Monopoly, Diversification through Adjacent Technologies, and Market Structure

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Abstract. The theoretical literature on technological competition has been mostly concerned with various aspects of innovative activity in a single market. By contrast, this paper studies the adoption of a sequence of product innovations in two markets characterized by a common technology base, and illustrates the effects of technological rivalry and preemption. Under a perfect information scenario, it is shown in a two incumbent model that if the innovation is drastic (total replacement of the old product), under certain conditions the fear of being preempted by the entrant forces the firms to diversify their product lines by adopting the innovations across each other's markets. On the other hand, with non-drastic innovation (partial replacement of the old product), it is more likely for the firms to diversify in their own product lines. Out of a class of equilibria characterized under non-drastic innovation, one is optimal in which innovations are adopted in the firms' own markets. In the Pareto inferior equilibria, the firms either adopt innovations in each other's market so that incumbency changes hands or jointly adopt both innovations in two separate product lines. Perfect Bayesian equilibria are characterized under an asymmetric information scenario where one of the firms is assumed to have complete information about the relevant costs of adopting an innovation in a separate product line. If the priors are based on pessimism, it is more often subject to exploitation by the informed firm leading to pooling equilibrium, while optimistism more often leads to diversification and to a competitive market structure in both product lines under a separating equilibrium. In all the cases considered, both innovations are adopted, and in most cases they are adopted by the high cost entrant. The former is socially desirable, but the latter is not. More competitiveness necessarily implies wasteful expenditure by the high cost firm. Lack of competitiveness and technological rivalry, on the other hand, imply that maximum product diversity may not be achieved.

Keywords: tehnological rivalry, preemption, adoption of innovations, upgrading.

1. Introduction

Almost all product innovations in recent times have been part of a sequence of innovations and upgrades in a certain product line rather than being a totally generic innovation that has never been materialized before. Once a product innovation has been first adopted in its most rudimentary form and has been successful in the market, it is almost certain that successive generations of upgraded versions with additional features will follow. As successive innovations generate rich product diversity in the marketplace, it is always those firms at the cutting edge of technology which persistently carry out R&D activity for upgrading and "perfecting" the product, to achieve or maintain a monopoly position that will skim the profits until a next generation product comes along. This, simply, is the Schumpeterian innovative process.

We also observe in the "sequences of innovations" process that the upgrading products will incorporate developments and/or findings of other product lines. While the underlying basic science required for a certain product innovation in a sequence of innovations could be very distinct from another innovation in a separate product line, it is becoming more likely that both product lines will share and contain the latest developments in some other technology and R&D base. Examples of these innovations and 'add ons' are all around us in cell phones, digital cameras, pocket pcs (PDAs), portable audio, video, digital imaging, communiation and storage devices. We observe many other examples of technological convergence in recent product generations especially in consumer electronics industries that is fuelled by the shrinking size of semiconductors .

I call the process in which composite technologies are increasingly embodied in generations of successive product innovations "technological convergence". For example, cameras and camcorders use the same optics technology base. As they become more 'sophisticated' we observe that microprocessors of various sort are being incorporated to increase their functions and capacity. VHS recorders followed by 8 mm camcorders which also function as players, are now being replaced with digital recording and digital cameras which all can be incorporated in a cell phone with still and movie camera features. This sequence illustrates the common practice of leading firms of technologically progressive industries diversifying their R&D base further from their initially established technological base, as the composite technology required to upgrade a product becomes increasingly diverse.

This aspect of increasing product diversification under technological convergence has been ignored in the theoretical R&D and innovation adoption literature. According to the prevailing theory firms may diversify into multiple product lines in response to excess capacity of productive factors (Montgomery and Wernerfelt, 1988). Accordingly, technological scope economies embodied in the excess heterogeneous productive factors create incentives to diversify[1]. In this article I attempt to explain firm diversification based on strategic adoption of innovations and add on features. Specifically, I explore the effects of the strategic adoption of product innovations by separate monopolists on market structure. I conjecture that most innovations which are subject to strategic behavior on the part of the innovating or adopting firms are part of a sequence of innovations in a product line. By a "sequence of innovations" in a product line I mean a series of either qualitative (better functioning and/or more

[1] In related literature Aron stresses the principal agent relationship between the owners and managers of firms and shows that diversification is an optimal response to the moral hazard problem facing firms' owners (Aron, 1988).

durable, etc.)[2] or quantitative (with additional functions/features incorporated)[3] , or some combination of both, upgrading opportunities. Rosen calls this process'add-on innovations,' (Rosen, 1991) but emphasises only the case of process innovations. I follow Reinganum's (Reinganum, 1985) market driven definition of innovation, but focus only on the product innovations instead of process innovations. Reinganum calls an innovation "drastic" if the innovator becomes a monopolist in the post adoption market. I, instead, call an innovation drastic if the add-on technology that upgrades the product makes the previous version obsolete by completely replacing it. Hence, a drastic innovation that embodies an add-on technology can be adopted by more than one firm. Consider the audio cassette tapes that almost totally replaced the earlier betamax tapes, or the color TV that in most part replaced the black and white TV which in turn will possibly be replaced by HDTV in the very near future. These are new products that use the same basic technology with the old products. Similarly, a non-drastic innovation that partly replaces the old product – in the sense of a demand shift – can make the adopting firm a monopolist in the upgraded product market. Sony, for example, used to be a monopoly in the 'recordable CD player' market in the early 1990's while the 'CD player' market was competitive. Consequently, a single success does not mean that the successful firm reaps monopoly profits forever (Reinganum, 1985). Rather, monopoly profits are earned only until the next, better innovation is developed and successfully adopted by the innovative firm and is accepted by the consumers [4]

[2] Examples to the idea of qualitative upgrading could be found in the innovations in pharmaceuticals industry (in headache pills and cold medicine market it would imply rapid effectiveness and fewer side effects), synthetics industry (in cassette/video tapes and photograph films/digital imagemaking industry it would imply higher resolution), and high definition TV (better picture quality).

[3] Examples to the quantitative upgrading would be certain products in consumer electronics industry (calculators, computers, DVD players, cell phones, cameras, and etc).

[4] Take the case of Home Video Revolution:Ampex in 1963 offered the first consumer version of a videotape recorder at an exorbitant price of $30,000; other iterations would follow, such as Sony's introduction of the videocassette recorder (VCR) in 1969, and the introduction of the U-Matic in 1972.In 1972, the AVCO CartriVision system was the first videocassette recorder to have pre-recorded tapes of popular movies (from Columbia Pictures) for sale and rental – three years before Sony's Betamax system emerged into the market. However, the company went out of business a year later The appearance of Sony's Betamax (the first home VCR or videocassette recorder) in 1975 offered a cheaper sales price of $2,000 and recording time up to one hour; this led to a boom in sales - it was a technically-superior format when compared to the VHS system that was marketed by JVC and Matsushita beginning in 1976 In 1976, Paramount became the first to authorize the release of its film library onto Betamax videocassettes. In 1977, 20th Century Fox would follow suit, and begin releasing its films on videotape In 1977, RCA introduced the first VCRs in the United States based on JVC's system, capable of recording up to four hours on 1/2" videotape.By the late 70s, Sony's market share in sales of Betamax VCRs was below that of sales of VHS machines; consumers chose the VHS' longer tape time and larger tape size, over Sony's smaller and shorter tape time (of 1 hour). Video sales - the first films on videotape were released by the Magnetic Video Corporation (a company founded in 1968 by Andre Blay in Detroit, Michigan, the first video distribution company) - it licensed fifty films for release from 20th Century

The idea of the models used in this paper is that each monopolist must choose either to preserve its own monopoly position by adopting an innovation in its product line or to challenge the incumbent monopolist in another market by adopting an innovation in that product line. Special features of the models arise from the differential costs in adopting innovations across separate product lines. Namely, if there is a product innovation to be adopted, any firm that is considering adoption is a potential entrant to the market which the adoption is expected to create. With the assumption that innovations have no patent protection and are common knowledge to all firms, the cost of entry to the potential market is the cost of adopting the innovation. I maintain that the cost of adoption would not be identical across existing firms considering undertaking R&D for this purpose. Unless the innovation is an original idea unrelated to any existing product market, the costs of adoption will be different across firms, depending on how suitable each firm's R&D program is to the requirements of adoption and how close its existing product line(s) is to the innovation under consideration. If the innovation is part of a sequence of innovations in a product line, the existing firm(s) in that product line would have a cost advantage in adoption. This is due to the experience in production and learning by doing in R&D as well as the already established and technologically substitutable cospecialized assets which they might simply alter for the new product. This paper seeks to shed light on the importance of cost advantages in adoption and answers the question why firms sometimes give up these advantages and choose to enter a new market.

The theoretical literature on R&D and the adoption of new technologies has been concerned with different aspects of innovative activity in a single market. One main body of research concentrates on the incentives and the process of bringing about inventions. To cite a few representative contributions among many, see: (Dasgupta and Stiglitz, 1980), (Dasgupta, 1986) on the industrial structure, uncertainty and the speed of R&D; (Harris and Vickers, 1985), on patent races and persistence of monopoly (Gilbert and Newberry, 1982); and on licensing of innovations and network externalities (Katz and Shapiro, 1985).

Another line of research concentrates on the strategic aspects of the adoption of new technologies, again among the many, the following are few representative contributions: on the timing of innovation and the diffusion of new technology

Fox for $300,000 in October, 1977; it began to license, market and distribute half-inch videotape cassettes (both Betamax and VHS) to consumers; it was the first company to sell pre-recorded videos; M*A*S*H (1970) was Magnetic's most popular title.Video rentals - in 1977, George Atkinson of Los Angeles began to advertise the rental of 50 Magnetic Video titles of his own collection in the Los Angeles Times, and launched the first video rental store, Video Station, on Wilshire Boulevard, renting videos for $10/day; within 5 years, he franchised more than 400 Video Station stores across the country.In 1978, Philips introduced the video laser disc (aka laserdisc and LD) – the first optical disc storage media for the consumer market; Pioneer began selling home LaserDisc players in 1980; eventually, the laserdisc systems would be replaced by the DVD ("digital versatile disc") format in the late 1990s.VHS video players, laser disc players and the release of films on videocassette tapes and discs multiplied as prices plummeted, creating a new industry and adding substantial revenue and profits for the movie studios. (15 April 2007 ¡http://www.filmsite.org/70sintro.html¿.)

(Reinganum, 1981); on rent dissipation (Fudenberg and Tirole, 1985, 1987); on market entry dynamics (Smirnov and Wait, 2007); on the sequence of innovations and industry evolution (Reinganum, 1985), (Vickers, 1986); and on divisionalization (Schwartz and Thompson, 1986).

The main effects governing R&D and technological rivalry have been mostly analyzed using game theoretical tools. The results, with a few exceptions (Arrow, 1962), generally support Schumpeter's (Schumpeter, 1942) thesis that monopoly situations and innovativeness are intimately related. Nevertheless, the focus has been on a single market where an incumbent and an entrant engage in some sort of technological supremacy game mostly for process innovations rather than product innovations.

The existing literature does not address the issues related to substitutability in basic science and technology when separate firms engage in strategic competition in R&D. That literature mostly treats an innovation as a generic idea unrelated to any existing product line. Furthermore, models which consider sequences of innovations (Reinganum, 1985), (Mclean and Riordan, 1989)) do not establish the links between the sequence of innovations in separate product lines. These links can be quite important depending on the technology and R&D base which is common knowledge to the firms sharing it. The knowledge of the common technology and R&D base enables firms considering adoption of an innovation to recognize their potential challengers and their relative strengths and weaknesses. The empirical work of Cockburn and Henderson (1994) is an exception to this. The authors studied research activity by 10 major pharmaceutical companies in pursuit of the discovery of ethical drugs over 17 years and have found the presence of complementarities and spillovers between firms leading to multiple prizes out of a single R&D race. Hence, the authors show that the implication of the early theoretical "racing" models are inconsistent with the causal facts and their empirical results.

Of the product innovations mentioned earlier, I first focus on the drastic innovations using the perfect information framework. Following the general structure of the basic model, strategic competition for the new product markets are analyzed under three separate cases. Using the competitive payoffs as a benchmark for classifying the type of innovations, the Nash equilibrium points (NEP) are characterized. Under high cost drastic innovations the model is shown to represent a prisoner's dilemma situation where the firms only diversify, and hence switch their product lines. Under medium cost drastic innovations where pure strategy equilibrium does not exist, the mixed strategy equilibrium is characterized using a theorem. Following this, an example of mixed strategy equilibrium is presented which satisfies the criteria developed in the theorem.

Next, the basic model is modified to the case of non-drastic innovations. The assumption of symmetry in payoffs is relaxed by allowing differential market growth in separate product lines. The model is shown to yield equilibria where product upgrading is the more preferred best reply strategy. High cost non-drastic innovations are shown to exhibit multiplicity of equilibria. Under medium cost non-drastic innovations equilibrium it is proved that firms prefer upgrading, whereas under low cost non-drastic innovations the equilibrium is characterized where the preferred strategy is upgrading and diversifying into both markets.

2. Drastic Innovation Under Perfect Information

The following model assumes that there is no uncertainty related with post-adoption market conditions, and that a firm will successfully replace the old product if it adopts the innovation in that product line. Total replacement of the old product by making it obsolete is the 'drastic' nature of the innovation. Secondly, it is assumed that all agents involved in the innovative process are perfectly informed about all payoff relevant parameters of all agents, and that this perfect information is common knowledge to all agents.[5]

Consider a two period game with two identical firms sharing the closest technology base in two separate markets. In period one, assume that two separate products are exclusively and successfully produced by the two firms. Let firm 1 be the monopolist producing a_1, and firm 2 be the monopolist producing a_2. Both firms are earning monopoly profits equal to π^{m}. Firm 1, (2) can adopt a_1', (a_2') (the innovation in its own product line) with a cost $\boldsymbol{c}$, and it can adopt a_2', (a_1') (the innovation in the other firm's product line) with a cost $\boldsymbol{kc}$ where $k > 1$, or adopt both innovations with a cost $c(1 + k)$, where $c(1 + k) \leq \pi^{m}$.

Firms can adopt four strategies in this non-cooperative, one shot game : adopt a_1', adopt a_2', adopt both a_1' & a_2', or adopt neither (stick with the existing product). If they both adopt either a_1' or a_2' they compete as Cournot duopolists in that product line. Both firms earn Cournot profits equal to π^{c} in this case. If only one firm adopts, it totally replaces the old product and becomes a monopolist earning monopoly profits equal to π^{m} in that product line. Symmetry assumptions about period two profits and markets are restrictive but they focus the analysis on the role that incumbency plays in the adoption of new technologies. The assumption of equal profits in period one is also due to the same consideration. I introduce the following notation:

π_{ij}^{m}: Per period monopoly profit of firm i in market j , $\{i,j = 1,2\}$
π_{ij}^{c}: Per period Cournot profit of firm i in market j, if it shares the market with the other firm.
c_{iu}: Cost to firm i of upgrading its product line by adopting the innovation in own market i
c_{id}: Cost to firm i of diversifying into another market by adopting the innovation in market j .

Following restrictions are consistent with our discussion above.

$$\pi_{\mathrm{ij}}^{\mathrm{m}} > \pi_{1\mathrm{j}}^{\mathrm{c}} + \pi_{2\mathrm{j}}^{\mathrm{c}} \quad ; \quad \pi_{\mathrm{ij}}^{\mathrm{m}} > \pi_{\mathrm{i1}}^{\mathrm{c}} + \pi_{\mathrm{i2}}^{\mathrm{c}} \quad ; \quad \pi_{1\mathrm{d}}^{\mathrm{c}} = \pi_{2\mathrm{d}}^{\mathrm{c}} \qquad \textbf{\textit{(i)}}$$

$$\mathrm{c_{id}} = k\mathrm{c_{iu}} \quad , \quad k > 1 \qquad \textbf{\textit{(ii)}}$$

$$\pi_{\mathrm{ij}}^{\mathrm{m}} \geq \mathrm{c_{id}} \quad \forall \quad i,j = 1,2 \qquad \textbf{\textit{(iii)}}$$

Inequalities in equation (i) are a general result of Cournot model (Tirole, 1988), (Shy, 2000), and symmetry assumptions imposed on the firms and markets. Accordingly, monopoly profits in one market strictly exceed the total Cournot profits of both firms in that market. Alternately, the total of Cournot profits any firm can

[5] See (Binmore, 1990).

earn in two separate markets is strictly less than monopoly profits it can earn in one market.

Equation (ii) states that the cost of adopting the innovation in any firm's product line is strictly less than the cost of adopting the innovation in the other product line. In other words, diversifying is costlier than upgrading.

Equation (iii) imposes the restriction that payoffs to being a monopolist in any market are non-negative. If the inequality holds for the incumbent but not for the entrant, lacking any threat of entry, there would be no incentive for the incumbent to adopt the innovation in its own product line since it would simply be replacing itself as a monopolist. Thus the restriction eliminates only a trivial case.

The following notation is introduced to simplify the characterization and manageability of the model. Accordingly the pure strategies for firm i are denoted by:

s_{i1} : no action (stick with the existing product)
s_{i2} : upgrade only (adopt the innovation in own product line)
s_{i3} : diversify only (adopt the innovation in the competitor's product line)
s_{i4} : upgrade and diversify (adopt both innovations)

Payoffs of the game are defined in terms of period 2 flow profits minus the cost of adoption. I use the following notation to denote the payoffs:

π_i^m : monopoly profits of firm i in own market i when no upgrading and entry has taken place
V_{id}^m : monopoly profits in other market j , (requires i to adopt an innovation that allows it to diversify into market j)
V_{iu}^m : monopoly profits in own market i , (requires i to adopt an innovation that allows it to upgrade its product)
V_{iu}^c : profits in own market i when product is upgraded but competitor has diversified and entered market i
V_{id}^c : profits in other market j when i has diversified but incumbent has upgraded.
π_{ii}^c : profits of firm i in own market i when no upgrading has taken place and entrant has diversified in the upgraded product, (by definition, $\pi_{\mathrm{ii}}^{\mathrm{c}} = 0$ for drastic innovations)

For example, suppose in period two, firm 1 decides to stick with its existing product a_1, and firm 2 diversifies by adopting a_1'. Since by definition of drastic innovations, a_1' totally replaces the market for a_1, firm 1 (the incumbent) is preyed upon by firm 2 (the entrant) in its own market. Firm 1 becomes a monopolist in both product lines producing a_1' and a_2. Thus, the payoffs to firm 1 and 2 respectively are $\{0\ ,\ \pi_{22}^{\mathrm{m}} + \mathrm{V}_{2\mathrm{d}}^{\mathrm{m}}\}$.

Clearly the best scenario for each firm is that the incumbent monopolist sticks with its old product while the entrant adopts the innovation. Under this scenario, the more aggressive firm preys upon the stagnant incumbent and becomes a monopoly in both markets. In the following section, I characterize the equilibrium strategies of the players, and classify the results under different restrictions on the Cournot profits and the cost of adoption in the two product lines. Table 1. depicts the normal form representation of the general model under drastic innovations.

		Firm 2			
		s_{21}	s_{22}	s_{23}	s_{24}
Firm 1	s_{11}	π_{11}^m, π_{22}^m	π_{11}^m, V_{2u}^m	0, $\pi_{22}^m + V_{2d}^m$	0, $V_{2u}^m + V_{2d}^m$
	s_{12}	V_{1u}^m, π_{22}^m	V_{1u}^m, V_{2u}^m	V_{1u}^c, $\pi_{22}^m + V_{2d}^c$	V_{1u}^c, $V_{2u}^m + V_{2d}^m$
	s_{13}	$\pi_{11}^m + V_{1d}^m$, 0	$\pi_{11}^m + V_{1d}^c$, V_{2u}^c	V_{1d}^m, V_{2d}^m	V_{1d}^c, $V_{2u}^c + V_{2d}^m$
	s_{14}	$V_{1u}^m + V_{1d}^m$, 0	$V_{1u}^m + V_{1d}^c$, V_{2u}^c	$V_{1u}^c + V_{1d}^m$, V_{2d}^c	$V_{1u}^c + V_{1d}^c$, $V_{2u}^c + V_{2d}^c$

Table 1. Strategic game with drastic innovations under perfect information.

The solution concept employed is payoff dominance. This method assumes economic agents behave rationally and it is common knowledge to the players that each player would play rationally. When players consider any two strategies , they would compare their payoffs in each cell of the corresponding strategies of the normal form. If in all pairwise comparisons one strategy yields payoffs that are strictly greater than the other, at least in one comparison and are equal in all the others, then it is said that the first strategy weakly dominates the second one. After players eliminate all dominated strategies, the game is then played on the remaining undominated strategies. Finally, Nash equilibrium point(s) (NEP) are searched.

Nash equilibrium points (NEP's) are strategy combinations s_{1i}^* , s_{2i}^*, that are best replies to each other, such that $\mathrm{E}_1(s_1^*\,,\ s_2^*,) = \max_{s_{11i}} E_1\,(s_1\,, s_2^*)$ *and* $E_2\,(s_1^*\,,\ s_2^*) = \max_{s_{2i}} E_2\,(s_1^*\,,\ s_2)$ where (s_i, s_j) is the combination of the ***i'*** th strategy of firm 1 and ***j'*** th strategy of firm 2.

Differences in the cost of adopting an innovation for the incumbent and the entrant result in three separate cases to consider.

