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# Optimal investment strategies in decentralized renewable power generation under uncertainty

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**Abstract**

This paper presents a method for evaluating investments in decentralized renewable power generation under price un certainty. The analysis is applicable for a client with an electricity load and a renewable resource that can be utilized for power generation. The investor has a deferrable opportunity to invest in one local power generating unit, with the objective to maximize the profits from the opportunity. Renewable electricity generation can serve local load when generation and load coincide in time, and surplus power can be exported to the grid. The problem is to find the price intervals and the capacity of the generator at which to invest. Results from a case with wind power generation for an office building suggests it is optimal to wait for higher prices than the net present value break-even price under price uncertainty, and that capacity choice can depend on the current market price and the price volatility. With low price volatility there can be more than one investment price interval for different units with intermediate waiting regions between them. High price volatility increases the value of the investment opportunity, and therefore makes it more attractive to postpone investment until larger units are profitable.


*Keywords:* Real options; Renewable electricity technologies; Electricity markets; Stochastic price; Distributed generation

## 1. Introduction

With increasing emissions and rising volatile oil prices, both large-scale and small-scale renewable power generation will be key ingredients in the electricity future. In the past decade there has been a trend towards liberalizing electricity markets, which has created exchanges for spot trading and financial markets. Driven by electricity market liberalization and cost improvements for small-scale power units, the future electricity system can include significant generation at end-users. This change increases the demand for market-based valuation and decision support tools for generation capacity for electricity customers. The following will present a method for finding optimal investment strategies in decentralized renewable power generation with an uncertain future electricity price, from the perspective of the developer. Finding optimal investment strategies includes finding both the optimal capacity and the timing of the investment. The setting of the analysis is in a liberalized power market with a market for trading electricity on spot and forward contracts (contracts for delivery in the future). The methodology can be applied to all types of decentralized renewable power generation, including wind power, photovoltaic power and hydropower. These technologies share some important properties such as the high initial investment cost and the intermittent uncontrollable power generation.

Distributed generation has many potential system benefits, such as reducing power losses from the grid, deferring grid capacity investments, reducing emissions and reducing the costs of electricity generation [1]. Much of the present literature on investment in distributed generation (e.g. [2,3]) takes the utility and societal perspective and focuses on wider system benefits. This paper instead takes the perspective of a building owner who wants to maximize

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**Nomenclature**

*Indices*

| | |
|---|---|
| $h$ | time (h) |
| $i$ | power generating unit considered in preliminary analysis (1..$N$) |
| $j$ | power generating unit considered for investment under uncertainty (1..$M$) |
| $m$ | indifference point where two net present value function of different power units have the same value |
| $t$ | time (y) |

*Endogenous variables and constants*

| | |
|---|---|
| $A_j$ | constant in option value function |
| $B_1$ | constant in option value function around and at difference point |
| $B_2$ | constant in option value function around and at difference point |
| $F_j(S)$ | value of the investment opportunity before investing in plant $j$ ($) |
| $G_{Di}$ | annual power generation that displaces the electricity load for unit $i$ (MWh/y) |
| $G_{Ei}$ | annual power generation that are exported for unit $i$ (MWh/y) |
| $K_{Di}$ | correction factor for the annual average price of displaced electricity for project $i$ |
| $K_{Ei}$ | correction factor for the average price exports receive for project $i$ |
| $NPV(S)$ | net present value of the most profitable capacity ($) |
| $P_{Di,t}(S)$ | annual effective price of displaced electricity load for unit $i$ ($/MWh) |
| $P_{Ei,t}(S)$ | annual effective export price for unit $i$ ($/MWh) |
| $S$ | electricity start price adjusted for short-term deviations ($/MWh) |
| $V_j(S)$ | value of unit $j$ after investment and in perpetuity ($) |
| $Z_j$ | optimal investment interval for power generating unit $j$ ($/MWh) |
| $g_{Di,h}$ | hourly substituted electricity load for unit $i$ (MWh) |
| $g_{Ei,h}$ | hourly power generation for exports for project $i$ (MWh) |
| $npv_i(S)$ | net present value of investment in power generating unit $i$ ($) |
| $u$ | control decision (invest or wait) |
| $v_i(S)$ | present value of a power plant during the lifetime of one unit ($) |
| $x_{i,t}$ | annual cash flow for project $i$ ($/y) |
| $z_j$ | optimal investment threshold for power plant $j$ ($/MWh) |
| $z^*$ | lowest price at which investment is optimal ($/MWh) |
| $\beta_1$ | positive solution to quadratic equation resulting from differential equation |
| $\beta_2$ | negative solution to quadratic equation resulting from differential equation |
| $\varepsilon_t$ | normally distributed random continuous process with a mean of zero and a standard deviation of one |
| $\Theta_i$ | simplifying variable in the equation for the annual cash flow $x_{i,t}$ |
| $\Upsilon_j$ | simplifying variable in equation for $V_j(S)$ |
| $\Phi_i$ | simplifying variable in equation for the annual cash flow $x_{i,t}$ |
| $\Omega_j$ | simplifying variable for the equation for $V_j(S)$ |

*Input data*

| | |
|---|---|
| $C_i$ | capacity of the power plant $i$ (kW) |
| $I_i$ | turn-key investment cost for power generating unit $i$ ($) |
| $O_i$ | annual operation and maintenance costs of power generating unit $i$ ($/y) |
| $S_0$ | electricity start price adjusted for short-term deviations at time of analysis ($/MWh) |
| $T_i$ | expected lifetime of power generating unit $i$ (y) |
| $d_h$ | hourly electricity load (kWh) |
| $g_{i,h}$ | hourly power generation for power generating unit $i$ (MWh) |
| $r$ | risk-free nominal interest rate (1/y) |
| $s_h$ | spot price in hour of the year ($/MWh) |
| $\bar{s}$ | annual average historic spot price ($/MWh) |
| $\alpha$ | expected annual risk-adjusted growth in the electricity price (1/y) |
| $\gamma$ | grid tariff ($/MWh) |
| $\delta$ | value added tax |
| $\lambda$ | supplier mark-up ($/MWh) |
| $\sigma$ | annual volatility in the electricity price (1/y) |

private profits, with a building that consumes electricity and has a renewable resource available. We compare different systems and find optimal timing for renewable power generation under electricity price uncertainty. The investment model developed is based on the real option literature, and uses the text book by Dixit and Pindyck [4] as the main reference. The real options methods can be used to find the value of flexible investment strategies under uncertainty, such as being able to postpone an investment, value that is not included in a now-or-never investment evaluation. Another recommended reference is the textbook by Trigeorgis [5].

In the model we assume that the plant is metered hourly in such a way that the electricity generated from a local power generating unit will displace electricity bought from the grid, and excess electricity can be sold back to the grid.

