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Ex-ante and *Ex-post* Strong Correlated Equilibria

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Abstract

A strong correlated equilibrium is a strategy profile that is immune to joint deviations. Different notions of strong correlated equilibria were defined in the literature. One major difference among those definitions is the stage in which coalitions can plan a joint deviation: before (*ex-ante*) or after (*ex-post*) the deviating players receive their part of the correlated profile. In this paper we prove that if deviating coalitions are allowed to use new correlating devices, then an *ex-ante* strong correlated equilibrium is also immune to deviations at the *ex-post* stage. Thus the set of *ex-ante* strong correlated equilibria of Moreno & Wooders (1996) is included in all other sets of strong correlated equilibria.

1 Introduction

The ability of players to communicate prior to the play, influences the set of self-enforcing outcomes of a non-cooperative game. The communication allows the players to correlate their play, and to implement a correlated strategy profile as a feasible non-biding agreement. For such an agreement to be self-enforcing, it has to be stable against "reasonable" coalitional deviations. Two notions in the literature describe such self-enforcing agreements: a *strong correlated equilibrium* is a profile that is stable against *all* coalitional deviations, and a *coalition-proof correlated equilibrium* is a profile that is stable against *self-enforcing* coalitional deviations (a deviation is self-enforcing if there is

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no further self-enforcing and improving deviation available to a proper sub-coalition).

Each notion has a few alternative definitions. One major difference among them, is the stage in which coalitions can plan a deviation from a correlated agreement. Assume that the correlated agreement is implemented by a mediator who privately recommends each player what to play. The definitions in Milgrom & Roberts (1996), Moreno & Wooders (1996), and Ray (1996) are *ex-ante* definitions: In their framework, players may plan deviations before receiving recommendations, and no further communication is possible after recommendations are issued. The definitions in Einy & Peleg (1995), Ray (1998) and Bloch & Dutta (2007) are *ex-post* [2] definitions: In their framework, players may plan deviations only after receiving recommendations.

However, in some frameworks it is reasonable to assume that coalitions can plan deviations at both stages: *ex-ante* and *ex-post*. One example for such framework is an extended game with *cheap-talk*: pre-play, unmediated, non-biding, non-verifiable communication among the players. [3] In such a framework, the players can "mimic" a mediator, and implement a large set of strong correlated equilibria as a strong Nash equilibria (Aumann, 1959) in the extended game (Heller, 2008). [4] A coalition can plan a deviation in the early phases of the cheap-talk when no player has received his recommendation yet (an *ex-ante* deviation), or in the late phases when all players have received their recommendations (an *ex-post* deviation).

A natural question is whether any of the existing notions is appropriate to such frameworks (in which deviations can be planned at both stages), or whether new definitions are needed. In this paper we prove that the existing *ex-ante* strong correlated equilibrium (à la Moreno & Wooders) is also resistant to *ex-post* deviations. The proof is based on three assumptions about the communication framework (which hold in the cheap-talk framework):

(1) A deviating coalition can use new correlating devices (play a joint *correlated* deviation).
(2) When a coalition decides to deviate, that decision is common knowledge among its members.
(3) The players share a common prior about the possible states of the world

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[2] Referred to as *"interim"* definitions in some of the literature.

[3] For a good nontechnical introduction to some of the main issues of cheap-talk, see the survey of Farrel & Rabin (1996).

[4] The implementation presented in Heller (2008) is only as a $\lfloor n/2 \rfloor$-strong correlated equilibrium (an equilibrium that is resistant to deviations of coalitions with less than $n/2$ players). If one assumes that the players are computationally restricted and "one-way" functions exist, then the implementation can be as a strong correlated equilibrium (as discussed in Lepinski et al., 2004 and Abraham et al., 2006).

in an incomplete information model à la Aumann (see Aumann, 1987, and the description in Subsection 3.3).

An immediate corollary is that the set of *ex-ante* strong correlated equilibria is included in all other sets of strong correlated equilibria, as defined in any of the existing definitions in the literature referred above, or in the new more general *ex-post* definition presented in this paper.

One could hope that similar results might be obtained for the coalition-proof notions. However, in Section 5 we demonstrate that the existing *ex-ante* coalition-proof notion is not appropriate to frameworks in which coalitions can plan deviations at both stages.

The paper is organized as follows: In Section 2 we show a few examples for *ex-ante* and *ex-post* strong correlated equilibria. Section 3 presents our model and definitions, Section 4 presents our main result. In Section 5 we discuss the coalition-proof notions, and we conclude in Section 6.

2 Examples for Strong Correlated Equilibria

In this Section we present two examples for strong correlated equilibria (both examples are adapted from Moreno & Wooders, 1996):

(1) An example for an *ex-ante* (and *ex-post*) strong correlated equilibrium, which is the only "reasonable" outcome of a game with pre-play communication.
(2) An example for an *ex-post* strong correlated equilibrium that is not an *ex-ante* strong correlated equilibrium.

2.1 Example 1 - Three Player Matching Pennies Game

Three players each simultaneously choose heads or tails. If all three faces match, then players 1 and 2 each win a penny while player 3 loses two pennies. Otherwise, Player 3 wins two pennies while players 1 and 2 each lose a penny. Table 1 presents the matrix representation of the game.

This game has three Nash equilibria:

- Two pure equilibria: (H_1, H_2, T_3), (T_1, T_2, H_3) that give a payoff of $(-1, -1, 2)$
- A totally mixed equilibrium in which every action is chosen with probability $1/2$, which gives an expected payoff of $(-0.5, -0.5, 1)$.

Table 1
The Three Player Matching Pennies Game

	H_3			T_3	
	H_2	T_2		H_2	T_2
H_1	1,1,-2	-1,-1,2	H_1	-1,-1,2	-1,-1,2
T_1	-1,-1,2	-1,-1,2	T_1	-1,-1,2	1,1,-2

None of those equilibria is a reasonable outcome of a game with pre-play communication, as players 1 and 2 can play together a correlated strategy profile that guarantees each of them a payoff of 0: playing (H_1, H_2) with probability 0.5 and (T_1, T_2) with probability 0.5.

