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Monetary Policy Analysis in a Small Open Economy: A Dynamic Stochastic General Equilibrium Approach

Francis Vitek

Abstract

This paper develops and estimates a dynamic stochastic general equilibrium model of a small open economy which approximately accounts for the empirical evidence concerning the monetary transmission mechanism, as summarized by impulse response functions derived from an estimated structural vector autoregressive model, while dominating that structural vector autoregressive model in terms of predictive accuracy. The model features short run nominal price and wage rigidities generated by monopolistic competition and staggered reoptimization in output and labour markets. The resultant inertia in inflation and persistence in output is enhanced with other features such as habit persistence in consumption, adjustment costs in investment, and variable capital utilization. Incomplete exchange rate pass through is generated by monopolistic competition and staggered reoptimization in the import market. Cyclical components are modeled by linearizing equilibrium conditions around a stationary deterministic steady state equilibrium, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. Parameters and trend components are jointly estimated with a novel Bayesian full information maximum likelihood procedure.

JEL Classification: C11; C13; C32; E52; F41; F47

Keywords: Monetary policy analysis; Inflation targeting; Small open economy; Dynamic stochastic general equilibrium model; Monetary transmission mechanism; Forecast performance evaluation

1. Introduction

Estimated dynamic stochastic general equilibrium or DSGE models have recently emerged as quantitative monetary policy analysis and inflation targeting tools. As extensions of real business cycle models, DSGE models explicitly specify the objectives and constraints faced by optimizing households and firms, which interact in an uncertain environment to determine equilibrium prices and quantities. The existence of short run nominal price and wage rigidities

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generated by monopolistic competition and staggered reoptimization in output and labour markets permits a cyclical stabilization role for monetary policy, which is generally implemented through control of the nominal interest rate according to a monetary policy rule. The persistence of the effects of monetary policy shocks on output and inflation is often enhanced with other features such as habit persistence in consumption, adjustment costs in investment, and variable capital utilization. Early examples of closed economy DSGE models incorporating some of these features include those of Yun (1996), Goodfriend and King (1997), Rotemberg and Woodford (1995, 1997), and McCallum and Nelson (1999), while recent examples of closed economy DSGE models incorporating all of these features include those of Christiano, Eichenbaum and Evans (2005), Altig, Christiano, Eichenbaum and Linde (2005), and Smets and Wouters (2003, 2005).

Open economy DSGE models extend their closed economy counterparts to allow for international trade and financial linkages, implying that the monetary transmission mechanism features both interest rate and exchange rate channels. Building on the seminal work of Obstfeld and Rogoff (1995, 1996), these open economy DSGE models determine trade and current account balances through both intratemporal and intertemporal optimization, while the nominal exchange rate is determined by an uncovered interest parity condition. Existing open economy DSGE models differ primarily with respect to the degree of exchange rate pass through. Models in which exchange rate pass through is complete include those of Benigno and Benigno (2002), McCallum and Nelson (2000), Clarida, Galí and Gertler (2001, 2002), and Gertler, Gilchrist and Natalucci (2001), while models in which exchange rate pass through is incomplete include those of Adolfson (2001), Betts and Devereux (2000), Kollman (2001), Corsetti and Pesenti (2002), and Monacelli (2005).

In an empirical investigation of the degree of exchange rate pass through among developed economies, Campa and Goldberg (2002) find that short run exchange rate pass through is incomplete, while long run exchange rate pass through is complete. This empirical evidence rejects both local currency pricing, under which the domestic currency prices of imports are invariant to exchange rate fluctuations in the short run, and producer currency pricing, under which the domestic currency prices of imports fully reflect exchange rate fluctuations in the short run. In response to this empirical evidence, Monacelli (2005) incorporates short run import price rigidities into an open economy DSGE model by allowing for monopolistic competition and staggered reoptimization in the import market. These import price rigidities generate incomplete exchange rate pass through in the short run, while exchange rate pass through is complete in the long run.

The economy is complex, and any model of it is necessarily misspecified to some extent. An operational substitute for the concept of a correctly specified model is the concept of an

empirically adequate model. A model is empirically adequate if it approximately accounts for the existing empirical evidence in all measurable respects, which as discussed in Clements and Hendry (1998) does not require that it be correctly specified. As argued by Diebold and Mariano (1995), a necessary condition for empirical adequacy is predictive accuracy, which must be measured in relative terms. Quantitative monetary policy analysis and inflation targeting should be based on empirically adequate models of the economy.

Thus far, empirical evaluations of DSGE models have generally focused on unconditional second moment and impulse response properties. While empirically valid unconditional second moment and impulse response properties are necessary conditions for empirical adequacy, they are not sufficient. Moreover, empirical evaluations of unconditional second moment properties are generally conditional on atheoretic estimates of trend components, while empirical evaluations of impulse response properties are generally conditional on controversial identifying restrictions. It follows that the empirical evaluation of predictive accuracy is a necessary precursor to a well informed judgment regarding the extent to which any DSGE model can and should contribute to quantitative monetary policy analysis and inflation targeting.

Existing DSGE models featuring long run balanced growth driven by trend inflation, productivity growth, and population growth generally predict the existence of common deterministic or stochastic trends. Estimated DSGE models incorporating common deterministic trends include those of Ireland (1997) and Smets and Wouters (2005), while estimated DSGE models incorporating common stochastic trends include those of Altig, Christiano, Eichenbaum and Linde (2005) and An and Schorfheide (2006). However, as discussed in Clements and Hendry (1999) and Maddala and Kim (1998), intermittent structural breaks render such common deterministic or stochastic trends empirically inadequate representations of low frequency variation in observed macroeconomic variables. For this reason, it is common to remove trend components from observed macroeconomic variables with deterministic polynomial functions or linear filters such as that described in Hodrick and Prescott (1997) prior to the conduct of estimation, inference and forecasting.

Decomposing observed macroeconomic variables into cyclical and trend components prior to the conduct of estimation, inference and forecasting reflects an emphasis on the predictions of DSGE models at business cycle frequencies. Since such decompositions are additive, given observed macroeconomic variables, predictions at business cycle frequencies imply predictions at lower frequencies. As argued by Harvey (1997), the removal of trend components from observed macroeconomic variables with atheoretic deterministic polynomial functions or linear filters ignores these predictions, potentially invalidating subsequent estimation, inference and forecasting. As an alternative, this paper proposes jointly modeling cyclical and trend

components as unobserved components while imposing theoretical restrictions derived from the approximate multivariate linear rational expectations representation of a DSGE model.

The development of empirically adequate DSGE models for purposes of quantitative monetary policy analysis and inflation targeting in a small open economy is currently an active area of research. Nevertheless, an estimated DSGE model of a small open economy which approximately accounts for the empirical evidence concerning the monetary transmission mechanism, as summarized by impulse response functions derived from an estimated structural vector autoregressive or SVAR model, while dominating that SVAR model in terms of predictive accuracy, has yet to be developed. This paper develops and estimates a DSGE model of a small open economy which satisfies these impulse response and predictive accuracy criteria. The model features short run nominal price and wage rigidities generated by monopolistic competition and staggered reoptimization in output and labour markets. The resultant inertia in inflation and persistence in output is enhanced with other features such as habit persistence in consumption, adjustment costs in investment, and variable capital utilization. Incomplete exchange rate pass through is generated by short run nominal rigidities in the import market, with monopolistically competitive importers setting the domestic currency prices of differentiated intermediate import goods subject to randomly arriving reoptimization opportunities. Cyclical components are modeled by linearizing equilibrium conditions around a stationary deterministic steady state equilibrium, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. Parameters and trend components are jointly estimated with a novel Bayesian full information maximum likelihood procedure.

The organization of this paper is as follows. The next section develops a DSGE model of a small open economy. Estimation, inference and forecasting within the framework of a linear state space representation of an approximate unobserved components representation of this DSGE model are the subjects of section three. Finally, section four offers conclusions and recommendations for further research.

2. Model Development

Consider two open economies which are asymmetric in size, but are otherwise identical. The domestic economy is of negligible size relative to the foreign economy.

2.1. The Utility Maximization Problem of the Representative Household

There exists a continuum of households indexed by $i \in [0,1]$. Households supply differentiated intermediate labour services, but are otherwise identical.

2.1.1. Consumption and Saving Behaviour

The representative infinitely lived household has preferences defined over consumption $C_{i,s}$ and labour supply $L_{i,s}$ represented by intertemporal utility function

$$U_{i,t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_{i,s}, L_{i,s}), \quad (1)$$

where subjective discount factor β satisfies $0 < \beta < 1$. The intratemporal utility function is additively separable and represents external habit formation preferences in consumption,

$$u(C_{i,s}, L_{i,s}) = v_s^C \left[\frac{(C_{i,s} - \alpha C_{s-1})^{1-1/\sigma}}{1-1/\sigma} - v_s^L \frac{(L_{i,s})^{1+1/\eta}}{1+1/\eta} \right], \quad (2)$$

where $0 \leq \alpha < 1$. This intratemporal utility function is strictly increasing with respect to consumption if and only if $v_s^C > 0$, and given this parameter restriction is strictly decreasing with respect to labour supply if and only if $v_s^L > 0$. Given these parameter restrictions, this intratemporal utility function is strictly concave if $\sigma > 0$ and $\eta > 0$.

