The environment and the long run: A comparison of different criteria

Chichilnisky, Graciela and Beltratti, Andrea and Heal, Geoffrey

1994

Ricerche Economiche (1994) **48**, 319–340

The environment and the long run: a comparison of different criteria

Andrea Beltratti
University of Torino and Fondazione ENI Enrico Mattei

Graciela Chichilnisky
Columbia University

Geoffrey Heal
Columbia Business School

Summary

We use growth models with natural resources to study the consequences of a ranking of intertemporal paths, due to Chichilnisky, which places weight on their very long run or limiting characteristics as well as on their characteristics over any finite period. This criterion shows more intertemporal symmetry or egalitarianism than the discounted utilitarian approach, which clearly emphasizes the immediate future at the expense of the long run. In this respect it captures the concerns of those who argue for sustainability and for a heightened sense of responsibility to the future. In some of the examples that we consider, the long-run characteristics of paths optimal by this criterion are a mixture of those of utilitarian paths and the "green golden rule" (the configuration which maximizes long-run sustainable utility from consumption and environment).

J.E.L. Classification: D90, Q20.
Keywords: Sustainable, resources, environment, growth.

1. Introduction

The literature on intergenerational equity is largely built around the discounted utilitarian approach to defining an optimal path, and many authors have expressed reservations about the balance this strikes between present and future, see, for example, Ramsey (1928), Solow (1974) and others (this literature is summarized in Dasgupta & Heal, 1974). Ramsey and Harrod were scathing about the ethical dimensions of discounting in a more general context,

0035–5054/94/040319 + 22 $08.00/0

commenting respectively that discounting "*is ethically indefensible and arises merely from the weakness of the imagination*" and that it is a "*polite expression for rapacity and the conquest of reason by passion*" (see Ramsey, 1928; Harrod, 1948; Heal, 1993a†).

It may be fair to say that discounted utilitarianism dominates our approach more for lack of convincing alternatives‡ than because of the conviction that it inspires. It has proven particularly controversial with non-economists concerned with environmental valuations. A positive discount rate forces a fundamental asymmetry between present and future generations, particularly those very far into the future. This asymmetry is troubling when dealing with environmental matters such as climate change, species extinction and disposal of nuclear waste, as many of the consequences of these may be felt only in the very long run indeed, a hundred or more years into the future. At any positive discount rate these consequences will clearly not loom large (or even at all) in project evaluations. If one discounts present world GNP over two hundred years at 5% per annum, it is worth only a few hundred thousand dollars, the price of a good apartment. Discounted at 10%, it is equivalent to a used car. On the basis of such valuations, it is irrational to be concerned about global warming, nuclear waste, species extinction, and other long-run phenomena. Yet societies *are* worried about these issues, and are actively considering devoting very substantial resources to them. So part of our concern about the future is not captured by discounted utilitarianism.

Here we take a new approach to this issue, using a welfare criterion developed by Chichilnisky (1993), and explore the implications of an alternative formulation of intertemporal welfare criteria which places positive weight on the very long run properties of a growth path. Technically speaking, it places positive weight on the limiting properties of a path. Chichilnisky (1993) notes that selecting an objective in intertemporal planning involves solving a social choice problem,§ the problem of combining the preferences of different generations. Difficulties arise because the distinctive feature of this problem is that there are infinitely many "voters". As Hotelling remarked, "*problems with exhaustible resources are peculiarly liable to become entangled with the infinite*". Chichilnisky presents a set of axioms for intergenerational social choice which imply that positive weight be placed on the limiting properties of alternative utility streams, as well as on their properties over

† Heal (1993a) has argued that a zero consumption discount rate can be consistent with a positive utility discount rate in the context of environmental projects.

‡ Other criteria, for example, the Ramsey criterion or the overtaking criterion, are incomplete orders, and fail to rank many reasonable alternatives.

§ An early precedent for using a social choice framework for studying intertemporal equity is Frerejohn and Page (1978).

finite horizons. Her approach builds on her earlier axiomatization of the social choice problem (Chichilnisky, 1980) combining continuity, equal treatment and respect of unanimity with its application to infinite populations by Lauwers (1993) and Lauwers and van Liederkierke (1993). Chichilnisky introduces two axioms which underlie our approach: these are that neither "the present" nor "the future" should be dictatorial. Non-dictatorship of the present means that it should not be possible to determine the ranking of any two utility streams by looking only at finite numbers of their components. Equivalently, it should always be possible to reverse the ordering of any two sequences by changing elements that are arbitrarily far along the sequences. Non-dictatorship of the future means that the ranking of two utility streams should not depend only on their limiting properties, but must be sensitive to their characteristics over finite horizons. Again, it must be possible to alter ratings of sequences by changing only "early" elements of the sequences. These axioms suffice to characterize the valuation of utility streams as the sum of two terms, one that is a discounted integral of utilities and one that depends on the limiting properties of the stream.†

In the following we use both the standard discounted and the Chichilnisky criteria in characterizing optimal use of environmental assets. We work with several different models of the economy as a step towards understanding when the implications of the two criteria are different, and whether such differences are somehow connected with the description of the economic structure. In a companion paper (Beltratti, Chichilnisky & Heal, 1994) we study in detail one of the models presented here and use it to formalize the concept of sustainable growth, an issue that in fact seems very related to the comparative evaluation of the welfare of current vs. future generations.

The plan of the paper is as follows: in Section 2 we introduce a model of growth with an environmental asset that enters the utility function. In Section 3 we formalize the Chichilnisky criterion function, and in Sections 4 and 5 we separately solve the model with both the utilitarian maximand and the long-run utility maximand. In Section 6 we apply the Chichilnisky criterion to two simplified versions of the general model, which describe cases

† Technically, this result builds on the Yosida–Hewitt theorem (1952) of functional analysis, which states that a continuous linear functional on a Banach space can be represented as the sum of an integral against a countably additive measure and an integral against a purely finitely additive measure. A measure is countably additive if the measure of a countable family of disjoint sets is the sum of their measures. For a purely finitely additive measure, this property holds only for finite families of disjoint sets. In the theory of general equilibrium with infinitely many commodities, trouble is taken to ensure that only countably additive measures occur naturally—see Chichilnisky and Heal (1993).

of exhaustible and renewable resources. Section 7 offers some concluding remarks.

