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# Bertrand-Edgeworth Equilibrium in Oligopoly

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## Abstract

This paper investigates a simultaneous move capacity constrained price competition game among three firms. I find that equilibria in an asymmetric oligopoly are substantially different from those in a duopoly and symmetric oligopoly. I characterize mixed strategy equilibria and show there exist possibilities of i) the existence of a continuum of equilibria ii) the smallest firm earning the largest profit per capacity and iii) non-identical supports of equilibrium mixed strategies, all of which never arise either in the duopoly or in the symmetric oligopoly. In particular, the second finding sheds light on a completely new pricing incentive in Bertrand competitions.

**JEL Classification Numbers:** L13 C72

**Key Words:** Price Competition, Oligopoly, Capacity Constraint, Homogeneous Goods.

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# 1 Introduction

While the notion of price competition is simple and of a long history, the research of homogeneous products has been limited, i.e., full characterizations of equilibria are not generally available. That is probably because of a mathematical difficulty, the discontinuity of payoff function, which cause non-existence of pure equilibrium. This paper investigates a capacity constrained price game among three firms in a homogeneous goods market with efficient rationing rule, and shows substantial differences between a duopoly and (asymmetric) oligopoly.

Kreps and Scheinkman (1983) and Osborne and Pitchik (1986) analyze the duopolistic version of my model and fully characterize the equilibrium. Although there exist a number of subsequent papers which examine oligopoly, all of them have some crucial additional assumptions and thus their scope is quite restricted. Brock and Scheinkman (1985) consider a repeated price game in a general  $N$  firms oligopoly and specify the equilibrium payoffs in the stage game (i.e., one shot price game), but they assume all firms have identical capacity. Vives (1986) proves that the support of equilibrium prices converges to the competitive price, but he also assumes symmetric capacity and takes the limit as the number of firms goes infinity. Bocard and Wauthy (2000) and De Francesco (2003) consider a two stage game á la Kreps and Scheinkman (1983) with finite number of asymmetric firms, but they examine only the largest firm's payoff and incentive in the price competition stage.<sup>1</sup>

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<sup>1</sup>The situation for price competition under convex costs is similar. Characterization of

Thus, this paper is the first attempt to characterize the equilibrium payoffs of all firms in a finite and asymmetric oligopoly. The reason why such an attempt has not been made may be as follows. The incentive of the largest firm is easy to characterise in the same way as a duopoly, and strong enough to investigate, for example, the subgame perfect equilibria of two stage models á la Kreps and Scheinkman. In other words, the largest firm has the strongest incentive to set a high price in either a duopoly or oligopoly, since its residual demand (market demand minus opponents' capacity) is the largest. Thus one might expect that other properties of the duopolistic equilibrium will also be extended to an oligopoly. In an asymmetric oligopoly, however, I show below that the smallest firm can have a special incentive to raise its price that never appears in the duopoly nor, of course, symmetric oligopoly. I also prove that a continuum of equilibria can exist whereas equilibrium is unique in the duopoly. In addition, the possibility that firms have heterogeneous supports of equilibrium strategies is also a departure from the duopoly. Three firms are enough to show the departures from the duopoly and gives important insights about the oligopoly.

The rest of this paper is organized as follows: we explain our model and introduce notations in Section 2, present the characterizations of equilibria in Section 3, and conclude in Section 4.

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equilibrium is not available, though existence of (mixed strategy) equilibrium is proved by Dixon (1984). See Dixon (1987) and Chowdhury (2007) for examples of research with additional assumptions.

## 2 The Model

I consider the following game. The set of the firms is  $I = \{1, 2, 3\}$ . Firm  $i$ 's strategy is its price, i.e., its strategy space is given by  $S_i = \mathbb{R}_+$  for all  $i \in I$ . For each firm  $i$ , let  $K_i > 0$  denote the production capacity and suppose  $K_1 \geq K_2 \geq K_3$  without loss of generality. I assume each firm has identical constant marginal cost, which is normalized to zero. The payoff function for player  $i$  is given by

$$\pi_i(p) = p_i \cdot \max \left( 0, \min \left\{ K_i, \left( D(p_i) - \sum_{j|p_j < p_i} K_j \right) \left( \frac{K_i}{\sum_{l|p_l = p_i} K_l} \right) \right\} \right),$$

where  $p = (p_1, p_2, p_3)$  and  $D$  is the demand function described below.<sup>2</sup> I impose two common assumptions on the demand function.

