Income Distribution, Market Structure, and Individual Welfare

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Abstract

This paper explores how income distribution influences market structure and affects the economic well-being of different groups. It shows that inequality may be good for the poor via a trickle-down effect operating through entry. I consider a general equilibrium model of monopolistic competition with free entry, heterogenous firms and consumers that share identical but non-homothetic preferences. The general model is solved. The case of two types of consumers, rich and poor, is considered in detail. I show that higher income inequality in the economy can benefit the poor. An increase in the personal income of the rich raises welfare of the poor, while an increase in the fraction of the rich has an ambiguous impact on the poor: welfare of the poor has an inverted U shape as a function of the fraction of the rich. At the same time, an increase in the personal income of the rich together with a decrease in the fraction of the rich keeping the aggregate income in the economy fixed raises the well-being of the poor. I also analyze the effect of changes in market size and entry cost. I show that the rich gain more from an increase in market size and lose more from an increase in the cost of entry than the poor.

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1 Introduction

What are the possible consequences of income redistribution for market structure, consumption allocation, and welfare? As Atkinson and Bourguignon (2000) argue, "it is difficult to think of economic issues without distributive consequences and it is equally difficult to imagine distributive problems without some allocational dimension." There is a large empirical and theoretical literature that relates income distribution and inequality to a number of social and economic outcomes[1]. Alesina and Rodrik (1994) show that an increase in income inequality has a negative impact on economic growth (see also Persson and Tabellini (1994)). Waldmann (1992) argues that the level of inequality is positively correlated with infant mortality. Glaeser, Scheinkman and Shleifer (2003) suggest that high inequality can negatively affect social and economic progress through the subversion of institutions in the economy.

This paper develops a simple model that establishes another link between income distribution and economic outcomes. I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. In traditional models of monopolistic competition, income distribution plays no role. This rests on two standard preference assumptions. First, when preferences are identical and homothetic, it is well understood that the distribution of income does not affect equilibrium: only aggregate income matters. Second, when preferences are quasi linear, the presence of a numeraire good eliminates the influence of income distribution on equilibrium outcomes.

I assume that all consumers share identical but non-homothetic preferences. I introduce income heterogeneity in the model by assuming that consumers differ in the efficiency units of labor they are endowed with. In models with homothetic preferences, any price change has the same impact on all consumers regardless of whether consumers are identical or not. Non-homothetic preferences and income heterogeneity imply that same price changes affect different groups differently. At the same time, the presence of market power induces endogenous heterogenous mark-ups across firms, which are in turn affected by income distribution. Thus, changes in income distribution have different consequences for different groups.

I adopt the preference structure from Murphy, Shleifer and Vishny (1989) and Matsuyama (2000). The basic idea is that goods are indivisible, and potential consumers want to buy only one unit of each good. This implies that, given prices, goods can be arranged so that consumers can be seen as moving down some list in choosing what to buy. For example, in developing countries, consumers first buy food, then clothing, then move up the chain of durables from kerosene stoves to refrigerators, to cars. Notice that the consumer utility can only be increased by the consumption of a greater number of goods. Moreover, consumers with high income consume the same set of goods as consumers with

[1] See Atkinson and Bourguignon (2000) for more substantial literature review.

low income, plus some others.

Only one firm produces each good. Goods differ in terms of the valuations that consumers attach to them. By the valuation of a particular good, I mean the utility delivered to consumers from the consumption of one unit of this good. Such differences between goods generate ex-post heterogeneity across firms, as firms enter the market before the valuations placed on their goods are realized. As in Melitz (2003)[2], there is free entry in the market. To enter the market, firms have to make costly investments that are sunk. Once firms enter, they know the valuations attached to their goods. Firms producing goods with high valuations stay in the market and compete in price, while firms with sufficiently low valuations choose to exit. This and the preference structure lead to an endogenous distribution of mark-ups, which is influenced not only by market size, but also by the distribution of income in the economy. Hence, the model incorporates two key features: imperfect competition and non-homothetic preferences, which allow analyzing the consequences of changes in the income distribution on pricing in the equilibrium, the market structure and, thereby, welfare of different groups of consumers.

While the general model is established and solved, the heart of the paper focuses on the case of two types of consumers: rich[3] and poor. Depending on the valuations attached to the goods they produce, firms are endogenously divided into three groups. Firms with high valuations choose to serve all consumers; firms with medium valuations decide to sell only to the rich, while firms with low valuations leave the market. I examine how this endogenous distribution of firms, firm mark-ups, and individual welfare are affected by income inequality, market size, and the cost of entry.

In particular, I analyze how income inequality influences the well-being of the poor. I show that higher income inequality in the economy can benefit the poor via a trickle-down effect operating through entry. Higher inequality can cause more entry in the market, inducing greater competition and lower mark-ups. As sources of income inequality, I consider changes in the income and the fraction of the rich consumers. Remember that inequality is increasing in the income of the rich and has an inverted U shape as a function of the fraction of the rich.

An increase in the income level of the rich has two effects: redistribution of firms across the groups and a higher number of firms entering the market, which results in tougher competition. As might be expected, the rich whose incomes rise are better off. Due to additional entry the poor also gains. In this manner, a higher income of the rich leads to higher welfare of the poor. This is reminiscent of the trickle-down effect in Aghion and Bolton (1997), who show that in the presence of imperfect capital markets, the accumulation of wealth by the rich may be good for the poor. Similar results can be

[2] See also Melitz and Ottaviano (2005).

[3] The fraction of the rich is an exogenous parameter.

observed, for example, in Melitz (2003) or Melitz and Ottaviano (2005). However, in these papers an increase in the income of the rich has the same or no impact on mark-ups of all firms. In the present paper I show that an increase in the income of the rich affects different firms differently. Mark-ups of firms that sell their goods to all consumers fall, while mark-ups of firms that serve only the rich rise.

Another intriguing issue is to compare welfare of the poor in countries with different fractions of the rich. What is better for the poor: tiny minority or vast majority of the rich? Keeping the same personal incomes and mass of the consumers, an increase in the fraction of the rich has two opposite implications for the poor. First, some firms that served all consumers choose to sell only to the rich. Second, a larger fraction of the rich attracts more firms to the market. The former effect hurts the poor, while the latter one helps them. I show that if the fraction of the rich is small, then the positive effect prevails otherwise the opposite happens. Thus, welfare of the poor has an inverted U shape as a function of the fraction of the rich. In contrast, in the previous literature[4] an increase in the fraction of the rich has an unambiguous impact on the welfare of the poor.

There is a common feature of both comparative statics mentioned above. An increase in the personal income of the rich as well as an increase in the fraction of the rich raises the aggregate income in the economy. To capture a pure redistribution effect, I consider an increase in the personal income of the rich together with a decrease in the fraction of the rich keeping the aggregate income in the economy fixed. Notice that these changes raise inequality in the economy. The previous results state that while an increase in the personal income of the rich has a positive effect on welfare of the poor, a decrease in the fraction of the rich has an ambiguous impact. If the fraction of the rich is large, both effects work in the same direction and the poor are better off. However, if the fraction of the rich is small, then one effect is positive and the other one is negative. I show that the overall impact on the welfare of the poor is still positive.

In this paper I also analyze the effect of changes in market size and entry cost. An increase in market size leads to tougher competition. Therefore, mark-ups of all firms fall and welfare of all consumers rises. These results are similar to those in Melitz (2003) and Melitz and Ottaviano (2005). However, there are some differences. In the present model, there is no "love for variety". Welfare of a particular individual depends only on her income and the valuation to price ratio of the goods consumed. Thus, an increase in welfare is caused by a purely competitive effect, which reduces firm mark-ups. Similarly, an increase in entry cost induces lower competition, raises mark-ups, and, thereby, decreases welfare of all consumers. What about relative welfare? Who gains more: the rich or the poor? I show that given some plausible assumption about the distribution function of valuations, the rich gain more from an increase in market size and lose more from an increase in entry cost than the poor.

[4]See the survey of literature below.

The related literature in this area can be divided into three strands. First, there are papers that consider monopolistic competition models with firm heterogeneity, but which assume homothetic or quasi-linear preferences. Melitz (2003) develops a general equilibrium model with firm heterogeneity and Dixit-Stiglitz preferences, which imply exogenous mark-ups. Melitz and Ottaviano (2005) examine a similar framework, but incorporate endogenous mark-ups considering a linear demand system. However, in both these papers, the distribution of income does not play any role. In contrast, the model presented here includes all the key features of the papers mentioned while also establishing a connection between income distribution and the market structure.

The second group of papers, for instance Flam and Helpman (1987), Stokey (1991), and Matsuyama (2000), explores the implications of non-homothetic preferences in a perfectly competitive environment for open economies. These papers mainly analyze the interaction between income distribution and trade patterns. Since in a the perfectly competitive framework prices are unaffected by income distribution, some important economic mechanisms (such as entry and exit of firms) related to income distribution, pricing, and market structure, are out of scope in these works.

Finally, the third group of papers deals with both monopolistic competition and non-homothetic preferences. There is a set of papers written by Krishna and Yavas[5], in which the role of indivisibilities and market distortions is investigated. However, the impact of income distribution on market structure is not considered in these papers. While Mitra and Trindade (2005) consider a model of monopolistic competition with non-homothetic preferences, the way they introduce non-homothetic preferences has the shortcoming that the share of income spent on a particular good is exogenous and depends on personal income.

Closer to this paper is the work of Foellmi and Zweimueller (2004) that develops a general equilibrium model with an exogenous mass of identical firms. In contrast, I consider heterogenous firms and free entry in the market, which in turn implies endogeneity of the mass of potential producers in equilibrium. Moreover, Foellmi and Zweimueller (2004) do not address welfare issues. They show that, depending on the parameters of the model, an increase in income inequality has either no impact on firm mark-ups or increases them. The present paper suggests that this is not necessarily the case; in fact, an increase in income inequality affects different firms differently. Due to free entry, greater income inequality may raise mark-ups for firms that sell their goods only to the rich and reduce mark-ups for firms that sell their goods to all consumers.

Murphy, Shleifer and Vishny (1989) study how income inequality affects the adoption of modern technologies. In their model, prices and mark-ups are exogenous. In fact, Murphy, Shleifer and Vishny

[5] See Krishna and Yavas (2001), Krishna and Yavas (2004), and Krishna and Yavas (2005).

(1989) leave the questions of competition, mark-ups, and welfare outside their analysis.

In the paper closest to this one, Foellmi and Zweimueller (2006) examine a dynamic variation of Murphy, Shleifer, and Vishny (1989). The structure of the model is similar to the present one. Assuming learning by R&D, they focus their analysis on the link between possible growth and inequality. In contrast, I do not consider the learning by R&D spillover and explore the impact of income distribution and inequality on the level of competition, mark-ups, and individual welfare.

