# All-Stage strong correlated equilbrium

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11 March 2008

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**Abstract**

A strong correlated equilibrium is a strategy profile that is immune to joint deviations. Different notions of strong correlated equilibria were defined in the literature. One major difference among those definitions is the stage in which coalitions can plan a joint deviation: before (*ex-ante*) or after (*ex-post*) the deviating players receive their part of the correlated profile. In this paper we prove that if deviating coalitions are allowed to use new correlating devices, then an *ex-ante* strong correlated equilibrium is immune to deviations at all stages. Thus the set of *ex-ante* strong correlated equilibria of Moreno & Wooders (1996) is included in all other sets of strong correlated equilibria.

## 1 Introduction

The ability of players to communicate prior to the play, influences the set of self-enforcing outcomes of a non-cooperative game. The communication allows the players to correlate their play, and to implement a correlated strategy profile as a feasible non-biding agreement. For such an agreement to be self-enforcing, it has to be stable against "reasonable" coalitional deviations. Two notions in the literature describe such self-enforcing agreements: a *strong correlated equilibrium* is a profile that is stable against *all* coalitional deviations, and a *coalition-proof correlated equilibrium* is a profile that is stable against *self-enforcing* coalitional deviations (a deviation is self-enforcing if no sub-coalition has further self-enforcing and improving deviation).

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[1] This work is in partial fulfillment of the requirements for the Ph.D. in mathematics at Tel-Aviv University. I would like to thank Eilon Solan for his careful supervision, and for the continuous help he offered. My deep gratitude is also given to Ehud Lehrer and to Yaron Azrieli for many useful comments, discussions and ideas.

Each notion has a few alternative definitions. One major difference among them, is the stage in which coalitions can plan a deviation from a correlated agreement. Assume that the correlated agreement is implemented by a mediator who privately recommends each player what to play. The definitions in Milgrom & Roberts (1996), Moreno & Wooders (1996), and Ray (1996) are *ex-ante* definitions: In their framework, players may plan deviations before receiving recommendations, and no further communication is possible after recommendations are issued. The definitions in Einy & Peleg (1995), Ray (1998) and Bloch & Dutta (2007) are *ex-post* [2] definitions: In their framework, players may plan deviations only after receiving recommendations.

However, in some frameworks coalitions can plan deviations at all stages. One example for such framework is an extended game with *cheap-talk*: pre-play, unmediated, non-biding, non-verifiable communication among the players. [3] In such a framework, the players can "mimic" a mediator, and implement a large set of strong correlated equilibria as a strong Nash equilibria (Aumann, 1959) in the extended game (Heller, 2008). [4] A coalition can plan a deviation in the early phases of the cheap-talk when no player has received his recommendation yet (*ex-ante* stage), in the late phases when all players have received their recommendations (*ex-post* stage), or in an intermediate stage when some of the players know their recommendations.

A natural question is whether any of the existing notions is appropriate to such frameworks, or whether new definitions are needed. In this paper we prove that the existing *ex-ante* strong correlated equilibrium (à la Moreno & Wooders) is resistant to deviations at all stages. The proof is based on three assumptions about the communication framework (which hold in the cheap-talk framework):

(1) A deviating coalition can use new correlating devices (play a joint *correlated* deviation).
(2) When a coalition decides to deviate, that decision is common knowledge among its members.
(3) The players share a common prior about the possible states of the world in an incomplete information model à la Aumann (1987).

An immediate corollary is that the set of *ex-ante* strong correlated equilibria

[2] Referred to as *"interim"* in some of the referred papers.
[3] For a good nontechnical introduction to some of the main issues of cheap-talk, see the survey of Farrel & Rabin (1996).
[4] The implementation presented in Heller (2008) is only as a $\lfloor n/2 \rfloor$-strong correlated equilibrium (an equilibrium that is resistant to deviations of coalitions with less than $n/2$ players). If one assumes that the players are computationally restricted and "one-way" functions exist, then the implementation can be as a strong correlated equilibrium (as discussed in Lepinski et al., 2004 and Abraham et al., 2006).

is included in all other sets of strong correlated equilibria, as defined in the literature referred above (as described in figure 1 in page 8).

One could hope that similar results might be obtained for the coalition-proof notions. However, in Section 5 we demonstrate that the existing *ex-ante* coalition-proof notion is not appropriate to frameworks in which coalitions can plan deviations at all stages.

The paper is organized as follows: Section 2 presents our model and the main result. The main result is demonstrated in Section 3, and proven in Section 4. We discuss the coalition-proof notion in Section 5, and conclude in Section 6.

## 2 Model and Definitions

### *2.1 Preliminary Definitions*

A game in strategic form $G$ is defined as: $G = \left(N, (A^i)_{i\in N}, (u^i)_{i\in N}\right)$, where $N$ is the finite (and non-empty) set of players with a size $n = |N|$, and for each $i \in N$, $A_i$ is player $i$'s finite (and non-empty) set of actions (or pure strategies), and $u^i$ is player $i$'s utility (payoff) function, a real-valued function on $A = \prod_{i\in N} A^i$. The multi-linear extension of $u^i$ to $\Delta(A)$ is is still denoted by $u^i$. A member of $\Delta(A)$ is called a (correlated) strategy profile. A coalition $S$ is a non-empty member of $2^N$. For simplicity of notation, the coalition $\{i\}$ is denoted as $i$. Given a coalition $S$, and let $A^S = \prod_{i\in S} A^i$, let $-S = \{i \in N \mid i \notin S\}$ denote the complementary coalition. A member of $\Delta(A^S)$ is called a (correlated) *$S$-strategy profile*. Given $q \in \Delta(A)$ and $a^S \in A^S$, we define $q_{|S} \subseteq \Delta(A^S)$ to be $q_{|S}(a^S) = \sum_{a^{-S}\in A^{-S}} q(a^S, a^{-S})$, and for simplicity we omit the subscript: $q(a^S) = q_{|S}(a^S)$. Given $a^S$ s.t. $q(a^S) > 0$, we define $q(a^{-S}|a^S) = \frac{q(a^S,a^{-S})}{q(a^S)}$.