**Assumption 1:**

There exists  $\bar{P}$  such that  $D(p) = 0$  if and only if  $p \geq \bar{P}$ .  $D$  is strictly decreasing on  $[0, \bar{P}]$ .

**Assumption 2:**

$D$  is twice continuously differentiable and concave on  $[0, \bar{P}]$ .

Here I introduce some more notations. Let  $K := K_1 + K_2 + K_3$ ,  $K_{-i} := K - K_i$ ,  $K_{ij} = K_i + K_j$ ,  $\tilde{p}_i := \operatorname{argmax}_p p(D(p) - K_{-i})$ , and  $\tilde{\pi}_i := \max_p p(D(p) - K_{-i})$ . Notice that  $\tilde{p}_i$  is uniquely determined because of Assumption 2.

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<sup>2</sup>The payoff function represents the so-called efficient rationing scheme. For examples of research with non-efficient rations, see Allen and Hellwig (1986, 1993) and Chowdhury (2003, 2007). Again, equilibria with these rations are characterized only in the duopoly or limit cases.

When there exists a pure strategy equilibrium, its properties are almost the same as those in the duopoly and symmetric oligopoly. That is, a pure equilibrium exists if and only if the largest firm has no incentive to raise its price at the competitive price ( $D^{-1}(\min\{K, D(0)\})$ ), and market demand is fully met at that price. Since the objective of this paper is to point out substantial differences between the duopoly and asymmetric oligopoly, I make another assumption in order to rule out pure equilibria.

**Assumption 3:**  $\tilde{p}_1 > D^{-1}(\min\{D(0), K\})$ .

Even when no pure equilibrium exists, existence of mixed strategy equilibrium is guaranteed by Theorem 5 of Dasgupta and Maskin (1986). Let  $(F_1(\cdot), F_2(\cdot), F_3(\cdot))$  denote an equilibrium triple of distribution functions (i.e., mixed strategies), and define  $a_i = \inf \text{supp } F_i$ ,  $b_i = \sup \text{supp } F_i$ ,  $a = \min_i a_i$ , and  $b = \max_i b_i$ . In the duopoly, Osborne and Pitchik (1986) show that i) the equilibrium is unique, ii) the equilibrium payoffs are given by  $(\pi_1^*, \pi_2^*) = (a^* \min\{D(a^*), K_1\}, a^* K_2)$ , where  $\pi_i^*$  is payoff of  $i$  at the equilibrium and  $a^*$  is the unique solution to  $\tilde{\pi}_1 = a^* \min\{D(a^*), K_1\}$ , and iii)  $F_1$  and  $F_2$  have identical supports  $[a, b] = [a^*, \tilde{p}_1]$  and  $\pi_i(p; F_j) = \pi_i^*$  ( $i, j = 1, 2$  and  $i \neq j$ ) for all  $p \in [a^*, \tilde{p}_1)$ . Hence the following conjecture will seem natural in a three firms oligopoly.

**Conjecture:**

- i) the equilibrium is unique,
- ii) the equilibrium payoffs are  $(\pi_1^*, \pi_2^*, \pi_3^*) = (a^* \min\{D(a^*), K_1\}, a^* K_2, a^* K_3)$  where  $a^*$  is the unique solution to  $\tilde{\pi}_1 = a^* \min\{D(a^*), K_1\}$ , and

iii)  $F_1, F_2$  and  $F_3$  have identical supports  $[a, b] = [a^*, \tilde{p}_1]$  and  $\pi_i(p; F_{-i}) = \pi_i^*$  ( $i = 1, 2, 3$ ) for all  $p \in [a^*, \tilde{p}_1]$ .

I show below that any part of the conjecture does *not* generally hold.

### 3 Results

First we show some useful lemmas which generally hold either in the duopoly or oligopoly. Similar results are frequently used in the literature.

**Lemma 1:**

There exists  $i$  such that  $b_i = \tilde{p}_i = b$  and  $\pi_i^* = \tilde{\pi}_i$ .

**Proof of Lemma 1:**

Let  $I^H := \{i \in I | b_i = b\}$ . If  $I^H = \{i\}$  for some  $i$ , the statement obviously holds for that  $i$ .