The rest of the paper is organized as follows. Section 2 introduces the basic concepts of the general model. Section 3 develops a special case with two types of consumers, rich and poor, and establishes existence and uniqueness of equilibrium for this case. It also derives the implications of the distribution of income on market structure and individual welfare. Section 4 extends the analysis to the general case with N types of consumers, and Section 5 concludes.

2 The Model

I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. The preference structure is adopted from Murphy, Shleifer and Vishny (1989) and Matsuyama (2000).

2.1 Production

The timing of the model is as follows: firms choose whether to incur a fixed cost, f_e, or not to do so. If a firm incurs the cost, then it obtains a draw b of the valuation of its good from the distribution $G(b)$ on $[0, A]$. This is meant to capture the idea that before they enter, firms do not know how well they will end up doing, as they do not know how highly consumers will value their products. I assume that $G'(b) = g(b)$ exists. The valuation, b, is interpreted as the utility delivered to consumers from the consumption of one unit of the good. Depending on the valuation they draw, firms choose to leave the market or to stay. Firms that decide to stay compete in price with other firms. The only factor of production is labor. I assume that marginal costs of production are the same for all firms and equal to c, i.e., it takes c effective units of labor (which are paid a wage of unity) to produce a unit of any good.

Consumers differ in the number of efficiency units of labor they are endowed with. I assume that there are N types of consumers, indexed by n. A consumer of type n is endowed with I_n efficiency units of labor. I choose indices so that $I_n > I_{n-1}$. Let α_n be the fraction of type n consumers in the aggregate mass L of consumers. Thus, the total labor supply in the economy in efficiency units is $L \sum_{i=1}^{N} \alpha_i I_i$.

2.2 Consumption

All consumers have the same non-homothetic preferences given by utility function

$$U = \int_{\omega\in\Omega} b(\omega)x(\omega)d\omega,$$

where Ω is the set of available goods in the economy, $b(\omega)$ is the valuation of good ω and $x(\omega) \in \{0, 1\}$ is the consumption of good ω. Each consumer owns a balanced portfolio of shares of all firms. Due to free entry, the total profit of all firms is equal to zero in the equilibrium. This implies that the value of any balanced portfolio is equal to zero. Thus, all consumers have the same wealth, while their incomes vary with their productivity. To simplify the notation, I assume that consumers have equal shares of all firms. Let π be the total profit of all firms in the economy. For given prices a type n consumer maximizes

$$\int_{\omega\in\Omega} b(\omega)x(\omega)d\omega$$

subject to the budget constraint

$$\int_{\omega\in\Omega} p(\omega)x(\omega)d\omega \leq I_n + \frac{\pi}{L},$$

where $p(\omega)$ is the price of good ω. It is clear that utility maximization merely involves moving down the list of products ordered by their valuation to price ratio, $\frac{b(\omega)}{p(\omega)}$, until all income is exhausted.

To build intuition, I first consider the simple case when consumers have one of two possible productivities.

3 A Special Case: Two Types of Consumers

There are two types of consumers: a high productivity and, thereby, high income type, and a low income type. The productivity of a high income type is defined by I_H, the productivity of a low income type is I_L. Given the preferences, all goods consumed by less productive consumers are also consumed by more productive ones. Thus, goods in the economy can be divided into three groups: the "poor" group includes goods that are consumed by both types of consumers; the "rich" group includes goods that are consumed by more productive type only; finally, there is the group of goods that are consumed by no one.

A firm that produces a good ω obtains profit of $(p(\omega) - c)Q(\omega)$, where $Q(\omega)$ is demand for good ω. If all consumers buy the good, then demand is L. If only the rich buy it, demand is $\alpha_H L$, where α_H is the fraction of a high income type. Thus, $Q(\omega) \in \{L, \alpha_H L, 0\}$.

Each firm takes the valuation to price ratio of all other firms as given and maximizes its profit. The following proposition holds.

Proposition 1 *Even though all goods have different valuation to marginal cost ratios, goods from the same group have the same valuation to price ratio in the equilibrium.*

Proof. Suppose not. In this case, there exists some group, in which there are at least two goods with different $\frac{b(\omega)}{p(\omega)}$ ratios. Since both goods belong to the same group, the firm that produces its good with higher $\frac{b(\omega)}{p(\omega)}$ can raise its $p(\omega)$ without affecting the demand. This in turn would increase its profit. ■

Define V_1 as the valuation to price ratio of goods from the "poor" group, i.e., goods that are consumed by both types of consumers, and V_2 as valuation to price ratio of goods from the "rich" group in the equilibrium. Here V_1, V_2 are endogenous parameters and V_1 is strictly greater than V_2. Thus, if a firm with valuation $b(\omega)$ sells to all consumers, then its price is equal to $\frac{b(\omega)}{V_1}$ and its profit is given by

$$(p(\omega) - c)\, L = \left(\frac{b(\omega)}{V_1} - c\right) L,$$

while if the firm sells only to the rich, its profit is given by

$$(p(\omega) - c)\alpha_H L = \left(\frac{b(\omega)}{V_2} - c\right) \alpha_H L.$$

As $V_1 > V_2$, the firm chooses between selling to more people at a lower price and selling to fewer of them, but at a higher price. Hence, the firm chooses $p(\omega) \in \{\frac{b(\omega)}{V_1}, \frac{b(\omega)}{V_2}\}$ to maximize its profit, taking V_1 and V_2 as given. It becomes obvious that in the equilibrium the price of good ω depends only on $b(\omega)$. Therefore, hereafter I omit the notation of ω and consider prices as a function of b.

Let b_M be the unique solution of the equation

$$\left(\frac{b}{V_1} - c\right) L = \left(\frac{b}{V_2} - c\right) \alpha_H L. \tag{1}$$

In the equilibrium the condition, $\frac{\alpha_H}{V_2} < \frac{1}{V_1}$, is satisfied; otherwise $\left(\frac{b}{V_2} - c\right) \alpha_H L > \left(\frac{b}{V_1} - c\right) L$ for any $b \geq 0$ and all firms would choose to sell only to high income consumers, but this is impossible in the equilibrium. This condition guarantees that

$$\begin{array}{lcll} \left(\frac{b}{V_1} - c\right) L & \geq & \left(\frac{b}{V_2} - c\right) \alpha_H L & \text{if } b \geq b_M \\ \left(\frac{b}{V_1} - c\right) L & < & \left(\frac{b}{V_2} - c\right) \alpha_H L & \text{otherwise.} \end{array}$$

This means that if a firm draws $b \geq b_M$, then in the equilibrium it sells to both types of consumers, otherwise it sells only to the rich or exits. A firm with valuation b_M of its good is indifferent between selling to all consumers and selling only to the rich (see *Figure 1*). Thus, even in the presence of market power products have a natural hierarchy: consumers at first buy goods with higher b.

Figure 1: Profit Function

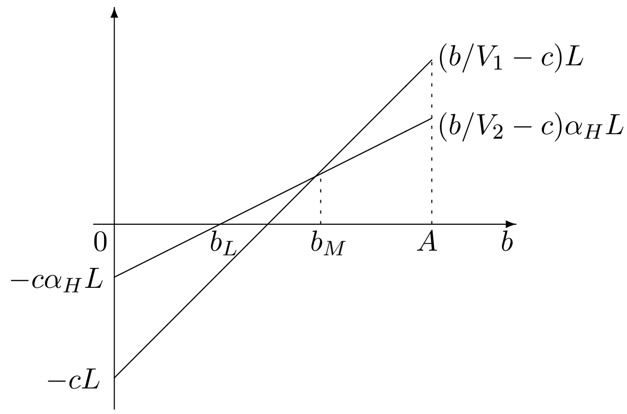
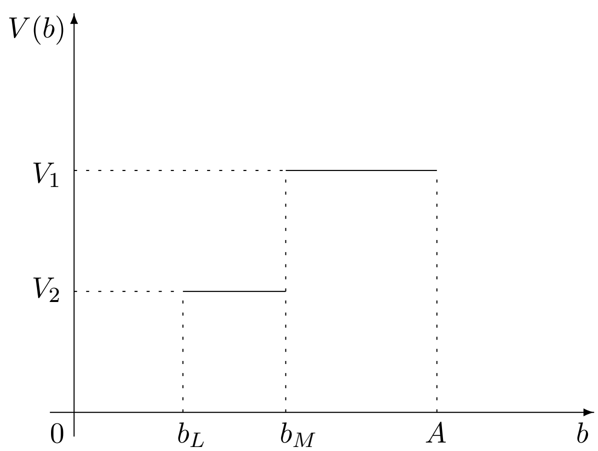


Figure 2: Valuation to Price Function: A Special Case



Hereafter, without loss of generality, I assume that a firm with valuation b_M sells to both types of consumers. Let a function $V(b)$ be defined by $\frac{b}{p(b)}$. Then, in the equilibrium $V(b)$ looks as in *Figure 2*, where $b_L \geq 0$ is a cutoff level such that firms drawn $b < b_L$ exit.

3.1 The Equilibrium

Let M_e be the mass of firms that enter the market. One can think of M_e as that there are $M_e g(b)$ different firms with a particular valuation b.

In the equilibrium, several conditions should be satisfied. First, as there is free entry in the market, the ex-ante expected profits of firms have to be equal to zero. Second, the goods market clears. Since the poor consume only goods from the "poor" group, the aggregate cost of the bundle of goods from the "poor" group should be equal to the income of a poor consumer. Similarly, the aggregate cost of the bundle of all available goods in the economy should be equal to the income of a rich consumer.

Definition 1 *The equilibrium of the model is defined by the price function $p(b)$ on $b \geq b_L$, the cutoff level $b_L \geq 0$, b_M, M_e, and the valuation to price ratios V_1, V_2 such that*

1) The ex-ante expected profits of firms are equal to zero.

2) The goods market clears.

Further, I derive equations that satisfy conditions mentioned above and prove that the equilibrium of the model always exists and is unique. Let $\pi(b)$ be the variable profit of a firm with valuation b. To find the equilibrium, I express $\pi(b)$ and $p(b)$ as functions of b, b_L, b_M and exogenous parameters. Firms with valuation b_L have zero profits, i.e., $\left(\frac{b_L}{V_2} - c\right)\alpha_H L = 0$. This implies that $b_L = cV_2$ or $V_2 = \frac{b_L}{c}$. From (1) one can easily find V_1 as a function of b_L and b_M. Thus, the following lemma holds.

