Productivity Differences in an Interdependent World

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Abstract

This paper studies cross country differences in productivity from an open economy perspective by using a Helpman-Krugman-Heckscher-Ohlin model. This allows to combine tools from development accounting and the trade literature. When simultaneously fitting data on income, factor prices and the factor content of trade, I find that rich countries have far higher productivities of human capital than poor ones, while differences in physical capital productivity are not sytematically related to income per worker. I estimate an aggregate elasticity of substitution between human and physical capital that is significantly below one, clearly rejecting a world that consists of a collection of Cobb-Douglas economies and also one where Heckscher-Ohlin trade is important.

Keywords: Heckscher-Ohlin, Productivity Differences, Development Accounting, Open Economy Growth.

JEL classification codes: F11, F43, O11, O41, O47.

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1 Introduction

Finding answers to the question why some countries are so much richer than others is one of the fundamental challenges in economics. While according to the consensus view cross country differences in factor endowments and differences in productivity are more or less equally important causes for the cross country variation in income per worker (Caselli (2005)), there is little evidence whether individual economies are actually well described by an aggregate Cobb-Douglas production function and whether differences in productivity across countries are really factor neutral as usually assumed in the quantitative growth literature.

Moreover, even though trade is empirically very important [1] and may also potentially affect the shape of countries' aggregate production possibility frontiers (Ventura (2005)) most research in growth and development still uses closed economy models when estimating cross country differences in productivity. This may not only be too restrictive for theoretical reasons but - since these models have by their very nature nothing to say about trade - it also leaves one of the best sources of cross country information - bilateral trade data - completely unexploited.

A second, independent line of investigation in international trade deals with the prediction of the Heckscher-Ohlin-Vanek (HOV) Theorem, that states that capital abundant countries should export capital (through trade in goods), while labor abundant countries should export labor. This research, which uses trade data to evaluate the validity of this theory, finds that cross country differences in productivity greatly help to explain factor flows embodied in trade.

The goal of this paper is to merge these two approaches by using a world equilibrium model - the Helpman-Krugman-Heckscher-Ohlin (1985) model - to estimate factor augmenting productivities, thereby providing a unified framework and exploiting the information contained both in income and in trade data. This model has been the workhorse of trade economists for more than two decades.[2] It combines inter-industry Heckscher-Ohlin trade with intra-industry trade due to increasing returns and love for variety. I augment the model for differences in the efficiencies with which factors are used across countries to introduce a potential role for productivity in generating cross country variation in income per worker.

The model encompasses two very popular views of the world as particular cases. The first one is the neoclassical one sector model with factor deepening that is the standard framework in the quantitative growth literature, while the second one is the Heckscher-Ohlin model with conditional factor price equalization, the canonical model for estimating productivities in the trade literature (Trefler (1993,1995)). Cases of intermediate integration are described by a world that separates into multiple cones of diversification, with different sets of countries specializing in the production of different sets of goods.

I simultaneously fit data on income, endowments, factor prices and the factor content of trade. This provides me with over-identifying restrictions that enable me to calibrate productivities and at the same time allow me

[1] In the early 1990ies trade already amounted to 38 per cent of world income and by the turn of the millennium it had reached 52 per cent of world output (Trade is measured as exports+imports, data are from the Penn World Tables 6.1.).

[2] While the original formulation of the model is due to Helpman (1981), Helpman and Krugman (1985) dedicate an entire book to the study of this model.

to evaluate the fit of the model and to estimate the values of underlying parameters. More specifically, I test the factor deepening case against cases with multiple cones of diversification and ones where conditional factor price equalization occurs and I estimate the elasticity of substitution between human and physical capital.

My main findings are that the factor deepening model with factor specific productivities and weak complementarity between human and physical capital vastly outperforms the other versions of the model considered in this paper. In particular, the elasticity of substitution between human and physical capital is estimated to be significantly lower than one, so that the Cobb-Douglas model is clearly rejected. Rich countries have far higher productivities of human capital than poor ones, while there is no clear relation between physical capital productivity and income per worker. Moreover, my results imply that the model best supported by the HOV Theorem has no Heckscher-Ohlin motiv for the exchange of goods and all trade is due to increasing returns and love for variety.

In terms of intellectual ancestors, this paper integrates two lines of investigation. The first one is the literature on development accounting, which uses income and endowment data to measure productivity differences. Some of the classical contributions are due to King and Levine (1994), Klenow and Rodriguez-Clare (1997), Prescott (1998) and Hall and Jones (1999). See Caselli (2005) for a survey. A stable result of these studies is that total factor productivity is strongly positively correlated with income per worker and accounts for at least half of the cross country variation of this variable. Caselli (2005) adds factor specific technology differences to this approach and discovers that rich countries have higher productivities of human capital than poor ones, whereas poor countries have higher productivities of physical capital than rich nations.

The second strand of research, that uses trade data to measure productivity differences, is the literature that tests the prediction of the Heckscher-Ohlin-Vanek (HOV) equations. They state that countries' trade structure is such that they are net exporters of the services of those factors, in which they are more abundant than the world average. A seminal contribution by Trefler (1993) shows how the HOV equations can be used to solve for the unknown factor specific productivities of each country that equalize measured and predicted factor flows under the assumption that differences in factor prices across countries are caused exclusively by variation in factor productivities. He finds that rich countries have both higher productivities of labor and physical capital than poor countries.

In another important paper Davis and Weinstein (2001) relax the assumption that differences in factor prices are caused only by differences in productivities. They show that both Hicks-neutral differences in total factor productivity, which they estimate using input-output data, and local factor abundance must be taken into account in order to improve the fit of the HOV equations. However, their sample is limited to ten large OECD countries, so that they have nothing to say about productivity differences between rich and poor countries.

This paper goes beyond the previous contributions because I allow both for factor productivities and local factor abundance to matter for income differences and I match data on income, factor prices and trade at the same time. Also, since I put more structure on the underlying model, I am able to estimate the value of underlying parameters and to test different special cases of the model.

Turning to the evidence on the aggregate elasticity of substitution between capital and labor, a long line of studies, summarized by Hamermesh (1986), have attempted to estimate this parameter at various levels of aggregation and using both cross section and time series data. Despite of this, the evidence on its value remains inconclusive, which may potentially reflect mis-specification because this body of research considers exclusively Hicks neutral technological change. Recently, Antràs (2004) discusses the bias that arises from this restriction and estimates the elasticity of substitution between capital and labor for the US economy allowing for factor augmenting productivity differences using time series data. In line with my results, most of his estimates are significantly lower than one.

Finally, Waugh (2007) performs a development accounting exercise in an open economy framework that extends Eaton and Kortum (2002). However, he restricts his analysis to Cobb-Douglas production technology and his main interest is to investigate the role of trade in accounting for cross country income differences.

Summing up, the main contributions of this paper are threefold: Firstly, it integrates development accounting with trade theory and methods. Secondly, the paper proposes to introduce formal over-identification using data from outside the model to evaluate the fit of the productivity calibrations. Thirdly, this buys me a very precise estimate of the elasticity of substitution between human and physical capital, a clear rejection of the aggregate production function being Cobb-Douglas and the possibility to test which model performs best in terms of fitting the HOV equations.

The outline of the paper is as follows: In the next section I develop a theoretical model of the world economy with trade due to factor proportions and love for variety that includes factor specific cross country productivity differentials. In section three I show how factor productivities can be recovered from the model when data on countries' endowments and factor prices are fed in and I discuss how the HOV equations can be used in this framework to evaluate the plausibility of the calibrated productivities and to estimate the values of underlying parameters. In section four I present the results of calibrating productivities in the Helpman-Krugman-Heckscher-Ohlin framework and use the calibrated productivities to perform a development accounting exercise. The last section concludes.

2 The Helpman-Krugman-Heckscher-Ohlin Model

2.1 Assumptions and Setup

The model presented in this section is a standard model of international trade. There are two reasons for trade in this environment. The first one is due to increasing returns. Consumers value variety and each variety is produced by a monopolist because increasing returns are internal to the firm and new varieties can be invented without cost. Since consumers want to consume all varieties each producer serves the world market for her particular variety, which leads to trade within sectors. The second motive for trade is factor proportions. There exist many sectors each of which uses factor inputs with distinct intensities and countries differ in the ratios of their endowments. This gives rise to Heckscher-Ohlin trade and countries produce on average more varieties in those sectors that use its relatively abundant factor intensively. I also introduce cross country differences in the productivities with which factors are used in production, so that a given amount of human or physical capital leads to a different amount of production, depending on the country where production is performed.

The Heckscher-Ohlin part of the model adds two main effects to the standard model used in the development accounting literature. The first one is structural change, that is the possibility for countries to adapt their production structure to their factor endowments. Countries which are abundant in physical capital will concentrate their production in sectors that are intensive in this factor. This tends to increase the role of variation in factor endowments in explaining cross country income differences because countries can employ their factors more efficiently. The other one is terms of trade. These work exactly in the opposite direction because they depress the income of those countries that produce goods which are intensive in the globally abundant factor, thereby reducing the importance of factor endowments in accounting for income differentials. The monopolistic competition part is introduced mainly to explain trade in the absence of differences in factor proportions, as is the case in the standard model for development accounting, but it has no important impact on countries' aggregate production possibility frontiers.

The flexible benchmark model of the world economy relies on the following main assumptions.

A.1: Countries are open to trade in goods and possess perfectly competitive factor markets.

A.2: Goods markets are monopolistically competitive.

A.3: Factors are immobile between countries and perfectly mobile within countries.[3]

A.4: Each country is endowed with human capital H_c and physical capital K_c.[4]

A.5: Productivity is specific to a factor located in a country.

A.6: Each country has access to technologies to produce in I sectors, that vary in their capital intensities.

A.7: Consumers in all countries have identical, homothetic preferences with fixed expenditure shares. [5]

The model is then easily described by Assumptions A.1-A.7 and the specification of demand and supply.

[3] The immobility of labor is probably not a very controversial assumption. Even though some mobility of people can be observed, there exist very large barriers to migration from poor to rich countries. Starting at least with Lucas (1990) a large literature in International Economics has been dealing with the question why capital does not flow from rich to poor countries. Caselli (2006) makes the interesting point that capital may actually be distributed quite efficiently across countries, so that there is no reason to observe large capital flows from rich to poor nations.

[4] Following the growth literature the factor "human capital" is measured as labor endowments in efficiency units, which is different from the convention used in the trade literature, where human capital is usually the amount of skilled labor. Because the model has only two factors this seems to be the adequate way to measure labor endowments.

[5] This together with **A.1** implies that the optimal price index of Gross Domestic Product has the same form in all countries.

2.2 Demand

Consider a world economy with countries indexed by $c \in C$ and sectors indexed by $i \in I$.[6] Assuming that trade is balanced, aggregate expenditure of country c equals its aggregate income.

$$E_c = P_c Y_c = \sum_{i=1}^{I} E_{ic} = \sum_{i=1}^{I} \beta_i E_c, \tag{1}$$

where $P_c Y_c$ is GDP of country c in dollars, and Y_c is GDP in purchasing power parities and is measured in aggregate consumption units. Aggregate spending is split across I sectors with fixed expenditure shares β_i.[7]

The ideal aggregate price index is Cobb-Douglas. It measures the minimum expenditure to buy one unit of the aggregate bundle of goods.

$$P_c = \prod_{i=1}^{I} \left(\frac{P_i}{\beta_i}\right)^{\beta_i}, \tag{2}$$

where P_i are the sectoral price indices.

Sectoral price indices are constant elasticity of substitution composites of the prices of sector specific varieties.

$$P_i = \left(\int_0^{N_i} p_i(n)^{1-\sigma} dn\right)^{\frac{1}{1-\sigma}}, \tag{3}$$

where $\sigma > 1$ is the elasticity of substitution between any two varieties and $N_i = \sum_{c=1}^{C} N_{ic}$ is the total number of varieties produced in sector i.

