Patent, Inequality and Innovation-Driven Growth

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Abstract

When people have hierarchic preferences inequality affects innovation-driven growth through the implied demand distribution over new goods. The paper examines the demand path of the firm through its life-cycle and analyzes the efficiency of dynamic resource allocation under different inequality scenarios. Unlike previous models of inequality and demand induced innovation, the innovators are protected by patents of finite length. Longer patents increase the profitability of an innovation because they reduce the effect of inequality by increasing the likelihood that the firms benefit from a future demand jump in sales to the poor. This result does not hold, however, when initial inequality is low or the purchasing power of the poor is high. Moreover, reducing inequality does not increase growth as long as the amount of redistribution is below a threshold level.

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1 Introduction

What is the effect of income distribution on incentives to innovate when innovators can not protect their monopoly position forever? In a static world, inequality affects market size for innovators because richer consumers purchase more of the new goods than poor consumers. In a dynamic setup, changes in inequality bring about demand increase and decreases for a particular item which affects its innovator's expected profit flow and its decision whether to invest in research and development. For instance, a decrease in inequality might push the profit flow earlier in the life cycle, thereby increasing the net present value of ongoing innovations. On the other hand, a decrease in inequality might lead to a lower return on innovation if a redistribution causes consumers to spend less on newer goods. While the former is a desirable outcome from a policy perspective, the innovator might still opt to stay out if limits on patent protection prevent gains from a reduction in inequality. Finally, the duration of patents might be irrelevant to the choice of entry if the purchasing power is high enough such that all new goods are already consumed by everyone.

This article analyzes the effect of inequality on innovation-driven growth when innovations are protected by patents of finite length. The effect of income or wealth inequality on incentives to innovate has been analyzed by a small group of researchers who have generally taken the monopoly duration to be infinite. Foellmi and Zweimuller (2006) and Zweimuller (2000) study the effect of inequality on innovation-based growth when people have non-homothetic preferences. In their setup, once the innovators enter the market, they maintain monopoly positions forever by virtue of infinite patents. The length of the patents in this paper, along with inequality and the growth rate, is one of the determinants of the regime in which the economy operates. Patents and the duration of the monopolistic state determine, for instance, who can afford an efficiently produced or a luxury product ex-ante.

This paper builds on previous work by relaxing the assumption that innovators are protected by patents of infinite length. In most of the quality-ladder models, this assumption is made for the purpose of tractability. However, within the set of models where inequality affects innovation-driven growth through the composition of demand, limited duration of monopolistic power has substantially different implications for the inequality-growth relationship. The market size for an innovation and its profitability might be affected by a change in inequality or a change in both the level and growth rate of income. However, when patents are finite the profitability of an innovation and incentives for a firm to innovate might be immune to fluctuations in inequality or income. In other words, when faced with increased or decreased demand, innovators might find themselves already in perfect competition if their patents expire early enough. Such a situation might arise either when patent duration is too short, inequality is too high or incomes are too low at the time of the innovator's entry into the market. In this setup there are several aspects of the relationship between inequality and incentives to innovate that are worth mentioning. First, it is not only the shape of the wealth distribution that is important to growth, but also the length of time that the innovator expects to survive as a monopolist, which affects its decision to enter the market. Second, it is not only the level of inequality that matters for the evolution of markets but also how fast the inequality changes . And finally, the growth rate and the duration of monopoly status jointly determine the nature of the relationship between inequality and innovation-driven growth.

Given a low initial level of inequality, an increase in the average income or a decrease in inequality might cause inefficiently produced luxuries to be consumed in equilibrium. Resources are thus tied up which would otherwise be used in more efficient production. This is an equilibrium with lower growth, which is most likely to occur if the average income is high. In such a situation, reducing inequality beyond a certain threshold reverses the inequality growth relationship. On the other hand, if the average income is too low, reducing inequality fosters growth as long as the innovator's product is already consumed by the rich. If the quality item is too expensive for both rich and the poor, then increasing inequality might foster growth by bringing the profit flow earlier so that the innovator can enjoy profits before his patent expires. In this setup, the position of the most recent innovator's entry in the hierarchy of needs and the duration of its status as a monopoly are two important determinants of the relationship between inequality and innovation-growth.

This article is also linked to the strand of literature that studies the effect of patent length on innovation rate. Judd (1985) shows in a general equilibrium framework that infinitely lived patents do not necessarily produce a socially optimal level of innovation. Horowitz and Lai(1996) find that longer patents increase the size, but decrease the frequency of innovations. This dichotomy gives rise to a situation in which social welfare and the rate of innovations can not be simultaneously maximized by optimally choosing a patent length. Laussel and Nyssena (1999) show that assuming a finite life time for patents results in multiple equilibria, a result also confirmed by our analysis. This article differs from the above papers in that it focuses on the effect of wealth inequality on the rate of innovations and analyzes the role of patent length as it pertains to the relationship between inequality and growth.

Demand side effects on innovation have been studied by Murphy, Shleifer and Vishny (1989) who take the menu of available goods to be fixed. As in Matsuyama (2002), our model ranks different goods according to priority and new goods are initially afforded only by the rich and finally become necessities. Greenwood and Mukoyama (2001) analyze how income distribution affects innovation incentives in a partial equilibrium context. Li (1996) has a premise which is similar to ours but assumes that consumers differ in their labour endowments which are uniformly distributed across households. The assumption of a uniform distribution has the disadvantage that only one dimension of inequality - the range of the distribution - can be studied. A similar paper by Chou and Talmain (1996) considers again a single type demand pattern in a Schumpeterian setup where all consumers, rich and poor, consume all products.

In the next section we introduce the model. In the third section, we characterize the equilibrium in this economy and we present the main arguments of the paper, and in section four we provide further arguments and some applications. Section five concludes.

2 The Model

2.1 Technology

The technology is similar to previous models of quality-ladders such as Romer (1987,1990), Grossman and Helpman's (1991) and Aghion and Howitt (1992) with slight modifications[1]. There are two types of technology, old and new. New technology is available only after research and development. Old technology, on the other hand, brings constant returns and access to it is free. New technology is either an innovation which is a new product replacing an existing product or a new method of production for an existing product. The innovator firms are granted patents which allow them to operate monopolistically for some constant amount of time η. In this economy the only factor of production is labour. The old firms require $a_{old}(t)$ units of labour per output. To make an innovation $a_{rd}(t)$ units of labour are needed. After innovation the firm can produce with

[1] Romer's (1987,1990) and Grossman and Helpman's (1991) models assume innovations take place in the intermediate goods sector and/or innovators introduce new but additional products. In Aghion and Howitt (1992) new products replace old ones but exist only until the next innovation.

$a_{new}(t)$ units of labour per output,where $a_{new}(t) < a_{old}(t)$. The growth is driven through innovations and $a_{rd}(t)$ is decreasing in the amount of past innovations due to learning experience. We will also assume that the labour input requirements in both final goods sectors are decreasing in the amount of past innovations a la Young (1993). Let $n(t)$ denote the number of innovations up to time t. The input coefficients at any time is then given by

$$a_i(t) = \frac{a_i}{n(t)} \text{ for } i = old, new, rd \tag{1}$$

This specification implies that any increase in the variable $n(t)$ is a proxy for the increase in the aggregate knowledge stock which spills over uniformly across all sectors[2].

2.2 Firms

The old firms operate in the competitive sector because of constant returns. At any time, the price of an old firm's product is given by $p_{old}(t) = w(t)a_{old}(t)$. where $w(t)$ is the wage rate determined in a perfectly competitive labour market and is the same for all sectors. For simplicity, we will set the price of competitive products to one for the rest of the article. We will assume that at steady state the economy grows at an endogenous rate of g. In other words, at steady state $n(t)$ is growing at a rate of g and all labour input coefficients are decreasing at rate g. Since the productivity growth is uniform across activities of research and production, one can easily show that the wages increase at the same rate and therefore that the marginal production costs for all sectors remain constant over time.

The new firms are granted patents that prevent entry by other firms as long as the patented firm charges the competitive price. The patent allows the firms to operate as monopolists for a certain amount of time. No monopolistic firm has an incentive to charge a price higher than one which will trigger entry from the competitive fringe. There is always a competitor out there who can use the older production technology therefore there is a limit to the price that the innovator can charge. For simplicity, it is assumed that the monopolistic firm does not charge a price lower than unity which translates to the monopolist not necessarily choosing the profit maximizing price. The monopolist earns a unit profit of $\pi = 1 - w(t)a_{new}(t)$. Using (6) and the wage condition $\pi = 1 - \frac{a_{old}}{a_{new}} > 0$ which is independent over time. The labour market is competitive

[2]Note that this differs from Grossman and Helpman (92) where the productivity grows only in research.

and the wage rate is given by $w(t)$ and is the same for all sectors.