Displaced electricity is valued at a retail price (including grid tariffs and taxes), and exported electricity is valued at a price close to the wholesale price. The model we develop can also be applied to cases with different metering regulations such as net metering, where the retail price is received also for the generated power that does not coincide with the building load. In a situation with hourly metering, the time correlations between generation, consumption and prices are important for the profitability of the power generating unit because displaced electricity and exports are valued at different prices. At the same time, power generation that is positively correlated with the electricity price variations will have a higher value. This can for example be the case for wind power in Norway because both wind speeds and spot prices are highest during the winter months.

We assume the owner of the plant can choose between different discrete capacity choices up to a maximum capacity, which is constrained by resource availability or regulation. With a low installed capacity a large portion of the power generation will be for building consumption, which has the retail electricity price value, but small units typically have a high investment cost per kilowatt. Larger systems have a lower investment cost per kilowatt but the added generation can for a large part be exported, and hence, is only valued at the export price that is lower than the retail price. Therefore, capacity choice is not straightforward. The optimal capacity is the capacity with the highest net present value. However, the optimal capacity can vary with the electricity price, and therefore, with time.

We derive an expression for the net present value of each investment alternative, using the price information from the forward market which directly reveals the value of future delivery of electricity. The long-term electricity price is assumed to be uncertain, while all other inputs are modeled deterministically. We assume capacity choice is a choice between mutually exclusive capacities, and we derive a method for valuing the investment opportunity for each capacity. If an investment opportunity for any capacity is worth more than the expected net present value, investment is postponed. Investment is optimal when the most valuable investment opportunity has the same value as the expected net present value of the underlying project. We illustrate the model using an example with small-scale wind power alternatives for an office building in Norway.

The paper is organized as follows; in Section 2 we present our stochastic long-term electricity price process, and in Section 3 we show how we model the expected net present value of the investment in power generating units. Section 4 introduces valuation under uncertainty and shows how to find optimal investment thresholds and capacity choice under uncertainty. Section 5 presents the input data used for the analysis, and Section 6 presents the results from the wind power example, which together with the limitations and potential applications of the research, are discussed in Section 7.

## 2. Stochastic long-term electricity price process

The choice of price description is important in an investment analysis. Stock prices are often described by random walk models where price changes are independent of the current price and, therefore, independent of historical movements. The most commonly used model is Brownian motion with a deterministic growth factor and a random term that depends on stock volatility. A typical characteristic of commodity prices is that they have a tendency to revert around a long-term average cost of generation. Therefore, prices that deviate from the long-term average cost will have a higher probability of moving towards the long-term average than away from it. The mean reversion can be due to varying renewable generation, such as in the hydropower dominated Nord Pool market in the Nordic countries, or due to mean reversion in fuel prices. Models that take this property into account are called mean-reverting models. Lucia and Schwartz [6] have studied the prices in the Nordic electricity market using one and two-factor models. In the one factor models, the prices are assumed to follow a mean reverting process. In the two factor models, the short term variations in the prices are assumed to follow a similar process, and the long-term variations are assumed to follow arithmetic or geometric Brownian motion. The two factor models have a better fit to the data. However, Schwartz and Smith [7] argue that when considering long-term investments, the long-term factor is the decisive one. Similarly, Pindyck [8] claims that when considering long-term commodity related investments, a geometric Brownian motion description of the price will not lead to large errors. Although using a geometric Brownian motion to model price dynamics ignores short term mean reversion, an investment in a renewable power generating unit should be regarded as a long-term investment, where the short-term mean reversion has minor influence on values and investment decisions. Especially in Nord Pool where the mean reversion in prices is driven by precipitation, prices are assumed to revert to normal levels after dry and wet years. A stochastic description of short-term deviations is more important for investments in power units with an operational flexibility such as natural gas units. Motivated by this, and due to the simple solutions obtainable for geometric Brownian motions, we assume the long-term electricity prices follow a geometric Brownian motion, where the change in price over a small time interval is written as

$$\mathrm{d}S = \alpha S\,\mathrm{d}t + \sigma S\,\mathrm{d}z, \tag{1}$$

where $\alpha$ is the annual risk-adjusted growth rate and $\sigma$ is the annual volatility. The last part, $\mathrm{d}z = \varepsilon_t\sqrt{\mathrm{d}t}$, is an increment of a standard Wiener process, where $\varepsilon_t$ is a normally distributed random variable with a mean of zero and a standard deviation of one. See for example [4] for a discussion about price processes.

The parameters of Eq. (1) are estimated from forward contracts with a long time to maturity, where the price is set ahead of time and, therefore, includes a risk-premium. Thus, Eq. (1) represents the risk-adjusted long-term price dynamics. An advantage of using a risk-adjusted price process is that the resulting cash flows can be discounted using the risk-free interest rate. Eq. (1) says that the current long-term price level is known, but future values are log-normally distributed. Even though information arrives over time with changes in futures and forward prices of electricity, future prices are always uncertain.

We are using annual cash flow estimates in which spot prices vary each hour over a year, hence seasonal variations do not have to be taken into account in the price model. With the price description in Eq. (1), the risk-adjusted expected price is given as [4]

$$E[S_t] = S_0 e^{\alpha t}, \tag{2}$$

where $S_0$ is the initial price adjusted for short-term deviations.

## 3. The value of the decentralized renewable power generation

We assume the owner of the property with the renewable resource has available $N$ different generators of different size—indexed $i$, from 1 to $N$. In the analysis we set a maximum capacity on the generator, even though we allow for sales back to the grid. The maximum capacity can be due to a limited space for a wind turbine, a limited space on a roof top for photovoltaics and due to limitations in water inflow for hydropower. Further, the concession to build a turbine may specify an upper limit to the developer, due to bounds on the intermittent capacity a decentralized grid can handle, or due to esthetic concerns or noise. The value of each generator, which depends on the amount of load that is displaced, is modeled assuming that the developer only invests in one unit. Only one unit is considered at a time because we study investments in small decentralized units, where a developer will invest in one larger unit instead of investing in two smaller units because of the reduced investment cost per kilowatt with size. Hence, choice of unit is assumed to be between mutually exclusive projects within a size range. Since we are interested in the value of the generating units at different market prices, we need to find the net present value of the units as a function of the electricity start price. In the calculation, it is necessary to adjust for seasonal and daily correlations between the expected electricity load, power generation and spot prices.

### *3.1. Modeling the electricity load, power generation and electricity prices*

In a situation with hourly metering, the time correlation between electricity load, power generation and prices is important for the profitability of the investment. First, if electricity is usually generated at the same time as the electricity load is high, a large share of the generated electricity will be valued at the end-user price, which includes grid tariffs and taxes, as opposed to the lower export price. Second, if electricity is usually generated at times when the electricity price is high, a large share of the power generation will be valued at a higher price than the annual average spot price. All three parameters have seasonal and daily variation patterns, and are correlated through the influence of varying weather. A simple approach to take into account the correlation, and in accordance with the discussion in [9], is to find the annual cash flows from available historical hourly data. In the following, at least one year of hourly data for the electricity price, climate data to estimate power generation (wind, radiation or water inflow) and the electricity load is available. If less than a year of hourly historic data is available, one must construct approximate data using available historic data and profiles or simulate the data.

