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Abstract

We introduce Cournot competition and endogenous entry in an otherwise neoclassical macroeconomic framework. First, we develop a model with exogenous savings *à la* Solow describing the dynamic path of business creation. Then, we develop a model *à la* Ramsey describing the dynamic interaction of consumption and business creation. Our models are able to explain why markups vary countercylically and profits are procyclical. The analysis of permanent and temporary technology and preference shocks and of the second moments suggests that our model can outperform the Real Business Cycle framework in many dimensions.

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1 Introduction

The neoclassical macroeconomic theory and the theory of Real Business Cycles are based on dynamic models with perfect competition, constant returns to scale and capital accumulation. In these frameworks there is no space for an analysis of imperfect market structures, of the dynamic of markups and extra profits and of the entry process, all elements that are quite important in real world markets.

In this paper we consider a infinite horizon economy characterized by a representative market for a standard homogenous good and without capital. Rather than assuming perfect competition between the firms, we assume that they compete in quantities and we characterize the Cournot equilibrium as a function of the number of active firms in each period.

Building on Bilbiee *et al* (2007) the number of firms is endogenized with the introduction of sunk costs of production such that in every period entry occurs until it guarantees a non-negative net value of expected profits. Strategic interaction together with the endogenous market structure allow our flexible prices economy to deliver business cycles with features consistent with a large body of empirical evidence. This is so whether the model is hit by demand or supply shocks.

First of all it shows procyclical profits, which is an uncontroversial empirical fact as argued by Bilbiie et al (2007b). Also it displays countercyclical mark ups, a pattern forcefully emphasized by Portier (1995), Chatterjee and Cooper (1993) and Rotemberg and Woodford (2000). Further it accounts for procyclical firms' entry as documented by Chatterjee and Cooper (1993) for the U.S., Portier (1995) for France and more recently by Jaimovich (2007) who updates and extends the evidence relative to the U.S.

These facts are hardly matched within the neoclassical framework, or even in its extensions to monopolistic competition a là Blanchard and Kiyotaki (1987), which neglects strategic interactions and entry.

Our first step is to develop a model with exogenous savings *à la* Solow (1956) describing the dynamic path of business creation. This allows us to obtain a model in which it is not investment in the physical capital to generate the accumulation of capital input over time, as in the original Solow model, but it is entry of new firms to generate the creation of new productive business. Since entry strengthens competition reducing the markups and the individual profits, entry also induces a sort of decreasing marginal productivity of business creation, just like capital accumulation reduces the marginal productivity of capital in the Solow model. Therefore both models generate a gradual convergence toward a steady state, in the standard Solow case through a decreasing rate of the growth of the capital stock, in our case through a decreasing rate of business creation.

In the main part of the paper we develop a model *à la* Ramsey (1928) describing the dynamic interaction of consumption and business creation. We show that our model is able to explain why markups vary countercyclically and profits are procyclical, and is also able to emphasize a new propagation mechanism associated with imperfect competition. To see why, consider a temporary technology shock that increases productivity and demand. On impact, such a shock increases the profits of the existing firms, attracting further entry. The increase in the number of firms enhances competition between them and reduces the markups and the prices. This, in turn, temporarily pushes consumption and propagates the shock. Of course, Cournot competition introduces an extra effect due to the change in the markups which is completely absent in a model with perfectly competitive markets (or even in a standard model with monopolistic competition where markups are constant and entry is not taken in consideration). The analysis of permanent and temporary technology shocks and preference shocks, and the analysis of the second moment properties suggests that our model can outperform the Real Business Cycle framework (Kydland and Prescott, 1982; King and Rebelo, 2000) in many dimensions.

The same mechanism working under Cournot competition is of course active under other forms of imperfect competition as well. We analyze two basic alternatives: a conjectural variation model, which nests Cournot competition and higher levels of collusion as particular cases, and a model of Stackelberg competition with one leader and endogenous entry of followers.

A closer attention to endogenous entry and endogenous mark ups has recently characterized the macroeconomic literature on general equilibrium models for closed and open economies. Ghironi and Melitz (2005) provide a relevant application to trade and business cycle, Etro (2007b) introduces strategic interactions with endogenous entry in basic models of trade, growth and business cycles. Most important for our purposes, after early attempts to endogenize entry with fixed costs of production in each period (notably Chattejee and Cooper, 1993, and Devereux *et al.*, 1996),[1] the recent work of Bilbiie *et al.* (2007a,b) has provided an important contribution on endogenous entry in a stochastic dynamic general equilibrium model. This line of research does not take in consideration the strategic interactions between firms and the impact of entry on them, but it focuses on the traditional case of constant mark ups

[1] Cooper (1999) surveys this early literature. Chatterjee *et al.* (1993) endogenize entry as well, but their focus is on sunspots equilibria in an OLG model.

due to monopolistic competition.[2] Recently, Jaimovich and Floetotto (2008) have augmented the model of Deveraux *et al.* (1996) with mark ups depending on the number of firms. That model endogenizes entry through fixed costs of production in each period, so that profits are again zero at all times, and not procyclical as in our framework, and it focuses on different issues, nevertheless it complements our work suggesting a crucial role for market structures in explaining the business cycle.

The remainder of the article is organized as follows. Section 2 develops the model with exogenous savings and the model with endogenous savings. Section 3 calibrates the latter and simulates the reaction to permanent and temporary supply shocks. Section 4 concludes.

2 The Model

We develop a very simple general equilibrium growth model without capital and with imperfect competition in quantities in a representative market for homogenous goods.[3] In our favorite specification firms compete *a là* Cournot, however below we analyze alternative competitive frameworks.

Consider a representative sector characterized by N_t firms in each period t, all producing the same consumption good under imperfect competition. Each firm i produces $y_t(i)$ according to a linear production function:

$$y_t(i) = A_t L\left(i\right)$$

where A_t is exogenous TFP common to all firms, and $L(i)$ is the labor input. Given the nominal wage W_t, the constant marginal cost of production is W_t/A. Total expenditure in the sector is:

$$E_t = p_t C_t = p_t \sum_{j=1}^{N_t} y_t(j)$$

where p_t is the price of the homogenous good and C_t its consumption in period t.

Assuming that all firms take total expenditure as given in each period,[4] their perceived inverse demand function must be $p_t = E_t / \sum_{j=1}^{N_t} y_t(j)$.

[2] Bilbiie *et al.* (2007a,b) and Bergin and Corsetti (2005) have introduced the translog preferences (due to Feenstra, 2003) to derive an elasticity of substitution between products that depends on the number of firms. As long as entry increases the substitutability between the existing goods, this generates mark ups depending on the number of goods, but such an *ad hoc* explanation is unrelated to endogenous motivations on the supply side, which are the focus of our paper.

[3] See Etro and Colciago (2007) for a related analysis of competition in prices with product differentiation in a macroeconomic model of the business cycle.

[4] In Etro and Colciago (2007) we have studied a related model with multiple markets where sectorial expenditure is constant in equilibrium.

Accordingly, the nominal profits of firm i are:

$$\Pi_t(i) = \left(p_t - \frac{W_t}{A}\right) y_t(i) =$$
$$= \frac{y_t(i)E_t}{\sum_{j=1}^{N_t} y_t(j)} - \frac{W_t y_t(i)}{A_t}$$

Assume that in each period, the N_t firms compete in quantities, choosing their individual production $y_t(i)$ to maximize profits taking as given the production of all the other firms. The Cournot equilibrium generates the symmetric individual output:

$$y_t = \frac{(N_t - 1)E_t A_t}{W_t N_t^2}$$

Substituting in the inverse demand, one obtains the equilibrium price:

$$p_t = \frac{N_t}{N_t - 1}\left(\frac{W_t}{A_t}\right) \tag{1}$$

which is associated with the equilibrium mark up $\mu(N_t) = N_t/(N_t - 1)$. This equilibrium generates individual profits $\Pi_t(N_t) = E_t/N_t^2$ in nominal terms. Since the equilibrium price of the consumption good is p_t, it is convenient to express all the variables in units of consumption, that is in real terms (alternatively one can use the consumption good as the *numeraire*). Then, real profits $\pi_t(N_t) = \Pi_t(N_t)/p_t$ become:

$$\pi_t(N_t) = \frac{C_t}{N_t^2} \tag{2}$$

and the real wage $w_t = W_t/p_t$ can be derived from the equilibrium pricing relation as:

$$w_t = \frac{(N_t - 1)}{N_t} A_t \tag{3}$$

When the number of firms increases, the equilibrium prices goes down and the wage goes up, with the former approaching the marginal cost and the latter approaching the marginal productivity of labor only for $N_t \to \infty$. However, the number of firms in the market is constrained by the presence of fixed costs of entry that endogenously rule the entry of new firms in the goods market.