Lemma 1 *In equilibrium*

$$p(b) = \begin{cases} \frac{b}{V_1} = cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) & \text{if } b \geq b_M \\ \frac{b}{V_2} = cb\frac{1}{b_L} & \text{if } b \in [b_L, b_M) \end{cases},$$

$$\pi(b) = \begin{cases} \left(cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - c\right)L & \text{if } b \geq b_M \\ \left(cb\frac{1}{b_L} - c\right)\alpha_H L & \text{if } b \in [b_L, b_M) \end{cases}.$$

Since firms with valuation b_M have the same profits from selling to all consumers and selling only to the rich, the price function has a jump at b_M; i.e., to compensate for lower demand, firms raise their prices (see *Figure 3*). This results in the nonmonotonicity of the price function.

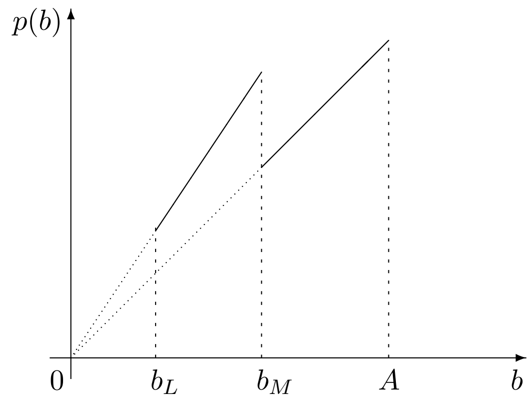
The ex-ante profit of firms is equal to zero in the equilibrium. Using the results from *Lemma 1* and taking into account that firms with $b < b_L$ exit, I obtain

$$\begin{aligned} f_e &= (G(b_M) - G(b_L))\left(\int_{b_L}^{b_M}\left(ct\frac{1}{b_L} - c\right)\alpha_H L dG_2(t)\right) + \\ &\quad (1 - G(b_M))\left(\int_{b_M}^{A}\left(ct\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - c\right)L dG_1(t)\right), \end{aligned}$$

where $G_2(t) = \frac{G(t)}{G(b_M) - G(b_L)}$ and $G_1(t) = \frac{G(t)}{1 - G(b_M)}$. Simple algebra shows that this equation can be rewritten as follows.

$$\frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1 - \alpha_H)H(b_M), \tag{2}$$

Figure 3: Price Function



where $H(x) = G(x) + \frac{\int_x^A x dG(x)}{x}$.

The goods market clearing condition implies that

$$\begin{cases} I_L + \frac{\pi}{L} = M_e \int_{b_M}^{A} p(t) dG(t) \\ I_H + \frac{\pi}{L} = M_e \int_{b_L}^{A} p(t) dG(t) \end{cases}. \tag{3}$$

At the same time, free entry in the market means that $\pi = 0$[6]. Thus, dividing the second line by the first one and using *Lemma 1*, I obtain

$$\frac{\int_{b_L}^{b_M} t dG(t)}{\int_{b_M}^{A} t dG(t)} = \left(\frac{I_H}{I_L} - 1 \right) \left(\alpha_H + \frac{b_L (1 - \alpha_H)}{b_M} \right).$$

Hence, given the exogenous parameters I_H, I_L, α_H, f_e, c, L, and the distribution of draws $G(\cdot)$, one can find endogenous b_M and b_L from the system of equations, which is given by

$$\begin{cases} \frac{\int_{b_L}^{b_M} t dG(t)}{\int_{b_M}^{A} t dG(t)} = \left(\frac{I_H}{I_L} - 1 \right) \left(\alpha_H + \frac{b_L (1-\alpha_H)}{b_M} \right) \\ \frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1 - \alpha_H) H(b_M) \end{cases}. \tag{4}$$

The following lemma states the existence and uniqueness of the equilibrium.

Lemma 2 *The system of equations* (4) *has a unique solution.*

Proof. In the appendix. ■

Once b_M and b_L are found, V_1 and V_2 can be derived from *Lemma 1.* Finally, the mass of firms can be found from (3).

[6] It can be easily checked, using (2).

3.2 Comparative Statics and Welfare

Before describing the comparative statics of the model, I focus on consumer welfare and income inequality in the economy.

3.2.1 Welfare

Welfare of a poor consumer is equal to $M_e \int_{b_M}^{A} t dG(t)$. At the same time, from (3) $M_e = \frac{I_L}{\int_{b_M}^{A} p(t) dG(t)}$. This implies that

$$W_p = I_L V_1.$$

Welfare of a poor consumer naturally rises with an increase in either her income or the valuation to price ratio of goods she consumes.

Similarly, welfare of a rich consumer is given by

$$W_r = I_L V_1 + (I_H - I_L) V_2.$$

As the rich consume the same bundle of goods as the poor plus some others, welfare of the rich is equal to welfare of the poor plus additional welfare from consumption of goods from the "rich" group, which is in turn equal to income spent on these goods multiplied by their valuation to price ratio.

Notice that all changes in individual welfare are divided into two components: an income effect and a price effect. The price effect is determined by changes in V_1 and V_2, which depend on the level of competition inside the groups of goods. The income effect is determined by changes in exogenous I_L and I_H.

3.2.2 Income Inequality

As the income inequality in the economy, I consider the variance of the income distribution. Define AG as the aggregate income per capita in the economy. Then,

$$AG = \alpha_H I_H + (1 - \alpha_H) I_L.$$

The variance of the income distribution, VAR, is given by

$$VAR = \alpha_H (1 - \alpha_H) (I_H - I_L)^2 = (\frac{1}{\alpha_H} - 1) (AG - I_L)^2 .$$

Thus, the income inequality is increasing in $I_H - I_L$ and has an inverted U shape as a function of α_H. Moreover, if one considers an increase in I_H together with a decrease in α_H keeping AG fixed, then this raises the income inequality.

In the next sections, I look at several comparative statics of the model and their impact on the market structure and individual welfare. In particular, I focus on changes in α_H and I_H that increase the income inequality in the model.

3.2.3 Changes In Individual Incomes

Consider an increase in the income of the rich I_H. Two opposite effects influence welfare of the poor. First, since I_H increases, some firms that sold their goods to all consumers find it more profitable to sell only to the rich. Second, a higher income of the rich results in higher expected profits of firms; this in turn implies that more firms enter the market, inducing tougher competition. While the former effect raises the prices of goods from the "poor" group, the latter one reduces them. I show that an overall impact is to decrease the prices of the goods from the "poor" group; as a result, V_1 rises. This implies that an increase in I_H raises welfare of the poor. Finally, both effects mentioned above raise b_M.

What about the prices of goods from the "rich" group? While a higher income of the rich allows firms that sell only to the rich to increase their prices, more entry causes more competition and, thereby, leads to a decrease of all prices. It can be shown that the former effect prevails over the latter one; that is, the prices of the goods from the "rich" group increase. This results in V_2 and b_L falling. Recall that welfare of the rich is given by $W_p + (I_H - I_L)V_2$. I show that in spite of a decrease in V_2, the rich are better off from an increase in I_H.

Similar logic works if one considers changes in I_L. An increase in I_L raises b_L, M_e, V_2 and decreases b_M, V_1. Again, all consumers are better off. The following proposition summarizes the results above.

Proposition 2 *An increase in the income of either type of consumers increases welfare of all consumers.*

Proof. In the appendix. ■

Thus, an increase in the income of either type of consumers has a positive spillover on welfare of the other consumers. To better understand the intuition, I think of short run and long run effects. Consider an increase in I_H. In the short run, when firms can not exit or enter, i.e., M_e is fixed, an increase in I_H obviously raises welfare of the rich but reduces welfare of the poor, as some firms switch from selling to all consumers to selling only to the rich. In the long run, more firms enter the market inducing tougher competition and lower prices. This raises welfare of the poor. As a result, in the long run the poor are better off, while in the short run the poor are worse off.

Similar results can be observed, for example, in Melitz (2003) or Melitz and Ottaviano (2005). However, in these papers an increase in the income of the rich has the same impact on mark-ups of all firms, while I show that an increase in the income of the rich affects different firms differently.

3.2.4 Changes In The Fraction of The Rich

Next, consider an increase in the fraction of the rich α_H. Since the mass of rich consumers in the economy rises, firms expect higher profits. This leads to an increase in the number of firms that entering the market, i.e., M_e rises. Because of tougher competition, firms with low valuations exit. Due to the higher fraction of the rich, there are some firms sold their goods to all consumers, which decide to sell only to the rich. Thus, b_L and b_M increase.

The impact on welfare of the poor is rather complicated. Two opposite effects influence welfare of the poor. The first one is an increase in M_e: it induces higher competition, reduces prices, and, thereby, positively affects W_p. The second effect is associated with the fact that some firms start selling only to the rich, while before these firms sold to the poor too. This effect reduces competition among firms that produce goods from the "poor" group, raising their prices; i.e., it has a negative impact on W_p. What effect is stronger is ambiguous. I prove that if α_H is close to 0, then welfare of the poor rises given an increase in α_H; while if α_H is close to 1, then W_p decreases.

The case, when α_H is inside $(0, 1)$ interval, is much more complicated to analyze. Therefore, I make numerical examples. As a distribution of draws, I take the power distribution[7]. For particular sets of exogenous parameters, I consider different $\alpha_H \in [0, 1]$ and calculate corresponding equilibrium. In all cases I consider, welfare of the poor has an inverted U shape as a function of α_H. This is also confirmed by the analytical results I derived. The next proposition summarizes these results.

Proposition 3 *Welfare of the poor has an inverted U shape as a function of the fraction of the rich.*

Proof. In the appendix. ■

3.2.5 Changes In The Income and The Fraction of The Rich Keeping The Aggregate Income Fixed

There is a common feature for both comparative statics mentioned above. An increase in the personal income of the rich as well as an increase in the fraction of the rich raises the aggregate income in the economy. To capture a pure redistribution effect, I consider an increase in the personal income of the rich together with a decrease in the fraction of the rich keeping the aggregate income in the economy fixed.

[7] The distribution function is $G(b) = \left(\frac{b}{A}\right)^k$, where $b \in [0, A]$ and $k > 0$.

Remember that while an increase in I_H always raises welfare of the poor, a decrease in α_H has an ambiguous impact on the poor. If α_H is high enough, then both effects increase welfare of the poor. If the fraction of the rich is small, then these effects work in the opposite directions. In this case, I show that if α_H is close to zero, then the overall impact on the welfare of the poor is still positive. The intuition is as follows. Given the changes in the income and fraction of the rich, there is a trade off: a lower number of the rich versus an increase in their income. It appears that this redistribution raises expected profits of firms and, thereby, increases the number of firms entering the market; that is, firms prefer a lower number of the rich, who have higher incomes. Due to more entry, welfare of the poor rises.