The representative household enters period s in possession of previously purchased domestic currency denominated bonds $B_{i,s}^{P,h}$ which yield interest at risk free rate i_{s-1} , and foreign currency denominated bonds $B_{i,s}^{P,f}$ which yield interest at risk free rate i_{s-1}^f . It also holds a diversified portfolio of shares $\{x_{i,j,s}^Y\}_{j=0}^1$ in domestic intermediate good firms which pay dividends $\{\Pi_{j,s}^Y\}_{j=0}^1$, and a diversified portfolio of shares $\{x_{i,k,s}^M\}_{k=0}^1$ in domestic intermediate good importers which pay dividends $\{\Pi_{k,s}^M\}_{k=0}^1$. The representative household supplies differentiated intermediate labour service $L_{i,s}$, earning labour income at nominal wage $W_{i,s}$. Households pool their labour income, and the government levies a tax on pooled labour income at rate τ_s . These sources of private wealth are summed in household dynamic budget constraint:

$$\begin{aligned}
B_{i,s+1}^{P,h} + \mathcal{E}_s B_{i,s+1}^{P,f} + \int_{j=0}^1 V_{j,s}^Y x_{i,j,s+1}^Y dj + \int_{k=0}^1 V_{k,s}^M x_{i,k,s+1}^M dk &= (1 + i_{s-1}) B_{i,s}^{P,h} + \mathcal{E}_s (1 + i_{s-1}^f) B_{i,s}^{P,f} \\
+ \int_{j=0}^1 (\Pi_{j,s}^Y + V_{j,s}^Y) x_{i,j,s}^Y dj + \int_{k=0}^1 (\Pi_{k,s}^M + V_{k,s}^M) x_{i,k,s}^M dk + (1 - \tau_s) \int_{l=0}^1 W_{l,s} L_{l,s} dl - P_s^C C_{i,s}. & \quad (3)
\end{aligned}$$

According to this dynamic budget constraint, at the end of period s , the representative household purchases domestic bonds $B_{i,s+1}^{P,h}$, and foreign bonds $B_{i,s+1}^{P,f}$ at price \mathcal{E}_s . It also purchases a diversified portfolio of shares $\{x_{i,j,s+1}^Y\}_{j=0}^1$ in intermediate good firms at prices $\{V_{j,s}^Y\}_{j=0}^1$, and a diversified portfolio of shares $\{x_{i,k,s+1}^M\}_{k=0}^1$ in intermediate good importers at prices $\{V_{k,s}^M\}_{k=0}^1$. Finally, the representative household purchases final consumption good $C_{i,s}$ at price P_s^C .

In period t , the representative household chooses state contingent sequences for consumption $\{C_{i,s}\}_{s=t}^\infty$, domestic bond holdings $\{B_{i,s+1}^{P,h}\}_{s=t}^\infty$, foreign bond holdings $\{B_{i,s+1}^{P,f}\}_{s=t}^\infty$, share holdings in intermediate good firms $\{\{x_{i,j,s+1}^Y\}_{j=0}^1\}_{s=t}^\infty$, and share holdings in intermediate good importers $\{\{x_{i,k,s+1}^M\}_{k=0}^1\}_{s=t}^\infty$ to maximize intertemporal utility function (1) subject to dynamic budget constraint (3) and terminal nonnegativity constraints $B_{i,T+1}^{P,h} \geq 0$, $B_{i,T+1}^{P,f} \geq 0$, $x_{i,j,T+1}^Y \geq 0$ and $x_{i,k,T+1}^M \geq 0$ for $T \rightarrow \infty$. In equilibrium, selected necessary first order conditions associated with this utility maximization problem may be stated as

$$u_C(C_t, L_{i,t}) = P_t^C \lambda_t, \quad (4)$$

$$\lambda_t = \beta(1 + i_t) E_t \lambda_{t+1}, \quad (5)$$

$$\mathcal{E}_t \lambda_t = \beta(1 + i_t^f) E_t \mathcal{E}_{t+1} \lambda_{t+1}, \quad (6)$$

$$V_{j,t}^Y \lambda_t = \beta E_t (\Pi_{j,t+1}^Y + V_{j,t+1}^Y) \lambda_{t+1}, \quad (7)$$

$$V_{k,t}^M \lambda_t = \beta E_t (\Pi_{k,t+1}^M + V_{k,t+1}^M) \lambda_{t+1}, \quad (8)$$

where $\lambda_{i,s}$ denotes the Lagrange multiplier associated with the period s household dynamic budget constraint. In equilibrium, necessary complementary slackness conditions associated with the terminal nonnegativity constraints may be stated as:

$$\lim_{T \rightarrow \infty} \frac{\beta^T \lambda_{t+T}}{\lambda_t} B_{t+T+1}^{P,h} = 0, \quad (9)$$

$$\lim_{T \rightarrow \infty} \frac{\beta^T \lambda_{t+T}}{\lambda_t} \mathcal{E}_{t+T} B_{t+T+1}^{P,f} = 0, \quad (10)$$

$$\lim_{T \rightarrow \infty} \frac{\beta^T \lambda_{t+T}}{\lambda_t} V_{j,t+T}^Y x_{j,t+T+1}^Y = 0, \quad (11)$$

$$\lim_{T \rightarrow \infty} \frac{\beta^T \lambda_{t+T}}{\lambda_t} V_{k,t+T}^M x_{k,t+T+1}^M = 0. \quad (12)$$

Provided that the intertemporal utility function is bounded and strictly concave, together with all necessary first order conditions, these transversality conditions are sufficient for the unique utility maximizing state contingent intertemporal household allocation.

Combination of necessary first order conditions (4) and (5) yields intertemporal optimality condition

$$u_C(C_t, L_{i,t}) = \beta E_t (1+i_t) \frac{P_t^C}{P_{t+1}^C} u_C(C_{t+1}, L_{i,t+1}), \quad (13)$$

which ensures that at a utility maximum, the representative household cannot benefit from feasible intertemporal consumption reallocations. Finally, combination of necessary first order conditions (4), (5) and (6) yields intratemporal optimality condition

$$E_t \frac{\beta u_C(C_{t+1}, L_{i,t+1}) \frac{P_t^C}{P_{t+1}^C} (1+i_t)}{u_C(C_t, L_{i,t}) \frac{P_t^C}{P_{t+1}^C}} = E_t \frac{\beta u_C(C_{t+1}, L_{i,t+1}) \frac{P_t^C}{P_{t+1}^C} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} (1+i_t^f)}{u_C(C_t, L_{i,t}) \frac{P_t^C}{P_{t+1}^C}}, \quad (14)$$

which equates the expected present discounted values of the gross real returns on domestic and foreign bonds.

2.1.2. Labour Supply and Wage Setting Behaviour

There exist a large number of perfectly competitive firms which combine differentiated intermediate labour services $L_{i,t}$ supplied by households in a monopolistically competitive labour market to produce final labour service L_t according to constant elasticity of substitution production function

$$L_t = \left[\int_{i=0}^1 (L_{i,t})^{\frac{\theta_t^L - 1}{\theta_t^L}} di \right]^{\frac{\theta_t^L}{\theta_t^L - 1}}, \quad (15)$$

where $\theta_t^L > 1$. The representative final labour service firm maximizes profits derived from production of the final labour service

$$\Pi_t^L = W_t L_t - \int_{i=0}^1 W_{i,t} L_{i,t} di, \quad (16)$$

with respect to inputs of intermediate labour services, subject to production function (15). The necessary first order conditions associated with this profit maximization problem yield intermediate labour service demand functions:

$$L_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\theta_t^L} L_t. \quad (17)$$

Since the production function exhibits constant returns to scale, in competitive equilibrium the representative final labour service firm earns zero profit, implying aggregate wage index:

$$W_t = \left[\int_{i=0}^1 (W_{i,t})^{1-\theta_t^L} di \right]^{\frac{1}{1-\theta_t^L}}. \quad (18)$$

As the wage elasticity of demand for intermediate labour services θ_t^L increases, they become closer substitutes, and individual households have less market power.

In an extension of the model of nominal wage rigidity proposed by Erceg, Henderson and Levin (2000) motivated by Smets and Wouters (2003, 2005), each period a randomly selected fraction $1-\omega^L$ of households adjust their wage optimally. The remaining fraction ω^L of households adjust their wage to account for past consumption price inflation according to partial indexation rule

$$W_{i,t} = \left(\frac{P_{t-1}^C}{P_{t-2}^C} \right)^{\gamma^L} \left(\frac{\bar{P}_{t-1}^C}{\bar{P}_{t-2}^C} \right)^{1-\gamma^L} W_{i,t-1}, \quad (19)$$

where $0 \leq \gamma^L \leq 1$. Under this specification, although households adjust their wage every period, they infrequently adjust their wage optimally, and the interval between optimal wage adjustments is a random variable.