In every period N_t^e new firms enter in the market, and, following Bilbiie *et al.* (2007a), we assume that a fraction $\delta \in (0, 1)$ of the (old and new) firms exits the market for exogenous reasons. Therefore, the number of firms follows the equation of motion:

$$N_{t+1} = (1-\delta)(N_t + N_t^e) \tag{4}$$

The real value of a firm V_t is the present discounted value of its future expected profits, or in recursive form:

$$\begin{aligned} V_t = &\beta(1-\delta)E_t\left[\frac{V_{t+1}+\pi_{t+1}(N_{t+1})}{1+r_{t+1}}\right] = \\ = &\beta(1-\delta)E_t\left[(1+r_{t+1})^{-1}\left(V_{t+1}+\frac{C_{t+1}}{N_{t+1}^2}\right)\right] \end{aligned}$$

where r_{t+1} is the real interest rate. The number of firm is endogenous in the sense that in each period entry occurs until the real value of the representative firm equates the fixed cost of entry. We assume that this fixed cost is equal to η/A_t units of labor, where $\eta > 0$. Given the wage $w_t = A_t(N_t - 1)/N_t$, the endogenous entry condition amounts to:

$$V_t = \frac{\eta(N_t - 1)}{N_t} \tag{5}$$

The aggregate resource constraint (in real terms), which must be satisfied in each period, is:

$$\begin{aligned} Y_t = &C_t + S_t = N_t\pi_t(N_t) + w_tL_t \\ = &\frac{C_t}{N_t} + w_tL_t \end{aligned}$$

where S_t are savings and L_t is total labor supply and we used the equilibrium expression for profits. Using the market clearing condition that equates savings and investments $S_t = I_t = N_t^eF_t = N_t^eV_t$, the budget constraint provides the following expression for savings:

$$S_t = N_t^eV_t = w_tL_t - \frac{N_t - 1}{N_t}C_t$$

By Walras' law labor supply equates labor demand in each period. For simplicity, we will now assume that labor supply is exogenous and normalized to unity.

To close the model we need to introduce a consumption function. For expository purposes we will approach the issue in two ways. First we will follow the Solow approach and assume that the savings rate is constant in every period, then we will augment the model with a standard utility maximizing agent choosing their savings in each period. The first approach will be useful to introduce the growth mechanism of this economy, the second one to examine in further details the reaction of this economy to exogenous shocks.

2.1 A Solow Model with Cournot Competition

In this section we will introduce the endogenous market structure characterized in the previous section in a dynamic framework that resembles the neoclassical growth model of Solow (1956), where savings are a constant fraction of income (which in our framework includes profits as well). This allows us to obtain a dynamic model in which it is not investment in the physical capital to generate the accumulation of capital input over time, as in the Solow model, but it is entry of new firms to generate the creation of new productive business. Since entry strengthens competition, entry also induces a sort of decreasing marginal productivity of business creation, just like capital accumulation reduces the marginal productivity of capital in the Solow model. Therefore both models generate a gradual convergence toward a steady state, in the Solow case through a decreasing rate of the growth of the capital stock, in our case through a decreasing rate of business creation.

Following the standard approach of Solow (1956) we now assume that savings are a fraction $s \in (0,1)$ of income, $S_t = sY_t$. From the budget constraint, these relationships imply:

$$\begin{aligned} Y_t &= \frac{(1-s)Y_t}{N_t} + w_t = \\ &= \frac{(1-s)Y_t}{N_t} + \frac{(N_t - 1)}{N_t} A \end{aligned}$$

where we used the equilibrium expression for the wage. Solving for income we obtain:

$$Y_t = \frac{(N_t - 1)A}{N_t - 1 + s} \tag{6}$$

which is an increasing function of the number of firms. Applying the equality of savings $S_t = sY_t$ and investment $I_t = N_t^e V_t$, we can solve for the equilibrium number of new firms:

$$N_t^e = \frac{sN_t A}{(N_t - 1 + s)\,\eta}$$

where we also used the equation (5) to substitute for, endogenous, value of the firm. Plugging the above expression in the equation of motion for the number of firms we have:

$$N_{t+1} = N_t\,(1-\delta) + \frac{s(1-\delta)A}{\eta - \frac{\eta(1-s)}{N_t}} \tag{7}$$

It is immediate to verify that the right hand side is increasing and concave in N_t, and when N_t approaches infinity its slopes tends to

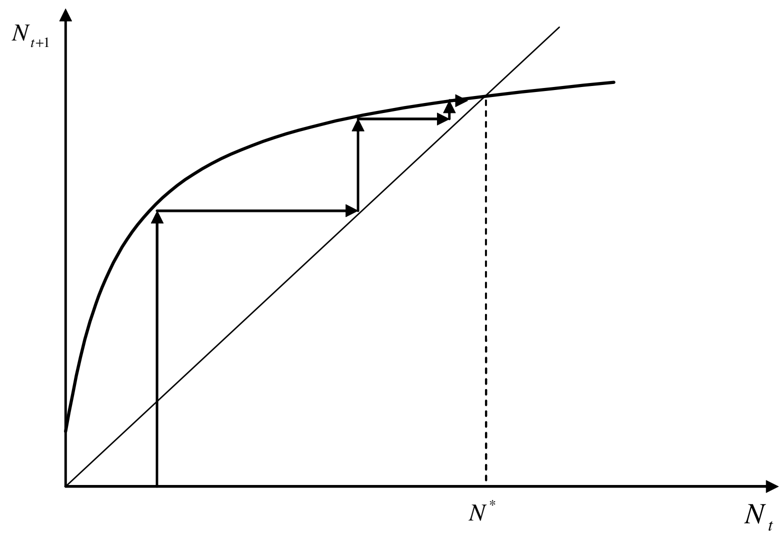


Figure 1: Dynamic creation of firms.

$(1-\delta) < 1$ (see Figure 1). This allows us to conclude that the dynamic of the economy is stable around its unique steady state. When the initial number of firms is low, savings contribute to create new firms, but new firms strengthen competition reducing the profits and the incentives to enter. The steady state number of firms is:[5]

$$N = 1 + s\left[\frac{A(1-\delta) - \delta\eta}{\delta\eta}\right] \tag{8}$$

which is increasing in the savings rate s and in the value of TFP A, and decreasing in the exit rate δ and in the relative size of the fixed costs η. The equilibrium endogenously generates imperfect competition between a positive but limited number of firms producing the homogenous good. Notice that dynamic inefficiency holds, since a better allocation of resources could be achieved reducing the number of firms and the waste in fixed costs of production of a homogenous good.

Of course, the dynamic path of output and consumption can be determined residually from that of the number of firms. When the latter increases toward its steady state value, output increases as well toward

[5] Notice that an increase in TFP has a positive impact on the steady state number of firms. This would disappear if the cost of entry parameter η was proportional to the size of the economy.

its steady state value. The latter is:

$$Y = A\left[1 - \frac{\delta\eta}{(1-\delta)\,A}\right] \tag{9}$$

Finally, notice that the model allows us to analyze the reaction of the economy to simple shocks. For instance, in case of an unexpected technological shock that moves the economy away from the steady state, the increase in profits attracts entry. Entry of new firms strengthens competition, which in turn depresses mark ups.

2.2 A Ramsey Model with Cournot Competition

Let us now move to a model with endogenous consumption.[6] Consider a representative agent with lifetime utility:

$$U = E_t \sum_{t=0}^{\infty} \beta^t \log C_t \tag{10}$$

where $\beta \in (0,1)$ is the discount factor and C_t is consumption of a homogenous good.