Thus, I show that in two extreme cases ($\alpha_H \approx 1$ and $\alpha_H \approx 0$) the poor are better off from the redistribution considered. Again, the case with α_H being inside $(0, 1)$ interval is much more complicated to analyze. To make numerical examples, I take the power distribution as the distribution of draws. For particular sets of exogenous parameters, keeping the same aggregate income, I consider different α_H, I_H and calculate corresponding equilibrium. In all cases I consider, W_p and M_e are decreasing functions with respect to α_H. Thus, the numerical examples are partially confirmed by the analytical results. The next proposition summarizes these findings.

Proposition 4 *An increase in the income of the rich together with a decrease in the fraction of the rich keeping the aggregate income fixed raises welfare of the poor and the number of firms entering the market.*

Proof. In the appendix. ■

3.2.6 Changes In The Cost of Entry and Market Size

An increase in the cost of entry f_e reduces the expected profits of firms. This in turn decreases the number of firms entering the market and reduces competitive pressure. Ex-post variable profits increase and firms that left the market before earn positive profits now and stay in the market. Because of less competition, some firms that sold their goods only to the rich find it more profitable now to sell to all consumers. Hence, the cutoff level b_L, b_M, and M_e decrease. Due to the reduction in competitive pressure, all firms, except firms that sold to the rich before but decide to sell to all consumers now, increase their prices. Both V_1 and V_2 fall, welfare of all individuals decreases. Thus, an increase in the cost of entry has an overall negative impact on welfare.

The opposite effect (but of the same form, see (4)) takes place if the size of the economy L increases. An increase in L results in an increase in the expected profits of firms. This leads to the higher number of firms entering the market and tougher competition. In spite of an increase in demand, firms with low valuations leave the market. All firms, except firms that switch to selling only to the rich and increase their prices, reduce their prices. The cutoff level b_L, b_M, and M_e rise. Welfare of all individuals increases. Thus, an increase in the mass of consumers positively affects individual welfare.

Finally, any changes in f_e and L such that entry cost per capita, $\frac{f_e}{L}$, remains the same do not cause any changes in b_L, b_M, M_e, and individual welfare. Two opposite effects completely compensate each other. The following proposition summarizes these findings.

Proposition 5 *Larger countries and countries with lower entry cost have higher individual welfare. That is, an increase in $\frac{f_e}{L}$ reduces welfare of all individuals.*

Proof. In the appendix. ■

These results are similar to those in Melitz (2003) and Melitz and Ottaviano (2005). At the same time, the present model implies that changes in market size or the cost of entry have different impacts on different types of consumers. Who gains or loses more from an increase in $\frac{f_e}{L}$: the rich or the poor? In the next section I consider an impact of an increase in $\frac{f_e}{L}$ on relative welfare of the rich with respect to the poor.

Relative Welfare Relative welfare of the rich with respect to the poor is given by $\frac{W_r}{W_p} = 1 + \frac{I_H - I_L}{I_L}\frac{V_2}{V_1}$. It can be rewritten as follows

$$\frac{W_r}{W_p} = 1 + \frac{I_H - I_L}{I_L}\left(\alpha_H + (1-\alpha_H)\frac{b_L}{b_M}\right). \tag{5}$$

Given an increase in $\frac{f_e}{L}$, b_L and b_M move in the same direction. From (5) one can see that changes in $\frac{f_e}{L}$ affect $\frac{W_r}{W_p}$ only through the ratio $\frac{b_L}{b_M}$. Moreover, any changes in $\frac{f_e}{L}$ have no impact on the goods market equilibrium condition (see below). Thus, exploring the impact of $\frac{f_e}{L}$ on relative welfare, one has to analyze the moving of $\frac{b_L}{b_M}$ along the goods market equilibrium curve

$$\frac{\int_{b_L}^{b_M} t dG(t)}{\int_{b_M}^{A} t dG(t)} - \frac{I_H - I_L}{I_L}\left(\alpha_H + (1-\alpha_H)\frac{b_L}{b_M}\right) = 0. \tag{6}$$

In the appendix I show that to determine the sign of $\left(\frac{b_L}{b_M}\right)'_{b_L}$ along (6), one needs to know the sign of $\left(\frac{x^2 g(x)}{\int_x^A t dG(t)}\right)'_x$. If $\left(\frac{x^2 g(x)}{\int_x^A t dG(t)}\right)'_x$ is always greater than zero, then $\left(\frac{b_L}{b_M}\right)'_{b_L}$ is always positive, otherwise the sign of $\left(\frac{b_L}{b_M}\right)'_{b_L}$ can be either and depends on the exogenous parameters of the model.

Recall that $\frac{f_e}{L}$ is negatively correlated with b_L. The following proposition formalizes the findings above.

Proposition 6 *If* $\left(\frac{x^2 g(x)}{\int_x^A t dG(t)}\right)'_x > 0$ *for any* $x \in [0, A]$, *then the rich gain more from an increase in market size and lose more from an increase in entry cost than the poor. That is, an increase in* $\frac{f_e}{L}$ *reduces* $\frac{W_r}{W_p}$.

Proof. In the appendix. ■

The sign of $\left(\frac{x^2 g(x)}{\int_x^A t dG(t)}\right)'_x$ has a strong economic interpretation. If it is positive, then $g(x)$ does not decrease too fast; i.e., the probability of getting higher values of b does not decrease too fast with b. Limiting the analysis to the cases when $\left(\frac{x^2 g(x)}{\int_x^A t dG(t)}\right)'_x$ is always positive[8], one derives that the rich lose more from an increase in $\frac{f_e}{L}$ than the poor. To better understand the intuition, I separately consider two markets. The first market is the market for goods from the "poor" group, while the second one is the market for goods from the "rich" group. Given an increase in $\frac{f_e}{L}$, fewer firms enter the both markets, this decreases b_L and b_M. Then, due to less competitive pressure, some firms that sold their goods only to the rich switch to selling to all consumers. This effect obviously decreases b_M and also reduces competition in the second market. This allows firms with low valuations to survive and results in b_L falling. Since firms that moved from the second market to the first one have relatively high valuations compared with firms that "survived", the prices of these goods are relatively high. This implies that b_L has to fall by more than b_M to compensate for the difference in the prices.

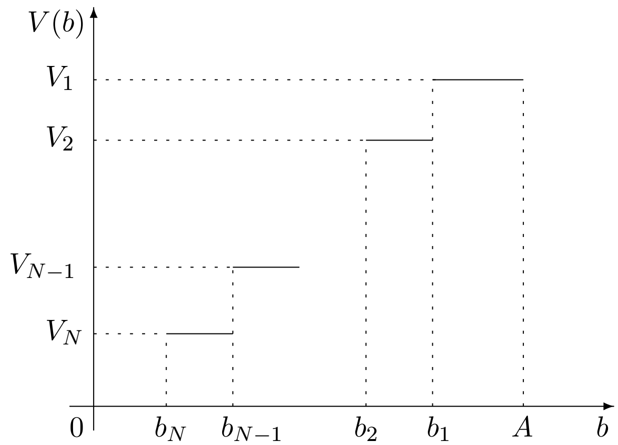
4 A General Model

Now, consider the general case with N types of consumers. Consumers differ in the number of efficiency units of labor they are endowed with. A consumer of type n is endowed with I_n efficiency units of labor. I choose indices so that $I_n > I_{n-1}$. Here α_n is the fraction of consumers of type n in the aggregate mass L of consumers.

The equilibrium of the general model is very similar to the equilibrium in the simple case above. All goods that are consumed by less productive type of consumers are also consumed by more productive type. Thus, goods in the economy are divided into $N+1$ groups. Goods belong to group $k = 1..N$ if they are only consumed by consumers with type $n \geq k$. Goods belong to group $N+1$ if they are consumed by nobody. Firms with the highest valuations b sell their goods to all types of consumers, while goods with less valuation are not available for the poorest consumers. Obviously, goods from the same group have the same valuation to price ratio. Let V_k be the valuation to price ratio of goods from group k. Then, in the equilibrium $V(b)$ looks as in *Figure 8,* where b_k is such that firms with b_k are indifferent between selling to consumers with type $n \geq k$ and selling to consumers with type

[8] For example, the set of power distributions satisfies this condition.

Figure 4: Valuation to Price Function: A General Model



$n \geq k+1$; for example, firms with b_1 are indifferent between selling to all consumers and selling to everyone except the poorest ones. This means that b_N is a cutoff level, i.e., firms with $b < b_N$ exit. Without loss of generality, I assume that firms with b_k choose to sell to consumers with type $n \geq k$.

As before, let M_e be the mass of firms that enter the market and draw valuation of their goods.

Definition 2 *The equilibrium of the model is defined by the price function $p(b)$ on $b \geq b_N$, M_e, the sequences $\{V_k\}_{k=1..N}$ and $\{b_k\}_{k=1..N}$ such that*

1) The ex-ante expected profits of firms are equal to zero.

2) The goods market clears.

Let $\pi_k(b)$ and $p_k(b)$ be the profit and the price of a firm with valuation $b \in [b_k, b_{k-1})$,[9] respectively. Then, the following lemma holds.

Lemma 3 *In equilibrium*

$$p_k(b) = \frac{b}{V_k} = bc\frac{\sum_{i=k}^N \frac{\alpha_i}{b_i}}{\sum_{i=k}^N \alpha_i}.$$

$$\pi_k(b) = cL \textstyle\sum_{i=k}^N \displaystyle\frac{\alpha_i(b-b_i)}{b_i}.$$

Proof. In the appendix. ■

[9] Let $b_0 = A$.

In the equilibrium, the expected profit of firms is equal to zero. This implies that

$$\begin{aligned} f_e &= \textstyle\sum_{k=1}^{N}(G(b_{k-1}) - G(b_k))E(\pi_k(b)|b \in [b_k, b_{k-1})) \iff \\ \frac{f_e}{cL} + 1 &= \textstyle\sum_{k=1}^{N} \alpha_k H(b_k). \end{aligned}$$

Moreover, the goods market clears. That is, the aggregate cost of the bundle of goods from group k should be equal to income of a consumer of type k. Taking into account that the aggregate profit of firms π is equal to zero in the equilibrium, I obtain

$$I_k = M_e \int_{b_k}^{A} p(t)dG(t) \quad k = 1..N.$$

Thus, there is the system of $N + 1$ equations

$$\begin{cases} I_k = M_e \int_{b_k}^{A} p(t)dG(t) \quad k = 1..N \\ \frac{f_e}{cL} + 1 = \sum_{k=1}^{N} \alpha_k H(b_k) \end{cases} \tag{7}$$

with $N + 1$ unknowns: $\{b_k\}_{k=1..N}$ and M_e.

