Uncertain future preferences and conservation

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3.4. Uncertain Future Preferences and Conservation

1. Introduction

An important problem in environmental economics arises from the irreversibility of consuming or destroying certain resources. Extractive resources like oil are a clear example. Even for environmental resources the same seems to be true in a number of important cases, for example biodiversity, current climate conditions, or complex ecological systems. Irreversibility imposes a severe externality across different generations; future generations will suffer from the destruction of a unique asset like Amazonia, and it is not clear how such a loss could be compensated in terms of other goods. If such an asset is destroyed, then it is not possible subsequently to restore it. In contrast, if the asset is preserved, then it is possible to "use" the asset at a subsequent date. If there is uncertainty about future preferences or valuations, then preservation provides a type of insurance which is not available if the irreversible decision is carried out.

Such an intuitive form of insurance policy has been related to the concept of option value. Amongst the earliest studies of these issues were [1, 7, 17, 18, 20, 26]. Subsequent works that built on these contributions include [4, 5, 11, 12, 21, 23, 24, 27, 28]. An extensive survey of the concept and the meaning of option value is provided in other chapters of this book (those by Chichilnisky and Heal and Vercelli) to which we refer the reader.

None of the previous papers, however, take into account the fact that generations yet to be borne may value natural resources and environmental assets quite differently from us. They may value them more because of an enhanced appreciation of the relationship between humanity and the rest of nature: they may value them less because their world may be more synthetic and created than ours, and they may be pleased with that. Casual empiricism suggests that in fact over the last few decades, the citizens of industrial countries have rediscovered the importance of environmental assets such as clean air and water, biodiversity, rain forests, and many others. The value placed on these appears to have increased substantially, suggesting the possibility of further

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changes in the future. The possibility of such changes has important implications for our current decisions about environmental preservation: many of the most difficult preservation decisions today concern very long-run issues such as climate change, species conservation and nuclear waste disposal. In all of these cases, most of the benefits and costs will fall on future generations: their valuations of environmental assets should therefore be central to our decisions, and uncertainty about their valuations, inevitable because they are not yet borne, must be explicitly recognized and modeled in our evaluations. This is the agenda of this paper. Solow [25] has argued that such uncertainty must be at the center of any analysis of sustainability. Asheim [2] has also made uncertainty, albeit of a different type, central to a theory of sustainability, as has Tucci in his papers in this volume.

We develop a simple continuous-time stochastic dynamic framework within which we can analyze the optimal preservation of an asset whose consumption is irreversible, in the face of uncertainty about future preferences. This framework is derived from that introduced by Dasgupta and Heal [8], who analyze optimal depletion of a fixed stock of an environmental resource. In a first version of the model utility depends only on the flow of consumption, so that the stock of the resource can be interpreted as an extractive resource which is used in production such as oil or gas. In a second version of the model, utility also depends directly on the stock of the resource, which can therefore be interpreted as an environmental asset such as a forest, a landscape or the biodiversity of a region. As the resource is available in a fixed total supply, its consumption is irreversible: any consumption today leads ineluctably to a reduction in the amount available for future consumption.

The innovation with respect to [8] lies in our using a dynamic optimization model in which current planners are uncertain about the preferences of future generations, and wish to respect the possibility of their having a stronger preference for an environmental good. We compare the optimal depletion paths and associated shadow prices for two cases, one where preferences for the consumption of the resource are known with certainty to be the same at all dates, and a second in which there is a possibility of a change in preferences in the future. We study the difference in optimal depletion (or, equivalently, conservation) policies, and their associated shadow prices, resulting from the introduction of the possible alteration in preferences.

The effect of uncertainty about future preferences on conservation decisions depends, naturally, on the probability distribution governing the evolution of preferences; the exact nature of this effect depends on whether the stock represents an extractive resource or an environmental asset, i.e. whether utility depends on the flow only or on stock and flow. In case of utility depending only on the flow, we show that the possibility of an increase in the intensity of preference for the consumption flow need not on its own have any effect on the optimal conservation policy. There is an impact on conservation policy if and only if the probability distribution is not neutral in a certain very intuitive sense. Roughly speaking, this non-neutrality means that the introduction of uncertainty is not mean-preserving in the sense of Rothschild and Stiglitz [22]: there is not only pure uncertainty about future preferences but also an expected drift.

In the case of an environmental asset valued both as a stock and as a flow, the optimal consumption policy will in general involve the preservation of a positive stock in perpetuity (see Heal's paper "Interpreting Sustainability", Chapter 1.1 of this book). In this case, we study the impact of uncertainty about future preferences on the size of the stock conserved. We take as a benchmark the stock which would be conserved if there were no uncertainty and it was known with certainty that preferences would continue for ever in their initial form. We show that whether preference uncertainty leads to the conservation of a greater or a lesser stock, depends in an intuitive way on the parameters describing the uncertainty. Although this problem is very complex, it is possible to give a complete characterization of how uncertainty affects the desirability of conserving the asset in the long run.

We conclude this introduction with some remarks about the relationship between our formulation and earlier papers (referenced above) which address the concepts of option and quasi-option value. Clearly, all the papers share a common motivation: that it is important, in present decisions, to recognize that the valuation of environmental assets may change over time and so has to be seen as uncertain. Our framework seems rather more general than that of most studies of option values, in that we consider an infinite horizon continuous time problem in which the choice variable is also continuous. The standard, though not uniform, paradigm for the option value literature is two periods and a zero-one choice variable, "conserve" or "don't conserve". We also work within a framework which is now standard in dynamic welfare economics, namely within the framework of a simple optimal growth model. In spite of the increased generality, we are able to obtain results which are at least comparable with earlier results in terms of their degree of detail.

The plan of the paper is as follows: in Section 2 we describe the model and its solution, while in Section 3 we describe a particular specification of the change in preferences and show that in this case pure uncertainty about the valuation of the flow of consumption of a resource requires consumption policies that do not deviate from those under certainty. In Section 4 we study a simpler version of the model involving uncertainty about the direction, but not about the timing, of the change in preferences. Section 5 extends the analysis to the case where uncertainty is about the relative importance of the flow and the stock in the utility function. Section 6 considers option values and the relationship to the previous literature. Section 7 concludes.