### *2.2 An Intuitive Description of Our Framework*

Assume that the players of a game $G$ (which will be played tomorrow), have agreed to play a correlated strategy profile $q \in \Delta(A)$. The players implement $q$ with the assistance of a mediator who chooses the action profile $a \in A$ with probability $q(a)$. Throughout the day, the mediator calls each player $i$ and privately gives him his recommendation: $a^i \in A^i$. The players do not necessarily know the order in which the mediator calls the players, or which players have already been called. During the day, the players can communicate, share information about their recommendations, and plan coalitional deviations from the agreement. If all the members of a coalition agree to use a deviation (each, with his own posterior information, believes that the deviation is profitable), then it is implemented with the assistance of *a deviating device* - a new mediator who receives (at the end of the day) the recommendations of the deviating players, and gives each of them a new recommendation. In the next day, each player simultaneously chooses an action in $G$. The profile $q$ is an *all-stage strong correlated equilibrium*, if for every calling order and every stage, there is no coalition with a profitable deviation, as will be formally defined in the next Subsection.

### *2.3 All-stage Strong Correlated Equilibrium*

A deviating device implements a mapping from $A^S$ (the original recommendations the players in $S$ have received) to $\Delta(A^S)$ (the set of correlated $S$-strategy profiles).

**Definition 1** Given a coalition $S \subseteq N$, *a deviating device* is a function $d^S : A^S \to \Delta(A^S)$.

When the members of $S$ consider to implement a deviating device, they are in a situation of incomplete information: each player may have the private value of his recommendation, and may have additional private information acquired when communicating with the other deviating players. We describe the information structure during the communication among the deviating players in a model based on Aumann (1976, 1987).

**Definition 2** Given a coalition $S \subseteq N$ , an *information structure of* $S$ is a 5-tuple: $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i \in S}, (a^i)_{i \in N})$ where:

*(1)* The 3-tuple $(\Omega, B, \mu)$ is a probability space.
*(2)* The $(\mathcal{F}^i)_{i \in S}$ are partitions of $\Omega$ whose join ($\bigwedge_{i \in S} \mathcal{F}^i$, the coarsest common refinement of $(\mathcal{F}^i)_{i \in S}$) consists of non-null events.
*(3)* The $(a^i)_{i \in N}$ are random variables in $(\Omega, B, \mu)$, where $a^i : \Omega \to A^i$.

We interpret $(\Omega, \mathcal{B})$ as the space of states of the world for the players of $S$ (at some stage of their consideration whether to use a deviating device), $\mu$ as the common prior (for the states of the world) for all the players in $S$, and $\mathcal{F}^i$ as the information partition of player $i$; that is, if the true state of the world is $\omega \in \Omega$ then player $i$ is informed of that element $F^i(\omega)$ of $\mathcal{F}^i$ that contains $\omega$. We interpret the random variable $a^i(\omega)$ as the recommendation of player $i$ (from the original agreement) in the state $\omega$.

**Remark 3** The state of the world $\omega \in \Omega$ includes a full description of the recommendation profile, the information each deviating player has acquired while communicating with the other deviating players, and the beliefs each of the deviating players has about the information and beliefs of the others. We assume that all the players share a common prior about the states of the world. The justification of this assumption is discussed in Aumann (1987).

Given a non-null event $E \in \mathcal{B}$, let $a^S(E) \in \Delta(A^S)$ be a random variable with the posterior distribution of $a^S(\omega)$ conditioned on that $\omega \in E$.

The information structure of the players must be consistent with the framework:

(1) The prior distribution of $a^S(\Omega)$ is equal to the agreement's distribution.
(2) The deviating players have no information about the recommendations of the non-deviating players, except the information that is induced by their information about their recommendations.

We formalize those requirements in the following definition.

**Definition 4** Given an agreement $q \in \Delta(A)$ and a coalition $S \subseteq N$ , we say that an information structure $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i\in S}, (a^i)_{i\in S})$ is a *consistent information structure* (of $S$) if the following conditions hold:

*(1)* $\forall b^S \in A^S,\ \Pr(a^S(\Omega) = b^S) = q(b^S)$
*(2)* $\forall \omega \in \Omega,\ \forall i \in S,\ \forall b \in A,$
$\Pr\left(a^N\left(F^i(\omega)\right) = b\right) = \Pr\left(a^S\left(F^i(\omega)\right) = b^S\right) \cdot q(b^{-S} \mid b^S)$

When each player considers whether the implementation of a deviating device is profitable to him, he compares his conditional expected payoff (given his information about the distribution of $(a^i)_{i\in S}$ ) when playing the original agreement and when implementing the deviating device. A player agrees to implement a deviating device only if the latter conditional expectation is larger. We now formally define the conditional expected payoffs of each player in each state of the world $\omega$, when following the agreement and when implementing a deviating device.

**Definition 5** Given an agreement $q \in \Delta(A)$, a coalition $S \subseteq N$, a player $i \in S$, a deviating device $d^S : A^S \to \Delta(A^S)$, and a consistent information structure $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i\in S}, (a^i)_{i\in S})$, let the c*onditional expected payoffs of player $i$ in* $\omega \in \Omega$ (given his information in $\omega$ and the assumption that the players in $-S$ follow the agreement $q$) be:

- The conditional expected payoff when the players in $S$ follow $q$:

$$u^i_f(\omega) = \sum_{b^S \in A^S} \Pr\left(a^S\left(F^i(\omega)\right) = b^S\right) \sum_{b^{-S} \in A^{-S}} q(b^{-S} \mid b^S) \cdot u^i(b^S, b^{-S})$$

- The conditional expected payoff when the players of $S$ deviate (by implementing $d^S$) :

$$u^i_d(\omega) = \sum_{b^S \in A^S} \Pr\left(a^S\left(F^i(\omega)\right) = b^S\right) \sum_{b^{-S} \in A^{-S}} q(b^{-S} \mid b^S) \sum_{c^S \in A^S} d^S(c^S \mid b^S) \cdot u^i(c^S, b^{-S})$$

If the players in $S$ decide to implement a deviating device in some state $\omega \in \Omega$, then it is common knowledge (in $\omega$) that each player expects to earn more from the deviation (conditioned on his information). We now present the formal definition of common knowledge (Aumann, 1976):

**Definition 6** Given a coalition $S \subseteq N$, an information structure $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i \in S}, (a^i)_{i \in N})$ and a state $\omega \in \Omega$, an event $E \in \mathcal{B}$ is *common knowledge* at $\omega$ if $E$ includes that member of the meet $\mathcal{F}^{meet} = \bigwedge_{i \in S} \mathcal{F}^i$ that contains $\omega$.