Proposition 7 *The model has a unique equilibrium.*

Proof. The proof is based on the fact that the system of equations (7) has a unique solution. Details are in the appendix. ■

Thus, I prove the existence and uniqueness of the equilibrium with N types of consumers. At the same time, the results above imply that there exists a unique equilibrium for the case, when there is a continuous distribution of efficiency units of labor among consumers. Since any continuous distribution can be approximated by the sequence of discrete distributions, one can interpret equilibrium in the continuous case as the limit of equilibria in the discrete cases. Thus, in the continuous case, the function $V(b) = \frac{b}{p(b)}$ is increasing on $[b_L^c, b_M^c)$ and flat on $[b_M^c, A]$, where b_L^c and b_M^c are endogenous and depend on the exogenous parameters of the model and the distribution of efficiency units of labor[10].

Unfortunately, due to mathematical difficulties, it is very hard to solve the model explicitly in the continuous case. To solve the problem explicitly, I need to make an assumption about the distribution of efficiency units of labor. I assume that this distribution has a constant hazard rate. That is, I consider the set of exponential distributions on $[s, \infty)$, where $s \geq 0$ is the minimum endowment of efficiency units of labor[11]. However, in this case, there are shortcomings. Since the maximum endowment of efficiency units of labor is infinity, there is no exit in the model, i.e., the cutoff b_L^c

[10] $0 \leq b_L^c < b_M^c \leq A$

[11] All details are available in the appendix.

is equal to zero. Moreover, the price function $p(b)$ is decreasing in b, when b is close to zero, and $p(0) = \infty$. At the same time, this partial case gives us a simple straightforward explanation of why some luxury goods, even with low valuations, are so expensive: they are bought only by very rich consumers.

5 Conclusion

In this paper, I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. The model incorporates two key features: imperfect competition and non-homothetic preferences, which allow us to analyze the consequences of changes in the income distribution on pricing in the equilibrium, the market structure and, thereby, welfare of different groups of consumers. The general model is set up and solved. Due to technical difficulties in exploring comparative statics in the general case, I focus on the case of two types of consumers: rich and poor.

This framework leads to interesting theoretical results that help to understand the impact of income inequality on individual well-being. In particular, I analyze how income inequality influences the well-being of the poor. I show how greater income inequality in the economy can benefit the poor. Greater inequality can cause more entry in the market inducing greater competition and lower mark-ups. This in turn benefits poor consumers. As sources of income inequality, I consider changes in the income and the fraction of the rich consumers. This model also allows us to analyze the effect of changes in market size and entry cost. An increase in market size leads to tougher competition. Therefore, mark-ups of all firms fall and welfare of all consumers rises. Similarly, an increase in entry cost induces lower competition, raises mark-ups, and, thereby, decreases welfare of all consumers. Moreover, I find that given plausible assumptions, the rich gain more from an increase in market size and lose more from an increase in entry cost than the poor.

There are a number of plausible extensions of this model. For example, it would be interesting to consider an open economy version of the model. In this case, the paper can be modified in two ways. First, one can explore a model of trade between two countries with different income distributions and examine how this difference affects the trade pattern. Second, it would be interesting to consider the case when income distribution is endogenous and, for example, affected by the level of openness. I leave these issues for future work.

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Appendix

Proof of Lemma 2

Consider

$$\frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{A} tdG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right). \tag{8}$$

Let $b_M = F_1(b_L)$ be an implicit solution of (8). Obviously, $F_1(b_L)$ is strictly increasing in b_L. Moreover, $A > F_1(b_L) > b_L$. Hence, if b_L tends to A, then $F_1(b_L)$ also tends to A.

Now, consider

$$\frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1-\alpha_H)H(b_M). \tag{9}$$

By analogy, let $b_M = F_2(b_L)$ be an implicit solution of (9). Because $H(\cdot)$ is strictly decreasing, $F_2(b_L)$ is also strictly decreasing in b_L. Moreover, $H(A) = 1$. This implies that $H(F_2(A)) = \frac{f_e}{cL(1-\alpha_H)} + 1 > 1$. Therefore, $F_2(A) < A$. Let b_L^A be such that $F_2(b_L^A) = A$, then $H(b_L^A) = \frac{f_e}{cL\alpha_H} + 1 > 1$, i.e., $b_L^A < A$.

Thus, the solution of (4) exists and is unique (see *Figure 5*).

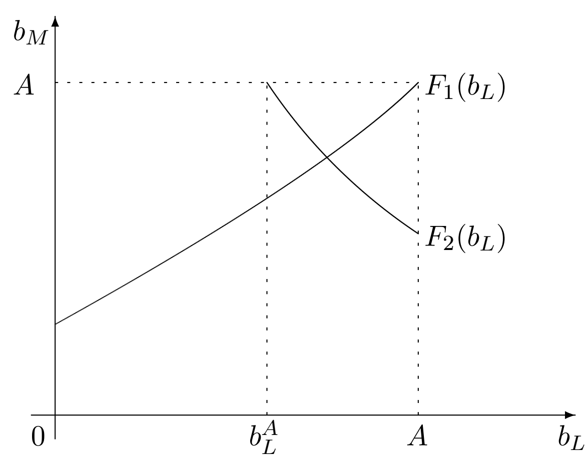
Proof of Lemma 3

Demand for a good from group k is equal to $L\sum_{i=k}^{N}\alpha_i$. From the definition of $\{b_k\}_{k=1..N}$: $\left(\frac{b_k}{V_k} - c\right)\sum_{i=k}^{N}\alpha_i = \left(\frac{b_k}{V_{k+1}} - c\right)\sum_{i=k+1}^{N}\alpha_i$.

By induction,

$$\frac{\sum_{i=k}^{N}\alpha_i}{V_k} = \frac{1}{V_1} - \sum_{i=1}^{k-1}\frac{\alpha_i}{b_i}c. \tag{10}$$

Figure 5: Equilibrium



Thus, from (10): $\pi_N(b) = \left(\frac{b}{V_N} - c\right)\alpha_N L = \frac{bL}{V_1} - bL\sum_{i=1}^{N-1}\frac{\alpha_i}{b_i}c - c\alpha_N L$. Since b_N is a cutoff level, $\pi_N(b_N) = 0$. This implies that $\frac{1}{V_1} = c\sum_{i=1}^{N}\frac{\alpha_i}{b_i}$. From (10): $\frac{1}{V_k} = \frac{c\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}$ $k = 1..N$. Finally, simple algebra shows that

$$\begin{aligned} p_k(b) &= bc\frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}, \\ \pi_k(b) &= cL\sum_{i=k}^{N}\frac{\alpha_i(b-b_i)}{b_i}. \end{aligned}$$

Proof of Proposition 7

Using *Lemma 3*, the system of equations (7) can be rewritten as follows[12]

$$\begin{cases} \frac{f_e}{cL} + 1 = \sum_{k=1}^{N}\alpha_k H(b_k) \\ \frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}\int_{b_k}^{b_{k-1}} t dG(t) \quad k = 1..N \end{cases}. \tag{11}$$

Consider $k = N$. Then,

$$\frac{I_N - I_{N-1}}{cM_e} = \frac{1}{b_N}\int_{b_N}^{b_{N-1}} t dG(t). \tag{12}$$

Given M_e and b_{N-1}, there exists a unique solution $b_N(b_{N-1}, M_e)$ of the equation (12). The function $b_N(b_{N-1}, M_e)$ is strictly increasing in M_e and b_{N-1}. Moreover, $\frac{M_e}{b_N(b_{N-1},M_e)} = \frac{I_N - I_{N-1}}{c\int_{b_N}^{b_{N-1}} t dG(t)}$ is strictly increasing in M_e. Consider $k = N-1$. Then,

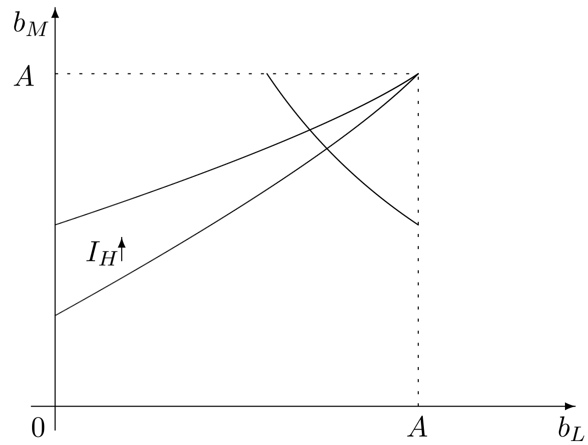
$$\frac{I_{N-1} - I_{N-2}}{cM_e} = \frac{\frac{\alpha_N}{b_N} + \frac{\alpha_{N-1}}{b_{N-1}}}{\alpha_N + \alpha_{N-1}}\int_{b_{N-1}}^{b_{N-2}} t dG(t). \tag{13}$$

Given M_e and b_{N-2}, there exists a unique solution $b_{N-1}(b_{N-2}, M_e)$ of the equation (13). The function $b_{N-1}(b_{N-2}, M_e)$ is strictly increasing in b_{N-2}. Since $\frac{M_e}{b_N(b_{N-1},M_e)}$ is strictly increasing in M_e, $b_{N-1}(b_{N-2}, M_e)$ is strictly increasing in M_e. Finally, $\frac{\left(\frac{\alpha_N}{b_N} + \frac{\alpha_{N-1}}{b_{N-1}}\right)M_e}{\alpha_N + \alpha_{N-1}} = \frac{I_{N-1} - I_{N-2}}{c\int_{b_{N-1}}^{b_{N-2}} t dG(t)}$ is strictly increasing in M_e. Using the backward induction, it can be proved that for any $k = 1..N$, there exists a unique solution $b_k(b_{k-1}, M_e)$ of the equation[13] $\frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}\int_{b_k}^{b_{k-1}} t dG(t)$ such that $b_k(b_{k-1}, M_e)$ is strictly increasing in b_{k-1} and M_e.

This implies that for any M_e, there exists a unique solution $\{b_k(M_e)\}_{k=1..N}$ of the system of equations $\frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}\int_{b_k}^{b_{k-1}} t dG(t)$ $k = 1..N$. And for any $k = 1..N$, $b_k(M_e)$ is strictly increasing

[12] Let I_0 be equal to zero.
[13] Recall that $b_0 = A$.

Figure 6: An Increase in I_H: Equilibrium



in M_e. Hence, (11) is equivalent to

$$\begin{aligned} \frac{f_e}{cL}+1 &= \textstyle\sum_{k=1}^{N}\alpha_k H(b_k(M_e)) \quad \text{and} \\ b_k &= b_k(M_e) \qquad k=1..N. \end{aligned} \tag{14}$$

Consider $D(M_e)=\sum_{k=1}^{N}\alpha_k H(b_k(M_e))$. $H(x)$ is a decreasing function. This means that $D(M_e)$ is decreasing in M_e. Moreover, if M_e is close to zero, then $b_N(M_e)$ is close to zero and, thereby, $D(M_e)$ is large enough[14]. At the same time, if M_e is large, then for any $k=1..N$, $b_k(M_e)$ is close to A and $D(M_e)\approx\sum_{k=1}^{N}\alpha_k H(A)=1<\frac{f_e}{cL}+1$. Thus, there exists a unique solution M_e of (14). This implies that there exists a unique solution of (7).