2. Conservation with Uncertain Future Preferences

2.1. *Modelling Uncertain Future Preferences*

Our aim is to model a situation where there is irreversible consumption of an asset in fixed supply. This could happen for example in the case of oil consumption. In order to keep the model as simple as possible we abstract from problems related to the production side of the economy, and imagine that consumption takes place directly by depletion of the asset.

The most natural counterpart to this kind of structure is a variant of the cake-eating problem, described, for example, in [9]. We consider an environmental good of which there is at time t a stock S_t. This good may be consumed at a rate c_t, so that the rate of change of the stock is given by

$$\frac{dS_t}{dt} \equiv \dot{S}_t = -c_t. \tag{1}$$

Feasibility requires that $S_t \geq 0\,\forall t$. At time zero society derives utility from the consumption of this good according to the function $u(c_t)$ which is assumed to be increasing, twice continuously differentiable and strictly concave. There is a possibility that at a random future date which we shall denote T the utility of consuming this good will change according to a multiplicative factor. For simplicity it is assumed that the function $u(c_t)$ will become equal to $(1+\alpha)u(c_t)$ with probability q or to $(1-\beta)u(c_t)$ with probability $(1-q)$, for $\alpha, \beta \geq 0$. The date T at which there is a switch of preferences is a random variable with marginal density ω_t. We also assume that the change in preferences is a once-for-all phenomenon. We can think of the change in preferences as representing a change of tastes from one generation to another: the current generation is uncertain about the preferences of its successors and wishes to allow for the fact that they may value more highly the environmental good. Alternatively, it might be the discovery that some aspect of the environment is medically important in ways not previously recognized, leading to an increase in its valuation.

We begin by formulating an ancillary problem. Following Dasgupta and Heal [8] we define a state valuation function $W_F(S_T)$ which values the stock S_T remaining at time T at which the change in preferences occurs. The valuation is according to the utility function $F(c_t)$, where F stands for the utility function that applies after T.

$$W_F(S_T) = \max \int_T^\infty F(c_t)e^{-\delta(t-T)}dt \text{ subject to } \int_T^\infty c_t dt = S_T. \tag{2}$$

δ is of course a discount rate applied to future utilities: for a discussion of the appropriateness of discounting in this context, see [16]. Let W_F be denoted $W_1(S_T)$ when $F(c) = (1+\alpha)u(c)$ and $W_2(S_T)$ when $F(c) = (1-\beta)u(c)$. Define

$$EW_F(S_T) = qW_1(S_T) + (1-q)W_2(S_T) \tag{3}$$

as the expected valuation of the stock at time T.

Given this, we may now define an overall problem as:

$$\max \int_0^\infty \omega_T \left\{ \int_0^T u(c_t)e^{-\delta t}dt + e^{-\delta T} EW_F(S_T) \right\} dT$$

$$\text{subject to } \dot{S}_t = -c_t \text{ and } S_t \geq 0 \; \forall t. \tag{4}$$

The interpretation of this problem is as follows. The date T at which preferences may change is a random variable. For any particular T, the expected utility of a consumption path is given by the expression in the parentheses $\{\,\}$. We then take as the maximand the expectation of this over all possible values of T. In other words, we maximize the expected discounted value of utility derived from consuming the environmental good, where the expectation is taken with respect to the probability distributions governing changes in preference for the environmental good. By integrating by parts, the maximand in (4) can be reformulated as

$$\int_0^\infty e^{-\delta t} \left\{ u(c_t)\Omega_t + \omega_t EW_F(S_t) \right\} dt, \tag{5}$$

where $\Omega_t = \int_t^\infty \omega_\tau d\tau$.

2.2. *Stochastic and Deterministic Solutions*

When solving the actual problem (4), it will be useful to consider as a standard of comparison the solution of a problem where a decision maker consumes a fixed stock of resources but ignores uncertainty about future preferences altogether, and assumes that the utility function is never going to change. Under these assumptions we face a standard cake-eating problem whose solution requires that consumption decline over time at a rate depending on the discount rate and the elasticity of the marginal utility of consumption. The problem can be formulated as

$$\max \int_0^\infty u(c_t)e^{-\delta t}dt \text{ subject to } \int_0^\infty c_t dt \leq S_0. \tag{6}$$

The solution to (6) is:

$$\frac{\dot{c}_t}{c_t} = \frac{\delta}{\eta}, \tag{7}$$

where η is the elasticity of the marginal utility of the function u with respect to consumption, i.e., $\eta = (u''c)/u' < 0$ and the single prime denotes the first and the double prime denotes the second derivative of u with respect to its argument. In the rest of the paper, we denote by $c_t(7)$ the consumption path that satisfies (7), and by $g_{ct}(7)$ the corresponding rate of growth.

In order to solve the general problem (4) we now introduce a shadow price or adjoint variable on the stock S_t, denoted p_t. Then a necessary condition for a consumption path and a shadow price path to solve (4) is that c_t and p_t satisfy the following equations:[1]

$$\begin{aligned} &u'(c_t)\Omega_t = p_t, \\ &\dot{p}_t - \delta p_t = -\omega_t E W'_F. \end{aligned} \tag{8}$$

A little manipulation allows us to condense these into the intuitive single equation

$$\frac{\dot{c}_t}{c_t} = \frac{\delta}{\eta} + \left\{\frac{u' - EW'_F}{u'}\right\} \frac{\omega_t}{\eta \Omega_t}. \tag{9}$$

According to Equation (9) the rate of growth of consumption is time-varying, and depends in a complicated way on the probabilities and on the marginal utilities that are in turn a function of the past consumption policy. In the rest of the paper we will denote by $c_t(9)$ the time path of consumption which satisfies (9), and by $g_{ct}(9)$ the corresponding rate of growth.

Equation (9) tells us that the rate of change of consumption depends on the discount rate, the elasticity of the marginal utility of consumption and the expectation of the increase in the marginal valuation of consumption conditional on the change in preferences not having yet occurred.[2] For a given elasticity of marginal utility, the rate of change of consumption depends negatively on the rate of time preference. The effect of the second term on the right-hand side of (9) is ambiguous, and depends on the difference between the immediate marginal utility of consumption at time t, and the expected marginal valuation of the remaining stock of the asset; for a given ratio ω/Ω, the larger is the marginal valuation of the remaining stock, the faster is its rate of depletion falls.