Case 1 : $\pi_{\mathrm{ij}}^{\mathrm{c}} \leq \mathrm{c}_{\mathrm{iu}} < \mathrm{c}_{\mathrm{id}}$, $\forall$ *i,j* = 1,2 $\{V_{\mathrm{iu}}^{\mathrm{c}} \leq 0\ ,\ V_{\mathrm{id}}^{\mathrm{c}} < 0\}$

Case 2 : $\mathrm{c}_{\mathrm{iu}} < \mathrm{c}_{\mathrm{id}} \leq \pi_{\mathrm{ij}}^{\mathrm{c}}$, $\forall$ *i,j* = 1,2 $\{V_{\mathrm{iu}}^{\mathrm{c}} > 0\ ,\ V_{\mathrm{id}}^{\mathrm{c}} \geq 0\}$

Case 3 : $\mathrm{c}_{\mathrm{iu}} < \pi_{\mathrm{ij}}^{\mathrm{c}} < \mathrm{c}_{\mathrm{id}}$, $\forall$ *i,j* = 1,2 $\{V_{\mathrm{iu}}^{\mathrm{c}} > 0\ ,\ V_{\mathrm{id}}^{\mathrm{c}} < 0\}$

Case 1 characterizes a situation where the net payoff to a firm when both firms undertake the same innovation in a given market is non-positive. I call this type of innovation as "high-cost" in this framework. Under Case 2 the net payoff to the firm, which is the Cournot profits net of adoption cost, is non-negative. I call this type of innovation "low-cost". Finally, Case 3 defines an intermediate scenario denoted as "medium cost". In the following sections the NEP's are characterized for each case.

2.1. High Cost Drastic Innovations

The conditions used in classifying an innovation as high-cost are (i) diversifying with an upgraded product is not profitable ($V^{\mathrm{c}}_{\mathrm{id}} < 0$), and ($ii$) competition in an upgraded product is not profitable either ($V^{\mathrm{c}}_{\mathrm{iu}} \leq 0$). Next, the payoff dominant strategies -if they exist- are searched for, and after eliminating all possible dominated strategies the NEP's in pure strategies are found. If pure strategy NEP's do not exist, equilibrium in mixed strategies is characterized.

Lemma 1. *Suppose* $\pi^{\mathrm{c}}_{\mathrm{ij}} \leq \mathrm{c}_{\mathrm{iu}} < \mathrm{c}_{\mathrm{id}}$*,* $\forall$ *i,j = 1,2* $\{V^{\mathrm{c}}_{\mathrm{iu}} \leq 0$ *;* $V^{\mathrm{c}}_{\mathrm{id}} < 0\}$*. Then,*

(a) $\pi_1(s_1, s) \geq \pi_1(s_2, s)$ *and* $\pi_2(s, s_1) \geq \pi_2(s, s_2)$ $\forall$ *s*

(b) $\pi_1(s_3, s) \geq \pi_1(s_4, s)$ *and* $\pi_2(s, s_3) \geq \pi_2(s, s_4)$ $\forall$ *s,*

where $\pi_{\mathrm{i}}(s_k, s_l)$ *is* ***i****' s payoff when firm 1 chooses* s_k *and firm 2 chooses* s_l*.*

(proof: see the appendix for all proofs not given in the main body of the text.)

Proposition 1. *Under drastic innovation where* $\pi^c_{ij} \leq {}_{iu} < c_{id}$*, the NEP is the strategy combination*$\{s_3, s_3\}$ *with payoffs* $(V^{\mathrm{m}}_{\mathrm{1d}}; V^{\mathrm{m}}_{\mathrm{2d}})$*. Thus the monopolists cross over and diversify into the other firm's market.*

Remark 1. Since innovations are drastic, there is a one-hundred percent replacement of the incumbent's product line. From the firms' point of view (s_1, s_1) may be the "better" outcome; yet it is not self-enforcing and therefore not "stable". They are facing a classic 'prisoner's dilemma' situation. Thus, each incumbent is preempted by the other monopolist, so that the monopolists switch markets. Although both innovations are adopted, the outcome is sub-optimal not only from the firms' viewpoint but also from a social standpoint since high cost entrants rather than the low cost incumbents are undertaking the adoption of innovations.

Example: $\pi^{\mathrm{m}}_{\mathrm{i}} = 10$, $\mathrm{c}_{\mathrm{iu}} = 4$, $\mathrm{c}_{\mathrm{id}} = 5$ **,** $V^{\mathrm{m}}_{\mathrm{iu}} = 6$, $V^{\mathrm{m}}_{\mathrm{id}} = 5$, $V^{\mathrm{c}}_{\mathrm{iu}} = -1$, $V^{\mathrm{c}}_{\mathrm{id}} = -2$**.** It is a simple exercise to plug in the values above into the generalized normal form given in table 1 and see that $\{s_3, s_3\}$ is the NEP with corresponding payoffs of (5,5). High Cost Drastic Innovations: $\pi^c_{ij}{<}c_{iu} < c_{id}$ ($V^c_{iu} < 0$; $V^c_{id} < 0$)

Example: π^m_i= 10 , c_{iu}= 4 , c_{id}= 5 , V^m_{iu} =6 , V^m_{id} =5 , V^c_{iu} =-1 , V^c_{iu} =-2 .

Firm 2

Firm 1	s_{21}	s_{22}	s_{23}	s_{24}
s_{11}	10 , 10	10 , 6	0 , 15	0 , 11
s_{12}	6 , 10	6 , 6	-1 , 8	-1 , 4
s_{13}	15 , 0	8 , -1	5 , 5*	-2 , 4
s_{14}	11 , 0	4 , -1	4 , 2	-3 , -3

Example

Fig. 1. High cost drastic innovations.

2.2. Low Cost Drastic Innovations

I say the innovations are low cost when both the diversification to compete with an upgraded product, and the competition in an upgraded product are profitable ($V^{c}_{iu} > 0$; $V^{c}_{id} \geq 0$). In other words, no matter what the rival firm does, undertaking an innovation is profitable for both the entrant firm and the incumbent firm.

Lemma 2. *Suppose* $c_{iu} < c_{id} \leq \pi^{c}_{ij}$, $\forall\, i, j = 1, 2$, $\{V^{c}_{iu} > 0$, $V^{c}_{id} \geq 0\}$. *Then*
(a) $\pi_1(s_3, s) \geq \pi_1(s_1, s)$, $\pi_2(s, s_3) \geq \pi_2(s, s_1)$ $\forall\, s$
(b) $\pi_1(s_4, s) \geq \pi_1(s_2, s)$, $\pi_2(s, s_4) \geq \pi_2(s, s_2)$ $\forall\, s$
where $\pi_i(s_k, s_l)$ *is* ***i****' s payoff when firm 1 chooses* s_k *and firm 2 chooses* s_l.

Proposition 2. *Under drastic innovation where* $c_{iu} < c_{id} \leq \pi^{c}_{ij}$ *, the NEP is the strategy combination* $\{s_4, s_4\}$ *with payoffs* $(V^{c}_{1u} + V^{c}_{1d}\ ;\ V^{c}_{2u} + V^{c}_{2d})$. *Thus, the monopolists upgrade and diversify in both product lines by adopting both innovations.*

Remark 2. Since competitive payoffs following a joint adoption of an innovation are greater than or equal to zero in each market, the monopolists upgrade and diversify in order to avoid being preempted by the entrant. The outcome is clearly pro competitive even though the monopolists spend greater portion of their resources for a product innovation in which they have a comparative disadvantage. Their monopoly positions are replaced with Cournot Competition. Notice that in the reduced game the firms face a prisoner's dilemma situation as in the previous case of high cost innovations.

Example: $\pi^{m}_{i} = 10$, $c_{iu} = 2$, $c_{id} = 3$ $V^{m}_{iu} = 8$, $V^{m}_{id} = 7$, $V^{c}_{iu} = 2$, $V^{c}_{id} = 1$
It is a simple exercise to plug in the values above into the generalized normal form given in table 1 and see that $\{s_4, s_4\}$ is the only NEP with corresponding payoffs of (3,3).

		Firm 2			
		s_{21}	s_{22}	s_{23}	s_{24}
Firm 1	s_{11}	10 , 10	10 , 8	0 , 17	0 , 15
	s_{12}	8 , 10	8 , 8	2 , 11	2 , 9
	s_{13}	17 , 0	11 , 2	7 , 7	1 , 9
	s_{14}	15 , 0	9 , 2	9 , 1	3 , 3*

Example

Fig. 2. Low cost drastic innovations.

2.3. Medium Cost Drastic Innovations

Innovations are classified as medium cost under the following assumptions. The first is the condition that diversifying to compete with an upgraded product is not profitable, i.e., $V^{c}_{id} < 0$. Second is the condition that competition in an upgraded product is profitable, i.e., $V^{c}_{iu} > 0$. These conditions. together with case 1 and case 2 exhaust the payoff spectrum under all possible strategy combinations chosen by the incumbent and the entrant. The idea of 'medium cost' embodies within it the concept of comparative advantage of being established in a product line. All else being equal, if the incumbent has to compete with an entrant in a product that it has upgraded, the incumbent will prevail and prey upon the entrant. Knowing its comparative disadvantage, the entrant will enter only if can secure a monopoly on the product line it is diversifying into. On the other hand, the incumbent monopolist is reasoning that if it does not adopt the innovation and upgrade its product, it will be preyed upon by a successful entrant -no matter how much a cost disadvantage exists. Because of the drastic nature of the innovation, no matter who adopts it, the existing product will be obsolete.

It is easy to see that there are not any payoff dominated strategies that can be eliminated to simplify the solution. It is also straightforward to verify that there are no NEP in pure strategies in this game either. Therefore, the NEP's has to be characterized using mixed strategies.

Let $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ denote firm 1's mixed strategy and $\beta = (\beta_1, \beta_2, \beta_3, \beta_4)$ denote firm 2's mixed strategy. Then, write the expected payoffs for each firm as follows:

$$\mathrm{E}_1(\alpha,\beta) = \alpha_1(\beta_1+\beta_2)\pi^{m}_{11} + \alpha_2\{(\beta_1+\beta_2)V^{m}_{1u} + (\beta_3+\beta_4)V^{c}_{1u}\}$$

$$+\ \alpha_3\{(\beta_1+\beta_2)\pi^{m}_{11} + (\beta_1+\beta_3)V^{m}_{1d} + (\beta_2+\beta_4)V^{c}_{1d}\}$$

$$+\ \alpha_4\{(\beta_1+\beta_2)V^{m}_{1u} + (\beta_1+\beta_3)V^{m}_{1d} + (\beta_2+\beta_4)V^{c}_{1d} + (\beta_3+\beta_4)V^{c}_{1u}\}$$

$$\mathrm{E}_2(\alpha,\beta) = \beta_1(\alpha_1+\alpha_2)\pi^{m}_{22} + \beta_2\{(\alpha_1+\alpha_2)V^{m}_{2u} + (\alpha_3+\alpha_4)V^{c}_{2u}\}$$

$$+\ \beta_3\{(\alpha_1+\alpha_2)\pi^{m}_{22} + (\alpha_1+\alpha_3)V^{m}_{2d} + (\alpha_2+\alpha_4)V^{c}_{2d}\}$$

$$+\ \beta_4\{(\alpha_1+\alpha_2)V^{m}_{2u} + (\alpha_1+\alpha_3)V^{m}_{2d} + (\alpha_2+\alpha_4)V^{c}_{2d} + (\alpha_3+\alpha_4)V^{c}_{2u}\}$$

Proposition 3. *Assume $V^{m}_{iu} < \pi^{m}_{ii}$, $V^{c}_{iu} > 0$, $V^{c}_{id} < 0 \quad \forall\ i = 1, 2$. Then, any NEP in mixed strategies must satisfy the following :*

$$(\beta_1+\beta_3)V^{m}_{1d} + (\beta_2+\beta_4)V^{c}_{1d} = 0 \tag{1}$$

$$(\alpha_1+\alpha_3)V^{m}_{2d} + (\alpha_2+\alpha_4)V^{c}_{2d} = 0 \tag{2}$$

Proposition 4. *Under the assumptions of Proposition 3, any NEP in mixed strategies must satisfy the following :*

$$(\alpha_1+\alpha_2)\pi^{m}_{22} = (\alpha_1+\alpha_2)V^{m}_{2u} + (\alpha_3+\alpha_4)V^{c}_{2u} \tag{3}$$

$$(\beta_1+\beta_2)\pi^{m}_{11} = (\beta_1+\beta_2)V^{m}_{1u} + (\beta_3+\beta_4)V^{c}_{1u} \tag{4}$$

Theorem 1. *$\forall\ \pi^{c}_{ij}$ and π^{c}_{ii} such that,* $c_{iu} < \pi^{c}_{ij} < c_{id}$, *$V^{m}_{iu} < \pi^{m}_{ii}$ {i,j=1,2} holds, the set of all mixed strategy NEP's satisfies:*

$$(\beta_1 + \beta_3)V^{m}_{1d} + (\beta_2 + \beta_4)V^{c}_{1d} = 0$$

$$(\beta_1 + \beta_2)(\pi^{m}_{11} - V^{m}_{1u}) - (\beta_3 + \beta_4)V^{c}_{1u} = 0$$

$$\beta_1 + \beta_2 + \beta_3 + \beta_4 = 1, \quad \beta_i \geq 0$$

$$(\alpha_1 + \alpha_3)V^{m}_{2d} + (\alpha_2 + \alpha_4)V^{c}_{2d} = 0$$

$$(\alpha_1 + \alpha_2)(\pi^{m}_{22} - V^{m}_{2u}) - (\alpha_3 + \alpha_4)V^{c}_{2u} = 0$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1, \quad \alpha_i \geq 0$$

where α and β are firm 1's and firm 2's mixed strategies respectively.

Proof of Theorem 1: This theorem is a consequence of the definition of medium cost drastic innovations, and of Propositions 3 and 4.

The corresponding expected payoffs are:

$$E_1(\alpha,\beta) = \alpha_1(\beta_1 + \beta_2)\pi^{m}_{11} + \alpha_2\{(\beta_1 + \beta_2)V^{m}_{1u} + (\beta_3 + \beta_4)V^{c}_{1u}\} + \alpha_3(\beta_1 + \beta_2)\pi^{m}_{11}$$

$$+ \alpha_4\{(\beta_1 + \beta_2)V^{m}_{1u} + (\beta_3 + \beta_4)V^{c}_{1u}\}$$

$$E_2(\alpha,\beta) = \beta_1(\alpha_1 + \alpha_2)\pi^{m}_{22} + \beta_2\{(\alpha_1 + \alpha_2)V^{m}_{2u} + (\alpha_3 + \alpha_4)V^{c}_{2u}\} + \beta_3(\alpha_1 + \alpha_2)\pi^{m}_{22}$$

$$+ \beta_4\{(\alpha_1 + \alpha_2)V^{m}_{2u} + (\alpha_3 + \alpha_4)V^{c}_{2u}\}.$$

Simplifying the equations in *NEP** we obtain:

$$(\alpha_1 + \alpha_2) = \frac{V^{c}_{2u}}{\pi^{m}_{22} - V^{m}_{2u} + V^{c}_{2u}} \tag{5}$$

$$(\alpha_3 + \alpha_4) = \frac{\pi^{m}_{22} - V^{m}_{2u}}{\pi^{m}_{22} - V^{c}_{2u}} \tag{6}$$

$$(\alpha_1 + \alpha_3) = \frac{V^{c}_{2d}}{V^{m}_{2d} - V^{c}_{2d}} \tag{7}$$

$$(\alpha_2 + \alpha_4) = \frac{V^{m}_{2d}}{V^{m}_{2d} - V^{c}_{2d}} \tag{8}$$

Remark 3. From (5) we note that the lower is the cost of upgrading , the more likely is that the incumbent will stay in own product line and upgrade its product . Similarly, from (6) we note that the higher is the cost of upgrading , the more likely is that the entrant will either cross over to a separate product line or diversify into both product lines.

In the following numerical example I find the range of individual probabilities for mixed strategy NEP's.

Example : $\pi^{\mathrm{m}}_{\mathrm{ij}}$=10, $\pi^{\mathrm{c}}_{\mathrm{ij}}$=4 $\forall$ *i,j* = 1,2 ; $\mathrm{c_{iu}}$=3, $\mathrm{c_{id}}$=5 $V^{\mathrm{m}}_{\mathrm{iu}} = 7$, $V^{\mathrm{m}}_{\mathrm{id}} = 5$, $V^{\mathrm{c}}_{\mathrm{iu}} = 1$, $V^{\mathrm{c}}_{\mathrm{id}} = -1$

The following table depicts the normal form representation of this example.

Firm 2

Firm 1	s_{21}	s_{22}	s_{23}	s_{24}
s_{11}	10 , 10	10 , 7	0 , 15	0 , 12
s_{12}	7 , 10	7 , 7	1 , 9	1 , 6
s_{13}	15 , 0	9 , 1	5 , 5	−1 , 6
s_{14}	12 , 0	6 , 1	6 , −1	0 , 0

Example

Fig. 3. Medium cost drastic innovations.

Let A1,...,A4 be the expected payoffs firm 1 will receive if firm 2 plays the mixed strategies $(\beta_1, ..., \beta_4)$ and let B1,...,B4 be the expected payoffs firm 2 will receive if firm 1 plays the mixed strategies $(\alpha_1, ..., \alpha_4)$. Then, using the payoffs in the example, the following linear equation systems are set up for firm 1 and firm 2 respectively.

$$\begin{aligned}
\mathrm{A}_1 &= \pi_1(s_1, \beta) = 10\beta_1 + 10\beta_2 \\
\mathrm{A}_2 &= \pi_1(s_2, \beta) = 7\beta_1 + 7\beta_2 + \beta_3 + \beta_4 \\
\mathrm{A}_3 &= \pi_1(s_3, \beta) = 15\beta_1 + 9\beta_2 + 5\beta_3 - \beta_4 \\
\mathrm{A}_4 &= \pi_1(s_4, \beta) = 12\beta_1 + 6\beta_2 + 6\beta_3
\end{aligned}$$

$$\begin{aligned}
\mathrm{B}_1 &= \pi_2(\alpha, s_1) = 10\alpha_1 + 10\alpha_2 \\
\mathrm{B}_2 &= \pi_2(\alpha, s_2) = 7\alpha_1 + 7\alpha_2 + \alpha_3 + \alpha_4 \\
\mathrm{B}_3 &= \pi_2(\alpha, s_3) = 15\alpha_1 + 9\alpha_2 + 5\alpha_3 - \alpha_4 \\
\mathrm{B}_4 &= \pi_2(\alpha, s_4) = 12\alpha_1 + 6\alpha_2 + 6\alpha_3
\end{aligned}$$

Equating A1=A4 , A2=A4 , A3=A4 and using the constraint that the sum of the probabilities of random strategies is equal to one $\sum_{\mathrm{i}=1}^{4} \beta_{\mathrm{i}} = 1$, I proceed to find the interior solution to the following system for firm 1.

$$
\begin{aligned}
2\beta_1 - 4\beta_2 + 6\beta_3 &= 0 \quad \text{(i)} \\
5\beta_1 - \beta_2 + 5\beta_3 - \beta_4 &= 0 \quad \text{(ii)} \\
-3\beta_1 - 3\beta_2 + \beta_3 + \beta_4 &= 0 \quad \text{(iii)} \\
\beta_1 + \beta_2 + \beta_3 + \beta_4 &= 0 \quad \text{(iv)}
\end{aligned}
$$

We also want to place the restriction that $\beta_1, \beta_2, \beta_3, \beta_4 \geq 0$. Note that (**i**) - (**ii**) = (**iii**). Therefore, we eliminate (**i**). From (**iii**) we have,

$$(\beta_1 + \beta_2) = \frac{(\beta_3 + \beta_4)}{3}.$$

Substituting this into (**iv**) we obtain,

$$\frac{(\beta_3 + \beta_4)}{3} + (\beta_3 + \beta_4) = 1$$

or,

$$(\beta_3 + \beta_4) = \frac{3}{4} \quad \left\{ = \frac{\pi_{11}^{\mathrm{m}} - V_{1\mathrm{u}}^{\mathrm{m}}}{\pi_{11}^{\mathrm{m}} - V_{1\mathrm{u}}^{\mathrm{m}} + V_{1\mathrm{u}}^{\mathrm{c}}} \right\} \tag{9}$$

It follows that,

$$(\beta_1 + \beta_2) = \frac{1}{4} \quad \left\{ = \frac{V_{1\mathrm{u}}^{\mathrm{c}}}{\pi_{11}^{\mathrm{m}} - V_{1\mathrm{u}}^{\mathrm{m}} + V_{1\mathrm{u}}^{\mathrm{c}}} \right\}. \tag{10}$$

From (**ii**) we have,

$$5(\beta_1 + \beta_3) = (\beta_2 + \beta_4).$$

Substituting this into (**iv**) we obtain,

$$\beta_1 + \beta_3 + 5(\beta_1 + \beta_3) = 1$$

or,

$$(\beta_1 + \beta_3) = \frac{1}{6} \quad \left\{ = -\frac{V_{1\mathrm{d}}^{\mathrm{c}}}{V_{1\mathrm{d}}^{\mathrm{m}} - V_{1\mathrm{d}}^{\mathrm{c}}} \right\}. \tag{11}$$

It follows that,

$$(\beta_2 + \beta_4) = \frac{5}{6} \left\{ = \frac{V_{1\mathrm{d}}^{\mathrm{m}}}{V_{1\mathrm{d}}^{\mathrm{m}} - V_{1\mathrm{d}}^{\mathrm{c}}} \right\}. \tag{12}$$

We have shown that equations (9) through (12) satisfy equations (5) through (8) respectively. It is also immediate that, from equations (9) through (12) we can write the following conditions:

$$0 \leq \beta_1 \leq \frac{1}{6}\,, \quad 0 \leq \beta_2 \leq \frac{1}{4}\,, \quad 0 \leq \beta_3 \leq \frac{1}{6}\,, \quad 0 \leq \beta_4 \leq \frac{3}{4} \tag{13}$$

Hence, it is seen that, in this example, any interior solution for the mixed strategy equilibria must satisfy (13). Simulations show that one solution that satisfies the non-negativity constraints are the following β_{i} values: $\beta_1 = 0$, $\beta_2 = 0.25$, $\beta_3 = 0.17$, $\beta_4 = 0.58$.