2. A model of growth with a natural asset

Dasgupta and Heal (1974) consider an economy that uses a stock of an asset as source for an instantaneous flow that enters the production function. We both extend and slightly modify that model. On the one hand, we add a regeneration process for the natural resource, so that it becomes a renewable rather than exhaustible resource, which could be interpreted as an environmental resource, such as a rain forest, the climate, species diversity, etc. To be coherent with this interpretation, we also include the stock of such a resource in the utility function (as in Krautkraemer, 1985).

The significance of these extensions is as follows. The presence of a *renewable* resource means that it is possible in principle for a positive stock of the resource to be maintained indefinitely. The fact that the resource is an argument of the utility function captures the concern that environmental resources are an important determinant of the quality of life, one of which our long-term successors may ultimately be deprived.

In our framework the social valuation of the state of the economy at time t depends on the level of consumption of a produced good and on the existing stock of an environmental good. Formally:

ASSUMPTION 1: *instantaneous utility is given by the strictly concave continuously differentiable real valued function $u(C_t, A_t)$ defined on consumption $C_t \in \mathscr{R}$ and on the stock of an environmental good $A_t \in \mathscr{R}$. We also assume, without any loss of generality, that $u(A,C)$ is bounded above. We assume that $\partial u/\partial C > 0$ always; u may show satiation in A so that $\partial u/\partial A$ may have either sign.*

ASSUMPTION 2: *production occurs according to the production function $F(K_t)$ where K_t is the stock of produced capital at time t. Capital accumulation is therefore described by the usual equation:*

$$\dot{K}_t = F(K_t) - C_t. \tag{1}$$

In another paper (Beltratti, Chichilnisky & Heal, 1994) we allow for the influence of the environmental stock in the production function; such an extension complicates the dynamics and is not essential to our main concern in this paper.

ASSUMPTION 3: *the stock of the environmental good has the ability to renew itself: the rate of renewal is given by the function $R(A)$,* *satisfying* $R(0)=0$. *However, the act of consuming output may deplete the environment, so that the net rate of change of the stock of the environment is*

$$\dot{A}_t = -\alpha C_t + R(A_t),\ \alpha \geq 0. \tag{2}$$

We assume that the renewal function R is bounded above (i.e. $\exists B: R(A) \leq B \forall A$*). R may exhibit a threshold effect, i.e.* $\exists H: R(A) = 0 \forall A \leq H$ *and R(A) is strictly concave for* $A \geq H$*. It is possible that above a certain level of A, R(A) may be decreasing, i.e.* $R'(A)<0$ *for* $A>A_m$*. This is always the case for the most commonly used reproduction function, the Pearl–Verhulst logistic model. In addition, it is assumed that the set of attainable values of A is bounded above, so that there is a limit to the amount of the environmental resource that can be accumulated.*

In addition, certain initial conditions and non-negativity constraints are imposed:

$$K_0 = \bar{K},\ A_0 = \bar{A},\ K_t \geq 0,\ A_t \geq 0,\ C_t \geq 0. \tag{3}$$

Because of (2) and the boundedness of $R(A)$, the depletion of the environment may exceed the environment's capacity to regenerate itself. It is possible to attain consumption levels that are not compatible with indefinite preservation of a positive stock of the environmental good.

3. Maximization criteria

Chichilnisky's axioms establish a criterion function representing the preferences of an infinite sequence of generations each of whom lives for an instant of time. This criterion function depends both on the sum of utilities over time and on the long-run behaviour of utility values and has the form:

$$\theta \int_0^\infty u(C_t, A_t)\mu(t)\mathrm{d}t + (1-\theta)\lambda(\bar{u}), \tag{4}$$

where $\theta \in [0,1]$, $\mu(t)$ is a countably additive measure, $\bar{u}$ represents the entire consumption path u_t, $t \in (0, \infty)$, and λ is a singular or purely finitely additive measure. A singular or purely finitely additive measure is a function of a sequence that depends only on the limiting properties of that sequence. Here without great loss of generality we take it that

$$\lambda(\bar{u}) = \lim_{t\to\infty} u(C_t, A_t). \tag{5}$$

The measure $\mu(t)$ is assumed to have the form $\mu(t)=e^{-\delta t}$, so that the criterion function as applied here takes the form

$$\theta\int_0^{\infty} u(C_t,A_t)e^{-\delta t}dt+(1-\theta)\lim_{t\to\infty} u(C_t,A_t). \tag{6}$$

In effect, what we are doing here is rather simple and intuitive: we are supplementing the conventional discounted utilitarian criterion with a term that depends only on the very long run behaviour of utility sequences. The value of the term $\lim_{t\to\infty} u(C_t,A_t)$ is not affected by changes in the values of C_t or A_t for any finite t. This term only depends on the very long run or limiting behaviour of utility values. The use in the criterion function of terms such as lim inf, lim and long-run average which depend on the very long run behaviour of the instantaneous payoff function is common in dynamic programming and dynamic games (see Dutta, 1991). These are conventional elements of an intertemporal criterion function in situations where the very long run matters. Returning to the perspective given by the sustainability debate, Chichilnisky's axioms allow us to capture a concern for sustainability—the capacity to generate welfare in the very long run, for our distant successors—by including in the maximand (6) a term commonly used for valuing long-run characteristics of payoff sequences in game theory and dynamic programming.