2.3 Consumers

Let $c_i(t)$ represent the most luxurious good that consumer i can afford. The lifetime utility of the consumer i can then be written as:

$$U_i = \int_0^\infty u_i(t)e^{-\rho t}dt = \int_0^\infty \left[1 + \ln(c_i(t))\right] e^{-\rho t}dt \tag{2}$$

where ρ represents the discount rate[3]. Consumers own the same wage rate $w(t)$ but differ in their asset holdings $W_i(t)$. We further assume there is a perfect capital market with interest rate r which is constant at the steady state. The steady state growth rate is g at which all variables grow including consumption. The consumer's problem can then be written as:

$$\max_{\{c_i\}} U_i \text{ s.t. } \int_0^\infty c_i(t)e^{-rt}dt \leq W_i(0) + \int_0^\infty w(0)(t)e^{-(r-g)t}dt$$

The optimal consumption path is governed by the following relations:

$$g = r - \rho \tag{3}$$

and

$$c_i(0) = w(0) + \rho W_i(0) \tag{4}$$

We assume the consumers have a hierarchy of wants $j \in [0, \infty)$. A low value of j is associated with basic needs and a high value of j is associated with luxury needs. Let $h_p(t)$ $(h_r(t))$ denote the highest ranked

[3] See Zweimuller (2000) for the derivation of this utility function.

want the poor (rich) can afford to satisfy. Then any good $j \,\epsilon \,[0, h_p(t)]$ is bought both by the rich and the poor whereas any good $j\epsilon\,(h_p(t), h_m(t)] \cup (h_m(t), h_r(t)]$is bought only by the rich where $h_m(t)$ is the highest ranked want satisfied by a monopolist firms product. The number of innovations up to date t ,$n(t)$, is equal to $h_m(t)$. Moreover, let the range $[0, h_c(t)]$ denote the firms whose patents have expired and who now operate competitively. The range where the monopolistic firms still operate is $(h_c(t), n(t)]$ and the competitive firms who serve the "fancies of the rich" operate in the range $(n(t), h_r(t)]$.

The ranking of the product ranges above relies on the assumption that any want can be satisfied by the products of old firms. The traditional technology is freely available and once there is demand the old (inefficient) producers can enter without any costs. The new producers on the other hand aim at markets which have the highest growth potential to justify the R&D costs. This implies that the "fancies of the rich" market is served by the competitive producers whereas the needs in the middle of the hierarchy is served partly by the monopolistic producers. The basic goods market is again served by competitive producers with expired patents. This situation is depicted unidimensionally in Figure 1. Everything moves at a constant rate towards right and the entry occurs at time t at point $n(t)$ and at time $t+$ 1 at $n(t+1)$. The patent length for a firm entering at date t is given by the distance between $h_c(t)$ and $n(t)$. The same firm has has $n(t) - h_c(t+1)$ time remaining at $t+1$ until its patent expires. At any time, there are three types of demand structure that will determine the size of these markets.

Case I: $h_p < h_c < n$ Only the rich can afford monopolistic a firm's products; in other words the firms patent expires before the poor can afford its product

Case II: $h_c < h_p < n$ At least some part of the monopolistic sector products can be afforded by the poor.

Case III : $h_c < n < h_p$ Both rich and the poor can afford all of the monopolistic firm's products.

Note that this specification also represents initial inequality levels in affordability or consumption.

FIGURE 1

2.4 Income Distribution and Demand Structure:

In this section we closely follow Zweimuller and Foellmi (2006) who consider a simple distribution of wealth in two groups: rich (r) and poor (p). Both rich and poor earn the same wage but own different wealth levels. Let γ_i be the ratio of consumer i's wealth to the average wealth, $\gamma_i = \frac{W_i}{W}$, $i = p, r$. If the population share of the poor is ϕ then the fraction of aggregate wealth held by the poor is $\phi\gamma_p$ whereas the rich hold a fraction of $(1-\phi)\gamma_r$. Since $\phi\gamma_p + (1-\phi)\gamma_r = 1$, one can write

$$\gamma_{r=} \frac{1-\phi\gamma_p}{1-\phi} \tag{5}$$

This relation represents the Lorentz curve. Note that an increase in γ_p and a decrease in ϕ (holding γ_p constant) leads to less inequality. In this economy, aggregate wealth refers to holdings of a firm's shares. Let $v_k(t)$ be the present value of firm k's profit. The aggregate value of the wealth at any time t is then given by $V(t) = \int_0^{n(t)} v_{k(t)}dk$. The value of asset holdings of each consumer is given by

$$W_i(t) = \frac{\gamma_i V(t)}{L} \quad i = p, r \tag{6}$$

where L is population.

2.5 The Resource Constraint

Note that the range $(h_c(t), n(t)]$will be determined by the growth rate, g, and the patent length, η, in the following manner; $n(t+\eta) = n(t)e^{-g\eta} = h_c(t)$. It follows that $\eta = -(1/g)\ln(h_c(t)/n(t))$. Note that the longer the patent length the smaller the ratio $h_c(t)/n(t)$. Moreover, in the steady state this ratio is determined solely by the patent length since the growth rate becomes a constant.

Let $Y_i(i = new, old)$ denote the total production in each sector. Since labour is the only factor of production, the employment in the R&D, new and old sectors are$L_{R\&D} = \dot{n}(t)a_{rd}(t)$, $L_{new} = Y_{new}(t)a_{new}(t)$ and $L_{old=}Y_{old}(t)a_{old}(t)$, respectively. The total employment is then

$$L = \dot{n}(t)a_{rd}(t) + Y_{new}(t)a_{new}(t) + Y_{old}(t)a_{old}(t) \tag{7}$$

In a steady state situation h_p, h_r and n grow at the same rate. Using the definitions $d_p = \frac{h_p(t)}{n}$, $d_r = \frac{h_r(t)}{n}$, $d_c = \frac{h_c(t)}{n}$, $g = \frac{\dot{n}}{n}$ and equation (7), we can write the resource constraint in each type of demand structure above respectively as[4]:

$$Case\ I: L = a_{rd}g + a_{new}\left[(1-\phi)(1-d_c)\right]L + a_{old}\left[(1-\phi)(d_r-1)L + d_p\right] \tag{8}$$

$$Case\ II: L = a_{rd}g + a_{new}\left[\phi d_p + (1-\phi)(1-d_c)\right]L + a_{old}\left[(1-\phi)(d_r-1)L + d_c\right]$$

$$Case\ III: L = a_{rd}g + a_{new}(1-d_c)L + a_{old}\left[(1-\phi)(d_r-1)L + \phi L(d_p-1) + d_c\right]$$

2.6 The Value of A Monopolistic Firm

At any time, the market size for each firm will depend on the number of people who can afford its good. Incomes grow over time and some firms anticipate a larger market because more people will be able to afford their good within the near future. Therefore the market size and expected profits for each firm will be determined by the overall growth rate of the economy, the patent length and the level of inequality. The entry decision is then made only if the expected profits are large enough to justify the R&D costs. With ongoing innovations and income growth a monopolist j decides whether or not to enter the market. Let η denote the length of the patent the monopolist is granted in case it enters the market. Let μ denote the length of time it takes until the poor can afford to buy good j which is consumed only by the rich at the time of decision. The value of the most recent innovator j at time t can then be written as

[4] Note that for each case the output can be written as follows:
Case 1: $Y_{new}(t) = (1-\phi)L(n(t)-c_c(t))$, $Y_{old}(t) = (1-\phi)L(c_r(t)-n(t))+c_p(t)$.Case 2: $Y_{new}(t) = \phi Lc_p(t)+(1-\phi)L(n(t)-c_c(t))$, $Y_{old}(t) = (1-\phi)L(c_r(t)-n(t))+c_c(t)$.Case 3: $Y_{new}(t) = L(n(t)-c_c(t))$, $Y_{old}(t) = (1-\phi)L(c_r(t)-n(t))+\phi L(c_p(t)-n(t))+c_c(t)$

$$v_j(t) = \pi \int_t^{t+\eta} D_j(\tau) e^{-r(\tau - t)} d\tau = \tag{9}$$

$$= \begin{cases} \pi \int_t^{t+\eta} (1-\phi) L e^{-r(\tau-t)} d\tau & \text{for } \mu > \eta \\ \pi \int_t^{t+\mu} (1-\phi) L e^{-r(\tau-t)} d\tau + \pi e^{-r\mu} \int_{t+\mu}^{t+\eta} L e^{-r(\tau-(t+\mu))} d\tau & \text{for } 0 < \mu \leq \eta \\ \pi \int_t^{t+\eta} L e^{-r(\tau-t)} d\tau & \text{for } \mu \leq 0 \end{cases}$$

where $D_j(\tau)$ is demand for firm j at time τ and L stands for population. Three possible values refer to the cases. 1) The monopolist serves only the rich until the patent expires. 2) The monopolist serves first the rich and then the whole population until the patent expires. 3) The monopolist serves the whole population until the patent expires. Note that as long as the patents do not expire before the poor can afford the firm's product there will be a jump in the expected demand for the product at date $t+$ from $(1-\phi)L$ to L. Solving the above integral yields the value of the most recent innovator j.