Comparative Statics

In this section I use simplifying notation: $\int_x^y$ means $\int_x^y tdG(t)$.

Proof of Proposition 2

An increase in I_H causes the curve $F_1(b_L)$ to shift up , while the curve $F_2(b_L)$ remains the same. Thus, b_L goes down and b_M goes up (see *Figure 6*). The impact on welfare is not so obvious. Rewrite (8) and (9) as follows

$$\begin{cases} J_1=(1-\alpha_H)cLH\left(b_M\right)+\alpha_H cLH\left(b_L\right)-f_e-cL=0 \\ J_2=I_L\int_{b_L}^{b_M}-\left(I_H-I_L\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^{A}=0 \end{cases}. \tag{15}$$

[14] $H(0)=\infty$.

Notice that equilibrium values of b_L and b_M solve (15). Thus,

$$\frac{\partial b_M}{\partial I_H} = \frac{\frac{\partial J_2}{\partial I_H}\frac{\partial J_1}{\partial b_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} > 0 \tag{16}$$

$$\frac{\partial b_L}{\partial I_H} = \frac{-\frac{\partial J_2}{\partial I_H}\frac{\partial J_1}{\partial b_M}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} < 0. \tag{17}$$

Consider $\frac{1}{V_1} = \frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}$. One can see that $\left(\frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}\right)'_{I_H} = \frac{-\alpha_H c}{(b_L)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{(b_M)^2}\frac{\partial b_M}{\partial I_H}$. From (16) and (17)

$$\frac{-\alpha_H c}{(b_L)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{(b_M)^2}\frac{\partial b_M}{\partial I_H} = \frac{c^2 L\alpha_H(1-\alpha_H)\frac{\partial J_2}{\partial I_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\left(\frac{H'(b_M)}{(b_L)^2} - \frac{H'(b_L)}{(b_M)^2}\right).$$

Recall that $H'(x) = -\frac{\int_x^A t dG(t)}{x^2} < 0$. Thus,

$$\frac{-\alpha_H c}{(b_L)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{(b_M)^2}\frac{\partial b_M}{\partial I_H} = \frac{c^2 L\alpha_H(1-\alpha_H)\frac{\partial J_2}{\partial I_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\frac{\int_{b_L}^A - \int_{b_M}^A}{(b_L)^2(b_M)^2}.$$

Since $\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L} > 0$ and $\frac{\partial J_2}{\partial I_H} < 0$, $\left(\frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}\right)'_{I_H} < 0$. Therefore, $(V_1)'_{I_H} > 0$. This implies that an increase in I_H causes an increase in $W_p = I_L V_1$. As $W_p = M_e\int_{b_M}^A$ and b_M increases, an increase in I_H raises M_e and $W_r = M_e\int_{b_L}^A$.

Similarly, an increase in I_L causes the curve $F_1(b_L)$ to shift down, while the curve $F_2(b_L)$ remains the same. Thus, b_L goes up and b_M goes down. To analyze the impact on welfare, I use the same technique as before. Since $W_r = M_e\int_{b_L}^A t dG(t)$ and $I_H = M_e\int_{b_L}^A p(t)dG(t)$, $W_r = \frac{I_H\int_{b_L}^A t dG(t)}{\int_{b_L}^A p(t)dG(t)} = \frac{I_H\int_{b_L}^A}{\frac{c}{b_L}\int_{b_L}^{b_M} + \left(\frac{c\alpha_H}{b_L} + \frac{c(1-\alpha_H)}{b_M}\right)\int_{b_M}^A}$. One can see that

$$\left(\frac{c}{b_L}\int_{b_L}^{b_M} + \left(\frac{c\alpha_H}{b_L} + \frac{c(1-\alpha_H)}{b_M}\right)\int_{b_M}^A\right)'_{I_L} =$$

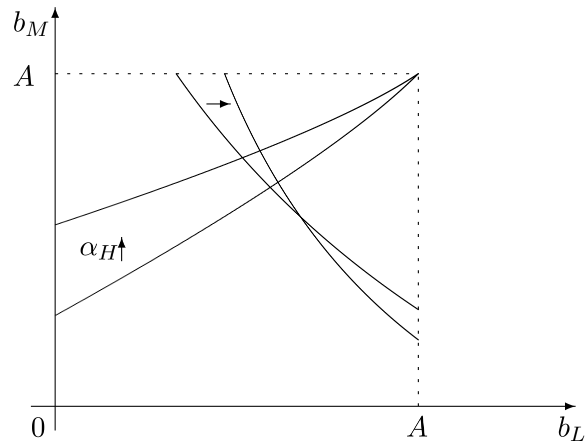
$$= c\left(\frac{\partial b_L}{\partial I_L}\left\{-g(b_L) - \frac{\int_{b_L}^{b_M} + \alpha_H\int_{b_M}^A}{(b_L)^2}\right\} + \frac{\partial b_M}{\partial I_L}(1-\alpha_H)\left\{g(b_M)b_M\left(\frac{1}{b_L} - \frac{1}{b_M}\right) - \frac{\int_{b_M}^A}{(b_M)^2}\right\}\right).$$

The sign of $(W_r)'_{I_L}$ is the same as the sign of $\left(\int_{b_L}^A t dG(t)\right)'_{I_L}\int_{b_L}^A p(t)dG(t) - \left(\int_{b_L}^A p(t)dG(t)\right)'_{I_L}\int_{b_L}^A t dG(t)$. Simple algebra shows that

$$\left(\int_{b_L}^A t dG(t)\right)'_{I_L}\int_{b_L}^A p(t)dG(t) - \left(\int_{b_L}^A p(t)dG(t)\right)'_{I_L}\int_{b_L}^A t dG(t) =$$

$$= \frac{c^2 L(1-\alpha_H)^2\frac{\partial J_2}{\partial I_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\frac{\int_{b_L}^{b_M}\int_{b_L}^A\int_{b_M}^A}{(b_L)^2(b_M)^2} + c(1-\alpha_H)(b_M - b_L)\left\{\frac{\partial b_L}{\partial I_L}\frac{g(b_L)}{b_M}\int_{b_M}^A - \frac{\partial b_M}{\partial I_L}\frac{g(b_M)}{b_L}\int_{b_L}^A\right\}.$$

Figure 7: An Increase in α_H: Equilibrium



Since $\frac{\partial b_L}{\partial I_L} > 0$, $\frac{\partial b_M}{\partial I_L} < 0$, and $\frac{\partial J_2}{\partial I_L} > 0$, $(W_r)'_{I_L} > 0$. As $M_e = \frac{W_r}{\int_{b_L}^{A} tdG(t)}$, an increase in I_L raises M_e and, thereby, W_p.

Proof of Proposition 3

An increase in α_H causes the curve $F_1(b_L)$ to shift up and the curve $F_2(b_L)$ to shift to the right around 45° degree line (see *Figure 7*). Thus, b_M goes up. The impact on b_L is not so obvious. There are two opposite effects: the upward shift of $F_1(b_L)$ decreases b_L, but the shift of the $F_2(b_L)$ increases b_L. Further, I explore which effect is stronger.

From (15)

$$\begin{aligned}
\frac{\partial b_M}{\partial \alpha_H} &= \frac{-\frac{\partial J_1}{\partial \alpha_H}\frac{\partial J_2}{\partial b_L} + \frac{\partial J_2}{\partial \alpha_H}\frac{\partial J_1}{\partial b_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} > 0 \\
\frac{\partial b_L}{\partial \alpha_H} &= \frac{-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H} + \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}.
\end{aligned}$$

To determine the sign of $\frac{\partial b_L}{\partial \alpha_H}$, I examine

$$-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H} + \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H} = cL\left(\left(H\left(b_L\right) - H\left(b_M\right)\right)\frac{\partial J_2}{\partial b_M} - \frac{(1-\alpha_H)}{\left(b_M\right)^2}\left(I_H - I_L\right)\left(1 - \frac{b_L}{b_M}\right)\left(\int_{b_M}^{A}\right)^2\right).$$

Consider

$$\frac{\partial J_2}{\partial b_M} = I_L b_M g\left(b_M\right) + \left(I_H - I_L\right)\left(\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right) b_M g\left(b_M\right) + \frac{b_L(1-\alpha_H)}{\left(b_M\right)^2}\int_{b_M}^{A}\right).$$

The second equation in (15) can be rewritten as $I_L = \frac{(I_H - I_L)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^A}{\int_{b_L}^{b_M}}$. Thus,

$$\frac{\partial J_2}{\partial b_M} = (I_H - I_L)\left(\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right) b_M g\left(b_M\right) \frac{\int_{b_L}^A}{\int_{b_L}^{b_M}} + \frac{b_L(1-\alpha_H)}{\left(b_M\right)^2}\int_{b_M}^A\right). \tag{18}$$

Therefore,

$$\begin{aligned}\frac{-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H} + \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H}}{cL\left(I_H - I_L\right)} &= \left(H\left(b_L\right) - H\left(b_M\right)\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right) b_M g\left(b_M\right)\frac{\int_{b_L}^A}{\int_{b_L}^{b_M}} + \\ &+ \frac{(1-\alpha_H)}{\left(b_M\right)^2}\int_{b_M}^A\left(G(b_L)b_L + \int_{b_L}^A - G(b_M)b_L - \int_{b_M}^A\right) > 0,\end{aligned}$$

since $b_M > b_L$ and $G(b_L)b_L + \int_{b_L}^A - G(b_M)b_L - \int_{b_M}^A$ is increasing in b_M and equal to zero when $b_M = b_L$. Thus, $\frac{\partial b_L}{\partial \alpha_H} > 0$.