Thus, consider the case where  $|I^H| > 2$ . We first show  $b$  is an atom of  $F_i$  for at most one firm. To see this, suppose  $b$  is an atom of  $F_i$  and  $F_j$  ( $i \neq j$ ). By definition of equilibrium  $\pi_i^* = \pi_i(b; F_{-i})$ , where  $\pi_i(p; F_{-i})$  is  $i$ 's expected profit given the distributions of opponents prices when it sets a price  $p$ . Firm  $i$  has an incentive to lower its price (i.e.  $\lim_{p \nearrow b} \pi_i(b; F_{-i}) > \pi_i^*$ ) if  $D(b) < K$  and to raise its price (i.e.  $\lim_{p \searrow b} \pi_i(b; F_{-i}) > \pi_i^*$ ) if  $D(b) > K$ . Thus  $D(b) = K$  is the only possible case. This contradicts, however, Assumption 3 that firm 1 has an incentive to raise its price at  $D^{-1}(K)$ .

If  $b$  is an atom only of  $F_i$ , that  $i$  must satisfy the statement. If  $b$  is not an atom for any  $F_i$ , the statement must hold for all  $i = 1, 2, 3$ . ■

**Lemma 2:**

If the conditions in Lemma 1 are satisfied for  $i$ , then  $a_i = a$ .

**Proof of Lemma 2:**

Suppose there exists  $j$  such that  $a_j < a_i$ . Since  $a_i \leq b_i = \tilde{p}_i$ , it must be that  $D(a_i) > K_{-i}$  and thus  $\pi_j(p; F_{-j})$  is increasing on  $[0, a_i)$ , a contradiction. ■

**Lemma 3:**

If the conditions in Lemma 1 are satisfied for  $i$ , then  $K_i = K_1$ .

**Proof of Lemma 3:**

Notice that  $K_i$  is unique by lemma 1 and the definition of  $\tilde{p}_i$ , though  $i$  may not be. Suppose that  $K_i < K_1$ , which implies  $D(a_i) \geq D(b_i) > K_{-i} > K_{-1}$ . Then,  $a = a_i = a_1$  must hold, because otherwise  $p_i \in [a_i, a_1)$  is strictly dominated by  $p_i = a_1$ . Since  $\pi_i(a; F_{-i}) = \pi_i^* = \pi_i(b; F_{-i})$ ,  $a = b(D(b) - K_{-i})/K_i$ . Then, however,

$$\begin{aligned} \pi_1^* - \tilde{\pi}_i &= \pi_1(a; F_{-1}) - \tilde{\pi}_1 \\ &\leq aK_1 - b(D(b) - K_{-1}) \\ &= (b/K_i)(D(b) - K)(K_1 - K_i) < 0, \end{aligned}$$

which is a contradiction to the equilibrium condition. ■

These Lemmas present the incentive of the largest firm that we discussed in the previous sections. In what follows, I assume firm 1 satisfies the conditions in Lemma 1 without loss of generality.

Next we characterize mixed strategy equilibria. Note that Lemmas 1-3



imply  $a = a^*$  and  $b = b^*$  in any equilibrium, where  $a^*$  is the unique solution to  $\tilde{\pi}_1 = a^* \min\{a^*, D(a^*)\}$  and  $b^* = \tilde{p}_1$ . It is clear that there exists  $i \neq 1$  such that  $a_i = a_1 = a^*$  so that firm 1 does not have an incentive to raise its price from  $a_1$ . This condition can pin down the unique equilibrium in the duopoly, but not in the oligopoly. We need to distribute cases by the relations among  $K_i$ 's and  $D(a^*)$  in our oligopolistic model.

The first is the case in which  $K_1$  is very large relatively to  $K_2$  and  $K_3$ .

**Claim 1:**

If  $D(a^*) < K_1$ , we can construct a continuum of equilibria, but the equilibrium payoff is unique  $(\pi_2^*, \pi_3^*) = (a^*K_2, a^*K_3)$ .