This game has a single *ex-ante* (and *ex-post*) strong correlated equilibrium: players 1 and 2 play the correlated profile mentioned above, and player 3 plays heads with probability 0.5 and tails with probability 0.5. The induced correlated profile, which is described in table 2, gives an expected payoff of $(0, 0, 0)$.

Table 2
The Ex-ante Strong Equilibrium of the Game

	H_3			T_3	
	H_2	T_2		H_2	T_2
H_1	1/4	0	H_1	1/4	0
T_1	0	1/4	T_1	0	1/4

The profile is a strong correlated equilibrium because no single player can deviate and improve upon this agreement: both players 1 and 2 loose by unilaterally deviating, and player 3 is indifferent between heads and tails. Moreover, since the interests of players 1 and 2 are completely opposed to those of player 3, no coalition involving player 3 can improve upon the given agreement. Finally, given player 3's strategy, players 1 and 2 obtain at most a payoff of 0. Hence, no coalition can gain by deviating from the agreement. In Moreno & Wooders (1998), an experimental study is presented to show that players tend to play this strong correlated equilibrium (and not any of the Nash equilibria) in the presence of pre-play (private) communication.

2.2 Example 2 - Two-Player Chicken *Game*

Table 3 describes a two player *Chicken* game and a strong *ex-post* correlated equilibrium (with an expected payoff of 5 for each player), which is not an *ex-ante* strong correlated equilibrium. The profile is not an *ex-ante* strong correlated equilibrium, because the players have a profitable joint *ex-ante* deviation: play the pure action profile (T, L), which gives both player a higher payoff of 6.

Table 3
Chicken Game and an *Ex-post* Strong Correlated Equilibrium

	L	R
T	6,6	2,7
B	7,2	0,0

	L	R
T	1/3	1/3
B	1/3	0

We now show that the profile is an *ex-post* strong correlated equilibrium. Observe first that the profile is a correlated equilibrium, thus no player has a profitable unilateral deviation. We have to show that there are no profitable *ex-ante* joint deviations. Assume to the contrary that there is a profitable *ex-post* joint deviation. When the players deviate, it must be common knowledge that both earn from the deviation. Observe that if player 1 (2) has received a recommendation B (R), then he expects to get his maximal payoff 7, and he can not earn from the use of any deviating device. Thus, when the players deviate, it must be common knowledge that the recommendation profile is (T, L), but then it is common knowledge that the payoff of each player is 6, and there is no deviation that can improve the payoffs of both players.

3 Model and Definitions

3.1 Preliminary Definitions

A game in strategic form G is defined as: $G = \left(N, (A^i)_{i\in N}, (u^i)_{i\in N}\right)$, where N is the finite (and non-empty) set of players with a size $n = |N|$, and for each $i \in N$, A_i is player i's finite (and non-empty) set of actions (or pure strategies), and u^i is player i's utility (payoff) function, a real-valued function on $A = \prod_{i\in N} A^i$. The multi-linear extension of u^i to $\Delta(A)$ is is still denoted by u^i. A member of $\Delta(A)$ is called a (correlated) strategy profile. A coalition S is a non-empty member of 2^N. For simplicity of notation, the coalition $\{i\}$ is denoted as i. Given a coalition S, and let $A^S = \prod_{i\in S} A^i$, let $-S = \{i \in N \mid i \notin S\}$ denote the complementary coalition. A member of $\Delta(A^S)$ is called a (correlated) *S-strategy profile.* Given $q \in \Delta(A)$ and $a^S \in A^S$, we define $q_{|S} \subseteq \Delta(A^S)$ to be $q_{|S}(a^S) = \sum_{a^{-S}\in A^{-S}} q(a^S, a^{-S})$, and for simplicity we omit the subscript: $q(a^S) = q_{|S}(a^S)$. Given a^S s.t. $q(a^S) > 0$, we define $q(a^{-S}|a^S) = \frac{q(a^S,a^{-S})}{q(a^S)}$.

3.2 An Ex-ante Strong Correlated Equilibrium

In this subsection, following the framework presented in Moreno & Wooders (1996), we conceive a framework in which deviations from an agreement are only planned at the *ex-ante* stage. Let the correlated strategy profile $q \in \Delta(A)$ be an agreement that have been reached by the players. The players implement the agreement q with the assistance of a correlating device (a mediator) who recommends the action profile $a \in A$ with probability $q(a)$. Each player i receives his recommendation a^i in a sealed envelope. At that stage (when all the envelopes are sealed) the players can communicate and possibly plan coalitional deviations from the agreement. If all the members of a coalition S agree to use a deviation, then it is implemented with the assistance of a deviating device (a new correlating device - another mediator), who receives the sealed envelopes from each player in S, privately reads their contents, and gives each player a new recommendation in a sealed envelope. Thus the deviating device implement a mapping from the set A^S (the original recommendations the players in S have received) to the set of $\Delta(A^S)$ of correlated S-strategy profiles. In the second stage, the players open their sealed envelopes, and each player simultaneously chooses an action (no further communication occurs).

Definition 1 Given a coalition $S \subseteq N$, *a deviating device* is a function $d^S : A^S \to \Delta(A^S)$.

We define the feasible deviations for a coalition S, as those correlated strategy profiles $p \in A$ that the coalition can induce with the use of deviating devices. Thus, a correlated strategy profile is a feasible *ex-ante* deviation by a coalition S from a given agreement if the members of S, using some plan to correlate their play (a deviating device), can induce the strategy profile when each member of the complementary coalition obeys his recommendation.

Definition 2 Let $q \in \Delta(A)$ be a strategy profile and let $S \subseteq N$ be a coalition. We say that $p \in \Delta(A)$ is a *feasible ex-ante deviation* by a coalition S from q if there is a deviating device $d^S : A^S \to \Delta(A^S)$ such that for all $a \in A$ we have $p(a) = \sum_{b^S \in A^S} q(b^S, a^{-S}) \cdot d^S(a^S | b^S)$. In that case we say that p is induced from q by the deviating device d^S. Let $D(q, S) \subseteq \Delta(A)$ denote the *set of all feasible ax-ante deviations* by a coalition S from q.