If the utility function is scaled up by $(1+\alpha)$ from T onwards, then $F(c) = (1+\alpha)u(c)$ and the appropriate state valuation function is $(1+\alpha)W(S_T)$, and if it is scaled down by $(1-\beta)$, then the state valuation function is $(1-\beta)W(S_T)$. Therefore, in this case the function (3) assumes the simple form:

$$EW_F(S_T) = q(1+\alpha)W(S_T) + (1-q)(1-\beta)W(S_T) = \Gamma W(S_T), \tag{10}$$

where $\Gamma = q(1+\alpha) + (1-q)(1-\beta)$.

By using such a description of uncertainty and the proposition proven in the Appendix, according to which the marginal valuation of the stock is equal to the marginal utility of consumption after the change in preferences, it is possible to rewrite Equation (9) as:

$$g_{ct}(9) = g_{ct}(7) + \left\{\frac{u'(c_t(9)) - \Gamma u'(c_t^*(7))}{u'(c_t(9))}\right\} \frac{\omega_t}{\eta \Omega_t}, \tag{11}$$

where $c_t^*(7)$ is optimal consumption after preferences have changed. Such a consumption level is different from the consumption which would hold under the solution to a standard cake-eating problem, which we have denoted by

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$c_t(7)$; $c_t^*(7)$ applies at time t if a solution to a certainty problem is followed from time t on, but not before time t, while $c_t(7)$ is the time t consumption that holds when the solution to the certainty problem has been followed since the very beginning. Formally:

$$c_t(7) = -\frac{\delta}{\eta} S_0 e^{\frac{\delta}{\eta} t},$$

$$c_t^*(7) = -\frac{\delta}{\eta}\left[S_0 - \int_0^t c_t(9)dt\right].$$

Clearly, $c_t^*(7) < c_t(7)$ whenever the actual policy prescribes an integral of consumption from time 0 to time t that is larger than the one required by the solution to the deterministic problem.

3. Specific Cases

Two specific cases of the general framework will be helpful in obtaining clearer results and building up intuition.

3.1. *Symmetric Uncertainty*

$\Gamma = 1$ provides an interesting benchmark, as in this case the probabilities of the states of nature compound with the magnitudes of the change in utility in such a way as to make the mathematical expectation of the future utility equal to the current utility; we describe this as symmetric uncertainty. In this case we can show that the decision maker may optimally ignore uncertainty about future preferences, and adopt the policy suggested by the certain problem. In this case, uncertainty about future preferences has no impact at all on optimal current consumption levels: it is not appropriate to consume more conservatively (with respect to the model that ignores uncertainty) in the face of a possible increase in future preferences for the good, if this is balanced by a possible decrease. So uncertainty about future preferences alone does not give rise to a conservation motive: the uncertainty has to be asymmetric, in the sense that the possibility of an increase in preferences outweighs that of a decrease.

Formally:

PROPOSITION 1. *The optimal consumption policy for the uncertain problems (4) with symmetric uncertainty about future preferences is identical to that for the certain problem (6) with unchanging preferences.*

Proof. In the case of symmetric uncertainty $EW_F(S_T) = W(S_T)$ and the maximand becomes:

$$\int_0^\infty \omega_T \left\{ \int_0^T u(c_t)e^{-\delta t}dt + e^{-\delta T} W(S_T) \right\} dT$$

$$= \int_0^\infty \omega_T \left\{ \int_0^\infty u(c_t) e^{-\delta t} dt \right\} dT$$

and the two problems are obviously identical. □

Intuitively, the result is due to the fact that risk aversion by itself does not necessarily imply aversion to uncertainty about future preferences. In our model the agent is risk averse for each given preference structure, but is risk neutral with respect to uncertainty about the structure of preferences. So for a risk averse decision maker who follows the standard paradigm of expected utility maximization, we obtain the strong result that uncertainty about preferences alone does not give rise to a more conservative policy for the use of the resource: there needs to be an asymmetry in the distribution of possible outcomes as well. This is also true of the earlier formulation of option value as in [1, 17]: this aspect of those results is reviewed by Chichilnisky and Heal in Chapter 1.2 of this volume.

3.2. *Preference Uncertainty Alone*

We now eliminate one source of uncertainty, the one due to timing shocks, and consider a decision-maker who knows that at a given time T her preferences will change, although in an uncertain way. Formally, the problem is now:

$$\max \int_0^T u(c_t) e^{-\delta t} dt + e^{-\delta T} EW_F(S_T) \text{ subject to} \dot{S}_t = -c_t, S_t \geq 0 \, \forall t, \quad (12)$$

where $EW_F(S_T)$ is given by Equation (10). Using the expression given in Equation (10), the maximand can be rewritten as:

$$\max \int_0^T u(c_t) e^{-\delta t} dt + e^{-\delta T} \Gamma W(S_T), \quad (13)$$

which shows that the particular specification we have chosen for the change in the utility function makes the problem with preference uncertainty only exactly equivalent from a formal point of view to a problem with a known change in preferences.

The necessary conditions of problem (12) are, apart from a transversality condition, the same as those of an infinite horizon problem:

$$u'(c_t) = p_t, \dot{p}_t - \delta p_t = 0, p_T = \Gamma W'(S_T).$$

The solution from time zero to T is:

$$c_T = -\frac{\delta}{\eta} S_T,$$

$$\frac{\dot{c}}{c} = \frac{\delta}{\eta}, \quad \frac{\dot{p}}{p} = \delta$$

Together with the transversality condition:

$$p_T = \Gamma W'(S_T), \quad (14)$$

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this yields an expression for the marginal utility of consumption at time 0:

$$u'(c_0) = e^{-\delta T}\Gamma W'(S_T). \tag{15}$$

Such an expression is useful in understanding the effect of uncertainty about future preferences on the optimal policy, as clearly it shows that *in the case of preference uncertainty and certain timing, initial consumption is larger than, less than, or equal to that obtaining under certainty according to whether* Γ *is less than, greater than, or equal to* 1.