2.4. Summary of Results

If the innovation is drastic, i.e., that it would totally replace the existing product in that product line, then the firms would: (*i*) diversify into the incumbent's product line only and in the process switch markets as monopolists if and only if $\pi^{\mathrm{c}}_{\mathrm{ij}} \leq \mathrm{c}_{\mathrm{iu}} < \mathrm{c}_{\mathrm{id}}$; (*ii*) upgrade in own product line and diversify into the competitor's product line and compete as Cournot competitors by adopting the innovations in both product lines if and only if $\mathrm{c}_{\mathrm{iu}} < \mathrm{c}_{\mathrm{id}} < \pi^{\mathrm{c}}_{\mathrm{ij}}$: and (*iii*) use a mixed strategy equilibrium in deciding which innovation(s) they would adopt if and only if, $\mathrm{c}_{\mathrm{iu}} < \pi^{\mathrm{c}}_{\mathrm{ij}} < \mathrm{c}_{\mathrm{id}}$.

Under the first type of equilibrium with "drastic" innovation we find that firms diversify their product lines by crossing over markets and totally replace the incumbents, if payoffs from the competitive outcome are non-positive (i.e., if the innovations are high cost) for both the entrant and for the incumbent. This type of equilibrium where the monopolists switch markets develops as a dominant 'defensive' strategy because under drastic innovation firms do not undertake adoptions in their own product lines, since it would only mean replacing themselves as incumbents. It can be concluded that, because competition reduces profits, each firm's incentive to become a monopolist is greater than its incentive to become a duopolist by jointly adopting the high cost innovation.

Under the second type of equilibrium, firms upgrade products not only in their own product line but also in the incumbent's product line. This type of total diversification arises when competitive payoffs from diversifying into the competitor's product line is non-negative, i.e., when the innovations are low cost.

We also observe that under some boundary values of cost of adoption and Cournot profits, firms may use mixed strategy equilibrium. We get mixed strategy equilibrium with drastic innovation if the profits from Cournot competition in any market strictly cover the cost of adoption for the incumbent, but are strictly less than the cost of adoption for the entrant, i.e., if the innovations are medium cost. Both innovations are adopted, however, either through switching of incumbency, or by the incumbent itself, or by joint adoption in both product lines, as demonstrated with a numerical example.

The main tendency is that if the firms are facing drastic innovations, then they would either diversify into the incumbent's product line only, or upgrade and diversify into both product lines. An interesting observation is that, lacking such a technological rivalry, monopolist firms would not undertake adoptions in their own product lines, since it would mean replacing themselves as incumbents. Thus, the outcome of this technological rivalry is socially desirable, since maximum product diversity is achieved through adoption of new innovations. On the other hand, the optimal "cooperative" strategy from the firms' standpoint would be not adopting the drastic innovations and sticking with the old product. Yet, this strategy can not be enforced as a credible commitment.

3. Non-drastic Innovation Under Perfect Information

With non-drastic innovation (or partial replacement of the old market) we mean that successful adoption of the innovation: (*i*) suppresses the demand for the old product but does not make it completely obsolete, and (*ii*) generates new demand so that the total demand in that product line is growing. In this model, up to 4 product markets (2 in each product line) can coexist in the second period. If both innovations are adopted exclusively, either by the incumbent or by the entrant, the payoff that can be earned from each new product market is $V_{\mathrm{i}}^{\mathrm{m}}$ The costs of adoption, suppressed in $V_{\mathrm{i}}^{\mathrm{m}}$, are $\mathrm{c_i}$ where $\mathrm{c_{id}} > \mathrm{c_{iu}}$, for all i=1,2.

The profits incumbent firms earn from their respective old markets, if the innovation is adopted, are denoted by a parameter $\mathrm{r_i}$, where $0 < \mathrm{r_i} \leq \pi_{\mathrm{i}}^{\mathrm{m}}$. $\mathrm{r_i} = 0$ would imply a drastic innovation where the old product market is totally replaced by the new one. On the other hand, $\mathrm{r_i} = \pi_{\mathrm{i}}^{\mathrm{m}}$ implies that adoption of the innovation has no effect on the old product market. While this is an extreme case of non-drastic innovation, no replacement indicates that the two products are possibly unrelated or not considered to be on the same product line on the demand side.

I modify the notation used for drastic innovations to simplify the characterization of the model.

The pure strategies for firm i are denoted by:

The pure strategies of firm i are denoted by:

s_{i1} : no action (stick with the existing product)
s_{i2} : upgrade only (adopt the innovation in own product line, and continue producing the old product if $r > 0$.
s_{i3} : diversify only (adopt the innovation in the competitor's product line)
s_{i4} : upgrade and diversify (adopt both innovations, and continue producing the old product if $r > 0$.

Payoffs to firm i are denoted by:

π_i^m : pre-innovation monopoly profits in own market i (when firm i sticks with its old product and no upgrading has taken place)
π_{ij}^c : Cournot profits of firm i when both firms jointly adopt the innovation in market j
r_i : post-innovation monopoly profits the incumbent earns from its old product market i, (requires upgrading by the incumbent and/or diversification through adoption of an innovation by the competitor into market i)
V_{id}^m : monopoly profits in other market j, (requires i to adopt an innovation that allows it to diversify into market j, but the incumbent does not upgrade)
V_{iu}^m : profits from having the monopoly of the new product in own market i, (requires i to adopt an innovation that allows it to upgrade its product)
V_{iu}^c : profits from the new product in own market i when product is upgraded but competitor has diversified and entered market i
V_{id}^c : profits in other market j when i has diversified by adopting an innovation but incumbent has upgraded.

Note that the following set of restrictions (i) through (iii), defined in section 2 before and repeated below for convenience is coupled with (iv). These are based on reasonable assumptions some of which follow directly from the economic theory.

$$
\begin{aligned}
\pi_{ij}^m &> 2\pi_{ij}^c \quad \forall \quad i,j = 1,2 && (i) \\
c_{id} &> c_{iu} \quad \forall \quad i,j = 1,2 && (ii) \\
V_{iu}^m,\ V_{id}^m &> 0 \quad \forall \quad i,j = 1,2 && (iii) \\
0 \leq r &\leq \pi^m. && (iv)
\end{aligned}
$$

Recall that (i) states that the total of Cournot profits any firm can earn in two separate markets is strictly less than the monopoly profits it can earn in a single market. Hence, monopoly is always a preferred status by both firms.

Equation (ii) states that the cost of adopting the innovation in any firm's product line is strictly less than the cost of adopting the innovation in the competitor's product line capturing the idea of comparative cost advantage obtained by being an established firm in a product line. It implies[6] $V_{iu}^m > V_{id}^m$, and $V_{iu}^c > V_{id}^c \ \forall\, i = 1,2$ meaning *'upgrading to keep monopoly is better than diversifying to obtain monopoly'* for the former, and *'divesifying to compete with an upgraded product is less profitable than upgrading to compete with an entrant that has diversified in the upgraded product'* for the latter.

Equation (iii) imposes the restriction that payoffs to being a monopolist in any market are non-negative. If not, it is either too costly for both the entrant and the incumbent to earn positive profits following an adoption, or it is too costly only for the entrant to earn positive profits even if it became a monopolist in the incumbent's product line. It implies that $\pi^m > c_{iu}, c_{id} \quad \forall\ i = 1,2$. If the inequality holds for the incumbent ($V_{iu}^m > 0$), but not for the entrant ($V_{id}^m < 0$), then, under drastic innovation ($r = 0$) there would be no reason for the incumbent to adopt the innovation in its own product line since it would be replacing itself as a monopolist ($V_{iu}^m = \pi^m$). Thus, this restriction not only eliminates a trivial case but also captures the idea of technological closeness and rivalry. When *(iii)* holds, firms find themselves within the technological boundaries of one another; hence, they see themselves as potential challengers and entrants in the incumbent's product line.

Note that (ii) and (iii) together imply the following: First, upgrading to obtain monopoly is better than diversifying into another market to obtain monopoly, $V_{iu}^m > V_{id}^m \ \forall\, i = 1,2$.

Second, diversifying to compete with an upgraded product is less profitable than upgrading to compete with an entrant that has diversified in the upgraded product, $V_{iu}^c > V_{id}^c \ \forall\, i = 1,2$.

Inequality (iv) captures the degree of replacement of the old product market by the adopted innovation. A drastic innovation where the old product market is totally replaced by the new one will be denoted by $r = 0$. On the other hand, $r = \pi^m$ implies that adoption of the innovation has no effect on the old product market since the incumbent can earn the same amount of profits from its old product market. While this is an extreme case of drastic innovation, no replacement indicates that the two products are possibly unrelated.

[6] However, it should be noted that Eq.(ii) does not imply either $c_{1u} > c_{2u}$, or $c_{1d} > c_{2d}$, or both necessarily hold.

Finally, note that using Equation (iv), together with (ii) and (iii) we require: $V_{id}^m + r > \pi^m \ \ \forall\, i = 1, 2 \ \& \ 0 \leq r \leq \pi^m$.

Hence, for a drastic innovation where $r = 0$, we require that $V_{iu}^m, V_{id}^m > \pi^m$. This defines the lower bound for profits so that the firms would not be worse off as monopolists if they considered upgrading and/or diversifying their product lines. I assume equal replacement in the two product lines, i.e., $r_1 = r_2$, without loss of generality, to simplify the notation, as this does not change the results.

Assume $V_{\mathrm{iu}}^{\mathrm{m}} + \mathrm{r_i} > \pi_{\mathrm{i}}^{\mathrm{m}}$ as a proxy that the total demand following adoption of an innovation in a product line is growing [7] Next assume $V_{\mathrm{iu}}^{\mathrm{m}} > V_{\mathrm{id}}^{\mathrm{m}}$ upgrading to obtain a monopoly is better than diversifying into another market to obtain monopoly. Finally, assume $V_{\mathrm{iu}}^{\mathrm{c}} > V_{\mathrm{id}}^{\mathrm{c}}$, diversifying to compete with an upgraded product is less profitable than upgrading to compete with an entrant that has diversified in the upgraded product.

Table 2 depicts the normal form representation of the general model. For example, if the firms exclusively adopt the innovations in their own product lines, the strategy combination for firm 1 and firm 2 would be denoted by (s_2, s_2), respectively. In this case, each firm maintains its monopoly position for both the old and the new markets in its product line. This strategy yields payoffs of $\pi_{\mathrm{i}}(s_2, s_2) = V_{\mathrm{iu}}^{\mathrm{m}} + \mathrm{r_i}$ and it is the maximum that can be earned as a monopolist in a single product line. The strategy combination (s_4, s_4) means that the firms both adopt the innovations in their product lines and across product lines while continuing to produce their original products. Thus, the firms become Cournot competitors in the new product markets and maintain their monopoly positions in the old product markets which is suppressed by the new products. In this case, payoffs to firm 1 and firm 2, in respective order, are as follows.

$$\pi_1(s_4, s_4) = V_{\mathrm{1u}}^{\mathrm{c}} + V_{\mathrm{1d}}^{\mathrm{c}} + \mathrm{r_1}; \pi_2(s_4, s_4) = V_{\mathrm{2u}}^{\mathrm{c}} + V_{\mathrm{2d}}^{\mathrm{c}} + \mathrm{r_2}.$$

Similar to the case of drastic innovations, differences in the cost of upgrading and diversification result in three separate cases to consider under this scenario also.

Case 1 : $\pi_{\mathrm{ij}}^{\mathrm{c}} < \mathrm{c_{iu}} < \mathrm{c_{id}}$, $\forall$ i,j = 1,2 $\{V_{\mathrm{iu}}^{\mathrm{c}} < 0\ ;\ V_{\mathrm{id}}^{\mathrm{c}} < 0\}$

Case 2 : $\mathrm{c_{iu}} < \mathrm{c_{id}} < \pi_{\mathrm{ij}}^{\mathrm{c}}$, $\forall$ i,j = 1,2 $\{V_{\mathrm{iu}}^{\mathrm{c}} > 0\ ;\ V_{\mathrm{id}}^{\mathrm{c}} A > 0\}$

Case 3 : $\mathrm{c_{iu}} \leq \pi_{\mathrm{ij}}^{\mathrm{c}} \leq \mathrm{c_{id}}$, $\forall$ i,j = 1,2 $\{V_{\mathrm{iu}}^{\mathrm{c}} \geq 0\ ;\ V_{\mathrm{id}}^{\mathrm{c}} \leq 0\}$

I label the above cases as high cost, low cost and medium cost respectively.

3.1. High Cost Non-drastic Innovations

If competitive payoffs to both the incumbent firm and the entrant firm following a joint adoption of an innovation in any market are strictly negative, I call them high cost innovations: [8] $(V_{\mathrm{iu}}^{\mathrm{c}} < 0\ ;\ V_{\mathrm{id}}^{\mathrm{c}} < 0)$.

[7] $V_{iu}^m + \mathrm{r_i} = \pi_i^m$, implies the innovation is non-drastic, but it has simply generated new demand and revenues enough to compensate exactly for the cost of its adoption.

[8] Recall that high cost innovations were defined as $(V_{\mathrm{iu}}^{\mathrm{c}} \leq 0\ ;\ V_{\mathrm{id}}^{\mathrm{c}} < 0)$ earlier. I ignore the discrepancy for the boundary values around zero and use the same terminology for both situations. This difference arises as a consequence of our concern for classifying types of innovations according to the outcomes they lead to in the solution of the game.

Firm 2

Firm 1	s_{21}	s_{22}	s_{23}	s_{24}
s_{11}	π_{11}^m π_{22}^m	π_{11}^m $V_{2u}^m + r$	r $\pi_{11}^m + V_{2d}^m$	r $V_{2u}^m + V_{2d}^m + r$
s_{12}	$V_{1u}^m + r$ π_{22}^m	$V_{1u}^m + r$ $V_{2u}^m + r$	$V_{1u}^c + r$ $\pi_{22}^m + V_{2d}^c$	$V_{1u}^c + r$ $V_{2u}^m + V_{2d}^m + r$
s_{13}	$\pi_{11}^m + V_{1d}^m$ r	$\pi_{11}^m + V_{1d}^c$ $V_{2u}^c + r$	$V_{1d}^m + r$ $V_{2d}^m + r$	$V_{1d}^c + r$ $V_{2u}^c + V_{2d}^m + r$
s_{14}	$V_{1u}^m + V_{1d}^m + r$ r	$V_{1u}^m + V_{1d}^c + r$ $V_{2u}^c + r$	$V_{1u}^c + V_{1d}^m + r$ $V_{2d}^c + r$	$V_{1u}^c + V_{1d}^c + r$ $V_{2u}^c + V_{2d}^c + r$

Table 2. Strategic game with non-drastic innovations under perfect information.

Lemma 3. *Suppose* $\pi_{\mathrm{ij}}^{\mathrm{c}} < \mathrm{c_{iu}} < \mathrm{c_{id}}$, $(V_{\mathrm{iu}}^{\mathrm{c}}$, $V_{\mathrm{id}}^{\mathrm{c}} < 0)$ $i = j$. *Then,*

(a) $\pi_1(s_1, s_4) > \pi_1(s, s_4)$ *and* $\pi_2(s_1, s_4) > \pi_2(s_1, s) \forall\ s$.

(b) $\pi_1(s_2, s_2) > \pi_1(s, s_2)$ *and* $\pi_2(s_2, s_2) > \pi_2(s_2, s) \forall\ s$.

(c) $\pi_1(s_3, s_3) > \pi_1(s, s_3)$ *and* $\pi_2(s_3, s_3) > \pi_2(s_3, s) \forall\ s$.

(d) $\pi_1(s_4, s_1) > \pi_1(s, s_1)$ *and* $\pi_2(s_4, s_1) > \pi_2(s_4, s) \forall\ s$.

where $\pi_{\mathrm{i}}(s_k, s_l)$ *is i"s payoff when firm 1 chooses* s_k *and firm 2 chooses* s_l.

Proposition 5. *Under high-cost non-drastic innovation where* $\pi_{\mathrm{ij}}^{\mathrm{c}} < \mathrm{c_{iu}} < \mathrm{c_{id}}$*, the pure strategy NEP's are the strategy combinations* $\{s_1, s_4\}$, $\{s_1, s_2\}$, $\{s_3, s_3\}$, *and* $\{s_4, s_1\}$ *with payoffs* $(\mathrm{r_1};\ V_{\mathrm{2u}}^{\mathrm{m}} + V_{\mathrm{2d}}^{\mathrm{m}} + \mathrm{r_2})$, $(V_{\mathrm{1u}}^{\mathrm{m}} + \mathrm{r_1};\ V_{\mathrm{2u}}^{\mathrm{m}} + \mathrm{r_2})$, $(V_{\mathrm{1d}}^{\mathrm{m}} + \mathrm{r_1};\ V_{\mathrm{2d}}^{\mathrm{m}} + \mathrm{r_2})$, *and* $(V_{\mathrm{1u}}^{\mathrm{m}} + V_{\mathrm{1d}}^{\mathrm{m}} + \mathrm{r_1};\ \mathrm{r_2})$ *respectively.*

Remark 4. Under two of the four equilibria, and , we observe a passive incumbent and an aggressive entrant. The entrant monopolist diversifies across both its own product line and the entrant's product line, whereas the incumbent sticks with the old product and is partly preyed upon and replaced by the aggressive entrant. In the other two equilibria, and , both firms are actively involved in diversification and specialization process. In the former equilibrium, firms stay in their own market and upgrade in their own product lines. In the latter equilibrium, they diversify only in the incumbent's product line; and in this process of switching, they partly replace the incumbent and are partly replaced by the entrant in their old markets.

Example: $\pi_{\mathrm{i}}^{\mathrm{m}} = 10$, $\mathrm{c_{iu}} = 4$, $\mathrm{c_{id}} = 5$, $V_{\mathrm{iu}}^{\mathrm{m}} = 6$, $V_{\mathrm{id}}^{\mathrm{m}} = 5$, $V_{\mathrm{iu}}^{\mathrm{c}} = -1$, $V_{\mathrm{id}}^{\mathrm{c}} = -2$, $\mathrm{r_1} = \mathrm{r_2} = 6$. It is a fairly easy exercise to plug in the values above into the generalized normal form given in table 2 and see that $\{s_1, s_4\}$,$\{s_2, s_2\}$,$\{s_3, s_3\}$, and

$\{s_4, s_1\}$ are the NEP's with corresponding payoffs of (6,17), (12,12), (11,11), and (17,6) respectively. r_1 and r_2, are given equal values to make the payoffs symmetric. Their equality is not necessary to drive the results.

		Firm 2			
		s_{21}	s_{22}	s_{23}	s_{24}
	s_{11}	10 , 10	10 , 12	6 , 15	6 , 17*
	s_{12}	12 , 10	12 , 12*	5 , 8	5 , 8
Firm 1	s_{13}	15 , 6	8 , 5	11 , 11*	4 , 10
	s_{14}	17 , 6*	10 , 5	10 , 4	3 , 3

Example

Fig. 4. High cost non-drastic innovations.

Lemma 4. *Suppose* $\mathrm{c_{iu}} \leq \pi^{\mathrm{c}}_{\mathrm{ij}}\ \ \forall\ i, j = 1, 2\ \ \{V^{\mathrm{c}}_{\mathrm{iu}} \geq 0\}$. *Then,*
(a) $\pi_1(s_2, s) \geq \pi_1(s_1, s)$ *and* $\pi_2(s, s_2) \geq \pi_2(s, s_1)\ \ \forall\ s$
(b) $\pi_1(s_4, s) \geq \pi_1(s_3, s)$ *and* $\pi_2(s, s_4) \geq \pi_2(s, s_3)\ \ \forall\ s$
where $\pi_{\mathrm{i}}(s_k, s_l)$ *is i's payoff when firm 1 chooses* s_k *and firm 2 chooses* s_l.

3.2. Low Cost Non-drastic Innovations

I classify the non-drastic innovations as low cost if the competitive payoffs the entrant and the incumbent would separately earn in any market following a joint adoption of an innovation are strictly positive ($V^c_{iu} > 0$, $V^c_{id} > 0$).

Proposition 6. *Under low-cost non-drastic innovations where* $\mathrm{c}_{iu} < \mathrm{c}_{id} < \pi^c_{ij}$, $\{\mathrm{V}^c_{iu} > 0\ ;\ \mathrm{V}^c_{id} > 0\}$ *the NEP is the strategy combination* $\{s_4, s_4\}$ *with payoffs* $(\mathrm{V}^c_{1u} + \mathrm{V}^c_{1d} + r_1\ ; \mathrm{V}^c_{2u} + \mathrm{V}^c_{2d} + r_2)$. *Thus, each monopolist upgrades both in its own product line and diversifies into the competitor's product line while keeping the monopoly position in the old product market which is partly replaced by the innovation.*

Remark 5. Since innovations are non-drastic each firm stays as a monopolist in the old product market and also guarantees a non-negative payoff by upgrading its own product even if the competitor diversifies. On the other hand, diversifying also yields a non-negative competitive payoff. Thus, each monopolist successfully enters the incumbent's market. Although the incumbent can not prevent entry, it avoids being partly preyed upon by the entrant, through upgrading its own product. Monopoly is replaced by competition in both of the new product markets since each monopolist both upgrades its own product line and diversifies into the competitor's product line. However, each firm maintains the monopoly position in the old product market which, in part, is replaced by the innovation. The costly diversification may be justified for the presumably lower prices that would result under competition.

Example : $\pi_{\mathrm{i}}^{\mathrm{m}} = 10$, $\mathrm{c_{iu}} = 2$, $\mathrm{c_{id}} = 3$, $V_{\mathrm{iu}}^{\mathrm{m}} = 8$, $V_{\mathrm{id}}^{\mathrm{m}} = 7$, $V_{\mathrm{iu}}^{\mathrm{c}} = 2$, $V_{\mathrm{id}}^{\mathrm{c}} = 1$, $\mathrm{r}_1 = \mathrm{r}_2 = 6$.

Firm 2

Firm 1	s_{21}	s_{22}	s_{23}	s_{24}
s_{11}	10 , 10	10 , 14	6 , 17	6 , 21
s_{12}	14 , 10	14 , 14	8 , 11	8 , 15
s_{13}	17 , 6	11 , 8	13 , 13	7 , 15
s_{14}	21 , 6	15 , 8	15 , 7	9 , 9*

Example

Fig. 5. Low cost non-drastic innovations.

When the above values are inserted into the generalized normal form given in table 2 we see that the NEP is $\{s_4, s_4\}$, and the corresponding payoffs are (9,9).