An *ex-ante* strong correlated equilibrium is a strategy profile from which no coalition has a profitable *ex-ante* deviation: a feasible deviation that makes every member of the coalition better off.

Definition 3 (definition 1.2 of Moreno & Wooders) A strategy profile $q \in \Delta(A)$ is an *ex-ante strong correlated equilibrium* if no coalition $S \subseteq N$ has a feasible deviation $p \in D(q, S)$, such that for each $i \subseteq S$, we have $u^i(p) > u^i(q)$.

Remark 4 Similar to the existing notions of a strong correlated equilibria, we assume that the deviating players are myopic: they do not take into account the possibility that there may be further deviations.

Different notions of *ex-ante* strong correlated equilibria have been presented in Ray (1996) and Milgrom & Roberts (1996). In the framework of Ray, deviating coalitions are not allowed to construct new correlating devices, and are limited to use only an uncorrelated deviating device - a function $d^S : A^S \to \prod_{i \in S} \Delta(A^i)$. [5] In the framework of Milgrom & Roberts only some of the coalitions can communicate and coordinate deviations. In both cases the sets of feasible coalitional deviations is included in our set of feasible coalitional deviations, and thus our set of *ex-ante* strong correlated equilibria is included in the sets of *ex-ante* correlated equilibria à la Ray and à la Milgrom & Roberts.

3.3 An Ex-post Strong Correlated Equilibrium

In this subsection, we conceive a different framework in which deviations from an agreement are only planned at the *ex-post* stage. As in the previous framework, the players implement an agreement $q \in \Delta(A)$ with the assistance of a correlating device who recommends the action profile $a \in A$ with probability $q(a)$. However, in this framework no communication is done before the players receive their recommendations. Only after each player i is informed of his recommendation a^i, then the players communicate and may plan joint deviations. As before, if the members of a coalition S unanimously agree to use a deviation, then it is implemented with the assistance of a deviating device, who receives the recommendations from each player in S, and gives each player a new recommendation. After all agreed deviations (if any) were implemented, each player simultaneously chooses an action.

We assume that a decision of the members of S to implement a deviating device, is *common knowledge* among the members of S. This is the case, for example, in the following setup:

- All the members of S meet face-to-face (after each of them received his original recommendation).
- Any player can suggest to implement a deviating device.
- Each of the players says if he agrees to use it.
- At any stage of the conversation, a player can change his mind.

[5] In Ray's setup, the original correlating device can also send an indirect signal to each player (which may hold more information than the recommendation itself). In that case, the uncorrelated deviating device is a function from the set of S-part of the signals to the set of uncorrelated S-strategy profiles.

- If and only if at the end of the conversation, all the players (in S) agree to use the deviating device (and no one changes his mind), then the deviating device is implemented.

When the members of S consider to implement a deviating device, they are in a situation of incomplete information: each player has the private value of his recommendation, and may have additional private information acquired when communicating with the other deviating players. We describe the information structure during the communication among the deviating players in a model based on Aumann (1976, 1987).

Definition 5 Given a coalition $S \subseteq N$, an *information structure of S* is a 5-tuple: $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i \in S}, (a^i)_{i \in N})$ where:

(1) The 3-tuple (Ω, B, μ) is a probability space.
(2) The $(\mathcal{F}^i)_{i \in S}$ are partitions of Ω whose join ($\bigwedge_{i \in S} \mathcal{F}^i$, the coarsest common refinement of $(\mathcal{F}^i)_{i \in S}$) consists of non-null events.
(3) The $(a^i)_{i \in N}$ are random variables in (Ω, B, μ), where $a^i : \Omega \to A^i$.

We interpret $(\Omega, \mathcal{B})$ as the space of states of the world for the players of S (at some stage of their consideration whether to use a deviating device), μ as the common prior (for the states of the world) for all the players in S, and $\mathcal{F}^i$ as the information partition of player i; that is, if the true state of the world is $\omega \in \Omega$ then player i is informed of that element $F^i(\omega)$ of $\mathcal{F}^i$ that contains ω. We interpret the random variable $a^i(\omega)$ as the recommendation of player i (from the original agreement) in the state ω.

Remark 6 The state of the world $\omega \in \Omega$ includes a full description of the recommendations all the players received in the original agreement, the information that were shared among the members of S, when they communicated and considered the use of the deviating device, and the beliefs each of the deviating players has about the information and beliefs of the other players. We assume that all the players share a common prior about the states of the world. The justification of this assumption is discussed in Aumann (1987).

Given a non-null event $E \in \mathcal{B}$, let $a^S(E) \in \Delta(A^S)$ be a random variable with the posterior distribution of $a^S(\omega)$ conditioned on that $\omega \in E$.

The players may only have information structures that are consistent with the setup described above:

(1) The prior distribution of $a^S(\Omega)$ is equal to the agreement's distribution.
(2) Each player knows his recommendation.
(3) The deviating players have no information about the recommendations of the non-deviating players, except the information that is induced by

their information about their recommendations.

We formalize those requirements in the following definition.

Definition 7 Given an agreement $q \in \Delta(A)$ and a coalition $S \subseteq N$, we say that an information structure $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i\in S}, (a^i)_{i\in S})$ is a *consistent information structure* (of S) if the following conditions hold:

(1)

$$\forall b^S \in A^S,\ \Pr(a^S(\Omega) = b^S) = q(b^S)$$

(2)

$$\forall \omega \in \Omega,\, \forall i \in S,\, \exists b^i \in A^i \text{ s.t. } \Pr\left(a^i\left(F^i(\omega)\right) = b^i\right) = 1$$

(3)

$$\begin{aligned}&\forall \omega \in \Omega,\, \forall i \in S,\, \forall b \in A,\\ &\qquad \Pr\left(a^N\left(F^i(\omega)\right) = b\right) = \Pr\left(a^S\left(F^i(\omega)\right) = b^S\right) \cdot q(b^{-S} \mid b^S)\end{aligned}$$

When each player considers whether the implementation of a deviating device is profitable to him, he compares his conditional expected payoff (given his information about the distribution of $(a^i)_{i\in S}$) when playing the original agreement and when implementing the deviating device. A player agrees to implement a deviating device only if the latter conditional expectation is larger. We now formally define the conditional expected payoffs of each player in each state of the world ω, when following the agreement and when implementing a deviating scheme.