The representative agent maximizes lifetime utility by choosing how much to invest in risk free bonds and risky stocks out of labor and profit income. The flow budget constraint expressed in real terms is:

$$B_{t+1} + V_t(N_t + N_t^e)x_{t+1} + C_t = w_t + (1+r_t)B_t + \left[\pi_t(N_t) + V_t\right] N_t x_t \tag{11}$$

where B_t is net bond holdings and x_t is the share of the stock market value of the firms that are owned by the agent. The first order conditions with respect to x_{t+1} and B_{t+1} are respectively:

$$V_t(N_t + N_t^e)C_t^{-1} = \beta E_t\left\{\left[\pi_{t+1}\left(N_{t+1}\right) + V_{t+1}\right] N_{t+1} C_{t+1}^{-1}\right\} \tag{12}$$

$$C_t^{-1} = \beta(1 + r_{t+1})E_t C_{t+1}^{-1} \tag{13}$$

Equation (12) is an asset pricing equation for stocks and equation (13) is a standard Euler equation for bonds. Using the equation of motion for the number of firms (4), the equilibrium real wage (3), the endogenous entry condition (5) and the definition of profits (2), we can rewrite (12) as

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(\frac{\eta(N_{t+1}-1)}{N_{t+1}} + \frac{C_{t+1}}{N_{t+1}^2}\right)\right] = \frac{\eta}{\beta(1-\delta)}\frac{N_t - 1}{N_t} \tag{14}$$

[6] To preserve continuity we the model outlined in the previous section, we do not consider endogenous labor choices here. However we do it in Appendix C, where we also allow for the possibility of taste shocks. Notice that part of the business cycle analysis conducted below draws on the model described in the Appendix.

The flow budget constraint under the equilibrium conditions $B_t = 1 - x_t = 0$ for any t, leads to the aggregate resource constraint of the economy:

$$C_t + V_t N_t^e = w_t + \pi_t(N_t)N_t \tag{15}$$

which, as usual, states that the sum of consumption and investment must be equal to total income from labor and profits. After simple manipulations we obtain from the latter the number of new entrants in each period:

$$N_t^e = \frac{(A_t - C_t)}{\eta}$$

Substituting this into the equation of motion for the number of firms we get:

$$N_{t+1} = (1-\delta)\left(N_t + \frac{A_t - C_t}{\eta}\right) \tag{16}$$

Given an exogenous processes for A_t, the system composed by equations (14) and (16) fully characterizes the dynamic behavior of C_t and N_t. As shown in appendix A, the deterministic steady state of the economy is characterized by the following two equations:[7]

$$N^* = \frac{(1-\delta)(A - C^*)}{\eta\delta}$$

$$C^* = \frac{\eta}{(1-\delta)} \frac{(1-\beta(1-\delta))}{\beta} (N^* - 1) N^*$$

where a variable without time index denotes a steady state value. The first steady state relation is a negative relation between consumption and the number of firms, the second one is a positive and convex relation. It follows that the steady state is unique and characterized by a positive number of firms as depicted in Figure 2.

Finally, notice that, as in the Solow model with imperfect competition, also in the Ramsey version dynamic inefficiency holds (see Etro and Colciago, 2007, for a generalization of this result).

2.3 Other forms of Competition

Until now we have focused our analysis of the endogenous market structures on Cournot competition. One of the main aim of this paper is to emphasize the need of a deeper investigation of the industrial organization of the markets in macroeconomic models. Therefore, in this section

[7] As in the previous model, also in this case an increase in TFP has a positive impact on the number of firms in steady state. Again, this would disappear if the cost of entry parameter η was proportional to the size of the economy.

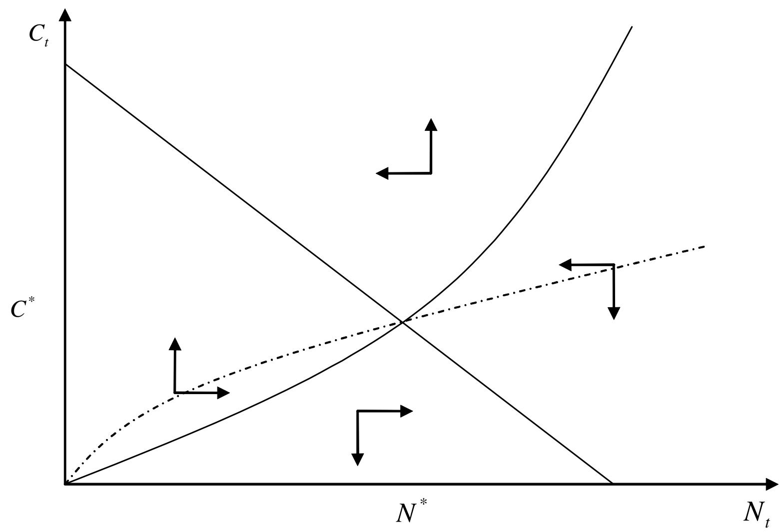


Figure 2: Steady state and saddle path stability in the Ramsey model.

we will import from the industrial organization literature[8] the analysis of other forms of competition. Further work in this direction, of course, is needed.

2.3.1 Conjectural Variations Approach

A simple extension of the Cournot model of competition can be obtained assuming general conjectural variations of the firms. Assuming that each firm takes as given the differential impact of its output choice on the output choice of the other firms $\lambda \equiv \partial x_t(j)/\partial x_t(i)$, the equilibrium price can be obtained as:

$$p_t(\lambda, N_t) = \frac{N_t}{(N_t - 1)(1 - \lambda)} \frac{W_t}{A_t}$$

which nests the case of Cournot competition in quantities for $\lambda = 0$ and tends to the (indeterminate) case of perfect collusion for $\lambda \to 1$. More importantly, intermediate situations with $\lambda \in (0, 1)$ describe cases of imperfect collusion between the firms which achieve mark ups above the Cournot level but below the perfect collusion level. Then, the real profits are:

$$\pi_t(\lambda, N_t) = \frac{[(N_t - 1)\lambda + 1] C_t}{N_t^2}$$

[8] See Etro (2007,a) on the industrial organization of endogenous market structures.

In the Ramsey model, the equilibrium system becomes:

$$N_{t+1} = (1-\delta)\left(N_t + \frac{A_t - C_t}{\eta}\right)$$

$$E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left[\frac{(N_{t+1}-1)}{N_{t+1}}\left(\frac{\eta(1-\lambda)N_{t+1}+\lambda C_{t+1}}{N_{t+1}}\right)+\frac{C_{t+1}}{N_{t+1}^2}\right]\right\}$$
$$= \frac{\eta(N_t-1)(1-\lambda)}{\beta(1-\delta)N_t}$$

which boils down to the previous one for $\lambda = 0$. Its dynamic properties will be examined in the next section.

2.3.2 Stackelberg Competition

Another relevant case emerges in the presence of asymmetries between firms, and is based on the theory of Stackelberg competition. Here we simply assume that the leader adopts the optimal static strategy in each period.

Let us assume that a single leader is always active in the representative sector and N_t followers are active in each period. The Stackelberg equilibrium can be derived with an equilibrium price:

$$p(N_t) = \frac{N_t}{(N_t - 1/2)}\frac{W_t}{A_t}$$

which is lower than under pure Cournot competition in quantities. The profits of the leader and the representative follower are respectively larger and smaller than the profits under Cournot competition:

$$\pi_t^L(N_t) = \frac{C_t}{4N_t} \qquad \pi_t^F(N_t) = \frac{C_t}{4N_t^2}$$

Since we assumed that entry and exit concerns only the followers, whose value is always pinned down by the endogenous entry condition:[9]

$$V_t^F = \frac{\eta(N_t-1)}{N_t}$$

the value of the leading firm must be calculated separately as the expected discounted value of its future profits:

$$V_t^L = \beta E_t\left[\frac{V_{t+1}^L + \pi_{t+1}^L(N_{t+1})}{1+r_{t+1}}\right] =$$
$$= \beta E_t\left[\frac{V_{t+1} + (C_{t+1})/4N_{t+1}}{1+r_{t+1}}\right]$$

[9] The assumption that also the leader cannot commit to a sequence of strategies is crucial here. If the leader could commit, it would engage in aggressive strategies aimed at reducing or deterring entry in the long run. See Etro (2008b) for a recent analysis of Stackelberg competition with commitment and endogenous entry.