We define a profitable deviating device, as a deviating device that in some consistent information structure, it is common knowledge in some state of the world, that each player expects to earn more if the deviating device is implemented.

**Definition 7** Given a strategy profile $q \in \Delta(A)$ and a coalition $S \subseteq N$, we say that a deviating device $d^S : A^S \to \Delta(A^S)$ is *profitable* (for $S$), if there exists a consistent information structure $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i \in S}, (a^i)_{i \in N})$ and a state $\omega_0 \in \Omega$ such that it is common knowledge in $\omega_0$ that $\forall i \in S$, $u^i_d(\omega) > u^i_f(\omega)$.

We can now define an *all-stage* strong correlated equilibrium as a strategy profile, from which no coalition has a profitable deviating device.

**Definition 8** A strategy profile $q \in \Delta(A)$ is an *all-stage strong correlated equilibrium* if no coalition $S \subseteq N$ has a profitable deviating device.

### *2.4 Ex-ante Strong Correlated Equilibrium*

A profile is an *ex-ante* strong correlated equilibrium, if no coalition has a profitable deviating device at the *ex-ante* stage (when the players have not received their recommends yet). It would be useful (following Moreno & Wooders, 1996), to define first the notion of a feasible *ex-ante* deviation for a coalition $S$, as a correlated strategy profile $p \in A$ that the coalition can induce with the use of a deviating device at the *ex-ante* stage.

**Definition 9** Let $q \in \Delta(A)$ be a strategy profile and let $S \subseteq N$ be a coalition. We say that $p \in \Delta(A)$ is a *feasible ex-ante deviation* by a coalition $S$ from $q$ if there is a deviating device $d^S : A^S \to \Delta(A^S)$ such that for all $a \in A$ we have $p(a) = \sum_{b^S \in A^S} q(b^S, a^{-S}) \cdot d^S(a^S | b^S)$. In that case we say that $p$ is induced from $q$ by the deviating device $d^S$. Let $D(q, S) \subseteq \Delta(A)$ denote the *set of all* *feasible ax-ante deviations* by a coalition $S$ from $q$.

An *ex-ante* strong correlated equilibrium is a strategy profile from which no coalition has a profitable *ex-ante* deviation.

**Definition 10** A strategy profile $q \in \Delta(A)$ is an *ex-ante strong correlated equilibrium* if no coalition $S \subseteq N$ has a feasible deviation $p \in D(q, S)$, such that for each $i \subseteq S$, we have $u^i(p) > u^i(q)$.

### *2.5 Main Result*

It is straightforward to see that an all-stage strong correlated equilibrium is also an *ex-ante* strong correlated equilibrium, as it is possible to choose a trivial consistent information structure, in which: $\forall i$, $\mathcal{F}^i = \Omega$. Our main result proves that the converse is also true, and thus the two sets of equilibria are equal.

**Theorem 11** *A correlated profile $q \in \triangle(A)$ is an <u>ex-ante</u> strong correlated equilibrium if and only if it is an <u>all-stage</u> strong correlated equilibrium.*

### *2.6 Relations With Other Notions of Strong Correlated Equilibria*

Other notions of *ex-ante* strong correlated equilibria have been presented in Ray (1996) and Milgrom & Roberts (1996). In the framework of Ray, deviating coalitions are not allowed to construct new correlating devices, and are limited to use only an uncorrelated deviating device - a function $d^S : A^S \rightarrow \prod_{i \in S} \Delta(A^i)$. [5] In the framework of Milgrom & Roberts only some of the coalitions can communicate and coordinate deviations. In both cases the sets of feasible coalitional deviations is included in our set of deviations, and thus our set of *ex-ante* strong correlated equilibria is included in the sets of *ex-ante* correlated equilibria à la Ray and à la Milgrom & Roberts.

An *ex-post* strong correlated equilibrium is a profile which is resistant to deviating devices at the *ex-post* stage (when each player knows his recommendation, i.e., $\forall \omega \in \Omega$, $\forall i \in S$, $\exists b^i \in A^i$ s.t. $\Pr\left(a^i\left(F^i(\omega)\right) = b^i\right) = 1$). Different notions of *ex-post* strong correlated equilibria are presented in Einy & Peleg (1995), Ray (1998) and Bloch & Dutta (2007). In the framework of Einy &

[5] In Ray's setup, the original correlating device can also send an indirect signal to each player (which may hold more information than the recommendation itself). In that case, the uncorrelated deviating device is a function from the set of $S$-part of the signals to the set of uncorrelated $S$-strategy profiles.

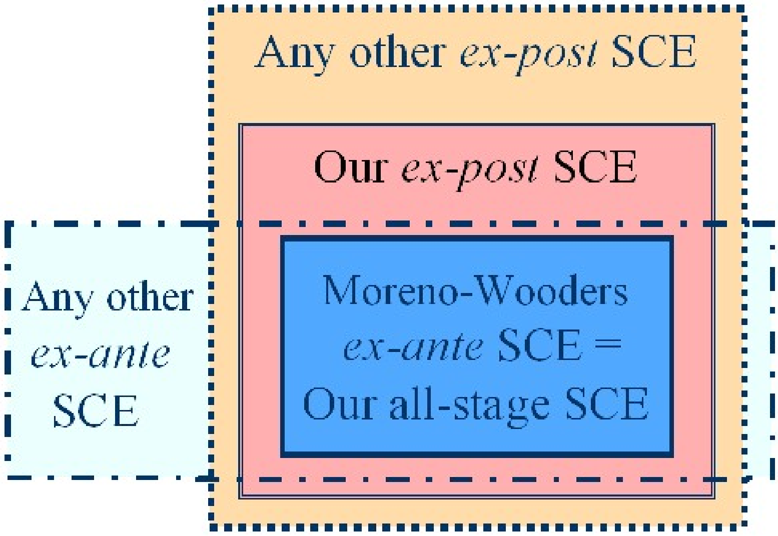
Peleg, a deviating coalition can only use deviating devices that improve the conditional utilities of all deviating players for *all possible* recommendation profiles.[6] In the framework of Ray, a deviating coalition $S$ can only use *pure* deviating devices - $d^S : A^S \to A^S$. In the framework of Bloch & Dutta, a deviating coalition $S$ can only use deviating devices that are implemented if and only if the recommendation profile $a^S$ is included in some set $E^S \subseteq A^S$ which satisfies:

(1) If $a^S \in E^S$, each player earns from implementing the deviating device.
(2) If $a^S \notin E^S$, then at least one player looses from implementing the deviation device (by falsely claiming that $a^S \in E^S$).

It can be shown that those conditions imply that there exists a consistent information structure of $S$ and a state in which it is common knowledge that $a^S(\omega) \in E^S$ and that $\forall i \in S$, $u^i_d(\omega) > u^i_f(\omega)$. Thus our set of *ex-post* strong correlated equilibria is included in the sets of *ex-post* correlated equilibria as defined in any of those papers.

The inclusion relations among the different notions of strong correlated equilibria is described in figure 1.

Figure 1. Inclusion Relations among Different Notions of Strong Correlated Equilibria




Examples for the different notions can be found in Moreno & Wooders (1996):

- An *ex-ante* strong correlated equilibrium in a 3-player matching pennies (which is also an all-stage strong correlated equilibrium due to our main

[6] In our formulation, it is equivalent to requiring that $\forall i \in S$, $u^i_d(\omega) > u^i_f(\omega)$ in every $\omega \in \Omega$, and not only in every $\omega \in F^{meet}(\omega_0)$ .

result), which is the only "reasonable" outcome of the game with pre-play communication (as experimentally demonstrated in Moreno & Wooders, 1998).

- An *ex-post* strong correlated equilibrium in a 2-player chicken game, which is not an *ex-ante* strong correlated equilibrium.

## 3 An Example of the Main Result

In the following example we present an *ex-ante* strong correlated equilibrium in a 3-player game, and a specific deviating device that is considered by the grand coalition at some intermediate stage. At first look, one may think that this deviating device is profitable to all the players (conditioned on their posterior information at that stage), but a more thorough analysis reveals that this is not true. The analysis in this example gives the intuition of the formal proof of the general case in the following Section.

Table 1 presents the matrix representation of a 3-player game, where player 1 chooses the row, player 2 chooses the column, and player 3 chooses the matrix.

Table 1
A 3-Player Game With An Ex-Ante Strong Correlated Equilibrium

| | $c_1$ | | | | $c_2$ | | | | $c_3$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $b_1$ | $b_2$ | $b_3$ | | $b_1$ | $b_2$ | $b_3$ | | $b_1$ | $b_2$ | $b_3$ |
| $a_1$ | 10,10,10 | 5, 20,5 | 0,0,0 | $a_1$ | 5,5,20 | 0,0,0 | 0,0,0 | $a_1$ | 0,0,0 | 0,0,0 | 0,0,0 |
| $a_2$ | 20,5,5 | 0,0,0 | 0,0,0 | $a_2$ | 0,0,0 | 0,0,0 | 0,0,0 | $a_2$ | 0,0,0 | 0,0,0 | 0,0,0 |
| $a_3$ | 0,0,0 | 0,0,0 | 0,0,0 | $a_3$ | 0,0,0 | 0,0,0 | 0,0,0 | $a_3$ | 0,0,0 | 0,0,0 | 7,11,12 |

Let $q$ be the following profile: $\left(\frac{1}{4}(a_1, b_1, c_1), \frac{1}{4}(a_2, b_1, c_1), \frac{1}{4}(a_1, b_2, c_1), \frac{1}{4}(a_1, b_1, c_2)\right)$ with an expected payoff of 10 to each player. Observe that $q$ is an *ex-ante* strong correlated equilibrium:

- No single player has a unilateral deviation ($q$ is a correlated equilibrium).
- No coalition of two players (say 1,2) has a profitable deviation (their uncertainty about the recommendation of player 3 prevents them from being able to earn together more than 20 by a joint deviation).
- The grand coalition cannot earn more than a total payoff of 30.

Now, consider an intermediate stage in which player 1 has received a recommendation $a_1$, player 2 has received a recommendation $a_2$, player 3 has not received his recommendation yet. Each player does not know whether the other players have received their recommendations. At first look, the implementation of the deviating device $d^S(\cdot) = (a_3, b_3, c_3)$ , which gives a payoff of $(7, 11, 12)$, may look profitable for all the players:

- Conditioned on his recommendation $(a_1)$, player 1 has an expected payoff of $6\frac{2}{3}$, and thus $d^S$ is profitable to him. The same is true for player 2 as well.
- Player 3 does not know his recommendation. His *ex-ante* expected payoff is 10, and he would earn a payoff of 12 by implementing $d^S$.

However, a more thorough analysis of player 3's information, reveals that $d^S$ is unprofitable for him. Player 1 can only earn from $d^S$ (which gives him a payoff of 7) if he has received a recommendation $a_2$. Thus, if player 1 agrees to implement $d^S$, then it is common knowledge that he has received $a_1$. The expected payoff of players 2 and 3, conditioned on that player 2 has received $a_1$, is $11\frac{2}{3}$. Thus, if player 2 agrees to implement $d^S$(with a payoff of 11), then he must have more information: that his recommendation is $a_2$. Therefore player 3 knows that the if the other players agree to implement $d^S$, then their part of the recommendation profile is$(a_1, a_2)$. Conditioned on that, his expected payoff is 15, and thus $d^S$ is unprofitable for him.