The form of the sectoral price indices implies that there is love for variety and aggregate consumption is increasing in the number of varieties available in each sector.

The demand function of country c for variety n produced in country c' in sector i implied by the price indices can be found from the sectoral price index by using Roy's law.

$$x_{icc'}(n) = \frac{p_i(n)^{-\sigma}}{P_i^{1-\sigma}} \beta_i E_c \tag{4}$$

2.3 Behavior of Firms and Technology

Final goods are freely traded and are produced by monopolistically competitive firms.

In each sector firms choose a variety and an optimal pricing decision taking as given the decisions of the other firms in the industry.[8] The output of an industry consists of a number of varieties that are imperfect substitutes for each other. Production of each variety is monopolistic because of economies of scale. In the model the invention of a new variety is costless, so firms always prefer to invent a new differentiated variety instead of entering in price competition with an existing firm.

In each country firms are homogeneous within a sector. Firms' technologies differ across sectors by the capital intensity of production for given wages, w_c, and rental rates, r_c. Varieties of final goods are produced

[6] I slightly abuse notation by denoting with C and I both the sets of countries and goods and their cardinalities.

[7] A possible interpretation for this setup is that each country has an aggregate Cobb-Douglas production function that produces a final good which can be used for consumption and investment.

[8] In what follows I will use the terms industry and sector interchangeably.

using both human capital $H_{ic}(n)$ and physical capital $K_{ic}(n)$ with constant marginal cost and a fixed cost, f. Production technology, represented by the homothetic total cost function TC (5), is CES in each sector[9] and varies across countries because of differences in factor productivities. Productivities are specific to factors located in country c, so that a country's productivity is described by the duple $\{A_{Kc}, A_{Hc}\}$, which are the productivity of physical capital and human capital in country c.

$$TC(q_{ic}) = \left[\alpha_i^{\epsilon}\left(\frac{r_c}{A_{Kc}}\right)^{1-\epsilon} + (1-\alpha_i)^{\epsilon}\left(\frac{w_c}{A_{Hc}}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} (f + q_{ic}), \tag{5}$$

where $\alpha_i \in [0,1]$ is the physical capital intensity of sector i and $\epsilon \in [0,\infty)$ is the elasticity of substitution between human capital and capital.

Monopolistic producers in sector i of country c maximize profits subject to the demand function

$$x_{ic'} = \frac{p_i^{-\sigma}}{P_i^{1-\sigma}}\beta_i \sum_{c=1}^{C} E_c. \tag{6}$$

Optimality implies that marginal revenue equals marginal cost. Note that given this constant elasticity demand function the solution to the firms' profit maximization problem implies that the price charged by a firm in sector i in country c is a constant markup over its marginal cost as long as firms are active in that sector in country c.

$$p_i = \frac{\sigma}{\sigma-1}\left[\alpha_i^{\epsilon}\left(\frac{r_c}{A_{Kc}}\right)^{1-\epsilon} + (1-\alpha_i)^{\epsilon}\left(\frac{w_c}{A_{Hc}}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{7}$$

If a sector is located in a country, free entry of firms drives profits to zero, so that firms price at their average cost. This determines the number of firms in each sector endogenously.

$$[\alpha_i^{\epsilon}\hat{r}_c^{1-\epsilon} + (1-\alpha_i)^{\epsilon}\hat{w}_c^{1-\epsilon}]^{\frac{1}{1-\epsilon}}\left(\frac{f}{q_{ic}} + 1\right) \geq p_i \tag{8}$$

The combination of the pricing rule, the free entry condition and the form of the fixed cost imply that firms' optimal output is the same in all sectors and countries (as long as it is positive).

$$q^* = (\sigma - 1)f \tag{9}$$

2.4 Equilibrium

It turns out to be useful to rewrite the model in terms of variables in efficiency units. Following Daniel Trefler (1993), let me define $\hat{H}_c \equiv A_{Hc}H_c$, $\hat{K}_c \equiv A_{Kc}K_c$, $\hat{w}_c \equiv \frac{w_c}{A_{Hc}}$ and $\hat{r}_c \equiv \frac{r_c}{A_{Kc}}$.

These are factor endowments in efficiency units and efficiency adjusted factor prices. So, for example, one unit of efficient physical capital is equivalent to A_{Kc} units of plain physical capital, and one unit of efficient physical capital, which is measured in common units across countries, costs $\frac{1}{A_{K,c}}$ as much as one unit of plain physical capital, that may differ in efficiency across countries. Hence, capital prices in country c may be higher

[9]The assumption that elasticities of substitution are the same across sectors rules out factor intensity reversals - the possibility that a sector i is more intensive in physical capital than sector i' for some combination of factor prices and more intensive in human capital for some other one.

than in country c' because buying one unit of capital in country c provides ownership of more efficient units of capital or because capital is scarcer in country c.

With this redefinition of variables I am able to describe the world economy as an ordinary Helpman-Krugman-Heckscher-Ohlin (1985) model without productivity differences in which factor endowments in each country are measured in efficiency units, while leaving the structure of the model formally equivalent to the one described by the production possibilities and demand structure listed above. The advantage of this formulation is that the extensive theory available on the factor proportions theory and its monopolistic competition hybrid, as discussed in Helpman and Krugman (1985), can be directly applied to this model.

In general, it may not be profitable to produce varieties in all sectors in every country because production in sectors that use the locally scarce factors intensively may be unprofitable. In this case, countries will be located in different cones of diversification. A cone of diversification is a set of countries that produce at least in two common sectors. Those countries have common efficient factor prices, and when the number sectors active in the cone is larger than the number of factors, individual countries' production of varieties is undetermined and only the aggregate number of varieties produced in each sector in the cone is unique. Consequently, let d be a set of countries with common efficient factor prices. Given this, one can define an equilibrium of the Helpman-Krugman-Heckscher-Ohlin model.

Definition 1: An **Equilibrium** is a collection of goods prices $\{p_i\}$, efficiency adjusted wages $\{\hat{w}_d\}$, efficiency adjusted rental rates $\{\hat{r}_d\}$, and numbers of sectoral varieties $\{N_{id}\}$ such that firms maximize profits, expenditure is minimized, factors are fully employed and goods markets clear in every set of countries d.

Since the general model is rather complex, let us instead take a look at some representative examples to learn something about the forces that determine countries' relative incomes in this world. The intuition gained in these specific cases will carry over to the general model.

Example 1: Factor Deepening

A.8 All sectors have identical factor intensities ($\alpha_i = \alpha$ for all $i \in I$).

This assumption eliminates both the role of structural change and terms of trade effects. From the pricing conditions (7) we have that goods in all sectors have the same price $p_i = p_{i'} = p$ and countries produce some varieties in all sectors. Taking this into account, it is not difficult to solve for the aggregate production function implicit in this model[10]

$$Y_c = p\left(\frac{\sigma}{\sigma-1}\right)[\alpha(A_{Kc}K_c)^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(A_{Hc}H_c)^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}, \tag{10}$$

where

$$p = (\prod_{i=1}^{I}(\frac{N_i}{\beta_i})^{\beta_i})^{\frac{1}{\sigma-1}}. \tag{11}$$

[10]To get this note that all prices are the same, and divide the factor market clearing conditions, which - by Shephard's Lemma - can be obtained by partially differentiating (5) with respect to factor prices, to solve for the wage/rental, $\frac{\hat{w}_c}{\hat{r}_c} = (\frac{1-\alpha}{\alpha})(\frac{A_{Kc}K_c}{A_{Hc}H_c})^{\frac{1}{\epsilon}}$. Use this together with the pricing condition (7) to solve for factor prices and substitute in the definition of aggregate income.

It can be seen from (10) that countries' aggregate production function is CES with an elasticity of substitution between physical and human capital equal to ϵ. This is the world with factor specific productivities described by Francesco Caselli (2005) in the Handbook of Economic Growth. The main features of this world are that countries experience decreasing returns to factor accumulation and that terms of trade do not matter for aggregate income because all relative goods prices are one. There is an aggregate scale effect due to love for variety, but it is irrelevant for relative incomes because all varieties are available in all countries.

In this world all trade is due to love for variety, as producers export their variety to all other countries. Imports by country c' of variety n produced in sector i in country c are a fraction of production that is proportional to the size of the importing country.[11]

$$x_{cc'i}(n) = \frac{p_i^{-\sigma}}{P_i^{1-\sigma}} \sigma_i E_c = \frac{Y_c}{\sum_{c=1}^{C} Y_c} q_{ci}(n) \tag{12}$$

Even though differences in factor endowments across countries do not constitute a reason for trade in this world, goods trade embodies factors, since countries that are abundant in efficient physical capital produce varieties much more capital intensively than efficient human capital abundant ones and therefore are net exporters of this factor.

An additional assumption leads us back to the Cobb-Douglas world, that has been the focus of the analysis in most of the development accounting literature.[12]

A.9: $\epsilon = 1$

Then the aggregate production function is Cobb-Douglas.

$$Y_c = p\left(\frac{\sigma}{\sigma-1}\right) A_{Kc}^{\alpha} A_{Hc}^{1-\alpha} K_c^{\alpha} H_c^{1-\alpha} = p\left(\frac{\sigma}{\sigma-1}\right) A_c K_c^{\alpha} H_c^{1-\alpha}, \tag{13}$$

where I have defined $A_c \equiv A_{Kc}^{\alpha} A_{Hc}^{1-\alpha}$. This view point of the world economy is similar to Caselli's, just that with a unit elasticity of substitution, only total factor productivity is identified. Countries experience decreasing returns and terms of trade effects are absent.

Example 2: Conditional Factor Price Equalization (CFPE)

A completely different picture of the world arises if we drop assumptions **A.8** and **A.9**, and instead maximize the role of structural change. We do this by assuming that trade integration is so strong, that trade in goods is able to make up for the immobility of efficient factors.

A.10: Conditional on measuring endowments in efficiency units factor prices are equalized in the world economy.

In this extreme case the equilibrium of the world economy is akin to the one of a hypothetical world, in which all impediments to movements of factors measured in efficiency units have been abolished. Call this equilibrium

[11] This follows from the definition of the sectoral price index and the fact that $\sum_{c=1}^{C} p_i N_{ic} q_{ic} = \beta_i \sum_{c=1}^{C} Y_c$

[12] See for example Mankiw, Romer and Weil (1991), Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999).

the integrated equilibrium. The Factor Price Equalization Set is the set of distributions of efficient factor endowments across countries, such that the world economy is able to replicate the integrated equilibrium.[13] This implies that the world economy has the allocations of the integrated equilibrium if every country can fully employ its resources when using the same capital to human capital ratios in each sector as in the integrated equilibrium given its endowments of efficient human and physical capital. The size of this set is larger if different sectors use very different capital to human capital ratios for given factor prices (α_i varies a lot across sectors) because even countries with extremely unbalanced factor endowments will be able to find production patterns such that they can employ their factors using the integrated equilibrium techniques. Still, in this world, marginal products of physical units of capital and human capital are not equalized across countries because of differences in factor productivities. Consequently, disparities in factor prices stem only from differences in factor productivities and not from variation in the abundance of human capital and physical capital across countries.