The first step in the analysis is to find the hourly power generation. For renewable power, this means converting historic climate data into expected electricity generation. For wind power this means historic wind speed data, for photovoltaic units, radiation data, and for hydropower, water inflow data. Manufacturers of generating units can usually supply a power curve that gives the relationship between energy inflow and power output. Using the hourly climate data as input to the power curve gives the expected hourly power generation profile $g_{i,h}$. With time series of the hourly expected power generation and the hourly expected load, $d_h$, we are able to find estimates of the annual displaced electricity load and the annual exported electricity. We find the annual displaced electricity for each unit as

$$G_{D\,i} = \sum_{h=1}^{8760} g_{D\,i,h} = \sum_{h=1}^{8760} \min(d_h, g_{i,h}), \tag{3}$$

where $g_{i,h}$ is the hourly displaced electricity load for unit $i$. Similarly, the exported electricity for each unit can be found as

$$G_{E\,i} = \sum_{h=1}^{8760} g_{E\,i,h} = \sum_{h=1}^{8760} \max(g_{i,h} - d_h, 0), \tag{4}$$

where $g_{E,i}$ is the hourly exported electricity for unit $i$.

The effects on profitability from the time correlation between load, price and power generation are gathered in two scalar parameters for each project $i$. One parameter adjusts the average wholesale price for displaced load compared to the annual average price, and a second parameter adjusts the average price of exported electricity. The factors will vary with the capacity of the unit. For example, in a power system like the Nordic, with high electricity prices, high electricity loads due to electricity-based heating and higher wind speeds in the winter, a small unit will primarily export electricity in the summer at low prices while a larger unit will export a larger share in the winter season at a higher price. The factor for adjusting the price for displaced electricity load is given as the ratio between the value of the displaced load on an hourly spot price and the value using the annual average price

$$K_{D\ i} = \frac{\sum_{h=1}^{8760} s_h g_{D\ i,h}}{\bar{s} G_{D\ i}}, \tag{5}$$

where $s_h$ is the hourly spot price and $\bar{s}$ is the annually average spot price.

The corresponding factor for adjusting the price that exports receive is given with a similar formula

$$K_{E\ i} = \frac{\sum_{h=1}^{8760} s_h g_{E\ i,h}}{\bar{s} G_{E\ i}}. \tag{6}$$

We are now able to find the annual average received price for displaced electricity and export price as a function of the annually average wholesale price. The end-user electricity price consists of several different parts, typically the wholesale price of electricity, taxes and grid tariffs. We assume a simple general description

$$P_{D\ i,t} = K_{D\ i} S e^{\alpha t}(1+\delta) + \gamma(1+\delta) + \lambda, \tag{7}$$

where $K_{Di}$ is the adjustment factor for the average wholesale price, $S$ is the annual average long-term market price, $\delta$ is the value added tax, $\lambda$ is a supplier mark-up and $\gamma$ is the grid tariff. The average electricity price relevant when exporting to the grid is assumed to be

$$P_{E\ i,t} = K_{E\ i} S e^{\alpha t} - \lambda, \tag{8}$$

where, $K_{Ei}$, is the adjustment factor for the average wholesale price and the supplier mark-up, $\lambda$, is assumed to be the same as when electricity is bought.

### 3.2. *Now-or-never investment evaluation*

With the given price description, the annual income from owning each power generating unit, i, can be calculated as

$$x_{i,t}(S) = G_{D\ i} P_{D\ i,t} + G_{E\ i} P_{E\ i,t} - O_i = \Phi_i + \Theta_i S e^{\alpha t}, \tag{9}$$

where $O_i$ is the annual operation and maintenance costs. The constants in Eq. (9) are abbreviated by $\Phi_i$ and $\Theta_i$ to simplify the equation.

The present value is the sum of all expected benefits less operational costs in the project life time. It is modeled as a function of the long-term annual average electricity price the first year

$$v_i(S) = \int_0^T (\Phi_i + \Theta_i S e^{\alpha t}) e^{-rt}\, dt = \frac{\Phi_i}{r}(1 - e^{-rT}) + \frac{\Theta_i}{r-\alpha}(1 - e^{-(r-\alpha)T}) S = \Upsilon_i + \Omega_i S. \tag{10}$$

The constants in Eq. (10) are abbreviated by $\Upsilon_i$ and $\Omega_i$ to simplify the equation. The net present value for each project is the present value of the benefits less the operational and investment cost

$$npv_i(S) = v_i(S) - I_i. \tag{11}$$

Only projects that maximize the net present will be considered for investment. Different projects have the highest net present value at different start prices, thus the maximal net present value is a function of the start price at the time of investment, and is given as

$$NPV(S) = \max(npv_i(S)\ \ i = 1..N), \tag{12}$$

where $j = 1..$M projects will be a part of the upper net present value function. An investor contemplating to invest now will choose the project with the highest positive net present value at the current price. This is the static net present value approach, or the Marshallian [4, p. 145] approach, to investment decisions.

## 4. Investment under uncertainty

If the owner of the property with the renewable resource has the exclusive right to invest, and if the price is expected to rise and/or there is uncertainty about future prices, there can be an added value associated with postponing the investment in a decentralized power system. The value of this option to postpone is not included in a static net present value analysis and can therefore affect the investment decision. First, if the electricity price is expected to rise, there is a positive value in postponing the investment if the discounted value of the future net present value is higher than the one today. Also, if there is uncertainty about the future price there can be a value in waiting because waiting will reveal new price information, and the developer always has the option to invest if the price moves in a favorable direction and the ability to not invest if the price is not favorable. Finally, there can be uncertainty about which capacity is most profitable, because the optimal capacity can be a function of the start price. By waiting, the developer can get new price information and invest in the most profitable generator.

When we consider postponing the investment, we could potentially consider a strategy consisting of investing in a sequence of units. For example, first buy a small generator, and if the price goes up, a larger generator. However, in this analysis we assume that the units are mutually exclusive, and that there can only be one system on the site at the same time. This might be the case for wind turbines if there is limited space to site a turbine, or a developer only has concession to build one turbine. Photovoltaic systems, on the other hand, are typically modular, and capacity could be added at a later stage. However, for all types of decentralized units, installation costs and reductions in costs per kilowatt with size can be a barrier to investment strategies that involve more than one phase.