## 4 A Proof of the Main Result

In this Section we prove our main result: (theorem 11) - A correlated profile $q \in \triangle(A)$ is an <u>ex-ante</u> strong correlated equilibrium if and only if it is an <u>all-stage</u> strong correlated equilibrium. As discussed earlier, one direction is straightforward, and we have to prove only the other direction:

**Theorem 12** *Every <u>ex-ante</u> strong correlated equilibrium is an <u>all-stage</u> strong correlated equilibrium.*

In other words: if a profitable deviating device from an agreement $q \in \triangle(A)$ exists, then there also exists a profitable *ex-ante* deviation from $q$.

The theorem immediately follows from the following two propositions:

(1) Proposition 14: If an agreement $q$ is not an all-stage strong correlated equilibrium, then there exists a "similar" agreement $\tilde{q}$ that is not an *ex-ante* strong correlated equilibrium. The "similarity" is in the sense that $\tilde{q}$ is absolute continues w.r.t. $q$ when restricted to $A^S$, and equal to $q$ when restricted to $A^{-S}$ and conditioned on $A^S$ (as formally defined below).
(2) Proposition 15: If such a "similar" agreement $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium, then $q$ itself is not an *ex-ante* strong correlated equilibrium.

We now present an auxiliary definition (which will be used in the proof of proposition 14) for the conditional expected payoffs of each player given the information that $\omega \in E$.

**Definition 13** Given an agreement $q \in \Delta(A)$, a coalition $S \subseteq N$, a deviating device $d^S : A^S \to \Delta(A^S)$, a consistent information structure $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i\in S},$ $(a^i)_{i\in S})$, and a non-null event $E \in \mathcal{B}$, let $\left(\left(\tilde{u}^i_f(E)\right)_{i\in S}, (\tilde{u}^i_d(E))_{i\in S}\right)$ denote the *conditional expected payoffs given the information that the state of the world is in* $E$ (and given that the players in $-S$ follow the agreement $q$):

- The conditional expected payoff of each player $i$ when the players in $S$ follow the agreement $q$ (given $\omega \in E$):

$$\tilde{u}^i_f(E) = \sum_{b^S \in A^S} \Pr\left(a^S(E) = b^S\right) \sum_{b^{-S}\in A^{-S}} q(b^{-S} \mid b^S) \cdot u^i(b^S, b^{-S})$$

- The conditional expected payoff when the players of $S$ deviate by implementing $d^S$ (given $\omega \in E$):

$$\tilde{u}^i_d(E) = \sum_{b^S \in A^S} \Pr\left(a^S(E) = b^S\right) \sum_{b^{-S}\in A^{-S}} q(b^{-S} \mid b^S) \sum_{c^S\in A^S} d^S(c^S \mid b^S) \cdot u^i(c^S, b^{-S})$$

Observe the difference between definition 5 and definition 13:

- In definition 5 $u^i_f(\omega)$ and $u^i_d(\omega)$ describe the conditional utility of player $i$ (in the state of the world $\omega \in \Omega$) *in the perspective of player* $i$, who is informed in $\omega$, that the state of the world in in $F^i(\omega)$.
- In definition 13 $\tilde{u}^i_f(E)$ and $\tilde{u}^i_d(E)$ describe the conditional utility of player $i$ *in the perspective of an outside observer,* who is informed that the state of the world is in $E$.

**Proposition 14** Let $q \in \Delta(A)$ be a strategy profile (the agreement) that is not an all-stage strong correlated equilibrium. Then there exists a strategy profile $\tilde{q} \in \Delta(A)$ that satisfies the following conditions:

(1) $\tilde{q}|_S$ is absolute continues with respect to $q|_S$ :

$$\forall b^S \in A^S,\ q(b^S) = 0 \Rightarrow \tilde{q}(b^S) = 0$$

(2) Conditioned on $S$-part of the recommendations: $\tilde{q}|_{-S} = q|_{-S}$:

$$\forall b^S \in A^S, \forall b^{-S} \in A^{-S},\ \tilde{q}(b^{-S} \mid b^S) = q(b^{-S} \mid b^S)$$

(3) $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium.

**PROOF.** Let $S \subseteq N$ be a coalition, let $d^S : A^S \to \Delta(A^S)$ be a deviating device, let $(\Omega, \mathcal{B}, \mu, (\mathcal{F}^i)_{i\in S}, (a^i)_{i\in N})$ be a consistent information structure, and let $\omega_0 \in \Omega$ be a state, such that it is common knowledge in $\omega_0$ that $\forall i,\ u^i_d(\omega) > u^i_f(\omega)$, i.e., $F^{meet}(\omega_0) \subseteq \left\{\omega \mid u^i_d(\omega) > u^i_f(\omega)\right\}$. Let $i \in S$ be a deviating player. Write $F^{meet} = F^{meet}(\omega_0) = \bigcup_j F^i_j$ where the $F^i_j$ are disjoint members of $\mathcal{F}^i$. Since $u^i_d(\omega) > u^i_f(\omega)$ throughout $F^{meet}$, then $\forall j,\ \tilde{u}^i_d(F^i_j) > \tilde{u}^i_f(F^i_j)$. Observe that if $E_1, E_2 \in \mathcal{B}$ are two disjoint non-null events then: $\tilde{u}^i_f(E_1 \bigcup E_2) = (\mu(E_1)\cdot \tilde{u}^i_f(E_1) + \mu(E_2)\cdot \tilde{u}^i_f(E_2))/\mu(E_1+E_2)$ and $\tilde{u}^i_d(E_1 \bigcup E_2) = (\mu(E_1)\cdot \tilde{u}^i_d(E_1) + \mu(E_2)\cdot \tilde{u}^i_d(E_2))/\mu(E_1+E_2)$. Thus, it follows that $\tilde{u}^i_d(F^{meet}) > \tilde{u}^i_f(F^{meet})$. This is true for every player, thus $\forall_{i\in S}\ \tilde{u}^i_d(F^{meet}) > \tilde{u}^i_f(F^{meet})$.