In this case, the equilibrium system becomes:

$$N_{t+1} = (1-\delta)\left(N_t + \frac{A_t - C_t}{\eta}\right)$$

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(\eta\frac{(N_{t+1}-1/2)}{N_{t+1}} + \frac{C_{t+1}}{4N_{t+1}^2}\right)\right] = \frac{\eta}{\beta(1-\delta)}\frac{(N_t - 1/2)}{N_t}$$

whose dynamic properties will be analyzed in the next section.

3 Business Cycle Analysis

As customary, we solve the model by log-linearization around the deterministic steady.[10] In the remainder we assume that the technology shock evolve according to an autoregressive process of the form

$$\log(A_t/A) = 0.9\log(A_{t-1}/A) + \varepsilon_t$$

where ε_t is an independently and identically distributed random variable with variance σ_ε^2;

In the next subsections we analyze the dynamic properties of the Ramsey model with imperfect competition. We will consider the behavior of the model in response to both technology (temporary and permanent) and demand shocks. Calibrations of parameters is standard. The time unit is meant to be a quarter. The discount factor, β, is set to 0.99, while we follow Bilbiie et al (2007b) and set the rate of business destruction, δ, equals 0.025 implying an annual rate equal to 10 percent. Also the baseline parameterization features $\eta = A = 1$.

3.1 Permanent technology shocks

To shed light on the dynamics implied by the model we initially consider the response to a permanent technology shock. Figure 3 reports the response of the main macroeconomic variables to a one percent permanent increase in technology under Cournot competition.[11] On the vertical axis we report percentage deviations of variables from the initial steady state. Time on the horizontal axis is in quarters. The steady

[10] Steady state analysis is carried out in Appendix A. Appendix B provides, instead, log-linear equilibrium conditions and an analysis of local stability and uniqueness of the rational expectation equilibrium.

[11] The model has been solved using DYNARE a software devoloped by Michele Juillard. In the case of permanent shocks the impulse response functions are actually deterministic simulations. For a given path of the exogenous variable DYNARE provides the response of the whole system assuming that variables are initially at the steady state. This is done by stucking all the equations of the system for all periods and solving the resulting system en bloc using the Newton-Raphson algorithm.

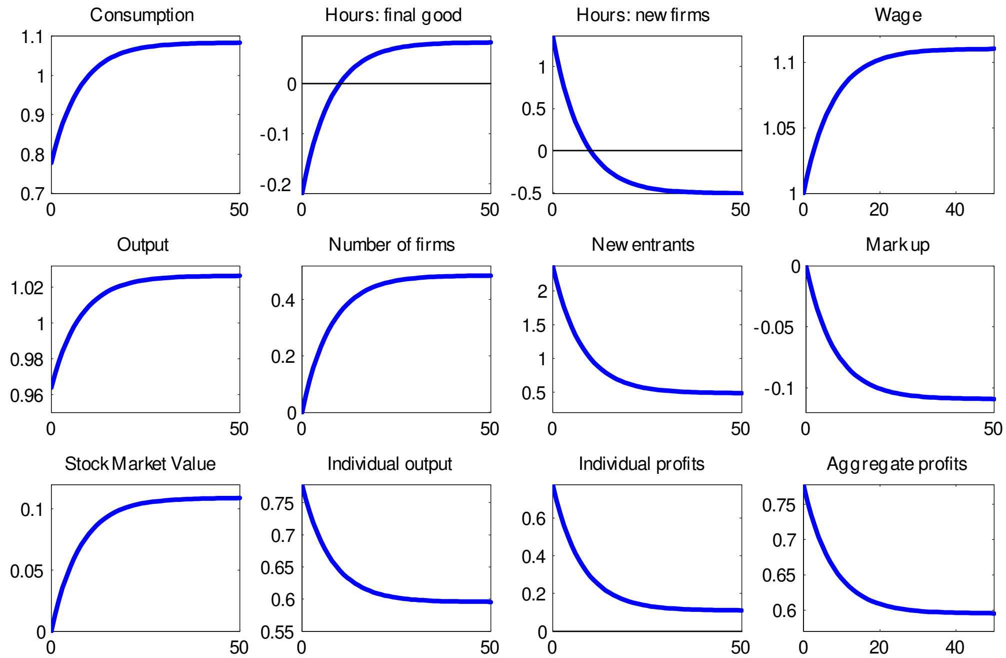


Figure 3: Transitional dynamics after a permanent technology shift under Cornot Competition

state mark up is endogenously determined and equals 22 % at the initial steady state.

Improved technological conditions create profits opportunities which, in turn, attract new firms into the market. This triggers stronger market competition and forces firms to decrease their mark ups. Despite lower markups, profits are above the initial level during the whole transition to the new steady state. This is so since the mark up reduction stimulates consumption demand which grants positive profits to producers. As the number of producers and the mark up converge to their new steady state levels the number of entrants settles down to a permanently higher value. Although a higher number of entrants requires a permanently higher level of investment on the side of the households the expansion in the production possibilities allows to attain permanently higher consumption.

At a micro-level the mechanism is as follows. Recall that the cost of entry equals the stock market value of the firm in each instant. Since the wage jumps on impact by the same amount of the technology shock, the entry cost, which is measured in unit of effective labor, is initially

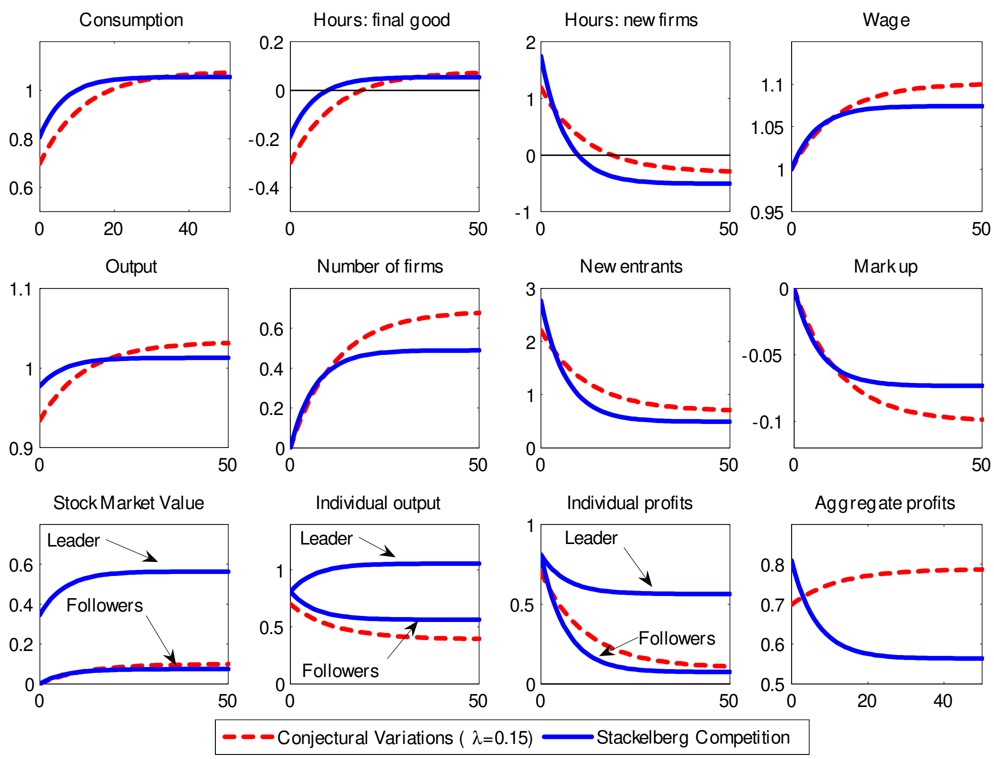


Figure 4: Transitional dynamics after a permanent technology shift under Stackelberg Competition and Conjectural Variations

unchanged. As the number of firms start increasing the mark up goes down leading the real wage to increase more than proportionally with respect to technology. This determines an increase in the cost of entry which is mirrored in the dynamic of the stock market value. The previous analysis justifies the reallocation of the fixed labor supply between the production of the final good and the creation of new firms. With an unchanged entry cost, there is a strong increase in the number of new entrants, which drives up the demand of hours for the creation of new firms. As the stock cost of entry reaches its new, higher, equilibrium value investment in new firms decreases and labor is reallocated to the production of the final good. Meanwhile, the number of firms has increased and the mark up has gone down.