4. The Stock as Source of Utility

It is now assumed that society derives utility both from the consumption of this good according to the function $u(c_t)$ and also from the stock according to the function $v(S_t)$. This extended specification is particularly useful when studying environmental resources, see, for example, [19]. Uncertainty in this case may be interpreted in terms of the relative valuation of consumption goods and the environmental asset. The scenario that will be described here is that at some future date society may increase its valuation of environmental assets relative to the flow of consumption which can be obtained by depleting the assets. Both utility functions $u(c)$ and $v(S)$ are assumed to be increasing, twice continuously differentiable and strictly concave. In order to consider a model with a steady state we assume that marginal utility of consumption evaluated at zero consumption is finite, $u'(0) < \infty$.

While at the beginning of the planning horizon the flow of consumption provides a certain utility $u(c_t)$ and the stock of the asset provides a certain utility of $v(S_t)$, there is a possibility that at a random future date T, the utility function is affected by an exogenous shock. The utility of the flow will not be affected, but the utility directly provided by the stock will change to either $(1+\alpha)v(S)$ (with probability q) or to $(1-\beta)v(S)$ (with probability $1-q$), for $\alpha, \beta \geq 0$. As before, the date T at which there is a switch of preferences is a random variable with marginal density ω_t.

The state valuation function $W(S_T)$ which values the stock S_T remaining at the time T at which the change in preferences occurs is now:

$$\begin{aligned} W_\alpha(S_T) &= \max \int_T^\infty \left[u(c_t) + (1+\alpha)v(S_t)\right] e^{-\delta(t-T)} dt \\ &\qquad \text{subject to } \int_T^\infty c_t dt = S_T, \end{aligned} \tag{16}$$

$$\begin{aligned} W_\beta(S_T) &= \max \int_T^\infty \left[u(c_t) + (1-\beta)v(S_t)\right] e^{-\delta(t-T)} dt \\ &\qquad \text{subject to } \int_T^\infty c_t dt = S_T. \end{aligned} \tag{17}$$

The overall problem can be written as the maximization of:

$$E\left\{\int_0^T e^{-\delta t}\left[u(c_t)+v(S_t)\right]dt+e^{-\delta T}\left[qW_\alpha(S_T)+(1-q)W_\beta(S_T)\right]\right\}$$

given the standard resource constraint. By integrating by parts, the maximand can be reformulated as:

$$\int_0^\infty e^{-\delta t}\left\{\left(u(c_t)+v(S_t)\right)\Omega_t+\omega_t\left[qW_\alpha(S_t)+(1-q)W_\beta(S_t)\right]\right\}dt, \quad (18)$$

where $\Omega_t=\int_t^\infty \omega_\tau d\tau$. Necessary conditions for a consumption path and a shadow price path to solve (18) are:

$$u'(c_t)\Omega_t=p_t,$$

$$\dot{p}_t-\delta p_t=-\Omega_t v'(S_t)-\omega_t\left[qW'_\alpha(S_t)+(1-q)W'_\beta(S_t)\right], \quad (19)$$

which can be used to derive:

$$\eta\frac{\dot{c}_t}{c_t}=\delta-\frac{v'(S_t)}{u'(c_t)}+\lambda\left\{\frac{u'(c_t)-qW'_\alpha(S_t)-(1-q)W'_\beta(S_t)}{u'(c_t)}\right\}, \quad (20)$$

where $\lambda=\omega/\Omega$.

Equation (20) admits a steady state at which consumption is constant. At any stationary state, the consumption of the flow must be zero and the stock must be constant. Hence at a stationary state of (20):

$$\delta u'(0)=v'(S)-\left[u'(0)-qW'_\alpha(S)-(1-q)W'_\beta(S)\right].$$

We shall consider three different steady states, each corresponding to a problem with no uncertainty about preferences. In one case the utility of the stock is constant at $v(S)$: this leads to stationary state value of the stock of S_D. In the other cases the utility of the stock is constant at $(1+\alpha)v(S)$ or $(1-\beta)v(S)$: these cases lead respectively to stationary states with stock levels S_α and S_β. Note that if preferences never change, $\lambda=0\ \forall t$. Hence:

PROPOSITION 2. *The steady state corresponding to the original preference structure of the agent is characterized by a level of the stock S_D such that $\delta u'(0)=v'(S_D)$, while the steady state corresponding to an increase (decrease) in preferences is characterized by a level of the stock S_α (S_β) such that $\delta u'(0)=(1+\alpha)v'(S_\alpha)$ ($\delta u'(0)=(1-\beta)v'(S_\beta)$).*

It is easy, simply by concavity of the valuation function, to relate the three stocks:

Remark 1. $S_\alpha > S_D > S_\beta$.

We now locate the level of the stock corresponding to a point of rest of the system before uncertainty is resolved, and compare it with the steady state which holds after resolution of uncertainty, which will be S_α or S_β.

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Equation (20) can be used to characterize a steady state of the system before uncertainty is resolved (S_S) by evaluating it at zero consumption and setting it equal to 0, while not setting λ to zero:

$$\delta - \frac{v'(S_S)}{u'(0)} + \lambda \left\{ \frac{u'(0) - qW'_\alpha(S_S) - (1 - q)W'_\beta(S_S)}{u'(0)} \right\} = 0. \quad (21)$$

To evaluate the general properties of such a function we define:

$$A_t \equiv \delta - \frac{v'(S_t)}{u'(c_t)} \equiv A(S_t, c_t)$$

and

$$B_t \equiv \lambda \left\{ \frac{u'(c_t) - qW'_\alpha - (1 - q)W'_\beta}{u'(c_t)} \right\} \equiv B(S_t, c_t)$$

and analyze these two components as a function of various values of the stock, for zero consumption, in order to find the equilibrium point.

The following remark, which derives directly from the very definition of the term S_D, takes care of the first term.

Remark 2. $A(S_D, 0) = 0$, $A(S, 0) > 0$ for $S > S_D$, $A(S, 0) < 0$ for $S < S_D$.

It follows that the function $A(.,.)$ is positive (negative) for large (small) values of the stock, and equals zero when the actual stock coincides with the stock representing the equilibrium of the deterministic system with the initial stock utility function $v(S)$.

The term B_t is more complicated to analyze. We now characterize the sign of $B(S_t, 0)$ at the stationary states S_α and S_β. The following lemma is used in this characterization:

LEMMA 3. $W'_\alpha(S_\alpha) = W'_\beta(S_\beta) = u'(0)$, $W'_\alpha(S_\beta) = ((1 + \alpha)/(1 - \beta))u'(0)$, $W'_\beta(S_\alpha) > ((1 - \beta)/(1 + \alpha))u'(0)$.