Let $\tilde{q}$ be the following strategy profile: $\forall b^S \in A^S$, $\forall b^{-S} \in A^{-S}$, $\tilde{q}(b) = \tilde{q}(b^S, b^{-S}) = \Pr\left(a^S(F^{meet}) = b^S\right)\cdot q(b^{-S} \mid b^S)$. We show that the three conditions are satisfied:

(1)

$$\forall b^S \in A^S,\ q(b^S) = 0 \Rightarrow \Pr\left(a^S(\Omega) = b^S\right) = 0$$
$$\Rightarrow \Pr\left(a^S(F^{meet}) = b^S\right) = 0 \Rightarrow \tilde{q}(b^S) = 0$$

(2)

$$\forall b^S \in A^S,\ \forall b^{-S} \in A^{-S},\ \tilde{q}(b^{-S} \mid b^S) = \frac{\tilde{q}(b^{-S}, b^S)}{\tilde{q}(b^S)}$$
$$= \frac{\Pr\left(a^S(F^{meet}) = b^S\right)\cdot q(b^{-S} \mid b^S)}{\Pr\left(a^S(F^{meet}) = b^S\right)} = q(b^{-S} \mid b^S)$$

(3) We have to show that $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium. Observe that:

$$\tilde{u}^i_f(F^{meet}) = \sum_{b^S\in A^S} \Pr\left(a^S\left(F^{meet}\right) = b^S\right) \sum_{b^{-S}\in A^{-S}} q(b^{-S} \mid b^S)\cdot u^i(b^S, b^{-S})$$
$$= \sum_{b\in A} \tilde{q}(b)\cdot u^i(b) = u^i(\tilde{q})$$

let $\tilde{p}$ be the *ex-ante* feasible deviation that is induced from $\tilde{q}$ by the deviating device $d^S$.

$$\begin{aligned}
\tilde{u}^i_d(F^{meet}) = & \sum_{b^S \in A^S} \Pr\left(a^S\left(F^{meet}\right) = b^S\right) \sum_{b^{-S} \in A^{-S}} q(b^{-S} \mid b^S) \\
& \sum_{c^S \in A^S} d^S(c^S \mid b^S) \cdot u^i(c^S, b^{-S}) \\
= & \sum_{b \in A} \tilde{q}(b) \sum_{c^S \in A^S} d^S(c^S \mid b^S) \cdot u^i(c^S, b^{-S}) = u^i(\tilde{p})
\end{aligned}$$

This implies that: $\forall i \in S$, $\tilde{u}^i_d(F^{meet}) > \tilde{u}^i_f(F^{meet}) \Rightarrow u^i(\tilde{p}) > u^i(\tilde{q})$, thus $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium. **QED.**

We finish our main result by the following proposition: If a "similar" agreement $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium, then $q$ itself is not an *ex-ante* strong correlated equilibrium.

**Proposition 15** Let $q, \tilde{q} \in \Delta(A)$ be two strategy profiles that satisfy the following conditions:

*(1)* $\tilde{q}|_S$ is absolute continues with respect to $q|_S$ *:*

$$\forall b^S \in A^S,\ q(b^S) = 0 \Rightarrow \tilde{q}(b^S) = 0$$

*(2)* Conditioned on $S$-part of the recommendations: $\tilde{q}|_{-S} = q|_{-S}$*:*

$$\forall b^S \in A^S, \forall b^{-S} \in A^{-S},\ \tilde{q}(b^{-S} \mid b^S) = q(b^{-S} \mid b^S)$$

*(3)* $\tilde{q}$ is not an *ex-ante* strong correlated equilibrium.

Then $q$ is not an *ex-ante* strong correlated equilibrium.

**PROOF.** For simplicity of notation, we assume w.l.o.g. that $\forall a^S \in A^S\ q(a^S) > 0$ (because $q(a^S) = 0 \Rightarrow \tilde{q}(a^S) = 0$, and those impossible action profiles do not affect any of the utilities functions and can be omitted). Let $\tilde{d}^S : A^S \to \Delta(A^S)$ be a deviating device, such that $\forall i,\ u^i(\tilde{p}) > u^i(\tilde{q})$, where $\tilde{p} \in \Delta(A)$ is the feasible deviation induced from $\tilde{d}^S$. Let $m = \max_{i \in S} |A^i|$ and let $\varepsilon = \frac{1}{m} \min_{a^S \in A^S, \tilde{q}(a^S) > 0} \frac{q(a^S)}{\tilde{q}(a^S)}$. We begin by constructing an auxiliary deviating device $\tilde{d}^S_\varepsilon : A^S \to \Delta(A^S)$:

$$\tilde{d}^S_\varepsilon(a^S|b^S) = \begin{cases} \varepsilon\tilde{d}^S(a^S|b^S) & a^S \neq b^S \\ 1 - \sum_{c^S \neq b^S} \varepsilon\tilde{d}^S(c^S|b^S) & a^S = b^S \end{cases}$$

In the deviating device $\tilde{d}^S_\varepsilon$ the players of $S$ follow the agreement with probability $1 - \varepsilon$, and deviate according to $\tilde{d}^S$ with probability $\varepsilon$. Let $\tilde{p}_\varepsilon \in D(\tilde{q}, S)$

be the feasible *ex-ante* deviation of $S$ from $\tilde{q}$ that is induced by $\tilde{d}^S_\varepsilon$. Observe that $\tilde{p}_\varepsilon$ is a profitable deviation for all the players in $S$: $\forall_{i\in S}$, $u^i(\tilde{q}) < u^i(\tilde{p}_\varepsilon)$ (because $u^i(\tilde{q}) - u^i(\tilde{p}_\varepsilon) = \varepsilon\left(u^i(\tilde{q}) - u^i(\tilde{p})\right) < 0$).