Assume that sectoral input ratios are sufficiently extreme and expenditure on sectors with extreme factor proportions is large enough in order for conditional factor price equalization to hold for the world economy, i.e. $\hat{w}_c = \hat{w}_{c'} = \hat{w}$ and $\hat{r}_c = \hat{r}_{c'} = \hat{r}$ for all c $\in C$. Then - as discussed - it is sufficient to analyze the equilibrium of the integrated economy. For analytical tractability let $\epsilon \to 1$ so that sectoral production functions are Cobb-Douglas. In this case one can show that the aggregate production function of the world economy is also Cobb-Douglas.[14]

$$Q_w = B\hat{K}_w^{\sum_{i\in I}\alpha_i\beta_i}\hat{H}_w^{(1-\sum_{i\in I}\alpha_i\beta_i)}, \tag{14}$$

where

$$B \equiv (\prod_{i=1}^{I}\left(\frac{N_i}{\beta_i}\right)^{\beta_i}\Bigg)^{\frac{\sigma}{\sigma-1}}\prod_{i=1}^{I}\left[\frac{(1-\alpha_i)\beta_i}{\sum_{i=1}^{I}(1-\alpha_i)\beta_i}\right]^{(1-\alpha_i\beta_i)}\left[\frac{\alpha_i\beta_i}{\sum_{i=1}^{I}\alpha_i\beta_i}\right]^{\alpha_i\beta_i} \tag{15}$$

and

$$\hat{H}_w = \sum_{c=1}^{C}\hat{H}_c, \qquad \hat{K}_w = \sum_{c=1}^{C}\hat{K}_c. \tag{16}$$

There are decreasing returns to factor accumulation in efficiency units at the world level. World factor prices are given by

$$\hat{w} = (1-\sum_{i\in I}\alpha_i\beta_i)B\left(\frac{\hat{K}_w}{\hat{H}_w}\right)^{\sum_{i\in I}\alpha_i\beta_i} \tag{17}$$

and

$$\hat{r} = (\sum_{i\in I}\alpha_i\beta_i)B\left(\frac{\hat{H}_w}{\hat{K}_w}\right)^{(1-\sum_{i\in I}\alpha_i\beta_i)}. \tag{18}$$

[13] For a thorough discussion of these concepts see, for example, Helpman and Krugman (1985).

[14] To obtain this, solve for sectoral factor shares and total factor shares and divide these equations to obtain sectoral factor use in terms of aggregate factor endowments. Then use the market clearing conditions to solve for goods prices and substitute them in the definition of the price indices to get the implicit aggregate production function of the world economy. Finally, use the definition of sectoral production functions, define $Q_{ic} = N_{ic}q_{ic}$ and substitute the sectoral factor use in terms of aggregate endowments to get equation (14).

Consequently, factor prices are determined at the world level and not at the country level. Income of country c is given by

$$Y_c = A_{Hc}H_c\hat{w} + A_{Kc}K_c\hat{r}. \tag{19}$$

To the extent that factor prices are given for individual countries, countries' aggregate production functions are linear and countries experience constant returns to factor accumulation. The infinite elasticity of substitution between factors reflects structural change. Countries absorb additional units of factor endowments by changing their production structure while holding constant sectoral production techniques, instead of using factor deepening like in Example 1 or in the closed economy.

In this world there is both intra-industry trade (because of love for variety and monopolistic competition) and inter-industry trade (because of differences in factor endowments across countries). Countries that are more abundant in an efficient factor than the average of the world economy are net exporters of this factor. Even though all countries produce in all sectors, it is not necessarily true that countries produce more in those sectors that use their abundant factor intensively because individual countries' production patterns are undetermined, when the number of sectors is larger than the number of factors,.

We have now seen two very diverse views of how countries' aggregate production possibilities may look like. In general, however, the world is likely to be somewhere between the two extremes of the Factor Deepening world and Conditional Factor Price Equalization and will combine features of both. If differences in efficient factor endowments are too large, efficient factor prices cannot be equalized in the whole world. Instead, there will be multiple cones of diversification. Between those cones, there will generically exist countries that specialize in the production of varieties in a single sector.

Example 3: Multiple Cones

Assume there are only two sectors, $i \in \{H, K\}$ with $\alpha_K > \alpha_H$. Since it is not possible to solve this model analytically, let us take goods prices as parameters. With only two sectors, sectoral production patterns are determined in each country. Countries with extremely high efficient physical to human capital ratios specialize in producing varieties in the K-sector. Countries with intermediate factor endowment ratios have diversified production structures and produce varieties in both sectors, while countries with very low efficient physical to human capital ratios specialize in the H-sector.

It is easy to show that for countries outside the cone of diversification the aggregate production function has the form of the sectoral production function of the sector in which they specialize. [15]

$$Y_c = p_i \frac{\sigma}{\sigma - 1}[\alpha_i(A_{Kc}K_c)^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha_i)(A_{Hc}H_c)^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} \tag{20}$$

In this case countries experience decreasing returns and the elasticity of substitution is ϵ because additional units of factors are absorbed by factor deepening. Terms of trade effects push up the income of countries that specialize in the sector which has a high relative price. Countries that lie in the cone of diversification, on the

[15] Divide the factor market clearing conditions to get the wage/rental and use this equation together with the pricing equation to solve for factor prices. Then substitute them in the definition of aggregate income.

other hand, have linear production technologies, reflecting again the fact that they are capable of absorbing additional units of factors through structural transformation.

$$Y_c = \hat{w}_d A_{Hc} H_c + \hat{r_d} A_{Kc} K_c \tag{21}$$

where

$$\hat{w}_d = \left(\frac{\sigma}{\sigma - 1}\right) \left[\frac{\alpha_H^\epsilon p_K^{1-\epsilon} - \alpha_K^\epsilon p_H^{1-\epsilon}}{\alpha_H^\epsilon (1-\alpha_K)^\epsilon - \alpha_K^\epsilon (1-\alpha_H)^\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{22}$$

$$\hat{r}_d = \left(\frac{\sigma}{\sigma - 1}\right) \left[\frac{(1-\alpha_H)^\epsilon p_K^{1-\epsilon} - (1-\alpha_K)^\epsilon p_H^{1-\epsilon}}{\alpha_K^\epsilon (1-\alpha_H)^\epsilon - \alpha_H^\epsilon (1-\alpha_K)^\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{23}$$

Factor prices are functions of goods prices only and consequently depend on the endowments of the world economy.[16] From the formulas (22) and (23) one can see that an increase in the price of a sector's output leads to a more than proportionate rise in the price of the factor that is used intensively in that sector. This is the Stolper-Samuelson effect. The intuition is that an increase in an industry's price shifts production towards that sector and thereby increases relative demand for the intensively used factor. Whether an increase in a sectoral price decreases or increases aggregate income depends on how much a country is producing in each sector (which in turn depends on its endowments).

This example provides all the mechanism present in the general model. The world sorts into cones of diversification between which lie countries that specialize in specific sectors. As a consequence the mapping between endowments, factor prices, income and productivity changes its shape depending on whether a country is located in a cone or specialized. Having discussed the properties of the model, let us now turn to calibrating productivities in this world.

3 A Method for Productivity Calibration

Given measures of factor productivities for every country I could test if the models described in the previous section are a reasonable representation of the real world. Unfortunately, I lack exactly these measures of productivities. Instead, in a first step I will follow the convention of the development accounting approach to assume that the model is specified correctly, and back out productivities from the model given some additional information about other endogenous variables. Subsequently, I will test the model fit using trade data.

The procedure is to use information on endowments $\{H_c\}$, $\{K_c\}$, wages, $\{w_c\}$, and rental rates, $\{r_c\}$, in order to back out the $2C$ unknowns $\{A_{Kc}\}$, $\{A_{Hc}\}$ from the equilibrium conditions of the model. This allows me to fit perfectly the cross section of income $\{Y_c\}$ and labor and capital income shares, $\{s_{Hc}\}$ and $\{s_{Kc}\}$, respectively, taken as given a combination of sectoral factor intensities $\{\alpha_i\}$, expenditure shares $\{\beta_i\}$ and an elasticity of substitution ϵ.

[16] To derive this use the pricing conditions for the relevant goods and solve for factor prices in terms of goods prices. These equations can be inverted if and only if the number of factors equals the number of sectors.

This method for calibrating productivities is analogous to the usual calibration exercise performed in the development accounting literature. The main complication is that productivities have to be determined simultaneously with the unknown specialization patterns, equilibrium prices and production levels in each country, because the relationship between a countries' inputs and outputs generally depends on endowments and the demand structure of the whole world economy. More formally, a **Productivity Calibration Problem** is defined as follows.

Definition 2: A **Productivity Calibration Problem** (**PCP**) is a collection of goods prices $\{p_i\}$, efficiency adjusted wages $\{\hat{w}_d\}$, efficiency adjusted rental rates $\{\hat{r}_d\}$, numbers of sectoral varieties $\{N_{id}\}$ and factor productivities $\{A_{Hc}\}$, $\{A_{Kc}\}$ such that given a cross section of human capital endowments $\{H_c\}$, physical capital endowments $\{K_c\}$, wages $\{w_c\}$, rentals $\{r_c\}$ and parameters $\{\alpha_i\}$, $\{\beta_i\}$, ϵ, σ and f, firms maximize profits, expenditure is minimized, factors are fully employed and goods markets clear for all d[17].

One can show that a solution to **PCP** is also an **Equilibrium** given efficient factor endowments, and that for given efficient factor endowments an **Equilibrium** also solves the **PCP** and measured productivity differences are zero, which is obviously necessary for the concept of **PCP** to make sense.

Solving the **PCP** requires the use of numerical methods. There are three main challenges. First, the large number of countries in the sample (96), which I use to have a representative picture of the world economy. Second, the fact that one cannot apply standard methods for computing equilibria because parameters and variables have been exchanged (since $\{A_{Hc}\}, \{A_{Kc}\}$ are unknown). Third, that I allow countries to specialize into multiple cones, which makes computation much more complex because corner solutions might occur.

I compute a solution to the **PCP** by imposing a specialization pattern, solving the resulting nonlinear system of equations and checking if the solution satisfies the non-negativity restrictions imposed on the variables. If it does, I accept the solution, otherwise I guess another specialization pattern until a solution is found. For reasons of computational tractability I restrict my attention to models with two sectors.[18]

As a next step I would like to find reasonable parameter values for production and demand $\{\alpha_i\}$, $\{\beta_i\}$ ϵ, for which to solve the model because results will be sensitive to the choice of these parameters[19]. In addition, I would like to test if certain restrictions imposed on the parameters by standard models, like $\alpha_i = \alpha$ or $\epsilon = 1$ or $A_{HC} = A_{KC}$ are realistic.

3.1 Using Trade Data to Evaluate the Model

To estimate these parameters I use the model's prediction on trade. The testable hypothesis of the Heckscher-Ohlin-Vanek (HOV) equations is that a country should export (through trade in goods) the services of those (efficient) factors with which it is abundantly endowed relative to the world average and import its relatively

[17]An exact mathematical definition of the **PCP** can be found in the appendix.

[18]As discussed in the appendix, the solution to **PCP**is unique under some restrictions.

[19]In the simulations I set $\sigma f = 1$ and define $\tilde{p}_i \equiv \frac{\sigma-1}{\sigma} p_i$ for convenience, since the productivities generated by the **PCP** do not depend on the values of these parameters.

scarce factors. The predictions on factor trade provide over-identifying restrictions that enable me to find the combination of parameter values that best match moments of the data and at the same time allow me to test, if certain constraints on parameters are valid.