Another consideration when a project is postponed is that also the reinvestment in a subsequent unit is postponed. We assume the most valuable investment opportunity on the occupied land is to build subsequent power generating units in perpetuity. After a generator is taken out of operation, one will usually have the option to invest in any of the units that can be considered. However, since one often will not build a small project (because the opportunity to invest in a large project is more valuable than investing in a small), and for analytic simplicity, we assume the only investment opportunity left after a project dies is to invest in the largest project available. It is also important to understand that the only decision we model is the initial, hence what happens after a project goes out of operation is just an estimate of the value at that time, and what is most important is that it is that same for both projects.

### 4.1. Mathematical description

We have M projects from which to choose—the generators that maximize net present value for different electricity start prices given by Eq. (12). We further denote the value of the investment possibility in the largest project $F_M(S)$ and the investment price threshold for the largest project $z_M$. The value functions $V_j(S)$, which represent the expected value of the first project and all later reinvestments, have two branches as functions of the start price. At the first branch, the expected price growth during the lifetime of the investment is not large enough to expect reinvestment in the large turbine immediately. This region is from $S$ equals zero to $S = \mathrm{e}^{-\alpha T} z_M$, and the value function is the sum of the present value of the first project and the expected present value of the option to reinvest in the large project

$$V_j(S) = \Upsilon_j + \Omega_j S + \mathrm{e}^{-rT} F_M(S\mathrm{e}^{\alpha T}). \quad (13)$$

From the start price $S = \mathrm{e}^{-\alpha T} z_M$, reinvestment in the large project is expected to happen immediately after the project dies; the value function is given as the discounted value in perpetuity less the investment cost for all later investments in perpetuity

$$V_j(S) = \Upsilon_j + \Omega_j S + \mathrm{e}^{-rT}\left(\frac{\Phi_M}{r} + \frac{\Theta_M}{(r-\alpha)} + S\mathrm{e}^{\alpha T}\right) - \frac{I_M}{\mathrm{e}^{rT} - 1}. \quad (14)$$

The two branches of the value functions meet tangentially at $S = \mathrm{e}^{-\alpha T} z_M$.

To find the value of the investment opportunities and the optimal investment thresholds, we first analyze each unit or strategy individually, and then afterwards choose the unit or strategy that is the most profitable. We assume the investment opportunity in project $j$, $F_j(S)$, yields no cash flows up to the time the investment is undertaken. By using the Bellman's principle of optimality, with no cash flow from the investment opportunity and in continuous time, the value of the investment opportunity can be stated as [4, p. 105]

$$F_j(S_t) = \max_u \left( \frac{1}{1 + r\,\mathrm{d}t} E^*\left[F_j\{S_{t+dt}\}|S_t, u\right]\right), \quad (15)$$

where $u$, is the control variable, here to invest or to wait, and $E^*$ denotes risk-adjusted expected value which must be used since we use the risk-free interest rate. By multiplying with $1 + r\,dt$ and rearranging the equation, the investment opportunity can be written

$$rF_j(S)\,\mathrm{d}t = E^*[\mathrm{d}F_j]. \quad (16)$$

Expanding $F_j(S)$, using Ito's lemma [4, p. 151] and taking the risk-adjusted expectations, leaves us with the following differential equation:

$$\tfrac{1}{2}\sigma^2 S^2 F_j + \alpha S F_j - rF_j = 0. \quad (17)$$

The differential equation is written independently of time; it only depends on the current start price in the market. A solution of the differential equation is $F_j(S) = A_j S^{\beta_1}$, where $A_j$ is a constant to be determined, and $\beta_1$ is given by the positive solution of the quadratic equation resulting from substituting the solution into the differential equation. To find the constant $A_j$, and the optimal investment thresholds $p_j$, we need two boundary conditions for each project [4, p. 183]. The first states that when it is optimal to invest, the investment opportunity must equal the expected net present value of the underlying project

$$F_j(z_j) = V_j(z_j) - I_j. \quad (18)$$

The second boundary condition means that the value of the investment opportunity and the net present value of the underlying project must meet tangentially at the investment threshold price

$$F_j'(z_j) = V_j'(z_j). \quad (19)$$

The value of the investment opportunity approaches the net present value of the project, and will be equal for all higher prices than the optimal investment threshold.

Now we can find optimal investment thresholds, $z_j$, for each project, which can be on any of the two value function branches, given in Eqs. (13) and (14), for the smaller projects but only on the higher branch for the largest alternative because one expects to invest in it forever if expected price growth is positive or zero. If there is only one relevant capacity ($M = 1$), the solution is complete, and one will invest for all prices over $z_M$. With more than one mutually exclusive strategy, we will not choose a project if another project has a higher option value. Choosing it means that opportunity to invest in the more valuable project is lost [10]. It is therefore optimal to wait until the price reaches a trigger level $z^*$ from below

$$z^* = \min_j p_j,$$
$$\text{s.t.} \quad F_j(z^*) = \max_j F_j(z^*) \ \wedge\ j = 1..M. \quad (20)$$

This can be interpreted as waiting for start prices below the lowest price trigger $z_j$, where the option to invest in that generator is worth more than the option to invest in any of the other projects. If the lowest threshold price satisfying Eq. (20) is $z^* = z_M$, the solution is complete and investment is optimal in the largest project for all higher start prices, and waiting is optimal for all prices below it.

However, if $z^* \leq z_M$ there can be an intermediate solution, where a smaller project is optimal for some prices and one or more larger projects are optimal for higher prices. Investment in the project, $j$, that is optimal for the lowest prices will then be optimal in a region from $z_{j,1}$ to $z_{j,2}$ where $z_{j,1} = z^*$.

The curve that consists of the value function with the highest value, can exhibit a kink where two electricity generating units of different sizes have the same value. Around this kink there is uncertainty about which project is optimal to invest in, and therefore, the opportunity to invest in both can be worth more than investing in one of the projects. The intuition in this can be understood by imagining a simple description of price uncertainty for a following period, where the price in the next period can go up or down. In this situation, the developer will invest in the large project if the price goes up, and in the small project if the price goes down. The expected discounted value of investing in the optimal project in the next period can be worth more than investing now.

There can hence be new waiting regions around the indifference point, from $z_{j,2}$ to $z_{j+1,1}$. Investment in the largest project will be optimal for all values over $z_{N,1}$. Now the solution consists of a set of one or more investment intervals, $Z_j = [z_{j,1}, z_{j,2}]$.