$$v_j = \begin{cases} \frac{\pi L}{r}(1-\phi)(1-e^{-r\eta}) & \text{for } \mu > \eta, \text{ or} \\ \frac{\pi L}{r}\left[(1-\phi) + \phi e^{-r\mu} - e^{-r\eta}\right] & \text{for } 0 < \mu \leq \eta \\ \frac{\pi L}{r}(1-e^{-r\eta}) \text{ for } \mu \leq 0 \end{cases} \tag{10}$$

Given $\mu \leq \eta$, it is clear that a shorter μ implies less discounting and a higher value for the firm. The value of the firm when the poor are able to afford the product becomes$\frac{\pi L}{r}\left[1 - e^{-r/\eta - \mu)}\right]$. This value will then drop until the patent expires. The amount of time, μ, that will pass until the jump in demand can be found by making use of the following relation; $h_p(t+\mu) = h_p(t)e^{g\mu} = j$.

$$\mu = -(1/g)\ln(h_p(t)/j) > 0 \tag{11}$$

A higher growth rate or a smaller distance between the most advanced good that the poor can afford and the monopolist firm's product j implies a shorter time until the demand jump. Substituting $d_p(j) = \frac{h_p(t)}{j}$ and equation (0) in (9) gives:

$$v_j(t) = \begin{cases} \frac{\pi L}{r}(1-\phi)(1-e^{-r\eta}) \quad \text{if } \mu > \eta \quad \text{, or} \\ \frac{\pi L}{r}\left[(1-\phi) + \phi d_p^{\frac{r}{g}}(j) - e^{-r\eta}\right] \text{if } 0 < \mu \ \leq \eta \\ \frac{\pi L}{r}(1-e^{-r\eta}) \text{ if } \mu \ \leq 0 \end{cases} \tag{12}$$

Proposition 1. *A higher relative consumption position of the poor*, d_p, *increases the profitability of an innovation if and only if* $0 < \mu \leq \eta$. *If the inequality is low enough such that* $h_p(t) > j, (i.e. \mu < 0)$ *a further decrease in inequality does not affect* the value of the monopolistic firm j

Fixing the patent length implies that for a sufficiently high μ the firms will never realize the increase in profit flow obtained by selling to the poor[5]. As long as $h_p(t) < j$ a shorter μ implies a higher growth rate and vice versa. Regardless of the inequality level a higher growth rate has two effects: 1) A higher interest rate. Future profits have to be discounted at a higher rate which reduces the firm's value. 2) A shorter time period until the poor can afford the firm's product. The sales to the poor are realized faster which increases the profitability of innovation, and hence the firm's value. If the inequality is sufficiently high then an increase in demand will never occur because the firm's patent will expire before the poor can afford to buy the product. In this case the second effect diminishes. Thus a higher growth rate means a lower value for the monopolist when it enters the market. On the other hand, if the inequality is sufficiently low, the second effect again diminishes, since there is no waiting time.

3 Equilibrium Analysis

3.1 Entry, Exit and the Partial Equilibrium

Let's consider the most recent innovator n. For a profitable entry the innovation costs $(a_{rd}(t)w(t) = \frac{a_{rd}}{a_{old}})$ should not exceed the reward for an innovation. Assuming free access to R&D technology implies there are zero profits in equilibrium .That is $\frac{a_{rd}}{a_{old}} \geq v_n(t)$ When the innovations take place this condition holds with equality. This zero profit condition implies that the current costs of an innovation must not be smaller than current returns which are simply the innovation costs discounted by the interest rate. Moreover, in equilibrium

[5] Setting $\eta = \infty$ produces the value of the monopolist in Zweimuller(2000) as a special case.

the interest rate is determined by the growth rate as in equation (2). The entry condition can be written as:

$$\frac{a_{rd}}{a_{old}}(g+\rho) = \begin{cases} \pi L(1-\phi)(1-e^{-(g+\rho)\eta}) \quad \text{if } \mu > \eta \quad , \text{or} \\ \pi L\left[(1-\phi)+\phi d_p^{\frac{g+\rho}{g}} - e^{-(g+\rho)\eta}\right] \text{if } 0 < \mu \ \leq \eta \\ \pi L(1-e^{-(g+\rho)\eta}) \text{ if } \mu \ \leq 0 \end{cases} \tag{13}$$

where $d_p = d_p(n) = \frac{h_p(t)}{n}$ is the poor's consumption position.

3.2 General Equilibrium

A general equilibrium in this model is a situation in which the following conditions hold simultaneously. 1) Consumers maximize their life-time utility subject to their temporal budget constraint and initial wealth. 2) Firms maximize profits. 3) The resource constraint holds. The firm's and the consumers optimal decisions and the zero profit equilibria were dealt with in previous sections. Equation (13) states the partial equilibrium condition on the firm's side. The optimal consumption choice in (3) and the resource constraint in (8) together with the partial equilibrium condition in (13) form a set of equations which describe the general equilibrium in this economy. To incorporate the wealth distribution into the general equilibrium framework we also make use of equation (5). Using algebra , which is omitted here, the system can be reduced to the following equations in the unknowns of d_p and g for each case.

$$d_p = \begin{cases} \frac{L}{a_{old}}\left[\frac{\gamma_p[1-(1-\phi)[a_{new}(1-d_c)-(a_{old}-1)]]+\gamma_r(1-\phi)}{\left(\gamma_r(1-\phi)L+\gamma_p\right)}\right] - g\frac{a_{rd}\gamma_p}{a_{old}\left(\gamma_r(1-\phi)L+\gamma_p\right)} & \text{if } \mu > \eta \\ \frac{1}{a_{old}} + \gamma_p\left[\frac{L-a_{old}d_c-(1-\phi)L[a_{new}(1-d_c)+1-a_{old}]-\frac{a_{new}}{a_{old}}L\phi}{L\left[a_{old}(1-\phi)\gamma_r+a_{new}\phi\gamma_p\right]}\right] - g\frac{a_{rd}\gamma_p}{L\left[a_{old}(1-\phi)\gamma_r+a_{new}\phi\gamma_p\right]} & \text{if } 0 < \mu \ \leq \eta \\ \frac{1}{a_{old}} + \gamma_p\left[\frac{L-a_{new}[L(1-d_c)]-a_{old}d_c-(1-a_{old})L}{a_{old}L}\right] - g\frac{a_{rd}\gamma_p}{a_{old}L((1-\phi)\gamma_r+\phi\gamma_p)} & \text{if } \mu \leq 0 \end{cases} \tag{14}$$

where $(1-\phi)\gamma_r + \phi\gamma_p = 1$. Equation (14) is simply a negative linear relation between g and d_p which holds in equilibrium. It reflects the optimal consumption choices and the resource constraints. Now, we can restate the no-profit condition in (13) as:

$$\frac{a_{rd}}{a_{old}}(g+\rho) = \begin{cases} \pi L(1-\phi)(1-d_c^{\frac{g+\rho}{g}}) \quad \text{if } \mu > \eta \quad \text{, or} \\ \pi L\left[(1-\phi)+\phi d_p^{\frac{g+\rho}{g}} - d_c^{\frac{g+\rho}{g}}\right] \text{if } 0 < \mu \ \leq \eta \\ \pi L(1-d_c^{\frac{g+\rho}{g}}) \text{ if } \mu \ \leq 0 \end{cases} \tag{15}$$

Equation (15) reflects the no-profit condition on the firm's side. Note that for $\mu > \eta$ and $\mu \leq 0$, the growth rate is independent of the position of the poor d_p which is as expected. Below we present the figures that represent the above conditions.

Figure 2 depicts the possible equilibria of the model[6]. The P curves represent equation (15) and Q curves represent equation (14). The numbers I, II and III refer to the initial demand patterns (cases) when $\mu > \eta$, $0 < \mu \leq \eta$ and $\mu \leq 0$, respectively. In case I, whatever the patent length, the poor can not afford to buy the monopolists product during its life-cycle. The poor's position, d_p, is irrelevant to the firm's decision. In other words, any change in growth rate which will change the poor's position will in turn not affect the firm's prospects in that it doesn't expect a demand jump in the future. This is shown by P_I.