The overall optimization problem that we study is the maximization of (6) subject to the constraints on capital accumulation (1), resource renewal (2) and to initial conditions (3). Our approach to solving this problem is to note that it is solvable by conventional methods in the extreme cases of $\theta=1$ (pure discounted utilitarianism) and $\theta=0$ (maximizing the long run value of utility), and then to base a general argument on the solution in these two cases. First, we consider the pure discounted utilitarian case in which $\theta=1$. Initially, we study the problem posed above in its full generality: subsequently, it will prove necessary to simplify it in order to derive precise results.

4. The utilitarian solution

Setting $\theta=1$, our problem is

$$\max\int_0^{\infty} u(C_t,A_t)e^{-\delta t}dt,\ \delta>0, \tag{7}$$

subject to (1), (2) and (3). The Hamiltonian is the following:

$$H = e^{-\delta t}u(C,A) + pe^{-\delta t}[F(K) - C] + qe^{-\delta t}[-\alpha C + R(A)]. \tag{8}$$

The first-order conditions are:

$$u_C = p + q\alpha, \tag{9}$$

$$\dot{p} - \delta p = -pF_K, \tag{10}$$

$$\dot{q} - \delta q = -u_A - qR_A, \tag{11}$$

$$\lim_{t \to \infty} e^{-\delta t} pK = 0, \tag{12}$$

$$\lim_{t \to \infty} e^{-\delta t} qA = 0. \tag{13}$$

A stationary solution to the necessary conditions (9)–(13) above must satisfy the following:

$$u_C(C,A) = p + q\alpha, \tag{14}$$

$$\delta = F_K(K), \tag{15}$$

$$\delta q = u_A(C,A) + qR_A(A), \tag{16}$$

$$F(K) = C, \tag{17}$$

$$\alpha C = R(A). \tag{18}$$

According to equation (14), the marginal utility of stationary consumption has to be equal to a linear combination of the prices of the two stocks, in order to take into account the fact that the consuming prevents capital accumulation and depletes the environment. According to equation (15), capital must be such as to make its marginal productivity equal to the rate of time preference. Equation (16) yields the shadow price of the environment as a present discounted value of marginal utility and marginal productivity, while (18) shows that in a stationary state the consumption of goods must be proportional to the regenerative capacity of the environment.

These conditions point to the following proposition:

PROPOSITION 1: *if $R(A)$ has a threshold and is strictly concave above this, then there may be zero, one or two stationary solutions with a positive level of A. These are characterized as solutions to:*

$$\alpha F[F_K^{-1}(\delta)] = R(A).$$

PROOF: a stationary solution can be shown to exist in the following way.

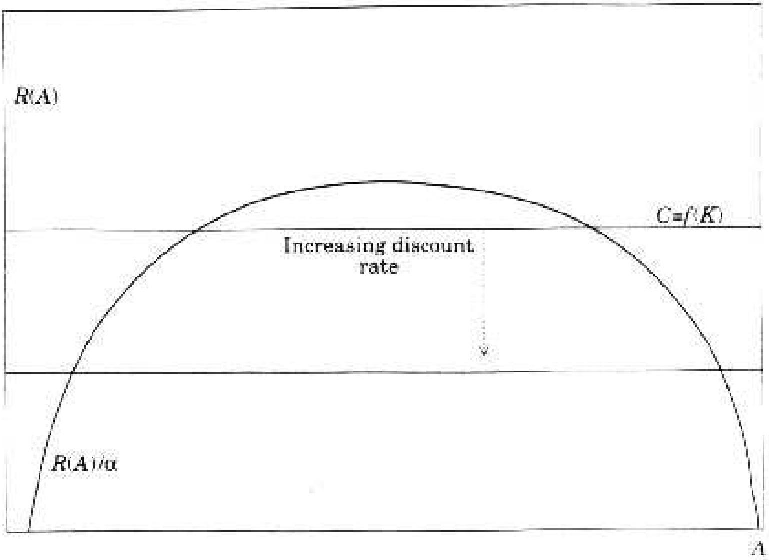

FIGURE A. Stationary solutions of the utilitarian problem are intersections of the line and the curve.

Given the rate of time preference, equation (15) determines the optimal stock of capital, K'; equation (17) then determines the level of consumption C' associated with such a level of capital. Equation (18) then determines the stock of environment A' that is necessary to sustain such a level of consumption in the steady state. Finally, from equations (14) and (16) it is possible to determine the shadow prices of the stocks. There will be a stationary solution if the various relationships are consistent, that is if the stock of capital that is dictated by the rate of time preference is associated with a level of consumption that does not exceed the maximum regenerative capacity of the environment.

The consistency or otherwise of these relationships is explored in Figure A, where we plot the relationship between C and K given by $C=F[F_k^{-1}(\delta)]$ and that given by $C=\alpha^{-1}R(A)$. Regardless of the existence of a threshold in the regeneration function, there may be zero, one or two intersections. This completes the proof. ■

Stability of stationary solutions to the system of differential equations given by (9)–(13) is studied in the Appendix. Here we summarize the results.

PROPOSITION 2: *consider a stationary solution at which $R_A<0$. It is possible to show that for small enough values of the discount rate δ the stationary solution is a saddle point.*

COROLLARY 1: *if $R_A>0$ at a stationary solution but the remaining conditions of proposition 2 are satisfied, then for small enough values of the discount rate δ (which have to satisfy $\delta<R_A$), the stationary solution is a saddle point. In general the bound on δ in this corollary will be tighter than that in the proposition.*

Intuitively, these results say that the right hand of the stationary solutions in Figure A will be locally stable in the saddle point sense. From the figure, one can also see that at the stationary solution there exists a positive relationship between the rate of time preference and the steady state stock of the environment. This is due to including in the model (through the regeneration function) a positive relationship between the steady state values of consumption and the environment. A larger rate of time preference is associated with a lower level of the capital stock and of consumption in the steady state, and this decreases the pressure that the economic system puts on the stock of environmental asset.

