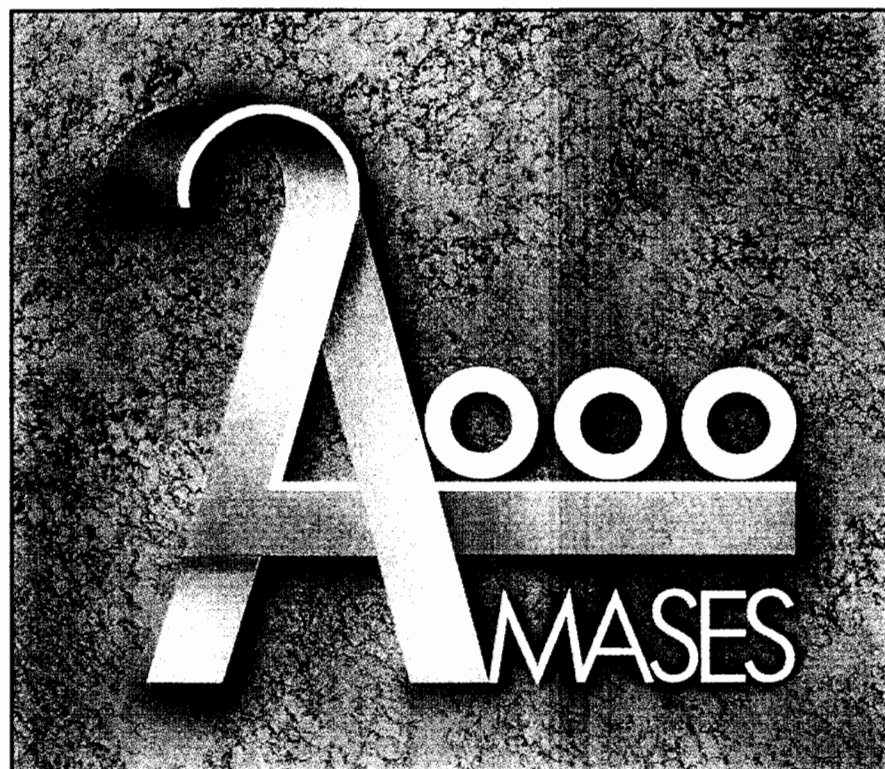
This paper, presented at the XXIV annual AMASES Conference in September 2000, presents a new notion of residual income, here called “Differential Systemic Value”. This notion has been extensively studied theoretically in later papers of mine, where it has been called “Systemic Value Added” and, more recently, “lost-capital paradigm”.

If you are interested in this topic and its implications for corporate finance, management accounting, cognitive psychology, visit my webpages at SSRN: <http://ssrn.com/author=812528> and IDEAS: <http://ideas.repec.org/f/pma506.html>.

Carlo Alberto Magni, February 21, 2008

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**ASSOCIAZIONE PER LA MATEMATICA APPLICATA  
ALLE SCIENZE ECONOMICHE E SOCIALI**



**ATTI  
DEL VENTIQUATTRESIMO  
CONVEGNO ANNUALE  
A.M.A.S.E.S.**

Padenghe sul Garda, 6-9 settembre 2000

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**DECOMPOSITION OF A CERTAIN CASH FLOW STREAM:  
DIFFERENTIAL SYSTEMIC VALUE AND NET FINAL VALUE  
EXTENDED ABSTRACT**

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1. This paper focuses on (nondeferrable) investments under certainty. The decision maker faces the opportunity of undertaking a project and she must decide whether accepting or rejecting it. A widely accepted evaluation index is the so-called Net Present Value (NPV), or Net Final Value (NFV) if compounded, which evaluates the differential profit between the two alternatives. A periodic decomposition of NPV (NFV) is proposed in Peccati ([2], [3], [4]). Hereafter we summarize it: Assume that a decision maker currently invests funds at a rate of interest  $i$  and that she faces the opportunity of a nondeferrable investment, say  $P$ : For the sake of simplicity we can assume that the project consists of an initial outlay  $-a_0$  at time 0, and equidistant cash flows  $a_s \in \mathbb{R}$  will be available at time  $s$ ,  $s=1, \dots, n$ . The evaluator's initial wealth is  $E_0$ , with  $0 < a_0 \leq E_0$ , and she aims at maximizing her terminal wealth at time  $T=n$ . We assume that she finances her investment with a loan contract, whose cash flows are  $f_0$  at time 0 and  $-f_s \leq 0$  at time  $s$ , with  $0 \leq f_0 \leq a_0$ . According to Peccati's model the Net Present Value's share for period  $s$  is given by

$$g_s = (w_{s-1}(x_s - i) + D_{s-1}(i - \delta_s))(1 + i)^{-s} \quad (1)$$

where

$$w_0 = a_0 \quad w_s = w_{s-1}(1 + x_s) - a_s \quad s = 1, 2, \dots, n.$$

represents the *outstanding capital (project balance)* at time  $s$  at the periodic rate of return  $x_s$ .