The value of the investment opportunity, $F_m(S)$, around each indifference point, $m$, is found using the same method as for individual projects. Hence, it is the solution to the differential equation in Eq. (17). Décamps et al. [11] have shown that the boundary conditions are also similar, but now investment can be optimal either if the price drops or grows. Both at the upper and at the lower investment price thresholds the investment opportunity must have the same value as the value function, and the value of the investment opportunity must meet the two value function tangentially at the price thresholds [11, p. 9]

$$\begin{aligned} F_m(z_{j,2}) &= V_j(z_{j,2}) - I_j, \\ F'_m(z_{j,2}) &= V'_j(z_{j,2}), \\ F_m(z_{j+1,1}) &= V_{j+1}(z_{j+1,1}) - I_{j+1}, \\ F'_m(z_{j+1,1}) &= V'_{j+1}(z_{j+1,1}). \end{aligned} \tag{21}$$

A solution to the differential equation that satisfies the boundary conditions is

$$F_m(S) = B_1 S^{\beta_1} + B_2 S^{\beta_2}, \tag{22}$$

where $\beta_2$ is the negative solution to the quadratic equation resulting from substituting Eq. (22) into Eq. (17) and $B_1$ and $B_2$ are constants to be determined. The four unknown parameters can be found from the four equations. There is no analytic solution, thus the solution must be found using numerical methods. The solution to the investment problem can be to invest in different capacities for different price regions.

## 5. Model parameters

In this section we present the model parameters used to model a case study of a wind turbine investment for an office building in Norway. The analysis requires price parameters, electricity load and different wind turbine characteristics.

The Nordic countries have a well-functioning spot and financial market called Nord Pool. Since we want a representation of a long-term price and not short-term deviations, we base the price parameters relevant for the investment decisions on the forward contract with the longest time (three years) to maturity. The volatility parameter, $\sigma$, which represents the uncertainty in prices is found as the historic annual standard deviation of price changes of this contract, the solid line in Fig. 1. Because Nord Pool only has contracts for up to three years ahead we used contracts traded between two parties, over-the-counter (OTC) contracts, to find an estimate for price growth from contracts with a longer time to delivery. In early December 2005 the 2008 contract sold at 40.9 \$/MWh. OTC contracts for 2009 are traded at 41.14 \$/MWh and for 2010 at 41.31 \$/MWh. This corresponds to a risk-adjusted price growth of 0.5 percent. In Fig. 1, the expected price growth with the upper and lower 66 percent confidence bound is plotted for the next 10 years.

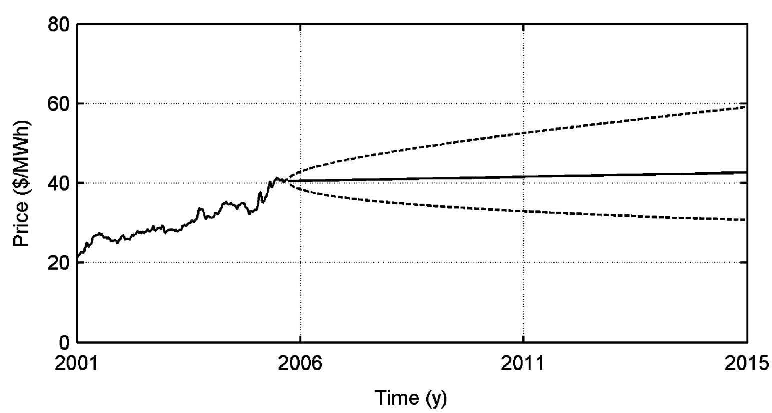


Fig. 1. Time series of the Nord Pool three-year-ahead forward contract price until late 2005, and projected prices with the upper and lower 66 percent confidence intervals until 2015.

The relevant start price is found by discounting the price of forward contracts back to the current year with a price growth of 0.5 percent. The estimated price parameters, end-user price adders and the assumed risk-free nominal interest rate are presented in Table 1, and are considered representative for a Norwegian setting.

For the electricity load we have one year of hourly data for an office building with a maximum load 99 kW and an annual load of 293 MWh. The hourly load, in the upper panel of Fig. 2, shows that there is a significant seasonal variation in consumption due to the fact that electricity is used for heating purposes, which is common in Norway. The middle panel of Fig. 2 shows the hourly wind power output; there is a large variation also in the power generation. In the winter and fall the wind power output is larger. In the lower panel, Fig. 2 displays the 2002 Nord Pool spot price. Because prices also are higher in winter and fall, there seems to be a positive correlation between load, generation and prices to be determined by the parameters $K_{Di}$ and $K_{Ei}$.

We assume the developer can choose among six different turbines with capacity, $C_i$, and costs shown in Table 2. We have assumed a significant drop in investment costs per kilowatt for wind turbines from 25 to 250 kW.

Table 1
Base case data used in the analysis

| Parameter | Unit | Value |
|---|---|---|
| $S$ | \$/MWh | 40.5 |
| $\alpha$ | | 0.005 |
| $\sigma$ | | 0.103 |
| $r$ | | 0.05 |
| $\delta$ | | 0.25 |
| $\gamma$ | \$/MWh | 35 |
| $\lambda$ | \$/MWh | 2 |

Table 2
Wind turbine data

| $i$ | $C_i$ (kW) | $I_i/C_i$ (\$/kW) | $O_i/I_i$ (1/y) | $T$ (y) |
|---|---|---|---|---|
| 1 | 25 | 2500 | 0.02 | 25 |
| 2 | 50 | 2200 | 0.02 | 25 |
| 3 | 100 | 2000 | 0.02 | 25 |
| 4 | 150 | 1900 | 0.02 | 25 |
| 5 | 200 | 1800 | 0.02 | 25 |
| 6 | 250 | 1700 | 0.02 | 25 |