Proof. The first two equalities follow from the definition of the steady state, which implies a marginal valuation of the stock equal to the marginal utility of consumption. The third equality comes from noting that if an extra unit of stock is given to an economy whose stock is currently S_β with utility of the stock $(1 + \alpha)v(S)$ then this unit will be preserved, as with these preferences a stock equal to S_β implies a corner solution with a marginal valuation of the stock larger than the marginal utility of consumption. Therefore, the utility value of an extra unit is equal to $((1 + \alpha)v'(S_\beta))/\delta$, which equals $((1 + \alpha)/(1 - \beta))u'(0)$ from using the definition of $v'(S_\beta)$ given in Proposition 2. Finally, the inequality describes the case where an extra unit of the stock is assigned to the decision maker in the case where he is already depleting the stock in order to reach a steady state with a lower stock level. Suppose that preferences are described by $(1 - \beta)v(S)$ but the stock is S_α. Given an extra unit of the stock ΔS if such extra unit is added to the stock S_α and maintained forever, the increment in total utility is $((1-\beta)v'(S_\alpha))/\delta$. But since the economy is not at the stationary state where $u'(c) = ((1-\beta)v'(S))/\delta$ then it is possible to increase utility by more than that so $W'_\beta(S_\alpha) > ((1-\beta)v'(S_\alpha))/\delta$. Using $v'(S_\alpha) = (\delta u'(0)/1+\alpha)$ one obtains $W'_\beta(S_\alpha) > ((1-\beta)u'(0)/1+\alpha)$, and this gives the inequality. □

We are now ready to show:

PROPOSITION 4. *There exists a positive stock preserved at the stochastic steady state defined by (21), which we denote S_S.*

Proof. Equation (21) defines a stochastic steady state. Multiplying both sides by $u'(0)$ and using Proposition 2 this can be rewritten as:

$$v'(S_D) - v'(S_S) = -\lambda\left[u'(0) - W'_\beta(S_S) - q\left(W'_\alpha(S_S) - W'_\beta(S_S)\right)\right],$$

according to which the difference in the marginal utility of the stock between the deterministic and the stochastic steady state is always positive. To see this, note that:

$$v'(S_D) - v'(S_S) = -u'(0)B(S_S, 0)$$

so that the sign of $v'(S_D) - v'(S_S)$ depends on the sign of $B(S_S, 0)$. To evaluate such a sign note that:

$$\begin{aligned} B(S,0) &= u'(0) - qW'_\alpha(S) + qW'_\beta(S) - W'_\beta(S) \\ &= u'(0) - W'_\beta(S) + q\left[W'_\beta(S) - W'_\alpha(S)\right]. \end{aligned}$$

The second term $W'_\beta(S) - W'_\alpha(S)$ is always negative. The term $u'(0) - W'_\beta(S)$ is equal to zero if $S = S_\beta$, is positive if $S > S_\beta$ and negative if $S < S_\beta$. Therefore at S_β $B(S_\beta, 0)$ is negative. At $S = S_\alpha$ instead $B(S_\alpha, 0) = u'(0) - W'_\beta(S_\alpha) + q\left[W'_\beta(S_\alpha) - u'(0)\right] = (1-q)\left[u'(0) - W'_\beta(S_\alpha)\right]$. To give a definite sign we need to show that $u'(0) - W'_\beta(S_\alpha) > 0$. We need therefore an upper bound for $W'_\beta(S_\alpha)$. To obtain such a bound, suppose that preferences are such that the utility function of the stock is $(1-\beta)v(S)$ and the stock is S_α. It follows that the best use is consumption, i.e. $W'_\beta(S_\alpha) = u'(\hat{c})$ where $\hat{c}$ is current consumption. But $u'(\hat{c}) \leq u'(0)$ by concavity of the utility function. Therefore, $W'_\beta(S_\alpha) \leq u'(0)$. Hence $u'(0) - W'_\beta(S_\alpha) \geq 0$. Therefore the function $B(S_t, c_t)$ is non-negative at S_α and the function $A(S_t, c_t) + B(S_t, c_t)$ is negative at S_β and positive at S_α and it has to change sign in the interval. □

The previous proposition also allows us to characterize the connection between the level of the stock in the stochastic steady state and the level of the stock in the deterministic steady state. In fact:

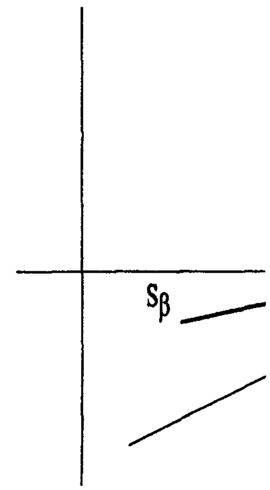

Figure 1. A is negative below
non-negative at S_α, so the sum m

PROPOSITION 5. *The lev may be larger than, equal deterministic steady state, S it is less than S_D, and vice*

Proof. The figure show $S = S_D$, positive for S $B(S,0)$ instead may be eit $S = S_D$. For example, whe follows that when $q = 0$ at that the intersection with t stock corresponding to the

4.1. *A Comparison between*

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Note that with utility de consequences of symmetric constancy of the expected the utility function in fact th