Notice that the dynamic of the model are governed by the same intertemporal substitution mechanisms which characterize the standard RBC model, with investment in new firms playing the role of investment in capital goods. Nevertheless it adds to the latter framework a new *competition effect* which, through its consequences for firms' strategic pricing decision, is capable of addressing basic empirical facts such as countercyclicality of mark ups and procyclicality of aggregate profits.

The qualitative dynamics described in the case of Cournot Competition are preserved also under alternative market arrangements. Figure 4 provides transitional dynamics for the Conjectural Variation model (dashed lines), with $\lambda = 0.15$, together with those delivered by the Stackelberg competition model (solid lines) in the face of a permanent technology shift. The initial mark up in the conjectural variation case amounts to 34 % while it is 15 % under Stackelberg competition.

As shown in the figure, the Stackelberg competition framework displays an asymmetric response between the individual variables of the leader and those of the representative follower. Consider, as an example, the dynamic of the stock market value. While the market value of follower is pinned down on a period-by-period basis by the entry cost, that of the leader is determined by the discounted value of its futures profits. Since the latter unambiguously undertake a positive variation, the leader's market value jumps on impact.

The larger variations in aggregate profits under imperfect collusion promotes a stronger long run change in investment which translates into a higher variation in the number of producers. The competition effect then implies a relatively stronger decrease in the markup with respect to that observed under Stackelberg competition leading to a higher variation in the long run level of consumption and output. Importantly, both market structures deliver procyclicality of profits and mark up countercyclicality.

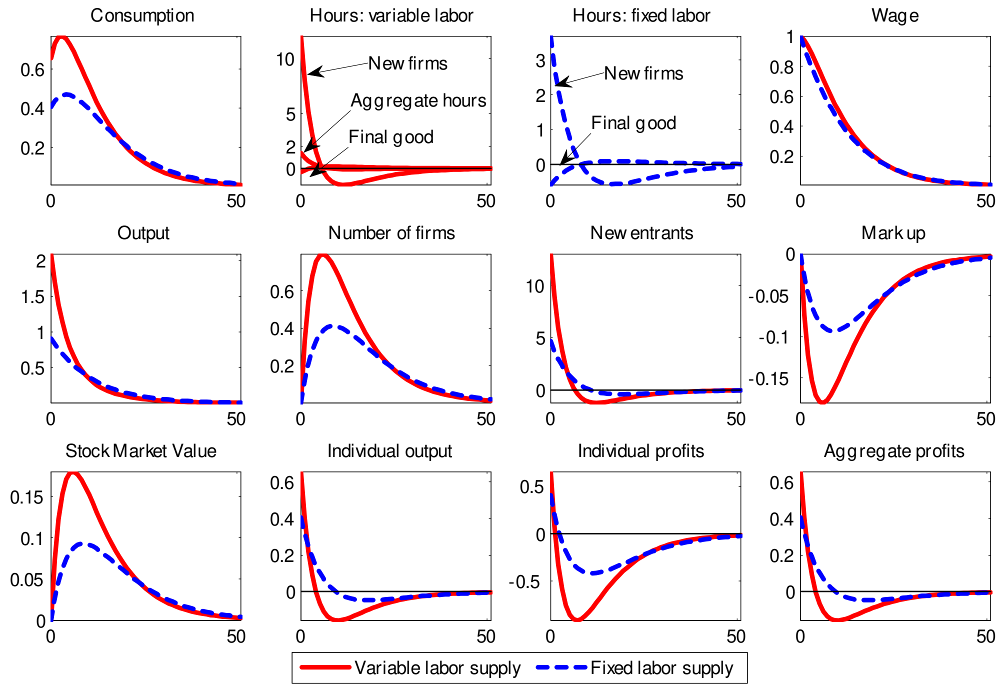


Figure 5: Impulse response to a temporary technology shock under Cournot competition

3.2 Temporary technology shocks

Let us now consider a temporary increase in technology. Since it is of interest to compare business cycles generated by our framework with those delivered by the prototype RBC model, we also report the response of variables in the case of endogenous labor supply.[12] Figure 5 depicts the impulse response of the economic system to a one percent temporary shock to A_t. Continuous lines refer to the case with fixed labor supply, while dashed line to that with endogenous labor.[13]

Similar mechanisms to those emphasized above are at work. The direction of movements of the variable is identical to that described in the case of a permanent shock, the main difference being the absence of a long run effect.

As in the standard RBC model the temporary technology shock in-

[12] Appendix C outlines the model with endogenous labor supply.

[13] Without loss of generality we set utility parameters such that steady state labor supply equals 1. In the latter case the Frish elasticity of labor supply reduces to φ to which we assign a value of 4 as in King and Rebelo (2000). Please see Appendix C for details.

creases consumption. However, differently from what happens in the afore mentioned framework, the impact of the shock on consumption is enhanced by deeper competition. Entry of new firms fosters competition and temporarily reduces the mark ups. This provides an extra boost to the intertemporal substitution of future for present consumption with respect to the case in which mark ups are constant or absent. Increased private demand of the final good fuels profits which, despite lower mark ups, stay above the baseline for several quarters.

The propagation of the shock is much stronger in the case of endogenous labor supply. The higher real wage boosts labor supply which translates into a substantial impact increase in output. This allows higher firms' entry and thus a stronger countercyclical mark up response.

To economize on space we do not report the dynamic response to a temporary shock for the Conjectural Variations and the Stackelberg competition models, however considerations made above extend to these cases.[14]

Figure 6 summarizes the sensitivity of our results to the value assumed by the entry cost parameter η in the baseline case of Cournot competition. To avoid excessive cluttering of the graphs we focus on the case with exogenous labor supply. Market concentration, defined as the inverse of the number of firms in the market, increases in the cost of entry.[15] This is mirrored in the value of markups which are relevantly different across the two specifications.

The mark up is 6 % when $\eta = 0.1$ (solid lines) and 34 % when $\eta = 2$ (dashed lines). While results are not qualitatively affected by the degree of market concentration, they are from a quantitative point of view. Deeper competition implies a stronger impact response of consumption and output, while dampens variability of stock market values. Profits increase more markedly on impact, but faster firms' entry brings them back to equilibrium, with a period of negative variation corresponding to the peak in the number of firms, after few quarters.

[14] Figures concerning temporary shocks for competitive frameworks other than Cournot are available from the authors.

[15] A common measure of market concentration is the Herfindhal Index, $HI = \sum_{i=1}^{N} s_i^2 \in (0,1)$, where s_i is the market share of firms i and N denotes the number of firms in the market. An HI above 0.18 is conventionally taken as an evidence of high market concentration. In a simmetric Cournot equilibrium, each firm holds the same market share. For this reson we take $1/N$ as a masure of market concentration.

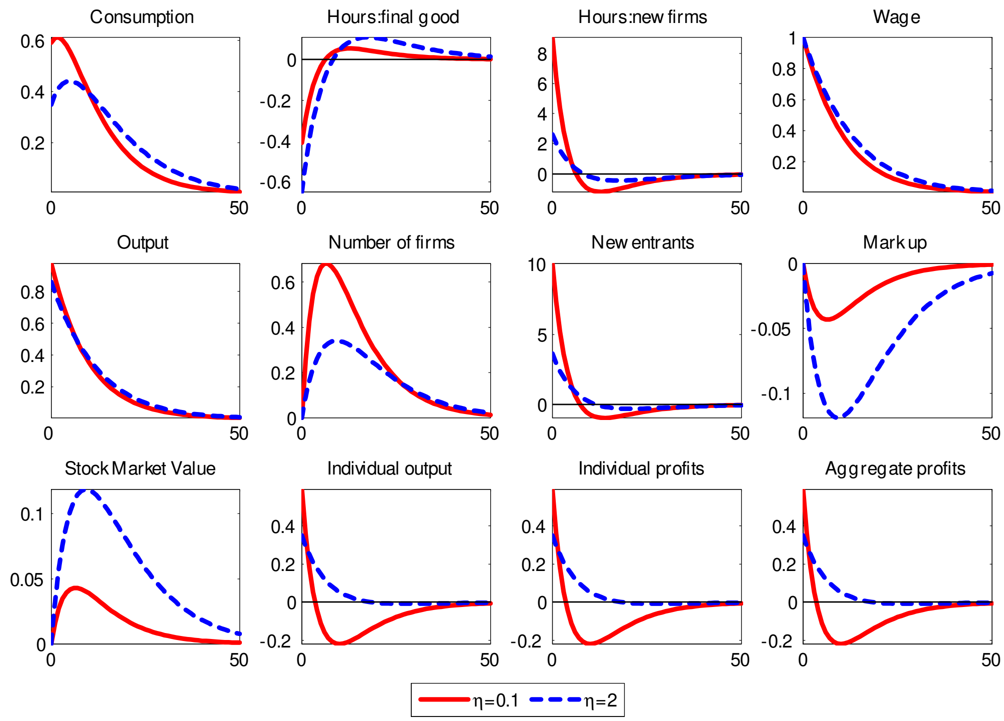


Figure 6: Impulse response to a temporary technology shock under Cournot competition. Sensitivity to η.