3.3. Medium Cost Non-drastic Innovations

Medium cost innovations are defined such that competition with an upgraded product is profitable ($V_{\mathrm{iu}}^{\mathrm{c}} \geq 0$) but diversifying to compete with an upgraded product is not ($V_{\mathrm{id}}^{\mathrm{c}} \leq 0$).

Proposition 7. *Under medium-cost non-drastic innovation where* $\mathrm{c}_{iu} \leq \pi_{ij}^{c} \leq \mathrm{c}_{id}$ $\{\mathrm{V}_{iu}^{c} > 0\ ;\ \mathrm{V}_{id}^{c} > 0\}$*, the NEP is the strategy combination* $\{s_2, s_2\}$ *with payoffs* $(\mathrm{V}_{1u}^{m} + r_1\ ;\ \mathrm{V}_{2u}^{m} + r_2)$*. Thus, the monopolists stay in their product lines and upgrade their own products.*

Remark 6. Since innovations are non-drastic each firm stays as a monopolist in the old product market and also guarantees a non-negative payoff by upgrading its own product even if the competitor diversifies. On the other hand, diversifying yields a non-positive competitive payoff. Thus, each incumbent effectively prevents entry by adopting only those innovations in its own product line and maintains its monopoly position. This is the optimal outcome from both the firms' and the society's standpoint. Both innovations are undertaken by the low cost firms established in those product lines. The threat of an incumbent prevents costly diversification which is desirable. Yet, both innovations are adopted through low cost upgrading by the incumbent firms which maintain their monopoly positions in their respective product lines.

Example: $\pi_{\mathrm{i}}^{\mathrm{m}} = 10$, $\mathrm{c}_{\mathrm{iu}} = 3$, $\mathrm{c}_{\mathrm{id}} = 5$, $V_{\mathrm{iu}}^{\mathrm{m}} = 7$, $V_{\mathrm{id}}^{\mathrm{m}} = 5$, $V_{\mathrm{iu}}^{\mathrm{c}} = 1$, $V_{\mathrm{id}}^{\mathrm{c}} = -1$, $\mathrm{r}_1 = \mathrm{r}_2 = 6$

Firm 2

Firm 1	s_{21}	s_{22}	s_{23}	s_{24}
s_{11}	10 , 10	10 , 13	6 , 15	6 , 18
s_{12}	13 , 10	13 , 13*	7 , 9	7 , 12
s_{13}	15 , 6	9 , 7	11 , 11	5 , 12
s_{14}	18 , 6	12 , 7	12 , 5	6 , 6

Example

Fig. 6. Medium cost non-drastic innovations.

A simple inspection upon plugging the values above into table 2 shows that (s_2, s_2) is the NEP, with the corresponding payoffs of (13,13).

3.4. Summary of Results

If the innovation is (*i*) non-drastic and high cost type, then the firms would find themselves in a multiple equilibria in pure strategies which ranges from sticking with their status quo to diversifying across both product lines; if the innovation is (*ii*) non-drastic and low cost type, they would upgrade and diversify across both product lines by adopting both of the innovations; and finally if the innovation is (*iii*) non-drastic and medium cost type, they would upgrade in their own product lines and maintain their monopoly positions.

Multiple equilibria arises with non-drastic innovation if competitive payoffs of both the entrant and the incumbent are strictly negative ($\pi_{ij}^{c}<c_{iu} < c_{id}$); in other words if the innovations are high cost type. Under this scenario a multiplicity of best reply strategies for each monopolist range from adopting both innovations, to not adopting any of the innovations. Both innovations are adopted, however, under any of the possible Nash equilibria. An interesting point is that, under two of the four possible equilibria, we observe a passive incumbent and an aggressive entrant. The entrant monopolist diversifies across both its own product line and the entrant's product line, whereas the incumbent sticks with the old product and is partly preyed upon and replaced by the aggressive entrant. In the other two multiple equilibria both firms are actively involved in diversification and specialization process. In one of the equilibrium, firms stay in their own market and diversify in their own product lines. In the other equilibrium, they diversify only in the incumbent's product line; and in this process of switching, they partly replace the incumbent and are partly replaced by the entrant in their old markets.

Under the second type of equilibrium with low cost innovations, firms upgrade products not only in their own product line but also in the incumbent's product line. This type of total diversification arises when competitive payoffs from diversifying into the competitor's product line is non-negative ($c_{iu} < c_{id} < \pi_{ij}^{c}$). This latter result is obtained under both the drastic and non-drastic low cost innovations.

Under the third type of Nash equilibrium, monopolists stay in their own markets and increase their specialization and upgrading of existing products. We get this result under Nash equilibrium with medium cost non-drastic innovations ($c_{iu} \leq \pi_{ij}^{c} \leq c_{id}$), i.e., when the cost of diversifying in another product line exceeds the flow profits of a possible competitive outcome.

The main tendency is that if the firms are facing non-drastic innovations, then, they would either upgrade in their own product lines only, or upgrade and diversify into both product lines. Unlike the case of drastic innovation, firms do have the incentive to upgrade their own products even without the technological rivalry. The existence of a potential threat of entry into the incumbent's product line enhances the process of diversification and the firms might find themselves with excessive diversification across all possible product lines within their technological reach. The optimal outcome from both the society's and the firms' standpoints dictates that the firms adopt innovations in their own product lines since the same maximum product diversity could be achieved by the least cost monopolist. However, not only that this can not be enforced as a credible commitment, but it would also imply that the incumbents' monopoly positions would have to remain unchallenged. Clearly, this process of strategic inventiveness is in accord with the Schumpeterian concept of "creative destruction".

4. A Model of Innovation Adoption under Asymmetric Information

Next I turn to an asymmetric information scenario. Asymmetry means that one of the firms has access to private information which the other firm does not, and can be justified by historical leadership of a firm in R&D activity and other reputation effects. Hence, the informed firm is recognized and its leadership is accepted, giving it the first-mover advantage. Yet, the resulting signal about its private information would enable the uninformed firm to form conjectures, update its prior beliefs and make assessments.

In this section I work with the non-drastic innovation case and consider the drastic innovation as a special case. *(Refer to Table 2. for the generalized normal form game.)*

Payoffs are as defined in the base model of section 3. I shall, however, suppress the first subscript of the above notation when writing the payoffs under different strategy combinations. For example, the payoff to firm 2 under s_{12} and s_{22} strategy combinations will be denoted by $\pi_2(s_2, s_2) = V_{2u}^m + r$, and the strategy combination (s_4, s_4) now means that the firms adopt the innovations both in their product lines (upgrading) and across product lines (diversifying) while continuing to produce the old products. Thus, the firms become Cournot competitors in the new product markets and maintain their monopoly position in the -now suppressed-old product markets. In this case, the payoffs to firm 1 and firm 2, in respective order, are as follows:

$$\pi_1(s_4, s_4) = V_{1u}^c + V_{2d}^c + r$$
$$\pi_2(s_4, s_4) = V_{2u}^c + V_{2d}^c + r$$

Next, recall that there are types of the players determined by 3 possible cost structures defined with respect to the competitive payoffs of each firm. I identify each cost structure with a possible firm type, denoted by N$_{ij}$(j^{th}type of firm i). These firm types are given as follows:

$$\begin{array}{lll} (i) & N_{i1} : V_{iu}^c \leq 0, V_{id}^c < 0 & (\pi^c \leq c_{iu} < c_{id}) \\ (ii) & N_{i2} : V_{iu}^c > 0, V_{id}^c < 0 & (c_{iu} < \pi^c < c_{id}) \\ (iii) & N_{i3} : V_{iu}^c > 0, V_{id}^c \geq 0 & (c_{iu} < c_{id} \leq \pi^c) \end{array}$$

From the incumbent's point of view, the competitor's type is an indicator of whether it is a low cost, medium cost, or a high cost entrant. From the entrant's point of view, the incumbent's type is an indicator of whether it would fight back to block entry, accommodate the entrant and share the new product market, or yield the monopoly position in its product line.

Suppose, firm 1 has private information about his own type and that of firm 2, but firm 2 does not.–throughout this paper we shall assume that firm 1 is the informed player, and first mover. Firm 1 observes both its own type (N_{1j}) and its competitor's type (N_{2j}).

Firm 2, on the other hand, knows only the adoption costs in its own product line,c_{2u} , which enables it to conclude whether $V_{2u}^c \leq 0$ or $V_{2u}^c > 0$. If, in fact, $c_{2u}$is too high such that $V_{2u}^c \leq 0$, then it follows that $V_{2u}^c < 0$since by initial condition(ii) we have $c_{2u} < c_{2d}$. This enables firm 2 to conclude that his type is (N_{21}). Firm 2 does not need additional information to decide whether its type is (N_{21})or not.

When firm 2 is of type (N_{21}), her best reply to the strategy played by firm 1 is based on only the information that she is a high cost firm. On the other hand, firm 2 has imperfect information about firm 1's type. Firm 2 knows that firm 1 is the informed player and has complete information about the types of both firms. This information advantage enables firm 1 to be the first mover regardless of his own type, and firm 2 accepts its leadership.

Lemma 5. *For the uninformed firm, its best reply to $s_{11}$is given as follows:*

$$s_{24} = b_2(s_{11}) \quad \forall \quad N_{2j}, j = 1, 2, 3.$$

Proof (of lemma). From Table 1 we immediately see that,

$$\begin{aligned} \pi_2(s_1, s_4 \mid N_{2j}) - \pi_2(s_1, s_3 \mid N_{2j}) &= (V_{2u}^m + V_{2d}^m + r) - \pi^m + V_{2d}^m) \\ &= V_{2u}^m \quad + \quad r - \pi^m \geq 0 \end{aligned}$$

$$\begin{aligned} \pi_2(s_1, s_4 \mid N_{2j}) - \pi_2(s_1, s_2 \mid N_{2j}) &= (V_{2u}^m + V_{2d}^m + r) - (V_{2u}^m + r) \\ &= V_{2u}^m > 0 \end{aligned}$$

$$\begin{aligned} \pi_2(s_1, s_4 \mid N_{2j}) - \pi_2(s_1, s_1 \mid N_{2j}) &= (V_{2u}^m + V_{2d}^m + r) - (\pi^m) \\ &= (V_{2u}^m \quad + \quad r - \pi^m) + V_{2d}^m > 0 \qquad \square \end{aligned}$$

We denote firm 2's best reply to the strategy s played by firm 1 by $b_2(s)$:

$$b_2(s) = \arg\max_{s'} \pi_2 \; \pi_2(s, s')$$

Then, a Stackelberg equilibrium is a pair of strategies $(s^*, b_2(s^*))$ such that,

$$s^* = \arg\max_{s} \pi_1 \, (s^*, b_2(s^*)).$$

Next, the following lemma is used to construct normalized strategies of firm 2 as a function of her type, N_{2j}

Lemma 6. *For the uninformed firm, its best replies given that it knows its type, are given as follows:*

$$\textbf{\textit{(a)}} \qquad \begin{aligned} s_{22} &= b_2(s_{12}) \quad iff \quad N_2 = N_{21}, N_{22} \\ s_{24} &= b_2(s_{12}) \quad iff \quad N_2 = N_{23} \end{aligned}$$

$$\textbf{\textit{(b)}} \qquad \begin{aligned} s_{23} &= b_2(s_{13}) \quad iff \quad N_2 = N_{21} \\ s_{24} &= b_2(s_{13}) \quad iff \quad N_2 = N_{22}, N_{23} \end{aligned}$$

$$\textbf{(c)}\qquad \begin{array}{lll} s_{21} = b_2(s_{14}) & iff & N_2 = N_{21} \\ s_{22} = b_2(s_{14}) & iff & N_2 = N_{22} \\ s_{24} = b_2(s_{14}) & iff & N_2 = N_{23}. \end{array}$$

Table 3 summarizes firm 2's best reply strategies as a function of her type and the corresponding payoffs to both firms.

$Firm\ \ 1's$ $Leader\ \ Strategy$	$Firm2's$ $Best\ \ Reply$	$Firm\ \ 1's$ $Payoff$	$Firm\ \ 2's$ $Payoff$
s_1	s_4	r	$V_{2u}^m + V_{2d}^m + r$
s_2	$s_2,\ \ if\ \ V_{2d}^c < 0$ $s_4,\ \ if\ \ V_{2d}^c > 0$	$V_{1u}^m + r$ $V_{1u}^c + r$	$V_{2u}^m + r$ $V_{2u}^m + V_{2d}^c + r$
s_3	$s_3,\ \ if\ \ V_{2u}^c < 0$ $s_4,\ \ if\ \ V_{2u}^c \geq 0$	$V_{1u}^m + r$ $V_{1d}^c + r$	$V_{2d}^m + r$ $V_{2u}^c + V_{2d}^m + r$
s_4	$s_1,\ \ if\ \ V_{2u}^c < 0,\quad V_{2d}^c < 0$ $s_2,\ \ if\ \ V_{2u}^c > 0,\quad V_{2d}^c < 0$ $s_4,\ \ if\ \ V_{2u}^c > 0,\quad V_{2d}^c > 0$	$V_{1u}^m + V_{1d}^m + r$ $V_{1u}^m + V_{1d}^c + r$ $V_{1u}^c + V_{1d}^c + r$	r $V_{2u}^c + r$ $V_{2u}^c + V_{2d}^c + r.$

Table 3. Stackelberg equilibria and payoffs under perfect information.

Lemma 7. *For the informed firm, the following payoff dominance relationships hold if the uninformed firm is of type* N_{2J} $\{J = 2, 3\}$

(a)

$$\textbf{(a)}\qquad \begin{array}{l} \pi_1(s_2, s_2 \mid N_{11}) > \pi_1(s_k, s_2 \mid N_{11}) \quad \forall \quad k = 1, 3, 4 \\ \pi_1(s_1, s_4 \mid N_{11}) > \pi_1(s_k, s_4 \mid N_{11}) \quad \forall \quad k = 2, 3, 4 \end{array}$$

$$\textbf{(b)}\qquad \begin{array}{l} \pi_1(s_2, s_2 \mid N_{12}) > \pi_1(s, s_2 \mid N_{12}) \\ \pi_1(s_2, s_4 \mid N_{12}) > \pi_1(s, s_4 \mid N_{12}) \end{array}$$

$$\textbf{(c)}\qquad \begin{array}{l} \pi_1(s_4, s_2 \mid N_{13}) > \pi_1(s, s_2 \mid N_{13}) \\ \pi_1(s_1, s_4 \mid N_{13}) > \pi_1(s, s_4 \mid N_{13}) \end{array}$$

4.1. Stackelberg Equilibria

It should be noted that only in the case where firm 1 leads by playing its s_{11}strategy, is firm 2's best reply not a function of its type; i.e., firm 2 does not need to know its type to play its best reply strategy, s_{24}. Consequently we can write the following proposition.

Proposition 8. *If firm 1 has perfect information, then* (s_4, s_1) *is a Stackelberg equilibrium if and only if firm 2's type is* N_{21}.

Proof (of proposition).

Proposition 8 can be easily proved by inspecting Table 2 :

If (s_4, s_1) is a Stackelberg equilibrium, then from the last part of table 3, $s_1 = b_2(s_4)$ if $V_{2u}^c > 0$, $V_{1d}^c < 0$, i.e., firm 2 s type is $N_{21.}$ Conversely, suppose firm 2's type is$N_{21.}$.

Then, consider each strategy of firm 1:

$$\begin{aligned}
s_4 = b_2(s_1) &\Rightarrow \pi_1(s_1, b_2(s_1)) = r \\
s_2 = b_2(s_2) &\Rightarrow \pi_1(s_2, b_2(s_2)) = V_{1u}^m + r \\
s_3 = b_2(s_3) &\Rightarrow \pi_1(s_3, b_2(s_3)) = V_{1u}^m + r \\
s_1 = b_2(s_4) &\Rightarrow \pi_1(s_4, b_2(s_4)) = V_{1u}^m + V_{1d}^m + r
\end{aligned}$$

From this we see that $s_4 = \arg\max_s \pi_1(s, b_2(s))$ and $s_1 = b_2(s_4)$.

Thus, $(s_4, s_1)$is a Stackelberg equilibrium. □

Remark 7. Under (N_{11}, N_{21}), $(N_{12}, N_{21})$and(N_{13}, N_{21}) the uninformed player can deduce that it is a high cost firm without further information or signaling by firm 1, the informed player. Under the above states of the world, firm 2 need not know which type of a competitor (high cost/medium cost/low cost) it is facing. Firm 2's best response solely depends on its own type,N_{21}. Firm 1 benefits from its information advantage only because it is the first mover. An interesting aspect of the NEP is that it is determined without any reference to the leader's type. Firm 1 might be a high cost or medium cost firm($N_{11} or N_{12}$). But this is irrelevant for the particular NEP obtained, as long as firm 1 is the first mover. Firm 1 uses the first mover advantage; it upgrades and diversifies to obtain monopoly position in both the old and the new product markets. Firm 2, on the other hand, continues to produce the old product and exploits the residual demand as a declining monopolist.

Proposition 9. *The following strategy combinations obtain under the following states of the world*[9] *when firm 1 is the Stackelberg leader, and firm 2 is the follower.*

If (N_{11}, N_{22}), *then the Stackelberg equilibrium is* (s_2, s_2);
If (N_{12}, N_{22}), *then the Stackelberg equilibrium is* (s_2, s_2);
If (N_{13}, N_{22}), *then the Stackelberg equilibrium is* (s_4, s_2);
If (N_{11}, N_{23}), *then the Stackelberg equilibrium is* (s_1, s_4);
If (N_{12}, N_{23}), *then the Stackelberg equilibrium is* (s_2, s_4);
If (N_{13}, N_{23}), *then the Stackelberg equilibrium is* (s_4, s_4);

Proof. Analogous to the proof of Proposition 8 as it is seen from Table 3.

[9] *A State of the World* is defined as the occurrence of a combination of the types of two firms such that $(N_{1i}, N_{2j}) \quad i, j = 1, 2, 3.$

4.2. Perfect Bayesian Equilibrium with a medium/low cost follower

The Solution Concept:

To firm 2, firm 2 is either type N_{22} or N_{23}; firm 1 can be any of N_{11},N_{12},N_{13}.

Being uninformed, firm 2 assigns prior probabilities $\Pr(N_{22}) = \phi$, $\Pr(N_{23}) = 1-\phi$; $\Pr(N_{11}) = \theta_1$; $\Pr(N_{12}) = \theta_2$; and $\Pr(N_{13}) = \theta_3$, $\phi \geq 0$, $\theta_j \geq 0$, $\theta_1+\theta_2+\theta_3$ =1

These prior beliefs are common knowledge, i.e., they are known to both firms. Firm 2 knows that firm 1 knows her prior beliefs, and firm 1knows that firm 2 knows that he knows her beliefs, and so on. Firm 2 has to update its beliefs after observing certain strategies played by firm 1.

Suppose for example, that firm 2 observes s_{12} being played by firm 1. Her type might be either N_{22} or N_{23}. Seeing s_{12} should make firm 2 update the posterior belief, $\Pr(N_{22} \mid s_{12})$. The natural method is to use Bayes's rule, which shows how to revise the prior belief in the light of data. It uses two pieces of information that firm 2 knows: the likelihood of seeing s_{12} given that the state of the world is N_{22}, $\Pr(s_{12} \mid N_{22})$, and the likelihood ofs_{12} given that the state of the world is, N_{23}, $\Pr(s_{12} \mid N_{23})$. Since there are only 2 alternatives to firm 2's type, the marginal likelihood of seeings_{12} as a result of one or another possible types of firm 2, (N_{22} *or* N_{23}) is given by,

$$\Pr(s_{12}) = \Pr(s_{12} \mid N_{22}) \Pr(N_{22}) + \Pr(s_{12} \mid N_{23}) \Pr(N_{23}).$$

The probability that both the strategy s_{12} played and the state of the world N_{22} occurs is:

$$\begin{aligned} \Pr(s_{12}, N_{22}) &= \Pr(s_{12} \mid N_{22}) \Pr(N_{22}) \\ &= \Pr(N_{22} \mid s_{12}) \Pr(s_{12}) \end{aligned}$$

Firm 2's new belief -its posterior- is calculated using $\Pr(s_{12})$, which yields the following Bayes Rule.

$$\Pr(N_{22} \mid s_{12}) = \frac{\Pr(s_{12} \mid N_{22}) \Pr(N_{22})}{\Pr(s_{12})}$$

The term Bayesian equilibrium is used to refer to Nash equilibrium when players update their beliefs according to Bayes's rule (Rasmussen, 1989). Perfect Bayesian equilibrium point (PBEP) is a Stackelberg equilibrium (s*, $\beta_2(s^*)$) where $\beta_2(s^*)$is the Bayesian best reply to to s* such that ,

$$\beta_2(s) = \arg\max_{s'} \pi_2 E\pi_2(s, s')$$

$$s^* = \arg\max_{s'} \pi_1 \left(s, \beta_2(s)\right)$$

4.3. Characterization of Equilibrium

The main focus of this section is Theorem 2. Its proof shall be accomplished in a series of lemmas and observations in Appendix 2.

Theorem 2. *Under asymmetric, imperfect and incomplete information where firm 1 is the informed player and where the states of the world are* (N_{1j}, N_{2l}), $\forall$ = 1, 2, 3, l = 2, 3, *the perfect Bayesian equilibrium points are the following strategy combinations:*

(a) : (s_1, s_4) *iff* $(-)\frac{(V_{2d}^c|, N_{22})}{(V_{2d}^c|, N_{23})} \leq \frac{\theta_2(1-\phi)}{(\theta_1+\theta_2)\phi}$ *and* $N_1 = N_{11}$

(b) : (s_2, s_4) *iff* $(-)\frac{(V_{2d}^c|, N_{22})}{(V_{2d}^c|, N_{23})} \leq \frac{\theta_2(1-\phi)}{(\theta_1+\theta_2)\phi}$ *and* $N_1 = N_{12}$

(c) : (s_4, s_4) *iff* $(-)\frac{(V_{2d}^c|, N_{22})}{(V_{2d}^c|, N_{23})} \leq \frac{(1-\phi)}{\phi}$ *and* $N_1 = N_{13}$

(d) : (s_2, s_2) *iff* $(-)\frac{(V_{2d}^c|, N_{22})}{(V_{2d}^c|, N_{23})} \geq \frac{\theta_2(1-\phi)}{(\theta_1+\theta_2)\phi}$ *and* $N_1 = N_{11}, N_{12}$

(e) : (s_4, s_2) *iff* $(-)\frac{(V_{2d}^c|, N_{22})}{(V_{2d}^c|, N_{23})} \leq \frac{(1-\phi)}{\phi}$ *and* $N_1 = N_{13}$

First, recall from weak dominance firm 2's dominant strategies are

s_{22} and $s_{24}$when it is either medium or low cost (see table 4.2). Hence, firm 1 effectively faces a normal form game of dimension (4x2) if firm 2 is of either N_{22} or N_{23} type (see Table 4. for the reduced normal form).