If the representative household can adjust its wage optimally in period t , then it does so to maximize intertemporal utility function (1) subject to dynamic budget constraint (3), intermediate labour service demand function (17), and the assumed form of nominal wage rigidity. Since all households that adjust their wage optimally in period t solve an identical utility maximization problem, in equilibrium they all choose a common wage W_t^* given by necessary first order condition:

$$\frac{W_t^*}{W_t} = - \frac{E_t \sum_{s=t}^{\infty} (\omega^L)^{s-t} \frac{\beta^{s-t} u_C(C_s, L_{t,s})}{u_C(C_t, L_{t,t})} \theta_s^L \frac{u_L(C_s, L_{t,s})}{u_C(C_s, L_{t,s})} \left[\left(\frac{P_{t-1}^C}{P_{s-1}^C} \right)^{\gamma^L} \left(\frac{\bar{P}_{t-1}^C}{\bar{P}_{s-1}^C} \right)^{1-\gamma^L} \frac{W_s}{W_t} \right]^{\theta_s^L} \left(\frac{W_t^*}{W_t} \right)^{-\theta_s^L} L_s}{E_t \sum_{s=t}^{\infty} (\omega^L)^{s-t} \frac{\beta^{s-t} u_C(C_s, L_{t,s})}{u_C(C_t, L_{t,t})} (\theta_s^L - 1)(1 - \tau_s) \frac{W_s}{P_s^C} \left[\left(\frac{P_{t-1}^C}{P_{s-1}^C} \right)^{\gamma^L} \left(\frac{\bar{P}_{t-1}^C}{\bar{P}_{s-1}^C} \right)^{1-\gamma^L} \frac{W_s}{W_t} \right]^{\theta_s^L - 1} \left(\frac{W_t^*}{W_t} \right)^{-\theta_s^L} L_s}. \quad (20)$$

This necessary first order condition equates the expected present discounted value of the consumption benefit generated by an additional unit of labour supply to the expected present discounted value of its leisure cost. Aggregate wage index (18) equals an average of the wage set by the fraction $1 - \omega^L$ of households that adjust their wage optimally in period t , and the average of the wages set by the remaining fraction ω^L of households that adjust their wage according to partial indexation rule (19):

$$W_t = \left\{ (1 - \omega^L) (W_t^*)^{1 - \theta_t^L} + \omega^L \left[\left(\frac{P_{t-1}^C}{P_{t-2}^C} \right)^{\gamma^L} \left(\frac{\bar{P}_{t-1}^C}{\bar{P}_{t-2}^C} \right)^{1 - \gamma^L} W_{t-1} \right]^{1 - \theta_t^L} \right\}^{\frac{1}{1 - \theta_t^L}}. \quad (21)$$

Since those households able to adjust their wage optimally in period t are selected randomly from among all households, the average wage set by the remaining households equals the value of the aggregate wage index that prevailed during period $t - 1$, rescaled to account for past consumption price inflation.

2.2. The Value Maximization Problem of the Representative Firm

There exists a continuum of intermediate good firms indexed by $j \in [0, 1]$. Intermediate good firms supply differentiated intermediate output goods, but are otherwise identical. Entry into and exit from the monopolistically competitive intermediate output good sector is prohibited.

2.2.1. Employment and Investment Behaviour

The representative intermediate good firm sells shares $\{x_{i,j,t+1}^Y\}_{i=0}^1$ to domestic households at price $V_{j,t}^Y$. Recursive forward substitution for $V_{j,t+s}^Y$ with $s > 0$ in necessary first order condition (7) applying the law of iterated expectations reveals that the post-dividend stock market value of

the representative intermediate good firm equals the expected present discounted value of future dividend payments:

$$V_{j,t}^Y = E_t \sum_{s=t+1}^{\infty} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \Pi_{j,s}^Y. \quad (22)$$

Acting in the interests of its shareholders, the representative intermediate good firm maximizes its pre-dividend stock market value, equal to the expected present discounted value of current and future dividend payments:

$$\Pi_{j,t}^Y + V_{j,t}^Y = E_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \Pi_{j,s}^Y. \quad (23)$$

The derivation of result (22) imposes transversality condition (11), which rules out self-fulfilling speculative asset price bubbles.

Shares entitle households to dividend payments equal to net profits $\Pi_{j,s}^Y$, defined as after tax earnings less investment expenditures:

$$\Pi_{j,s}^Y = (1 - \tau_s)(P_{j,s}^Y Y_{j,s} - W_s L_{j,s}) - P_s^I I_s. \quad (24)$$

Earnings are defined as revenues derived from sales of differentiated intermediate output good $Y_{j,s}$ at price $P_{j,s}^Y$ less expenditures on final labour service $L_{j,s}$. The government levies a tax on earnings at rate τ_s , and negative dividend payments are a theoretical possibility.

The representative intermediate good firm utilizes capital K_s at rate $u_{j,s}$ and rents final labour service $L_{j,s}$ given labour augmenting technology coefficient A_s to produce differentiated intermediate output good $Y_{j,s}$ according to constant elasticity of substitution production function

$$\mathcal{F}(u_{j,s} K_s, A_s L_{j,s}) = \left[(\varphi)^{\frac{1}{\vartheta}} (u_{j,s} K_s)^{\frac{\vartheta-1}{\vartheta}} + (1-\varphi)^{\frac{1}{\vartheta}} (A_s L_{j,s})^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta-1}}, \quad (25)$$

where $0 < \varphi < 1$, $\vartheta > 0$ and $A_s > 0$. This constant elasticity of substitution production function exhibits constant returns to scale, and nests the production function proposed by Cobb and Douglas (1928) under constant returns to scale for $\vartheta = 1$.¹

In utilizing capital to produce output, the representative intermediate good firm incurs a cost $\mathcal{G}(u_{j,s}, K_s)$ denominated in terms of output:

¹ Invoking L'Hospital's rule yields $\lim_{\vartheta \rightarrow 1} \ln \mathcal{F}(u_{j,s} K_s, A_s L_{j,s}) = \varphi \ln(u_{j,s} K_s) + (1-\varphi) \ln(A_s L_{j,s}) - \varphi \ln \varphi - (1-\varphi) \ln(1-\varphi)$, which implies that $\lim_{\vartheta \rightarrow 1} \mathcal{F}(u_{j,s} K_s, A_s L_{j,s}) = \varphi^{-\varphi} (1-\varphi)^{-(1-\varphi)} (u_{j,s} K_s)^{\varphi} (A_s L_{j,s})^{1-\varphi}$.

$$Y_{j,s} = \mathcal{F}(u_{j,s}K_s, A_sL_{j,s}) - \mathcal{G}(u_{j,s}, K_s). \quad (26)$$

Following Christiano, Eichenbaum and Evans (2005), this capital utilization cost is increasing in the rate of capital utilization at an increasing rate,

$$\mathcal{G}(u_{j,s}, K_s) = \mu \left[e^{\kappa(u_{j,s}-1)} - 1 \right] K_s, \quad (27)$$

where $\mu > 0$ and $\kappa > 0$. In deterministic steady state equilibrium, the rate of capital utilization is normalized to one, and the cost of utilizing capital equals zero.

Capital is endogenous but not firm-specific, and the representative intermediate good firm enters period s with access to previously accumulated capital stock K_s , which subsequently evolves according to accumulation function

$$K_{s+1} = (1 - \delta)K_s + \mathcal{H}(I_s, I_{s-1}), \quad (28)$$

where depreciation rate parameter δ satisfies $0 \leq \delta \leq 1$. Following Christiano, Eichenbaum and Evans (2005), effective investment function $\mathcal{H}(I_s, I_{s-1})$ incorporates convex adjustment costs,

$$\mathcal{H}(I_s, I_{s-1}) = v_s^I \left[1 - \frac{\chi}{2} \left(\frac{I_s - I_{s-1}}{I_{s-1}} \right)^2 \right] I_s, \quad (29)$$

where $\chi > 0$ and $v_s^I > 0$. In deterministic steady state equilibrium, these adjustment costs equal zero, and effective investment equals actual investment.

In period t , the representative intermediate good firm chooses state contingent sequences for employment $\{L_{i,s}\}_{s=t}^{\infty}$, capital utilization $\{u_{j,s}\}_{s=t}^{\infty}$, investment $\{I_s\}_{s=t}^{\infty}$, and the capital stock $\{K_{s+1}\}_{s=t}^{\infty}$ to maximize pre-dividend stock market value (23) subject to net production function (26), capital accumulation function (28), and terminal nonnegativity constraint $K_{T+1} \geq 0$ for $T \rightarrow \infty$. In equilibrium, demand for the final labour service satisfies necessary first order condition

$$\mathcal{F}_{AL}(u_{j,t}K_t, A_tL_{j,t})\Phi_{j,t} = (1 - \tau_t) \frac{W_t}{P_t^Y A_t}, \quad (30)$$

where $P_s^Y \Phi_{j,s}$ denotes the Lagrange multiplier associated with the period s production technology constraint. This necessary first order condition equates real marginal cost $\Phi_{j,t}$ to the ratio of the after tax real wage to the marginal product of labour. In equilibrium, the rate of capital utilization satisfies necessary first order condition

$$\mathcal{F}_{uK}(u_{j,t}, K_t, A_t L_{j,t}) = \frac{\mathcal{G}_u(u_{j,t}, K_t)}{K_t}, \quad (31)$$

which equates the marginal product of utilized capital to its marginal cost. In equilibrium, demand for the final investment good satisfies necessary first order condition

$$Q_t \mathcal{H}_1(I_t, I_{t-1}) + E_t \frac{\beta \lambda_{t+1}}{\lambda_t} Q_{t+1} \mathcal{H}_2(I_{t+1}, I_t) = P_t^I, \quad (32)$$

which equates the expected present discounted value of an additional unit of investment to its price, where $Q_{j,s}$ denotes the Lagrange multiplier associated with the period s capital accumulation function. In equilibrium, this shadow price of capital satisfies necessary first order condition

$$Q_t = E_t \frac{\beta \lambda_{t+1}}{\lambda_t} \left\{ P_{t+1}^Y \Phi_{j,t+1} \left[u_{j,t+1} \mathcal{F}_{uK}(u_{j,t+1}, K_{t+1}, A_{t+1} L_{j,t+1}) - \mathcal{G}_K(u_{j,t+1}, K_{t+1}) \right] + (1 - \delta) Q_{t+1} \right\}, \quad (33)$$

which equates it to the expected present discounted value of the sum of the future marginal cost of capital, and the future shadow price of capital net of depreciation. In equilibrium, the necessary complementary slackness condition associated with the terminal nonnegativity constraint may be stated as:

$$\lim_{T \rightarrow \infty} \frac{\beta^T \lambda_{t+T}}{\lambda_t} Q_{t+T} K_{t+T+1} = 0. \quad (34)$$

Provided that the pre-dividend stock market value of the representative intermediate good firm is bounded and strictly concave, together with all necessary first order conditions, this transversality condition is sufficient for the unique value maximizing state contingent intertemporal firm allocation.