We continue by constructing the following deviating device (of $S$) $d^S : A^S \to \Delta(A^S)$:

$$d^S(a^S|b^S) = \begin{cases} \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(a^S|b^S) & a^S \neq b^S \\ 1 - \sum\limits_{c^S \neq b^S} \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(c^S|b^S) & a^S = b^S \end{cases}.$$

We first show that $d^S$ is a valid deviating device by validating that $\forall a^S, b^S \in A^S$ $0 \le d^S(a^S|b^S) \le 1$.

$$\begin{aligned}\forall a^S \neq b^S,\ d^S(a^S|b^S) &= \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(a^S|b^S) = \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \varepsilon\tilde{d}^S(a^S|b^S) \\ &= \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \frac{1}{m}\left(\min_{a^S\in A^S, \tilde{q}(a^S)>0} \frac{q(a^S)}{\tilde{q}(a^S)}\right) \cdot \tilde{d}^S(a^S|b^S) \\ &\le \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \frac{1}{m} \cdot \frac{q(b^S)}{\tilde{q}(b^S)} \cdot \tilde{d}^S(a^S|b^S) = \frac{1}{m} \cdot \tilde{d}^S(a^S|b^S) \le 1\end{aligned}$$

And using the inequality (which is a part of the last chain of inequalities):

$$\frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(a^S|b^S) \le \frac{1}{m} \cdot \tilde{d}^S(a^S|b^S)$$

We get:

$$d^S(a^S|a^S) = 1 - \sum_{a^S \neq b^S} \frac{\tilde{q}(b^S)}{q(b^S)} \cdot \tilde{d}^S_\varepsilon(a^S|b^S) \ge 1 - \frac{1}{m}\sum_{a^S\neq b^S} \tilde{d}^S(a^S|b^S) \ge 1 - 1 \ge 0.$$

Let $p \in D(q, S)$ be the feasible *ex-ante* deviation that is induced by $d^S$. We finish the proof by showing that $p$ is a profitable deviation from $q$: i.e.,
$\forall i \in S$, $u^i(q) < u^i(p)$), and thus $q$ is not an *ex-ante* strong correlated equilibrium. Let $i \in S$. We show: $u^i(p) - u^i(q) \stackrel{?}{=} u^i(\tilde{p}_\varepsilon) - u^i(\tilde{q}) > 0$. Observe that:

$$u^i(q) = \sum_{a\in A} q(a) \cdot u^i(a) = \sum_{a^S\in A^S} q(a^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|a^S) \cdot u^i(a)$$

$$u^i(p) = \sum_{a\in A} p(a)\cdot u^i(a) = \sum_{a^S\in A^S}\sum_{b^S\in A^S} q(b^S)\cdot d^S(a^S|b^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|b^S)\cdot u^i(a)$$

Therefore:

$$\begin{aligned}
u^i(p) - u^i(q) &= \sum_{a^S\in A^S}\sum_{b^S\in A^S} q(b^S)\cdot d^S(a^S|b^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|b^S)\cdot u^i(a) \\
&\quad - \sum_{a^S\in A^S} q(a^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|a^S)\cdot u^i(a) \\
&= \sum_{a^S\in A^S}\sum_{b^S\neq a^S} q(b^S)\cdot d^S(a^S|b^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|b^S)\cdot u^i(a) \\
&\quad - \sum_{a^S\in A^S} q(a^S)\cdot \Big(1 - d^S(a^S|a^S)\Big) \sum_{a^{-S}\in A^{-S}} q(a^{-s}|a^S)\cdot u^i(a) \\
&= \sum_{a^S\in A^S}\sum_{b^S\neq a^S} \tilde{q}(b^S)\cdot \tilde{d}^S_\varepsilon(a^S|b^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|b^S)\cdot u^i(a) \\
&\quad - \sum_{a^S\in A^S} \tilde{q}(a^S)\cdot \Big(1 - \tilde{d}^S_\varepsilon(a^S|a^S)\Big) \sum_{a^{-S}\in A^{-S}} q(a^{-s}|a^S)\cdot u^i(a)
\end{aligned}$$

Last equality is due to the following two equalities:

$$\forall a^S \neq b^S,\ q(b^S)\cdot d^S(a^S|b^S) = q(b^S)\frac{\tilde{q}(b^S)}{q(b^S)}\cdot \tilde{d}^S_\varepsilon(a^S|b^S) = \tilde{q}(b^S)\cdot \tilde{d}^S_\varepsilon(a^S|b^S)$$

$$\begin{aligned}
q(a^S)\cdot\Big(1 - d^S(a^S|a^S)\Big) &= q(a^S) \sum_{c^S\neq a^S} \frac{\tilde{q}(a^S)}{q(a^S)}\cdot \tilde{d}^S_\varepsilon(c^S|a^S) \\
&= \tilde{q}(a^S) \sum_{c^S\neq a^S} \frac{q(a^S)}{\tilde{q}(a^S)}\cdot\frac{\tilde{q}(a^S)}{q(a^S)}\cdot \tilde{d}^S_\varepsilon(c^S|a^S) \\
&= \tilde{q}(a^S) \sum_{c^S\neq a^S} \tilde{d}^S_\varepsilon(c^S|a^S) \\
&= \tilde{q}(a^S)\cdot\Big(1 - \tilde{d}^S_\varepsilon(a^S|a^S)\Big)
\end{aligned}$$

We finish the proof by showing that the last expression is equal to $u^i(\tilde{p}_\varepsilon) - u^i(\tilde{q})$. Observe that:

$$\begin{aligned}
u^i(\tilde{p}_\varepsilon) &= \sum_{a\in A} \tilde{p}_\varepsilon(a)\cdot u^i(a) \\
&= \sum_{a^S\in A^S}\sum_{b^S\in A^S} \tilde{q}(b^S)\cdot \tilde{d}^S_\varepsilon(a^S|b^S) \sum_{a^{-S}\in A^{-S}} q(a^{-S}|b^S)\cdot u^i(a)
\end{aligned}$$

$$u^i(\tilde{q}) = \sum_{a \in A} \tilde{q}(a) \cdot u^i(a) = \sum_{a^S \in A^S} \tilde{q}(a^S) \sum_{a^{-S} \in A^{-S}} q(a^{-S}|a^S) \cdot u^i(a)$$