		Firm 2	
		s_{22}	s_{24}
Firm 1	s_{11}	π_{11}^m , $V_{2u}^m + r$	r , $V_{2u}^m + V_{2d}^m + r$
	s_{12}	$V_{1u}^m + r$, $V_{2u}^m + r$	$V_{1u}^c + r$, $V_{2u}^m + V_{2d}^m + r$
	s_{13}	$\pi_{11}^m + V_{1d}^c$, $V_{2u}^c + r$	$V_{1d}^c + r$, $V_{2u}^c + V_{2d}^m + r$
	s_{14}	$V_{1u}^m + V_{1d}^c + r$, $V_{2u}^c + r$	$V_{1u}^c + V_{1d}^c + r$, $V_{2u}^c + V_{2d}^c + r$

Table 4. Reduced strategic game with non-drastic innovations under imperfect information.

I assume that firm 1, being an informed player, will always strive to reach the Stackelberg equilibrium that corresponds with its observed state. This assumption allows us to derive firm 2's conjectures of observing a strategys_{1k}conditional on her type N_{2j}, i.e., $\Pr(s_{1k} \mid N_{2j})\ \forall\ J = 2, 3$ **&** $k = 1, ..., 4$.

Lemma 8. *For the uninformed firm the following conjectures hold.*

$$
\begin{aligned}
(a): \quad Pr(s_{11}, N_{22}) &= \sum_{i=1}^{3} \Pr(s_{11} \mid N_{1i}, N_{22}) \Pr(N_{1i}) \\
&= \Pr(s_{11} \mid N_{11}, N_{22}) \Pr(N_{11}) + \Pr(s_{11} \mid N_{12}, N_{22}) \Pr(N_{12}) \\
&\quad + \Pr(s_{11} \mid N_{13}, N_{22}) \Pr(N_{13}) \\
&= 0 \\
\Pr(s_{11}, N_{23}) &= \sum_{i=1}^{3} \Pr(s_{11} \mid N_{1i}, N_{23}) \Pr(N_{1i}) \\
&= \Pr(s_{11} \mid N_{11}, N_{23}) \Pr(N_{11}) + \Pr(s_{11} \mid N_{12}, N_{23}) \Pr(N_{12}) \\
&\quad + \Pr(s_{11} \mid N_{13}, N_{23}) \Pr(N_{13}) \\
&= \Pr(N_{11}) = \theta_1 \\
(b): \quad \Pr(s_{12}, N_{22}) &= \sum_{i=1}^{3} \Pr(s_{12} \mid N_{1i}, N_{22}) \Pr(N_{1i}) \\
&= \Pr(s_{12} \mid N_{11}, N_{22}) \Pr(N_{11}) + \Pr(s_{12} \mid N_{12}, N_{22}) \Pr(N_{12}) \\
&\quad + \Pr(s_{12} \mid N_{13}, N_{22}) \Pr(N_{13}) \\
&= \Pr(N_{11}) + \Pr(N_{12}) = \theta_1 + \theta_2 \\
\Pr(s_{12}, N_{23}) &= \sum_{i=1}^{3} \Pr(s_{12} \mid N_{1i}, N_{23}) \Pr(N_{1i}) \\
&= \Pr(s_{12} \mid N_{11}; N_{23}) \Pr(N_{11}) + \Pr(s_{12} \mid N_{12}, N_{23}) \Pr(N_{12}) \\
&\quad + \Pr(s_{12} \mid N_{13}, N_{23}) \Pr(N_{13}) \\
&= \Pr(N_{12}) = \theta_2 \\
(c): \quad \Pr(s_{14}, N_{22}) &= \sum_{i=1}^{3} \Pr(s_{12} \mid N_{1i}, N_{22}) \Pr(N_{1i}) \\
&= \Pr(s_{14} \mid N_{11}, N_{22}) \Pr(N_{11}) + \Pr(s_{14} \mid N_{12}, N_{22}) \Pr(N_{12}) \\
&\quad + \Pr(s_{14} \mid N_{13}, N_{22}) \Pr(N_{13}) \\
&= \Pr(N_{13}) = \theta_3 \\
\Pr(s_{14}, N_{23}) &= \sum_{i=1}^{3} \Pr(s_{14} \mid N_{1i}, N_{23}) \Pr(N_{1i}) \\
&= \Pr(s_{14} \mid N_{11}, N_{23}) \Pr(N_{11}) + \Pr(s_{14} \mid N_{12}, N_{23}) \Pr(N_{12}) \\
&\quad + \Pr(s_{14} \mid N_{13}, N_{23}) \Pr(N_{13}) \\
&= \Pr(N_{13}) = \theta_3
\end{aligned}
$$

Remark on Separating and Pooling Equilibria:

Suppose part **(d)** of Theorem 2 holds, so that firm 2's best reply to s_{12} is s_{22} We note that under **(d)** firm 1 can be either a high cost type or a medium cost type. Clearly, by deciding on a best reply of s_{22} under **(d)** firm 2 can not differentiate between the two types of competitors.

From lemma 7(b), recall that a medium cost firm 1's best reply to either s_{22} or s_{24} is s_{12}. From lemma 7(a), also recall that a high cost type firm 1's best reply to s_{22} is also s_{12}. This indicates that under **(d)** an informed high cost firm successfully pretends that it is medium cost type. Under **(d)** we obtain a pooling equilibrium.

Next, suppose **(a)** holds. Notice that **(a)** is a perfectly symmetric condition to **(d).** In this case, firm 2's best reply to $s_{12}$is s_{24} . But **(a)** holds only if firm 1 is high cost type so that his best reply to s_{24} is s_{11}. In this case, a high cost firm 1 is successfully differentiated from a medium cost one. Under **(a)** we obtain a separating equilibrium.

Rational Priors at a Boundary Payoff : $V_{id}^c = 0$

Recall that a firm is defined as medium cost -type N_{i2}- if $V_{iu}^c > 0$ and $V_{id}^c < 0$ hold; and it is defined as low cost-type N_{i3}- if $V_{iu}^c > 0$ and $V_{id}^c \geq 0$ hold. Hence, note that the lower bounds of V_{id}^c under the two types are given by the following equations:

$$(V_{id}^c \mid N_{i2}) = -c_{id}$$
$$(V_{id}^c \mid N_{i3}) = 0$$

Corollary 1. *Suppose the uninformed firm's competitive payoff is given by,* $(V_{id}^c \mid N_{23}) = 0$*. Then, a separating equilibrium is obtained if and only if the following priors hold:*
$1 - \phi = 1$,*and/or* $\theta_3 = 1$

Proof. Suppose firm 2 observes s_{12}. Then, from the proof of Theorem 2 part (b) we note that firm 2's best reply to s_{12} is s_{24} if and only if:

$$0 \leq \theta_2(1-\phi)(V_{2d}^c \mid N_{23}) + (\theta_1 + \theta_2)\phi(V_{2d}^c \mid N_{22})$$

Substituting $(V_{2d}^c \mid N_{23}) = 0$, into the above equation we obtain,

$$0 \leq (\theta_1 + \theta_2)\phi(V_{2d}^c \mid N_{11}, N_{22}). \tag{14}$$

But, since $(V_{2d}^c \mid N_{22}) < 0$ then, we require either $\theta_1 + \theta_2 = 0$, *and/or* $\phi = 0$ for (14) to hold. Hence, it is easily seen that:
$1 - \phi = 1$, and/or $\theta_3 = 1$. □

Remark 8. If $(V_{2d}^c \mid N_{23})$ then from theorem 2**(b)** we require (14) to hold for a separating equilibrium (s_2, s_4). But, it is immediate that if there is a slight deviation in the priors so that the reverse inequality to (14) holds then, we obtain the condition in part**(d)** which gives s_{22} as a Bayesian best reply to s_{12}. Therefore, a pooling equilibrium shall be obtained under part **(d)**. Obviously, firm 2 would prefer a separating equilibrium to a pooling equilibrium as opposed to firm 1 who would rather have a pooling equilibrium. This clearly requires (14) to hold, enabling firm 2 to derive the range of her rational priors as defined in the corollary.

4.4. A Proposed Equilibrium Using "Equally Likely" Assumption

A reasonable way to form priors for firm 2 is to conjecture that her type being N_{22} or N_{23} is equally likely (conditioned on not observing N_{21}), Pr(N_{22}) =Pr(N_{23}) =0.5

Further, suppose that she conjectures her facing a competitor of type N_{11}, N_{12} or N_{13} is also equally likely, $\Pr(N_{11})$ =$\Pr(N_{12})$ =$\Pr(N_{13})$ =0.333 .

Based on these common priors, firm 2 would update her beliefs. Accordingly, using the conjecture equations from lemma 8, firm 1 would play s_{12} 66.7 percent of the time, and play s_{14} 33.3 percent of the time, when firm 2's type is N_{22}. Hence, firm 2's conjecture upon observing s_{12} would be $\Pr(s_{11} \mid N_{22}) = 0.667$ and $\Pr(s_{14} \mid N_{22}) = 0.333$.

Similarly when firm 2's type is N_{23}, firm 1 would plays_{11}, s_{12} and s_{14}33.3 percent of the time, leading to conjectures $\Pr(s_{11} \mid N_{23})$ =$\Pr(s_{12} \mid N_{23})$ =$\Pr(s_{14} \mid N_{23}) =$ 0.333

Suppose firm 1 plays his s_{12} strategy. Recall that this situation is characterized under parts **(b)** and **(d)** of theorem 2. Firm 2 would update her beliefs using her priors and Bayes's rule upon observing s_{12}. Then, we have:

$$\Pr(N_{22} \mid s_{12}) = \frac{\{\ \Pr(s_{12} \mid N_{22})\ \}\,\phi}{\{\ \Pr(s_{12} \mid N_{22})\ \}\,\phi + \{\Pr(s_{12} \mid N_{23})\ \}\,(1-\phi)}.$$

Substituting the conjectures,

$$= \frac{(\theta_1+\theta_2)\phi}{(\theta_1+\theta_2)\phi + \theta_2(1-\phi)}$$

and finally, substituting the values of priors,

$$\Pr(N_{22} \mid s_{12}) = \frac{(0.667)(0.5)}{(0.667)(0.5) + (0.333)(0.5)} = 0.667.$$

Also,

$$\Pr(N_{23} \mid s_{12}) = 1 - \Pr(N_{22} \mid s_{12}) = 1 - 0.667 = 0.333$$

Thus, using the notation of theorem 2 we obtain,

$$\frac{\theta_2(1-\phi)}{(\theta_1+\theta_2)\phi} = \frac{0.333}{0.667} = 0.5$$

Hence, from the theorem 2, part **(d)** if the condition $(-)\frac{(V^c_{2d} \mid N_{1i}, N_{22})}{(V^c_{2d} \mid N_{1i}, N_{23})} \geq 0.5$ holds, then firm 2's equilibrium strategy upon observing s_{22} is to play s_{22}.

On the other hand, if, instead, firm 2's conjectures satisfy part **(b)** of theorem 2, such that $(-)\frac{(V^c_{2d} \mid N_{1i}, N_{22})}{(V^c_{2d} \mid N_{1i}, N_{23})} \leq 0.5$ holds, then firm 2's best reply to s_{12} is s_{24}.

Next, suppose firm 1 plays his s_{14} strategy. This situation is characterized under parts **(c)** and **(e)** of theorem 2. Firm 2's updated beliefs would be,

$$\Pr(N_{22} \mid s_{12}) = \frac{\{\ \Pr(s_{12} \mid N_{22})\ \}\,\phi}{\{\ \Pr(s_{12} \mid N_{22})\ \}\,\phi + \{\Pr(s_{12} \mid N_{23})\ \}\,(1-\phi)}$$

substituting the conjectures we have,

$$= \frac{(\theta_1+\theta_2)\phi}{(\theta_1+\theta_2)\phi + \theta_2(1-\phi)}.$$

Finally, substituting the values of priors we obtain,

$$\Pr(N_{22} \mid s_{12}) = \frac{(0.667)(0.5)}{(0.667)(0.5) + (0.333)(0.5)} = 0.667$$

and,

$$\Pr(N_{22} \mid s_{12}) = 1 - \Pr(N_{22} \mid s_{12}) = 1 - 0.667 = 0.333.$$

Thus, using the notation of theorem 2 we obtain,

$$= \frac{\theta_2(1-\phi)}{(\theta_1+\theta_2)\phi} = \frac{0.333}{0.667} = 0.5$$

Hence, from the theorem 1, part (d) if the condition $(-)\frac{(V_{2d}^c \mid N_{1i}, N_{22})}{(V_{2d}^c \mid N_{1i}, N_{23})} \geq 0.5$ holds, firm 2's equilibrium strategy upon observing s_{22} is to play s_{22}.

On the other hand, if, instead, firm 2's conjectures satisfy part (b) of theorem 2, such that $(-)\frac{(V_{2d}^c \mid N_{1i}, N_{22})}{(V_{2d}^c \mid N_{1i}, N_{23})} \leq 0.5$ holds, then firm 2's best reply to s_{12} is s_{24}.

Next, suppose firm 1 plays his s_{14}strategy. This situation is characterized under parts (c) and (e) of theorem 2. Firm 2's updated beliefs would be,

$$\Pr(N_{22} \mid s_{14}) = \frac{\{\ \Pr(s_{14} \mid N_{22})\ \}\,\phi}{\{\ \Pr(s_{14} \mid N_{22})\ \}\,\phi + \{\Pr(s_{14} \mid N_{23})\ \}\,(1-\phi)}.$$

Substituting the conjectures,

$$= \frac{\theta_3\phi}{\theta_3\phi + \theta_3(1-\phi)}$$

And finally, substituting the values of priors we get:

$$\Pr(N_{22} \mid s_{14}) = \frac{(0.333)(0.5)}{(0.333)(0.5) + (0.333)(0.5)} = 0.5.$$

It follows that,

$$\Pr(N_{23} \mid s_{14}) = 1 - \Pr(N_{22} \mid s_{14}) = 1 - 0.5 = 0.5$$

Substituting the notation of theorem 2 we obtain,

$$= \frac{(1-\phi)}{\phi} = \frac{0.5}{0.5} = 1$$

Hence, from the theorem 2, part (e) if the condition $(-)\frac{(V_{2d}^c \mid N_{1i}, N_{22})}{(V_{2d}^c \mid N_{1i}, N_{23})} \geq 1$ holds, upon observing s_{14}, firm 2's equilibrium strategy is to play s_{22}. Also, from theorem 1, part (c) if the condition $(-)\frac{(V_{2d}^c \mid N_{1i}, N_{22})}{(V_{2d}^c \mid N_{1i}, N_{23})} \leq 1$ holds, then upon observing s_{14}firm 2's equilibrium strategy is to play s_{24}. On the other hand firm 1's best reply does no depend on firm 2's strategy since under type N_{13} his best reply to boths_{22} and s_{24} is to play s_{14}. Nevertheless, the equilibrium (s_4, s_2)under (e) is preferred to the equilibrium (s_4, s_4) under (c) by firm 1, since it yields higher payoffs (see table 2).

Last, suppose firm 1 playss_{11} , then from lemma 5 we know that firm 2's best reply is to play s_{24}. But, this does not suffice for (s_1, s_4) to be a perfect Bayesian equilibrium, as I have argued in the proof of theorem 2. We check for the binding condition which causes s_{11} to be firm 1's equilibrium strategy.

$$\Pr(N_{22} \mid s_{12}) = \frac{\{\ \Pr(s_{12} \mid N_{22})\ \}\,\phi}{\{\ \Pr(s_{12} \mid N_{22})\ \}\,\phi + \{\Pr(s_{12} \mid N_{23})\ \}\,(1-\phi)}$$

Substituting for the values of the priors, and the "passive conjectures" we get,

$$\Pr(N_{22} \mid s_{12}) = \frac{(0.667)(0.5)}{(0.667)(0.5) + (0.333)(0.5)} = 0.667,$$

also,

$$\Pr(N_{22} \mid s_{12}) = 1 - \Pr(N_{22} \mid s_{12}) = 1 - 0.667 = 0.333.$$

Substituting the notation of theorem 2 we obtain,

$$= \frac{\theta_2(1-\phi)}{(\theta_1+\theta_2)\phi} = \frac{0.333}{0.667} = 0.5$$

Hence, we get the condition **(a),** $(-)\frac{(V^c_{2d} \mid N_{1i}, N_{22})}{(V^c_{2d} \mid N_{1i}, N_{23})} < 0.5$ for (s_1, s_4) to be an equilibrium strategy combination. That is, firm 1 decides to play his best reply strategy ofs_{11}, only after being certain that firm 2's Bayesian best reply to s_{12} is s_{24}. In return, firm 2's best reply to s_{11} is s_{24}. Firm 2 does not require to update her beliefs about her own type in order to decide on a best reply strategy, since from lemma 5 we know that s_{24} is her best reply under both N_{22} and N_{23}. In this case, firm 1 successfully recognizes a high cost competitor and we obtain a separating equilibrium (s_1, s_4) under **(a)**

Thus, proposed priors and the "passive conjectures" support the set of perfect Bayesian equilibria characterized in theorem 2.

4.5. Concluding Remarks

It is most likely and plausible to define the initial conditions for an interior solution, i.e., that for a low-cost firm competitive payoff from diversification would be strictly positive, whereas for a high-cost firm it would be strictly negative. Hence, the uninformed firm will have to form prior beliefs unconstrained by any rationality criteria which is discussed in the corollary. Theorem 2 allows us to make comparative static conjectures about the likelihood of certain equilibrium outcomes with respect to a change in the uninformed firm's prior beliefs.

Suppose firm 1 is considering to play s_1 strategy, knowing from theorem 2

That firm 2's best reply is s_4. Then, he is more likely to play s_1 the higher is firm 2's prior belief that she is a low-cost firm, ceteris paribus. Also, he is more likely to play s_1 the higher is firm 1's prior belief that she is facing a medium-cost competitor, ceteris paribus, thereby attaining the (s_1, s_4) equilibrium strategy combination. Hence, the higher are the values of $1-\phi$and/or $\theta_2$are, the more likely it is to obtain a separating equilibrium. If the likelihood of firm 2's prior beliefs is such that she strongly believes she is a high-cost firm and that she is facing either a high-cost and/or medium-cost competitor, then the more likely is it that the equilibrium outcome would be (s_1, s_4) supporting a pooling equilibrium, ceteris paribus.

An interesting observation is that it is less likely for a high-cost firm to exploit his information advantage against an optimistic firm 2 who strongly believes that she is a low-cost type, ceteris paribus. With an optimistic firm 2 and a high cost firm 1 it is more likely to observe (s_1, s_4) than it is to observe (s_2, s_2). On the other hand a pessimistic firm 2 can be more easily forced to stay in her product line by a high cost firm 1 who would stay in his product line himself. In this case we observe that firm 1 would be forced to maintain his monopoly position by upgrading his product line even though it would have been less costly for firm 2 to do so if she had chosen to diversify.

Another important observation on this point is that the efficiency choice from the society's standpoint is not one between having monopoly or competition, but it is one between a high-cost monopolist (firm 1) and a low-cost monopolist (firm 2). Hence, from an efficiency standpoint the society is losing from firm 2's ignorance, and more so if she does not have an optimistic outlook. This, possibly, is a supportive argument for having government research labs (or joint research projects coordinated by government institutions, like the MITI in Japan) provide and/or coordinate the flow of scientific information across companies which will enable them assess their potential strengths and weaknesses.

Next, suppose firm 1 plays his s_4 strategy. We know from theorem 2 that firm 1 would do so only when he is a low-cost firm. In other words, firm 1 would stick to s_4 if he is low -cost, no matter what firm 2's prior beliefs are. Firm 2's response, in this case, is a function of her prior beliefs about her own type only. She will tend to play s_4 the higher are her prior beliefs that she is a low-cost firm, enabling her to upgrade and diversify into both markets following her competitor, ceteris paribus. This requires firm 2 to be optimistic about her own type, whereas firm 1 would prefer her to be pessimistic and stick to her product line.

Hence, it is more likely to get a competitive outcome (s_4, s_4)in both markets the more optimistic firm 2 is about her own type. On the other hand, it is more likely that a pessimistic firm 2 would lose her monopoly position in her product line by sharing it with firm 1, while firm 1continues to be a monopolist in his product line.

From an efficiency standpoint, the choice is between having monopoly or competition in firm 1's product line. Since with a more optimistic firm 2 we tend to have competition in both markets, then, policies that encourage firm 2 to increase her optimism about diversification should be devised. Such policies would possibly provide additional incentives that would enable downgrading of the uninformed firm's prior beliefs about the costs of diversification.

5. Conclusion

This paper has investigated strategic behavior of technologically progressive monopolist firms facing a choice of upgrading or diversifying their product lines by adopting product innovations which embody increasing composition of technology. The analysis starts with formalizing a set of plausible assumptions of the basic game theoretical model at the beginning of each chapter. First, the case of perfect information is taken up under drastic innovations. The assumptions of symmetry in payoffs and no market growth enabled us to focus on the role of pure strategic behavior on the part of the incumbent and the entrant firms considering adoption of innovations. Using the competitive payoffs as a benchmark for classifying the type of innovations, the Nash equilibrium points (NEP's) are characterized under three separate cases.