3.3 Temporary Preference Shock

In this section we consider the impact of a temporary change in demand determined by a preference shock.[16] We assume that the preference shifter Θ_t follows the first order autoregressive process

$$\log(\Theta_t/\Theta) = 0.9\log(\Theta_t/\Theta) + \varepsilon_t$$

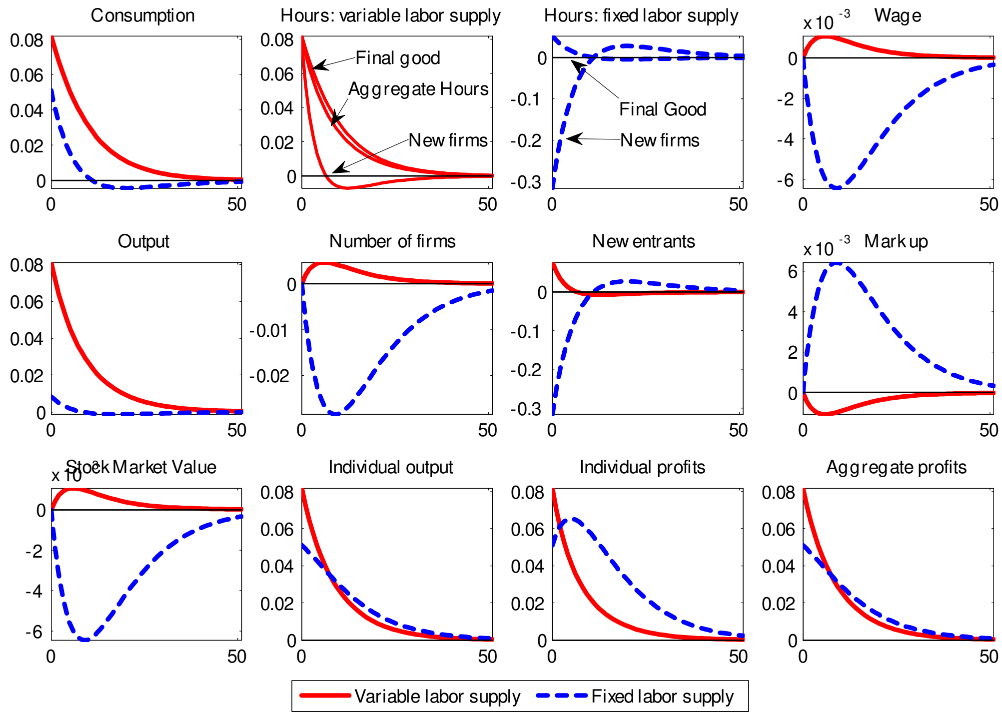
where Θ is a parameter and ε_t is an independently and identically distributed random variable. We follow Wen (2006) and assume that at the steady state $\frac{\Theta}{C} = 0.1$.[17] Figure 7 depicts the response of key variables to a one percent increase in Θ_t under Cournot competition. As in the previous section, continuous lines refer to the case with endogenous labor supply, while dashed line to that with fixed labor. As emphasized by Baxter and King (1992) and more recently by Wen (2006), taste shocks in standard general equilibrium models generates countercyclical investment dynamics due to the crowding out effect.[18]

This is not the case in our framework, as long as we allow for endogenous labor supply. In that case the model replicates the positive comovement between output, consumption and investment which characterize a typical business cycle. Notice also that the comovement between output, aggregate profits and the mark up has the "right" sign. Our intuition for this result goes as follows. When labor supply is fixed an increase in investment in the aftermath of the shock would require a shift of the fixed quantity of labor from the production of the final good to the making of new firms. In this case, however, the household would not be able to satisfy her desire to consume. Thus, the increase in demand can be satisfied uniquely through disinvestment. This generates a perverse cycle which by reducing the number of new entrants and the overall number of firms ultimately leads to a higher mark up. On the contrary when labor supply is endogenous agents have and additional channel through which they can react to the shock. By increasing total hours worked they can set up new firms without decreasing the produc-

[16] Appendix C introduces preference shocks into the model. In a nutshell, the felicity from consumption reads now as $log(C_t - \Theta_t)$, thus a temporary positive variation in Θ_t leads to an urgency to consume by temporarily increasing marginal utility of consumption.

[17] Empirical evidence cannot say much about the steady state ratio $\frac{\Theta}{C}$. For this reason we follow the literature and give it a, conservative, small value. As the value of the ratio increases the impact response of variables, as well as their variability, in the face of a demand shock are amplified.

[18] Investment increases on impact in the baseline RBC model if one assumes that the taste shock has a near randow walk behavior. Clearly, however, this *ad hoc* solution does not settle the issue.



tion of the final good. The increase in hours allows a contemporaneous positive impact variation of investment and consumption.[19]

Clearly the volatility of variables in response to the shock is low. However our intent is that of proving the capability of the model to generate a standard business cycle in the face of a demand shock and not that of explaining business cycle variability resorting solely to demand shocks.[20]

Before concluding this section we wish to remark that procyclicality of aggregate profits and mark ups countercyclicality are preserved in the face of both demand and supply shocks, and under all the competitive frameworks we have considered. In our view this is a strong argument in favor of a competition-based explanation of these evidence we want to address and, as a consequence, of the relevance of market structure endogeneity for business cycle modelling.

[19]Similar results apply in the cases of Stackelberg competition and Conjectural Variations. Results are available from the authors.

[20]Notice also that the initial shock has a small magnitude in terms of steady state consumption. It amounts to 1 percent of the steady state value of Θ, which in turn represents 10 percent of steady state consumption.

3.4 A comparison with standard RBC

To further assess the implications of endogenous market structures for the business cycle, we compute second moments of the key macroeconomic variables. In this exercise we follow the RBC literature by focusing on the case with endogenous labor supply and assuming that the only source of random fluctuations are technology shocks. Moreover, to compare the performance of our model to that of the mainstream RBC model, we take the calibration of the parameters characterizing the technology process from King and Rebelo (2000). Namely we set the shock persistence to $\rho_A = 0.979$ and its standard deviation to $\sigma_A = 0.0072$.[21]

$Variables$	$\sigma(X)$	$\sigma(X)/\sigma(Y)$	$E(X_t, X_{t-1})$	Corr(X,Y)
Y	$1.66, 1.39$	1	$0.84, 0.72$	1
C	$1.24, 0.60$	$0.75, 0.43$	$0.80, 0.78$	$0.76, 0.94$
I	$5.00, 4.09$	$3.01, 2.59$	$0.88, 0.70$	$0.79, 0.98$
L	$1.82, 0.67$	$1.10, 0.48$	$0.90, 0.70$	$0.88, 0.97$
Π	$8.08, n.a$	$4.87, n.a$	$0.76, n.a$	$0.67, n.a$
μ	$1.87, n.a$	$1.13, n.a$	$0.85, n.a$	$-0.27, n.a$

Table 1: Second moments. Left: US data. Right: RBC model

For future reference we report in Table 1 the performance of the standard RBC model, under the specified calibration, with respect to the statistics on US data (1947–1/2007-3) for output Y, consumption C, investment I, labor force L, aggregate profits Π and the mark up μ.[22] As well known, the main problems of the RBC model are the limited variability of output and especially consumption and hours, and the lack of explanations for the cyclical movement of profits and mark ups.