The definition of d_p implies that above a certain level ($d_p = 1$) the entering firm's products will be bought by the poor regardless of the growth rate or the patent length during its life-cycle. This case is shown by P_{III}. For given levels of patent length and the poor's population share equation (15) implies that P_{III} lies to the right of P_I. If there is a possibility of a demand jump during the life-cycle of the firm, then the poor's position will determine the entry rate. The poor's position and the growth rate will be linked in a non-linear way as in the second line of equation(15). This situation is shown by P_{II}. The economy will operate in I for a low g and d_p combination, in II for a medium g and d_p combination and in III for a high g and d_p combination. For all levels of d_p the Q curve is a negatively sloped, straight line. The reason that a higher d_p implies a lower growth rate in each case is due to more resources being are diverted towards production of consumer goods instead to R&D. Using equation (14) it is possible to show that the d_p- intercept of Q becomes larger when moving from I to III. The intuition is simply that when there is no growth all resources

[6] We use baseline parameters of $\phi = 0.7$, $\rho = 0.02$, $\eta = 20$, $\pi = 1$, $L = 1$, $\frac{a_{rd}}{a_{old}} = 0.7$.

are used for consumption which solely determines the poor's position d_p.The thick line combining P_I,P_{II} and P_{III} represents the set of unique equilibria. For all levels of d_p the Q curve is a negatively sloped, straight line. The reason that a higher d_p implies a lower growth rate in each case is due to more resources being are diverted towards production of consumer goods instead to R&D.

FIGURE 2

FIGURE 3

For given parameters, the slope of Q becomes larger when switching from I to II, but becomes smaller again when switching from II to III with III still being higher than I. This deserves some discussion. The slope of Q is determined by the fraction of resources diverted to production of consumption goods and, except in case III, by the inequality level. If more growth can be achieved for a unit of consumption forgone, than it must be the case that initially few resources are diverted to R&D. The slope of Q is small, as in case I. Note that the fraction of resources diverted to production of consumption goods is higher in case I and III than in II to achieve the same level of increase in $d_{p.}$ The intuition here is twofold: first, for both the poor and the rich it is optimal to smooth consumption whenever there arises a possibility of an increase in wealth and consumption in the near future by doing so. In such a case the slope of the Q curve will be higher. Second, diverted resources are increasingly used in the new goods sector, which is more efficient. This reduces the R&D resources which have to be sacrificed in order to achieve the same level of consumption. In case III the first effect is absent, and in case I both effects are absent. As a result, the slope of Q_{II} is higher than Q_{III} both of which are higher than Q_I.

The equilibrium (d_p, g) pair is found at the intersection of P and Q curves.

3.3 Uniqueness

A unique general equilibrium exists if and only if the conditions on time preference are such that d_p is continuous and strictly increasing in growth rate in the no-profit condition (15). Such a situation is presented by the thick line in Figure 2. As the growth rate changes, it is possible to switch from one case to another.

Uniqueness of equilibrium requires that the switch, which is shown by the arrows, be continuous without any jumps in growth rate. In other words, a (d_p, g) pair should exist above which the $\mathrm{P}_I(\mathrm{P}_{II})$ curve smoothly connects to $\mathrm{P}_{II}(\mathrm{P}_{III})$.

In case I and case III the equilibrium growth rate is solely determined by the positions of P_I and P_{III}. This is evident in equation 15 where the first and the third line does not include the term d_p.

Each equilibrium situation depends on the length of the time period μ which is determined not only by the growth rate but also the poor's purchasing power and the patent length η.If we relax the conditions on the time preference the model exhibits multiple equilibria as shown in Figure 3. An increase in the time preference, ρ, changes the intercept and the curvature of P_{II} curve whereas it shifts P_I and P_{III} to the left as shown by the arrows. Multiple equilibria occur only in case II where for a sufficiently large ρ there exists more than one zero-profit equilibrium.

4 Applications

4.1 Inequality and Growth1

Since an analytical solution is not attainable, we provide the arguments based on the numerical methods and associated graphs. We distinguished earlier between the three initial conditions (I, II, III) and their implications for the model's behavior. Below, the effects of inequality on growth in each of those cases are analyzed. Case I and case III deal with very high and very low inequality respectively, whereas case II deals with a wide range of inequalities from high to low.

Since the patent length determines the state of this economy it is initially kept fixed to isolate the effects of inequality on growth. Later, this assumption is relaxed to consider the effects of patent length on growth at each level of inequality. Both the poor's share of wealth γ_p and the poor's share of population, ϕ, are used as a proxy to inequality. Note that whether looking effects of γ_p or ϕ the Lorentz relation given in (5) has to hold. For instance, any increase in γ_p is coupled by an appropriate decrease in γ_r keeping ϕ fixed. We summarize the main arguments in the following proposition.

Proposition 2. *i) If* $\mu > \eta$*, a lower population share of the poor* ϕ*, holding poor's share of wealth* γ_p *increases the growth rate* g*, whereas an increase in* γ_p*, holding* ϕ *constant , leads to a higher* g *if and only if* $\Delta\gamma_p$ *is positive and sufficiently large. ii) If* $0 < \mu < \eta$ *, a lower* ϕ *increases* g *only if the initial consumption position of the poor relative to the entry position of the monopolist firm* d_p *is sufficiently low, whereas an increase in* γ_p *leads to an unambiguous increase in* g*. iii) If* $\mu \leq 0$*, a lower population share of the poor(*ϕ*) or an increase in the wealth of the poor(*γ_p*) does not have any effect on the growth rate.*

Proof. See appendix.

When waiting time until demand jump is bounded below by the patent duration, transferring wealth to the poor might not affect growth because innovators might have a 'too short' life. On the other hand, when the poor can already afford most of the new goods, reducing inequality by increasing the population share of the rich might increase the entering firms market size less than it does the incumbent, luxuries producers. The production becomes less efficient and less resources are diverted to R&D reducing growth. Obviously, the first result is obtained by virtue of fixed patent length. Second result would hold even if patent length is infinite, however, compared to an infinitely lived monopolist, the marginal effect of a reduction in ϕ on the profit flow, and therefore on the entry rate, is higher for a finitely lived monopolist, This is because the profit flow is spread over a fixed horizon.

Proposition 3. *If* $0 < \mu < \eta$ *,the effect of a change in* ϕ *on the entry rate is decreasing in patent length,* η*.*

Proof. See appendix.

4.2 Patent Length, Growth and Redistribution

The effects of patent policies on growth has attracted a lot of interest in the literature. On the one hand patents create mark-ups, distorting relative prices and thus reducing welfare; on the other hand they stimulate R&D by increasing profitability of innovations and cause dynamic efficiency gains. For tractability purposes, in this model it is assumed that the firms do not mark-up during their finite monopolistic life, which implies that the optimal patent length is infinite. By imposing finite patent lengths, however, one is able to distinguish between different levels of inequality which will matter to the firm's decision to innovate.

Figure 4 shows the patent length as proxied inversely by d_c on y-axis and growth on x-axis. The figure is drawn after the model is adjusted to the feasible inequality parameters for case II. The growth is maximized at $d_c = 0$ which amounts to infinite patent length. An increase in patent length unambiguously increases the growth rate and the poor's consumption. The decreasing slope as we Longer patents reduce the effect of inequality by increasing the likelihood that the firms benefit from a future demand jump from the sales to the poor.

One interesting issue is how the patent length and growth relate at different levels of wealth inequality. As the wealth inequality increases the curve shifts outward and its slope becomes flatter. At high levels of inequality a unit increase in patent length increases growth more than it does at low levels of inequality. When the poor's population share is high the potential increase in profitability is higher in case of a demand jump. An increase in patent length makes such higher gains more likely.

FIGURE 4

Limited monopoly duration introduces threshold effects in redistributive policies. In a dynamic context, the size of the markets is determined by the patent lengths and with the hierarchic preferences not every redistribution is high enough to make the poor purchase the efficient product during the product's lifetime. For instance, in a high inequality situation, a sufficient redistribution to increase growth is the one which increases the wealth of the poor by at least 232% when the patent length parameter, d_c, is equal to 0.2[7]

5 Conclusion

We presented above a general equilibrium model which accounted for the nonlinear relationship between inequality and innovation-driven growth. Unlike previous models of demand-induced growth the patent length is taken to be fixed which introduces some interesting equilibrium dynamics. At high and low levels of

[7] When the growth rate is 5% this is equivalent of a patent length of 32 years. With a 3% growth rate it is equivalent to 53 years.

inequality, an increase in inequality might have no effect or opposite effects on growth depending on the patent length. The mechanics of the model is such that the entering firm's potential market size determines the level of growth, whereas the market size is determined by the inequality level, the patent length and the growth rate. Furthermore, it is shown that in this setup an increase in patent length increases growth, a result also confirmed by previous studies. The magnitude of a change in inequality on growth, on the other hand, differs with respect to the duration of patents.