$$D_0 = f_0 \quad D_s = D_{s-1}(1 + \delta_s) - f_s \quad s = 1, 2, \dots, n$$

represents the *outstanding debt (debt balance)* at time  $s$  at the periodic rate of cost  $\delta_s$ .  $i$  is the (constant) opportunity cost of capital. Compounding (1) until  $n$  we find the Net Final Value's share for period  $s$ , which we denote by  $G_s$ .

2. Now we propose a different decomposition model based on an accounting-like perspective. The evaluator compares two lines of action: (i) undertaking the project; (ii) investing funds in an asset yielding the opportunity cost of capital  $i$ . Let us begin to construct prospective (cash) balance sheets for (ii). Let  $C$  be the asset yielding interest at the rate  $i$ . We call it the "opportunity" account.

As the decision maker invests her funds in  $C$  her net worth  $E^s$  at time  $s$  is given by the sum  $C^s$ , representing the value of “opportunity” account  $C$ :

$$\begin{array}{c|c} \underline{\text{Assets}} & \underline{\text{Equities}} \\ \hline C^s & E^s \end{array} \quad (2a)$$

for  $s = 0, 1, 2, \dots, n$ . For case (i), our investor/accountant will record two accounts in the debit side and two accounts in the credit side, expressing the fact that she holds an asset  $C$  (whose rate of return is  $i$ ), an asset  $P$  whose periodic rate of return is  $x_s$ , a loan contract  $D$  whose periodic rate (of cost) is  $\delta_s$  and her own net worth  $E$ : At time  $s$  we have

$$\begin{array}{c|c} \underline{\text{Assets}} & \underline{\text{Equities}} \\ \hline C_s & D_s \\ w_s & E_s \end{array} \quad (2b)$$

where  $C_s, w_s, D_s, E_s$  are the values of accounts  $C, w, D, E$  respectively. For (2a) and (2b), respectively, we state the following relations:

$$C^0 = E_0 \quad C^s = C^{s-1}(1+i) \quad (3a)$$

$$C_0 = E_0 - a_0 + f_0 \quad C_s = C_{s-1}(1+i) + a_s - f_s \quad (3b)$$

for  $s \geq 1$ . As for the income statement at time  $s, s \geq 1$ , (3a) leads to the net profit  $iC^{s-1}$ , whereas (3b) leads to the net profit  $x_s w_{s-1} + iC_{s-1} - \delta_s D_{s-1}$ . The sum

$$E_n - E^0 = \sum_{s=1}^n \left( x_s w_{s-1} - \delta_s D_{s-1} - i(C^{s-1} - C_{s-1}) \right) \quad (4)$$

shows the global profitability of  $P$ . We call (4) the Differential Systemic Value (DSV) as it is the difference between the two alternative financial systems. It is easy to see that the DSV coincides with the NFV, but the former's decomposition is different from Peccati's. In fact, the periodic share  $\overline{M}_s$  of  $P$ 's DSV is given by the difference between net earnings *sub* (i) and net earnings *sub* (ii), that is

$$\overline{M}_s := (E_s - E_{s-1}) - (E^s - E^{s-1}). \quad (5)$$

(5) is nothing but  $x_s w_{s-1} - \delta_s D_{s-1} - i(C^{s-1} - C_{s-1})$ . Letting  $M_s := G_s(1+i)^{n-s}$  it is easy to show that

$$\overline{M}_s = M_s + \sum_{k=1}^{s-1} i M_k (1+i)^{s-k-1}. \quad (6)$$

Summing for  $s$ , after simple algebraic manipulations, we obtain

$$DSV = \sum_{s=1}^n \overline{M}_s = \sum_{s=1}^n M_s(1+i)^{n-s} = NFV.$$

While coinciding in overall terms, the DSV model and the NFV model give rise to different partitions. The  $s$ -th share of the NFV is the compound amount of  $M_s$ , i.e.  $G_s$ , whereas the “accounting-flavored” partition provides us with  $\overline{M}_s$ , with  $\overline{M}_s \neq G_s$ . The DSV model is grounded on a systemic/accounting way of reasoning which makes no use of NFVs nor compounding processes, whereas Peccati’s model rests on financial arguments, in particular on the concept of Net Final (Present) Values and on capitalization processes. It is easy to see that  $\overline{M}_s$  can be rewritten as an  $s$ -th order difference equation

$$\overline{M}_s = M_s + i \left( \sum_{h=1}^{s-1} \overline{M}_h \right). \quad (7)$$