**Proof of Claim 1:**

First I show how to construct a continuum of equilibria in which  $(\pi_2^*, \pi_3^*) = (a^*K_2, a^*K_3)$ . Note that, if  $D(p) \leq K_1$  and  $p$  is not an atom of  $F_1$ ,

$$\pi_i(p; F_{-i}) = (1 - F_1(p))pK_i \quad (i \neq 1) \quad (1)$$

which depends only on  $F_1$ . Hence, taking  $F_1(p) = 1 - (a^*/p)$  for  $p \in [a^*, b^*)$ , equilibrium conditions for firms 2 and 3 are obviously satisfied. Any pair of non-atomic  $F_2$  and  $F_3$  which satisfies

$$\pi_1(p; F_2, F_3) = \pi_1^*(= a^*D(a^*))$$

for all  $p \in [a^*, b^*)$  forms an equilibrium. Since only one condition is imposed on two variables, we can take a continuum of  $(F_2(\cdot), F_3(\cdot))$ .

Next we prove  $(\pi_2^*, \pi_3^*) = (a^*K_2, a^*K_3)$  in any equilibrium. Notice that  $F_1$  must be non-atomic on  $[a^*, b^*)$ . To see why, suppose  $\bar{p} \in [a^*, b^*)$  is an atom of  $F_1$ . It implies that, for  $i \in \{2, 3\}$ ,  $\lim_{p \nearrow \bar{p}} \pi_i(p; F_{-i}) < \pi_i(\bar{p}; F_{-i})$  and thus  $F_i(\bar{p} + \epsilon) = F_i(\bar{p})$  for sufficiently small but strictly positive  $\epsilon$ . Then, however, it follows by Assumption 2 that  $\pi_1(p; F_{-1}) > \pi_1(\bar{p}; F_{-1})$  for  $p \in (\bar{p}, \bar{p} + \epsilon)$  which contradicts the assumption  $\bar{p}$  is an atom of equilibrium strategy  $F_1$ . Therefore  $F_1$  is non-atomic on  $[a^*, b^*)$  and  $\pi_2^*/K_2 = \pi_3^*/K_3$  since  $\text{supp } F_2 \cap \text{supp } F_3$  must not be empty by the same logic as above. Moreover, there must exist  $i \in \{2, 3\}$  such that  $a_i = a^*$  and  $\pi_i^* = a^*K_i$ , and the statement on the payoffs must hold obviously. ■

Notice that exactly the same logic will hold even when there are more than three firms. That is, our three firms setting is not restrictive in this result. Note that we can construct equilibria so that supports of  $F_i$ 's are heterogeneous, i.e., not only part i) but also part iii) of Conjecture fails. See appendix for example.

The second is the case where  $K_1, K_2$  and  $K_3$  are relatively close to each other. The symmetric capacities case, in which  $K_1 = K_2 = K_3$ , must be included here.

**Claim 2:**

If  $K_1 + K_2 \leq D(a^*)$ ,  $a_2 = a_3 = a^*$  and  $(\pi_2^*, \pi_3^*) = (a^*K_2, a^*K_3)$ .

**Proof of Claim 2:**

If there exists  $i \in I \setminus \{1\}$  such that  $a_i > a^*$ , firm 1 has an incentive to raise

its price from  $a^*$ .<sup>3</sup> Thus  $a_i = a^*$  for all  $i$ .

It is obvious that  $\pi_i^* = a^*K_i$  if  $a^*$  is an atom of  $F_i$ . Even if not, combining the equilibrium condition

$$\lim_{p \searrow a^*} \pi_i(p; F_{-i}) = \pi_i^* \geq \lim_{p \nearrow a^*} \pi_i(p; F_{-i})$$

and the fact

$$\lim_{p \searrow a^*} \pi_i(p; F_{-i}) \leq a^*K_i = \lim_{p \nearrow a^*} \pi_i(p; F_{-i})$$

also yields  $\pi_i^* = a^*K_i$ . ■

Notice that this result is also expendable to a general  $N$  firms oligopoly. Suppose there are  $N$  firms with  $K_1 \geq K_2 \geq \dots \geq K_N$ . If  $D(a^*) > K_{-N}$ ,  $a_i = a^*$  and  $\pi_i^* = a^*K_i$  for all  $i$ . Claim 2 exhibits a natural extension of the duopoly, but equilibria quite different from Conjecture can arise even when  $D(a^*) > K_1$ .

The next is the case in which  $K_3$  is very small relatively to  $K_1$  and  $K_2$ . Parts ii) and iii) of Conjecture fail here.