Next, I explore the impact of an increase in α_H on welfare of the poor. Recall that welfare of the poor is given by $\frac{I_L}{c\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)}$. To determine the sign of $\left(W_p\right)'_{\alpha_H}$, one needs to know the sign of $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \frac{1}{b_L} - \frac{1}{b_M} - \left(\frac{\alpha_H}{(b_L)^2}\frac{\partial b_L}{\partial \alpha_H} + \frac{(1-\alpha_H)}{(b_M)^2}\frac{\partial b_M}{\partial \alpha_H}\right)$. Consider

$$\frac{\alpha_H}{\left(b_L\right)^2}\frac{\partial b_L}{\partial \alpha_H} + \frac{(1-\alpha_H)}{\left(b_M\right)^2}\frac{\partial b_M}{\partial \alpha_H} = \frac{\left(\frac{\partial J_1}{\partial \alpha_H}\left(\frac{\alpha_H\frac{\partial J_2}{\partial b_M}}{(b_L)^2} - \frac{(1-\alpha_H)\frac{\partial J_2}{\partial b_L}}{(b_M)^2}\right) + \frac{\partial J_2}{\partial \alpha_H}\left(\frac{(1-\alpha_H)\frac{\partial J_1}{\partial b_L}}{(b_M)^2} - \frac{\alpha_H\frac{\partial J_1}{\partial b_M}}{(b_L)^2}\right)\right)}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}.$$

From (15)

$$\begin{aligned}\frac{\partial J_2}{\partial b_L} &= -I_L b_L g\left(b_L\right) - \left(I_H - I_L\right)\frac{(1-\alpha_H)}{b_M}\int_{b_M}^A \\ &= -\left(I_H - I_L\right)\left(\frac{b_L g\left(b_L\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^A}{\int_{b_L}^{b_M}} + \frac{(1-\alpha_H)}{b_M}\int_{b_M}^A\right).\end{aligned} \tag{19}$$

Thus, from (18) and (19)

$$\frac{\alpha_H\frac{\partial J_2}{\partial b_M}}{\left(b_L\right)^2} - \frac{(1-\alpha_H)\frac{\partial J_2}{\partial b_L}}{\left(b_M\right)^2} = \left(I_H - I_L\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)P_1,$$

where $P_1 = \left(\frac{\alpha_H b_M g(b_M)\int_{b_L}^A}{(b_L)^2\int_{b_L}^{b_M}} + \frac{(1-\alpha_H)b_L g(b_L)\int_{b_M}^A}{(b_M)^2\int_{b_L}^{b_M}} + \frac{(1-\alpha_H)}{b_L(b_M)^2}\int_{b_M}^A\right)$. Moreover,

$$\frac{(1-\alpha_H)\frac{\partial J_1}{\partial b_L}}{\left(b_M\right)^2} - \frac{\alpha_H\frac{\partial J_1}{\partial b_M}}{\left(b_L\right)^2} = -\frac{(1-\alpha_H)\alpha_H cL}{\left(b_M\right)^2\left(b_L\right)^2}\int_{b_L}^{b_M}.$$

Thus,

$$\frac{\alpha_H}{(b_L)^2}\frac{\partial b_L}{\partial \alpha_H} + \frac{(1-\alpha_H)}{(b_M)^2}\frac{\partial b_M}{\partial \alpha_H} = \frac{\frac{\partial J_1}{\partial \alpha_H}(I_H - I_L)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)P_1 - \frac{\partial J_2}{\partial \alpha_H}\frac{(1-\alpha_H)\alpha_H cL}{(b_M)^2(b_L)^2}\int_{b_L}^{b_M}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}.$$

Using the expressions for $\frac{\partial J_1}{\partial \alpha_H}$ and $\frac{\partial J_2}{\partial \alpha_H}$, one can easily derive that

$$\frac{\alpha_H}{(b_L)^2}\frac{\partial b_L}{\partial \alpha_H} + \frac{(1-\alpha_H)}{(b_M)^2}\frac{\partial b_M}{\partial \alpha_H} = \frac{cL(I_H - I_L)\left((H(b_L) - H(b_M))\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)P_1 + \left(1 - \frac{b_L}{b_M}\right)P_2\right)}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}},$$

where $P_2 = \frac{(1-\alpha_H)\alpha_H}{(b_M)^2(b_L)^2}\int_{b_L}^{b_M}\int_{b_M}^{A}$.

Consider $\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}$. From (18) and (19)

$$\frac{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}{cL(I_H - I_L)} = \frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3 + \frac{(1-\alpha_H)\int_{b_M}^{A} t dG(t)}{(b_M)^2}P_4,$$

where $P_3 = \frac{(1-\alpha_H)b_L g(b_L)\left(\int_{b_M}^{A}\right)^2}{(b_M)^2} + \frac{\alpha_H b_M g(b_M)\left(\int_{b_L}^{A}\right)^2}{(b_L)^2}$ and $P_4 = \frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M} + \frac{\alpha_H\int_{b_L}^{A}}{b_L}$. Therefore,

$$\begin{aligned}\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} &= \frac{1}{b_L} - \frac{1}{b_M} - \frac{(H(b_L) - H(b_M))\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)P_1 + \left(1 - \frac{b_L}{b_M}\right)P_2}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3 + \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}P_4} \\ &= \frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\left(P_3\left(\frac{1}{b_L} - \frac{1}{b_M}\right) - (H(b_L) - H(b_M))P_1\int_{b_L}^{b_M}\right)}{\left(\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3 + \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}P_4\right)\int_{b_L}^{b_M}} \\ &\quad + \frac{\left(\frac{1}{b_L} - \frac{1}{b_M}\right)\left(\frac{(1-\alpha_H)P_4\int_{b_M}^{A} - b_L(b_M)^2 P_2}{(b_M)^2}\right)}{\left(\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3 + \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}P_4\right)}.\end{aligned}$$

Simple arithmetic shows that $(1-\alpha_H)P_4\int_{b_M}^{A} - b_L(b_M)^2 P_2 = \left(\int_{b_M}^{A}\right)^2\frac{(1-\alpha_H)}{b_L}\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)$. Thus,

$$\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3 + \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}P_4}P_5,$$

where $P_5 = \left(\frac{1}{b_L} - \frac{1}{b_M}\right)\left(\int_{b_M}^{A}\right)^2\frac{(1-\alpha_H)}{b_L(b_M)^2} + \left(\frac{P_3\left(\frac{1}{b_L} - \frac{1}{b_M}\right)}{\int_{b_L}^{b_M}} - (H(b_L) - H(b_M))P_1\right)$. Using the expressions for P_1 and P_3, it is not difficult to obtain that

$$\begin{aligned}P_5 &= \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}\left(\frac{1}{b_L}+\frac{b_L g(b_L)}{\int_{b_L}^{b_M}}\right)\left(G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_L}\right)\\ &\quad+\frac{\alpha_H\int_{b_L}^{A}}{(b_L)^2}\frac{b_M g(b_M)}{\int_{b_L}^{b_M}}\left(G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_M}\right).\end{aligned}$$

One can see that the sign of $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$ is the same as the sign of P_5. As $b_M > b_L$, $G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_L} < 0$ and $G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_M} > 0$. Hence, if α_H is close enough to zero, then $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} < 0$; that is, $(W_p)'_{\alpha_H} > 0$. However, if α_H is close enough to one, then $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} > 0$. This implies that $(W_p)'_{\alpha_H} < 0$. It appears much more complicated to determine the sign of P_5 for all other values of $\alpha_H \in (0,1)$. It is difficult to examine the strict monotonicity of P_5 with respect to α_H, as the equilibrium values of b_M and b_L depend on α_H.

Now, I examine the impact of an increase in α_H on welfare of the rich given by $\frac{1}{c}\left(\frac{I_L}{\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)}+(I_H-I_L)\,b_L\right)$. Thus,

$$c\,(W_r)'_{\alpha_H} = \frac{(I_H-I_L)\frac{\partial b_L}{\partial\alpha_H}\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}}{\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2}.$$

To find the sign of $(W_r)'_{\alpha_H}$, one needs to find the sign of $(I_H-I_L)\frac{\partial b_L}{\partial\alpha_H}\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$. Using the previous results,

$$\begin{aligned}&(I_H-I_L)\frac{\partial b_L}{\partial\alpha_H}\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} =\\ &= (I_H-I_L)\frac{\partial b_L}{\partial\alpha_H}\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3+\frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}P_4}P_5.\end{aligned}$$

Using the expressions for $\frac{\partial b_L}{\partial\alpha_H}$ and P_5 derived above, one can obtain that to prove the positive sign of $(W_r)'_{\alpha_H}$, it is enough to prove that

$$(I_H-I_L)\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)(H(b_L)-H(b_M)) - \frac{I_L}{b_L}\left(G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_M}\right) > 0.$$

This is equivalent to

$$\frac{I_L}{b_L}\left(\frac{\int_{b_L}^{b_M}}{\int_{b_M}^{A}}\left(H\left(b_L\right)-H\left(b_M\right)\right)-\left(G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_M}\right)\right) \quad > \quad 0 \iff$$
$$\frac{I_L\int_{b_L}^{A}}{b_L}\left(\frac{1}{\int_{b_M}^{A}}\left(H\left(b_L\right)-H\left(b_M\right)\right)-\frac{1}{b_L}+\frac{1}{b_M}\right) \quad > \quad 0.$$

For any $b_L < b_M$, $\frac{(H(b_L)-H(b_M))}{\int_{b_M}^{A}} - \frac{1}{b_L} + \frac{1}{b_M} > 0$. Thus, $(W_r)'_{\alpha_H}$ is always greater than zero. Since $(W_r)'_{\alpha_H} > 0$, $(b_L)'_{\alpha_H} > 0$ and $W_r = M_e\int_{b_L}^{A}$, the mass of firms entering the market goes up, i.e., $(M_e)'_{\alpha_H} > 0$.