The HOV-equations hold for a class of trade models that satisfy a consumption similarity condition (see Trefler and Zhu (2005)) and perfect competition in factor markets. In particular, they apply to the Helpman-Krugman-Heckscher-Ohlin model, and all its versions considered in this paper.[20]

Because the HOV-equations are a statement about factor flows embodied in trade and not directly about trade in goods, one needs to define the factor content of trade. Let $f \in \{H, K\}$ denote factors, V_{fc} denote endowments of factor f in country c, $i = 1, ..., I$ denote goods and let D_c be the $F \times I$ factor use matrix in country c, with elements b_{fic} denoting the use of factor f in the production of one unit of good i in country c and rows D_{fc}, that state the use of factor f per unit of output in each sector. Let B_c be country c's input-output matrix and denote factor prices by $\pi_c = (w_c, r_c)$. Then, for example, the direct use of human capital measured in efficiency units in the production of one unit of good i in country c in the above models is

$$d_{Hic} = \frac{\sigma}{\sigma - 1} p_i^{\epsilon} w_c^{-\epsilon} (1 - \alpha_i)^{\epsilon} A_{Hc}^{\epsilon}. \tag{24}$$

In addition, the fth row of the direct factor use matrix of the United States in efficiency units is

$$D_{fUS} = \pi_{US}^{-\epsilon} A_{fUS}^{\epsilon} D_f, \tag{25}$$

where D_f is common to all countries because of free and costless trade and the absence of industry specific (Ricardian) technology differences across countries. The factor use matrix measured in efficiency units of every country c can be expressed as a function of the one of the US,

$$D_{fc} = \pi_c^{-\epsilon} D_f A_{fc}^{\epsilon} = \left(\frac{\pi_{US}}{\pi_c}\right)^{\epsilon} \left(\frac{A_{fc}}{A_{fUS}}\right)^{\epsilon} D_{fUS}. \tag{26}$$

Following Trefler and Zhu (2006), in the presence of trade in intermediate goods, the measured factor content of trade in efficient factor f by country c is defined as

$$F_{fc}^{*} = E_{fc} X_c - \sum_{c' \neq c} E_{fc'} M_{cc'}, \tag{27}$$

where E_{fc} is an $1 \times I$ vector that converts trade in goods into trade in factors measured in efficiency units, X_c is an $I \times 1$ vector of country c's exports and $M_{cc'}$ are $I \times 1$ vectors of bilateral imports of goods by country c from country c'. The total net efficient factor content of country c's trade is computed as the amount of efficient factor f embodied in country c's exports, $E_{fc}X_c$, minus the quantity of efficient factor f that country c imports from other countries, $\sum_{c' \neq c} E_{fc'} M_{cc'}$. Here, E_{fc} is the c'th column of $E_f = \tilde{D}_f (I - B)^{-1}$, which is a complicated function of the efficient factor use matrices D_{fc}, and the input-output matrices B_c of all countries in the world. [21]

[20] Interestingly, they apply to the Heckscher-Ohlin model with perfect competition only in special cases. One is (conditional) factor price equalization, and the other one is complete specialization of all countries, a borderline case. The fact that many models imply the HOV equations means also that one cannot use them to test a particular trade model against an alternative, unless one has an underlying structural model that generates testable data.

[21] For the exact definitions of $\tilde{D}$ and B I refer the interested reader to Trefler and Zhu (2005).

The reason why one needs to consider the input-output relations of the whole world to compute the factor content of a single country's trade is trade in intermediate goods. For example, if the US imports cars from Germany that have been produced using Chinese steel, the factors embodied in the Chinese steel must be evaluated using the Chinese factor use matrix, and not the German one.

To obtain the HOV equations in efficiency units, one needs to make the assumption that each country c consumes a fraction of the world's total consumption of all goods produced in a country c' which is proportional to the importing country's size.

A.11: $C_{cc'} = s_c \sum_{c' \in C} C_{c'}$

This condition is met by the class of models considered in this paper, since - because of love for variety and monopolistic competition - each country imports a fraction of every good produced in each country and the entire production of every good is consumed.

As Trefler and Zhu (2005) show **A.11** is sufficient (and in general also necessary) for the HOV-equations to hold in efficiency units. Hence, we have that

$$F^*_{fc} = A_{fc} V_{fc} - s_c \sum_{c \in C} A_{fc} V_{fc} = \hat{V}_{fc} - s_c \hat{V}_{fw}. \tag{28}$$

Equation (28) states that F^*_{fc}, the measured efficient factor content of trade, equals the difference between a country's endowments of factor f in efficiency units, $\hat{V}_{fc}$ and the world endowments of this factor in efficiency units, $\hat{V}_{fw}$, multiplied by country c's share in world GDP. The right hand side of (28) is usually called the predicted factor content of trade.

Consequently, a country exports an efficient factor f in net terms (i.e. $F^*_{fc} > 0$) whenever $\hat{V}_{fc} > s_c \hat{V}_{fw}$ because it consumes a fraction of the world efficient endowments that is proportional to its size.

Let us now turn back to the question how to determine the parameter values of the **PCP**. As we noted, the HOV-equations provide restrictions on the factor productivities computed from the model for given parameter values $\theta = (\{\alpha_i\}, \{\beta_i\}, \epsilon)$. Let $F^*_{fc}(\theta)$ be the factor content of trade constructed using the factor use matrices that have been generated by the model, $D_{fc}(\theta)$. Dividing (28) by the equation for the US and normalizing A_{fUS} to one, we obtain the following relation.

$$A_{fc}(\theta) = \frac{Y_c}{Y_{US}} \frac{V_{fUS}}{V_{fc}} + \frac{F^*_{fc}(\theta)}{V_{fc}} - \frac{Y_c}{Y_{US}} \frac{F^*_{fUS}(\theta)}{V_{fUS}} + u_{fc}, \tag{29}$$

where I have augmented the equation for an i.i.d error term, u_{fc}. Hence, a country has a high factor productivity relative to the US if its relative factor-output-ratio is low and if it exports more of a factor relative to its endowments compared to the US controlling for its relative size. If $\{A_{fc}(\theta)\}$ are good estimates of factor productivities, equation (29) should hold roughly with equality.

This test requires - in addition to calibrated values of $A_{fc}(\theta)$ - information on countries' input-output matrices, on bilateral trade at the sector level and on the factor use matrix of the US (or any other reference country).

To make this more formal, I use a GMM estimation procedure to choose the vector of parameters θ.

I use the following L=4 orthogonality conditions to estimate θ: $E(u_{fc}) = 0$ and $E(u_{fc}\frac{Y_c}{Y_{US}}) = 0$ for $f \in \{H, K\}$. These conditions exactly identify θ in the two sector multiple cone case and over-identify it in the factor deepening and the CFPE case.

Let $Z = [1, \frac{Y}{Y_{US}}]$ be the matrix of instruments, let $g_c(\theta) = Z_c' u_c(\theta)$ and let $g(\theta) = 1/C \sum_{c=1}^{C} g_c(\theta)$ be the L vector stacking the orthogonality conditions. Then I choose θ to minimize the quadratic form

$$\min_{\theta} J = \min_{\theta} g(\theta)' W_i g(\theta), \tag{30}$$

where $\theta = (\{\alpha_i\}, \{\beta_i\}, \epsilon)$ is the vector of parameters to be estimated and W^i is a weighting matrix.

The procedure is to

1) choose $W_0 = I$,

2) solve **PCP** for a given θ_n,

3) evaluate (30) at θ_n and update using an optimization routine to get θ_{n+1} and

4) repeat steps 2) and 3) until $||\theta_{n+1} - \theta_n||$ is small enough and obtain the preliminary estimate θ_i

5) update $W_i = C[\sum_{c=1}^{C} g_c(\theta(i)) g_c(\theta(i))']^{-1}$

6) iterate on 2) - 5) until $||W_{i+1} - W_i||$ is small.

The GMM estimator $\hat{\theta}$ has an asymptotic normal distribution with $E(\hat{\theta}) = \theta$ and variance covariance matrix $\Sigma = 1/C[(1/C \sum_{c=1}^{C} \nabla g_c(\theta))' W_i^{-1} (1/C \sum_{c=1}^{C} \nabla g_c(\theta))]^{-1}$.

The data set used for implementing this approach consists of a cross section of endowments, income and factor prices in PPPs for 96 countries in 2001. Data on human capital, $\{H_c\}$, measured as efficient labor, are constructed following Caselli (2005) using data from Barro and Lee (2001) and the Penn World Table (PWT) version 6.2. Data on income in PPPs are also taken from the PWT and capital stocks are constructed from PWT investment data using the perpetual inventory method. Finally, factor prices in PPPs, $\{w_c\}$ and $\{r_c\}$, are computed using the previous data and information on labor income shares $\{s_{Hc}\}$, from Bernanke and Guerkaynak (2001) and my own calculations, by making use of the fact that $w_c = s_{Hc} Y_c / H_c$ and $r_c = (1 - s_{Hc}) Y_c / K_c$. The required data on factor use matrices, input-output matrices and bilateral trade at the sector level for 53 countries are from the Global Trade Analysis Project (GTAP) Version 6.[22]

4 Results

In this section I provide the results of calibrating productivities and estimating parameters within the Helpman-Krugman-Heckscher-Ohlin model. I start by discussing the examples considered in section two and then compare the fit of the different models in terms of the HOV equations.

[22] Even though I compute productivities for 96 countries, because of data availability on input-output matrices only a subset of 53 countries can be used in evaluating the model fit. For a detailed description of the data and their construction see the data appendix.

Example 1: Factor Deepening

As a starting point assume that all sectors have identical factor intensities (**A.8**). In this case it is straightforward to derive analytical solutions for the **PCP**, because the possibility of structural change has been eliminated and terms of trade effects are absent. Hence, a country's aggregate income is independent of foreign variables. [23]

$$A_{fc} = \left(\frac{\sigma}{\sigma-1}\right)\frac{1}{p}\left(\frac{1}{1-\alpha}\frac{\pi_c V_{fc}}{Y_c}\right)^{\frac{\epsilon}{\epsilon-1}}\frac{Y_c}{V_{fc}} \tag{31}$$

Consequently, relative factor productivities are given by the following expression,

$$\frac{A_{fc}}{A_{fUS}} = \left(\frac{s_{fc}}{s_{fUS}}\right)^{\frac{\epsilon}{\epsilon-1}}\frac{\left(\frac{Y_c}{V_{fc}}\right)}{\left(\frac{Y_{US}}{V_{fUS}}\right)}. \tag{32}$$

This is Caselli's (2005) formula for calibrating productivities with factor augmenting productivity differences. If factors are substitutes ($\epsilon > 1$), relative factor productivities are increasing in relative factor shares. The intuition is that when inputs are good substitutes, factor demand shifts towards the productive factor, raising its income share. When factors are complements ($\epsilon < 1$) the opposite is true. Since the unproductive factor is required in production, a high income share is a sign of inefficiency.

Moreover, relative factor productivities are linearly decreasing in relative factor-output ratios. Holding constant factor income shares, and the technology parameter, α, a high output per unit of factor implies that the factor must be productive.

To estimate ϵ, the only parameter of interest in this specification, combine (32) with the HOV-equations. The efficient factor use relative to the US can then be written as $D_{fc} = \left(\frac{s_{fc}}{s_{fUS}}\right)^{\frac{1}{\epsilon-1}} D_{fUS}$, while the HOV-equations relative to the US become

$$\left(\frac{s_{fc}}{s_{fUS'}}\right)^{\frac{\epsilon}{\epsilon-1}}\frac{Y_c}{Y_{US}}\frac{V_{fUS}}{V_{fc}} = \frac{Y_c}{Y_{US}}\frac{V_{fUS}}{V_{fc}} + \frac{F^*_{fc}(\epsilon)}{V_{fc}} - \frac{Y_c}{Y_{US}}\frac{F^*_{fUS}(\epsilon)}{V_{fUS}} + u_{fc}. \tag{33}$$

The first rows of table 1 present the estimation results for the factor deepening case. The first specification uses all four orthogonality conditions. The estimate of the elasticity of substitution between factors, $\hat{\epsilon}$, is 0.836 and extremely precise. The J-statistic implies that the imposed orthogonality conditions are valid. In addition, the null that $\epsilon = 1$ is rejected at the one percent level. To check whether these estimates are robust, I re-estimate ϵ using first only the first two orthogonality conditions (zero mean of the error term). This alternative estimate is 0.833, again estimated very precisely, and extremely close to the first estimate. Subsequently, I redo the estimation using only one factor content of trade at a time. Again, the estimates remain very stable and precise and are surprisingly similar independently whether the human or physical capital content of trade is used.

Consequently, if the factor deepening case is assumed to be the true model, the HOV-equations imply that physical and human capital are weak complements, with point estimates of the elasticity of substitution lying in the interval [0.816,0.837].

[23] To derive this divide the factor market clearing conditions and solve for factor prices using the pricing condition.