Definition 8 Given an agreement $q \in \Delta(A)$, a coalition $S \subseteq N$, a player $i \in S$, a deviating device $d^S : A^S \to \Delta(A^S)$, and a consistent information structure $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i\in S}, (a^i)_{i\in S})$, let the c*onditional expected payoffs of player i* in $\omega \in \Omega$ (given his information in ω and the assumption that the players in $-S$ follow the agreement q) be:

- The conditional expected payoff when the players in S follow q:

$$u^i_f(\omega) = \sum_{b^S \in A^S} \Pr\left(a^S\left(F^i(\omega)\right) = b^S\right) \sum_{b^{-S} \in A^{-S}} q(b^{-S} \mid b^S) \cdot u^i(b^S, b^{-S})$$

- The conditional expected payoff when the players of S deviate (by implementing d^S) :

$$u^i_d(\omega) = \sum_{b^S \in A^S} \Pr\left(a^S\left(F^i(\omega)\right) = b^S\right) \sum_{b^{-S} \in A^{-S}} q(b^{-S} \mid b^S) \sum_{c^S \in A^S} d^S(c^S \mid b^S) \cdot u^i(c^S, b^{-S})$$

Observe that u^i_f, u^i_d are measurable in the partition $\mathcal{F}^i$, i.e., $\omega, \omega' \in \Omega \Rightarrow u^i_f(\omega) = u^i_f(\omega')$, $u^i_d(\omega) = u^i_d(\omega')$. In our framework, if the players in S decide to implement a deviating device in some state $\omega \in \Omega$, then it is common knowledge (in ω) that each player expects to earn more from the deviation (conditioned on his information). We now present the formal definition of common knowledge (Aumann, 1976):

Definition 9 Given a coalition $S \subseteq N$, an information structure $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i\in S}, (a^i)_{i\in N})$ and a state $\omega \in \Omega$, an event $E \in \mathcal{B}$ is *common knowledge* at ω if E includes that member of the meet $\mathcal{F}^{meet} = \bigwedge_{i\in S} \mathcal{F}^i$ that contains ω.

We can now define a profitable *ex-post* deviating device, as a deviating device that in some consistent information structure, it is common knowledge in some state of the world, that each player expects to earn more if the deviating device is implemented.

Definition 10 Given a strategy profile $q \in \Delta(A)$ and a coalition $S \subseteq N$, we say that a deviating device $d^S : A^S \to \Delta(A^S)$ is a *profitable ex-post deviating device* (for S), if there exists a consistent information structure $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i\in S}, (a^i)_{i\in N})$ and a state $\omega_0 \in \Omega$ such that it is common knowledge in ω_0 that $\forall i \in S$, $u^i_d(\omega) > u^i_f(\omega)$.

We can now define an *ex-post* strong correlated equilibrium as a strategy profile, from which no coalition has a profitable *ex-post* deviating device.

Definition 11 A strategy profile $q \in \Delta(A)$ is an *ex-post strong correlated equilibrium* if no coalition $S \subseteq N$ has a profitable *ex-post* deviating device.

We now define the conditional expected payoffs of each player given the information that $\omega \in E$.

Definition 12 Given an agreement $q \in \Delta(A)$, a coalition $S \subseteq N$, a deviating device $d^S : A^S \to \Delta(A^S)$, a consistent information structure $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i\in S}, (a^i)_{i\in S})$, and a non-null event $E \in \mathcal{B}$, let $\left(\left(\tilde{u}^i_f(E)\right)_{i\in S}, \left(\tilde{u}^i_d(E)\right)_{i\in S}\right)$ denote the *conditional expected payoffs given the information that the state of the world is in E* (and given that the players in $-S$ follow the agreement q):

- The conditional expected payoff of each player i when the players in S follow the agreement q (given $\omega \in E$):

$$\tilde{u}^i_f(E) = \sum_{b^S\in A^S} \Pr\left(a^S(E) = b^S\right) \sum_{b^{-S}\in A^{-S}} q(b^{-S} \mid b^S) \cdot u^i(b^S, b^{-S})$$

- The conditional expected payoff when the players of S deviate by implementing d^S (given $\omega \in E$):

$$\tilde{u}^i_d(E) = \sum_{b^S\in A^S} \Pr\left(a^S(E) = b^S\right) \sum_{b^{-S}\in A^{-S}} q(b^{-S} \mid b^S) \sum_{c^S\in A^S} d^S(c^S \mid b^S)\cdot u^i(c^S, b^{-S})$$

Observe the difference between definition 8 and definition 12:

- In definition 8 $u^i_f(\omega)$ and $u^i_d(\omega)$ describe the conditional utility of player i (in the state of the world $\omega \in \Omega$) *in the perspective of player* i, who is informed in ω, that the state of the world in in $F^i(\omega)$.
- In definition 12 $\tilde{u}^i_f(E)$ and $\tilde{u}^i_d(E)$ describe the conditional utility of player i *in the perspective of an outside observer,* who is informed that the state of the world is in E.

Different notions of *ex-post* strong correlated equilibria are presented in Einy & Peleg (1995), Ray (1998) and Bloch & Dutta (2007). In the framework of Einy & Peleg, a deviating coalition can only use deviating devices that improve the conditional utilities of all deviating players for *all possible* recommendation profiles.[6] In the framework of Ray, a deviating coalition S can only use *pure* deviating devices - $d^S : A^S \rightarrow A^S$. In the framework of Bloch & Dutta, a deviating coalition S can only use deviating devices that are implemented if and only if the recommendation profile a^S is included in some set $E^S \subseteq A^S$ which satisfies:

(1) If $a^S \in E^S$, each player earns from implementing the deviating device.
(2) If $a^S \notin E^S$, then at least one player looses from implementing the deviation device (by falsely claiming that $a^S \in E^S$).