[21] We use the same process as in King and Rebelo (2000) for comparison purposes with the RBC literature and with Bilbiie et al. (2007a,b). Notice that Jaimovich and Floetotto (2008) have prepared a measure of the TFP based on U.S. data taking into account the mark up variability and then fitting an AR (1) process to the constructed series. While they do not find significant differences from King and Rebelo (2000) in the estimated autoregressive coefficient, they estimate a lower standard deviation of TFP innovations. Although our results are marginally affected by a change in σ_A, the main message of the analysis is not altered.

[22] Variables have been logged. We report theoretical moments of HP filtered variables with a smoothing parameter equal to 1600. Profits include both the remunaration of capital and the extra-profits due to market power: while we could not distinguish between the two, future research may try to do it. Please refer to Etro and Colciago (2007) for a full description of the data set.

Table 2 reports second moments of Y, C, $I \equiv N^e V$, L, Π, and mark up μ for our model with Cournot competition. The figures suggest that this competitive frameworks can reach a more realistically high volatility of consumption than the RBC. This is obtained at the expenditure of volatility of aggregate output and labor supply. However, these are still in line with those resulting form the standard RBC. More importantly, the model performs quite well in matching the contemporaneous correlation with output of mark ups and profits, on which the neoclassical model is completely silent.

$Variables$	$\sigma(X)$	$\sigma(X)/\sigma(Y)$	$E(X_t, X_{t-1})$	$Corr(X,Y)$
Y	1.36	1	0.67	1
C	0.87	0.64	0.78	0.94
I	5.86	4.31	0.63	0.92
L	0.57	0.42	0.60	0.93
Π	0.70	0.51	0.63	0.99
μ	0.09	0.07	0.93	-0.29

Table 2: Second moments under Cournot Competition

Table 3 accounts for the statistics delivered by the model with Stackelberg Competition (left) and for the Conjectural Variations approach when $\lambda = 0.15$ (right). Stackelberg competition, while displaying a poor performance, with respect to the RBC, in terms of output volatility delivers a remarkably high (relative and absolute) variability of consumption, suggesting a considerable propagation mechanism of the shock. The Conjectural Variations approach reproduces a volatility of variables closer to that found in the data. We do not further emphasize the good performance of this model because of its somehow *ad hoc* nature. However, we interpreter it as suggesting that competitive frameworks involving a degree of collusion stronger than that implied by the Cournot model could help improving the performance of dynamic general equilibrium models at replicating the features of the business cycle. Both table 2 and 3 show that variables' autocorrelation is lower with respect to that which shows up in the data. This could be accommodated, as suggested by Bilbiie *et al.* (2007a,b), by imposing a longer time to build up a new firm.[23]

[23] However, this would come together with the cost of imposing more exogenous features on the neat framework presented.

$Variables$	$\sigma(X)$	$\sigma(X)/\sigma(Y)$	$E(X_t, X_{t-1})$	$Corr(X, Y)$
Y	$1.29, 1.51$	1	$0.66, 0.68$	1
C	$0.88, 0.78$	$0.68, 0.51$	$0.77, 0.78$	$0.95, 0.93$
I	$6.72, 5.91$	$5.21, 3.91$	$0.57, 0.65$	$0.90, 0.97$
L	$0.47, 0.84$	$0.36, 0.56$	$0.57, 0.65$	$0.90, 0.96$
Π	$0.71, 0.71$	$0.55, 0.46$	$0.60, 0.71$	$0.98, 0.99$
μ	$0.07, 0.07,$	$0.05, 0.05$	$0.93, 0.94$	$-0.32, -0.17$

Table 3: Second moments. Left: Stackelberg Competition; Right: Conjectural Variations with lambda=0.15

Overall we regard the performance of the plain vanilla Cournot model with homogeneous good as an improvement with respect to that of the standard flexible prices model of the business cycle. It delivers moment of output, hours and investment which are in line with those of the RBC model and it provides a better approximation of the variability of consumption in front of real shocks. The little analytical complication involved in the supply side of the model is rewarded by its ability of partially reproducing the cyclical properties of markups and aggregate profits on which the neoclassical model is silent.

4 Conclusions

In this article we have studied a dynamic model with flexible prices and homogenous goods where the structures of the markets is endogenous and accounts for strategic interactions of different kinds. We considered both a Solow-type economy with exogenous saving and a Ramsey-type economy with maximizing consumers.

Our approach belongs to the emerging literature on endogenous entry in the macroeconomy (Bilbiie *et al.*, 2007a,b; Lewis, 2007; Etro and Colciago, 2007) and, as others, it provides some improvements in the explanation of the business cycle compared to the standard real flexible prices framework.

In particular our Ramsey model with Cournot competition reproduces standard business cycles in the face of both demand and supply shocks. When tested against a temporary technology shock it delivers second moments of macroeconomic variables which are in line with those provided by the standard RBC model. Further, it adds to the latter framework an endogenous characterization of the market structure which allows to explain the procyclical variability of profits together with the countercyclical variability of mark ups found in the data.

The model could be easily extended in various directions. For example in Etro and Colciago (2007) we account for a multisector economy with imperfectly sustitutable goods. Also it could be employed for the analysis of open economy issues to study international business cycle as in Ghironi and Melitz (2005) and optimal policy coordination in an open economy context

Finally we wish to remark that the approach taken in this paper constitutes a very simple and neat framework for the introduction of market structure endogeneity and strategic interactions in a business cycle model and, as such, could be a valuable teaching tool.

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Appendix A: steady state

This appendix shows that the model with endogenous savings displays, for a given number of firms, constant steady state output shares of consumption, investment, profits and labor income. We provide details relative to the Cournot case, but identical steps apply to the other competitive frameworks we have considered in the text.

The steady state counterpart of the aggregate resource constraint is $Y = C+I$. Since $I = N^e V$ the latter can equivalently be written as $1 = \frac{C}{Y} + \frac{N^e V}{Y}$. Evaluating at the steady state the Euler equation for bonds delivers $\beta = \frac{1}{1+r}$, where r is the steady state interest rate. The steady state counterpart of the Euler equation for stock holdings yields the steady state value of a firm as:

$$V = \frac{1-\delta}{r+\delta}\pi$$

or, using steady state profits:

$$V = \left(\frac{1-\delta}{r+\delta}\right)\left(\frac{C}{N^2}\right)$$

The share of profits over consumption is $\frac{\pi N}{C} = \frac{1}{N}$ while the share of investment over aggregate consumption reads as:

$$VN^e = \frac{1-\delta}{r+\delta}\pi N^e = \frac{\delta}{r+\delta}\pi N$$

where we considered that the number of new entrants at the steady state is $\frac{\delta}{1-\delta}N$. Using previous results, the ratio of investment over total consumption reads as:

$$\frac{N^e V}{C} = \frac{\delta}{r+\delta}\frac{\pi N}{C} = \frac{\delta}{r+\delta}\frac{1}{N}$$

To compute the output share of aggregate consumption notice that:

$$1 = \frac{C}{Y} + \frac{N^e V}{C}\frac{C}{Y} = \frac{C}{Y}\left(1 + \frac{\delta}{r+\delta}\frac{1}{N}\right)$$

thus the share of private consumption over output is:

$$\frac{C}{Y} = \left[1 + \frac{\delta}{r+\delta}\frac{1}{N}\right]^{-1}$$

The output share of investment is instead:

$$\frac{I}{Y} = \frac{N^e V}{C}\frac{C}{Y} = \frac{\delta}{r+\delta}\frac{1}{N}\frac{C}{Y} = \frac{\delta}{(r+\delta)N + \delta}$$

while the share of profits over aggregate output is:

$$\frac{N\pi}{Y} = \frac{r+\delta}{1-\delta}\frac{1-\delta}{\delta}\frac{N^e V}{Y} = \frac{r+\delta}{\delta}\frac{N^e V}{Y} = \frac{r+\delta}{(r+\delta)N + \delta}$$

Finally notice that the output share of labor income, $\frac{wL}{Y}$, depends on the steady state number of firms as follows:

$$\frac{wL}{Y} = 1 - \frac{r+\delta}{(r+\delta)N + \delta}$$

To fully characterize the steady state it remains to determine the number of producing firms. This can be obtained by substituting equation (14) into equation (16), and evaluating the resulting expression at the steady state. The steady state number of firms is the value of N which solves the following equation:

$$N = \left[\frac{\beta}{\delta\beta + (N-1)(1-\beta(1-\delta))}\right]\frac{(1-\delta)}{\eta}A$$

which clearly has a unique solution.