It is also important to notice that the solution exists only as long as the discount rate is larger than a positive value δ_m, due to the assumption of a bounded reproduction function. For example, in the case of the logistic function $R(A)=rA-(rA^2/A^s)$, which will again be used in what follows, the largest flow of consumption that can be deducted from the environment each instant in a steady state (the maximum sustainable yield) corresponds to $A=A^s/2$, and is equal to $rA^s/4$. In this case δ_m is the solution to:

$$\alpha F[F_K^{-1}(\delta_m)]=r\frac{A^s}{4}.$$

That the discount rate cannot be too low can also be seen from the first-order conditions (14)–(18), which do not admit any solution for $\delta\to 0$, as a discount rate tending to zero would imply an infinite capital stock from (15), and this would be associated with an infinite consumption from (17), which contrasts with the constraint provided by (18).

The non-existence of the solution for a low discount rate points out that for some models one cannot think of improving on an equilibrium which discriminates against the future generations simply by lowering the effective discount rate. Maximizing long-run utility in the way described in the next section is a way to find a solution which gives more weight than the utilitarian to the welfare of future generations.

5. Maximizing long-run utility

Now we consider the case in which, in (6), $\theta=0$ and society is only concerned with the very long run values of consumption and environment. We seek a path of consumption and capital accumulation that maximizes $\lim_{t\to\infty} u(C_t,A_t)$ over the set of feasible paths. Then the solution admits a straightforward characterization given in the following proposition. Note that in the model used here, the maximization of lim inf, lim sup, lim, or any increasing function of any of these would lead to the same characterization.

PROPOSITION 3: *there exist values* (A^*,K^*,C^*) *such that*

$$\lim_{t\to\infty}\{A_t,K_t,C_t\}=\{A^*,K^*,C^*\}$$

is a necessary and sufficient condition for a feasible path $\{A_t,K_t,C\}\forall t$ *to be a solution of the problem "maximize* $\lim_{t\to\infty}$ $u(C_t,A_t)$ *over all feasible paths".* (A^*,K^*,C^*) *is characterized by* $u_C/u_A=-\chi/R_A$.

PROOF: the maximand is independent of the values of A and C at any finite dates. The solution of the problem therefore requires that we find the indefinitely maintainable values of C and A which give the maximum utility level over all such levels. As indefinitely maintainable values of C and A satisfy $R(A)=\chi C$, this means that the problem

$$\max \lim_{t\to\infty} u(C_t,A_t) \text{ over feasible paths satisfying (1)–(3)}$$

reduces to:

$$\max u(C,A) \text{ subject to } R(A)=\chi C. \tag{19}$$

The stock of capital is not a concern in this situation because any stock of capital can be accumulated given a sufficiently long period of time. The set of $\{C,A\}$ pairs satisfying the constraint in (19) is compact, so this problem is well-defined. The maximum is characterized by the first order condition:

$$\frac{u_A}{u_C}=-\frac{R_A}{\chi}. \tag{20}$$

This completes the proof. ∎

The solution given in the proposition involves equality between the marginal rate of transformation and the marginal rate of substitution between consumption and environment across steady states. It can be depicted in graphical terms as the point of tangency between the indifference curve and the renewal function (see

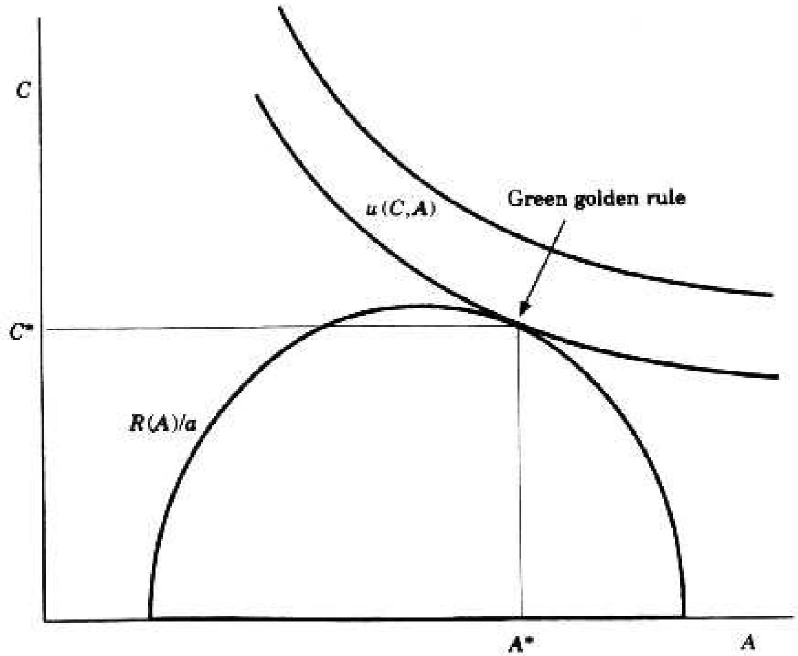

FIGURE B. The path which maximizes lim $u(C,A)$ approaches (A^*, C^*), the "green golden rule" and the point giving the highest sustainable utility level.

Figure B). It is clear that the solution to the problem of maximizing the limiting utility value does not define a growth path for the economy: it merely defines a long-run or limiting configuration. There are many paths that will lead to this, some efficient, others inefficient. Amongst the efficient paths, some will give higher values of the integral of discounted utilities than others.

There is a close connection between optimality according to the Chichilnisky criterion with $\theta=0$ (all weight on the long run) and the Meade–Phelps–Robinson golden rule of economic growth (see Phelps, 1961). Such a connection is not surprising: Phelps described the golden rule as the growth path that gives the highest indefinitely maintainable level of consumption per head. Clearly, there is an implicit concept of sustainability here: the golden rule path is the best sustainable path. Our green golden rule gives the highest indefinitely maintainable level of instantaneous utility, in a framework where environmental goods are valued in their own rights, i.e. are a source of utility. It is a generalization of the earlier concept. It is an easily-defined concept which is an essential element in the task of making operational the Chichilnisky criterion. Our green golden rule has points in common with Brock's "polluted golden age" (Brock, 1977), although he models a pollution stock rather than an environmental asset.