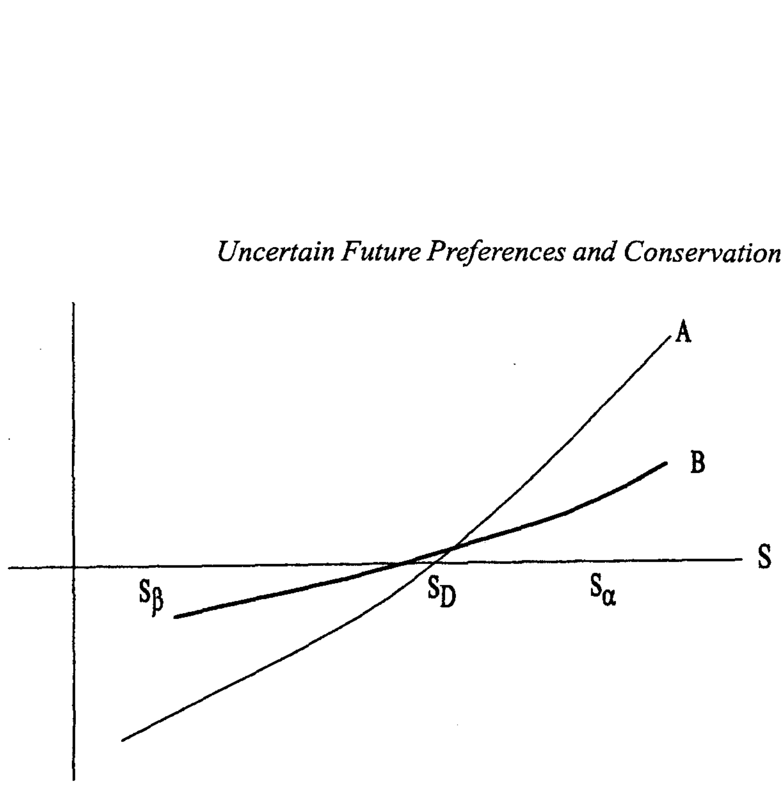

Figure 1. A is negative below S_D, zero at S_D and positive above. B is negative at S_β and non-negative at S_α, so the sum must equal zero somewhere between S_α and S_β.

PROPOSITION 5. *The level of the stock in the stochastic steady state, S_S may be larger than, equal to or smaller than the level of the stock in the deterministic steady state, S_D. For a small probability of a preference increase it is less than S_D, and vice versa.*

Proof. The figure shows that the function $A(S, 0)$ is equal to zero for $S = S_D$, positive for $S > S_D$ and negative for $S < S_D$. The function $B(S, 0)$ instead may be either larger or smaller than zero when evaluated at $S = S_D$. For example, when $q = 0$ one has $u'(0) = W'_\beta(S_\beta) > W'_\beta(S_D)$. It follows that when $q = 0$ at $S = S_D$ one has $A(S_D, 0) + B(S_D, 0) > 0$ so that the intersection with the horizontal axis, which defines the level of the stock corresponding to the stochastic steady state, is to the left of S_D. □

4.1. *A Comparison between the Two Models*

In the model with environmental assets as source of utility, uncertainty about future preferences may give rise to a path of consumption of resources which implies an increase or a decrease in the total use of resources with respect to the total consumption under the certainty of no change. No equivalent result could be obtained in the previous model, where all the stock was eventually consumed due to the absence of a steady state.

Note that with utility depending on the stock it is difficult to analyze the consequences of symmetric uncertainty, which can again be defined as the constancy of the expected utility of wealth. In the model with the stock in the utility function in fact the two marginal valuation functions do not have a simple relation to each other given that the total use of the stock is different in the two states of the world.

Notice also a fundamental difference in the specification of the two models: when the stock is in the utility function it is always possible to consume at a future date with a utility function which is not affected by uncertainty, as it is assumed that there is no change in the direct utility of consumption. This provides a hedge against the bad state of the world, to the extent that a low utility of the stock in itself simply suggests to the policy-maker the optimality of consuming a larger fraction of the remaining stock, deriving from this a utility that has not been affected by the change. Our conjecture, which cannot be proven formally at this stage, is that this characteristic promotes a more conservative usage policy, in the same way as the introduction of a riskless asset always increases savings on the part of a risk-averse agent.

5. Shadow Prices and Option Values

This section discusses the connections of our model with the definitions of option value given in the environmental resources literature.[3] Such definitions involve both taxes aimed at inducing a myopic decision-maker to take an optimal decision and comparisons of value functions of optimal and sub-optimal problems.

The effects of uncertainty about preferences on the consumption policies are reflected in the effects on the shadow prices associated with consumption. The first order conditions can be evaluated at the equilibrium quantities in order to find the market price that would induce the agent to follow a policy leading exactly to those quantities. It follows that at each point of the planning horizon one can compare the market price for the case where the change in preferences is ignored with the market price for the case where the possibility of new preferences is taken into account.

In the specification where uncertainty about future preferences only involves the flow of utility, there may or may not be any effects on competitive prices. It has been shown that there are no effects in the case of symmetric uncertainty, as the optimal solution under uncertainty corresponds to that under certainty, with no change in shadow prices. In the case of uncertain preferences but certain timing instead it was observed that the optimal policy changes in an intuitive way, depending on the direction of the shift in preferences, anticipating (postponing) consumption in the case that a worsening (improving) of utility is anticipated. In this case the shadow price is also different from the one holding under certainty.

The difference in the prices can then be interpreted as a tax that should be imposed on the myopic decision-maker in order to induce him to adopt the optimal policy which takes into account the possibility of a change in preferences. A way to interpret such a change is in terms of the price that a benevolent and fully-informed dictator should impose on myopic private agents in order to induce them to internalize the possibility of a change in preferences.

Consider as an example a case of asymmetric uncertainty in the model for the environmental asset, where the optimal stochastic policy deviates from the optimal deterministic policy assuming no change of preferences. Assume, for the sake of the argument, that it is optimal to decrease current (time 0) consumption so as to reach a stochastic steady state with a stock which is larger than the stock which would obtain in the deterministic case. Then the (positive) difference in the prices of the stochastic and the deterministic problems may be considered a tax that must be imposed on the private agents in order to take into account the dynamic externality when deciding their consumption path. A positive tax should be put on consumption of environmental assets to induce myopic decision-makers to slow down the rate of deletion of the stock. Such a tax resembles the "development tax" that is sometimes suggested in the environmental literature in cases of irreversible consumption.

The difference in the shadow prices between the solution which ignores and the solution which takes into account uncertainty is also connected with the definition of option value given in the environmental literature. This literature, as was pointed out in the introduction, has largely been based on very simple two-period discrete choice models. In the context of such a models there is a one-to-one relationship between the value function of the problem and the shadow prices of the problem. In fact, in these models the tax on development is also the measure which equates the value functions of a problem under full information and a problem with less information.