In this paper, growth is driven by innovations which are protected by patents of finite length. Innovators are subject to displacement only after their patents expire and they are displaced by other more efficient competitive producers, if any. A range of equilibria is characterized by a situation where the poor can not afford all the products available on the market and the innovators face a demand jump but not until the poor are rich enough to purchase the new product. The waiting time, in turn, depends both on the growth rate and the inequality level. The length of time that a monopolist sells its product to the whole population instead of just to the rich is not determined solely by the inequality level but also by the patent length and the growth rate of the economy.

The model presented here is open to modifications and further enhancements. For instance, allowing for mark-ups creates a possibility for a finite optimal patent length due to static price distortions. The negative welfare effect of distorted relative prices would then counterbalance the dynamic efficiency gain of infinite patents. A general solution of the model will considerably be more difficult, however. First the price structure would not reflect the cost structure and consumer purchases would not be efficiently spread over varieties. The equilibrium could then be described by non-linear delayed differential equations.

APPENDIX

Proof of Proposition 2.

Case I $: \mu > \eta$

1) *Lower population share of the poor* ϕ : Keeping the poor's consumption position fixed, a lower population share of the poor unambiguously increases the growth rate as in Figure A1. P curve shifts right ($\mathrm{P}_I \rightarrow \mathrm{P}'_I$).

The demand is higher for the entering firms because more people can afford their products. This increases profitability and induces more entry. At the same time the demand for the consumption goods by the poor falls but the demand for all consumption goods increase. This is simply due the change in the composition of the population. The composition of demand shifts towards new and more efficient goods instead of 'luxuries' leaving more resources for R&D. For a unit the poor forgo more growth can be achieved because of this increased efficiency. This means that the new Q curve (Q'_I) has a lower slope. At the new equilibrium, the growth rate (g'_1) is higher. The effect on the poor's consumption depends on the efficiency of the new technology. The more efficient the new technology the higher is the poor's consumption.

2) *Higher wealth share of the poor* γ_p : A small increase of the poor's share of wealth does not affect the growth rate. A small wealth increase is insufficient to make the poor's consumption choices differ enough to include the new goods before their patent expiration. As long as the poor's consumption stays within a certain range, the economy will operate as in case I and any effects on profitability are ruled out. As a result P_I curve does not shift. The range of wealth for which an increase does not effect growth can be found by making use of the equilibrium conditions and the Lorentz equation (5). A sufficient increase in wealth which would switch the model from I to II and increase the growth rate must be at least 232% using the parameters specified as above for $d_p \leq d_c = 0.2$, In Figure A2 a small increase in wealth rotates the curve Q_I clockwise around (g_1,d_{p1}) but does not affect the P_I curve. It can be shown that P_I lies to the left of (g_1,d_{p1}) for the parameters above[8] and the increase in wealth actually improves poor's consumption.

Case II : $0 < \mu < \eta$

1) *Lower population share of the poor* ϕ : When inequality is high, a lower population share of the poor shifts and rotates the Q curve ($Q_{IIa} \rightarrow Q'_{IIa}$) and rotates the P_{II} curve around (g_2, d_{p2})in Figure A3. The shift in both curves implies that up to a certain level of d_p a lower population share of the poor increases growth. The increase in the growth rate is diminishing as a result of two opposite effects. First, as the growth rate increases so does the interest rate. The future profits have to be discounted at a higher rate which reduces the entering firm's value. Fewer firms enter the market. This has a negative effect on growth. Second, the sales to the poor are realized faster due to improving purchasing power which increases profitability. The returns to innovation are therefore higher, which has a positive effect on growth. Fewer resources are spent on the

[8] It turns out that (g, d_p)=(0.32, 0.18) and $g_1 = 0.08$ in Figure A2

production of luxuries improving efficiency and enhancing growth. When the inequality is low, the Q curve sits above (g_2, d_{p2}) and is shown by Q_{IIb}. A lower population share of the poor reduces growth. A decrease in the poor's population increases the demand for the luxuries $(j > n)$ more than it does the demand for new goods.

2) *Higher wealth share of the poor* γ_p :

An increase in wealth of the poor rotates the Q_{II} curve around $(g_{2,}d_{p2})$ in Figure A3. It can be shown that P curve lies to right of $(g_{2,}d_{p2})$ labeled as P_{IIa}. An increase in the wealth of the poor then unambiguously increases the growth rate when the inequality is high $(g_{2a} \rightarrow g_{2a}^{'})$. An increase in the wealth of the poor increases the poor's consumption and speeds up the anticipated demand jump for the entering firms. A lower population share of the poor rotates the P_{II} curve around (g_2, d_{p2}) .

Case III: $\mu \leq 0$

A lower population share of the poor does not have any effect on growth or poor's consumption when the inequality is sufficiently small. Note that the last part of (14) and (15) are independent of the poor's population share, ϕ. An increase in the wealth of the poor has no effect on the P_{III} curve in Figure A4. It rotates the Q curve counter-clockwise.

FIGURE A1-A4

Proof of Proposition 3.

Consider the case II. Let $v_{fin}(t)$ denote the value of a monopolist with a patent length of $\eta < \infty$ and $v_{\inf}(t)$ denote the value of a monopolist with a patent length of $\eta = \infty$. One can show by substituting d_p from the equilibrium condition (14) in (12), taking the derivative with respect to ϕ and after some algebra that $\frac{\partial v_{fin}(t)}{\partial \phi} > \frac{\partial v_{\inf}(t)}{\partial \phi}$.

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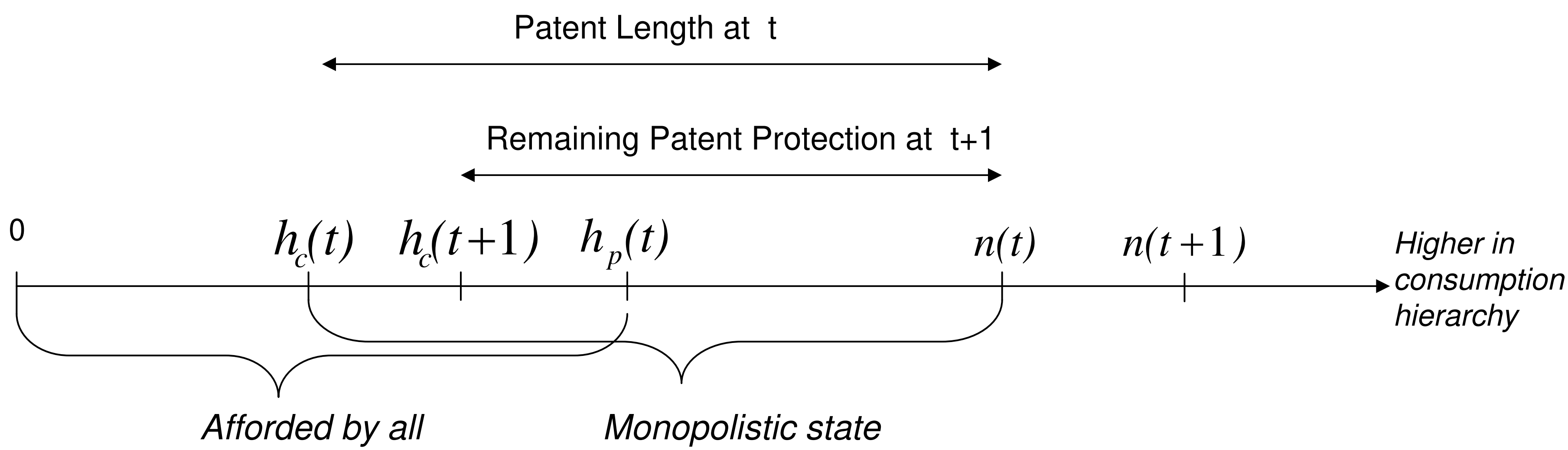