2.2.2. Output Supply and Price Setting Behaviour

There exist a large number of perfectly competitive firms which combine differentiated intermediate output goods $Y_{j,t}$ supplied by intermediate good firms in a monopolistically competitive output market to produce final output good Y_t according to constant elasticity of substitution production function

$$Y_t = \left[\int_{j=0}^1 (Y_{j,t})^{\frac{\theta_t^Y - 1}{\theta_t^Y}} dj \right]^{\frac{\theta_t^Y}{\theta_t^Y - 1}}, \quad (35)$$

where $\theta_t^Y > 1$. The representative final output good firm maximizes profits derived from production of the final output good

$$\Pi_t^Y = P_t^Y Y_t - \int_{j=0}^1 P_{j,t}^Y Y_{j,t} dj, \quad (36)$$

with respect to inputs of intermediate output goods, subject to production function (35). The necessary first order conditions associated with this profit maximization problem yield intermediate output good demand functions:

$$Y_{j,t} = \left(\frac{P_{j,t}^Y}{P_t^Y} \right)^{-\theta_t^Y} Y_t. \quad (37)$$

Since the production function exhibits constant returns to scale, in competitive equilibrium the representative final output good firm earns zero profit, implying aggregate output price index:

$$P_t^Y = \left[\int_{j=0}^1 (P_{j,t}^Y)^{1-\theta_t^Y} dj \right]^{\frac{1}{1-\theta_t^Y}}. \quad (38)$$

As the price elasticity of demand for intermediate output goods θ_t^Y increases, they become closer substitutes, and individual intermediate good firms have less market power.

In an extension of the model of nominal output price rigidity proposed by Calvo (1983) motivated by Smets and Wouters (2003, 2005), each period a randomly selected fraction $1 - \omega^Y$ of intermediate good firms adjust their price optimally. The remaining fraction ω^Y of intermediate good firms adjust their price to account for past output price inflation according to partial indexation rule

$$P_{j,t}^Y = \left(\frac{P_{t-1}^Y}{P_{t-2}^Y} \right)^{\gamma^Y} \left(\frac{\bar{P}_{t-1}^Y}{\bar{P}_{t-2}^Y} \right)^{1-\gamma^Y} P_{j,t-1}^Y, \quad (39)$$

where $0 \leq \gamma^Y \leq 1$. Under this specification, optimal price adjustment opportunities arrive randomly, and the interval between optimal price adjustments is a random variable.

If the representative intermediate good firm can adjust its price optimally in period t , then it does so to maximize to maximize pre-dividend stock market value (23) subject to net production function (26), capital accumulation function (28), intermediate output good demand function (37), and the assumed form of nominal output price rigidity. Since all intermediate good firms that adjust their price optimally in period t solve an identical value maximization problem, in equilibrium they all choose a common price $P_t^{Y,*}$ given by necessary first order condition:

$$\frac{P_t^{Y,*}}{P_t^Y} = \frac{E_t \sum_{s=t}^{\infty} (\omega^Y)^{s-t} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \theta_s^Y \Phi_{j,s} \left[\left(\frac{P_{t-1}^Y}{P_{s-1}^Y} \right)^{\gamma^Y} \left(\frac{\bar{P}_{t-1}^Y}{\bar{P}_{s-1}^Y} \right)^{1-\gamma^Y} \frac{P_s^Y}{P_t^Y} \right]^{\theta_s^Y} \left(\frac{P_t^{Y,*}}{P_t^Y} \right)^{-\theta_s^Y} P_s^Y Y_s}{E_t \sum_{s=t}^{\infty} (\omega^Y)^{s-t} \frac{\beta^{s-t} \lambda_s}{\lambda_t} (\theta_s^Y - 1)(1 - \tau_s) \left[\left(\frac{P_{t-1}^Y}{P_{s-1}^Y} \right)^{\gamma^Y} \left(\frac{\bar{P}_{t-1}^Y}{\bar{P}_{s-1}^Y} \right)^{1-\gamma^Y} \frac{P_s^Y}{P_t^Y} \right]^{\theta_s^Y - 1} \left(\frac{P_t^{Y,*}}{P_t^Y} \right)^{-\theta_s^Y} P_s^Y Y_s}. \quad (40)$$

This necessary first order condition equates the expected present discounted value of the after tax revenue benefit generated by an additional unit of output supply to the expected present discounted value of its production cost. Aggregate output price index (38) equals an average of the price set by the fraction $1 - \omega^Y$ of intermediate good firms that adjust their price optimally in period t , and the average of the prices set by the remaining fraction ω^Y of intermediate good firms that adjust their price according to partial indexation rule (39):

$$P_t^Y = \left\{ (1 - \omega^Y) (P_t^{Y,*})^{1-\theta_t^Y} + \omega^Y \left[\left(\frac{P_{t-1}^Y}{P_{t-2}^Y} \right)^{\gamma^Y} \left(\frac{\bar{P}_{t-1}^Y}{\bar{P}_{t-2}^Y} \right)^{1-\gamma^Y} P_{t-1}^Y \right]^{1-\theta_t^Y} \right\}^{\frac{1}{1-\theta_t^Y}}. \quad (41)$$

Since those intermediate good firms able to adjust their price optimally in period t are selected randomly from among all intermediate good firms, the average price set by the remaining intermediate good firms equals the value of the aggregate output price index that prevailed during period $t-1$, rescaled to account for past output price inflation.

2.3. The Value Maximization Problem of the Representative Importer

There exists a continuum of intermediate good importers indexed by $k \in [0,1]$. Intermediate good importers supply differentiated intermediate import goods, but are otherwise identical. Entry into and exit from the monopolistically competitive intermediate import good sector is prohibited.

2.3.1. The Real Exchange Rate and the Terms of Trade

The representative intermediate good importer sells shares $\{x_{i,k,t+1}^M\}_{i=0}^1$ to domestic households at price $V_{k,t}^M$. Recursive forward substitution for $V_{k,t+s}^M$ with $s > 0$ in necessary first order condition (8) applying the law of iterated expectations reveals that the post-dividend stock market value of the representative intermediate good importer equals the expected present discounted value of future dividend payments:

$$V_{k,t}^M = E_t \sum_{s=t+1}^{\infty} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \Pi_{k,s}^M. \quad (42)$$

Acting in the interests of its shareholders, the representative intermediate good importer maximizes its pre-dividend stock market value, equal to the expected present discounted value of current and future dividend payments:

$$\Pi_{k,t}^M + V_{k,t}^M = E_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \Pi_{k,s}^M. \quad (43)$$

The derivation of result (42) imposes transversality condition (12), which rules out self-fulfilling speculative asset price bubbles.

Shares entitle households to dividend payments equal to gross profits $\Pi_{k,s}^M$, defined as earnings less fixed costs:

$$\Pi_{k,s}^M = P_{k,s}^M M_{k,s} - \mathcal{E}_s P_s^{Y,f} M_{k,s} - \Gamma_s. \quad (44)$$

Earnings are defined as revenues derived from sales of differentiated intermediate import good $M_{k,s}$ at price $P_{k,s}^M$ less expenditures on foreign final output good $M_{k,s}$. The representative intermediate good importer purchases the foreign final output good at domestic currency price $\mathcal{E}_s P_s^{Y,f}$ and differentiates it, generating zero gross profits on average.

The law of one price asserts that arbitrage transactions equalize the domestic currency prices of domestic imports and foreign exports. Define the real exchange rate,

$$Q_s = \frac{\mathcal{E}_s P_s^{Y,f}}{P_s^Y}, \quad (45)$$

which measures the price of foreign output in terms of domestic output. Also define the terms of trade,

$$\mathcal{T}_s = \frac{P_s^M}{P_s^X}, \quad (46)$$

which measures the price of imports in terms of exports. Violation of the law of one price drives a wedge $\Psi_s = \mathcal{E}_s P_s^{Y,f} / P_s^M$ between the real exchange rate and the terms of trade,

$$Q_s = \Psi_s \mathcal{T}_s, \quad (47)$$

where the domestic currency price of exports satisfies $P_s^X = P_s^Y$. Under the law of one price $\Psi_s = 1$, and the real exchange rate and terms of trade coincide.