6. The Chichilnisky criterion

We have developed a methodology for analysing paths which are optimal with respect to the very long run properties of a growth path. Now we combine this with the utilitarian approach and then apply this methodology to two problems, each a special case of the general problem above. In one case, there is a solution to the overall problem which is intermediate between the discounted utilitarian and the green golden rule solutions. In the second case, there appears to be no solution to the overall optimization problem, although there are solutions to the two components taken separately. These special cases of the earlier problem are obtained by dropping the production side of that problem and considering only a natural resource that is consumed and whose stock is a source of utility. In one case the resource is taken to be exhaustible: in the other, renewable. The most general form of the problem, with two state variables, remains to be solved.

6.1. AN EXHAUSTIBLE RESOURCE

In this case, the overall optimization problem is

$$\max \theta \int_0^{\infty} \{u_1(C_t)+u_2(A_t)\} e^{-\delta t} dt + (1-\theta) \lim_{t\to\infty} \{u_1(C_t)+u_2(A_t)\},$$

$$\text{subject to} \int_0^{\infty} C_t \, dt \le A_0.$$

The discounted utilitarian part of this is closely related to a problem considered in Krautkraemer (1985), except that the choice of an additively separable utility function makes it possible to characterize a solution with greater precision than was possible in his case.

It is obvious that the "green golden rule" solution in this case involves setting $C_t = 0 \forall t$, and maintaining the initial stock intact for ever, $A_t = A_0 \forall t$. Complete preservation of the resource is required.

The following equations give necessary conditions for a solution to the discounted utilitarian problem:

$$u_{1C}(C) = q,$$

$$\frac{dq}{dt} - \delta q = -u_{2A}(A),$$

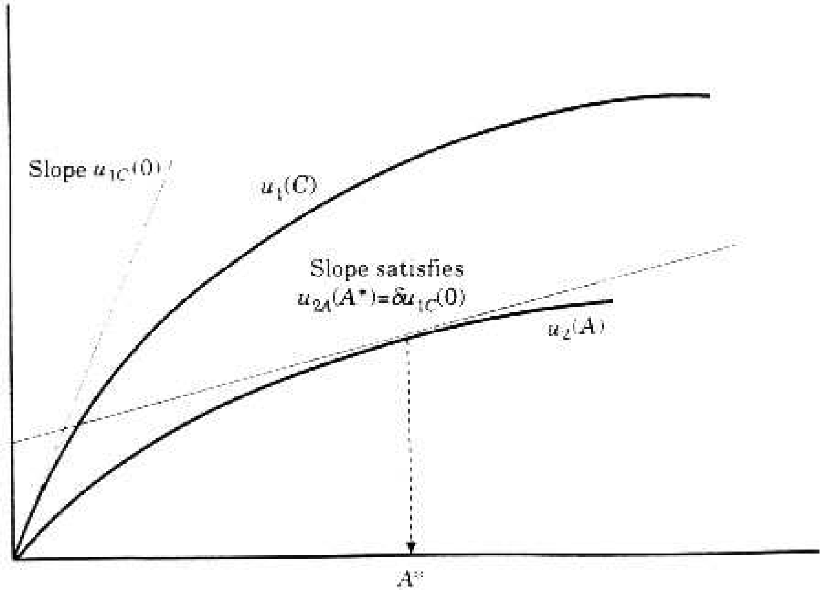

FIGURE C. Determining the stationary stock of the environmental asset.

where q is as before the shadow price associated with the resource constraint, u_{1C} is the marginal utility of consumption and u_{2A} the marginal utility of the stock. These equations have the following stationary solution:

$$C_t = 0,\ A = A^*;\ \text{where } \delta u_{1C}(0) = u_{2A}(A^*). \tag{21}$$

The determination of A^* is illustrated in Figure C: this also shows that there may be no stationary solution satisfying (21) if the derivative of u_2 is everywhere less than $\delta u_{1C}(0)$. Assuming that a stationary solution exists, the optimal path in the discounted utilitarian case is to follow a consumption path that goes to zero as the remaining stock goes to A^*. The solution in this case is very intuitive: the stock which is preserved for ever, A^*, is smaller, the higher the discount rate, and the relationship between the marginal utilities in (21) equates the marginal utility of consumption with the present value of the marginal utility of a permanent incremental addition to the capital stock at a stock level of A^*. If there is no value of A which satisfies (21), then the discounted utilitarian optimum involves running the stock asymptotically to zero. Also, there is a value of the discount rate δ_m which implies optimal preservation of the initial stock even under the utilitarian criterion; such a value is defined by $\delta_m u_{1C}(0) = u_{2A}(A_0)$. For $\delta < \delta_m$ the relevant