The upper left panel of figure 1 plots the productivity of human capital against income per worker for $\epsilon = 0.836$. There is a strong positive correlation between the productivity of human capital and income per worker, which is easily explained by the fact that empirically there is no clear correlation between the income share of labor and income per worker and a positive relation between output per unit of human capital and income per worker. On the other hand, there is no obvious relation between the productivity of physical capital and income per worker (see right panel of figure 1).

If in addition to assuming that factor intensities are identical across sectors we presuppose that productivities are not factor augmenting but Hicks neutral and that the elasticity of substitution is one[24], we obtain the standard model for development accounting that has been used by Klenow and Rodriguez-Clare (1997), Hall and Jones (1999) and many others.

$$A_c = \frac{Y_c}{p\left(\frac{\sigma}{\sigma-1}\right) K_c^{\alpha} H_c^{1-\alpha}} \tag{34}$$

$$\frac{A_c}{A_{US}} = \left(\frac{Y_c}{Y_{US}}\right)\left(\frac{K_{US}}{K_c}\right)^{\alpha}\left(\frac{H_{US}}{H_c}\right)^{1-\alpha} \tag{35}$$

In this case one data point per country is sufficient to determine A_c, so I follow the tradition to take aggregate income, Y_c, as given. Hall and Jones (and others using this approach) find that cross country differences in A_c are large and strongly positively correlated with income per worker. This can be seen clearly from the lower panel of figure 1, which plots countries' calibrated productivities against their incomes per worker for $\alpha = 0.33$, the average capital income share in my sample.[25]

Since the Cobb-Douglas case is not directly a limiting case of the general factor deepening model[26], I estimate α using the HOV-equations to check if a plausible value for this parameter can be obtained.

With a Cobb-Douglas production function the efficient factor use relative to the US is $D_{fc} = \frac{V_{fc}}{V_{fUS}}\left(\frac{K_{US}}{K_c}\right)^{\alpha}\left(\frac{H_{US}}{H_c}\right)^{1-\alpha} D_{fUS}$ and substituting the expression for productivities, the HOV-equations relative to the US become

$$\frac{Y_c}{Y_{US}}\left(\frac{K_{US}}{K_c}\right)^{\alpha}\left(\frac{H_{US}}{H_c}\right)^{1-\alpha} = \frac{Y_c}{Y_{US}}\frac{V_{fUS}}{V_{fc}} + \frac{F^*_{fc}(\alpha)}{V_{fc}} - \frac{Y_c}{Y_{US}}\frac{F^*_{fUS}(\alpha)}{V_{fUS}} + u_{fc}. \tag{36}$$

Using all 4 orthogonality conditions, $\hat{\alpha} = 0.54$, which is implausibly large (the null that α=0.33 is rejected at the 1 percent level). However, the J-statistic lets me reject the validity of the orthogonality conditions at the one percent level. As a consequence, I reestimate the model using only the first two orthogonality conditions. Now $\hat{\alpha} = 0.69$ (again the null that α=0.33 is rejected at the 1 percent level), which is even more implausible

[24] The assumption of a unit elasticity of substitution is necessary if productivity is Hicks neutral, if one wants to match the fact that there is no correlation between factor income shares and income per worker.

[25] This calibration is Caselli's (2005) and is an accounting view point of explaining income differences, because some part of differences in capital stocks may actually be due to differences in productivity, since in any neoclassical growth model an increase in A induces capital accumulation. Hall and Jones (1999) control for this by writing output as a function of the capital output ratio which is invariant to total factor productivity in the steady state. Results are not very sensitive to the particular approach taken.

[26] The efficient factor use (33)is not well defined if ϵ=1.

and once more the orthogonality conditions are unlikely to be satisfied. Next, I estimate α using only the moment conditions for the human capital content of trade. Now $\hat{\alpha} = 0.23$, which is somewhat more realistic but still the null that $\alpha = 0.33$ is rejected, while the orthogonality conditions seem to be valid now. Furthermore, using only the moment conditions for the physical capital content of trade, $\hat{\alpha} = 1$, which does not make much sense. Summing up, the Cobb-Douglas model with factor deepening performs poorly in terms of fitting the HOV-equations, especially in the case of physical capital.

Example 2: Conditional Factor Price Equalization (CFPE) & Trefler's Productivities

If one assumes instead that conditional on measuring endowments in efficiency units, factor prices are equalized across countries, relative factor productivities can be directly read off from relative factor prices,

$$\hat{\pi}_c = \hat{\pi}_{US} = \hat{\pi} = \frac{\pi_c}{A_{fc}} = \frac{\pi_{US}}{A_{fUS}}. \tag{37}$$

Since $w_c = \frac{s_{Hc} Y_c}{H_c}$ and $r_c = \frac{s_{Kc} Y_c}{K_c}$, I obtain a relationship between factor productivities, factor shares and factor-income ratios that is similar to Example 1.

$$\frac{A_{fc}}{A_{fUS}} = \frac{s_{fc}}{s_{fUS}} \frac{\left(\frac{Y_c}{V_{fc}}\right)}{\left(\frac{Y_{US}}{V_{fUS}}\right)} \tag{38}$$

Relative factor productivities are depicted in the upper panels of figure 2. Hence, if conditional factor price equalization is assumed to hold, rich countries are again more productive in the use of human capital, while poor countries make more efficient use of physical capital.

The efficient factor use relative to the US is now $D_{fc} = \left(\frac{s_{fc}}{s_{fUS}}\right) D_{fUS}$ and the HOV-equations relative to the US are $\left(\frac{s_{fc}}{s_{fUS'}}\right) \frac{Y_c}{Y_{US}} \frac{V_{fUS}}{V_{fc}} = \frac{Y_c}{Y_{US}} \frac{V_{fUS}}{V_{fc}} + \frac{F^*_{fc}}{V_{fc}} - \frac{Y_c}{Y_{US}} \frac{F^*_{fUS}}{V_{fUS}} + u_{fc}$, so that they are independent of θ.[27] Note also that as $\epsilon \to \infty$ the HOV equations relative to the US for the factor deepening case (33) converge to CFPE. Hence, the hypothesis of the aggregate elasticity of substitution being infinite is strongly rejected by the previous estimates. More evidence against CFPE can be obtained by checking directly the factor use matrices in efficiency units, which by hypothesis should be equal across countries, i.e. $D_{fUS} = \frac{A_{fc}}{A_{fUS}} D_{fc}$. In fact, for both factors there is a significant negative correlation between the average factor use in efficiency units and income per worker, so that poor countries use more efficient factors per unit of output than rich ones.[28]

At this point it seems adequate to relate my procedure to Trefler's (93) paper. His approach is to find a set of factor productivities that makes the HOV-equations hold exactly under the assumption of CFPE and then to compare productivity estimates with factor prices. To be more specific, he assumes that there are no Ricardian technology differences and that conditional factor price equalization holds at the world level (**A.10**), so that the factor use matrices of all countries are a simple transformation of the one of the US, $D_{fc} = A^{-1}_{fc} D_{fUS}$ and that all countries have identical input-output matrices, $B_c = B_{US}$. Then one can write the factor content of trade

[27] This reflects the fact that the values of θ such that CFPE holds at the world level is not unique. The implicit aggregate elasticity is ∞.

[28] The correlation is -0.26 (P-value 0.06) for human capital and -0.48 (P-value 0.0003) for physical capital.

in efficiency units as

$$F^*_{fc} = D_{US}(I - B_{US})^{-1}(X_c - \sum_{c' \neq c} M_{cc'}). \tag{39}$$

Normalizing $A_{fUS} = 1$ and dropping the equation for the US, the HOV equations in efficiency units (28) form a system of $C - 1$ independent linear equations in A_{fc}, which can be solved for the unknown factor productivities.

From (29) we see that if F^*_{fc} is small, relative productivities equal relative average products. In fact this is the case in the data if the factor content of trade is computed with the US factor use matrix and as a consequence productivities computed with Trefler's method are similar to the ones obtained from (38), which also explains why Trefler (1993) finds that relative productivities are similar to relative factor prices.[29] Rich countries are measured to have much higher human capital productivities than poor nations, while poor countries tend to have higher productivities of physical capital.[30]

Example 3: Multiple Cones

If there are multiple cones of diversification, the picture is quite different because the mapping between endowments, factor prices and factor productivities changes its shape, depending on whether a country specializes or lies in a cone. Again, let us take goods prices as parameters for now.

For countries that specialize in sector $i \in \{H, K\}$ the mapping from endowments, factor prices and income to factor productivities looks similar to Caselli's.

$$A_{fc} = \left(\frac{\sigma}{\sigma - 1}\right) \frac{1}{p_i} \left(\frac{1}{1 - \alpha_i} s_{fc}\right)^{\frac{\epsilon}{\epsilon - 1}} \frac{Y_c}{V_{fc}} \tag{40}$$

There are, however, some important differences. First of all, terms of trade effects matter. If goods prices in the sector in which a country specializes are higher, a lower factor productivity is sufficient to reach a given income. In addition, if $\epsilon > 1$, factor productivities are decreasing in the weight of capital in production, α_i, which varies across industries, because holding constant factor income shares an increase (decrease) in α_i would increase income per unit of physical capital (human capital). Since it is held constant, factor productivity must decrease. If $\epsilon < 1$, factor productivities are increasing in the weight of factors in production because an increase (decrease) in α_i would cause a decrease in output per unit of factor input for given factor income shares. Holding it constant, factor productivity must increase. Consequently, a high α_K implies that - holding everything else constant - capital abundant countries that specialize in the capital intensive good have high (low) capital productivities if factors are complements (substitutes).

[29] Productivities are not reported, but very similar to figure 2. These results are robust to using the technology matrix of other countries as reference and to using the true input-output tables of each country in computing the factor content of trade.

[30] These results differ from Trefler's. He finds that rich countries tend to use both labor and physical capital more efficiently than poor ones. The main reasons seem to be his small sample and his choice of very high depreciation rates of 15% (instead of 6%, as common in the development accounting literature), which imply capital-output ratios that are higher in rich countries.

For countries that lie within the cone of diversification the mapping between endowments, income, prices and parameters has another form.

$$A_{Hc} = \left(\frac{\sigma}{\sigma - 1}\right) \left[\frac{\alpha_H^{\epsilon} p_K^{1-\epsilon} - \alpha_K^{\epsilon} p_H^{1-\epsilon}}{\alpha_H^{\epsilon}(1-\alpha_K)^{\epsilon} - \alpha_K^{\epsilon}(1-\alpha_H)^{\epsilon}}\right]^{\frac{1}{\epsilon-1}} s_{Hc} \frac{Y_c}{H_c} \tag{41}$$

$$A_{Kc} = \left(\frac{\sigma}{\sigma - 1}\right) \left[\frac{(1-\alpha_H)^{\epsilon} p_K^{1-\epsilon} - (1-\alpha_K)^{\epsilon} p_H^{1-\epsilon}}{\alpha_K^{\epsilon}(1-\alpha_H)^{\epsilon} - \alpha_H^{\epsilon}(1-\alpha_K)^{\epsilon}}\right]^{\frac{1}{\epsilon-1}} s_{Kc} \frac{Y_c}{K_c} \tag{42}$$

Factor productivities are again linear functions of factor-output ratios and also of factor income shares. Terms of trade effects are at work too, but they are more complex than for countries which specialize. Now productivities are decreasing in the price of the sector that uses the factor intensively. The explanation is again the Stolper-Samuelson effect. An increase in the price of an industry shifts production towards that sector, and increases the income share of that factor. If the income share and income per unit of factor are held constant, the factor must have lower productivity.

The lower panels of figure 2 plot A_{Hc} and A_{Kc} against income per worker for a model with two sectors and multiple cones, in which goods prices have been solved endogenously for the optimal θ.[31] Again, human and physical capital are estimated to be complements. 39 poor countries specialize in the human capital intensive sector, while the rest of the world lies in a common cone of diversification. The correlation between A_{Hc} and income per worker is again strongly positive and poor countries are still estimated to be more productive in the use of physical capital.

