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Abstract

This paper suggests that the difference in the Theil indices of inequality between two economies approximately measures the relative loss of aggregate productivity caused by distortions in labor allocation. Moreover, the Theil index itself can be interpreted approximately as the possible maximum loss of aggregate productivity caused by these distortions.

Keywords: inequality; marginal productivity theory; misallocation; productivity; Theil index

JEL classifications: D24; D33; D61; E25

1 Introduction

The Theil index is a widely used measure of economic inequality. While the information-theoretic aspects of this index have been extensively analyzed (e.g., Cowell, 2003), there are very few analyses on its economic aspects. This paper analyzes an economic aspect of the Theil index and shows a connection between this index and productivity.

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There exists an age-old controversy surrounding the relation between inequality and productivity (see Samuelson, 1966; Sen and Foster, 1997). The marginal productivity theory states that inequality is an efficient outcome of competition and is a consequence of rewards for high productivity workers. One counterargument to this – which is also the focus of this paper – is the existence of distortions in labor allocation. For example, a monopoly right granted to industry insiders (e.g., Parente and Prescott, 2000; Rajan and Zingales, 2003) prevents competition and maintains a low number of workers in the protected industry. Unequal opportunities for education (Glomm and Ravikumar, 1992) can result in a shortage of skilled workers. These labor misallocations cause wage inequalities, and this is not an efficient outcome. Labor misallocations also cause the productivity losses in the economy.

This paper clarifies the relation between the Theil index and the loss in aggregate productivity caused by the distortions in labor allocation. My interpretation is that the difference in the Theil indices of inequality between two economies approximately measures the relative loss in aggregate productivity caused by these distortions. Moreover, the Theil index itself can be interpreted approximately as the possible maximum loss of aggregate productivity caused by these distortions.

2 Model and Result

The production function of the economy is given by

$$Y = F(K, n_1, \ldots, n_I),$$

where F is a well-behaved function, K is the aggregate capital (or any inputs, except for labor), and n_i is the number of type i workers. The type can be interpreted in terms of occupation (e.g., farmer, lawyer, etc.) or skill level (skilled and unskilled labor). The prices of capital and labor inputs are determined based on the marginal products:

$$F_K = r, \quad F_{n_i} = w_i,$$

where F_K and F_{n_i} are the marginal products and r and w_i are the prices of the inputs.

Let the total number of workers be N; then,

$$N = \sum_i n_i. \tag{1}$$

I define w and $\widehat{w}_i$ as follows:

$$w \equiv \frac{1}{N}\sum_i w_i n_i, \tag{2}$$

$$\widehat{w}_i \equiv \frac{w_i}{w}. \tag{3}$$

Using (1), (2), and (3), we can obtain the following identity:

$$\begin{aligned} n_i &= \frac{w_i n_i/\widehat{w}_i}{w\sum_j n_j} N \\ &= \frac{w_i n_i/\widehat{w}_i}{\sum_j w_j n_j/\widehat{w}_j} N \\ &= \frac{w_i n_i}{wN}\widehat{\lambda}_{n_i} N, \end{aligned} \tag{4}$$

where

$$\widehat{\lambda}_{n_i} \equiv \frac{\frac{1}{\widehat{w}_i}}{\sum_j \frac{w_j n_j}{wN}\frac{1}{\widehat{w}_j}}.$$

Under this setting, I compare the outputs under different states – Y^d and Y^n. (Hereafter, I add the superscripts d and n to the variables in order to denote the different states.)[1] Here, I assume that the production function is the

[1] d implies distortions and n implies no distortions.

same between the states.[2] By applying the mean value theorem, I obtain

$$\ln\left(\frac{Y^d}{Y^n}\right) = \frac{\partial \ln Y}{\partial \ln K} \ln\left(\frac{K^d}{K^n}\right) + \sum_i \frac{\partial \ln Y}{\partial \ln n_i} \ln\left(\frac{n_i^d}{n_i^n}\right)$$
$$\simeq \frac{rK}{Y} \ln\left(\frac{K^d}{K^n}\right) + \frac{wN}{Y} \ln\left(\frac{N^d}{N^n}\right) + \frac{wN}{Y} \sum_i \frac{w_i n_i}{wN} \ln\left(\frac{\widehat{\lambda}_{n_i}^d}{\widehat{\lambda}_{n_i}^n}\right). \tag{5}$$