Under the first type of equilibrium with drastic innovation we find the interesting possibility that firms diversify their product lines by crossing over markets and totally replace the incumbents, if the innovations are high cost. This type of equilibrium where the monopolists switch markets develops as a dominant 'defensive' strategy because under drastic innovation firms do not undertake adoptions in their own product lines, since it would mean replacing themselves as incumbents. It can be concluded that, because competition reduces profits, each firm's incentive to become a monopolist is greater than its incentive to become a duopolist by jointly adopting the high cost innovation.

We also observe that under some boundary values of cost of adoption and Cournot profits, firms may use mixed strategy equilibrium. We get this type of equilibrium with medium cost drastic innovations. We characterize the range of mixed strategy equilibria using a theorem. Both innovations are adopted, however, either through switching of incumbency, or by the incumbent itself, or by joint adoption in both product lines, as we have demonstrated with a numerical example.

Under the third type of equilibrium, firms upgrade products not only in their own product line but also in the incumbent's product line. This type of total diversification arises when the innovations are low cost.

The main tendency is that if the firms are facing drastic innovations, then, they would either diversify into the incumbent's product line only, or upgrade and diversify into both product lines. An interesting observation is that, lacking such a technological rivalry, monopolist firms would not undertake adoptions in their own product lines, since it would mean replacing themselves as incumbents. Thus, the outcome of this technological rivalry is socially desirable, since maximum product diversity is achieved through new innovations. On the other hand, the optimal 'cooperative' strategy from the firms' standpoint would be not adopting the drastic innovations and sticking with the old product. Yet, this strategy can not be enforced as a credible commitment.

The basic model is later modified to the case of non-drastic innovations. The assumption of symmetry in payoffs is relaxed by allowing differential market growth in separate product lines. The model is shown to yield equilibria where product upgrading is the more often preferred best reply strategy. High cost non-drastic innovations is shown to exhibit multiple equilibria. Under this scenario a multiplicity of best reply strategies for each monopolist range from adopting both innovations, to not adopting any of the innovations. Both innovations are adopted, however, under any of the possible Nash equilibria. An interesting point is that, under two of the four possible equilibria, we observe a passive incumbent and an aggressive entrant. The entrant monopolist diversifies across both its own product line and the entrant's product line, whereas the incumbent sticks with the old product and is partly preyed upon and replaced by the aggressive entrant. In the other two multiple equilibria both firms are actively involved in diversification and specialization process. In one of the equilibrium, firms stay in their own market and diversify in their own product lines. In the other equilibrium, they diversify only in the incumbent's product line;

and in this process of switching , they partly replace the incumbent and be partly replaced by the entrant in their old markets.

Under the second type of Nash equilibrium, monopolists stay in their own markets and increase their specialization and upgrading of existing products. We get this result under Nash equilibrium with medium cost non-drastic innovations, i.e., when the cost of diversifying in another product line exceeds the flow profits of a possible competitive outcome.

Under the third type of equilibrium with low cost innovations, firms upgrade products not only in their own product line but also in the incumbent's product line. This latter result is obtained under both the drastic and non-drastic low cost innovations.

The main tendency under non-drastic innovations is found to be either upgrading in own product line only, or upgrading and diversifying into both product lines. Unlike the case of drastic innovation, firms do have the incentive to upgrade their own products even without the technological rivalry. The existence of a potential threat of entry into the incumbent's product line enhances the process of diversification and the firms might find themselves with excessive diversification across all attainable product lines. The optimal outcome from both the society's and the firms' standpoints dictates that the firms adopt innovations in their own product lines since the same maximum product diversity could be achieved by the least cost monopolist. However, not only that this can not be enforced as a credible commitment, but it would also imply that the incumbents' monopoly positions would have to remain unchallenged. When put together, these models imply a process of strategic inventiveness that is in accord with the Schumpeterian concept of "creative destruction".

Finally, the perfect information framework is extended to the case of asymmetric information. Three separate types are defined for each player. It is shown that if the uninformed firm is a high cost type, then, Stackelberg equilibrium is the appropriate equilibrium concept. Under a low or a medium cost type follower, perfect Bayesian equilibria are characterized using a theorem. It is shown that equilibrium requires the uninformed firm should derive assessments which are best replies to the strategy chosen by the informed firm in response.

Following this, it is argued that the prior beliefs of the uninformed firm can support two different types of equilibrium. Under pessimistic prior beliefs, the outcome will be a pooling equilibrium in which the uninformed firm can not differentiate between a high cost competitor from a medium cost one. This allows a high cost informed firm to successfully pretend that it is a medium cost type. Alternatively, it is less likely for a high cost firm to exploit his information advantage against an optimistic firm who has strong prior beliefs that she is a low cost type, leading to a separating equilibrium. Exploiting the information advantage under a pooling equilibrium implies that a low cost uninformed firm would be forced to stay in her product line by a high cost informed firm who would stay in his product line himself. In this case we also observed that a high cost informed firm would be forced to maintain his monopoly position by upgrading his product line even though it would have been less costly for the uninformed firm to do so if she had chosen to diversify.

An important observation on this point is that the efficiency choice from the society's standpoint is not one between having monopoly or competition, but it is one between a high-cost monopolist (the informed firm) and a low-cost monopolist (the uninformed firm). Hence, from an efficiency standpoint the society is losing from not only the uninformed firm's ignorance, but also from her pessimistic outlook. This, possibly, is a supportive argument for a centrally coordinated industrial policy that is augmented by government research labs (or joint research projects coordinated by government institutions) providing and/or coordinating the flow of scientific information across companies which will enable them assess their potential strengths and weaknesses.

Hence, it is argued that a competitive outcome in both markets is more likely the more optimistic the uninformed firm is about her own type. On the other hand, it is more likely that a pessimistic uninformed firm would lose her monopoly position in her product line by sharing it with the informed firm, while he continues to be a monopolist in his product line.

From an efficiency standpoint, since with a more optimistic ignorance we tend to have competition in both markets, then, policies that encourage the uninformed firm to increase her optimism about diversification should be devised. Such policies would possibly provide additional incentives that would enable downgrading of the uninformed firm's prior beliefs about the costs of diversification.

The range of rational prior beliefs for a boundary payoff value of a low cost follower is characterized under a corollary. It is shown that these priors are the necessary conditions that satisfy the criteria for a separating equilibrium. Finally, an example of sensible equilibrium is presented using passive conjectures under 'equally likely' assumption.

An obviously restrictive assumption of the models in this paper is that only 2 incumbent firms seeking diversification were allowed to be challengers to each other and choose between only two innovations to adopt under certainty. While introducing more firms and more innovations would complicate the analysis, it is our belief that the qualitative results would not change. However, an intriguing extension for future research would be to explore the equilibrium under post adoption market uncertainty. An adoption will be either a success or a failure[10] with a two point probability distribution. Uncertainty of a successful adoption might be another factor why firms want to diversify across markets. Finally, although the simple model in this paper provides some insights about the relationship of technological closeness and market structure, it is but a small step towards understanding the more complex dynamic process where market structure evolves with increasing product diversity, and where incumbent firms identify their potential competitors and try to enhance their comparative advantages in the basic research of their product lines.

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[10] Glazer (1985) considers possibility of entry and failure in this sense and gives a brief summary of the empirical literature on product failures,

Appendix

1. First Appendix

Proof (of Lemma 1). From Table 1 we immediately see that, $\pi_1(s_1,s_1) > \pi_1(s_2,s_1)$, $\pi_1(s_1,s_2) > \pi_1(s_2,s_2)$, $\pi_1(s_1,s_3) \geq \pi_1(s_2,s_3)$ and $\pi_1(s_1,s_4) \geq \pi_1(s_2,s_4)$. Therefore,

$\pi_1(s_3,s) \geq \pi_1(s_4,s)\ \forall\ s$. Similarly, $\pi_2(s,s_1) \geq \pi_2(s,s_2)\ \forall\ s$. From Table 1 we see that, $\pi_1(s_3,s_1) - \pi_1(s_4,s_1) = \mathrm{c}_{1\mathrm{u}} > 0$,$\pi_1(s_3,s_2) - \pi_1(s_4,s_2) = \mathrm{c}_{1\mathrm{u}} > 0$, $\pi_1(s_3,s_3) - \pi_1(s_4,s_3) = -V^{\mathrm{c}}_{1\mathrm{u}} \geq 0$

and $\pi_1(s_3,s_4) - \pi_1(s_4,s_4) = -V^{\mathrm{c}}_{1\mathrm{u}} \geq 0$. Therefore, $\pi_1(s_3,s) \geq \pi_1(s_4,s)\ \forall\ s$. Similarly, by symmetry we obtain $\pi_2(s,s_3) \geq \pi_2(s,s_4)\ \forall\ s$. □

Proof (of proposition 1). From Lemma 1. we can replace the original game by the (2×2) game with the dominant strategies s_1 and s_3 for each firm. Notice that the resulting reduced game represents a 'prisoner's dilemma' situation. Firms play their maximin strategies in order to avoid the worst outcome of being totally preyed upon by their competitors. Hence, the NEP is the strategy combination (s_3,s_3). □

Proof (of Lemma 2). From Table 1 we see that, $\pi_1(s_3,s_1) - \pi_1(s_1,s_1) = V^{\mathrm{m}}_{1\mathrm{d}} > 0$; $\pi_1(s_3,s_1) - \pi_1(s_1,s_2) = V^{\mathrm{c}}_{1\mathrm{d}} \geq 0$; $\pi_1(s_3,s_3) - \pi_1(s_1,s_3) = V^{\mathrm{m}}_{1\mathrm{d}} > 0$, and

$\pi_1(s_3,s_4) - \pi_1(s_1,s_4) = V^{\mathrm{c}}_{1\mathrm{d}} \geq 0$. Therefore, $\pi_1(s_3,s) \geq \pi_1(s_1,s)\ \forall\ s$. Similarly, $\pi_2(s,s_3) \geq \pi_2(s,s_1)\ \forall\ s$. From Table 1 we see that, $\pi_1(s_4,s_1) - \pi_1(s_2,s_1) = V^{\mathrm{m}}_{1\mathrm{d}} > 0$, $\pi_1(s_4,s_2) - \pi_1(s_2,s_2) = V^{\mathrm{c}}_{1\mathrm{d}} \geq 0$, $\pi_1(s_4,s_3) - \pi_1(s_2,s_3) = V^{\mathrm{m}}_{1\mathrm{d}} > 0$, and $\pi_1(s_4,s_4) - \pi_1(s_2,s_4) = V^{\mathrm{c}}_{1\mathrm{d}} \geq 0$. Therefore, $\pi_1(s_4,s) \geq \pi_1(s_2,s)\ \forall\ s$. Similarly, by symmetry we obtain $\pi_2(s,s_3) \geq \pi_2(s,s_1)\ \forall\ s$. □

Proof (of proposition 2). From Lemma 2 we can replace the original game by the (2×2) reduced game with the dominant strategies s_2 and s_4 for each firm. Notice that this reduced game represents a "prisoner's dilemma" situation. Hence, the only self-enforcing and stable strategy combination is (s_4,s_4) which is the only NEP. □

Proof (of proposition 3). To prove (1) it is sufficient to show that $(\beta_1+\beta_3)V^{\mathrm{m}}_{1\mathrm{d}}+(\beta_2+\beta_4)V^{\mathrm{c}}_{1\mathrm{d}}$ can not be either strictly positive or strictly negative in a Nash Equilibrium.

Case 1. : Suppose

$(\beta_1+\beta_3)V^{\mathrm{m}}_{1\mathrm{d}} + (\beta_2+\beta_4)V^{\mathrm{c}}_{1\mathrm{d}} > 0$ (i)

Then, upon inspection of the expression for $\mathrm{E}_1(\alpha,\beta)$ above, it is immediate that firm 1's best reply to β such that (i) holds must satisfy $\alpha_1 = 0\ ,\ \alpha_2 = 0$. Then $\mathrm{E}_2(\alpha,\beta)$ becomes:

$\mathrm{E}_2(\alpha,\beta) = \beta_2(\alpha_3+\alpha_4)V^{\mathrm{c}}_{2\mathrm{u}}+\beta_3\{\alpha_3V^{\mathrm{m}}_{2\mathrm{d}}+\alpha_4V^{\mathrm{c}}_{2\mathrm{d}}\}+\beta_4\{\alpha_3V^{\mathrm{m}}_{2\mathrm{d}}+\alpha_4V^{\mathrm{c}}_{2\mathrm{d}}+(\alpha_3+\alpha_4)V^{\mathrm{c}}_{2\mathrm{u}}\}$ (ii)

If β is firm 2's best reply, it will maximize (ii) given α. If $\alpha_3V^{\mathrm{m}}_{2\mathrm{d}} + \alpha_4V^{\mathrm{c}}_{2\mathrm{d}} > 0$, then a best reply β will consist of $\beta_1 = 0$, $\beta_2 > 0$, $\beta_3 = 0$, $\beta_4 > 0$. But these values of β contradict (i). If $\alpha_3V^{\mathrm{m}}_{2\mathrm{d}} + \alpha_4V^{\mathrm{c}}_{2\mathrm{d}} < 0$, then a best reply β will consist of $\beta_1 = 0$, $\beta_2 = 0$, $\beta_3 = 0$, $\beta_4 = 0$. But these values of β also contradict (i). Finally, if $\alpha_3V^{\mathrm{m}}_{2\mathrm{d}} + \alpha_4V^{\mathrm{c}}_{2\mathrm{d}} = 0$, then, a best reply β will consist of $\beta_1 = 0$, $\beta_2 > 0$, $\beta_3 = 0$, $\beta_4 > 0$, which again contradicts (i). Therefore, no NEP can exist which satisfies (i).

Case 2. : Suppose,

$(\beta_1+\beta_3)V^{\mathrm{m}}_{1\mathrm{d}} + (\beta_2+\beta_4)V^{\mathrm{c}}_{1\mathrm{d}} < 0$ (iii)

Upon inspection of $\mathrm{E}_1(\alpha,\beta)$ above, the inequality (iii) implies that firm 1's best reply must satisfy $\alpha_3 = 0$, $\alpha_4 = 0$. This implies that $\mathrm{E}_2(\alpha,\beta)$ becomes:

$\mathrm{E}_2(\alpha,\beta) = \beta_1(\alpha_1+\alpha_2)\pi^{\mathrm{m}}_{22} + \beta_2(\alpha_1+\alpha_2)V^{\mathrm{m}}_{2\mathrm{u}} + \beta_3\{(\alpha_1+\alpha_2)\pi^{\mathrm{m}}_{22} + \alpha_1 V^{\mathrm{m}}_{2\mathrm{d}} + \alpha_2 V^{\mathrm{c}}_{2\mathrm{d}}\}$

$+\ \beta_4(\alpha_1+\alpha_2)V^{\mathrm{m}}_{2\mathrm{u}} + \alpha_1 V^{\mathrm{m}}_{2\mathrm{d}} + \alpha_2 V^{\mathrm{c}}_{2\mathrm{d}}\}$ (iv)

A best reply for firm 2 will maximize (iv) given α. Since $\pi^{\mathrm{m}}_{22} > V^{\mathrm{m}}_{2\mathrm{u}}$, then, $\beta_2 = 0$ and $\beta_4 = 0$. But then (iii) can never hold when $\beta_2 = 0$, $\beta_4 = 0$. Therefore, it follows that (iii) cannot hold under a NEP.

This proves that under a NEP, (1) must be true. Clearly, by symmetry, it follows that under a NEP, (2) must also be true. □

Proof (of proposition 4). We first prove (4). Substituting 1) from Proposition 3 into $\mathrm{E}_1(\alpha,\beta)$ we get:

$$\begin{aligned}\mathrm{E}_1(\alpha,\beta) = {} & \alpha_1(\beta_1+\beta_2)\pi^{\mathrm{m}}_{11} + \alpha_2\{(\beta_1+\beta_2)V^{\mathrm{m}}_{1\mathrm{u}} + (\beta_3+\beta_4)V^{\mathrm{c}}_{1\mathrm{u}}\} \\ & +\ \alpha_3(\beta_1+\beta_2)\pi^{\mathrm{m}}_{11} + \alpha_4\{(\beta_1+\beta_2)V^{\mathrm{m}}_{1\mathrm{u}} + (\beta_3+\beta_4)V^{\mathrm{c}}_{1\mathrm{u}}\}. \qquad (i)\end{aligned}$$

We will prove that, $(\beta_1+\beta_2)\pi^{\mathrm{m}}_{11} = (\beta_1+\beta_2)V^{\mathrm{m}}_{1\mathrm{u}} + (\beta_3+\beta_4)V^{\mathrm{c}}_{1\mathrm{u}}$ is impossible. First, consider the inequality,

$$(\beta_1+\beta_2)\pi^{\mathrm{m}}_{11} > (\beta_1+\beta_2)V^{\mathrm{m}}_{1\mathrm{u}} + (\beta_3+\beta_4)V^{\mathrm{c}}_{1\mathrm{u}}. \qquad (ii)$$

But, maximizing (i) under (ii) implies that firm 1's best reply must satisfy $\alpha_1 > 0$, $\alpha_3 > 0$, $\alpha_2 = 0$, $\alpha_4 = 0$. But this strategy violates Proposition 3-2. Hence, (ii) can not hold in a Nash Equilibrium.

Next, consider the inequality,

$(\beta_1+\beta_2)\pi^{\mathrm{m}}_{11} < (\beta_1+\beta_2)V^{\mathrm{m}}_{1\mathrm{u}} + (\beta_3+\beta_4)V^{\mathrm{c}}_{1\mathrm{u}}$. (iii)

Maximizing (i) under (iii) implies that firm 1's best reply must satisfy $\alpha_1 = 0$, $\alpha_2 > 0$, $\alpha_3 = 0$, $\alpha_4 > 0$. This also contradicts Proposition 3-2. Thus, we conclude that (4) must hold under a Nash Equilibrium. By symmetry we can write the expected payoff of firm 2 using Proposition 3-2 as follows:

$$\mathrm{E}_2(\alpha,\beta) = \beta_1(\alpha_1+\alpha_2)\pi^{\mathrm{m}}_{22} + \beta_2\{(\alpha_1+\alpha_2)V^{\mathrm{m}}_{2\mathrm{u}} + (\alpha_3+\alpha_4)V^{\mathrm{c}}_{2\mathrm{u}}\}$$

$+\ \beta_3(\alpha_1+\alpha_2)\pi^{\mathrm{m}}_{22} + \beta_4\{(\alpha_1+\alpha_2)V^{\mathrm{m}}_{2\mathrm{u}} + (\alpha_3+\alpha_4)V^{\mathrm{c}}_{2\mathrm{u}}$ (iv)

In a symmetric manner it is easy to see that a NEP that maximizes (iv) requires that (3) hold. □

Proof (of Lemma 3). From Table 2 we immediately see that,

$$\begin{aligned}\pi_2(s_1,s_4) - \pi_2(s_1,s_3) &= (V^{\mathrm{m}}_{2\mathrm{u}} + V^{\mathrm{m}}_{2\mathrm{d}} + r_2) - (\pi^{\mathrm{m}}_{2} + V^{\mathrm{m}}_{2\mathrm{d}}) \\ &= (V^{\mathrm{m}}_{2\mathrm{u}} + r_2 - \pi^{\mathrm{m}}_{2}) > 0. \\ \pi_2(s_1,s_4) - \pi_2(s_1,s_2) &= (V^{\mathrm{m}}_{2\mathrm{u}} + V^{\mathrm{m}}_{2\mathrm{d}} + r_2) - (V^{\mathrm{m}}_{2\mathrm{u}} + r_2) \\ &= V^{\mathrm{m}}_{2\mathrm{d}} > 0. \\ \pi_2(s_1,s_4) - \pi_2(s_1,s_1) &= (V^{\mathrm{m}}_{2\mathrm{u}} + V^{\mathrm{m}}_{2\mathrm{d}} + r_2) - (\pi^{\mathrm{m}}_{2}) \\ &= (V^{\mathrm{m}}_{2\mathrm{u}} + r_2 - \pi^{\mathrm{m}}_{2}) + V^{\mathrm{m}}_{2\mathrm{d}} > 0.\end{aligned}$$

From Table 2 we see that,

$$\begin{aligned}
\pi_1(s_1,s_4)-\pi_1(s_2,s_4) &= r_1-(V^{\mathrm{c}}_{1\mathrm{u}}+r_1)=-V^{\mathrm{c}}_{1\mathrm{u}}>0.\\
\pi_1(s_1,s_4)-\pi_1(s_3,s_4) &= r_1-(V^{\mathrm{c}}_{1\mathrm{d}}+r_1)=-V^{\mathrm{c}}_{1\mathrm{d}}>0.\\
\pi_1(s_1,s_4)-\pi_1(s_4,s_4) &= r_1-(V^{\mathrm{c}}_{1\mathrm{u}}+V^{\mathrm{c}}_{1\mathrm{d}}+r)\\
&= -(V^{\mathrm{c}}_{1\mathrm{u}}+V^{\mathrm{c}}_{1\mathrm{d}})>0.
\end{aligned}$$

This proves **(a).**

From Table 2 we see that,

$$\begin{aligned}
\pi_2(s_2,s_2)-\pi_2(s_2,s_1) &= (V^{\mathrm{m}}_{2\mathrm{u}}+r_2)-\pi^{\mathrm{m}}_2>0.\\
\pi_2(s_2,s_2)-\pi_2(s_2,s_3) &= (V^{\mathrm{m}}_{2\mathrm{u}}+r_2)-(V^{\mathrm{c}}_{2\mathrm{d}}+\pi^{\mathrm{m}}_2)\\
&= (V^{\mathrm{m}}_{2\mathrm{u}}+r_2-\pi^{\mathrm{m}}_2)-V^{\mathrm{c}}_{2\mathrm{d}}>0.\\
\pi_2(s_2,s_2)-\pi_2(s_2,s_4) &= (V^{\mathrm{m}}_{2\mathrm{u}}+r_2)-(V^{\mathrm{m}}_{2\mathrm{u}}+V^{\mathrm{c}}_{2\mathrm{d}}+r_2)=-V^{\mathrm{c}}_{2\mathrm{d}}>0
\end{aligned}$$

Similarly, from symmetry we see that, $\pi_1(s_2,s_2)-\pi_1(s,s_2)>0\ \forall\ s$.