Figure 1. Inequality and Innovation Dynamics

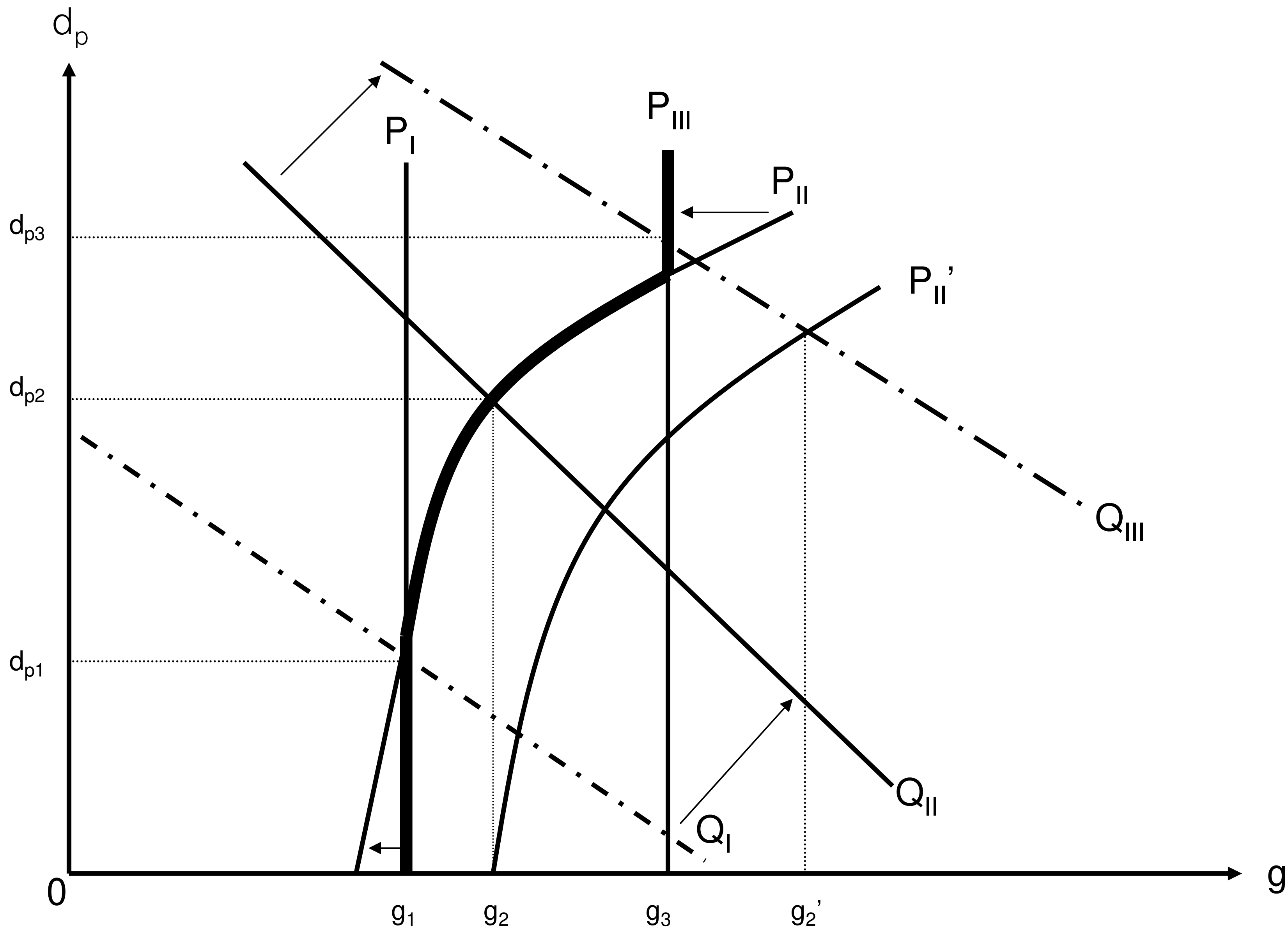


Figure 2. Set of Unique Equilibria.

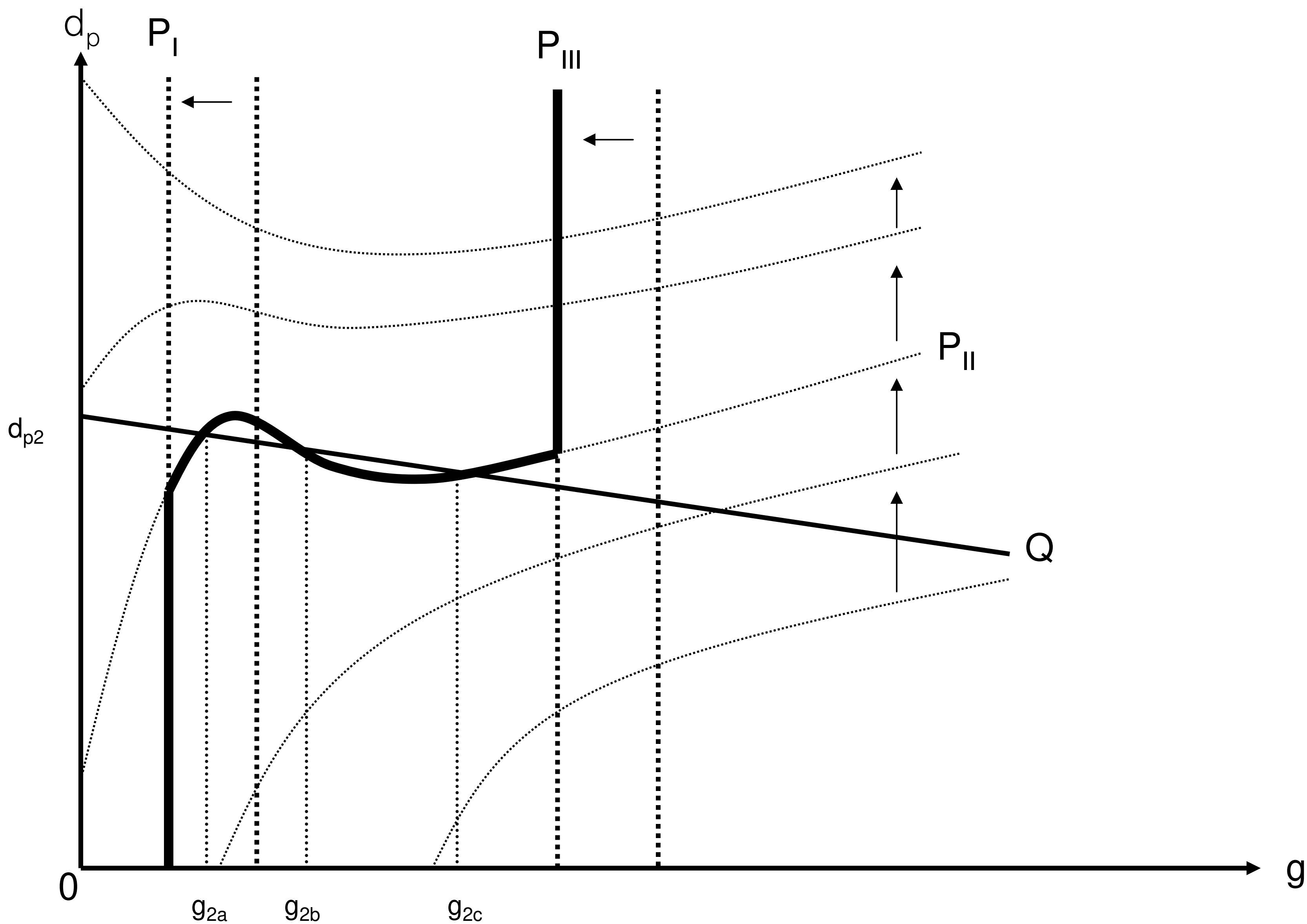


Figure 3. Multiple Equilibria.