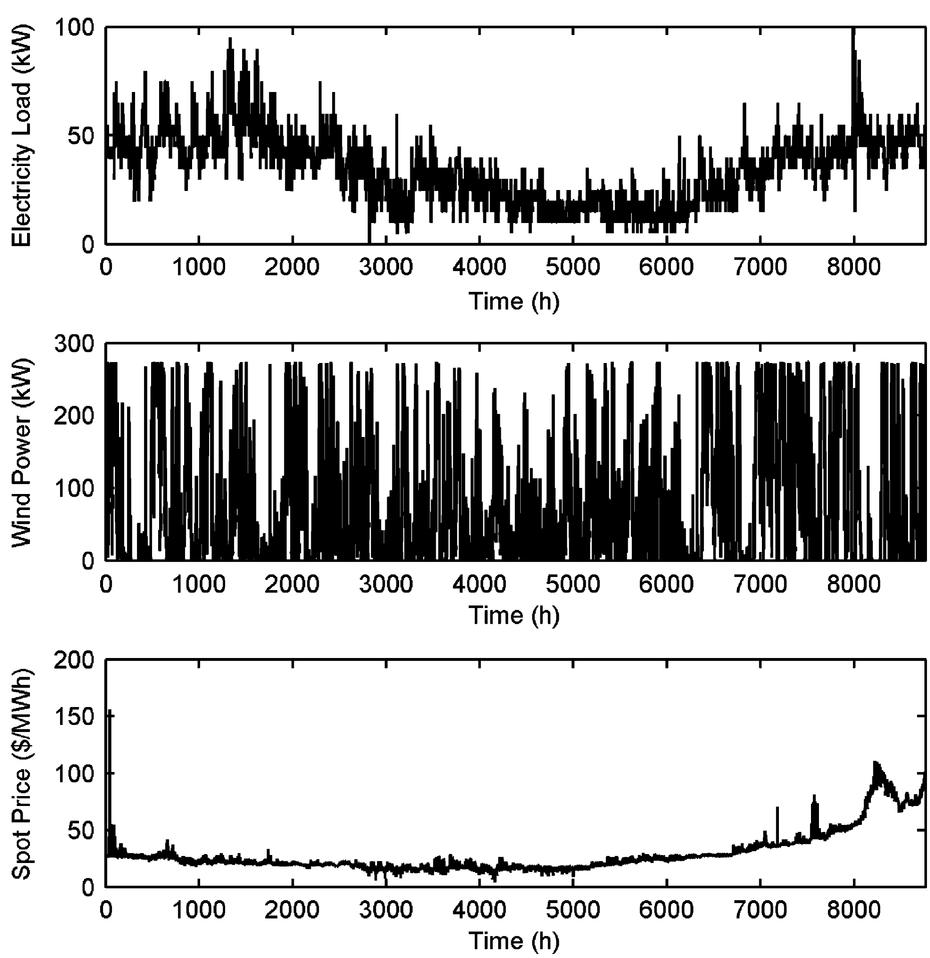


Fig. 2. Office electricity load, wind power generation for the 250 kW unit, and Nord Pool spot prices in a representative year.

## 6. Model Results

The first step in the investment analysis is to find the amount of displaced electricity load and how much electricity is exported for each turbine. Fig. 3 shows the quantities of displaced and exported electricity from generation units with different capacities. From this, it can be seen that the generated power from the 25 kW turbine is used almost solely for its own load. When the size of the turbine is increased, an increasing share of generation is exported.

The correlation between load, generation and prices for the different turbines are captured by the values of $K_{Di}$ and $K_{Ei}$ (Table 3). They are found using the data displayed in Fig. 2. They show that the average prices received for displaced load and exports varies significantly with size. Displaced load receives a price that is on average 103 percent of the average price. Generation for exports shows a larger variation because the price is adjusted from 89 percent of the average price for the 25 kW to 103 percent for the 250 kW turbine. The small turbine receives a low export price because most exports occur at summer time and at times of the day when there is a low electricity load, namely at off-peak hours. As the capacity increases, electricity is also exported at peak hours because the turbine generates more electricity, which result in a higher average price for exports.

Now we have all the data we need to find the expected net present value of the six different turbines. The current long-term start price is estimated to be 40.5$/MW h. The three smallest turbines, the 25 kW, the 50 kW and the 100 kW units all have positive net present values. The net present value is highest for the 50 kW turbine, as can be seen in Fig. 4. In a now-or-never deterministic net present value analysis the building owner would invest in the 50 kW turbine now because it has the highest positive net present value.

However, we also have the option to postpone the investment and we consider postponing the investment because we know that the electricity price can change. Therefore, we are interested in the net present value as a function of the start price. Fig. 5 plots the net present value as a function of the start price for the six turbines under consideration. Each of the six linear lines in Fig. 5 corresponds to the net present value of one of the six projects from Table 2. An increase in project size, results in a steeper net present value function. The 50 kW project has the net present value break-even at the lowest price, 32$/MWh. However, the largest project has the highest net present value for high prices, because the export price is high enough to recover the investment cost of the additional capacity, and the largest project has the lowest investment costs per kilowatt and generates the most electricity. Someone considering an investment on a now-or-never basis would choose the project with the highest positive net present value at the current start price. Only two turbines, the 50 kW and the 250 kW turbines, are ever optimal. For all other turbines, another turbine is worth more at all start prices.

As indicated above, to invest in the 50 kW turbine, even when it maximizes net present value, is not necessarily the optimal solution under uncertainty and price growth. It might not have sufficient return on investment to justify investment, and in addition the investment opportunity in a larger project can be worth more. At a price of 47.5$/MWh the upper net present value exhibits a kink, where investment in the 50 kW unit maximizes net present value for lower prices and the 250 kW unit for higher prices. Therefore, there is uncertainty regarding which turbine to invest in at this price. A net present value analysis that does

Table 3
The factors that decide the relationship between the annual average price and the average price of displaced and exported electricity

| $i$ | $K_{Di}$ | $K_{Ei}$ |
|---|---|---|
| 1 | 1.032 | 0.885 |
| 2 | 1.041 | 0.958 |
| 3 | 1.036 | 1.018 |
| 4 | 1.034 | 1.026 |
| 5 | 1.033 | 1.028 |
| 6 | 1.032 | 1.029 |

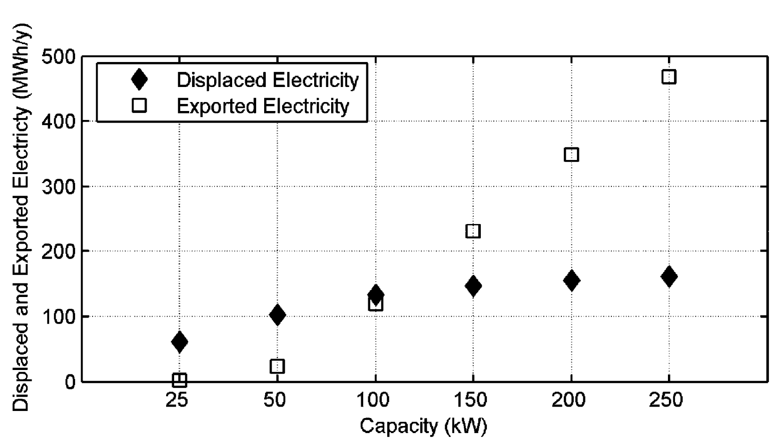


Fig. 3. Annual displaced and exported electricity for the six turbines of different capacities.

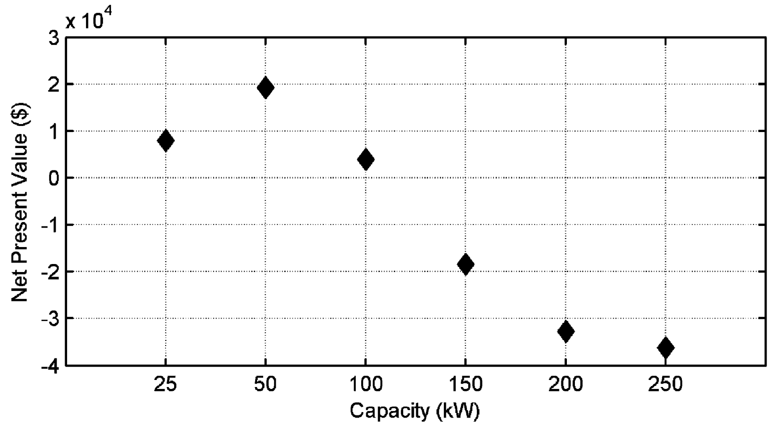


Fig. 4. Static net present value analysis at current market price of 40.5$/MWh.