This proves **(b).**

From Table 2 we see that,

$$\begin{aligned}
\pi_2(s_3,s_3)-\pi_2(s_3,s_1) &= (V^{\mathrm{m}}_{2\mathrm{d}}+r_2)-r_2=V^{\mathrm{m}}_{2\mathrm{d}}>0\\
\pi_2(s_3,s_3)-\pi_2(s_3,s_2) &= (V^{\mathrm{m}}_{2\mathrm{d}}+r_2)-(V^{\mathrm{c}}_{2\mathrm{u}}+r_2)=V^{\mathrm{m}}_{2\mathrm{d}}-V^{\mathrm{c}}_{2\mathrm{d}}>0\\
\pi_2(s_3,s_3)-\pi_2(s_3,s_4) &= (V^{\mathrm{m}}_{2\mathrm{d}}+r_2)-(V^{\mathrm{c}}_{2\mathrm{u}}+V^{\mathrm{m}}_{2\mathrm{d}}+r_2)=-V^{\mathrm{c}}_{2\mathrm{u}}>0
\end{aligned}$$

Similarly, from symmetry it is immediate that, $\pi_1(s_3,s_3)-\pi_1(s,s_3)>0\ \ \forall\ \ s$.

This proves **(c).**

Also note from Table 2 that the strategy combination (s_4,s_1) implied by **(d)** is symmetrically opposite to the strategy combination (s_1,s_4) implied by **(a).** To prove **(d)** it is sufficient to see that the payoffs are also distributed symmetrically. Hence, using **(a)** we get, $\pi_1(s_4,s_1)-\pi_1(s,s_1)>0\ \forall\ s$ and $\pi_2(s_4,s_1)-\pi_2(s_4,s)>0\ \forall\ s$. This proves **(d)**. □

Proof (of proposition 5). Suppose firm 1 plays its s_1 strategy. Then, firm 2's best reply strategy is s_4. Since from Lemma 3 (a) we have, $\pi_2(s_1,s_4)>\pi_2(s_1,s)\ \ \forall\ \ s$. Alternately, suppose firm 2 plays its s_4 strategy. Then, firm 1's best reply strategy is s_1. Since from Lemma 3 (a) we have, $\pi_1(s_1,s_4)>\pi_1(s,s_4)\ \ \forall\ \ s$. Hence, $\{s_1,s_4\}$ is a NEP.

Suppose firm 1 plays its s_2 strategy. Then, firm 2's best reply strategy is s_2. Since from Lemma 3 (b) we have, $\pi_2(s_2,s_2)>\pi_2(s_2,s)\ \ \forall\ \ s$. Alternately, suppose firm 2 plays its s_2 strategy. Then, firm 1's best reply strategy is s_2. Since from Lemma 3 (b) we have, $\pi_1(s_2,s_2)>\pi_1(s,s_2)\ \ \forall\ \ s$. Hence, $\{s_2,s_2\}$ is another NEP.

Suppose firm 1 plays its s_3 strategy. Then, firm 2's best reply strategy is s_3. Since from Lemma 3 (c) we have, $\pi_2(s_3,s_3)>\pi_2(s_3,s)\ \ \forall\ \ s$. Alternately, suppose firm 2 plays its s_3 strategy. Then, firm 1's best reply strategy is s_3. Since from Lemma 3 (c) we have, $\pi_1(s_3,s_3)>\pi_1(s,s_3)\ \ \forall\ \ s$. Hence, $\{s_3,s_3\}$ is another NEP.

Suppose firm 1 plays its s_4 strategy. Then, firm 2's best reply strategy is s_1. Since from Lemma 3 (d) we have, $\pi_2(s_4, s_1) > \pi_2(s_4, s) \;\; \forall \;\; s$. Alternately, suppose firm 2 plays its s_1 strategy. Then, firm 1's best reply strategy is s_4. Since from Lemma 3 (d) we have, $\pi_1(s_4, s_1) > \pi_1(s, s_1) \;\; \forall \;\; s$. Hence, $\{s_4, s_1\}$ is another NEP. □

Proof (of Lemma 4). From Table 2. we immediately see that, $\pi_1(s_2, s_1) > \pi_1(s_1, s_1)$, $\pi_1(s_2, s_2) > \pi_1(s_1, s_2)$, $\pi_1(s_2, s_3) \geq \pi_1(s_1, s_3)$ and $\pi_1(s_2, s_4) \geq \pi_1(s_1, s_4)$. Therefore, $\pi_1(s_2, s) \geq \pi_1(s_1, s) \, \forall \, s$. Similarly, $\pi_2(s, s_2) \geq \pi_2(s, s_1) \, \forall \, s$.

From Table 3 we see that, $\pi_1(s_4, s_1) > \pi_1(s_3, s_1)$, $\pi_1(s_4, s_2) > \pi_1(s_3, s_2)$, $\pi_1(s_4, s_3) \geq \pi_1(s_3, s_3)$ and $\pi_1(s_4, s_4) \geq \pi_1(s_3, s_4)$. Therefore, $\pi_1(s_4, s) \geq \pi_1(s_3, s) \, \forall \, s$. Similarly, by symmetry we obtain,$\pi_2(s, s_4) \geq \pi_2(s, s_3) \, \forall \, s$. □

Proof (of proposition 6). From Lemma 4. we can replace the original game by the reduced form (2×2) game with the dominant strategies s_2 and s_4 for each firm. Notice that the resulting reduced game can be further iterated for payoff dominance. From Table 2 it is immediate that, $\pi_1(s_4, s_2) - \pi_1(s_2, s_2) = V^c_{1d} > 0 \, and \, \pi_1(s_4, s_4) - \pi_1(s_2, s_4) = V^c_{1d} > 0$. Therefore, $\pi_1(s_4, s) > \pi_1(s_2, s) \;\; \forall \; s$. Similarly, $\pi_2(s, s_4) > \pi_2(s, s_2) \;\; \forall \, s$. Hence, NEP is the strategy combination (s_4, s_4). □

Proof (of proposition 7). From Lemma 4. we can replace the original game 3. by the (2×2) game with the dominant strategies s_2 and s_4 for each firm. Notice that the resulting reduced game can be further iterated for payoff dominance. To see this note from Table 2. that, $\pi_1(s_2, s_2) - \pi_1(s_4, s_2) = V^{\mathrm{c}}_{1\mathrm{d}} \leq 0$, $\pi_1(s_2, s_4) - \pi_1(s_4, s_4) = V^{\mathrm{c}}_{1\mathrm{d}} \leq 0$. Therefore, $\pi_1(s_2, s) \geq \pi_1(s_4, s) \; \forall \; s$. Similarly, $\pi_2(s, s_2) \geq \pi_2(s, s_4) \; \forall \; s$. Hence, the NEP is the strategy combination $\{s_2, s_2\}$. □

Proof (of Lemma 6). From Table 1 we immediately see that,

$$\begin{aligned}
\pi_2(s_2, s_2 \mid N_{2j}) - \pi_2(s_2, s_1 \mid N_{2j}) &= V^m_{2u} + r + \pi^m > 0 \\
\pi_2(s_2, s_2 \mid N_{2j}) - \pi_2(s_2, s_3 \mid N_{2j}) &= (V^m_{2u} + r + \pi^m) - V^d_{2c} > 0 \\
\pi_2(s_2, s_2 \mid N_{2j}) - \pi_2(s_2, s_4 \mid N_{2j}) &= -V^c_{2d} > 0 \quad \forall \quad j = 1, 2.
\end{aligned}$$

And,

$$\begin{aligned}
\pi_2(s_2, s_4 \mid N_{23}) - \pi_2(s_2, s_1 \mid N_{23}) &= V^m_{2u} + r - \pi^m + V^c_{2d} > 0 \\
\pi_2(s_2, s_4 \mid N_{23}) - \pi_2(s_2, s_2 \mid N_{23}) &= V^c_{2d} \geq 0 \\
\pi_2(s_2, s_4 \mid N_{23}) - \pi_2(s_2, s_3 \mid N_{23}) &= V^m_{2u} + r - \pi^m > 0
\end{aligned}$$

This proves **(a).**

From table 1. we see that,

$$\begin{aligned}
\pi_2(s_3, s_3 \mid N_{21}) - \pi_2(s_3, s_1 \mid N_{21}) &= V^m_{2d} > 0 \\
\pi_2(s_3, s_3 \mid N_{21}) - \pi_2(s_3, s_2 \mid N_{21}) &= V^m_{2d} - V^c_{2d} > 0 \\
\pi_2(s_3, s_3 \mid N_{21}) - \pi_2(s_3, s_4 \mid N_{21}) &= -V^C_{2u} \geq 0 \\
\pi_2(s_3, s_4 \mid N_{2j}) - \pi_2(s_3, s_1 \mid N_{2j}) &= V^C_{2u} + V^m_{2d} > 0 \\
\pi_2(s_3, s_4 \mid N_{2j}) - \pi_2(s_3, s_2 \mid N_{2j}) &= V^m_{2d} > 0 \\
\pi_2(s_3, s_4 \mid N_{2j}) - \pi_2(s_3, s_3 \mid N_{2j}) &= V^C_{2u} \geq 0 \quad \forall \quad j = 2, 3
\end{aligned}$$

This proves **(b).**

From Table 1 we immediately see that,

$$\begin{aligned}
\pi_2(s_4, s_1 \mid N_{21}) - \pi_2(s_4, s_2 \mid N_{21}) &= -V^C_{2u} \geq 0\\
\pi_2(s_4, s_1 \mid N_{21}) - \pi_2(s_4, s_3 \mid N_{21}) &= -V^C_{2d} > 0\\
\pi_2(s_4, s_1 \mid N_{21}) - \pi_2(s_4, s_4 \mid N_{21}) &= -(V^c_{2u} + V^C_{2d}) > 0
\end{aligned}$$

We also see that,

$$\begin{aligned}
\pi_2(s_4, s_2 \mid N_{22}) - \pi_2(s_4, s_1 \mid N_{22}) &= -V^C_{2u} > 0\\
\pi_2(s_4, s_2 \mid N_{22}) - \pi_2(s_4, s_3 \mid N_{22}) &= V^C_{2u} - V^c_{d} > 0\\
\pi_2(s_4, s_2 \mid N_{22}) - \pi_2(s_4, s_4 \mid N_{22}) &= -V^C_{2d} > 0
\end{aligned}$$

and,

$$\begin{aligned}
\pi_2(s_4, s_4 \mid N_{23}) - \pi_2(s_4, s_1 \mid N_{23}) &= V^c_{2u} + V^C_{2d} > 0\\
\pi_2(s_4, s_4 \mid N_{23}) - \pi_2(s_4, s_2 \mid N_{23}) &= V^C_{2d} \geq 0\\
\pi_2(s_4, s_4 \mid N_{23}) - \pi_2(s_4, s_3 \mid N_{23}) &= V^c_{2u} > 0
\end{aligned}$$

This proves **(c).** □

Proof (of Lemma 7). From Table 3 we note that,

$$\begin{aligned}
\pi_1(s_2, s_2 \mid N_{11}) - \pi_1(s_1, s_2 \mid N_{11}) &= (V^m_{1u} + r) - \pi^m > 0\\
\pi_1(s_2, s_2 \mid N_{11}) - \pi_1(s_3, s_2 \mid N_{11}) &= (V^m_{1u} + r) - (\pi + V^c_{1d})\\
&= (V^m_{1u} + r - \pi^m) - V^c_{1d} > 0\\
\pi_1(s_2, s_2 \mid N_{11}) - \pi_1(s_4, s_2 \mid N_{11}) &= (V^m_{1u} + r) - (V^m_{1u} + V^c_{1d} + r) = -V^c_{1d} > 0
\end{aligned}$$

We also note that,

$$\begin{aligned}
\pi_1(s_1, s_4 \mid N_{11}) - \pi_1(s_2, s_4 \mid N_{11}) &= r - (V^c_{1u} + r) = -V^c_{1u} > 0\\
\pi_1(s_1, s_4 \mid N_{11}) - \pi_1(s_3, s_4 \mid N_{11}) &= r - (V^c_{1d} + r) = -V^c_{1d} > 0\\
\pi_1(s_1, s_4 \mid N_{11}) - \pi_1(s_4, s_4 \mid N_{11}) &= r - (V^c_{1u} + V^c_{1d} + r)\\
&= -(V^c_{1u} + V^c_{1d}) > 0
\end{aligned}$$

This proves **(a).**

From Table 3 we see that,

$$\begin{aligned}
\pi_1(s_2, s_2 \mid N_{12}) - \pi_1(s_1, s_2 \mid N_{12}) &= (V^m_{1u} + r) - \pi^m > 0\\
\pi_1(s_2, s_2 \mid N_{12}) - \pi_1(s_1, s_2 \mid N_{12}) &= (V^m_{1u} + r) - (\pi^m + V^c_{1d})\\
&= (V^m_{1u} + r - \pi^m) - V^c_{1d}) > 0\\
\pi_1(s_2, s_2 \mid N_{12}) - \pi_1(s_4, s_2 \mid N_{12}) &= (V^m_{1u} + r) - (V^m_{1u} \quad + \quad V^c_{1d} + r)\\
&= -V^c_{1d} > 0
\end{aligned}$$

We also note that,

$$\begin{aligned}
\pi_1(s_2, s_4 \mid N_{12}) - \pi_1(s_1, s_4 \mid N_{12}) &= (V^c_{1u} + r) - r > 0 \\
\pi_1(s_2, s_4 \mid N_{12}) - \pi_1(s_3, s_4 \mid N_{12}) &= (V^c_{1u} + r) - (V^c_{1d} + r) \\
&= V^c_{1u} - V^c_{1d} > 0 \\
\pi_1(s_2, s_4 \mid N_{12}) - \pi_1(s_4, s_4 \mid N_{12}) &= (V^c_{1u} + r) - (V^c_{1u} \quad + \quad V^c_{1d} + r) \\
&= -V^c_{1d} > 0
\end{aligned}$$

This proves **(b).**
From Table 3 we see that,

$$\begin{aligned}
\pi_1(s_4, s_2 \mid N_{13}) - \pi_1(s_1, s_2 \mid N_{13}) &= (V^m_{1u} + V^c_{1d} + r) - \pi^m > 0 \\
\pi_1(s_4, s_2 \mid N_{13}) - \pi_1(s_2, s_2 \mid N_{13}) &= (V^m_{1u} + V^c_{1d} + r) - (V^m_{1u} + r) = V^c_{1d} > 0 \\
&= (V^m_{1u} + r - \pi^m) > 0
\end{aligned}$$

We also note that,

$$\begin{aligned}
\pi_1(s_4, s_4 \mid N_{13}) - \pi_1(s_1, s_4 \mid N_{13}) &= (V^c_{1u} + V^c_{1d} + r) - r > 0 \\
\pi_1(s_4, s_4 \mid N_{13}) - \pi_1(s_2, s_4 \mid N_{13}) &= (V^c_{1u} + V^c_{1d} + r) - (V^c_{1u} + r) \\
&= V^c_{1d} > 0 \\
\pi_1(s_4, s_4 \mid N_{13}) - \pi_1(s_3, s_4 \mid N_{13}) &= (V^c_{1u} + V^c_{1d} + r) - (V^c_{1d} + r) \\
&= V^c_{1d} > 0
\end{aligned}$$

This proves **(c).**

Proof (of Lemma 8). First, we prove lemma 8(c). Recall from lemma 7(c) that s_{14} is the dominant leading strategy if firm 1 is of type N_{13}. Hence, firm 2 uses the following joint distribution of the states of the world and firm 1's dominant strategies to derive her

conjectures, $\Pr(s_{1k} \mid N_{2j}) \forall j = 2, 3 \& k = 1, ..., 4$. In particular, when strategy s_{14} observed, then, the following equations must hold.

$$\begin{aligned}
\Pr(s_{14} \mid N_{11}, N_{22}) = 0 \, \Pr(s_{14} \mid N_{11}, N_{23}) = 0 \\
\Pr(s_{14} \mid N_{12}, N_{22}) = 0 \, \Pr(s_{14} \mid N_{12}, N_{23}) = 0 \\
\Pr(s_{14} \mid N_{13}, N_{22}) = 1 \, \Pr(s_{14} \mid N_{13}, N_{23}) = 1
\end{aligned}$$

Therefore, it follows that,

$$\begin{aligned}
\Pr(s_{14}, N_{22}) &= \sum_{i=1}^{3} \Pr(s_{14} \mid N_{1i}, N_{22}) \Pr(N_{1i}) \\
&= \Pr(s_{14} \mid N_{13}; N_{22}) \Pr(N_{13}) = \Pr(N_{13}) = \theta_3
\end{aligned}$$

and,

$$\begin{aligned}\Pr(s_{14}, N_{23}) &= \sum_{i=1}^{3} \Pr(s_{14} \mid N_{1i}, N_{23}) \Pr(N_{1i}) \\ &= \Pr(s_{14} \mid N_{13}; N_{23}) \Pr(N_{13}) = \Pr(N_{13}) = \theta_3\end{aligned}$$

This proves part **(c)** of lemma 8.

Also, note that under $(N_{11}, N_{12})$the Stackelberg equilibrium is (s_1, s_2).This allows firm 2 to use the following joint distribution to derive her conjectures.

$$\Pr(s_{12} \mid N_{11}, N_{22}) = 1 \Pr(s_{12} \mid N_{11}, N_{23}) = 0$$

$$\Pr(s_{12} \mid N_{12}, N_{22}) = 1 \Pr(s_{12} \mid N_{12}, N_{23}) = 1$$

$$\Pr(s_{12} \mid N_{13}, N_{22}) = 0 \Pr(s_{12} \mid N_{13}, N_{23}) = 0$$

It follows that,

$$\begin{aligned}\Pr(s_{12}, N_{22}) &= \sum_{i=1}^{3} \Pr(s_{12} \mid N_{1i}, N_{22}) \Pr(N_{1i}) \\ &= \Pr(s_{12} \mid N_{11}; N_{22}) \Pr(N_{11}) + \Pr(s_{12} \mid N_{12}, N_{22}) \Pr(N_{12}) \\ &= \Pr(N_{11}) + \Pr(N_{12}) = \theta_1 + \theta_2\end{aligned}$$

and,

$$\begin{aligned}\Pr(s_{12}, N_{23}) &= \sum_{i=1}^{3} \Pr(s_{12} \mid N_{1i}, N_{23}) \Pr(N_{1i}) \\ &= \Pr(s_{12} \mid N_{11}; N_{23}) \Pr(N_{12}) + \Pr(s_{12} \mid N_{13}, N_{23}) \Pr(N_{13}) \\ &= \Pr(N_{12}) = \theta_2.\end{aligned}$$

This proves part **(b)** of lemma 8.

Last, we note that $(s_1, s_4)$is a Stackelberg equilibrium only if the state of the world is (N_{11}, N_{23}). In other words, s_{11} can only be observed if (i) firm 1 is of type N_{11} (recall Lemma 7 (a)), and (ii) firm 2 is of type N_{23} (recall Lemma 6). Hence, we conclude that if and when s_{11} is observed the following distribution should hold:

$$\Pr(s_{11} \mid N_{11}, N_{22}) = 0 \Pr(s_{11} \mid N_{11}, N_{23}) = 1$$

$$\Pr(s_{11} \mid N_{12}, N_{22}) = 0 \Pr(s_{11} \mid N_{12}, N_{23}) = 0$$
$$\Pr(s_{11} \mid N_{13}, N_{22}) = 0 \Pr(s_{11} \mid N_{13}, N_{23}) = 0$$

which are used by firm 2 to calculate her conjectures of her type upon observing s_{11}.

$$\begin{aligned}\Pr(s_{11}, N_{22}) &= \sum_{i=1}^{3} \Pr(s_{11} \mid N_{1i}, N_{22}) \Pr(N_{1i}) \\ &= \Pr(s_{11} \mid N_{11}, N_{22}) \Pr(N_{11}) + \Pr(s_{11} \mid N_{12}, N_{22}) \Pr(N_{12}) \\ &\quad + \Pr(s_{11} \mid N_{13}, N_{22}) \Pr(N_{13}) = 0\end{aligned}$$

and,

$$\begin{aligned}\Pr(s_{11}, N_{23}) &= \sum_{i=1}^{3} \Pr(s_{11} \mid N_{1i}, N_{23}) \Pr(N_{1i}) \\ &= \Pr(s_{11} \mid N_{11}, N_{23}) \Pr(N_{11}) + \Pr(s_{11} \mid N_{12}, N_{23}) \Pr(N_{12}) \\ &\quad + \Pr(s_{11} \mid N_{13}, N_{23}) \Pr(N_{13}) \\ &= \Pr(s_{11} \mid N_{11}; N_{23}) \Pr(N_{11}) = \Pr(N_{11}) = \theta_1\end{aligned}$$

This proves part **(a)** of Lemma 8.

2. Second Appendix

Proof (of Theorem 2).