There exist a large number of perfectly competitive firms which combine a domestic intermediate good $Z_{h,t} \in \{C_{h,t}, I_{h,t}, G_{h,t}\}$ and a foreign intermediate good $Z_{f,t} \in \{C_{f,t}, I_{f,t}, G_{f,t}\}$ to produce final good $Z_t \in \{C_t, I_t, G_t\}$ according to constant elasticity of substitution production function

$$Z_t = \left[(\phi^Z)^{\frac{1}{\psi}} (Z_{h,t})^{\frac{\psi-1}{\psi}} + (1-\phi^Z)^{\frac{1}{\psi}} (v_t^M Z_{f,t})^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}, \quad (48)$$

where $0 < \phi^Z < 1$, $\psi > 1$ and $v_t^M > 0$. The representative final good firm maximizes profits derived from production of the final good

$$\Pi_t^Z = P_t^Z Z_t - P_t^Y Z_{h,t} - P_t^M Z_{f,t}, \quad (49)$$

with respect to inputs of domestic and foreign intermediate goods, subject to production function (48). The necessary first order conditions associated with this profit maximization problem imply intermediate good demand functions:

$$Z_{h,t} = \phi^Z \left(\frac{P_t^Y}{P_t^Z} \right)^{-\psi} Z_t, \quad (50)$$

$$Z_{f,t} = (1-\phi^Z) \left(\frac{P_t^M}{v_t^M P_t^Z} \right)^{-\psi} \frac{Z_t}{v_t^M}. \quad (51)$$

Since the production function exhibits constant returns to scale, in competitive equilibrium the representative final good firm earns zero profit, implying aggregate price index:

$$P_t^Z = \left[\phi^Z (P_t^Y)^{1-\psi} + (1-\phi^Z) \left(\frac{P_t^M}{V_t^M} \right)^{1-\psi} \right]^{\frac{1}{1-\psi}}. \quad (52)$$

Combination of this aggregate price index with intermediate good demand functions (50) and (51) yields:

$$Z_{h,t} = \phi^Z \left[\phi^Z + (1-\phi^Z) \left(\frac{T_t}{V_t^M} \right)^{1-\psi} \right]^{\frac{\psi}{1-\psi}} Z_t, \quad (53)$$

$$Z_{f,t} = (1-\phi^Z) \left[(1-\phi^Z) + \phi^Z \left(\frac{T_t}{V_t^M} \right)^{\psi-1} \right]^{\frac{\psi}{1-\psi}} \frac{Z_t}{V_t^M}. \quad (54)$$

These demand functions for domestic and foreign intermediate goods are directly proportional to final good demand, with a proportionality coefficient that varies with the terms of trade.

2.3.2. Import Supply and Price Setting Behaviour

There exist a large number of perfectly competitive firms which combine differentiated intermediate import goods $M_{k,t}$ supplied by intermediate good importers in a monopolistically competitive import market to produce final import good M_t according to constant elasticity of substitution production function

$$M_t = \left[\int_{k=0}^1 (M_{k,t})^{\frac{\theta_t^M - 1}{\theta_t^M}} dk \right]^{\frac{\theta_t^M}{\theta_t^M - 1}}, \quad (55)$$

where $\theta_t^M > 1$. The representative final import good firm maximizes profits derived from production of the final import good

$$\Pi_t^M = P_t^M M_t - \int_{k=0}^1 P_{k,t}^M M_{k,t} dk, \quad (56)$$

with respect to inputs of intermediate import goods, subject to production function (55). The necessary first order conditions associated with this profit maximization problem yield intermediate import good demand functions:

$$M_{k,t} = \left(\frac{P_{k,t}^M}{P_t^M} \right)^{-\theta_t^M} M_t. \quad (57)$$

Since the production function exhibits constant returns to scale, in competitive equilibrium the representative final import good firm earns zero profit, implying aggregate import price index:

$$P_t^M = \left[\int_{k=0}^1 (P_{k,t}^M)^{1-\theta_t^M} dk \right]^{\frac{1}{1-\theta_t^M}}. \quad (58)$$

As the price elasticity of demand for intermediate import goods θ_t^M increases, they become closer substitutes, and individual intermediate good importers have less market power.

In an extension of the model of nominal import price rigidity proposed by Monacelli (2005) motivated by Smets and Wouters (2003, 2005), each period a randomly selected fraction $1 - \omega^M$ of intermediate good importers adjust their price optimally. The remaining fraction ω^M of intermediate good importers adjust their price to account for past import price inflation according to partial indexation rule

$$P_{k,t}^M = \left(\frac{P_{t-1}^M}{P_{t-2}^M} \right)^{\gamma^M} \left(\frac{\bar{P}_{t-1}^M}{\bar{P}_{t-2}^M} \right)^{1-\gamma^M} P_{k,t-1}^M, \quad (59)$$

where $0 \leq \gamma^M \leq 1$. Under this specification, the probability that an intermediate good importer has adjusted its price optimally is time dependent but not state dependent.

If the representative intermediate good importer can adjust its price optimally in period t , then it does so to maximize to maximize pre-dividend stock market value (43) subject to intermediate import good demand function (57), and the assumed form of nominal import price rigidity. Since all intermediate good importers that adjust their price optimally in period t solve an identical value maximization problem, in equilibrium they all choose a common price $P_t^{M,*}$ given by necessary first order condition

$$\frac{P_t^{M,*}}{P_t^M} = \frac{E_t \sum_{s=t}^{\infty} (\omega^M)^{s-t} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \theta_s^M \Psi_s \left[\left(\frac{P_{t-1}^M}{P_{s-1}^M} \right)^{\gamma^M} \left(\frac{\bar{P}_{t-1}^M}{\bar{P}_{s-1}^M} \right)^{1-\gamma^M} \frac{P_s^M}{P_t^M} \right]^{\theta_s^M} \left(\frac{P_t^{M,*}}{P_t^M} \right)^{-\theta_s^M} P_s^M M_s}{E_t \sum_{s=t}^{\infty} (\omega^M)^{s-t} \frac{\beta^{s-t} \lambda_s}{\lambda_t} (\theta_s^M - 1) \left[\left(\frac{P_{t-1}^M}{P_{s-1}^M} \right)^{\gamma^M} \left(\frac{\bar{P}_{t-1}^M}{\bar{P}_{s-1}^M} \right)^{1-\gamma^M} \frac{P_s^M}{P_t^M} \right]^{\theta_s^M - 1} \left(\frac{P_t^{M,*}}{P_t^M} \right)^{-\theta_s^M} P_s^M M_s}, \quad (60)$$

where $\Psi_s = \mathcal{E}_s P_s^{Y,f} / P_s^M$ measures real marginal cost. This necessary first order condition equates the expected present discounted value of the revenue benefit generated by an additional unit of import supply to the expected present discounted value of its production cost. Aggregate import price index (58) equals an average of the price set by the fraction $1 - \omega^M$ of intermediate good importers that adjust their price optimally in period t , and the average of the prices set by the remaining fraction ω^M of intermediate good importers that adjust their price according to partial indexation rule (59):

$$P_t^M = \left\{ (1 - \omega^M)(P_t^{M,*})^{1-\theta_t^M} + \omega^M \left[\left(\frac{P_{t-1}^M}{P_{t-2}^M} \right)^{\gamma^M} \left(\frac{\bar{P}_{t-1}^M}{\bar{P}_{t-2}^M} \right)^{1-\gamma^M} P_{t-1}^M \right]^{1-\theta_t^M} \right\}^{\frac{1}{1-\theta_t^M}}. \quad (61)$$

Since those intermediate good importers able to adjust their price optimally in period t are selected randomly from among all intermediate good importers, the average price set by the remaining intermediate good importers equals the value of the aggregate import price index that prevailed during period $t-1$, rescaled to account for past import price inflation.

2.4. Monetary and Fiscal Policy

The government consists of a monetary authority and a fiscal authority. The monetary authority implements monetary policy, while the fiscal authority implements fiscal policy.

2.4.1. The Monetary Authority

The monetary authority implements monetary policy through control of the nominal interest rate according to monetary policy rule

$$i_t - \bar{i}_t = \xi^\pi (\pi_t^C - \bar{\pi}_t^C) + \xi^Y (\ln Y_t - \ln \bar{Y}_t) + v_t^i, \quad (62)$$

where $\xi^\pi > 1$ and $\xi^Y > 0$. As specified, the deviation of the nominal interest rate from its deterministic steady state equilibrium value is a linear increasing function of the contemporaneous deviation of consumption price inflation from its target value, and the contemporaneous proportional deviation of output from its deterministic steady state equilibrium

value. Persistent departures from this monetary policy rule are captured by serially correlated monetary policy shock v_t^i .

2.4.2. The Fiscal Authority

The fiscal authority implements fiscal policy through control of nominal government consumption and the tax rate applicable to the pooled labour income of households and the earnings of intermediate good firms. In equilibrium, this distortionary tax collection framework corresponds to proportional output taxation.

The ratio of nominal government consumption to nominal output satisfies fiscal expenditure rule

$$\ln \frac{P_t^G G_t}{P_t^Y Y_t} - \ln \frac{\bar{P}_t^G \bar{G}_t}{\bar{P}_t^Y \bar{Y}_t} = \zeta^G \left[\ln \left(-\frac{B_{t+1}}{P_t^Y Y_t} \right) - \ln \left(-\frac{\bar{B}_{t+1}}{\bar{P}_t^Y \bar{Y}_t} \right) \right] + v_t^G, \quad (63)$$

where $\zeta^G < 0$. As specified, the proportional deviation of the ratio of nominal government consumption to nominal output from its deterministic steady state equilibrium value is a linear decreasing function of the contemporaneous proportional deviation of the ratio of net foreign debt to nominal output from its target value. This fiscal expenditure rule is well defined only if the net foreign debt is positive. Persistent departures from this fiscal expenditure rule are captured by serially correlated fiscal expenditure shock v_t^G .

The tax rate applicable to the pooled labour income of households and the earnings of intermediate good firms satisfies fiscal revenue rule

$$\ln \tau_t - \ln \bar{\tau}_t = \zeta^\tau \left[\ln \left(-\frac{B_{t+1}^G}{P_t^Y Y_t} \right) - \ln \left(-\frac{\bar{B}_{t+1}^G}{\bar{P}_t^Y \bar{Y}_t} \right) \right] + v_t^\tau, \quad (64)$$

where $\zeta^\tau > 0$. As specified, the proportional deviation of the tax rate from its deterministic steady state equilibrium value is a linear increasing function of the contemporaneous proportional deviation of the ratio of net government debt to nominal output from its target value. This fiscal revenue rule is well defined only if the net government debt is positive. Persistent departures from this fiscal revenue rule are captured by serially correlated fiscal revenue shock v_t^τ .