For example, Henry [17] defines the option value, for the case of a discrete control variable, as the amount of consumption good that a person would pay to have the possibility of making decisions taking into account all the information. Paying this amount and then making the decision with full information would leave her as well off as in the case where she has to make all the decisions at the beginning of the planning period, before the uncertainty is resolved. He shows that considering the value of information in this sense tilts the decision towards a more conservative policy. Likewise, omitting this value leads to under-conservation. As another example, Hanemann interprets the option value as the difference between the value of the problem when this is solved with a closed loop (CL) policy and the value of the problem solved with an open loop policy, which may be a simple open loop (OL) policy or a feedback open loop (FOL). He shows that in cases of discrete control variables, such a value is always positive, because a CL policy takes into account the state of the system while the OL policy does not. An OL policy is therefore in general a sub-optimal policy when compared with a CL policy.

The common characteristics of these examples are: (1) a comparison between a model with full information and a model with partial information;

(2) a comparison between the value functions of such problems; and (3) a comparison between the shadow prices so as to induce a myopic decision-maker to take a rational solution. In the more general models we have worked with, (1) and (2) are still closely connected: we can compare optimal and sub-optimal decision structures and then evaluate the value functions. However, (3) is not the same as (2): the change in the shadow prices at each point in time is in general different from the difference in the value functions. It is, therefore, necessary to distinguish more clearly what is meant by option value. We believe (see also [13] for a similar opinion) that the definition of option value connected with the difference in shadow prices is the most relevant, at least from the point of view of environmental policy, because affecting market prices is an important instrument for promoting sustainable policies.

The relevance of shadow prices for environmental policy is well described by the following example proposed by Graham-Tomasi [13]: "A resource problem that illustrates most of the issues is the depletion of moist tropical rainforest. The resource stock has value both for its timber and the agricultural land (or other uses) it may be converted to, as well as for the ecosystem services it provides and the biodiversity it contains. The values of the goods and services provided by a tropical forest in its natural state are not well known in comparison with our understanding of the value of harvested timber and agricultural products... The basic idea of quasi-option value, then, is that the mere prospect of improved research programs on the value of moist tropic forest ecosystems, even allowing for the possibility that they may find that such forests are less valuable than we now believe, should lead to greater conservation of such forests".

6. Conclusions

We have analyzed the problem of making irreversible decisions in a situation where preferences may change in the future, after the irreversible decision has been made. Following [1, 7, 20, 26], we have used this as a framework for thinking about the conservation of environmental assets and deriving an option value. Contrary to most of the papers in the literature, we have worked with a general model in which there is a possibility of a quantum increase in the intensity of preference for environmental goods at an unknown future date. The present generation does not know the preferences of its successors and wishes to allow for the possibility of them having a greater valuation of the environmental good. We have studied the change in the shadow price of the environmental asset as a result of the possibility of a preference change.

Perhaps the most interesting conclusion is that in the present framework uncertainty about future preferences, including the possibility of an increase in the strength of the preference for the environmental good, is not on its own

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A. Appendix

A.1. *Marginal Utility of W*

In the next Proposition we
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function which applies fro

PROPOSITION 6. *Let W*
$(du(c_T^*)/dc_T^*) = p_T$, *whe*
onwards and p_T *is the sha*

Proof. Without any loss

$$\frac{dW}{dS_0} = \frac{d}{dS_0}\left\{\max \int_0 \right.$$

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$$= \frac{d}{dS_0}\left\{\int_0^\infty u \right.$$

$$= -\int_0^\infty \frac{du(c_t)}{dc}$$

$$= \frac{du(c_0)}{dc}\frac{\delta}{\eta}\int_0$$

The equality with p_T f
the problem (4). This con

Notes

1. For mathematical details, s
2. ω/Ω is of course the prob
has not previously occurre
3. See [13] for a general disc

a source of a conservation motive. It is only asymmetric uncertainty that leads to these conclusions. In other words, it is not sufficient for a positive option value that we believe that future generations may have a stronger preference for environmental goods: we have to believe that on average they will do so.

A. Appendix

A.1. *Marginal Utility of Wealth*

In the next Proposition we establish a simple identity that was used in earlier computations. It shows that the derivative of the state valuation function with respect to the remaining stock at time T, is equal to the marginal utility of consumption at time T along a path which is optimal according to the utility function which applies from T on.

PROPOSITION 6. *Let $W_C(S_T)$ be defined as in (6). Then $(dW/dS_T) = (du(c_T^*)/dc_T^*) = p_T$, where c_T^* is the optimal consumption path from time T onwards and p_T is the shadow price of the stock S_T at time T.*

Proof. Without any loss of generality we shall set $T = 0$ and $W_C = W(S_0)$.

$$\begin{aligned}
\frac{dW}{dS_0} &= \frac{d}{dS_0}\left\{\max \int_0^\infty u(c_t)e^{-\delta t}dt \text{ subject to } \int_0^\infty c_t \leq S_0\right\} \\
&= \frac{d}{dS_0}\left\{\int_0^\infty u(c_t)e^{-\delta t}dt \text{ where } \frac{\dot{c}}{c} = \frac{\delta}{\eta} \text{ and } c_0 = \frac{-S_0\delta}{\eta} \text{ by (7)}\right\} \\
&= \frac{d}{dS_0}\left\{\int_0^\infty u\left(\frac{-S_0\delta}{\eta}e^{(\delta/\eta)t}\right)e^{-\delta t}dt\right\} \\
&= -\int_0^\infty \frac{du(c_t)}{dc}\, e^{-\delta t+(\delta/\eta)t}\frac{\delta}{\eta}dt \\
&= \frac{du(c_0)}{dc}\,\frac{\delta}{\eta}\int_0^\infty e^{(\delta/\eta)t}dt \text{ using (9) for the problem in (6).}
\end{aligned}$$

The equality with p_T follows from the first order condition (9), applied to the problem (4). This completes the proof. □

Notes

1. For mathematical details, see [9].
2. ω/Ω is of course the probability of the change in preferences occurring at t given that it has not previously occurred.
3. See [13] for a general discussion of this point.