Appendix B: local stability and uniqueness

This appendix studies local uniqueness and non-explosiveness of the rational expectation equilibrium of the model with endogenous savings and Cournot competition, whose steady state was provided in Appendix A. Given our purpose we can restrict our analysis to a deterministic setting . In this case log-linear approximations around the steady state of equations (16) and (14) read respectively as

$$n_{t+1} = (1-\delta)n_t - (\delta + r)(N-1)\, c_t \tag{17}$$

and

$$E_t c_{t+1} = \frac{(1-\delta) - 2(\delta + r)(N-1)}{(N-1)(1-\delta)} n_{t+1} - \frac{(1+r)}{(N-1)(1-\delta)} n_t + \frac{(1+r)}{(1-\delta)} c_t \tag{18}$$

Rearranging yields a system of two equations into two variables with the following matrix form

$$\begin{bmatrix} E_t c_{t+1} \\ n_{t+1} \end{bmatrix} = B \begin{bmatrix} c_t \\ n_t \end{bmatrix}$$

where B is a 2×2 matrix of coefficients, c_t is a forward looking variable and n_{t+1} is a state variable. In general, the characteristic polynomial of matrix B reads as $H(\lambda) = \lambda^2 - Trace(B)\lambda + \det(B)$, and the condition for stability and uniqueness of the equilibrium is $H(1)H(-1) < 0$. Matrix B is characterized by the following coefficients:

$$B(1,1) = \tfrac{(1+r)}{(1-\delta)} - (\delta + r) + 2\tfrac{(\delta+r)(\delta+r)}{(1-\delta)}(N-1)$$

$$B(1,2) = -\tfrac{1}{N-1}\tfrac{(1+r)}{(1-\delta)} + \tfrac{(1-\delta)}{N-1} - 2(\delta + r)$$

$$B(2,1) = -(\delta + r)(N-1)\,; B(2,2) = (1-\delta)$$

It follows that:

$$trace(B) = \frac{(1+r)}{(1-\delta)} - (\delta + r) + 2\frac{(\delta + r)(\delta + r)}{(1-\delta)}(N-1) + (1-\delta)$$

and

$$det(B) = (1+r) - \frac{(1+r)(\delta + r)}{(1-\delta)}$$

In this case:

$$H(1) = \frac{(\delta + r)}{(1-\delta)}[r(1-2N) - 2N\delta] < 0$$

and

$$H(-1) = 4 + 2r - \frac{(\delta + r)}{(1-\delta)}[r(1-2N) - 2N\delta] \; > 0$$

since $N > 1$. Therefore the equilibrium is always unique and locally stable. The same result holds under the alternative competitive frameworks we have considered in the text.

Appendix C: the Ramsey model with endogenous labor supply and taste shocks

The Ramsey model can be easily extended to endogenous labor supply and taste shocks. In this case the representative agent has expected lifetime utility given by:

$$U = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(C_t - \Theta_t\right) - \upsilon \frac{L_t^{1+1/\varphi}}{1+1/\varphi} \right\} \quad \upsilon, \varphi \geq 0$$

where L_t represents hours worked and Θ_t is a random variable which permits us to analyze demand shocks. As described in Baxter and King (1991) and more recently in Wen (2006), a positive innovation to Θ_t by increasing the marginal utility of consumption leads to an urgency to consume. The first order conditions with respect to x_{t+1} and B_{t+1} read respectively as:

$$V_t(N_t+N_t^e)\left(C_t - \Theta_t\right)^{-1} = \beta E_t \left\{ \left[\pi_{t+1}\left(N_{t+1}\right) + V_{t+1}\right] N_{t+1} \left(C_{t+1} - \Theta_{t+1}\right)^{-1} \right\}$$

and

$$\left(C_t - \Theta_t\right)^{-1} = \beta(1 + r_{t+1}) E_t \left(C_{t+1} - \Theta_{t+1}\right)^{-1}$$

Notice that with respect to the analysis in the main text we also we have an extra first order condition with respect to L_t:

$$L_t = \left(\frac{1}{\left(C_t - \Theta_t\right)} \frac{w_t}{\upsilon} \right)^{\varphi}$$

Substituting this into the aggregate resource constraint delivers the number of new entrants as

$$N_t^e = \frac{1}{\eta} \left[A_t^{1+\varphi} \left(\frac{1}{\left(C_t - \Theta_t\right)} \frac{\left(N_t - 1\right)}{\upsilon N_t} \right)^{\varphi} - C_t \right]$$

The latter in conjunction with the dynamic equation describing the evolution of the number of firms delivers

$$N_{t+1} = (1-\delta) N_t + \frac{1-\delta}{\eta} \left[A_t^{1+\varphi} \left(\frac{1}{\left(C_t - \Theta_t\right)} \frac{\left(N_t - 1\right)}{\upsilon N_t} \right)^{\varphi} - C_t \right] \quad (19)$$

The equilibrium equation for the dynamic of consumption is now

$$E_t \left[\left(\frac{C_{t+1} - \Theta_{t+1}}{C_t - \Theta_t} \right)^{-1} \left(\frac{\eta\left(N_{t+1} - 1\right)}{N_{t+1}} + \frac{C_{t+1}}{N_{t+1}^2} \right) \right] = \frac{\eta}{\beta(1-\delta)} \frac{N_t - 1}{N_t} \quad (20)$$

Equations (19) and (20) together with the exogenous processes for A_t and Θ_t specified in the text determine the equilibrium path of consumption and the number of firms, from which we can obtain the dynamics of the other relevant variables. Assuming that the steady state ratio $\frac{\Theta}{C} = \phi$ (= 0.1 in our baseline calibration, as in Wen, 2006), steady state analysis is the same as before with labor supply satisfying.

$$L = \left(\frac{1}{(1-\phi)\,\upsilon} \frac{w}{C} \right)^{\frac{1}{1+\frac{1}{\varphi}}}$$

The steady state number of firms is the value of N that solves the following equation:

$$N = \frac{1-\delta}{\delta\eta} A^{1+\varphi} \frac{\left(\frac{\beta(1-\delta)}{(1-\phi)\upsilon\eta(1-\beta(1-\delta))N^2} \right)^{\varphi}}{\left[1 + \frac{(N-1)}{\delta} \left(\frac{1-\beta(1-\delta)}{\beta} \right) \right]}$$

which, given the right hand side is decreasing in N, has a unique solution.

Equation (19) takes the following log-linear form:

$$n_{t+1} = \Phi_n n_t - \Phi_c c_t + \Phi_a a_t + \Phi_\theta \theta_t$$

with the associated coefficients defined as $\Phi_n = \frac{(1-\delta)(N-1)+\varphi(\delta+\gamma_n)}{(N-1)}$, $\Phi_a = (1+\varphi)(\delta+\gamma_n)$, $\Phi_c = [\varphi\delta + (1+\varphi)\gamma_n]$ and $\Phi_\theta = \frac{\phi(\delta+\gamma_n)}{(1-\phi)}$ where $\gamma_n = (\delta + r)(N-1)$.

The log-linear version of equation (20) is:

$$E_t c_{t+1} = \digamma_{n+1} n_{t+1} - \digamma_n n_t + \digamma_c c_t + \digamma_\theta E_t (\theta_{t+1} - \theta_t)$$

with coefficients defined as $\digamma_{n+1} = \frac{\gamma_c - 2\gamma_n}{\gamma_c(N-1)-\phi\gamma_n}$, $\digamma_n = \frac{(1-\phi)(1+r)}{\gamma_c(N-1)-\phi\gamma_n}$, $\digamma_c = \frac{(1+r)}{\gamma_c - \phi(\delta+r)}$ and $\digamma_\theta = \frac{\phi(1+r)}{\gamma_c-\phi(\delta+r)}$ where $\gamma_c = (1-\phi)(1-\delta)$.