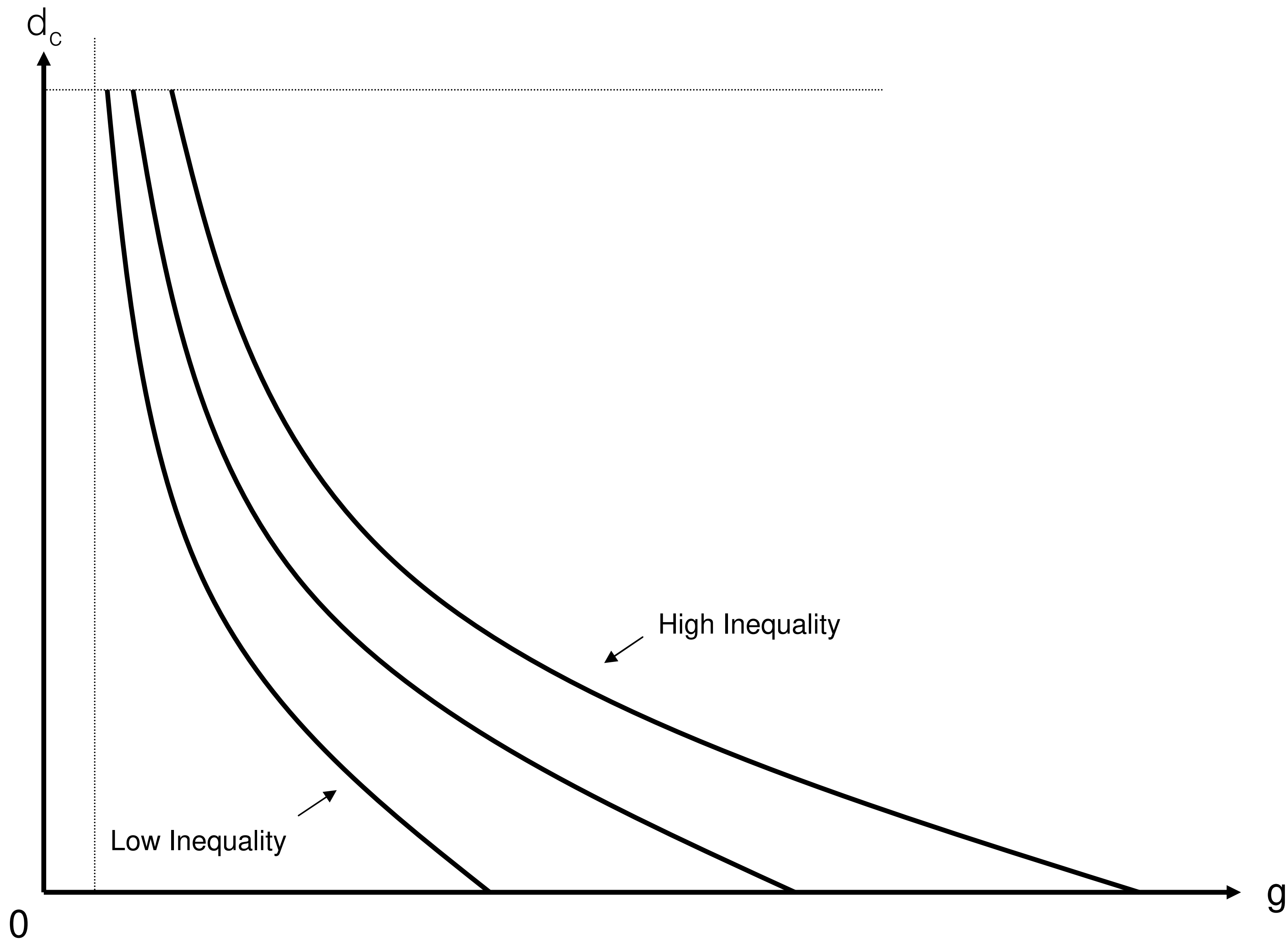
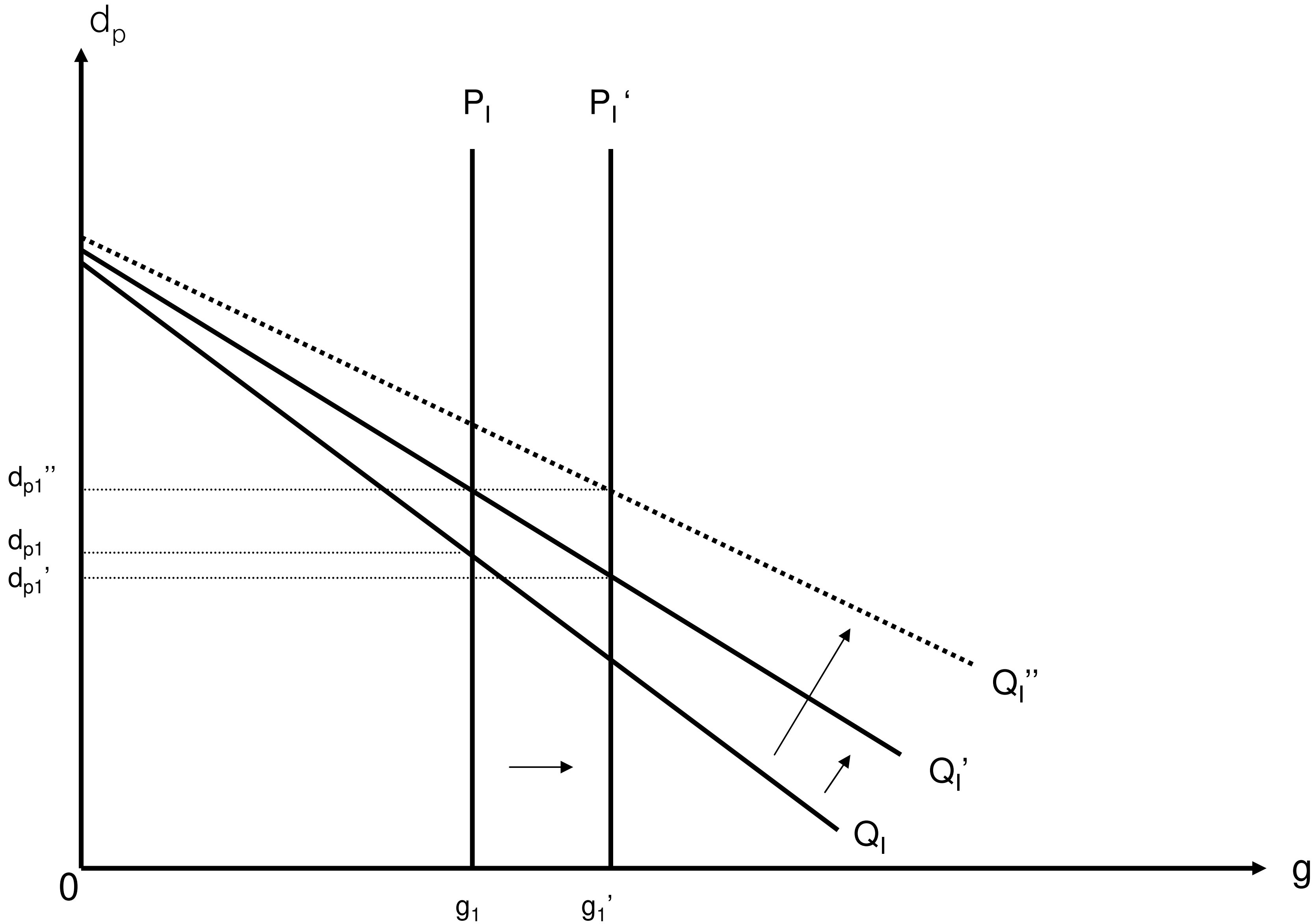
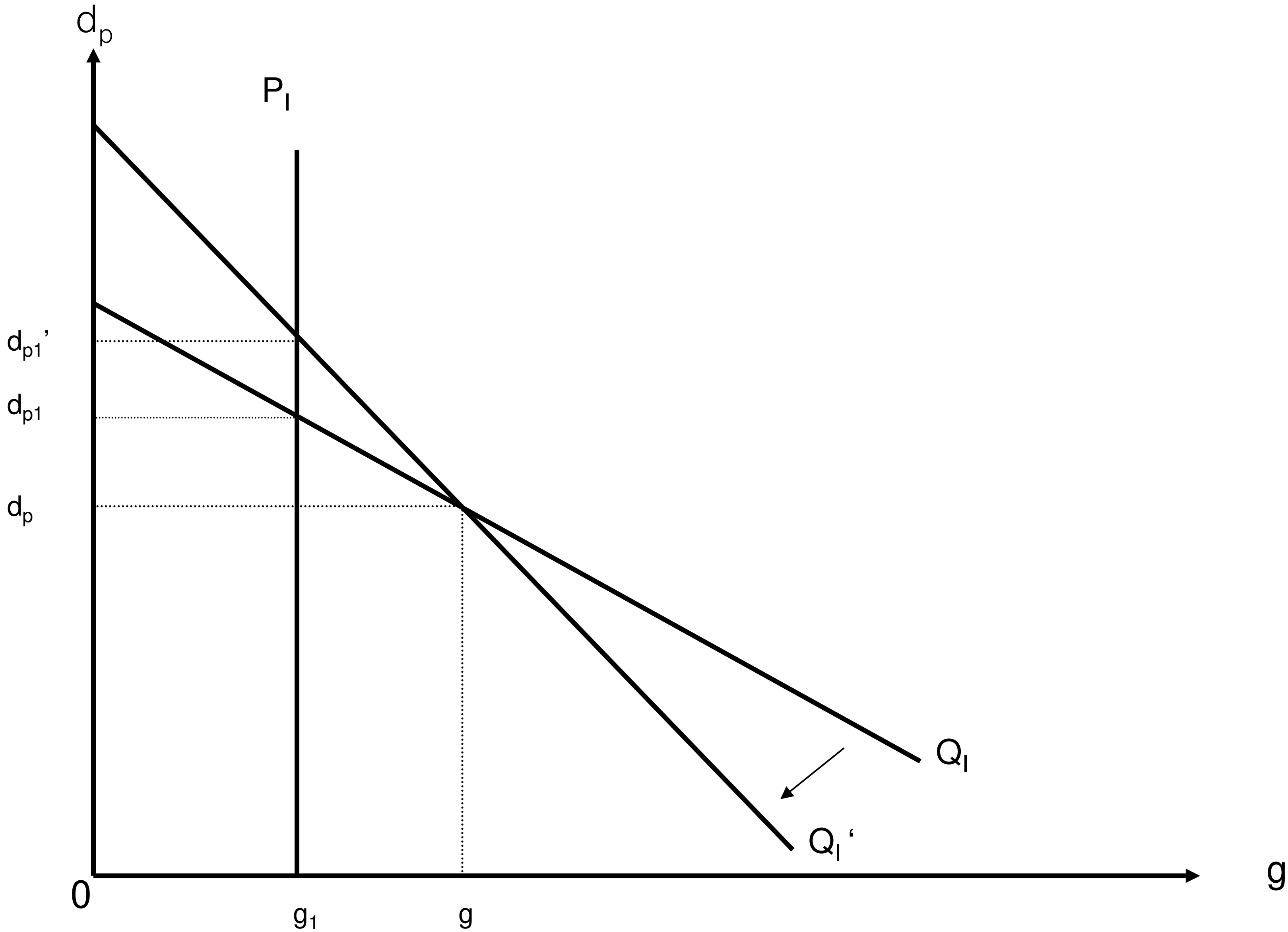