Using the HOV equations to Compare Model Fit

To see which of the different versions of the model performs best I use an economic measure of performance - I evaluate the fit of the HOV-equations in efficiency units at $\hat{\theta}$. I provide the results of the following classical tests. First the "sign test" that reports the fraction of observations for which the left hand side (measured factor content) and the right hand side (predicted factor content) of the HOV-equations (28) have the same sign. Second the "weighted sign test" that weights observations by the magnitude of factor flows, third the slope coefficient, β, of a regression of the measured on the predicted factor content, with a theoretical value of one. Fourth, the R-squared from this regression and finally the ratio of the variances of the measured and the predicted factor content, a measure known as the "missing trade" statistic.

Table 2 reports the results of these tests. It is quite obvious that the factor deepening model with complementary factors ($\epsilon = 0.836$) easily outperforms all its competitors - the Cobb-Douglas model ($\alpha = 0.33$), the CFPE model and also the - admittedly overly simplistic - two sector multiple cone model in virtually all tests. For example, the weighted sign statistic is 0.97 for physical capital and 0.96 for human capital for the factor deepening model, which is by far closer to the theoretical value of one than for any of the other models. It is also the only model that gets βs of the right sign and roughly correct magnitudes and that does not suffer from "missing trade"

[31] $\alpha_H = 0.06$, $\alpha_K = 0.77$, $\epsilon = 0.5$ and $\beta_H = 0.84$. Meaningful standard errors for these estimates are hard to obtain, since J is not continuously differentiable in θ at $\hat{\theta}$.

Figure 3 plots the measured factor content against the predicted factor content of trade for the factor deepening case. The good fit of the model, especially for human capital, is clearly visible, while measured factor trade is somewhat too large for physical capital. Hence, I conclude that the model by far best supported by the HOV-equations is the factor deepening model with factor augmenting productivities and weak complementarity between human and physical capital.

Let me therefore discuss the features of this world in somewhat more detail. Coming back to the upper panels of figure 1, we see that rich countries are much more productive in the use of human capital than poor ones. The correlation between A_{HC} and income per worker is 0.432 and strongly significant (P-value: 0.000). There are some outliers, like Botswana and Singapore, which have extremely low labor income shares in the data and therefore very high human capital productivities. Since the data quality on labor income shares is not very good, this should be taken with some caution. The ratio of human capital productivity of the 90th to the 10th percentile is 10.72. In the case of physical capital, there is a no clear relation between factor productivity and income per worker. The correlation between A_{Kc} and income per worker is slightly negative (-0.031) but insignificant (P-value: 0.767). The ratio of physical capital productivity of the 90th to the 10th percentile is 10.72. A number of very poor African economies are measured to have very high capital productivities, which is due to their extremely low capital output ratio. When we disregard these countries, there is a positive relation between capital productivity and income per worker, with Sweden, the UK, Ireland, Switzerland, France, Belgium and the US measured to have very high capital productivities. While it is quite intuitive that rich countries are much more efficient in their use of human capital, it is less clear, why a number of very poor African countries should be so productive in the use of physical capital. The fact that some of the poorest countries in the world use so little physical capital in production could well reflect distortions in capital markets, like high tariffs on capital goods and malfunctioning of credit markets instead of high capital productivities. In this world there are incentives for human capital to move to rich countries and for capital to move to poor ones because returns in physical units are not equalized.

A further feature of the factor deepening world is that the physical to human capital ratios in efficiency units are quite similar across countries. This is due to the fact that rich countries, which have large physical to human capital ratios, have very high human capital productivities and that there is no clear relation between capital productivity and income per worker.

Turning to the estimate of the elasticity of substitution, note that my estimates are similar to those of Antràs (2004), who estimates the elasticity of substitution between labor and capital for the US aggregate production function from time series data allowing for biased technological change. He finds values in the range of 0.5 to 0.9, so my estimates are consistent with time series evidence for the US.

Another point worth mentioning is the relation of my findings to the extensive literature on the HOV-equations. Even though it is well known that these relations apply to a wide class of models, it seems interesting that the model that actually best fits the HOV-equations (at least in the relatively restrictive class of models considered in this paper) is a one sector economy, in which any Heckscher-Ohlin style trade is absent.

4.1 Development Accounting

Having said this, let me now perform the typical development accounting exercise which is to ask why some countries are so much richer than others. The first question I pose is: What would the world income distribution look like if all countries had the same per capita factor endowments given their factor productivities? The experiment is to endow each country at a time with the per worker endowments of human and physical capital of the US and to compute its counterfactual income per worker for given productivities A_{Hc} and A_{Kc}[32].

In the factor deepening world, the counterfactual income per worker is given by

$$\tilde{y_c} = [\alpha(A_{Kc}k_{US})^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(A_{Hc}h_{US})^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}. \tag{43}$$

The upper left panel of figure 4 plots predicted income per worker given US per capita endowments against income per worker in the case of factor deepening.[33] The ratio of income per worker of the 90th to the 10th percentile is reduced from 25 to 4.5. The lower left panel plots the output gain (the ratio of predicted to actual income per worker) for this case. Obviously, income gains are largest for poor countries.

Alternatively, I ask the question what the world income distribution would look like if all countries had the US factor productivities but their own factor endowments. In this case, counterfactual income is the following,

$$\tilde{y_c} = [\alpha(A_{KUS}k_c)^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(A_{HUS}h_c)^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}. \tag{44}$$

The upper right panel plots predicted against actual income per worker, while the lower right panel plots the predicted output gain. The ratio of income per worker of the 90th to the 10th percentile is reduced from 25 to 6.96, which is a smaller reduction in inequality than in the first case, where all countries have the same endowments per worker. This can also be seen from output gains which are smaller for most countries in the second case. [34] The reason is that the US has some of the largest per capita human capital and physical capital endowments and a very high human capital productivity, whereas its physical capital productivity is rather low, so that in efficiency units poor countries have higher and more balanced per capita endowment levels when endowed with the US per capita endowments than when given the US factor productivities.

[32] This experiment differs somewhat from the one performed by Caselli (2005), who asks the question: How much dispersion of the income distribution could we observe if all countries had the same A_K and A_H? He defines 100% success as a model that can generate the actual dispersion of the cross country income distribution without productivity differences. However, with factor augmenting productivities, this statistic is not very meaningful because the effect of productivities and endowments on the variance of income cannot be separated. I ask the question how compressed the income distribution would be if countries had their own productivities but the same endowments, which seems more natural to me because it addresses the question which policy would help to increase the income of poor countries.

[33] For $\epsilon = 0.836$.

[34] Caselli (2005) notes that the factor deepening model is able to replicate the cross country variance in income per capita even if all countries have the same productivities (those of the US) when ϵ is sufficiently low (around 0.5). In that sense the whole cross country variation in income per worker is "explained" by factor endowments. However, this does not imply that factor accumulation would help much in reducing cross country income differences given that we know that productivities differ. In fact, the lower the elasticity of substitution, the less powerful is factor accumulation in reducing income differences.

5 Conclusion

This paper has developed a quantitative Krugman-Helpman-Heckscher-Ohlin (1985) model of the world economy in order to estimate cross country differences in factor productivities using an approach that integrates the development accounting literature and the research on the Heckscher-Ohlin-Vanek (HOV) Theorem. This has enabled me to simultaneously fit data on income, factor prices, endowments and the factor content of trade to calibrate productivities, which in turn has allowed me to evaluate the fit of the model and to estimate the elasticity of substitution between human and physical capital with great precision.

My main findings can be summarized as follows: The model best supported by the data features an aggregate neoclassical production function with an elasticity of substitution between human and physical capital that is significantly lower than one. This implies that human and physical capital are (weak) complements and productivities are factor augmenting, while the standard Cobb-Douglas model used in the quantitative growth literature is clearly rejected. Rich countries have much higher productivities of human capital than poor ones, while there is no clear relation between the productivity of physical capital and income per worker. My results also show that this one sector economy, where differences in factor prooportions do not constitute a reason for trade, fares far better in terms of explaining cross country flows of efficient factor services (i.e. fits the HOV-equations better) than a simple generalized Heckscher-Ohlin model, where this motive is prominently present.

Although this paper has taken us a small step further in the estimation of cross country differences in productivity, it has also made evident some of the limits of the Helpman-Krugman-Heckscher-Ohlin model. Specifically, since the model has no trade costs, within the conditional factor price equalization set there is no direct connection between local factor abundance and export shares in sectors that are intensive in abundant factors, while outside this set predicted specialization patterns are too extreme to be realistic. These disproportionate predictions may have potentially lead to a rejection of a Heckscher-Ohlin style world in favor of a one sector economy. An interesting alternative approach has recently been taken by Romalis (2004), who modifies a version of the Heckscher-Ohlin model without productivity differences to get clear predictions on trade in goods instead of trade in factors. This enables him to use very disaggregated trade data and to show the existence of strong Rybczynski effects. Another option is to extend the Eaton and Kortum (2002) model to Heckscher-Ohlin trade, as this model is more tractable in multi-country general equilibrium.

A further restriction of the present work is that I have abstracted from sectoral (Ricardian) productivity differences and income differences due to increasing returns. Ricardian productivity differences shift production towards those sectors in which countries have high productivities, while increasing returns in combination with trade costs tend to increase the income of countries with large markets. Both mechanisms are worth further investigation.

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6 Appendix

6.1 Data

To make my results comparable with the development accounting literature I follow Caselli (2005) as closely as possible in the construction of the data. Data are from two main sources: The first one is the Penn World Table (Version 6.2), which provides the data for income per worker and physical capital stocks in purchasing power parities. The other main source is the Global Trade Analysis Project (GTAP) Version 6, which has information on input-output tables and bilateral sectoral trade data at 57 sector aggregation for the year 2001.[35]. Data on income and endowments are available for 96 countries, while the sample size for input-output tables and trade data is 53.

Capital stocks for 2001 are computed from the PWT using the perpetual inventory method, ie $K_{ct} = I_{ct} + (1-\delta)K_{ct-1}$. Here, I_{ct} is real aggregate investment in PPP[36]. Following Caselli, I choose a depreciation rate, δ, of 6% per year, and $K_0 = I_0/(g+\delta)$ where I_0 is investment in the first year with available data and g is the average geometric growth rate for investment between that year and 1970.

Human capital is constructed from average years of schooling in the population over 25 in the year 1999. Data on average years of schooling are from Barro and Lee (2001). These are converted into human capital following Caselli (2005) using the formula $h = e^{\phi(s)}$, where $\phi(s)$ is piecewise linear with slope 0.134 for $s <= 4$, 0.101 for $4 < s <= 8$ and 0.068 for $s > 8$. Aggregate human capital is computed as $Hc = h_c L_c$, where L_c is the number of workers computed from the Penn World Tables as RGDPPCH*POP/RGDPWOK. Here, RGDPCH is real GDP per capita using the chain series method and RGDPWOK is real GDP per worker constructed with the same method.

Aggregate income for the year 2001, Y_c, is real GDP in PPP computed with the chain method, defined as RDGPCH*POP.

Since I need an additional data point per country in order to calibrate factor productivities, I construct estimates of average unskilled wages for all countries in the sample. To obtain wage data, I proceed in the following way. As a first step I use data on country labor income shares from Bernanke and Guerkaynak (2001). Following a procedure suggested by Gollin (2002), they have adjusted raw data on labor shares for the labor of self-employed workers, who make up a large fraction of the labor force in most developing countries. Because their dataset includes only 54 countries of my sample, I regress these labor shares on controls and predict labor shares out of sample for the rest of the countries. Right hand side variables include real trade openness from

[35]The sectors are: paddy rice, wheat, other grains, vegetables, oil seeds, sugar cane, plant based fibres, other crops, cattle, animal products, raw milk, wool, forestry, fishing, coal, oil, gas, other minerals, cattle and sheep meat, other meat, vegetable oils, dairy products, processed rice, sugar, other food, beverages and tobacco, textiles, wearing apparel, leather products, wood products, paper and publishing, petroleum, chemicals and rubber, mineral products, ferrous metals, other metals, metal products, motor vehicles, transport equipment, electronic equipment, machinery, other manufactures, electricity, gas distribution, water, construction, trade, other transport, water transport, air transport, communication, financial services, insurance, business services, recreational services, education and health, dwellings

[36]Computed as RGDPL*POP*KI, where RDGPL is real GDP per capita computed with the Laspeyres index, POP is population and KI is the investment share of RGDPL.

the PWT averaged over 15 years, and regional dummies.