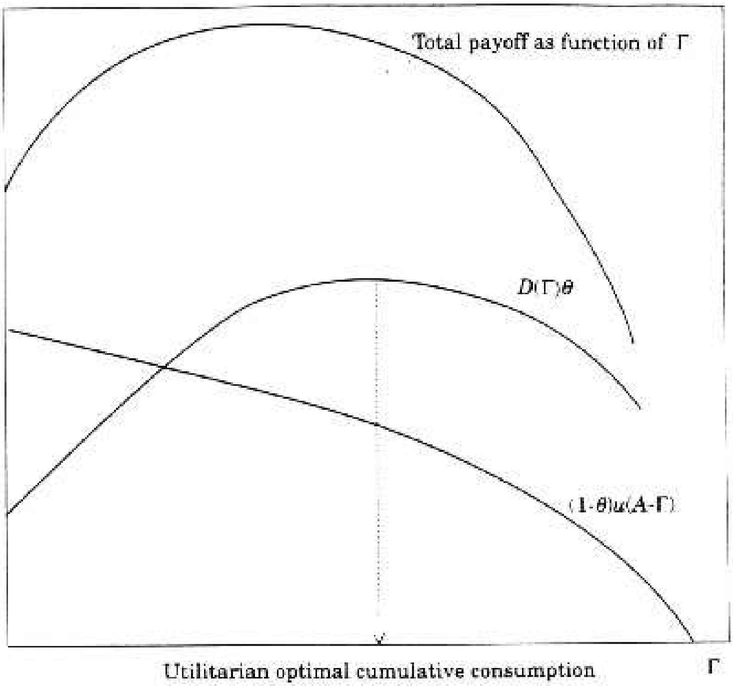

FIGURE D. The overall objective as a function of cumulative consumption Γ.

first-order condition becomes an inequality, $\delta u_{1C}(0)<u_{2A}(A_0)$.

Now consider the solution to the overall problem. Let

$$D(\Gamma)=\max\left[\int_0^{\infty}\{u_1(C_t)+u_2(A_t)\}e^{-\delta t}\,\mathrm{d}t\right]$$

subject to

$$\int_0^{\infty}C_t\,\mathrm{d}t\le\Gamma\le A_0.$$

The constraints here allow the consumption of only a part, Γ, of the initial stock of the resource. Of course, if $\Gamma\ge A_0-A^*$, then this additional constraint is not binding, as the fully optimal amount can be consumed. We can now express the overall problem as $\max_{\Gamma\le A_0}[\theta D(\Gamma)+(1-\theta)u_2(A_0-\Gamma)]$. The solution to the overall problem is obtained by choosing Γ to maximize this expression, both terms of which are plotted against Γ in Figure D. In general the solution will be between $\Gamma=0$, which is the "green golden rule" solution, and $\Gamma = A_0 - A^*$, which is the discounted utilitarian solution.

6.2. A RENEWABLE RESOURCE

Again we have only one good, the resource, but we now consider this to be renewable. The optimization problem is now:

$$\text{Max}\ \theta \int_0^{\infty} u(C_t, A_t) e^{-\delta t} \mathrm{d}t + (1-\theta) \lim_{t\to\infty} u(C_t, A_t)$$

subject to

$$\dot{A}_t = -C_t + R(A_t).$$

To be even more specific, we assume

$$u(C_t, A_t) = \ln C_t + \gamma \ln A_t,$$
$$R(A_t) = rA_t - \frac{rA_t^2}{A^s}.$$

This reproduction function is logistic with A^s the carrying capacity of the environment. In this case the Hamiltonian of the system is:

$$H = [\ln C + \gamma \ln A] + q\left[-C + rA - \frac{r}{A^S} A^2\right].$$

The first-order conditions for a utilitarian optimum are now:

$$\frac{1}{C} = q, \tag{22}$$

$$\dot{q} - \delta q = -\frac{\gamma}{A} - qr\left(1 - \frac{2A}{A^S}\right), \tag{23}$$

$$\dot{A} = -C + rA - \frac{r}{A^S} A^2, \tag{24}$$

$$\lim_{t\to\infty} e^{-\delta t} qA = 0.$$

The steady states of the two variables implied by these necessary conditions are:

$$A = \frac{A^S(\gamma r - \delta + r)}{2r + \gamma r},$$
$$C = \left[\frac{A^S(\gamma r - \delta + r)}{r(2+\gamma)}\right]\left[\frac{r+\delta}{2+\gamma}\right].$$

When $\gamma = 0$, corresponding to the case of no utility of the environment, we have:

$$A = \frac{A^s}{2}\left(\frac{r-\delta}{r}\right) < \frac{A^s}{2}$$

and this means that the steady state level of environment is lower than the one that gives the maximum sustainable yield. This result resembles the one obtained in standard growth theory, in which impatience prevents society from accumulating the stock of capital that maximizes steady state consumption. The lines along which values of A and C are constant are given respectively by

$$C = R(A),$$
$$R'(A) = \delta - \frac{\gamma C}{A}.$$

The characterization of the dynamics of the discounted utilitarian solution is given in Figure E, which shows that the utilitarian stationary solution is a saddle point.

We can also characterize the green golden rule: in that case the steady state stock of environment is

$$A = A^s\left(\frac{1+\gamma}{2+\gamma}\right).$$

This is in general larger than the value at the utilitarian stationary solution, although the two stationary states converge as the discount rate is reduced to zero.

For this framework, the solution to the overall optimization problem is not well-defined, unless $\delta = 0$. To see this, suppose that the initial stock is A_0 in Figure E. Pick an initial value of C below the path leading to the saddle-point, follow the path satisfying the utilitarian necessary conditions (22)–(24) until it leads to the resource stock corresponding to the green golden rule, and then

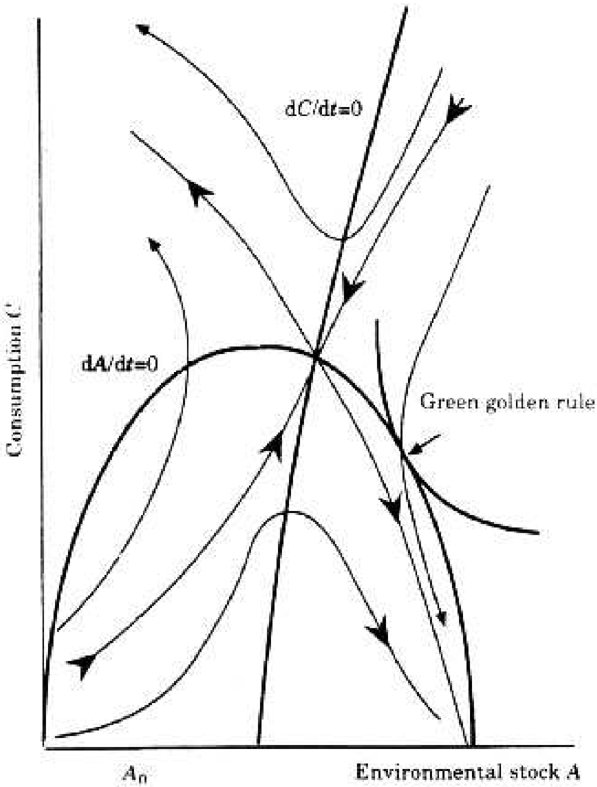

FIGURE E. Dynamics of the utilitarian solution for the simple case of one state variable A and no production.