First, we prove **(e)**

We will prove that $s_{22}$is a best reply to s_{14}iff

$$(-)\frac{(V_{2d}^{c} \mid N_{22})|}{(V_{2d}^{c} \mid N_{23})} \geq \frac{(1-\phi)}{\phi}. \tag{1}$$

Recall from weak dominance that firm 2's dominant strategies ares_{22} and s_{24} under either N_{22} or N_{23}. Therefore, it follows that s_{22} is a best reply tos_{14} if and only if,

$$E\pi_2(s_4 s_2) \geq E\pi_2(s_4, s_4) \tag{2}$$

where,

$$E\pi_2(s_4 s_2) = \Pr(N_{22} \mid s_{14})\pi_2(s_4, s_2 \mid N_{22}) + \Pr(N_{23} \mid s_{14})\pi(s_4, s_2 \mid N_{23}.) \tag{3}$$

$$E\pi_2(s_4 s_4) = \Pr(N_{22} \mid s_{14})\pi_2(s_4, s_4 \mid N_{22}) + \Pr(N_{23} \mid s_{14})\pi(s_4, s_4 \mid N_{23}.) \tag{4}$$

We use Bayes's rule to calculate $\Pr(N_{22} \mid s_{14})$ and $\Pr(N_{23} \mid s_{14})$:

$$\begin{aligned}\Pr(N_{22} \mid s_{14}) &= \frac{\Pr(s_{14} \mid N_{22}) \Pr(N_{22})}{\Pr(s_{14})} \\ &= \frac{\{\ \Pr(s_{14} \mid N_{22})\ \}\,\phi}{\{\ \Pr(s_{14} \mid N_{22})\ \}\,\phi + \{\Pr(s_{14} \mid N_{23})\ \}\,(1-\phi)}\end{aligned} \tag{5}$$

where $\phi$and $1-\phi$ are the priors for $\Pr(N_{22})$ and $\Pr(N_{23})$, respectively. But from Lemma **8(c)** we have

$$\Pr(s_{14} \mid N_{22}) = \Pr(N_{13}) = \theta_3$$
$$\Pr(s_{14} \mid N_{23}) = \Pr(N_{13}) = \theta_3$$

which upon substitution into (5) yields:

$$\Pr(N_{22} \mid s_{14}) = \phi$$

It follows that,

$$\Pr(N_{23} \mid s_{14}) = 1 - \phi$$

From Table 2. and the definitions of N_{22}, N_{23} we observe that:

$$\pi_2(s_4, s_2 \mid N_{22}) = \pi_2(s_4, s_2 \mid N_{23}) = V_{2u}^c + r > 0$$

and,

$$\pi_2(s_4, s_4 \mid N_{2j}) = (V_{2u}^c \mid N_{2j}) + (V_{2d}^c \mid N_{2j}) + r$$

where, $(V_{2u}^c \mid N_{2j}) = V_{2u}^c > 0$ for $j = 2, 3$
and,

$$(V_{2d}^c \mid N_{2j}) = \begin{Bmatrix} V_{2d}^c < 0 \ \ for \ \ j = 2 \\ V_{2d}^c \geq 0 \ \ for \ \ j = 3 \end{Bmatrix}$$

Therefore, substituting the expressions for $\Pr(N_{2j} \mid s_{14}), \pi_2(s_4, s_2 \mid N_{2j})$ and $P\pi_2(s_4, s_2 \mid N_{2j}), \quad j = 2, 3$ into (3) and (4), we can write the inequality (2) as,

$$V_{2u}^c + r \geq V_{2u}^c + r + (V_{2d}^c \mid N_{22})\phi + V_{2d}^c \mid N_{23}(1-\phi) \qquad or,$$
$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \geq \frac{(1-\phi)}{\phi}$$

What we have shown so far is that s_{22} is a best reply to s_{14}if and only if the inequality (1) holds independently of player 1's type. The arguments also show that s_{24} is a best reply to s_{14} if the reverse inequality to (1) holds independently of player 1's type.

Next, we will prove that $s_{24}$is a best reply to s_{12}iff,

$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \leq \frac{\theta_2(1-\phi)}{(\theta_1 + \theta_2)\phi} \tag{6}$$

We know from weak dominance that firm 2's dominant strategies are s_{22} and s_{24} under either N_{22} orN_{23} . Therefore, it follows that s_{24} is a best reply to s_{12} if and only if,

$$E\pi_2(s_4 s_4) \geq E\pi_2(s_2, s_2) \tag{7}$$

where,

$$E\pi_2(s_2 s_4) = \Pr(N_{22} \mid s_{12})\pi_2(s_2, s_4 \mid N_{22}) + \Pr(N_{23} \mid s_{12})\pi_2(s_2, s_4 \mid N_{23}) \tag{8}$$

$$E\pi_2(s_2 s_2) = \Pr(N_{22} \mid s_{12})\pi_2(s_2, s_2 \mid N_{22}) + \Pr(N_{23} \mid s_{12})\pi_2(s_2, s_2 \mid N_{23}) \tag{9}$$

We use Bayes's rule to calculate $\Pr(N_{22} \mid s_{12})$ and $\Pr(N_{23} \mid s_{12})$:

$$\Pr(N_{22} \mid s_{12}) = \frac{\Pr(s_{12} \mid N_{22}) \Pr(N_{22})}{\Pr(s_{12})}$$

$$= \frac{\{\Pr(s_{12} \mid N_{22})\}\phi}{\{\Pr(s_{12} \mid N_{22})\}\phi + \{\Pr(s_{12} \mid N_{23})\}(1-\phi)} \tag{10}$$

where ϕ and $1-\phi$ are the priors for $\Pr(N_{22})$ and $\Pr(N_{23})$ respectively. But from Lemma **8 (b)** we have

$$\Pr(s_{12} \mid N_{22}) = \Pr(N_{11}) + \Pr(N_{12}) = \theta_1 + \theta_2$$

$$\Pr(s_{12} \mid N_{23}) = \Pr(N_{12}) = \theta_2$$

which upon substitution into (10) yields:

$$\Pr(N_{22} \mid s_{12}) = \frac{(\theta_1 + \theta_2)\phi}{\theta_1 \phi + \theta_2}$$

Then, it follows that,

$$\begin{aligned} \Pr(N_{23} \mid s_{12}) &= 1 - \Pr(N_{22} \mid s_{12}) \\ &= \frac{\theta_2(1-\phi)}{\theta_1 \phi + \theta_2} \end{aligned}$$

From Table 2. and the definitions of N_{22}, N_{23} we observe that:

$$\pi_2(s_2, s_4 \mid N_{2j}) = V_{2u}^m + (V_{2d}^c \mid N_{2j}) + r > 0$$

and,

$$\pi_2(s_2, s_2 \mid N_{22}) = \pi_2(s_2, s_2 \mid N_{23}) = V_{2u}^m + r$$

where,

$$(V_{2d}^c \mid N_{2j}) = \left\{ \begin{matrix} V_{2d}^c < 0 \ \ for \ \ j = 2 \\ V_{2d}^c \geq 0 \ \ for \ \ j = 3 \end{matrix} \right\}$$

Therefore, substituting the expressions for $\Pr(N_{2j} \mid s_{12})$,$\pi_2(s_2, s_4 \mid N_{2j})$ and $\pi_2(s_2, s_2 \mid N_{2j}), j = 2, 3$into (8) and (9), we can write the inequality (7) as

$$0 \leq \frac{(\theta_1 + \theta_2)\phi}{\theta_1\phi + \theta_2}(V_{2d}^c \mid N_{22}) + \frac{\theta_2(1-\phi)}{\theta_1\phi + \theta_2}(V_{2d}^c \mid N_{23})$$

or,

$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \leq \frac{\theta_2(1-\phi)}{(\theta_1 + \theta_2)\phi}$$

Hence, we have shown that s_{24} is a best reply to s_{12}if and only if the inequality (6) holds independently of player 1's type. The arguments also show that s_{22} is a best reply to s_{12} if and only if the reverse inequality to (6) holds independently of player 1's type.

What we have shown so far is $s_{22} = b_2(s_{24})$ if and only if (1) holds; and $s_{24} = b_2(s_{12})$ if and only if (6) holds.

Conversely, suppose (1) holds. Then we know that

$$s_{24} \neq b_2(s_{14})$$

.

Similarly, suppose (6) holds. Then we know that

$$s_{22} \neq b_2(s_{14})$$

.

To see this we reproduce (1) and (6) below for convenience,

$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \geq \frac{(1-\phi)}{\phi}$$

$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \leq \frac{\theta_2(1-\phi)}{(\theta_1 + \theta_2)\phi}$$

It is easily seen that,

$$\frac{\theta_2(1-\phi)}{(\theta_1 + \theta_2)\phi} \leq \frac{(1-\phi)}{\phi} \forall 0 \leq \theta_i \leq 1 \tag{11}$$

From (11) it is obvious that (1) and (6) can not hold together.

Now, if (s_4, s_2) is a perfect Bayesian equilibrium point (PBEP), then we know s_{22} is a best reply to s_{14} so that inequality (1) holds. Conversely, suppose (1) holds. Then we know s_{22} is a best reply to s_{14}. However, in order for (s_4, s_2) to be a PBEP we must have,

$$\pi_1(s_4, s_2) \geq \pi_1(s_i, b_2(s_i)) \quad \forall \quad i = 1, 2, 3, 4 \qquad where \qquad s_{22} = b_2(s_{14}).$$

Recall that under Lemma 7 s_{11} , s_{12} and s_{14} are identified as best replies to s_{22} and s_{24} under N_{1j} for all $j = 1, 2, 3$. Therefore it follows that s_{14} is a Bayesian best reply to s_{22} if and only if,

$$\begin{array}{ll} (a) & \pi_1(s_4, s_2) \geq \pi_1(s_1, s_4) \\ (b) & \pi_1(s_4, s_2) \geq \pi_1(s_2, s_2) \\ (c) & \pi_1(s_4, s_2) \geq \pi_1(s_2, s_4) \end{array} \tag{12}$$

But we have shown that $s_{24} \neq b_2(s_{12})$ when (1) holds. Therefore (12.c) drops out.

From Table 2 and the definitions of N_{11},$N_{12}$and N_{13}we observe that,

$$\pi_1(s_4, s_2) = V_{1u}^m \mid V_{1d}^c + r \geq = \left\{ \begin{array}{ll} \pi_1(s_1, s_4) = r & always \quad holds \\ \pi_1(s_2, s_2) = V_{1u}^m + r & holds \;\; if \;\; N_1 = N_{13} \end{array} \right\} \tag{13}$$

$(s_2, s_4)$is a PBEP iff,

$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \geq \frac{(1-\phi)}{\phi} \; and \quad N_1 = N_{13}.$$

This proves part (e) of Theorem 2.

If (s_2, s_4) is a PBEP then we know s_{24} is a best reply to s_{12} so that inequality (6) holds.

Conversely, suppose (6) holds. Then, we know s_{24} is a best reply to s_{12}. However, in order for (s_2, s_4) to be a PBEP we must have

$\pi_1(s_2, s_4) \geq \pi_1(s_i, b_2(s_i)) \; \forall \; i = 1, 2, 3, 4$ where $s_{24} = b_2(s_{12})$.

Since from Lemma 7 s_{11}, s_{12}, s_{14} are identified as best replies to s_{22} and s_{24} under N_{1j} for all $j = 1, 2, 3$, it follows that s_{12} is a Bayesian best reply to s_{24} if and only if,

$$\begin{array}{ll} (a) & \pi_1(s_2, s_4) \geq \pi_1(s_1, s_4) \\ (b) & \pi_1(s_2, s_4) \geq \pi_1(s_4, s_4) \\ (c) & \pi_1(s_2, s_4) \geq \pi_1(s_4, s_2) \end{array} \tag{14}$$

But we have shown that $s_{22} \neq b_2(s_{14})$ when (6) holds. Therefore, (14.c) drops out.

From Table 4 and the definitions of N_{11},$N_{12}$and N_{13} we observe that,

$$\pi_1(s_2, s_4) = V_{1u}^c + r \geq$$

$$\left\{ \begin{array}{ll} \pi_1(s_1, s_4) = r & holds \;\; if \;\; N_1 = N_{12}, N_{13} \\ \pi_1(s_4, s_4) = V_{1u}^c + V_{1d}^c + r & holds \;\; if \;\; N_1 = N_{11}, N_{12} \end{array} \right\} \tag{15}$$

$(s_2, s_4)$is a PBEP iff,

$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \geq \frac{\theta_2(1-\phi)}{(\theta_1+\theta_2)\phi} \quad and \quad N_1 = N_{12}\ .$$

This proves part **(b)** of Theorem 2

.

We have already proved that s_{22} is a best reply to s_{12}if and only if,

$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \geq \frac{\theta_2(1-\phi)}{(\theta_1+\theta_2)\phi} \tag{16}$$

Thus, $(s_2, s_2)$is a PBEP we s_{22}know is a best reply to s_{12} so that (16) must hold.

Conversely, suppose (16) holds. Then we know s_{22} is a best reply to s_{12}. But in order for (s_2, s_2) to be a PBEP, we must have,

$\pi_1(s_2, s_2) \geq \pi_1(s_i, b_2(s_i))\ \forall\ i = 1, 2, 3, 4$ where $s_{22} = b_2(s_{12})$.

Since from Lemma 7 s_{11}, s_{12} and s_{14} are best replies to s_{22} and s_{22}under , N_{1j} for all $j = 1, 2, 3$, it follows that s_{12} is a Bayesian best reply to s_{24} if and only if,

$$\begin{array}{cccc} (a) & \pi_1(s_2, s_2) & \geq & \pi_1(s_1, s_4) \\ (b) & \pi_1(s_2, s_2) & \geq & \pi_1(s_4, s_2) \\ (c) & \pi_1(s_2, s_2) & \geq & \pi_1(s_4, s_4). \end{array}$$

From Table 2 and the definitions of N_{11},$N_{12}$and N_{13} we observe that,

$$\pi_1(s_2, s_2) = V_{1u}^m + r \geq$$

$$\left\{ \begin{array}{ll} \pi_1(s_1, s_4) = r & always \quad holds \\ \pi_1(s_4, s_2) = V_{1u}^m + V_{1d}^c + r & holds\ \ if \quad N_1 = N_{11}, N_{12} \\ \pi_1(s_4, s_4) = V_{1u}^c + V_{1d}^c + r & always\ \ holds \end{array} \right\} \tag{17}$$

$(s_2, s_2)$is a PBEP iff,

$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \geq \frac{\theta_2(1-\phi)}{(\theta_1+\theta_2)\phi} \qquad and \qquad N_1 = N_{11}, N_{12}.$$

This proves part **(d)** of Theorem 2.

We now consider the necessary and sufficient conditions for(s_4, s_4) to be a PBEP.

We have already proved that s_{24} is a best reply to s_{14}iff,

$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \leq \frac{(1-\phi)}{\phi} \tag{18}$$

Thus, if (s_4, s_4) is a PBEP we know s_{24} is a best reply to s_{14} so that inequality (18) must hold. Conversely, suppose (18) holds. Then we know s_{24} is a best reply to s_{14}. But, in order for (s_4, s_4) to be a PBEP, we must have,

$$\pi_1(s_4, s_4) \geq \pi_1(s_i, b_2(s_i)) \qquad \forall \qquad i = 1, 2, 3, 4$$

where

$$s_{24} = b_2(s_{14}).$$

Since from Lemma 7 s_{11}, s_{12} and s_{14} are identified as best replies to s_{22} and s_{24} under N_{1j} for all $j = 1, 2, 3$, it follows that s_{14} is a Bayesian best reply to s_{24} if and only if,

$$\begin{aligned} (a) &\qquad \pi_1(s_4, s_4) \geq \pi_1(s_1, s_4) \\ (b) &\qquad \pi_1(s_4, s_4) \geq \pi_1(s_2, s_4) \\ (c) &\qquad \pi_1(s_4, s_4) \geq \pi_1(s_2, s_2). \end{aligned} \tag{19}$$

But we have shown under part (d) that (s_2, s_2) cannot be a PBEB if $N_1 = N_{13}$. Hence $s_{22} \neq b_2(s_{14})$ when (18) holds. Therefore, (19.c) drops out.

From Table 2 and the definitions of N_{1j} for all $j = 1, 2, 3$we observe that,

$$\pi_1(s_4, s_4) = V_{1u}^c + V_{1d}^c + r \geq$$

$$\left\{ \begin{array}{ll} \pi_1(s_1, s_4) = r & holds\ \ if\ \ V_{1u}^c + V_{1d}^c + \geq 0 \\ & \quad and\ \ N_1 = N_{13} \\ \pi_1(s_2, s_4) = V_{1u}^c + r & \quad holds\ \ if\ \ N_1 = N_{13} \end{array} \right\} \tag{20}$$

$(s_4, s_4)$is a PBEP iff,

$$(-)\frac{(V_{2d}^c \mid N_{22})}{(V_{2d}^c \mid N_{23})} \leq \frac{(1-\phi)}{\phi} \qquad and \qquad N_1 = N_{13}.$$

This proves part **(c)** of Theorem 2.

Finally, we consider the necessary and sufficient conditions for (s_1, s_4) to be a PBEP.

First recall from Lemma 5 that $s_{24} = b_2(s_{11})$ under N_{2j} for all $j = 1, 2, 3$ independent of Player 1's type. But this is an ex-post condition, i.e., s_{24} is a best reply to s_{11} for all N_{2j} $j = 1, 2, 3$once player 1 chooses s_{11} strategy. However, Player 1 choosess$_{11}$ as a Bayesian best reply to s_{24} if and only if, $E\pi_2(s_1, s_4) \geq E\pi_2(s_1, s_2)$ and $N_1 = N_{11}$.

Recall that firm 2's dominant strategies are s_{22} and s_{24} under either N_{22} or N_{23}. Hence, we will first prove that, ex-ante, s_{24} is a best reply s_{11} if and only if,

$$E\pi_2(s_1, s_4) \geq E\pi_2(s_1, s_2) \tag{21}$$

where,

$$E\pi_2(s_1 s_4) = \Pr(N_{22} \mid s_{11})\pi_2(s_1, s_4 \mid N_{22}) + \Pr(N_{23} \mid s_{11})\pi_2(s_1, s_4 \mid N_{23}) \tag{22}$$

$$E\pi_2(s_1 s_2) = \Pr(N_{22} \mid s_{11})\pi_2(s_1, s_2 \mid N_{22}) + \Pr(N_{23} \mid s_{11})\pi_2(s_1, s_2 \mid N_{23}) \tag{23}$$

We use Bayes's rule to calculate $\Pr(N_{22} \mid s_{11})$and $\Pr(N_{23} \mid s_{11})$:

$$\Pr(N_{22} \mid s_{11}) = \frac{\Pr(s_{11} \mid N_{22}) \Pr(N_{22})}{\Pr(s_{11})} =$$

$$\frac{\{ \Pr(s_{11} \mid N_{22}) \} \phi}{\{ \Pr(s_{11} \mid N_{22}) \} \phi + \{\Pr(s_{11} \mid N_{23}) \} (1 - \phi)} \tag{24}$$

where $\phi$and $1 - \phi$are the priors for $\Pr(N_{22})$ and $\Pr(N_{23})$ respectively. But from Lemma **8 (a)** we have,

$$\Pr(s_{11} \mid N_{22}) = 0$$

$$\Pr(s_{11} \mid N_{22}) = \Pr(N_{11}) = \theta_1$$

which upon substitution into (24) yields:

$$\Pr(N_{22} \mid s_{11}) = 0.$$

It follows that,

$$\Pr(N_{23} \mid s_{11}) = 1.$$

From Table 2 and the definitions of N_{22}, N_{23}we observe that:

$$\pi_2(s_1, s_4 \mid N_{22}) = \pi_2(s_1, s_2 \mid N_{23}) = V_{2u}^m + V_{2d}^m + r > 0$$

and,

$$\pi_2(s_1, s_2 \mid N_{22}) = \pi_2(s_1, s_2 \mid N_{23}) = V_{2u}^m + r > 0$$

Therefore, substituting the expressions for $\Pr(N_{2j} \mid s_{11})$, $\pi_2(s_1, s_4 \mid N_{2j})$ and $\pi_2(s_1, s_4 \mid N_{2j})$, $j = 2, 3$ into (22) and (23), we can write the inequality (21) as,

$$V_{2u}^m + V_{2d}^m + r \geq V_{2u}^m + r \quad \Rightarrow \quad V_{2d}^m \geq 0.$$

But, $V_{2d}^m \geq 0$ is always true by the definition of payoffs. Therefore (21) always holds. Hence, we have shown that s_{14} is always a best reply to s_{11}independent of player 1's and player 2's types.

Now, if (s_1, s_4) is a PBEP then we know $s_{24}$is a best reply to s_{11} so that (21) holds. Conversely, suppose (21) holds. Then we know s_{24} is a best reply to s_{11}. However, in order for (s_1, s_4) to be a PBEP we must have:

$\pi_1(s_1, s_4) \geq \pi_1(s_i, b_2(s_i)) \; \forall \; i = 1, 2, 3, 4$ where $s_{24} = b_2(s_{11})$.

But from Table 3 this can only occur if,

$$
\begin{aligned}
&(a) \qquad \pi_1(s_1, s_4) \geq \pi_1(s_2, s_4) \\
&(b) \qquad \pi_1(s_1, s_4) \geq \pi_1(s_2, s_2) \\
&(c) \qquad \pi_1(s_1, s_4) \geq \pi_1(s_4, s_2) \\
&(d) \qquad \pi_1(s_1, s_4) \geq \pi_1(s_4, s_4)
\end{aligned}
\tag{25}
$$

Recalling from part **(b)** that we have shown $s_{24} = b_2(s_{12})$ iff (6) holds.

Also recall from part **(e)**that we have shown $s_{22} = b_2(s_{12})$ iff (1) holds.

But, we have proved that (1) and (6) can not hold together. Therefore, (25.c) drops out since $s_{22} \neq b_2(s_{12})$ when (6) holds. We also noted in proving part **(b)** that $s_{22} \neq b_2(s_{14})$ when (6) holds. Therefore, (25.b) also drops out.

This argument shows that in order fors_{24} to be a best reply to either s_{12}or s_{14} must hold as the binding condition. From Table 2. and the definitions N_{11}, N_{12} and N_{13}we observe that:

$$
\pi_1(s_1, s_4) = r \geq \left\{ \begin{array}{lll} \pi_1(s_2, s_4) = V_{1u}^c + r & holds & if \; N_1 = N_{11} \\ \pi_1(s_4, s_4) = V_{1u}^c + V_{1d}^c + r & holds & if \; N_1 = N_{11} \end{array} \right\} \tag{26}
$$

Therefore, we conclude that (s_1, s_4) is a PBEP iff,

$(-)\frac{(V_{2d}^c | N_{22})}{(V_{2d}^c | N_{23})} \leq \frac{\theta_2(1-\phi)}{(\theta_1+\theta_2)\phi}$ and $N_1 = N_{11}$.

This proves part **(a)** of Theorem 2. □

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