Once labor shares are constructed for all countries, PPP wages are computed as $w_c = \frac{S_{Hc}Y_c}{H_c}$, where S_{Hc} is the labor share in country c. Rental rates are then backed out using the formula $r_c = \frac{Y_c - w_c H_c}{K_c}$.[37]

Direct factor use by industry, V_{ic}, is computed by assuming that sectoral factor use is proportional to payments to the factor by industry. These are scaled such as to fit aggregate factor endowments H_c and K_c. Sectoral payments to capital and labor are from the GTAP (version 6) input-output accounts. Factor use per unit of output V_{ic}/Q_{ic} is computed by converting sectoral gross output from GTAP into international dollars using price indices from the PWT and dividing sectoral factor use by deflated gross output.

Input-output tables $\bar{B}_c$ as well as bilateral sectoral trade data X_c and $M_{cc'}$ are taken from GTAP. Input-output tables are converted into international dollars using PWT price indices. The B-matrix is constructed from the input-output tables, following Trefler and Zhu (2005).

6.2 The Productivity Calibration Problem (PCP)

Definition 2: A **Productivity Calibration Problem** (**PCP**) is a collection of goods prices $\{p_i\}$, efficiency adjusted wages $\{\hat{w}_d\}$, efficiency adjusted rental rates $\{\hat{r}_d\}$, numbers of sectoral varieties $\{N_{id}\}$ and factor productivities $\{A_{Hc}\}$, $\{A_{Kc}\}$ such that given a cross section of human capital endowments $\{H_c\}$, physical capital endowments $\{K_c\}$, wages $\{w_c\}$, rentals $\{r_c\}$ and parameters $\{\alpha_i\}$, $\{\beta_i\}$, ϵ, σ and f the following system of equations holds for all $d \in D$:

$$\frac{\sigma}{\sigma-1}[\alpha_i^\epsilon \hat{r}_d^{1-\epsilon} + (1-\alpha_i)^\epsilon \hat{w}_d^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \geq p_i \tag{A-1}$$

with

$$\{p_i - \frac{\sigma}{\sigma-1}[\alpha_i^\epsilon \hat{r}_d^{1-\epsilon} + (1-\alpha_i)^\epsilon \hat{w}_d^{1-\epsilon}]^{\frac{1}{1-\epsilon}}\}N_{id} = 0 \tag{A-2}$$

$$\sum_{i\in I}[\alpha_i^\epsilon \hat{r}_d^{1-\epsilon} + (1-\alpha_i)^\epsilon \hat{w}_d^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}}(1-\alpha_i)^\epsilon \hat{w}_d^{-\epsilon} f\sigma N_{id} = \sum_{c\in d} A_{Hc}H_c \tag{A-3}$$

$$\sum_{i\in I}[\alpha_i^\epsilon \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^\epsilon \hat{w}_c^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}}(\alpha_i)^\epsilon \hat{r}_d^{-\epsilon} f\sigma N_{id} = \sum_{c\in d} A_{Kc}K_c \tag{A-4}$$

$$p_i(\sigma-1)f\sum_{d\in D} N_{id} = \beta_i \sum_{c\in C} Y_c \quad i = 1, ..., I-1 \tag{A-5}$$

$$\prod_{i=1}^{I}\left(\frac{N_i^{\frac{1}{1-\sigma}} p_i}{\beta_i}\right)^{\beta_i} = 1; \tag{A-6}$$

$$A_{Hc} = \frac{w_c}{\hat{w}_d} \tag{A-7}$$

[37] Caselli and Feyrer (2006) stress that poor countries have a large fraction of capital income that goes to non-reproducible capital (land and natural resources) and that this tends to upward-bias measured rental rates in these countries if this factor is not considered separately. Since my model has only two factors and all income must be payed to some factor, the above way to calculate rentals is consistent, even though it might exacerbate differences in rentals.

$$A_{Kc} = \frac{r_c}{\hat{r}_d} \tag{A-8}$$

Regarding the connection between the **PCP** and an **Equilibrium**, one can establish the following relationships.

Lemma 1: If given $\{H_c\}$, $\{K_c\}$, $\{w_c\}$, $\{r_c\}$, parameters $\{\alpha_i\}$, $\{\beta_i\}$, σ and ϵ and f we have that $\{p_i\}$, $\{\hat{w}_d\}$, $\{\hat{r}_d\}$, $\{N_{id}\}$, $\{A_{Hc}\}$, $\{A_{Kc}\}$ are a solution to the **PCP** then $\{p_i\}$, $\{\hat{w}_d\}$, $\{\hat{r}_d\}$, $\{N_{id}\}$ are also an **Equilibrium** given $\{A_{Hc}H_c\} = \{\hat{H}_c\}$, $\{A_{Kc}K_c\} = \{\hat{K}_c\}$.

Proof: Follows from inspecting the equations of **PCP**.

Lemma 2: If given $\{H_c\}$, $\{K_c\}$, $\{\alpha_i\}$, $\{\beta_i\}$, ϵ, σ and f we have that $\{p_i\}$, $\{w_d\}$, $\{r_d\}$, $\{N_{id}\}$ are an **Equilibrium** then they also solve the **PCP** given $\{H_c\}$, $\{K_c\}$, $\{w_d\}$ and $\{r_d\}$ with $\{A_{Hc}\} = \{A_{Kc}\} = \{1\}$.

Proof: Follows from inspecting the equations of **PCP**.

6.3 Uniqueness of Solution to PCP

Simsek, Ozdaglar and Acemoglu (2006) have derived a sufficient condition for the uniqueness of the solution to nonlinear complementarity problems. If this condition is met it guarantees uniqueness of the solution to **PCP** for the particular parameter values considered.

Let $F : R^n \to R^n$ be a function. The nonlinear complementarity problem is to find a vector x that satisfies the following

$$x \geq 0, F(x) \geq 0 \tag{A-9}$$

$$x^T F(x) = 0 \tag{A-10}$$

Note that **PCP** has the structure of a nonlinear complementarity problem.

Denote the set of solutions to (A-9), (A-10) by NCP(F). Define the index sets

$$I^{NB}(x) = \{i \in \{1, ..., n\} | x_i > 0\} \tag{A-11}$$

$$I^F(x) = \{i \in \{1, ..., n\} | F_i(x) = 0\} \tag{A-12}$$

A.A1: There exists a compact set $C \subset R^n_+$ such that for all x $\in R^n_+ - C$ there exists some $y \in C$ and $i \in 1, ..., n$ such that $(y_i - x_i)F_i(x) < 0$.

A.A2: Let $U^n_+ \subset R^n$ be an open set containing R^n_+ and $F : U^n_+ \to R^n$ be a continuous function that is continuously differentiable at every $x \in NCP(F)$. We have $det(\bigtriangledown F(x)|_J) > 0$ for every $x \in$ NCP(F) and for every index set J such that $I^{NB}(x) \subseteq J \subseteq I^F(x)$.

Theorem (Simsek, Ozdaglar, Acemoglu): Let $U_+^n \subset R^n$ be an open set containing R_+^n and $F : U_+^n \to R^n$ be a continuous function which is continuously differentiable at every x $\in NCP(F)$. Let Assumptions **A.A1** and **A.A2** hold. Then NCP(F) has a unique element.

As noted above the **PCP** is a nonlinear complementarity problem. Continuity can be checked by inspection. In addition, the **PCP** satisfies **A.A1**.

Proof: Let $p^{max} \in R_+ \equiv \beta_i \sum_{c \in C} Y_c$. Let C be the rectangle $(0, (p^{max}, ..., p^{max}))$. Let $F_i(x) = p_i \sum_{d \in D} Q_{id} - \beta_i \sum_{c \in C} Y_c$. Then for all $x \in R^n - C$ we have that $F_i(x) > 0$. Hence, it follows that by choosing $y = 0 \in C$ the condition $(y_i - x_i)F_i(x) < 0$ is satisfied.

The condition that the determinant of the Jacobian must be positive in NCFP(x), can be checked numerically. It is satisfied for all the examples with $\epsilon > 1$. When $\epsilon < 1$ multiple solutions may exist. However, the solution is unique provided that the following additional assumptions are made: 1) Countries are ranked by K_c/H_c and 2) Factor price equalization holds, when a set of countries can have equalized efficient factor prices.

Figure 1: Factor deepening case. The upper panels plot factor augmenting productivities against income per worker for the factor deepening case ($\epsilon = 0.836$). The lower panel plots TFP against income per worker in the Cobb-Douglas case ($\epsilon = 1$).

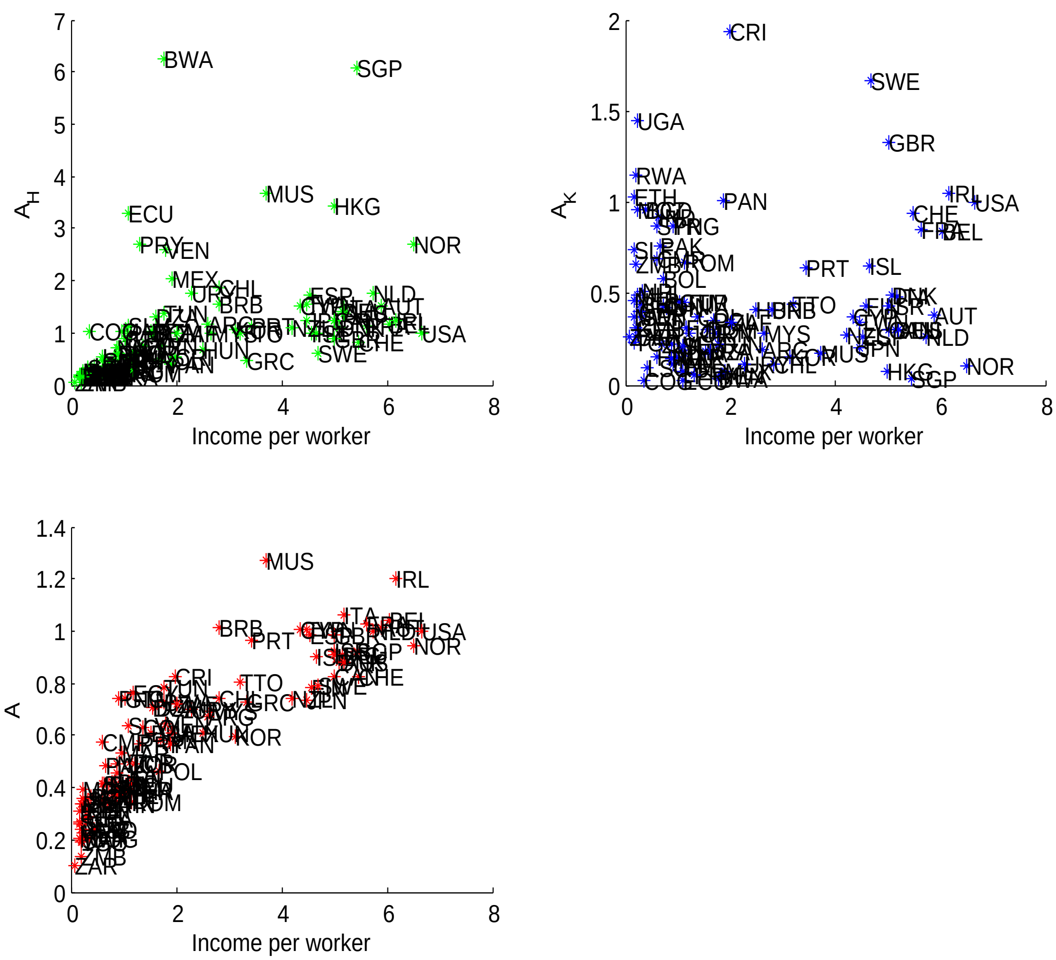

Figure 2: Heckscher-Ohlin case. The upper panels plot factor augmenting productivities against income per worker for the case of conditional factor price equalization. The lower panels show the case of multiple cones ($\epsilon = 0.5$).