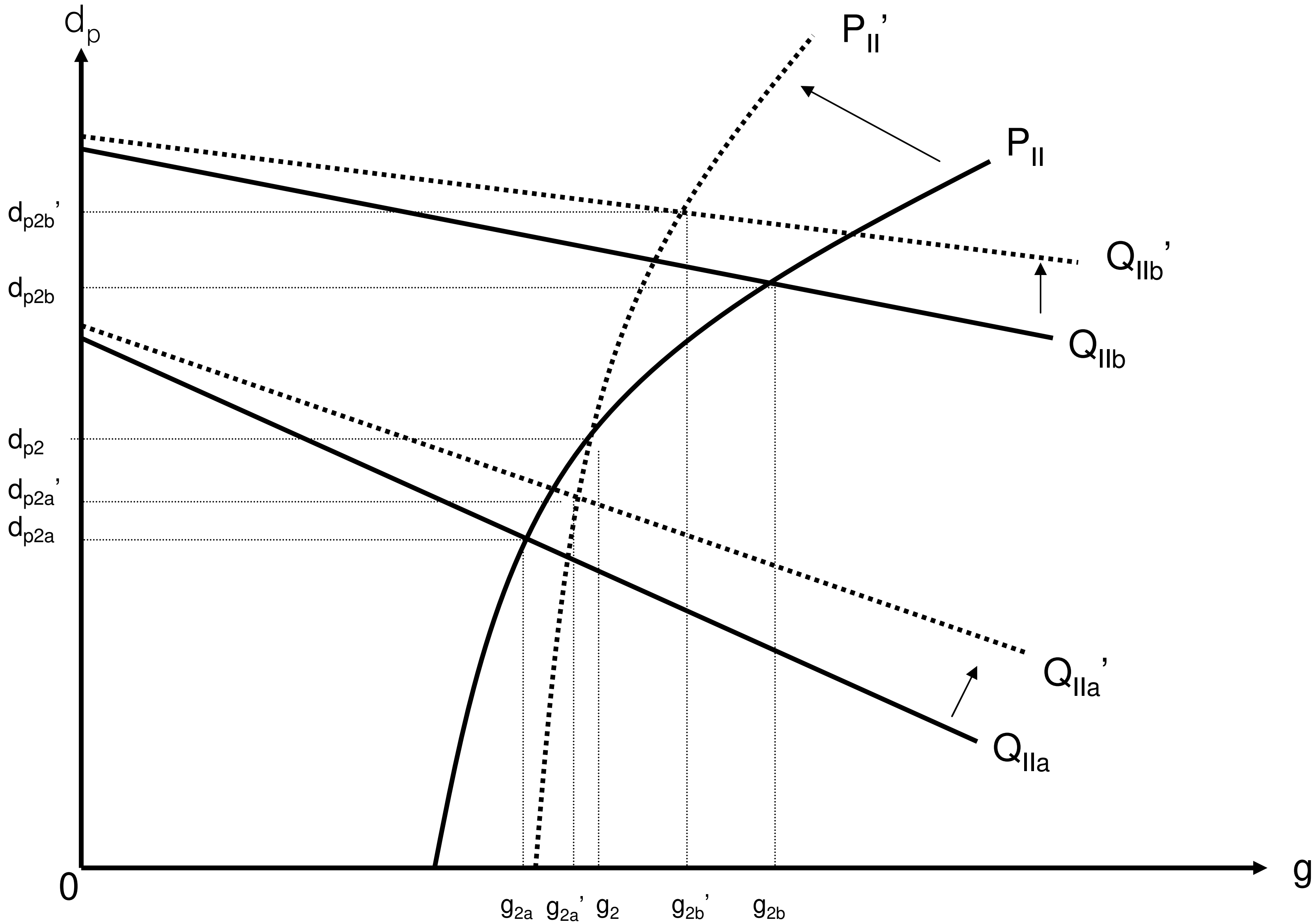
Figure 4. Patent Length and Growth. Effect of Inequality.



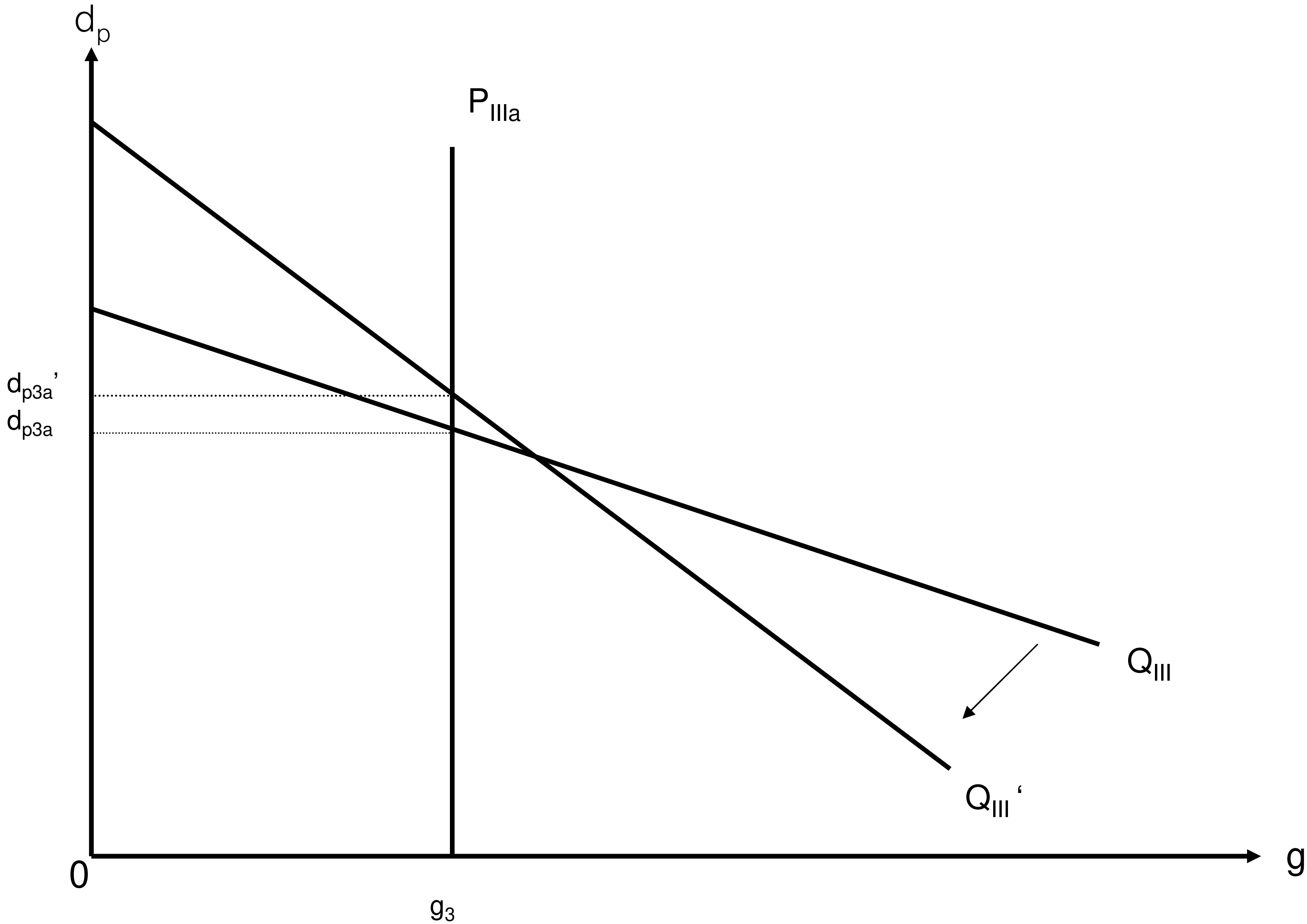
A1. Effect of Lower Population Share of The Poor When $\mu > \eta$.



A2. Effect of Higher Wealth Share of The Poor When $\mu > \eta$.



A3. Comparative Statics when $0 < \mu \leq \eta$.



A4. Comparative Statics When $\mu \le 0$.