Proof of Proposition 4

In this section I consider an increase in I_H together with a decrease in α_H keeping the aggregate income fixed. The aggregate income per capita, AG, is given by $\alpha_H I_H + (1-\alpha_H)I_L$. This implies that $I_H = \frac{AG-(1-\alpha_H)I_L}{\alpha_H}$ or $\alpha_H\left(I_H - I_L\right) = AG - I_L$. Using the expression for I_H, I rewrite (15)

$$\begin{cases} J_1 = (1-\alpha_H)cLH\left(b_M\right) + \alpha_H cLH\left(b_L\right) - f_e - cL = 0 \\ J_2 = I_L\int_{b_L}^{b_M} - (AG - I_L)\left(1 + \frac{b_L(1-\alpha_H)}{\alpha_H b_M}\right)\int_{b_M}^{A} = 0 \end{cases}. \tag{20}$$

Hence, it is necessary to find the impact of a decrease in α_H on welfare of the poor given new equilibrium equations (20). I use the same technique as in the previous case. The only difference is that $\frac{\partial J_2}{\partial \alpha_H}$ changes. Thus,

$$\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \frac{1}{b_L}-\frac{1}{b_M}-\frac{\frac{\partial J_1}{\partial \alpha_H}\left(I_H-I_L\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)P_1 - \frac{\partial J_2}{\partial \alpha_H}\frac{(1-\alpha_H)\alpha_H cL}{(b_M)^2(b_L)^2}\int_{b_L}^{b_M}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}.$$

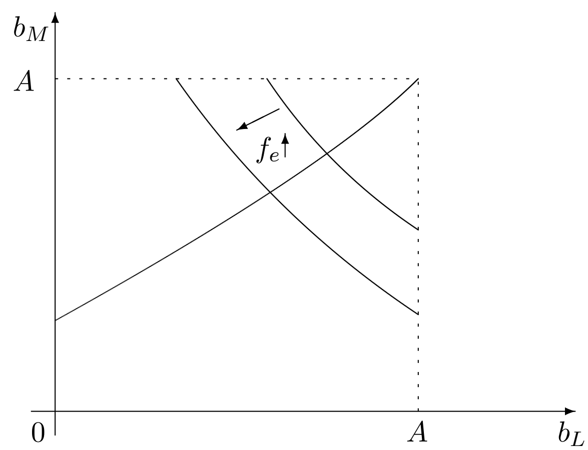
Using the expressions for $\frac{\partial J_1}{\partial \alpha_H}$ and $\frac{\partial J_2}{\partial \alpha_H}$, one can derive

$$\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \frac{1}{b_L}-\frac{1}{b_M}-\frac{(H(b_L)-H(b_M))\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)P_1 - \frac{(1-\alpha_H)}{(b_M)^3 b_L}\int_{b_M}^{A}\int_{b_L}^{b_M}}{\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3+\frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}P_4}.$$

Using the expressions for P_1, P_3, P_4 and making some simplifications, one can obtain

$$\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \frac{\left(\frac{\alpha_H}{b_L}+\frac{b_L(1-\alpha_H)}{b_M}\right)\left(\frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}\left(G(b_M)-G(b_L)\right)+P_6\right)}{\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}tdG(t)}P_3+\frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}P_4},$$

Figure 8: An Increase in f_e: Equilibrium



where $P_6 = \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}\frac{b_L g(b_L)}{\int_{b_L}^{b_M}}\left(b_L\left(G(b_M)-G(b_L)\right)-\int_{b_L}^{b_M}\right)+\frac{\alpha_H\int_{b_L}^{A}}{b_L}\frac{b_M g(b_M)}{\int_{b_L}^{b_M}}\left(G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_M}\right).$
Thus, if α_H is high enough, then $P_6 > 0$ and $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} > 0$. That is, welfare of the poor rises with a decrease in α_H. This is also confirmed by the fact that given high α_H both an increase in I_H and a decrease in α_H have the positive impact on welfare of the poor. However, if α_H is close to zero, then these two effects work in the opposite direction. Consider $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$ when α_H is close to zero. Notice that from (20) $\lim_{\alpha_H\to 0} b_L(\alpha_H) = 0$ and $\lim_{\alpha_H\to 0}\frac{b_L(\alpha_H)}{\alpha_H}$ is some positive constant. Since for any density function $g(\cdot)$, $\lim_{x\to 0} xg(x) = 0$, $\lim_{\alpha_H\to 0} P_6 > 0$. This implies that $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H=0} > 0$; that is, welfare of the poor goes up with a decrease in α_H.

Finally, consider $(b_M)'_{\alpha_H} = \frac{-\frac{\partial J_1}{\partial \alpha_H}\frac{\partial J_2}{\partial b_L}+\frac{\partial J_2}{\partial \alpha_H}\frac{\partial J_1}{\partial b_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}$.

The numerator is equal to $\frac{cL(I_H-I_L)\int_{b_M}^{A}}{b_M}\left(-\frac{\int_{b_L}^{A}}{b_L}+\left(H(b_L)-H(b_M)\right)\left(\frac{b_L g(b_L)(\alpha_H b_M + b_L(1-\alpha_H))}{\int_{b_L}^{b_M}}+1-\alpha_H\right)\right).$

Thus, the sign of $(b_M)'_{\alpha_H=0}$ is equal to the sign of $H(b_L)-H(b_M)-\frac{\int_{b_L}^{A}}{b_L}$, which is negative. Recall that $W_p = M_e\int_{b_M}^{A}$, this implies that $(M_e)'_{\alpha_H=0} < 0$.

Proof of Proposition 5

An increase in f_e causes the curve $F_2(b_L)$ (see the proof of *Lemma 2*) to shift down and to the left, while the curve $F_1(b_L)$ remains unchanged. Therefore, b_L and b_M go down (see *Figure 8).* Since $W_p = \frac{I_L}{\frac{\alpha_H c}{b_L}+\frac{(1-\alpha_H)c}{b_M}}$ and $W_r = W_p + \frac{I_H-I_L}{c}b_L$, one can see that W_p and W_r decrease. M_e, which is equal to $\frac{W_p}{\int_{b_M}^A}$, also falls.

Similarly, an increase in L causes the curve $F_2(b_L)$ to shift up and to the right, and the curve $F_1(b_L)$ remains unchanged. Thus, b_L and b_M go up. M_e, W_p, and W_r increase.

Finally, any changes in f_e and L such that $\frac{f_e}{L}$ remains the same do not affect $F_2(b_L)$ and $F_1(b_L)$. This implies that b_L and b_M do not change. As a result, M_e, W_p, and W_r remain unchanged.

Proof of Proposition 6

I show that if $\left(\frac{x^2 g(x)}{\int_x^A tdG(t)}\right)'_x > 0$ for any x, then $\left(\frac{b_M}{b_L}\right)'_{b_L} < 0$ along

$$\int_{b_L}^{b_M} - \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^A = 0.$$

Simple algebra shows that

$$\frac{\partial b_M}{\partial b_L} = \frac{b_L g(b_L) + \frac{\int_{b_L}^{b_M}}{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}\frac{(1-\alpha_H)}{b_M}}{\frac{\int_{b_L}^A}{\int_{b_M}^A} b_M g(b_M) + \frac{b_L(1-\alpha_H)}{(b_M)^2}\frac{\int_{b_L}^{b_M}}{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}}.$$

$\left(\frac{b_M}{b_L}\right)'_{b_L} = \frac{\frac{\partial b_M}{\partial b_L} b_L - b_M}{(b_L)^2}$ is less than zero if and only if $\frac{\partial b_M}{\partial b_L} b_L - b_M < 0$. One can derive that

$$\begin{aligned} \frac{\partial b_M}{\partial b_L} b_L - b_M &< 0 \iff \\ \frac{b_L g(b_L) + \frac{\int_{b_L}^{b_M}}{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}\frac{(1-\alpha_H)}{b_M}}{\frac{\int_{b_L}^A}{\int_{b_M}^A} b_M g(b_M) + \frac{b_L(1-\alpha_H)}{(b_M)^2}\frac{\int_{b_L}^{b_M}}{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}}\frac{b_L}{b_M} &< 1 \iff \frac{(b_L)^2 g(b_L)}{\int_{b_L}^A} < \frac{(b_M)^2 g(b_M)}{\int_{b_M}^A}. \end{aligned}$$

The Continuous Distribution of Efficiency Units of Labor

I assume that there is a distribution $F(\cdot)$ on $[s, S]$ (with a density function $f(\cdot)$) of efficiency units of labor. That is, given the mass L of consumers, there are $F(x)L$ consumers with income less or equal to x. Define $V(b) = \frac{b}{p(b)}$. From the main body of the paper one knows that $V(b)$ is increasing on $[b_L^c, b_M^c)$ and flat on $[b_M^c, A]$, where b_L^c and b_M^c are endogenous and depend on the exogenous parameters of the model and the distribution of efficiency units of labor. I assume that $V(b)$ is differentiable on $[b_L^c, b_M^c)$. Finally, without loss of generality, I also assume that $L = 1$.

Consider a particular firm with valuation b. If $b \in [b_M^c, A]$, then demand for this good is equal to one and $p(b) = \frac{b}{V(b_M^c)}$, since $V(b)$ is flat on $[b_M^c, A]$. Suppose $b \in [b_L^c, b_M^c)$ and this firm imposes the price p of its good. Then, given $V(b)$ in the equilibrium, $s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)$ is total spending on goods, which are bought before the considered good, goods that have higher valuation to price ratios. This implies that demand for this good is equal to $1-F\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)$. Thus, in the equilibrium firms with $b\in[b_L^c,b_M^c)$ solve the following maximization problem

$$\max_p (p-c)\left(1-F\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)\right).$$

The first order condition implies that

$$\frac{1-F\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)}{f\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)}=(p-c)\frac{bM_e p\left(V^{-1}\left(\frac{b}{p}\right)\right)g\left(V^{-1}\left(\frac{b}{p}\right)\right)}{p^2V'\left(V^{-1}\left(\frac{b}{p}\right)\right)}.$$

This equation should be satisfied for any $b\in[b_L^c,b_M^c)$ in the equilibrium. That is, the price function $p(b)$ on $[b_L^c,b_M^c)$ solves the following differential equation

$$\frac{1-F\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)}{f\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)}=(p(b)-c)\frac{bM_e g\left(b\right)}{p(b)V'\left(b\right)}, \tag{21}$$

where $V(b)=\frac{b}{p(b)}$. Using the solution of (21), free entry condition, and the goods market equilibrium, one can find b_L^c, b_M^c, and M_e in the equilibrium.

However, it is very hard to find the solution of (21) in general. To simplify the problem, I assume that $F(x)=1-e^{-\alpha(x-s)}$ on $[s,\infty)$. This implies that $\frac{1-F\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)}{f\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)}=\frac{1}{\alpha}$.Thus,

$$\begin{aligned}\frac{1}{\alpha} &= (p(b)-c)\frac{bM_e g\left(b\right)}{p(b)V'\left(b\right)} \iff \\ V'\left(b\right) &= \alpha M_e\left(b-cV(b)\right)g\left(b\right).\end{aligned} \tag{22}$$

Since the maximum endowment of efficiency unit of labor is infinity, there is no exit and $b_L^c=0$. Using the initial condition $V(0)=0$ and (22), one can derive

$$V(b)=\frac{1}{c}\left(b-e^{-\alpha M_e cG(b)}\int_0^b e^{\alpha M_e cG(t)}dt\right).$$

Finally,

$$p(b)=\frac{cb}{b-e^{-\alpha M_e cG(b)}\int_0^b e^{\alpha M_e cG(t)}dt}.$$

From the goods market equilibrium one can obtain that $s = \int_{b_M^c}^{A} M_e p(t) dG(t) = \frac{M_e}{V(b_M^c)} \int_{b_M^c}^{A} t dG(t)$. Using this equation and free entry condition, one can find M_e and b_M^c. In the simplest case with $s = 0$, $b_M^c = A$ and M_e can be found from

$$f_e = \int_0^A (p(b) - c)\, e^{-\alpha M_e \int_b^A p(t) dG(t)} dG(b).$$

Notice that $\lim_{b \to 0} p(b) = \infty$. This means that the goods with lowest valuations have the highest prices.