This reformulation enables us to interpret  $\overline{M}_s$  as the sum of a direct factor  $M_s$  (generated by the capital invested  $w_{s-1}$  and by the residual debt  $D_{s-1}$ ) and the periodic interest on the  $(s-1)$  indirect factors  $\overline{M}_h$ : the latter represent the gain generated in period  $s$  by those shares referring to the previous periods  $\overline{M}_1, \overline{M}_2, \dots, \overline{M}_{s-1}$ , which yield returns at the rate  $i$ . These returns are borne in the  $s$ -th period: Therefore, each share depends on all the preceding ones, which periodically bear interest at the rate  $i$ . Such an imputation collides with the NFV-based imputation. To see why, let us assume, for the sake of convenience,  $n=3$ , and let us decompose both DSV and NFV. We have the following decomposition table:

$$\begin{array}{lll} G_1 = M_1 + (iM_1) + (iM_1 + i^2M_1) & G_2 = M_2 + (iM_2) & G_3 = M_3 \\ \overline{M}_1 = M_1 & \overline{M}_2 = M_2 + (iM_1) & \overline{M}_3 = M_3 + (iM_1 + i^2M_1) \\ & & + (iM_2) \end{array} \quad (8)$$

where the first row decomposes the DSV, the second one decomposes the NFV. As we can see, the NFV decomposition accomplishes a two-step evaluation. The idea is the following:  $M_1, M_2, M_3$  are the three shares for period 1, 2, 3 respectively. As this is money referred to the dates 1, 2, 3, respectively, the basic principles of financial calculus force the evaluator to compound (or discount) flows to take time into consideration. After capitalization (and only after) the evaluator may sum the three shares. Conversely, in the light of our

systemic perspective the decision maker can construct, in a gradual way, the three shares of the DSV. The first share is  $M_1$ , which exactly represents the difference between what the investor receives in the first period and what she would receive should she decide to forego the project opportunity and invest her funds at the opportunity cost of capital  $i$ . In the second period the difference between what she receives and what she would have received must take into account that, in addition to  $M_2$ , the first share does not disappear, but yields interest equal to  $iM_1$ . This is natural, for if in the first period the investor gains, say, a differential 100\$, in order to calculate the second differential gain, we must necessarily consider that 100\$ produce, just in the second period (not in the first one) a differential return of  $100i$ . Iterating the argument, the third share must consider the return on the two first shares  $\overline{M}_1$  and  $\overline{M}_2$ , which is produced just in the third period, and is equal to  $100i$  and  $(100i+100i^2)$  respectively. You can see that this line of argument is not obeyed by the “financial-flavored” decomposition.  $G_1$  embodies the term  $iM_1$  which, as we have seen, is to be ascribed to the second share, since it is generated in the second period. In addition, it comprehends the term  $iM_1+i^2M_1$  which in turn is related to the third period. At the same time  $G_2$  includes  $iM_2$ , which is pertinent to the third period, but lacks the term  $iM_1$  (previously embodied in  $G_1$ ). Finally, the third share  $G_3$  forgets the return on previous periods’ shares.

3. Now we look at the implicit assumptions of the two decomposition models. For the sake of convenience, we will assume  $D_s=0$  for every  $s$ . Both the NFV and the DSV decomposition aim at answering the following question: “*What is the differential gain of (i) over (ii) that we are to ascribe to the s-th period?*”(\*). The answer to (\*) is the difference  $(E_s - E_{s-1}) - (E^s - E^{s-1})$ , which is reached under the DSV outlook by directly drawing up two sequences of (cash) balance sheets for alternative (i) and (ii) respectively (we call it the *systemic* argument). According to the NFV approach (the *financial* argument) the decision maker should evaluate a (fictitious) uniperiodic sub-project whose initial outlay is the project balance at time  $(s-1)$  and the receipt is given by the sum of  $a_s$  and the project balance at time  $s$ . Let  $\text{NFV}(s)$  denote this sub-project’s Net Final Value, calculated at time  $s$  (or, which is the same, its Net Present Value compounded until time  $s$ ): we have