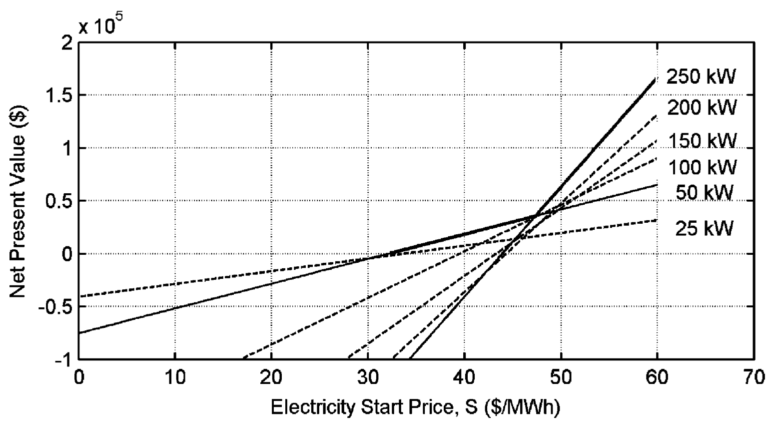


Fig. 5. Net present value as a function of the long-term electricity start price, $S$, for the six wind turbines.

not consider postponement of the investment will ignore these points.

Fig. 6 shows the solution when the investor has the possibility to postpone the investment. Because only the 50 kW and the 250 kW turbines maximize net present value at any prices they are the only two turbines considered. The solid lines are the expected net present value functions of the investment in the two different projects in perpetuity. Note that they are no longer linear on the lower branch because they include the option to invest in the largest project after a unit has been taken out of operation, and the option value is not a linear function of the start price. The upper dashed line is the value of the option to invest. With the base case data, it is never optimal to invest in the 50 kW turbine, because the investment opportunity regarding the larger turbine is more valuable for all start prices. The optimal strategy is to wait for start prices under 61$/MWh, and invest in the 250 kW turbine for all higher prices.

Less uncertainty about the level of future prices reduces the value of the investment opportunity. This is because there is a lower probability of high prices, and therefore, a lower value associated with waiting. Fig. 7 shows the solution with an uncertainty parameter reduced from $\sigma = 0.103$ to $\sigma = 0.04$. The investment opportunity in the 250 kW unit is no longer worth more than investment in 50 kW for all start prices. Now, we have one interval from $z_{1,1} = 38.5$\$/MWh to $z_{1,2} = 43.7$\$/MWh where investment is optimal in the 50 kW turbine and a second interval for all prices above $z_{2,1} = 50.4$\$/MWh where investment is optimal in the 250 kW unit. For all other start prices, it is optimal to wait for new price information.

Fig. 8 shows the optimal investment intervals for the two turbines with changing values of the uncertainty parameter, $\sigma$. As expected, increased uncertainty leads to optimal investment at higher threshold prices. The intermediate waiting region gets larger and larger, until only investment in the 250 kW turbine is optimal at $\sigma = 0.046$.

## 7. Discussion

With the provided example we have presented a method for analysis of investment in decentralized renewable power generation under uncertainty, when the investor can choose between mutually exclusive capacities and chose investment timing to maximize benefits. As expected, the

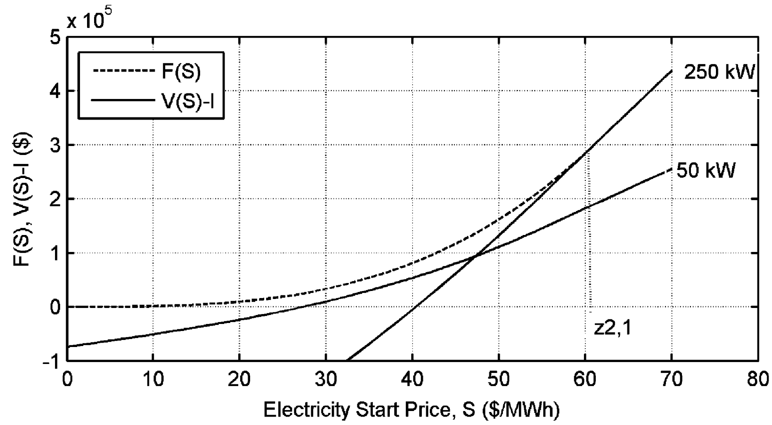


Fig. 6. The value of the investment opportunity $F(S)$, and the expected net present value of the 50 and 250 kW turbine after investment.

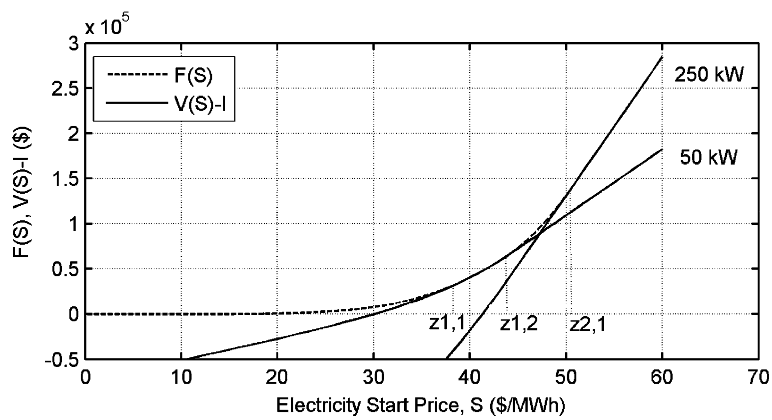


Fig. 7. The value of the investment opportunity $F(S)$, and the expected net present value of the 50 and 250 kW turbine after investment with a reduced price uncertainty ($\sigma = 0.04$).