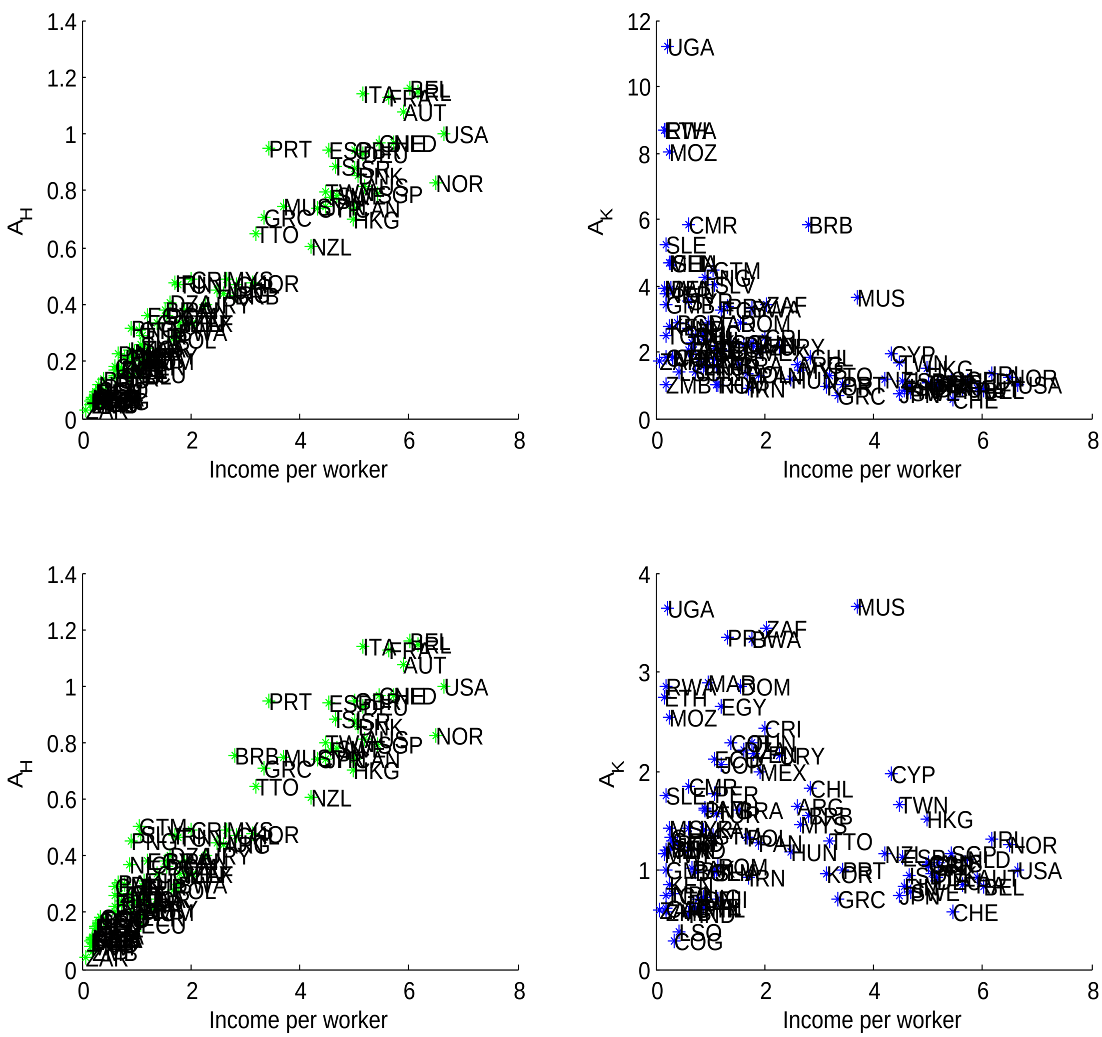

Figure 3: Fit of the HOV equations. The panels show a plot of the measured efficient factor content of trade (vertical axis) against the predicted efficient factor content of trade (horizontal axis) for the factor deepening case with $\epsilon = 0.836$.

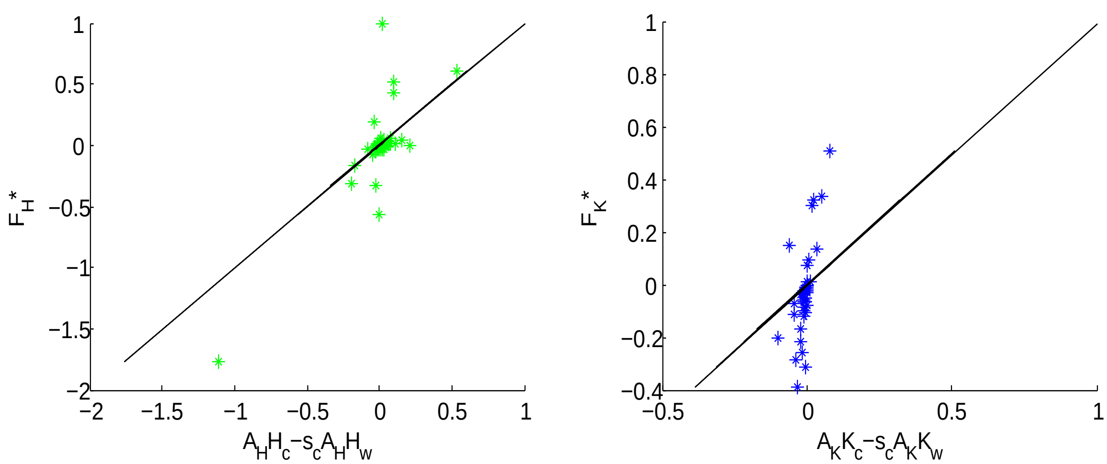

Figure 4: Development Accounting. The upper panels plot the predicted income per worker (vertical axis) against actual income per worker (horizontal axis) for the case in which all countries have the US endowments of factors per worker (left panel) or the US factor productivities (right panel) for the factor deepening world with $\epsilon = 0.836$. The lower panels plot the output gain (vertical axis) against income per worker (horizontal axis) for the same experiments.

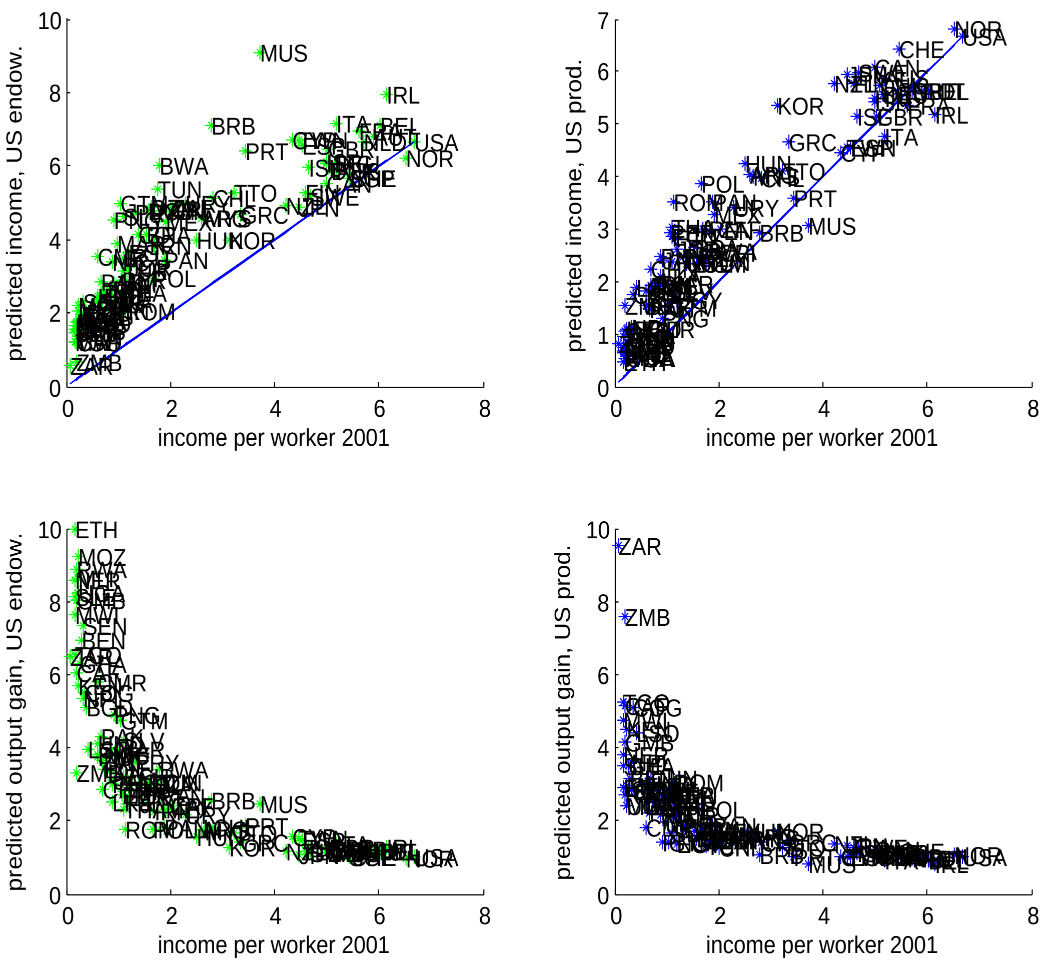

Table 1: Model fit 1 - Parameters. GMM estimates of the elasticity of substitution between human and physical capital (ϵ) (the capital income share (α) in the Cobb-Douglas case) using different sets of moment conditions from the HOV equations (29). The last column is the P-value for the validity of the overidentifying restrictions.

Case	Moment Cond.	ϵ	Std(ϵ)	t-statistic	J-statistic	P-value
Factor	all	0.8363	0.009	-18.29	0.234	0.972
Deepening	$E(u) = 0$	0.8342	0.012	-13.23	0.179	0.672
	F_H	0.8158	0.046	-3.99	0.006	0.9392
	F_K	0.8373	0.009	-17.75	0.028	0.868
Cobb-	all	0.5407	0.035	6.08	16.123	0.001
Douglas (α)	$E(u) = 0$	0.6868	0.04	8.79	2.617	0.106
	F_H	0.2311	0.064	-1.53	0.098	0.755
	F_K	1	0	-	356.067	0
Multi Cone	all	0.5	-	-	-	-

Table 2: Model fit 2 - HOV-equations. Fit of the HOV equations for the factors human capital H and physical capital K of different specifications. *Sign* is the fraction of observations for which measured and predicted factor content of trade have the same sign, *W. Sign* weights the signs with the magnitude of factor flows, β is the slope coefficient in a regression of the measured on the predicted factor content, R^2 is the R-square from this regression and *Missing Trade* is the ratio of the variances of the measured relative to the predicted factor content of trade.

Model	Factor	Sign	W. Sign	β	R^2	Missing Trade
Factor	H	0.698	0.967	1.446	0.656	3.184
Deepening	K	0.925	0.936	3.211	0.693	14.88
Cobb	H	0.34	0.073	-0.341	0.495	0.23
Douglas	K	0.34	0.412	-0.0003	0.121	0
CFPE	H	0.738	0.459	-0.324	0.148	0.7
	K	0.34	0.467	-0.0009	0.096	0
Multi	H	0.623	0.369	-0.077	0.038	0.154
Cone	K	0.509	0.8	-0.0007	0.197	0