**Claim 3:**

If  $K_1 + K_3 \leq D(a^*) < K_1 + K_2$ ,  $a_3 > a_2 = a^*$  and  $\pi_3^*/K_3 > \pi_2^*/K_2 = a^*$ .

**Proof of Claim 3:**

Notice that  $a_1 = a_2 = a^*$  must be satisfied in the equilibrium so that firm 1 does not strictly prefer  $a^* + \epsilon$  to  $a^*$ . In addition,  $a^*$  is not an atom of  $F_1$  or  $F_2$ , since, if  $a^*$  is an atom of  $F_i$  ( $i = 1, 2$ ),  $\pi_j^* = \lim_{p \searrow a^*} \pi_j(p; F_{-j}) <$

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<sup>3</sup>Even when  $K_1 + K_2 = D(a^*)$ , Assumption 2 guarantees this.

$\lim_{p \nearrow a^*} \pi_j(p; F_{-j})$  ( $j \in \{1, 2\} \setminus \{i\}$ ).

These imply that  $a_2 = a^*$  and  $\pi_2^* = a^*K_2$ . If  $a_3 = a^*$  and  $F_3$  is right-increasing at  $a^*$ ,

$$\begin{aligned}\pi_2(p; F_{-2}) = \pi_2^* &\Leftrightarrow (1 - F_1) + F_1(1 - F_3) \frac{D - K_1}{K_2} + F_1 F_3 \frac{D - K_{12}}{K_2} = \frac{a^*}{p} \\ \pi_3(p; F_{-3}) = \pi_3^* &\Leftrightarrow (1 - F_1 F_2) = \frac{a^*}{p}\end{aligned}$$

must be satisfied for  $p \in (a^*, a^* + \epsilon)$  where  $\epsilon$  is a small positive number. Thus, solving the second equation, we get

$$F_1 = \frac{1 - (a^*/p)}{F_2},$$

for  $p \in (a^*, a^* + \epsilon)$ . Substituting this into the first equation,

$$F_3 = \frac{K_2}{K_3} F_2 + \frac{D - K_{12}}{K_3}.$$

Since  $F_2 \searrow 0$  as  $p \searrow a^*$ , however,

$$\lim_{p \searrow a^*} F_3 = \frac{D(a^*) - K_{12}}{K_3} < 0,$$

a contradiction to the definition of distribution function. Therefore we can conclude that  $a_3 > a^*$  or  $F_3$  is not right-increasing at  $a^*$ .

Next suppose that  $a_3 > a^*$  and  $\pi_3^* = a^*K_3$ . Solving  $\pi_1(a; F_{-1})/K_1 =$

$\pi_2(a; F_{-2})/K_2 = a^*$ , we get

$$F_1 F_2 = K_1 K_2 \left( \frac{1 - (a^*/p)}{K_{12} - D} \right)^2,$$

for  $p \in (a^*, a^* + \epsilon)$ . If firm 3 sets  $p_3 = p$ , it can earn

$$\pi_3(p; F_{-3}) = (1 - F_1 F_2) p K_3.$$

Equilibrium condition  $\pi_3(p; F_{-3}) \leq \pi_3^* = a^* K_3$  implies that

$$1 - F_1 F_2 \leq (a^*/p)$$

must be satisfied for  $p \in (a^*, a_3)$ . Since  $1 - (a^*/p) > 0$  for  $p > a^*$ , the inequality is equivalent to

$$1 \leq \frac{F_1 F_2}{1 - (a^*/p)} = K_1 K_2 \frac{1 - (a^*/p)}{(K_{12} - D)^2},$$

which cannot hold as  $p \searrow a^*$ . That is, if  $\pi_3^* = a^* K_3$  firm 3 has an incentive to set a price lower than  $a_3$ , a contradiction. Thus  $a_3 > a^*$  and  $\pi_3^* > a^* K_3$ , or  $a_3 = a^*$  and  $F_3$  is not right-increasing at  $a^*$ .

If  $a_3 = a^*$  and  $F_3$  is not right-increasing at  $a^*$ , a similar contradiction (firm 3 has a strict incentive to set higher price than  $a^*$ ) occurs since  $F_1$  and  $F_2$  will be smaller than the above specification. ■

The intuition behind Claim 3 is simple, but completely new in the liter-

ature. The smallest firm cannot sell a lot even if it charges the lowest price. Thus the ratio of residual demand ( $D - K_{-i}$ ) to capacity ( $K_i$ ) is the largest for the smallest firm. This property gives the smallest firm an incentive to raise its price despite raising price also raises the probability of being undercut. Such an incentive can never appear in a duopoly, because the smaller firm's behavior is determined solely by the equilibrium condition for the larger firm.