It can be shown that those conditions imply that there exists a consistent information structure of S and a state in which it is common knowledge that $a^S(\omega) \in E^S$ and that $\forall i \in S$, $u^i_d(\omega) > u^i_f(\omega)$. Thus our set of *ex-post* strong correlated equilibria is included in the sets of *ex-post* correlated equilibria as defined in any of those papers (we require resistance against a wider range of *ex-post* profitable deviating devices).

4 Main Result

In this Section we prove our main result:

Theorem 13 *Every ex-ante strong correlated equilibrium is also an ex-post strong correlated equilibrium.*

In other words: if a profitable *ex-post* deviating device from an agreement $q \in \triangle(A)$ exists, then there also exists a profitable *ex-ante* deviation from q.

The theorem immediately follows from the following two propositions:

[6] In our formulation, it is equivalent to requiring that $\forall i \in S$, $u^i_d(\omega) > u^i_f(\omega)$ in every $\omega \in \Omega$, and not only in every $\omega \in F^{meet}(\omega_0)$.

(1) Proposition 14: If an agreement q is not an *ex-post* strong correlated equilibrium, then there exists a "similar" agreement $\tilde{q}$ that is not an *ex-ante* strong correlated equilibrium. The "similarity" is in the sense that $\tilde{q}$ is absolute continues w.r.t. q when restricted to A^S, and equal to q when restricted to A^{-S} and conditioned on A^S (as formally defined below).
(2) Proposition 15: If such a "similar" agreement $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium, then q itself is not an *ex-ante* strong correlated equilibrium.

Proposition 14 Let $q \in \Delta(A)$ be a strategy profile (the agreement) that is not an *ex-post* strong correlated equilibrium. Then there exists a strategy profile $\tilde{q} \in \Delta(A)$ that satisfies the following conditions:

(1) $\tilde{q}|_S$ is absolute continues with respect to $q|_S$:

$$\forall b^S \in A^S,\ q(b^S) = 0 \Rightarrow \tilde{q}(b^S) = 0$$

(2) Conditioned on S-part of the recommendations: $\tilde{q}|_{-S} = q|_{-S}$:

$$\forall b^S \in A^S, \forall b^{-S} \in A^{-S},\ \tilde{q}(b^{-S} \mid b^S) = q(b^{-S} \mid b^S)$$

(3) $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium.

PROOF. Let $S \subseteq N$ be a coalition, let $d^S : A^S \to \Delta(A^S)$ be a deviating device, let $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i\in S}, (a^i)_{i\in N})$ be a consistent information structure, and let $\omega_0 \in \Omega$ be a state, such that it is common knowledge in ω_0 that $\forall i,\ u^i_d(\omega) > u^i_f(\omega)$, i.e., $F^{meet}(\omega_0) \subseteq \left\{\omega \mid u^i_d(\omega) > u^i_f(\omega)\right\}$. Let $i \in S$ be a deviating player. Write $F^{meet} = F^{meet}(\omega_0) = \bigcup\limits_j F^i_j$ where the F^i_j are disjoint members of $\mathcal{F}^i$. Since $u^i_d(\omega) > u^i_f(\omega)$ throughout F^{meet}, then $\forall j,\ \tilde{u}^i_d(F^i_j) > \tilde{u}^i_f(F^i_j)$. Observe that if $E_1, E_2 \in \mathcal{B}$ are two disjoint non-null events then: $\tilde{u}^i_f(E_1 \bigcup E_2) = (\mu(E_1)\cdot \tilde{u}^i_f(E_1)+\mu(E_2)\cdot\tilde{u}^i_f(E_2))/\mu(E_1+E_2)$ and $\tilde{u}^i_d(E_1 \bigcup E_2) = (\mu(E_1)\cdot\tilde{u}^i_d(E_1)+\mu(E_2)\cdot \tilde{u}^i_d(E_2))/\mu(E_1+E_2)$. Thus, it follows that $\tilde{u}^i_d(F^{meet}) > \tilde{u}^i_f(F^{meet})$. This is true for every player, thus $\forall_{i\in S}\ \tilde{u}^i_d(F^{meet}) > \tilde{u}^i_f(F^{meet})$.

Let $\tilde{q}$ be the following strategy profile: $\forall b^S \in A^S, \forall b^{-S} \in A^{-S},\ \tilde{q}(b) = \tilde{q}(b^S, b^{-S}) = \Pr\left(a^S(F^{meet}) = b^S\right) \cdot q(b^{-S} \mid b^S)$. We show that the three conditions are satisfied:

(1)

$$\forall b^S \in A^S,\ q(b^S) = 0 \Rightarrow \Pr\left(a^S(\Omega) = b^S\right) = 0$$
$$\Rightarrow \Pr\left(a^S(F^{meet}) = b^S\right) = 0 \Rightarrow \tilde{q}(b^S) = 0$$

(2)

$$\forall b^S \in A^S,\ \forall b^{-S} \in A^{-S},\ \tilde{q}(b^{-S} \mid b^S) = \frac{\tilde{q}(b^{-S}, b^S)}{\tilde{q}(b^S)}$$
$$= \frac{\Pr\left(a^S(F^{meet}) = b^S\right) \cdot q(b^{-S} \mid b^S)}{\Pr\left(a^S(F^{meet}) = b^S\right)} = q(b^{-S} \mid b^S)$$

(3) We have to show that $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium. Observe that:

$$\tilde{u}^i_f(F^{meet}) = \sum_{b^S \in A^S} \Pr\left(a^S\left(F^{meet}\right) = b^S\right) \sum_{b^{-S} \in A^{-S}} q(b^{-S} \mid b^S) \cdot u^i(b^S, b^{-S})$$
$$= \sum_{b \in A} \tilde{q}(b) \cdot u^i(b) = u^i(\tilde{q})$$

let $\tilde{p}$ be the *ex-ante* feasible deviation that is induced from $\tilde{q}$ by the deviating device d^S.

$$\tilde{u}^i_d(F^{meet}) = \sum_{b^S \in A^S} \Pr\left(a^S\left(F^{meet}\right) = b^S\right) \sum_{b^{-S} \in A^{-S}} q(b^{-S} \mid b^S)$$
$$\sum_{c^S \in A^S} d^S(c^S \mid b^S) \cdot u^i(c^S, b^{-S})$$
$$= \sum_{b \in A} \tilde{q}(b) \sum_{c^S \in A^S} d^S(c^S \mid b^S) \cdot u^i(c^S, b^{-S}) = u^i(\tilde{p})$$

This implies that: $\forall i \in S,\ \tilde{u}^i_d(F^{meet}) > \tilde{u}^i_f(F^{meet}) \Rightarrow u^i(\tilde{p}) > u^i(\tilde{q})$, thus $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium. **QED.**

We finish our main result by the following proposition: If a “similar” agreement $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium, then q itself is not an *ex-ante* strong correlated equilibrium.