increase consumption to the level corresponding to the green golden rule. Any such path will satisfy the necessary conditions and lead to the green golden rule in finite time, so the utility integral which constitutes the first part of the maximand can be improved by picking a slightly higher value of C_0 and reaching the green golden rule slightly later. This does not detract from the second term in the maximand. By this process it will be possible to approximate the independent maximization of both terms in the maximand: the discounted utilitarian term, by staying long enough on the stable manifold leading to the utilitarian stationary solution, and the purely finitely additive term, by moving very far into the future to the green golden rule. Although it is possible to approximate the maximization of both terms in the maximand independently, there is no feasible path that actually achieves this maximum. The supremum of the values of the maximand over feasible paths is approximated arbitrarily closely by paths which reach the green golden rule at later and later dates, but the limit of these paths never reaches the green golden rule.

We cannot currently give a sharp characterization of the class of problems for which a solution to the overall problem exists, although we can, of course, do this for the two component problems viewed as independent problems. This issue is explored further in Heal (1993*c*).

7. Conclusions

We have explored the implications of an axiomatization of the ranking of intertemporal utility sequences that places weight both on the characteristics of the sequence over any finite period and its very long run or limiting characteristics. The criterion shows more intertemporal symmetry than the discounted utilitarian approach, which clearly emphasizes the immediate future at the expense of the long run. In this respect the criterion captures some of the concerns of those who argue for sustainability and for a heightened sense of responsibility to the future. An exploration of this criterion has led us to define the "green golden rule" configuration, which plays an important role in characterizing the long-run behaviour of paths optimal according to this criterion.

The characterization of optimal paths that emerge from this criterion is eminently intuitive. Their long-run characteristics are a mixture of utilitarianism and the green golden rule: locally, they always satisfy the utilitarian first-order conditions familiar from optimal growth theory. It is also intuitive that they cannot be supported by the maximization of present value profits and by consumer maximizing behaviour with present value budget constraints.

Our objective function (6) has some point of contact with the Rawlsian approach to optimal resource use, so some comparative comments are in order. The key distinction is that the Rawlsian approach ranks paths only by the lowest of their utility values: we replace the lowest value of utility by the limiting value and supplement this by the discounted integral of utilities along the path. This has two great advantages. One is that it avoids trapping an economy into low consumption levels because it has poor initial endowments (see Solow, 1974; Dasgupta & Heal, 1979). The other is that it ensures that any solution path is dynamically locally optimal because the path satisfies the local optimality conditions (see Heal, 1973).

Acknowledgements

Paper presented at the Workshop on Economic Growth, Venice, 13–15 January, 1994. This work was funded in part by grants from the Fondazione ENI Enrico Mattei and NSF grants 92 16028 and 91 10460.

References

Beltratti, A., Chichilnisky, G. & Heal, G.M. (1994). The green golden rule and sustainable growth. In I. Goldin & A. Winters, Eds. *Approaches to Sustainable Economic Development*. Paris: OECD, forthcoming.

Brock, W. (1977). A polluted golden age. In V. L. Smith, Ed. *Economics of Natural and Environmental Resources*. New York: Gordon & Breach.

Chichilnisky, G. (1980). Social choice and the topology of preference spaces. *Advances in Mathematics*, **37**, 165–176.

Chichilnisky, G. (1993). *What is sustainable development?* Paper presented at the 1993 workshop of the Stanford Institute for Theoretical Economics.

Chichilnisky, G. & Heal, G.M. (1993). Competitive equilibrium in Sobolev spaces without bounds on short sales. *Journal of Economic Theory*, **59**, 364–384.

Dasgupta, P.S. & Heal, G.M. (1974). The optimal depletion of exhaustible resources. *Review of Economic Studies, Symposium on the Economics of Exhaustible Resources*, 3–28.

Dasgupta, P. & Heal, G.M. (1979). *Economic Theory and Exhaustible Resources*. London: Cambridge University Press.

Dutta, P. (1991). What do discounted optima converge to? A theory of discount rate asymptotics in economic models. *Journal of Economic Theory*, **55**, 64–94.

Frerejohn, J. & Page, T. (1978). On the foundations of intertemporal choice. *Journal of Agricultural Economics*, **May**, 15–21.

Harrod, R. (1948). *Towards a Dynamic Economy*. London: Macmillan.

Heal, G.M. (1973). *The Theory of Economic Planning*. Amsterdam: North Holland.

Heal, G.M. (1985). Depletion and discounting: a classical issue in the economics of exhaustible resources. *American Mathematical Society, Proceedings of Symposia in Applied Mathematics*, **32**, 33–43.

Heal, G.M. (1993*a*). The optimal use of exhaustible resources. In A. Kneese & J. Sweeney, Eds. *Handbook of Natural Resource and Energy Economics*, Vol. III. Amsterdam, New York and Oxford: North Holland.

Heal, G.M. (1993*b*). *The Economics of Exhaustible Resources*. International Library of Critical Writings in Economics, Edward Elgar.

Heal, G.M. (1993*c*). *Valuing the very long run*. Working paper, Columbia Business School.