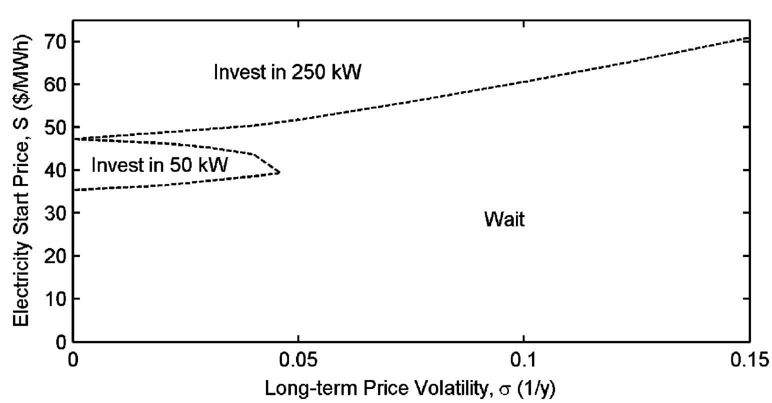


Fig. 8. Investment and waiting regions as a function of price volatility and the long-term electricity start price.

method results in a recommendation to postpone the investment beyond the net present value break-even price, because of price uncertainty. Also the optimal investment decision varies with the start price. For each capacity that possibly can be optimal, there is a price region where investment is recommended. For the largest capacity the investment threshold is a trigger price where investment is optimal for all higher prices. The results reveal intermediate waiting regions similar to those in [11,12]. This paper does not, however, assume that the projects have an infinite lifetime. Studying a sequence of investments in perpetuity reduces the intermediate waiting region and the values at stake, because the capacity choice is not as irreversible, considering that one can choose another capacity at the end of the current project's useful life. In terms of the graphs in Figs. 4, 5 the kink in the net present value functions is smoother. Considering only the value in the lifetime is the same as assuming that one can invest only once in perpetuity. It leads to higher investment thresholds and could fail to realize that investing in a small project can be optimal if one can invest in a larger project later. Further, the model only analyzes a discrete number of capacities. This is realistic for most cases; there are usually a limited number of fairly cost effective offers to compare for investment, and units are usually not available in a continuous range of capacity. The results regarding capacity choice with more sizes to choose between would not necessarily be very different, as there can still be a kink in the net present value function where a large unit sized for exports cuts off the net present value function for a smaller unit sized mainly to satisfy the load. This indication is supported in Fig. 5 by the fact that some turbines are never optimal.

The method we used is based on some minor simplifications. First, we use a relatively simple model of price uncertainty, although it is justifiable for long-term projects. Second, we assume that after a project dies only the option to invest in the largest project is available. In reality, the option to invest in any project is available. Therefore, the model can fail to give accurate results if the value of the investment opportunity in the largest project is not important at the price ranges relevant for choosing between two smaller projects. If a preliminary analysis reveals such a situation, a smaller project can be used instead of the largest. Similarly if the price is expected to decline, one can compare another investment sequence. It is possible to find the accurate optimal row of investments based on the expected price, and optimize a sequence of different projects in perpetuity. However, estimates of all of the input parameters more than a lifetime ahead is bound to be uncertain, and taking it into account would probably complicate the analysis more than it would improve it. When we assume that one can only invest in the large project after the lifetime of the first, at least both strategies have the same value after they are taken out of operation, which is important for a fair comparison of the different capacity alternatives.

As we assume that the investment decision is a choice between mutually exclusive projects, we do not allow for modular investments in the model. In cases where capacity is considered to be added in many phases, one will have many different strategies to choose from. A possible method can be to choose some discrete strategies and compare them within this framework. Yet, transaction costs for adding capacity in many phases can be high both due to the actual construction and due to new investment analysis and market monitoring. Adding the capability to the model would increase the number of investment strategies considerably and, therefore, complicate the analysis significantly. For many applications the investment in different capacities is truly mutually exclusive (e.g. in the case of wind turbines, when there is a limited area, and building more than one plant is not possible because of the required distance between turbines). Regulation can potentially also reduce the number of installations allowed.

The results are based on an example of a customer with only one year of hourly data for consumption and wind speeds. Given these limitations however, the data sets are representative enough to provide some insight into the problem. Further, the price parameters, based on Nord Pool financial data, are always only approximate. Very few contracts with a time to maturity exceeding three years are sold in the market, such that good forecasting of risk adjusted prices for a long period is very challenging.

The model does not include inflation in future investment costs and operation and maintenance cost, nor income tax effects, subsidies or a turbine construction time. Including these additions to the model is straightforward, but was not done here to make the equations simpler. In a real application of the model one would also model electricity generation for the different turbine alternatives more accurately. One would have a specific power curve for each turbine and analyze e.g. wind speeds at different heights.

Some of the distributed renewable technologies are immature, and reductions in investment costs are expected. We have assumed a constant investment cost over time. To allow for a reduction would complicate the model because of the time dependency, and would increase the value of postponing the investment. This expected reduction in costs can be a further reason to postpone an investment.

We do not analyze uncertainty in the climatic data because we assume that their average values will not change significantly in the future, and yearly variations will even out over the lifetime. Hence, the analysis assumes that the developer maximizes profits and is not intimidated by annual variations in the cash flow. Very often there will be publicly available climate data for a nearby location that the local data can be compared to. If the developer has good climate data there will hence not be a reason to wait for new information. Of course, if there are insufficient climatic measurements available, making it difficult to assess their distribution accurately, such measurements are worth paying and/or waiting for. A method to analyze risk specifically is to simulate the price, power generation and load as stochastic processes, and calculate risk measures such as standard deviation of return or electricity costs and value-at-risk (the maximum simulated loss or electricity cost within a confidence level, typically 95 percent). In a risk-perspective, the cost risk is what matters for many developers, and the cost risk can be lower with renewable generation because most of the costs are initial costs, hence the price risk is less important. Awerbush [13] claims that investors often undervalue renewable generation due to neglection of potential reductions in portfolio cost risk from renewables.

It should, however, be noted that there can be uncertainty in the governmental policy, for example, in whether green tags, that credit renewable generation, or carbon taxation will be introduced. Such uncertainty can be important but difficult to quantify and incorporate in a model. Although there is uncertainty in other parameters, the price uncertainty is likely to be a dominating uncertain factor.

Among proponents of distributed generation, there is a desire to allow for net metering over a longer period, effectively letting the owner of the generator receive the higher end-user price for all generated electricity. This is the case in many states in the US for example. It increases the value of the investment in renewable distributed generation and would make capacity choice simpler if, as is often the case, electricity generation that exceeds the annual load would have no value. Under such policies one would choose the size that generates the amount closest to the annual electricity load. Then one could use this model with one alternative capacity. But if there is an upper limit on capacity to qualify for net metering, and a larger turbine can be sized also for sales back to the grid, capacity choice is not necessarily straightforward.

## 8. Conclusions

Motivated by the continued restructuring of the electricity sector and increased interest in renewable energy, we have presented a market-based tool for project evaluation under uncertainty for investments in decentralized renewable power generation. In our setting, the developer has the option to postpone the investment and can choose the capacity among discrete alternatives. Optimal investment strategies in decentralized renewable power generation depend on several factors, including electricity load, climatic data and electricity prices. We have assumed that the most important uncertainty factor is the future electricity price, and have therefore included a stochastic price process. The analysis is based on data from the Nord Pool financial market, with an expected growth in the electricity price, and an evident uncertainty in forward prices. In our case, the optimal investment decision is to invest at a price considerably over the net present value break-even price. The optimal strategy is to invest in different capacities at different prices ranges. Increased price volatility increases the investment price thresholds, and can increase the value of the investment opportunity for larger projects so much that the only optimal strategy is to wait until investment in the largest project is optimal.

## Acknowledgements

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