In the last expression, I apply an approximation on $w_i n_i/(wN)$.[3] If $w_i n_i/(wN)$ is constant, then the expression is exact.[4]

I define the difference in aggregate productivity $d \ln \mathrm{TFP}$ between states d and n as follows:

$$\Delta \ln \mathrm{TFP} \equiv \ln\left(\frac{Y^d}{Y^n}\right) - \frac{rK}{Y} \ln\left(\frac{K^d}{K^n}\right) - \frac{wN}{Y} \ln\left(\frac{N^d}{N^n}\right).$$

This is a standard definition of aggregate productivity. Then, by rewriting (5), we obtain

$$\Delta \ln \mathrm{TFP} \simeq \frac{wN}{Y} \sum_i \frac{w_i n_i}{wN} \ln\left(\frac{\widehat{\lambda}_{n_i}^d}{\widehat{\lambda}_{n_i}^n}\right)$$
$$= \frac{wN}{Y}\left[\sum_i \frac{w_i n_i}{wN} \ln\left(\frac{w_i^n}{w^n}\right) - \sum_i \frac{w_i n_i}{wN} \ln\left(\frac{w_i^d}{w^d}\right)\right]. \tag{6}$$

[2] We can extend the analysis to consider the difference in technologies between the states. Then, the production function is given by

$$Y = F(K, n_1, \ldots, n_I, T),$$

where T indexes technology. In this case, the term on the difference in technology is added in (5) and (6). Otherwise, the result remains the same.

[3] Then, $\sum_i \frac{w_i n_i}{wN} \ln\left(\frac{w_i^d n_i^d}{w^d N^d} \Big/ \frac{w_i^n n_i^n}{w^n N^n}\right)$ becomes approximately zero because

$$\begin{aligned}\sum_i \sigma_i \ln\left(\frac{\sigma_i^d}{\sigma_i^n}\right) &= \sum_i \sigma_i \Delta \ln \sigma_i \\ &\simeq \sum_i \sigma_i \frac{\Delta \sigma_i}{\sigma_i} \\ &= 1 - 1 \\ &= 0,\end{aligned}$$

where $\sigma_i \equiv w_i n_i/(wN)$ and Δ denotes the difference.

[4] $w_i n_i/(wN)$ becomes constant if

$$F(K, n_1, \ldots, n_I) = G\left(K,\ \prod_i n_i^{\sigma_i}\right),$$

where G is a well-behaved function.

The last equation is obtained by substituting (4) into $\widehat{\lambda}_{n_i}$. The term in the last equation, which is in parentheses, is approximately equal to the difference between the two Theil indices.[5] This claim holds exactly (and (6) also holds without approximation) if $w_i n_i/(wN)$ is constant. (6) suggests that a higher Theil index under state d compared with state n is related to the lower aggregate productivity of state d.

We can interpret the result as follows. Distortions in labor allocation, such as monopoly right granted to industry insiders or unequal opportunities for education, increase wage differences. These wage differences increase the Theil index as well as the aggregate productivity loss.[6] (6) shows that under the interpretation, the difference between the two Theil indices approximately corresponds to the relative loss in aggregate productivity caused by these distortions. Moreover, since the minimum value of the Theil index is zero, the Theil index itself can be interpreted as the (approximate) possible maximum loss of aggregate productivity caused by these distortions.

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[5]The definition of the Theil index is given by

$$\text{Theil index} \equiv \frac{1}{N}\sum_{n}\frac{w_n}{w}\ln\left(\frac{w_n}{w}\right),$$

where n denotes the individual worker. In this model, since workers of the same type receive the same wage, we can rewrite this equation as follows:

$$\text{Theil index} = \frac{1}{N}\sum_{i}\frac{w_i n_i}{w}\ln\left(\frac{w_i}{w}\right),$$

where i denotes the worker type. This expression is approximately the same as that in (6) (and exactly the same if $w_i n_i/(wN)$ is constant).

[6]For the mechanism between distortions on allocation and aggregate productivity loss, see Restuccia and Rogerson (2007).

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