References

1. Arrow, K. J. and A. C. Fisher. "Environmental Preservation, Uncertainty and Irreversibility", *Quarterly Journal of Economics* 88, 1974, 312–319.
2. Asheim, G. "Sustainability When Resource Management Has Stochastic Consequences", Discussion Paper, Norges Handelshøyskole, 1193.
3. Beltratti, A., G. Chichilnisky and G. Heal. "Sustainable Growth and the Green Golden Rule", in *Approaches to Sustainable Economic Development*, I. Goldin and A. Winters (eds.), Paris, OECD, pp. 147–156.
4. Bishop, R. "Option Value: An Exposition and Extension", *Land Economics* 58, 1982, 1–15.
5. Bohm, P. "Option Demand and Consumers' Surplus: Comment", *American Economic Review* 65(4), 1975, 733–736.
6. Chichilnisky G. and G. M. Heal. "Global Environmental Risks", *Journal of Economic Perspectives* 7, 1993, 65–86.
7. Cicchetti, C. J. and A. M. Freeman. "Option Demand and Consumer Surplus: Further Comment", *Quarterly Journal of Economics* 85, 1971, 528–539.
8. Dasgupta, P. S. and G. M. Heal. "The Optimal Depletion of Exhaustible Resources", *Review of Economic Studies*, Special Issue on Symposium on the Economics of Exhaustible Resources, 1974, 3–28.
9. Dasgupta, P. S. and G. M. Heal. *Economic Theory and Exhaustible Resources*, Cambridge, U.K., Cambridge University Press, 1979.
10. Fisher, A. C. and W. M. Hanemann. "Quasi-Option Value: Some Misconceptions Dispelled", *Journal of Environmental Economics and Management* 15, 1987, 183–190.
11. Fisher, A. C. and W. M. Hanemann. "Option Value and the Extinction of Species", in *Advances in Applied Microeconomics*, Vol. 4, V. K. Smith (ed.), Greenwich, CT, JAI Press, 1979.
12. Fisher, A. C., J. V. Krutilla and C. J. Cicchetti. "The Economics of Environmental Preservation: A Theoretical and Empirical Perspective", *American Economic Review* 57, 1972, 605–619.
13. Graham-Tomasi, T., "Quasi-Option Value", in *Handbook of Environmental Economics*, D. W. Bromley (ed.), Oxford (U.K.) and Cambridge (U.S.A.), Blackwell, 1995.
14. Hanemann, W. M. "Information and the Concept of Option Value", *Journal of Environmental Economics and Management* 16, 1989, 23–37.
15. Heal, G. M. "Uncertainty and the Optimal Supply Policy for an Exhaustible Resource", in *Advances in the Economics of Energy and Resources*, Vol. 2, R. Pindyck (ed.), Greenwich, CT, JAI Press, 1979, pp. 119–147.
16. Heal, G. M. "Depletion and Discounting", in *Proceedings of Symposia in Applied Mathematics*, Vol. 32, R. McKelvey (ed.), American Mathematical Society, 1985, pp. 33–43.
17. Henry, C. "Option Values in the Economics of Irreplaceable Assets", *Review of Economic Studies*, Special Issue on Symposium on the Economics of Exhaustible Resources, 1974, 89–104.
18. Henry, C. "Investment Decisions under Uncertainty: The Irreversibility Effect", *American Economic Review* 64, 1974, 1006–1012.
19. Krautkraemer, J. A. "Optimal Growth, Resource Amenities and the Preservation of Natural Environments", *Review of Economic Studies* LII, 1985, 153–170.
20. Krutilla, J. V. "Conservation Reconsidered", *American Economic Review* 57, 1967, 777–786.
21. Mitchell, R. C. and R. Carson, *Using Surveys to Value Public Goods: the Contingent Valuation Method*, Washington DC, Resources for the Future, 1989.
22. Rothschild, M. and J. E. Stiglitz. "Increasing Risk 1: A Definition", *Journal of Economic Theory* 2(3), 1970, 225–243.
23. Schmalensee, R. "Optio under Uncertainty", *Ame*
24. Smith, V. K. "A Bound f
25. Solow, R. M. "Sustainab Johnson Lecture, Woods
26. Weisbrod, B. A. "Collect *Quarterly Journal of Ec*
27. Viscusi, W. K. "Environn ty", *Journal of Environm*
28. Viscusi, W. K. "Irreversil *Journal of Environmenta*
29. World Commission on "Brundtland Report"), O

ɔn, Uncertainty and Irreversibil-

Has Stochastic Consequences",

Growth and the Green Golden
ment, I. Goldin and A. Winters

ɪn", *Land Economics* 58, 1982,

:omment", *American Economic*

al Risks", *Journal of Economic*

and Consumer Surplus: Further
528–539.
ion of Exhaustible Resources",
posium on the Economics of

d Exhaustible Resources, Cam-

lue: Some Misconceptions Dis-
igement 15, 1987, 183–190.
d the Extinction of Species", in
mith (ed.), Greenwich, CT, JAI

e Economics of Environmental
American Economic Review 57,

k of Environmental Economics,
S.A.), Blackwell, 1995.
ion Value", *Journal of Environ-*

for an Exhaustible Resource", in
2, R. Pindyck (ed.), Greenwich,

s of Symposia in Applied Math-
atical Society, 1985, pp. 33–43.
le Assets", *Review of Economic*
of Exhaustible Resources, 1974,

rreversibility Effect", *American*

nities and the Preservation of
, 1985, 153–170.
conomic Review 57, 1967, 777–

Public Goods: the Contingent
ture, 1989.
efinition", *Journal of Economic*

23. Schmalensee, R. "Option Demand and Consumers' Surplus: Valuing Price Changes under Uncertainty", *American Economic Review* 62, 1972, 813–824.
24. Smith, V. K. "A Bound for Option Value", *Land Economics* 60(3), 1984, 292–296.
25. Solow, R. M. "Sustainability: An Economist's Perspective", The Eighteenth J. Seward Johnson Lecture, Woods Hole Oceanographic Institution, Woods Hole, MA, 1992.
26. Weisbrod, B. A. "Collective Consumption Services of Individual Consumption Goods", *Quarterly Journal of Economics* 77, 1964, 71–77.
27. Viscusi, W. K. "Environmental Policy Choice with an Uncertain Chance of Irreversibility", *Journal of Environmental Economics and Management* 12, 1985, 28–44.
28. Viscusi, W. K. "Irreversible Environmental Investments with Uncertain Benefit Levels", *Journal of Environmental Economics and Management* 15, 1988, 147–157.
29. World Commission on Environment and Development. *Our Common Future*, (the "Brundtland Report"), Oxford University Press, 1987.