$$\text{NFV}(s) = -w_{s-1}(1 + i) + w_s + a_s = w_{s-1}(x_s - i). \quad (9)$$

If this is to be the answer to (\*), then we must have

$$\text{NFV}(s) = (E_s - E_{s-1}) - (E^s - E^{s-1}). \quad (10)$$

But we know that a Net Final Value represents the difference between alternative final net worths. We have must therefore have  $\text{NFV}(s) = E_s - E^s$ . The



latter and (10) yield

$$E_{s-1} = E^{s-1} \quad (11)$$

for every  $s$ . This tells us that if project  $P$  is undertaken the net worth at time  $(s-1)$  (left-hand side) coincides with the net worth produced if the project is not undertaken (right-hand side). We distinguish two exhaustive cases: (a)  $x_s \neq i$  for at least one  $s$ ; (b)  $x_s = i$  for all  $s$ . If (a) holds, we have two kinds of contradiction: the *mathematical* and the *factual* contradiction. As for the *mathematical* contradiction, let  $s^*$  be an index such that  $x_{s^*} \neq i$ . As (10) and (11) must hold,  $\text{NFV}(s^*)$  is zero. This implies  $x_{s^*} = i$ , due to (9), but this contradicts the assumption. As for the *factual* contradiction, it is due to a vitiated interpretation of facts. In fact, the *financial* argument accomplishes the decomposition by calculating the  $\text{NFV}(s)$  for period  $s$ , which presupposes that the following assumption is made: *at time 0 the investor invests her net worth  $E_0$  in asset  $C$  at the opportunity cost of capital until time  $(s-1)$ . At time  $(s-1)$  the sum  $w_{s-1}$  is withdrawn by account  $C$  and invested in a uniperiodic project with rate of return  $x_s$ . At time  $s$ , the investor holds the final amount  $w_s$  alongside the value of account  $C$ .* As the latter is assumed to hold for every period  $s=1, 2, \dots, n$ , then it boils down to a set of  $n$  incompatible assumptions. As for (b), it causes the decision process to be an idle issue, as alternative (i) coincides, from a *mathematical-financial* point of view, with alternative (ii): There is no difference, financially speaking, in investing at the opportunity cost of capital the whole net worth or only a part of it, if the remainder is invested in a project whose rate of return is the opportunity cost of capital itself. In this case the two arguments lead to the same obvious (and uninteresting) result. In this situation (i) and (ii) are different only under a *factual* perspective, for they, though financial equivalent, represent distinct courses of action. Further, the *factual* contradiction persists, as the aforementioned assumption holds regardless of (a) and (b). No such contradictions, mathematical or factual, arise in the *systemic* argument, which presupposes the following hypothesis: *at time 0 the investor invests the sum  $(C^0 - w_0)$  in asset  $C$  at the opportunity cost of capital and the sum  $w_0$  in project  $P$ .* While the *factual* contradiction cannot be repaired, we might ask if we are able to heal the mathematical contradiction inherent in the *financial* argument. In other terms, can the *systemic* perspective be incorporated in the *financial* argument so as to provide a correct partition of the  $\text{NFV}$  on the basis of the *financial* argument? Let  $\bar{w}_s$  be the value of  $w_s$  obtained by replacing each  $x_s$  with  $i$ :

$$\bar{w}_s := a_0(1+i)^s - a_1(1+i)^{s-1} - \dots - a_s \quad s = 1, 2, \dots, n.$$

Note that the following then hold:

$$\bar{w}_s = C^s - C_s \quad \text{and} \quad \bar{w}_s = \bar{w}_{s-1}(1+i) - a_s.$$

The *systemic*  $C_s$  can be then rewritten as

$$C_0 = C^0 - \bar{w}_0, \quad C_s = C_{s-1}(1+i) + a_s = (C^{s-1} - \bar{w}_{s-1})(1+i) + a_s, \quad s = 1, \dots, n \quad (12)$$

where  $\bar{w}_0 := a_0$ . Now let

$$\bar{a}_0 := a_0 \quad \text{and} \quad \bar{a}_s := x_s w_{s-1} - i \bar{w}_{s-1} + a_s \quad s = 1, \dots, n. \quad (13)$$