Proposition 15 Let $q, \tilde{q} \in \Delta(A)$ be two strategy profiles that satisfy the following conditions:

(1) $\tilde{q}|_S$ is absolute continues with respect to $q|_S$ *:*

$$\forall b^S \in A^S,\ q(b^S) = 0 \Rightarrow \tilde{q}(b^S) = 0$$

.

(2) Conditioned on S-part of the recommendations: $\tilde{q}|_{-S} = q|_{-S}$:

$$\forall b^S \in A^S, \forall b^{-S} \in A^{-S}, \tilde{q}(b^{-S} \mid b^S) = q(b^{-S} \mid b^S)$$

(3) $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium.

Then q is not an *ex-ante* strong correlated equilibrium.

PROOF. For simplicity of notation, we assume w.l.o.g. that $\forall a^S \in A^S \, q(a^S) > 0$ (because $q(a^S) = 0 \Rightarrow \tilde{q}(a^S) = 0$, and those impossible action profiles do not affect any of the utilities functions and can be omitted). Let $\tilde{d}^S : A^S \to \Delta(A^S)$ be a deviating device, such that $\forall i$, $u^i(\tilde{p}) > u^i(\tilde{q})$, where $\tilde{p} \in \Delta(A)$ is the feasible deviation induced from $\tilde{d}^S$. Let $m = \max_{i \in S} |A^i|$ and let $\varepsilon = \frac{1}{m} \min_{a^S \in A^S, \tilde{q}(a^S) > 0} \frac{q(a^S)}{\tilde{q}(a^S)}$. We begin by constructing an auxiliary deviating device $\tilde{d}^S_\varepsilon : A^S \to \Delta(A^S)$:

$$\tilde{d}^S_\varepsilon(a^S|b^S) = \begin{cases} \varepsilon \tilde{d}^S(a^S|b^S) & a^S \neq b^S \\ 1 - \sum_{c^S \neq b^S} \varepsilon \tilde{d}^S(c^S|b^S) & a^S = b^S \end{cases}$$

In the deviating device $\tilde{d}^S_\varepsilon$ the players of S follow the agreement with probability $1 - \varepsilon$, and deviate according to $\tilde{d}^S$ with probability ε. Let $\tilde{p}_\varepsilon \in D(\tilde{q}, S)$ be the feasible *ex-ante* deviation of S from $\tilde{q}$ that is induced by $\tilde{d}^S_\varepsilon$. Observe that $\tilde{p}_\varepsilon$ is a profitable deviation for all the players in S: $\forall_{i \in S}$, $u^i(\tilde{q}) < u^i(\tilde{p}_\varepsilon)$ (because $u^i(\tilde{q}) - u^i(\tilde{p}_\varepsilon) = \varepsilon \left(u^i(\tilde{q}) - u^i(\tilde{p}) \right) < 0$).

We continue by constructing the following deviating device (of S) $d^S : A^S \to \Delta(A^S)$:

$$d^S(a^S|b^S) = \begin{cases} \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(a^S|b^S) & a^S \neq b^S \\ 1 - \sum_{c^S \neq b^S} \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(c^S|b^S) & a^S = b^S \end{cases}.$$

We first show that d^S is a valid deviating device by validating that $\forall a^S, b^S \in A^S \; 0 \leq d^S(a^S|b^S) \leq 1$.

$$\begin{aligned}\forall a^S \neq b^S,\ d^S(a^S|b^S) &= \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(a^S|b^S) = \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \varepsilon \tilde{d}^S(a^S|b^S) \\ &= \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \frac{1}{m}\left(\min_{a^S\in A^S,\, \tilde{q}(a^S)>0} \frac{q(a^S)}{\tilde{q}(a^S)}\right) \cdot \tilde{d}^S(a^S|b^S) \\ &\leq \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \frac{1}{m} \cdot \frac{q(b^S)}{\tilde{q}(b^S)} \cdot \tilde{d}^S(a^S|b^S) = \frac{1}{m} \cdot \tilde{d}^S(a^S|b^S) \leq 1\end{aligned}$$

And using the inequality (which is a part of the last chain of inequalities):

$$\frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(a^S|b^S) \leq \frac{1}{m} \cdot \tilde{d}^S(a^S|b^S)$$

We get:

$$d^S(a^S|a^S) = 1 - \sum_{a^S\neq b^S} \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(a^S|b^S) \geq 1 - \frac{1}{m}\sum_{a^S\neq b^S} \tilde{d}^S(a^S|b^S) \geq 1 - 1 \geq 0.$$

Let $p \in D(q,S)$ be the feasible *ex-ante* deviation that is induced by d^S. We finish the proof by showing that p is a profitable deviation from q: i.e., $\forall i \in S,\ u^i(q) < u^i(p))$, and thus q is not an *ex-ante* strong correlated equilibrium. Let $i \in S$. We show: $u^i(p) - u^i(q) \overset{?}{=} u^i(\tilde{p}_\varepsilon) - u^i(\tilde{q}) > 0$. Observe that:

$$u^i(q) = \sum_{a\in A} q(a) \cdot u^i(a) = \sum_{a^S\in A^S} q(a^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|a^S) \cdot u^i(a)$$

$$u^i(p) = \sum_{a\in A} p(a) \cdot u^i(a) = \sum_{a^S\in A^S}\sum_{b^S\in A^S} q(b^S) \cdot d^S(a^S|b^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|b^S) \cdot u^i(a)$$