Hotelling, H. (1931). The economics of exhaustible resources. *Journal of Political Economy*, **39**, 137–175.

Krautkraemer, J.A. (1985). Optimal growth, resource amenities and the preservation of natural environments. *Review of Economic Studies*, **52**, 153–170.

Lauwers, L. (1993). *Equity among infinitely many generations*. Discussion Paper, Katholik Universtaet Leuven, Belgium.

Lauwers, L. & van Liederkierke, L. (1993). *Monotonic Chichilnisky rules with infinite populations*. Discussion Paper, Katholik Universtaet Leuven, Belgium.

Phelps, E.S. (1961). The golden rule of accumulation: a fable for growthmen. *American Economic Review*, **51**, 638–643.

Ramsey, F. (1928). A mathematical theory of saving. *Economic Journal*, **38**, 543–559.

Solow, R.M. (1974). Intergenerational equity and exhaustible resources. *Review of Economic Studies, Symposium on the Economics of Exhaustible Resources*, 29–45.

Yosida, K. (1974). *Functional Analysis*, Fourth edition. Berlin, Heidelberg, New York: Springer Verlag.

Yosida, K. & Hewitt, E. (1952). Finitely additive measures. *Transactions of the American Mathematical Society*, **72**, 46–66.

Appendix

In order to study stability we linearize the system of four differential equations governing prices and state variables. We first invert $u_C = p + q\alpha$ to obtain:

$$C = \Phi[p + q\alpha, A], \text{ where } \Phi_1 < 0.$$

We then rewrite the linearized system as:

$$\dot{p} = \delta p - pF_{KK}K - F_K p, \tag{25}$$

$$\dot{q} = \delta q - u_{AC}\Phi_p p - u_{AC}\alpha\Phi_q q - u_{AA}A - qR_{AA}A - R_A q - u_{AC}\Phi_A A, \tag{26}$$

$$\dot{K} = F_K K - \Phi_p p - \alpha\Phi_q q - \Phi_A A, \tag{27}$$

$$\dot{A} = -\alpha\Phi_p p - \alpha^2\Phi_q q - \alpha\Phi_A A + R_A A, \tag{28}$$

which can be summarized as:

$$\dot{z} = Sz, \tag{29}$$

where $z' = (p, q, K, A)$. After imposing separability in the utility function, the determinant of the matrix $S - \lambda I$, necessary to find the eigenvalues of the system, may be written as:

$$-\lambda \begin{vmatrix} \delta - R_A - \lambda & 0 & -u_{AA} - qR_{AA} \\ -\alpha\Phi_q & F_K - \lambda & 0 \\ -\alpha^2\Phi_q & 0 & R_A - \lambda \end{vmatrix} +$$

$$-pF_{KK} \begin{vmatrix} 0 & \delta - R_A - \lambda & -u_{AA} - qR_{AA} \\ -\Phi_p & -\alpha\Phi_q & 0 \\ -\alpha\Phi_p & -\alpha^2\Phi_q & R_A - \lambda \end{vmatrix}.$$

By solving one obtains:

$$\lambda^4+\Gamma_1\lambda^2+\Gamma_2\lambda+\Gamma_3=0, \tag{30}$$

where:

$$\begin{aligned}
\Gamma_1 &= \delta F_K+\delta R_A-2\delta\alpha F_K-R_A^2+\alpha^2\Phi_1 u_{AA}+\alpha^2\Phi_1 qR_{AA}-pF_{KK}\Phi_1,\\
\Gamma_2 &= -\delta R_A F_K+R_A^2F_K-\alpha^2\Phi_1 u_{AA}F_K-\alpha^2\Phi_1 qR_{AA}F_K+\delta pF_{KK}\Phi_1,\\
\Gamma_3 &= \Phi_1 pF_{KK}R_A^2-\delta pF_{KK}\Phi_1 R_A.
\end{aligned}$$

PROPOSITION 4: *consider a stationary solution at which $R_A<0$. For small enough values of the discount rate δ the quartic equation (30) has four real roots, two positive and two negative, so that the stationary solution is a saddle point.*

PROOF: in this case the quartic equation is

$$\lambda^4+\Gamma_1\lambda^2+\Gamma_2\lambda+\Gamma_3=0,$$

where

$$\begin{aligned}
\Gamma_1 &= \delta^2+R_A(\delta-R_A)-pF_{KK}\Phi_1,\\
\Gamma_2 &= \delta R_A(R_A-\delta)+\delta pF_{KK}\Phi_1,\\
\Gamma_3 &= pF_{KK}\Phi_1 R_A(R_A-\delta).
\end{aligned}$$

Noting that $\Phi_1<0$, we see that if $\Gamma_1<0$, $\Gamma_3>0$ but small, and Γ_2 is small, then there are four real roots, two positive and two negative. Under the assumptions of the proposition, Γ_3 is always positive. The second two terms in Γ_1 are negative, and the only positive term goes to zero with the square of δ. This establishes the proposition. ∎

In this case a stationary solution is a saddle point and so is locally stable in the sense that, for any initial stocks of capital and the environment, there exist prices which if chosen will lead the system to the stationary point. In terms of Figure A, we have established that any stationary solution on the downward-sloping part of the $R(A)$ curve is stable in the saddle-point sense.

COROLLARY 2: *if $R_A>0$ but the remaining conditions of the proposition are satisfied, then for small enough values of the discount rate δ (which have to satisfy $\delta<R_A$) there are four real roots, two positive and two negative. In general the bound on δ in the corollary will be tighter than that in the proposition.*

The proof follows immediately. ∎

For completeness, note that if $\Gamma_1>0$, then there will be at most two real roots, and there will be two if and only if $\Gamma_3<0$. Otherwise (i.e. if $\Gamma_3>0$) there will be no real roots. In these cases there may still be a stationary solution that is saddle point stable, but it will be approached by a damped spiral.