Suppose that the investor undertakes a project  $\bar{P}$  consisting of the cash flow stream  $(-\bar{a}_0, \bar{a}_1, \dots, \bar{a}_n)$ . It easy to demonstrate that we can correctly decompose  $P$  by applying Peccati's decomposition to the *shadow* project  $\bar{P}$  provided we avoid compounding the Net Final Valus so obtained. From (13) we obtain

$$\bar{w}_0 = \bar{a}_0, \quad \bar{w}_s = \bar{w}_{s-1}(1 + \bar{x}_s) - \bar{a}_s \quad s = 1, \dots, n \quad (14)$$

where  $\bar{x}_s := x_s w_{s-1} / \bar{w}_{s-1}$ . We can then interpret  $\bar{w}_s$  as the *project balance* of  $\bar{P}$  at the rate  $\bar{x}_s$ , and the  $\bar{a}_s$  as withdrawals from (if positive) or investments in (if negative) an account yielding interest at the periodic rate  $\bar{x}_s$ ,  $s=1, 2, \dots, n$ .

Let us decompose the *shadow* project  $\bar{P}$  by using the *financial* argument: the investor invests  $\bar{w}_{s-1}$  at the beginning of the  $s$ -th period and receives the sum  $\bar{w}_s + \bar{a}_s$  at the end of the period: At time  $s$  the value of the project is

$$\begin{aligned} \overline{\text{NFV}}(s) &= -\bar{w}_{s-1}(1+i) + \bar{w}_s + \bar{a}_s \\ &= -\bar{w}_{s-1}(1+i) + (\bar{w}_{s-1}(1 + \bar{x}_s) - \bar{a}_s) + \bar{a}_s \\ &= \bar{w}_{s-1}(\bar{x}_s - i). \end{aligned} \quad (15)$$

Denoting with  $\bar{E}_s$  and  $\bar{E}^s$  the net worths at time  $s$  for (i) and (ii) respectively, we have

$$\overline{\text{NFV}}(s) = E_s - E^s = [\text{for (11)}] = (E_s - E_{s-1}) - (E^s - E^{s-1}) \quad (16)$$

(16) tells us that  $\overline{\text{NFV}}(s)$  is the  $s$ -th share of the Net Final Value of  $P$  and, as we expect, it coincides with  $\bar{M}_s$ , previously found by following a *systemic* argument:

$$\overline{\text{NFV}}(s) = \bar{w}_{s-1}(\bar{x}_s - i) = x_s w_{s-1} - i (C^{s-1} - C_{s-1}) = \bar{M}_s.$$

Hence,

$$\text{DSV} = \sum_{s=1}^n \bar{w}_{s-1}(\bar{x}_s - i) = \sum_{s=1}^n \overline{\text{NFV}}(s) = \text{NFV} \quad (17)$$

and the two models are, to a certain extent, reconciled with no need of compounding.

4. We now generalize the systemic model by allowing a portfolio of  $q$  projects,  $p$  “opportunity” accounts and  $m$  loan contracts, with respective periodic rates  $i_s^j$ ,  $\delta_s^l$ ,  $x_s^r$ ,  $j = 1, \dots, p$ ,  $l = 1, \dots, m$ ,  $r = 1, \dots, q$ .<sup>1</sup> The latter generalization forces the evaluator to select one or more “opportunity” accounts  $K^j$  to be activated for withdrawals and reinvestment of the cash flows released by the projects  $w^r$  and the debts  $D^l$ . Referring to time  $s$ , denote with  $a_s^{rj}$  the quota of project  $r$ 's cash flow invested in (if positive) or withdrawn from (if negative) account  $K^j$ . Likewise, denote with  $f_s^{lj}$  the quota of debt  $l$ 's cash flow withdrawn from account  $K^j$ ,  $j=1,2,\dots,p$ . We must obviously have

$$\sum_{r=1}^q \sum_{j=1}^p a_s^{rj} = a_s \quad \text{and} \quad \sum_{l=1}^m \sum_{j=1}^p f_s^{lj} = f_s.$$

Let us give the following notation:

$$\begin{aligned} \bar{w}_0^{rj} &:= w_0^{rj} := a_0^{rj} & \bar{D}_0^{lj} &:= D_0^{lj} := f_0^{lj} \\ w_s^{rj} &:= w_{s-1}^{rj}(1 + x_s^r) - a_s^{rj} & \bar{w}_s^{rj} &:= \bar{w}_{s-1}^{rj}(1 + i_s^j) - a_s^{rj} \\ D_s^{lj} &:= D_{s-1}^{lj}(1 + \delta_s^l) - f_s^{lj} & \bar{D}_s^{lj} &:= \bar{D}_{s-1}^{lj}(1 + i_s^j) - f_s^{lj} \end{aligned}$$