The fiscal authority enters period t holding previously purchased domestic currency denominated bonds $B_t^{G,h}$ which yield interest at risk free rate i_{t-1} , and foreign currency denominated bonds $B_t^{G,f}$ which yield interest at risk free rate i_{t-1}^f . It also levies taxes on the

pooled labour income of households and the earnings of intermediate good firms at rate τ_t . These sources of public wealth are summed in government dynamic budget constraint:

$$B_{t+1}^{G,h} + \mathcal{E}_t B_{t+1}^{G,f} = (1 + i_{t-1})B_t^{G,h} + \mathcal{E}_t(1 + i_{t-1}^f)B_t^{G,f} + \tau_t \int_{i=0}^1 \int_{l=0}^1 W_{l,t} L_{l,t} dldi + \tau_t \int_{j=0}^1 (P_{j,t}^Y Y_{j,t} - W_t L_{j,t}) dj - P_t^G G_t. \quad (65)$$

According to this dynamic budget constraint, at the end of period t , the fiscal authority purchases domestic bonds $B_{t+1}^{G,h}$, and foreign bonds $B_{t+1}^{G,f}$ at price \mathcal{E}_t . It also purchases final government consumption good G_t at price P_t^G .

2.5. Market Clearing Conditions

A rational expectations equilibrium in this DSGE model of a small open economy consists of state contingent intertemporal allocations for domestic and foreign households and firms which solve their constrained optimization problems given prices and policy, together with state contingent intertemporal allocations for domestic and foreign governments which satisfy their policy rules and constraints given prices, with supporting prices such that all markets clear. Since the domestic economy is of negligible size relative to the foreign economy, in equilibrium $P_t^{Y,f} = P_t^{C,f} = P_t^{I,f} = P_t^{G,f} = P_t^{X,f}$ and $\frac{X_t^f}{Y_t^f} = \frac{M_t^f}{Y_t^f} = \frac{B_{t+1}^f}{P_t^f Y_t^f} = 0$.

Clearing of the final output good market requires that exports X_t equal production of the domestic final output good less the cumulative demands of domestic households, firms, and the government,

$$X_t = Y_t - C_{h,t} - I_{h,t} - G_{h,t}, \quad (66)$$

where $X_t = M_t^f$. Clearing of the final import good market requires that imports M_t satisfy the cumulative demands of domestic households, firms, and the government for the foreign final output good,

$$M_t = C_{f,t} + I_{f,t} + G_{f,t}, \quad (67)$$

where $M_t = X_t^f$. In equilibrium, combination of these final output and import good market clearing conditions yields aggregate resource constraint:

$$P_t^Y Y_t = P_t^C C_t + P_t^I I_t + P_t^G G_t + P_t^X X_t - P_t^M M_t. \quad (68)$$

The trade balance equals export revenues less import expenditures, or equivalently nominal output less domestic demand.

Let B_{t+1} denote the net foreign asset position of the economy, which in equilibrium equals the sum of the domestic currency values of private sector bond holdings $B_{t+1}^P = B_{t+1}^{P,h} + \mathcal{E}_t B_{t+1}^{P,f}$ and public sector bond holdings $B_{t+1}^G = B_{t+1}^{G,h} + \mathcal{E}_t B_{t+1}^{G,f}$, since domestic bond holdings cancel out when the private and public sectors are consolidated:

$$B_{t+1} = B_{t+1}^P + B_{t+1}^G. \quad (69)$$

The imposition of equilibrium conditions on household dynamic budget constraint (3) reveals that the expected present discounted value of the net increase in private sector asset holdings equals the expected present discounted value of private saving less domestic investment:

$$E_{t-1} \frac{\beta \lambda_t}{\lambda_{t-1}} (B_{t+1}^P - B_t^P) = E_{t-1} \frac{\beta \lambda_t}{\lambda_{t-1}} [i_{t-1} B_t^P + (1 - \tau_t) P_t^Y Y_t - P_t^C C_t - P_t^I I_t]. \quad (70)$$

The imposition of equilibrium conditions on government dynamic budget constraint (65) reveals that the expected present discounted value of the net increase in public sector asset holdings equals the expected present discounted value of public saving:

$$E_{t-1} \frac{\beta \lambda_t}{\lambda_{t-1}} (B_{t+1}^G - B_t^G) = E_{t-1} \frac{\beta \lambda_t}{\lambda_{t-1}} (i_{t-1} B_t^G + \tau_t P_t^Y Y_t - P_t^G G_t). \quad (71)$$

Combination of these household and government dynamic budget constraints with aggregate resource constraint (68) reveals that the expected present discounted value of the net increase in foreign asset holdings equals the expected present discounted value of the sum of net international investment income and the trade balance, or equivalently the expected present discounted value of national saving less domestic investment:

$$E_{t-1} \frac{\beta \lambda_t}{\lambda_{t-1}} (B_{t+1} - B_t) = E_{t-1} \frac{\beta \lambda_t}{\lambda_{t-1}} (i_{t-1} B_t + P_t^X X_t - P_t^M M_t). \quad (72)$$

In equilibrium, the current account balance is determined by both intratemporal and intertemporal optimization.

2.6. The Approximate Linear Model

Estimation, inference and forecasting are based on a linear state space representation of an approximate unobserved components representation of this DSGE model of a small open economy. Cyclical components are modeled by linearizing equilibrium conditions around a stationary deterministic steady state equilibrium which abstracts from long run balanced growth, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path.

In what follows, $E_t x_{t+s}$ denotes the rational expectation of variable x_{t+s} , conditional on information available at time t . Also, \hat{x}_t denotes the cyclical component of variable x_t , while \bar{x}_t denotes the trend component of variable x_t . Cyclical and trend components are additively separable, that is $x_t = \hat{x}_t + \bar{x}_t$.

2.6.1. Cyclical Components

The cyclical component of output price inflation depends on a linear combination of past and expected future cyclical components of output price inflation driven by the contemporaneous cyclical components of real marginal cost and the tax rate according to output price Phillips curve:

$$\hat{\pi}_t^y = \frac{\gamma^y}{1 + \gamma^y \beta} \hat{\pi}_{t-1}^y + \frac{\beta}{1 + \gamma^y \beta} E_t \hat{\pi}_{t+1}^y + \frac{(1 - \omega^y)(1 - \omega^y \beta)}{\omega^y (1 + \gamma^y \beta)} \left[\ln \hat{\Phi}_t + \frac{\tau}{1 - \tau} \ln \hat{\tau}_t - \frac{1}{\theta^y - 1} \ln \hat{\theta}_t^y \right]. \quad (73)$$

The persistence of the cyclical component of output price inflation is increasing in indexation parameter γ^y , while the sensitivity of the cyclical component of output price inflation to changes in the cyclical components of real marginal cost and the tax rate is decreasing in nominal rigidity parameter ω^y and indexation parameter γ^y . This output price Phillips curve is subject to output price markup shocks.

The cyclical component of output depends on the contemporaneous cyclical components of utilized capital and effective labour according to approximate linear net production function

$$\ln \hat{Y}_t = \left(1 - \frac{\theta^y}{\theta^y - 1} \frac{WL}{PY} \right) \ln(\hat{u}_t \hat{K}_t) + \frac{\theta^y}{\theta^y - 1} \frac{WL}{PY} \ln(\hat{A}_t \hat{L}_t), \quad (74)$$

where $\frac{K}{Y} = \frac{\beta(1-\tau)}{1-\beta(1-\delta)} \left(\frac{\theta^y - 1}{\theta^y} - \frac{WL}{PY} \right)$. This approximate linear net production function is subject to output technology shocks.

The cyclical component of consumption, investment, or government consumption price inflation depends on a linear combination of past and expected future cyclical components of consumption, investment, or government consumption price inflation driven by the contemporaneous cyclical components of real marginal cost and the tax rate according to Phillips curves:

$$\begin{aligned} \hat{\pi}_t^Z = & \frac{\gamma^y}{1 + \gamma^y \beta} \hat{\pi}_{t-1}^Z + \frac{\beta}{1 + \gamma^y \beta} E_t \hat{\pi}_{t+1}^Z + \frac{(1 - \omega^y)(1 - \omega^y \beta)}{\omega^y (1 + \gamma^y \beta)} \left[\ln \hat{\phi}_t + \frac{\tau}{1 - \tau} \ln \hat{\tau}_t - \frac{1}{\theta^y - 1} \ln \hat{\theta}_t^y \right] \\ & - \frac{\gamma^y (1 - \phi^Z)}{1 + \gamma^y \beta} \Delta \ln \frac{\hat{T}_{t-1}}{\hat{v}_{t-1}^M} + (1 - \phi^Z) \Delta \ln \frac{\hat{T}_t}{\hat{v}_t^M} - \frac{\beta (1 - \phi^Z)}{1 + \gamma^y \beta} E_t \Delta \ln \frac{\hat{T}_{t+1}}{\hat{v}_{t+1}^M}. \end{aligned} \quad (75)$$

Reflecting the entry of the price of imports into the aggregate consumption, investment, or government consumption price index, the cyclical component of consumption, investment, or government consumption price inflation also depends on past, contemporaneous, and expected future proportional changes in the cyclical component of the terms of trade. These Phillips curves are subject to output price markup and import technology shocks.