Therefore:

$$\begin{aligned}u^i(p) - u^i(q) &= \sum_{a^S\in A^S}\sum_{b^S\in A^S} q(b^S) \cdot d^S(a^S|b^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|b^S) \cdot u^i(a) \\ &\quad - \sum_{a^S\in A^S} q(a^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|a^S) \cdot u^i(a) \\ &= \sum_{a^S\in A^S}\sum_{b^S\neq a^S} q(b^S) \cdot d^S(a^S|b^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|b^S) \cdot u^i(a) \\ &\quad - \sum_{a^S\in A^S} q(a^S) \cdot \Big(1 - d^S(a^S|a^S)\Big) \sum_{a^{-S}\in A^{-S}} q(a^{-s}|a^S) \cdot u^i(a) \\ &= \sum_{a^S\in A^S}\sum_{b^S\neq a^S} \tilde{q}(b^S) \cdot \tilde{d}^S_\varepsilon(a^S|b^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|b^S) \cdot u^i(a) \\ &\quad - \sum_{a^S\in A^S} \tilde{q}(a^S) \cdot \Big(1 - \tilde{d}^S_\varepsilon(a^S|a^S)\Big) \sum_{a^{-S}\in A^{-S}} q(a^{-s}|a^S) \cdot u^i(a)\end{aligned}$$

Last equality is due to the following two equalities:

$$\forall a^S \neq b^S,\; q(b^S) \cdot d^S(a^S|b^S) = q(b^S)\frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(a^S|b^S) = \tilde{q}(b^S) \cdot \tilde{d}^S_\varepsilon(a^S|b^S)$$

$$\begin{aligned} q(a^S) \cdot \Big(1 - d^S(a^S|a^S)\Big) &= q(a^S) \sum_{c^S \neq a^S} \frac{\tilde{q}(a^S)}{q(a^S)} \cdot \tilde{d}^S_\varepsilon(c^S|a^S) \\ &= \tilde{q}(a^S) \sum_{c^S \neq a^S} \frac{q(a^S)}{\tilde{q}(a^S)} \cdot \frac{\tilde{q}(a^S)}{q(a^S)} \cdot \tilde{d}^S_\varepsilon(c^S|a^S) \\ &= \tilde{q}(a^S) \sum_{c^S \neq a^S} \tilde{d}^S_\varepsilon(c^S|a^S) \\ &= \tilde{q}(a^S) \cdot \Big(1 - \tilde{d}^S_\varepsilon(a^S|a^S)\Big) \end{aligned}$$

We finish the proof by showing that the last expression is equal to $u^i(\tilde{p}_\varepsilon) - u^i(\tilde{q})$. Observe that:

$$\begin{aligned} u^i(\tilde{p}_\varepsilon) &= \sum_{a \in A} \tilde{p}_\varepsilon(a) \cdot u^i(a) \\ &= \sum_{a^S \in A^S} \sum_{b^S \in A^S} \tilde{q}(b^S) \cdot \tilde{d}^S_\varepsilon(a^S|b^S) \sum_{a^{-S} \in A^{-S}} q(a^{-S}|b^S) \cdot u^i(a) \end{aligned}$$

$$u^i(\tilde{q}) = \sum_{a \in A} \tilde{q}(a) \cdot u^i(a) = \sum_{a^S \in A^S} \tilde{q}(a^S) \sum_{a^{-S} \in A^{-S}} q(a^{-S}|a^S) \cdot u^i(a)$$

Therefore:

$$\begin{aligned} u^i(p_\varepsilon) - u^i(\tilde{q}) &= \sum_{a^S \in A^S} \sum_{b^S \in A^S} \tilde{q}(b^S) \cdot \tilde{d}^S_\varepsilon(a^S|b^S) \sum_{a^{-S} \in A^{-S}} q(a^{-S}|b^S) \cdot u^i(a) \\ &\quad - \sum_{a^S \in A^S} \tilde{q}(a^S) \sum_{a^{-S} \in A^{-S}} q(a^{-S}|a^S) u^i(a) \\ &= \sum_{a^S \in A^S} \sum_{b^S \neq a^S} \tilde{q}(b^S) \cdot \tilde{d}^S_\varepsilon(a^S|b^S) \sum_{a^{-S} \in A^{-S}} q(a^{-S}|b^S) \cdot u^i(a) \\ &\quad - \sum_{a^S \in A^S} \tilde{q}(a^S) \Big(1 - \tilde{d}^S_\varepsilon(a^S|a^S)\Big) \sum_{a^{-S} \in A^{-S}} q(a^{-s}|a^S) u^i(a) \end{aligned}$$

QED.

5 An Ex-ante Coalition-Proof Correlated Equilibria

In the last Section we show that strong correlated equilibrium *à* la Moreno & Wooders is also appropriate to frameworks in which players can communicate both at *ex-ante* and at *ex-post* stages. A natural question is whether a similar result holds for their notion of coalition-proof correlated equilibrium. In this Section we show that the answer is negative, by presenting an example in which there is an *ex-ante* coalition-proof correlated equilibrium that is not a self enforcing agreement in a framework in which communication is possible at both stages.

5.1 A Definition of an Ex-ante Coalition-Proof Equilibrium

In this Sub-section we formally define an *ex-ante* coalition-proof correlated equilibrium. We begin by defining a self-enforcing *ex-ante* deviation as a feasible *ex-ante* deviation, such that no proper sub-coalition has a further self-enforcing and improving deviation:

Definition 16 Let $q \in \Delta(A)$ be a strategy profile and let $S \subseteq N$ be a coalition. The set of self enforcing deviations by the coalition S from q, $SED(q, S)$ is defined, recursively, as follows:

(1) If $|S| = 1$, then $SED(q, S) = D(q, S)$.
(2) If $|S| > 1$, then $SED(q, S) =$

$$\Big\{p \in D(q, S) \mid \forall R \subset S, \forall r \in SED(p, R), \exists i \in R, :\ u^i(r) \leq u^i(p)\Big\}$$

We now define an *ex-ante* coalition-proof correlated equilibrium as a strategy profile from which no coalition has a self-enforcing and improving deviation:

Definition 17 (definition 1.4 of Moreno & Wooders) A strategy profile $q \in \Delta(A)$ is a *coalition-proof correlated equilibrium* if no coalition $S \subseteq N$ has a self-enforcing deviation $p \in SED(q, S)$, such that for each $i \subseteq S$, we have $u^i(p) > u^i(q)$.