Following Claims 4 and 5 on intermediate cases are just combinations of Claims 1-3.

**Claim 4:**

If  $K_1 < D(a^*) < K_3$  and  $K_2 > K_3$ ,  $a_3 > a_2 = a^*$  and  $\pi_3^*/K_3 > \pi_2/K_2 = a^*$  or there exists a continuum of equilibria in any of which  $\pi_2^*/K_2 = \pi_3^*/K_3 = a^*$ .

**Proof of Claim 4:**

First we show that  $a_3 > a^*$  if  $a_2 = a^*$ . By way of contradiction, suppose  $a_2 = a_3 = a^*$ . Notice that  $a^*$  cannot be an atom of any  $F_i$  in this case. The same as in the proof of Claim 3,

$$\begin{aligned}\pi_2(p; F_{-2}) = \pi_2^* &\Leftrightarrow (1 - F_1) + F_1(1 - F_3) \frac{D - K_1}{K_2} = \frac{a^*}{p} \\ \pi_3(p; F_{-3}) = \pi_3^* &\Leftrightarrow (1 - F_1) + F_1(1 - F_2) \frac{D - K_1}{K_2} = \frac{a^*}{p}\end{aligned}$$

must hold for  $p \in (a^*, a^* + \epsilon)$ . Solving the second equation and substituting into the first, we get

$$F_3 = \frac{K_2}{K_3} F_2 + \frac{K_2}{D - K_1} \left( \frac{K_1 - D}{K_3} + \frac{D - K_1}{K_2} \right).$$

Again, as  $p \searrow a^*$ ,

$$F_3 \rightarrow \frac{K_2}{D(a^*) - K_1} \left( \frac{K_1 - D(a^*)}{K_3} + \frac{D(a^*) - K_1}{K_2} \right) < 0,$$

a contradiction.

Next we consider the case in which  $a_2 > a^*$ . Then it must be satisfied that  $a_3 = a^*$  and  $\pi_3^* = a^* K_3$ . By way of contradiction, suppose  $D(a_2) < K_1$ . Equilibrium condition for firm 3 implies

$$\pi_3(a_2; F_{-3}) = a_2[(1 - F_1(a_2))K_3 + F_1(D(a_2) - K_1)] \leq a^* K_3 = \lim_{p \nearrow a^*} \pi_3(p; F_{-3}).$$

However,

$$\pi_2(a_2; F_{-2}) < a_2[(1 - F_1(a_2))K_2 + F_1(D(a_2) - K_1)] < a^* K_2 = \lim_{p \nearrow a^*} \pi_2(p; F_{-2}).$$

That is, firm 2 has a strict incentive to set  $a^*$  (or slightly below), a contradiction. Therefore we can conclude that if  $a_2 > a^*$  in equilibrium,  $D(a_2) \leq K_1$ . This implies that  $\pi_2^*/K_2 = \pi_3^*/K_3 = a^*$  by the same logic as Claim 1. Furthermore, if  $D^{-1}(K_1)(K_1 - K_3) < \pi_1^*$ , we can construct a continuum of equilibria in the same way as Claim 1. <sup>4</sup> ■

**Claim 5:**

If  $K_1 < D(a^*) < K_3$  and  $K_2 = K_3$ , there can exist a continuum of equilibrium, but  $(\pi_2^*, \pi_3^*) = (a^* K_2, a^* K_3)$  in any equilibrium.

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<sup>4</sup>If  $D^{-1}(K_1)(K_1 - K_3) = \pi_1^*$ , we can construct only one equilibrium in which  $a_2 > a^*$ , but this is a degenerate case and a continuum of equilibrium generically exists.