The cyclical component of consumption depends on a linear combination of past and expected future cyclical components of consumption driven by the contemporaneous cyclical component of the real interest rate according to approximate linear consumption Euler equation:

$$\ln \hat{C}_t = \frac{\alpha}{1 + \alpha} \ln \hat{C}_{t-1} + \frac{1}{1 + \alpha} E_t \ln \hat{C}_{t+1} - \sigma \frac{1 - \alpha}{1 + \alpha} \left[\hat{r}_t^C + E_t \ln \frac{\hat{v}_{t+1}^C}{\hat{v}_t^C} \right]. \quad (76)$$

The persistence of the cyclical component of consumption is increasing in habit persistence parameter α , while the sensitivity of the cyclical component of consumption to changes in the cyclical component of the real interest rate is increasing in intertemporal elasticity of substitution parameter σ and decreasing in habit persistence parameter α . This approximate linear consumption Euler equation is subject to preference shocks.

The cyclical component of investment depends on a linear combination of past and expected future cyclical components of investment driven by the contemporaneous cyclical component of the relative shadow price of capital according to approximate linear investment demand function:

$$\ln \hat{I}_t = \frac{1}{1 + \beta} \ln \hat{I}_{t-1} + \frac{\beta}{1 + \beta} E_t \ln \hat{I}_{t+1} + \frac{1}{\chi(1 + \beta)} \ln \left(\hat{v}_t^I \frac{\hat{Q}_t}{\hat{P}_t^I} \right). \quad (77)$$

The sensitivity of the cyclical component of investment to changes in the cyclical component of the relative shadow price of capital is decreasing in investment adjustment cost parameter χ . This approximate linear investment demand function is subject to investment technology shocks.

The cyclical component of the ratio of nominal government consumption to nominal output depends on the contemporaneous cyclical component of the ratio of net foreign debt to nominal output according to fiscal expenditure rule:

$$\ln \frac{\hat{P}_t^G \hat{G}_t}{\hat{P}_t^Y \hat{Y}_t} = \zeta^G \ln \left(-\frac{\hat{B}_{t+1}}{\hat{P}_t^Y \hat{Y}_t} \right) + \hat{v}_t^G. \quad (78)$$

This fiscal expenditure rule ensures convergence of the level of the ratio of net foreign debt to nominal output to its target value, and is subject to fiscal expenditure shocks.

The cyclical component of import price inflation depends on a linear combination of past and expected future cyclical components of import price inflation driven by the contemporaneous cyclical component of the deviation of the domestic currency price of foreign output from the price of imports according to import price Phillips curve:

$$\hat{\pi}_t^M = \frac{\gamma^M}{1 + \gamma^M \beta} \hat{\pi}_{t-1}^M + \frac{\beta}{1 + \gamma^M \beta} E_t \hat{\pi}_{t+1}^M + \frac{(1 - \omega^M)(1 - \omega^M \beta)}{\omega^M (1 + \gamma^M \beta)} \left[\ln \frac{\hat{\mathcal{E}}_t \hat{P}_t^{Y,f}}{\hat{P}_t^M} - \frac{1}{\theta^M - 1} \ln \hat{\theta}_t^M \right]. \quad (79)$$

The persistence of the cyclical component of import price inflation is increasing in indexation parameter γ^M , while the sensitivity of the cyclical component of import price inflation to changes in the cyclical component of real marginal cost is decreasing in nominal rigidity parameter ω^M and indexation parameter γ^M . This import price Phillips curve is subject to import price markup shocks.

The cyclical component of exports depends on the contemporaneous cyclical components of foreign consumption, investment, government consumption, and the terms of trade according to approximate linear export demand function

$$\begin{aligned} \frac{X}{Y} \frac{Y}{Y^f} \ln \hat{X}_t &= (1 - \phi^{C,f}) \frac{C^f}{Y^f} \ln \frac{\hat{C}_t^f}{\hat{v}_t^{M,f}} + (1 - \phi^{I,f}) \frac{I}{Y} \ln \frac{\hat{I}_t^f}{\hat{v}_t^{M,f}} + (1 - \phi^{G,f}) \frac{G}{Y} \ln \frac{\hat{G}_t^f}{\hat{v}_t^{M,f}} \\ &\quad - \psi \left[\phi^{C,f} (1 - \phi^{C,f}) \frac{C^f}{Y^f} + \phi^{I,f} (1 - \phi^{I,f}) \frac{I}{Y} + \phi^{G,f} (1 - \phi^{G,f}) \frac{G}{Y} \right] \ln \frac{\hat{T}_t^f}{\hat{v}_t^{M,f}}, \end{aligned} \quad (80)$$

where $\frac{X}{Y} = 1 - \phi^C \frac{C}{Y} - \phi^I \frac{I}{Y} - \phi^G \frac{G}{Y}$. The sensitivity of the cyclical component of exports to changes in the cyclical component of the foreign terms of trade is increasing in elasticity of

substitution parameter ψ . This approximate linear export demand function is subject to foreign import technology shocks.

The cyclical component of imports depends on the contemporaneous cyclical components of consumption, investment, government consumption, and the terms of trade according to approximate linear import demand function

$$\begin{aligned} \frac{M}{Y} \ln \hat{M}_t &= (1 - \phi^C) \frac{C}{Y} \ln \frac{\hat{C}_t}{\hat{v}_t^M} + (1 - \phi^I) \frac{I}{Y} \ln \frac{\hat{I}_t}{\hat{v}_t^M} + (1 - \phi^G) \frac{G}{Y} \ln \frac{\hat{G}_t}{\hat{v}_t^M} \\ &- \psi \left[\phi^C (1 - \phi^C) \frac{C}{Y} + \phi^I (1 - \phi^I) \frac{I}{Y} + \phi^G (1 - \phi^G) \frac{G}{Y} \right] \ln \frac{\hat{T}_t}{\hat{v}_t^M}, \end{aligned} \quad (81)$$

where $\frac{M}{Y} = (1 - \phi^C) \frac{C}{Y} + (1 - \phi^I) \frac{I}{Y} + (1 - \phi^G) \frac{G}{Y}$. The sensitivity of the cyclical component of imports to changes in the cyclical component of the terms of trade is increasing in elasticity of substitution parameter ψ . This approximate linear import demand function is subject to import technology shocks.

The cyclical component of the real wage depends on a linear combination of past and expected future cyclical components of the real wage driven by the contemporaneous cyclical component of the deviation of the marginal rate of substitution between leisure and consumption from the after tax real wage according to wage Phillips curve:

$$\begin{aligned} \ln \frac{\hat{W}_t}{\hat{P}_t^C} &= \frac{1}{1 + \beta} \ln \frac{\hat{W}_{t-1}}{\hat{P}_{t-1}^C} + \frac{\beta}{1 + \beta} E_t \ln \frac{\hat{W}_{t+1}}{\hat{P}_{t+1}^C} + \frac{\gamma^L}{1 + \beta} \hat{\pi}_{t-1}^C - \frac{1 + \gamma^L \beta}{1 + \beta} \hat{\pi}_t^C + \frac{\beta}{1 + \beta} E_t \hat{\pi}_{t+1}^C \\ &+ \frac{(1 - \omega^L)(1 - \omega^L \beta)}{\omega^L (1 + \beta)} \left[\frac{1}{\eta} \ln \hat{L}_t + \frac{1}{\sigma} \frac{\ln \hat{C}_t - \alpha \ln \hat{C}_{t-1}}{1 - \alpha} + \frac{\tau}{1 - \tau} \ln \hat{\tau}_t - \ln \frac{\hat{W}_t}{\hat{P}_t^C} - \frac{1}{\theta^L - 1} \ln \hat{\theta}_t^L \right]. \end{aligned} \quad (82)$$

Reflecting the existence of partial wage indexation, the cyclical component of the real wage also depends on past, contemporaneous, and expected future cyclical components of consumption price inflation. The sensitivity of the cyclical component of the real wage to changes in the cyclical component of consumption price inflation is increasing in indexation parameter γ^L , to changes in the cyclical component of the deviation of the marginal rate of substitution between leisure and consumption from the after tax real wage is decreasing in nominal rigidity parameter ω^L , and to changes in the cyclical component of employment is decreasing in elasticity of substitution parameter η . This wage Phillips curve is subject to wage markup shocks.

The cyclical component of real marginal cost depends on the contemporaneous cyclical component of the deviation of the after tax real wage from the marginal product of labour according to approximate linear implicit labour demand function:

$$\ln \hat{\Phi}_t = \ln \frac{\hat{W}_t}{\hat{P}_t^Y \hat{A}_t} - \frac{\tau}{1-\tau} \ln \hat{\tau}_t - \frac{1}{\mathcal{G}} \left(1 - \frac{\theta^Y}{\theta^Y - 1} \frac{WL}{PY} \right) \ln \frac{\hat{u}_t \hat{K}_t}{\hat{A}_t \hat{L}_t}. \quad (83)$$

The sensitivity of the cyclical component of real marginal cost to changes in the cyclical component of the ratio of utilized capital to effective labour is decreasing in elasticity of substitution parameter \mathcal{G} . This approximate linear implicit labour demand function is subject to output technology shocks.