5.2 An Example of an Ex-ante Coalition-Proof Correlated Equilbirum

In the following example we present a two-player game and an *ex-ante* coalition-proof correlated equilibrium, and explain why it is not a self-enforcing agreement in a framework in which the players can communicate at both stage (*ex-ante* and *ex-post*). The example is adapted from Bloch & Dutta (2007). Table 4 presents the Matrix representation of the game and an *ex-ante* coalition-proof correlated equilibrium in this game.

Table 4
A Two-Player Game and an *Ex-ante* Coalition-Proof Correlated Equilibrium

	b_1	b_2	b_3
a_1	6,6	-2,0	0,7
a_2	2,2	2,2	0,0
a_3	0,0	0,0	3,3

	b_1	b_2	b_3
a_1	1/2	0	0
a_2	1/4	1/4	0
a_3	0	0	0

We first show that the profile presented in table 4 is an *ex-ante* coalition-proof equilibrium. First, observe that the profile is a correlated equilibrium: no player has a profitable unilateral deviation. Moreno & Wooders (1996) have proved that in a two-player game, every correlated profile which is not Pareto-dominated by another correlated equilibrium is a coalition-proof correlated equilibrium. Our profile gives each player a payoff of 4. Thus we prove that the profile is an *ex-ante* coalition-proof correlated equilibrium, by showing that any correlated equilibrium q gives player 1 a payoff of at most 4. Let $x = q(a_1, b_1)$. Observe that $q(a_2, b_1) \geq x/2$ because otherwise player 1 would have a profitable deviation: playing b_3 when recommended b_1. This implies $q(a_2, b_2) \geq x/2$, because otherwise player 2 would have a profitable deviation (playing a_1 when recommended a_2). Thus the payoff of q conditioned on that the recommendation profile is in $A = ((a_1, b_1), (a_2, b_1), (a_2, b_2))$ is at most 4, and because the payoff of q conditioned on that the recommendation profile is not in A is at most 3, then the total payoff of q is at most 4.

We now explain why this profile is not a self-enforcing agreement in a framework in which the players can communicate and plan deviations also at the *ex-post* stage.[7] Assume that the players have agreed to play the profile, and player 1 has received a recommendation a_2. In that case, he can communicate with player 1 at the *ex-post* stage, tell him that he received a_2 (and thus if the players follow the recommendation profile they would get a payoff of 2), and suggest a joint deviation - playing (a_3, b_3). As player 1 has no incentive to lie (to make a false claim that his recommendation is a_2 when it is a_1), then player 2 would believe player 1, and they would both play (a_3, b_3). This *ex-post* deviation is self-enforcing: (a_3, b_3) is a Nash equilibrium, and thus no player has a profitable sub-deviation.

[7] The profile is not an *ex-post* coalition-proof correlated equilibrium according to the definitions of Bloch & Dutta (2007) and Ray (1998), and according to our informal notion (which have not formally defined) of an *ex-post* coalition-proof correlated equilibrium in the framework of consistent information structures. The profile is an *ex-post* coalition-proof correlated equilibrium according to the definition of Einy & Peleg, due to their requirement that an *ex-post* deviation would be strictly profitable to each player given all recommendations he may receive.

Observe that the same deviation is not self–enforcing in the *ex-ante* stage. If the players agree at the *ex-ante* stage to implement a deviating device that changes (a_2, b_1) into (a_3, b_3), then player 2 will have a profitable sub-deviation - playing b_3 when recommended b_1. Similarly, if they agree to implement a deviating device that changes (a_2, b_2) into (a_3, b_3), then player 1 will have a profitable sub-deviation - playing a_1 when recommended a_2.

6 Concluding Remarks

(1) *Bayesian games:* Moreno & Wooders (1996) present a notion of *ex-ante* strong communication equilibrium in Bayesian games. Our result can be extended to this framework as well, to show that an *ex-ante* [8] strong communication equilibrium is also resistant to deviations at the *ex-post* stage.

(2) *Intermediate stage*: In some frameworks, the process in which players receive their recommendations from the correlating device may be long. Thus, coalitions may also plan deviations during this process, when some of them know their recommendations, and other do not. [9] One can alter our definition of consistent information structure (definition 7), by omitting the second requirement (that each player knows his recommendation). This requirement is not used in the proof of our main theorem, and thus an *ex-ante* strong correlated equilibrium is also resistant to deviations in such an intermediate stage.

(3) *k-strong equilibria*: In Heller (2008) an *ex-ante* notion of k-strong correlated equilibrium is defined as a strategy profile that is resistant to all coalitional deviations of up to k players. Our result can be directly extended to this notion as well: an *ex-ante* k-strong correlated equilibrium is resistant to deviations of up to k players at all stages.

(4) *Related Literature*:
   (a) The question of existence of strong and coalition-proof correlated equilibria is discussed in Moreno & Wooders (1996), Milgrom & Roberts (1996), Ray (1996), Holzman & Law-Yone (1996), Bloch & Dutta (2007).
   (b) Applications of strong and coalition-proof equilibria are presented and discussed in Bernheim & Whinston (1986, 1987), Einy & Peleg (1996) and Delgado & Moreno (2004).

[8] The stage is *ex-ante* with respect to the recommendations of the correlated agreement. However, all possible deviations are planned only after each player knows his type (i.e., it is *interim* stage with respect to the types of the players).

[9] An example for such a framework is the implementation of a strong correlated equilibrium by a polite protocol in cheap-talk (Heller, 2008). A polite protocol is a strategy profile in the cheap talk extended game, in which in every talk phase at most one player sends a message (players do not send messages simultaneously).

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