The value of  $K^j$  is

$$K_s^j = \left( K_0^j \prod_{k=1}^{s-1} (1 + i_k) - \sum_{r=1}^q \bar{w}_{s-1}^{rj} + \sum_{l=1}^m \bar{D}_{s-1}^{lj} \right) (1 + i_s^j) + \sum_{r=1}^q a_s^{rj} - \sum_{l=1}^m f_s^{lj}. \quad (18)$$

Let us focus on a generic account  $K^j$ . We have for it a portfolio of  $q$  *shadow* projects with initial outlays  $\bar{w}_0^{rj}$ . It is easy to see that this portfolio's periodic Net Final Value is

$$\sum_{r=1}^q \bar{w}_{s-1}^{rj} (\bar{x}_s^r - i_s^j) - \sum_{l=1}^m \bar{D}_{s-1}^{lj} (\bar{\delta}_s^l - i_s^j) \quad (19)$$

<sup>1</sup>The financial system's structure is now articulated as

<u>Assets</u>	<u>Equities</u>
$K_s^1$	$D_s^1$
$K_s^2$	$D_s^2$
...	...
$K_s^p$	...
$w_s^1$	...
$w_s^2$	...
...	$D_s^m$
$w_s^q$	$E_s$ .

where  $\bar{x}_s^r = x_s w_{s-1}^r / \bar{w}_{s-1}^r$  and  $\bar{\delta}_s^r = \delta_s D_{s-1}^l / \bar{D}_{s-1}^l$ . (19) is therefore the share of the  $q$  projects' portfolio generated in period  $s$  by account  $j$ . We can rearrange (19) so as to decompose the share according to the source of funds used. Let  $\alpha_s^{rj} := \bar{w}_{s-1}^{rj} / (\sum_{r=1}^q \bar{w}_{s-1}^{rj})$ ; then  $\alpha_s^{rj} \bar{D}_{s-1}^{lj}$  is that part borrowed from creditor  $l$  financing the initial outlay  $\bar{w}_{s-1}^{rj}$ . Rearranging terms and manipulating we obtain

$$\sum_{r=1}^q \left[ \left( \sum_{l=1}^m \alpha_s^{rj} \bar{D}_{s-1}^{lj} (\bar{x}_s^r - \bar{\delta}_s^l) \right) + \left( (\bar{w}_{s-1}^{rj} - \sum_{l=1}^m \alpha_s^{rj} \bar{D}_{s-1}^{lj}) (\bar{x}_s^r - i_s^j) \right) \right] \quad (20)$$

Let  $A_s^{lrj} := \alpha_s^{rj} \bar{D}_{s-1}^{lj} (\bar{x}_s^r - \bar{\delta}_s^l)$ ,  $l=1, \dots, m$  and denote with  $A_s^{m+1,rj}$  the last addend in (20). Summing for  $j$  and  $s$  we obtain the portfolio's Net Final Value

$$\text{NFV} = \sum_{s=1}^n \sum_{j=1}^p \sum_{r=1}^q \sum_{l=1}^{m+1} A_s^{lrj} \quad (21)$$

$A_s^{lrj}$  is the quota of the portfolio's NFV to be ascribed to source  $l$ , to project  $r$ , to account  $j$ , to period  $s$ . The evaluation we have arrived to provides us with four types of decomposition: (I) periodic decomposition (the share of portfolio's NFV generated in period  $s$ ), obtained by summing  $A_s^{lrj}$  for all variables except  $s$ ; (II) opportunity account decomposition (the share of portfolio's NFV generated by the use of account  $K^j$ ), obtained by summing  $A_s^{lrj}$  for all variables except  $j$ ; (III) project decomposition (the share of portfolio's NFV generated by project  $r$ ), obtained by summing  $A_s^{lrj}$  for all variables except  $r$ ; (IV) financing decomposition (the share of portfolio's NFV generated by the use source  $l$ ), obtained by summing  $A_s^{lrj}$  for all variables except  $l$ . We further point out that the portfolio's NFV in (21) coincides, as we expect, with the portfolio's DSV we would have obtained by directly using the *systemic* argument, that is

$$\sum_{s=1}^n \left( \sum_{r=1}^q x_s^r w_{s-1}^r - \sum_{l=1}^m \delta_s^l D_{s-1}^l - \sum_{j=1}^p i_s^j (K_0^j \prod_{k=1}^{s-1} (1 + i_k) - K_{s-1}^j) \right)$$

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