**Proof of Claim 5:**

By the same logic as in the proof of Claim 4, a contradiction occurs if  $a_i > a_j$  and  $D(a_i) > K_1$  ( $i, j = 2, 3$   $i \neq j$ ). Thus we can conclude that  $a_2 = a_3 = a^*$  or  $a_i > a_j$  and  $D(a_i) \leq K_1$ . In either case, it is obvious that  $\pi_2^* = \pi_3^* = a^*K_2 = a^*K_3$ . If  $D^{-1}(K_1)(K_1 - K_3) < \pi_1^*$ , we can construct a continuum of equilibria again, by the same way as Claims 1 and 4. ■

## 4 Concluding Remarks

This paper investigates a model of price competition in an oligopolistic homogeneous goods market. Our main contributions are presented in Claims 1 and 3, which clearly exhibit the differences between a duopoly and asymmetric oligopoly. I find the possibilities of i) the existence of a continuum of equilibria, ii) the smallest firm earning the highest per capacity profit, and iii) heterogeneous supports of equilibrium strategies. All of them seem interesting in the sense that they cannot arise in the duopoly. In particular, the second possibility (Claim 3) sheds new light on pricing incentives in Bertrand competition. That is, small capacity has relatively small loss in demand by setting a higher price and thus relatively higher incentive to set a higher price.

In addition, Claim 3 implies an interesting comparative statics. Suppose the condition of Claim 3 holds. That is, in the equilibrium, firm 3 earns the greatest per capacity profit. Then, it will not be strange that firm 2



considers to merge firm 3 in order to compete with firm 1. However, when they actually merge the joint profit will be strictly smaller than before the merger if  $K_1 > K_2 + K_3$ .

Finally we note on the limitation of our setting. We only consider the oligopoly among three firms. As mentioned in the previous section, Claims 1 and 2 can be easily extended to a general  $N$  firms oligopoly. However, it is hard to generalize Claims 3-5, because how to distribute cases depends on each specific  $N$ . Though it remains for future research to explore a new logic to characterize equilibria in a more general setting, three is enough to show the departure from the duopoly.

## A. 1 Example of A Continuum of Equilibria

$D(p) = 1 - p$ ,  $K_1 = 1$  and  $K_2 = K_3 = 1/4$ . Then  $(a^*, \tilde{p}_1) = ((2 - \sqrt{3})/4, 1/4)$  and  $\tilde{\pi}_1 = 1/16$ . For example, a triple  $(F_1, F_2, F_3)$  such that

$$F_1(p) = \begin{cases} 1 - \frac{a_1}{p} & \text{for } p \in [\frac{2-\sqrt{3}}{4}, \frac{1}{4}) \\ 1 & \text{for } p = \frac{1}{4} \end{cases}$$

$$F_2(p) = \begin{cases} \frac{1}{4p}(-16p^2 + 16p - 1) & \text{for } p \in [\frac{2-\sqrt{3}}{4}, \frac{3-\sqrt{5}}{8}] \\ 1 & \text{for } p \in [\frac{3-\sqrt{5}}{8}, \frac{1}{4}] \end{cases}$$

$$F_3(p) = \begin{cases} 0 & \text{for } p \in [\frac{2-\sqrt{3}}{4}, \frac{3-\sqrt{5}}{8}] \\ \frac{1}{4p}(-16p^2 + 12p - 1) & \text{for } p \in [\frac{3-\sqrt{5}}{8}, \frac{1}{4}] \end{cases}$$

consists an equilibrium.

[INSERT FIGURE 1 HERE]

For another example, the same  $F_1$  as above and  $F_2 = F_3 = F$  such that

$$F(p) = \frac{1}{8p}(-16p^2 + 16p - 1) \quad \text{for } p \in [\frac{2-\sqrt{3}}{4}, \frac{1}{4}],$$

i.e., the solution to the equilibrium condition for firm 1

$$p \left[ (1-p)(1-F)^2 + 2 \left( \frac{3}{4} - p \right) F(1-F) + \left( \frac{1}{2} - p \right) F^2 \right] = \frac{1}{16},$$

consist another equilibrium.