Therefore:

$$\begin{aligned} u^i(p_\varepsilon) - u^i(\tilde{q}) = & \sum_{a^S \in A^S} \sum_{b^S \in A^S} \tilde{q}(b^S) \cdot \tilde{d}^S_\varepsilon(a^S|b^S) \sum_{a^{-S} \in A^{-S}} q(a^{-S}|b^S) \cdot u^i(a) \\ & - \sum_{a^S \in A^S} \tilde{q}(a^S) \sum_{a^{-S} \in A^{-S}} q(a^{-S}|a^S) u^i(a) \\ = & \sum_{a^S \in A^S} \sum_{b^S \neq a^S} \tilde{q}(b^S) \cdot \tilde{d}^S_\varepsilon(a^S|b^S) \sum_{a^{-S} \in A^{-S}} q(a^{-S}|b^S) \cdot u^i(a) \\ & - \sum_{a^S \in A^S} \tilde{q}(a^S) \left(1 - \tilde{d}^S_\varepsilon(a^S|a^S)\right) \sum_{a^{-S} \in A^{-S}} q(a^{-s}|a^S) u^i(a) \end{aligned}$$

**QED**.

## 5 Coalition-Proof Correlated Equilibria

In the last Section we show that an *ex-ante* strong correlated equilibrium *à* la Moreno & Wooders is also appropriate to frameworks in which players can plan deviations at all stages. A natural question is whether a similar result holds for their notion of coalition-proof correlated equilibrium.[7] In this Section we show that the answer is negative, by presenting an example (adapted from Bloch & Dutta, 2007), in which there is an *ex-ante* coalition-proof correlated equilibrium that is not self enforcing agreement in a framework in which communication is possible at all stages. Table 2 presents the matrix representation of a two-player game and an *ex-ante* coalition-proof correlated equilibrium.

Table 2
A Two-Player Game and an *Ex-ante* Coalition-Proof Correlated Equilibrium

| | $b_1$ | $b_2$ | $b_3$ |
|---|---|---|---|
| $a_1$ | 6,6 | -2,0 | 0,7 |
| $a_2$ | 2,2 | 2,2 | 0,0 |
| $a_3$ | 0,0 | 0,0 | 3,3 |

| | $b_1$ | $b_2$ | $b_3$ |
|---|---|---|---|
| $a_1$ | 1/2 | 0 | 0 |
| $a_2$ | 1/4 | 1/4 | 0 |
| $a_3$ | 0 | 0 | 0 |

We first show that the profile presented in table 2 is an *ex-ante* coalition-proof equilibrium. First, observe that the profile is a correlated equilibrium:

[7] Recall (Moreno & Wooders, 1996) that an *ex-ante* coalition-proof correlated equilibrium is a strategy profile from which no coalition has a self-enforcing and improving *ex-ante* deviation. An *ex-ante* deviation is self enforcing, if no proper sub-coalition has a further self-enforcing and improving deviation.

no player has a profitable unilateral deviation. Moreno & Wooders (1996) have proved that in a two-player game, every correlated profile which is not Pareto-dominated by another correlated equilibrium is a coalition-proof correlated equilibrium. Our profile gives each player a payoff of 4. Thus we prove that the profile is an *ex-ante* coalition-proof correlated equilibrium, by showing that any correlated equilibrium $q$ gives player 1 a payoff of at most 4. Let $x = q(a_1, b_1)$. Observe that $q(a_2, b_1) \geq x/2$ because otherwise player 1 would have a profitable deviation: playing $b_3$ when recommended $b_1$. This implies $q(a_2, b_2) \geq x/2$, because otherwise player 2 would have a profitable deviation (playing $a_1$ when recommended $a_2$). Thus the payoff of $q$ conditioned on that the recommendation profile is in $A = ((a_1, b_1), (a_2, b_1), (a_2, b_2))$ is at most 4, and because the payoff of $q$ conditioned on that the recommendation profile is not in A is at most 3, then the total payoff of $q$ is at most 4.

We now explain why this profile is not a self-enforcing agreement in a framework in which the players can communicate and plan deviations also at the *ex-post* stage.[8] Assume that the players have agreed to play the profile, and player 1 has received a recommendation $a_2$. In that case, he can communicate with player 1 at the *ex-post* stage, tell him that he received $a_2$ (and thus if the players follow the recommendation profile they would get a payoff of 2), and suggest a joint deviation - playing $(a_3, b_3)$. As player 1 has no incentive to lie (to make a false claim that his recommendation is $a_2$ when it is $a_1$), then player 2 would believe player 1, and they would both play $(a_3, b_3)$. This *ex-post* deviation is self-enforcing: $(a_3, b_3)$ is a Nash equilibrium, and thus no player has a profitable sub-deviation.

Observe that the same deviation is not self–enforcing in the *ex-ante* stage. If the players agree at the *ex-ante* stage to implement a deviating device that changes $(a_2, b_1)$ into $(a_3, b_3)$, then player 2 will have a profitable sub-deviation: playing $b_3$ when recommended $b_1$. Similarly, if they agree to implement a deviating device that changes $(a_2, b_2)$ into $(a_3, b_3)$, then player 1 will have a profitable sub-deviation - playing $a_1$ when recommended $a_2$.

## 6 Concluding Remarks

(1) *Bayesian games:* Moreno & Wooders (1996) present a notion of *ex-ante* strong communication equilibrium in Bayesian games. Our result can be

[8] The profile is not an *ex-post* coalition-proof correlated equilibrium according to the definitions of Bloch & Dutta (2007) and Ray (1998). The profile is an *ex-post* coalition-proof correlated equilibrium according to the definition of Einy & Peleg, due to their requirement that an *ex-post* deviation would be strictly profitable to each player given all recommendations he may receive.

extended to this framework as well, to show that an *ex-ante*[9] strong communication equilibrium is also resistant to deviations at all stages.

(2) *k-strong equilibria*: In Heller (2008) an *ex-ante* notion of $k$-strong correlated equilibrium is defined as a strategy profile that is resistant to all coalitional deviations of up to $k$ players. Our result can be directly extended to this notion as well: an *ex-ante* $k$-strong correlated equilibrium is resistant to deviations of up to $k$ players at all stages.

(3) *Related Literature*:
   (a) The question of existence of strong and coalition-proof correlated equilibria is discussed in Moreno & Wooders (1996), Milgrom & Roberts (1996), Ray (1996), Holzman & Law-Yone (1996), and Bloch & Dutta (2007).
   (b) Applications of strong and coalition-proof equilibria are presented and discussed in Bernheim & Whinston (1986, 1987), Einy & Peleg (1996) and Delgado & Moreno (2004).

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