[INSERT FIGURE 2 HERE]

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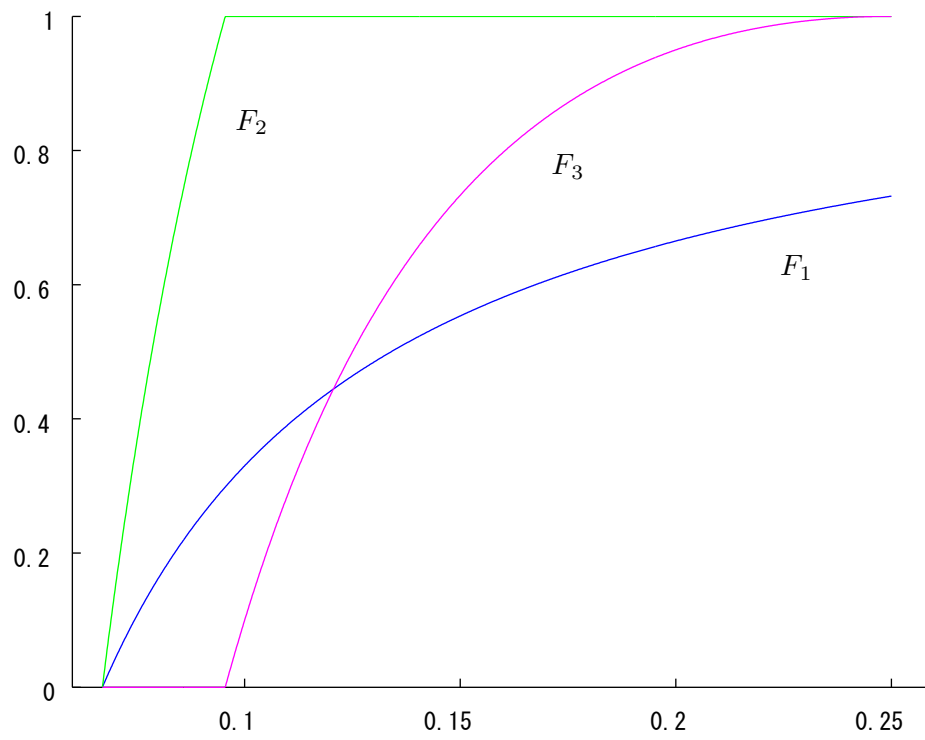


Figure 1: Distribution functions of the first equilibrium.

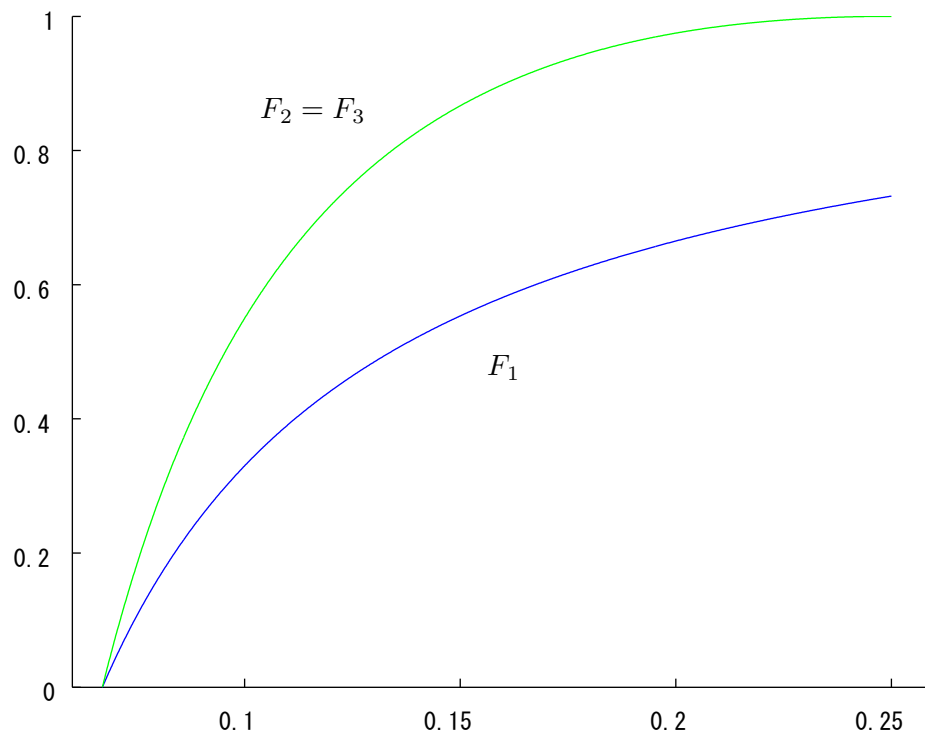


Figure 2: Distribution functions of the second equilibrium.