The cyclical component of the relative shadow price of capital depends on the expected future cyclical component of the relative shadow price of capital, the contemporaneous cyclical component of the real interest rate, the expected future cyclical component of real marginal cost, and the expected future cyclical component of the marginal product of capital according to approximate linear investment Euler equation:

$$\begin{aligned} \ln \frac{\hat{Q}_t}{\hat{P}_t^Y} &= \beta(1-\delta) E_t \ln \frac{\hat{Q}_{t+1}}{\hat{P}_{t+1}^Y} - \hat{r}_t^Y \\ &+ [1 - \beta(1-\delta)] E_t \ln \hat{\Phi}_{t+1} - \frac{1 - \beta(1-\delta)}{\mathcal{G}} \frac{\theta^Y}{\theta^Y - 1} \frac{WL}{PY} E_t \ln \frac{\hat{u}_{t+1} \hat{K}_{t+1}}{\hat{A}_{t+1} \hat{L}_{t+1}}. \end{aligned} \quad (84)$$

The sensitivity of the cyclical component of the relative shadow price of capital to changes in the cyclical component of the ratio of utilized capital to effective labour is decreasing in elasticity of substitution parameter \mathcal{G} . This approximate linear investment Euler equation is subject to output technology shocks.

The cyclical component of the rate of capital utilization depends on the contemporaneous cyclical component of the ratio of capital to effective labour according to approximate linear implicit capital utilization function:

$$\ln \hat{u}_t = - \frac{\theta^Y}{\theta^Y - 1} \frac{WL}{PY} \left(\kappa \mathcal{G} + \frac{\theta^Y}{\theta^Y - 1} \frac{WL}{PY} \right)^{-1} \ln \frac{\hat{K}_t}{\hat{A}_t \hat{L}_t}. \quad (85)$$

The sensitivity of the cyclical component of the rate of capital utilization to changes in the cyclical component of the ratio of capital to effective labour is decreasing in capital utilization cost parameter κ and elasticity of substitution parameter \mathcal{G} . This approximate linear implicit capital utilization function is subject to output technology shocks.

The cyclical component of the capital stock depends on the past cyclical component of the capital stock and the contemporaneous cyclical component of investment according to approximate linear capital accumulation function

$$\ln \hat{K}_{t+1} = (1 - \delta) \ln \hat{K}_t + \delta \ln(\hat{v}_t^I \hat{I}_t), \quad (86)$$

where $\frac{I}{K} = \delta$. This approximate linear capital accumulation function is subject to investment technology shocks.

The cyclical component of the nominal interest rate depends on the contemporaneous cyclical components of consumption price inflation and output according to monetary policy rule:

$$\hat{i}_t = \xi^\pi \hat{\pi}_t^C + \xi^Y \ln \hat{Y}_t + \hat{v}_t^i. \quad (87)$$

This monetary policy rule ensures convergence of the level of consumption price inflation to its target value, and is subject to monetary policy shocks. The cyclical component of the output based real interest rate satisfies $\hat{r}_t^Y = \hat{i}_t - E_t \hat{\pi}_{t+1}^Y$, while the cyclical component of the consumption based real interest rate satisfies $\hat{r}_t^C = \hat{i}_t - E_t \hat{\pi}_{t+1}^C$.

The cyclical component of the tax rate depends on the contemporaneous cyclical component of the ratio of net government debt to nominal output according to fiscal revenue rule:

$$\ln \hat{\tau}_t = \zeta^\tau \ln \left(-\frac{\hat{B}_{t+1}^G}{\hat{P}_t^Y \hat{Y}_t} \right) + \hat{v}_t^\tau. \quad (88)$$

This fiscal revenue rule ensures convergence of the level of the ratio of net government debt to nominal output to its target value, and is subject to fiscal revenue shocks.

The cyclical component of the nominal exchange rate depends on the expected future cyclical component of the nominal exchange rate and the contemporaneous cyclical component of the nominal interest rate differential according to approximate linear uncovered interest parity condition:

$$\ln \hat{\mathcal{E}}_t = E_t \ln \hat{\mathcal{E}}_{t+1} - (\hat{i}_t - \hat{i}_t^f). \quad (89)$$

The cyclical component of the real exchange rate satisfies $\ln \hat{Q}_t = \ln \hat{\mathcal{E}}_t + \ln \hat{P}_t^{Y,f} - \ln \hat{P}_t^Y$, while the cyclical component of the terms of trade satisfies $\ln \hat{T}_t = \ln \hat{P}_t^M - \ln \hat{P}_t^X$, where $\ln \hat{P}_t^X = \ln \hat{P}_t^Y$.

The cyclical component of nominal output depends on the contemporaneous cyclical components of nominal consumption, investment, government consumption, exports, and imports according to approximate linear aggregate resource constraint:

$$\ln(\hat{P}_t^Y \hat{Y}_t) = \frac{C}{Y} \ln(\hat{P}_t^C \hat{C}_t) + \frac{I}{Y} \ln(\hat{P}_t^I \hat{I}_t) + \frac{G}{Y} \ln(\hat{P}_t^G \hat{G}_t) + \frac{X}{Y} \ln(\hat{P}_t^X \hat{X}_t) - \frac{M}{Y} \ln(\hat{P}_t^M \hat{M}_t). \quad (90)$$

In equilibrium, the cyclical component of output is determined by the cumulative demands of domestic and foreign households, firms, and governments.

The cyclical component of the net government debt depends on the past cyclical component of the net government debt, the past cyclical component of the nominal interest rate, the contemporaneous cyclical component of tax revenues, and the contemporaneous cyclical component of nominal government consumption according to approximate linear government dynamic budget constraint

$$E_{t-1} \ln(-\hat{B}_{t+1}^G) = \frac{1}{\beta} \left[\ln(-\hat{B}_t^G) + \hat{i}_{t-1} \right] + \left(\frac{B^G}{PY} \right)^{-1} E_{t-1} \left[\tau \ln(\hat{\tau}_t \hat{P}_t^Y \hat{Y}_t) - \frac{G}{Y} \ln(\hat{P}_t^G \hat{G}_t) \right], \quad (91)$$

where $\frac{B^G}{PY} = -\frac{\beta}{1-\beta} \left(\tau - \frac{G}{Y} \right)$. This approximate linear government dynamic budget constraint is well defined only if the level of the net government debt is positive.

The cyclical component of the net foreign debt depends on the past cyclical component of the net foreign debt, the past cyclical component of the nominal interest rate, the contemporaneous cyclical component of export revenues, and the contemporaneous cyclical component of import expenditures according to approximate linear national dynamic budget constraint

$$E_{t-1} \ln(-\hat{B}_{t+1}) = \frac{1}{\beta} \left[\ln(-\hat{B}_t) + \hat{i}_{t-1} \right] + \left(\frac{B}{PY} \right)^{-1} E_{t-1} \left[\frac{X}{Y} \ln(\hat{P}_t^X \hat{X}_t) - \frac{M}{Y} \ln(\hat{P}_t^M \hat{M}_t) \right], \quad (92)$$

where $\frac{B}{PY} = -\frac{\beta}{1-\beta} \left(\frac{X}{Y} - \frac{M}{Y} \right)$. This approximate linear national dynamic budget constraint is well defined only if the level of the net foreign debt is positive.

Variation in cyclical components is driven by ten exogenous stochastic processes. The cyclical components of the preference, output technology, investment technology, import technology, output price markup, import price markup, wage markup, monetary policy, fiscal expenditure, and fiscal revenue shocks follow stationary first order autoregressive processes:

$$\ln \hat{v}_t^C = \rho_{v^c} \ln \hat{v}_{t-1}^C + \varepsilon_t^{v^c}, \quad \varepsilon_t^{v^c} \sim \text{iid } \mathcal{N}(0, \sigma_{v^c}^2), \quad (93)$$

$$\ln \hat{A}_t = \rho_A \ln \hat{A}_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim \text{iid } \mathcal{N}(0, \sigma_A^2), \quad (94)$$

$$\ln \hat{v}_t^I = \rho_{v^I} \ln \hat{v}_{t-1}^I + \varepsilon_t^{v^I}, \quad \varepsilon_t^{v^I} \sim \text{iid } \mathcal{N}(0, \sigma_{v^I}^2), \quad (95)$$

$$\ln \hat{v}_t^M = \rho_{v^M} \ln \hat{v}_{t-1}^M + \varepsilon_t^{v^M}, \quad \varepsilon_t^{v^M} \sim \text{iid } \mathcal{N}(0, \sigma_{v^M}^2), \quad (96)$$

$$\ln \hat{\theta}_t^Y = \rho_{\theta^Y} \ln \hat{\theta}_{t-1}^Y + \varepsilon_t^{\theta^Y}, \quad \varepsilon_t^{\theta^Y} \sim \text{iid } \mathcal{N}(0, \sigma_{\theta^Y}^2), \quad (97)$$

$$\ln \hat{\theta}_t^M = \rho_{\theta^M} \ln \hat{\theta}_{t-1}^M + \varepsilon_t^{\theta^M}, \quad \varepsilon_t^{\theta^M} \sim \text{iid } \mathcal{N}(0, \sigma_{\theta^M}^2), \quad (98)$$

$$\ln \hat{\theta}_t^L = \rho_{\theta^L} \ln \hat{\theta}_{t-1}^L + \varepsilon_t^{\theta^L}, \quad \varepsilon_t^{\theta^L} \sim \text{iid } \mathcal{N}(0, \sigma_{\theta^L}^2), \quad (99)$$

$$\hat{\nu}_t^i = \rho_{\nu^i} \hat{\nu}_{t-1}^i + \varepsilon_t^{\nu^i}, \quad \varepsilon_t^{\nu^i} \sim \text{iid } \mathcal{N}(0, \sigma_{\nu^i}^2), \quad (100)$$

$$\hat{\nu}_t^G = \rho_{\nu^G} \hat{\nu}_{t-1}^G + \varepsilon_t^{\nu^G}, \quad \varepsilon_t^{\nu^G} \sim \text{iid } \mathcal{N}(0, \sigma_{\nu^G}^2), \quad (101)$$

$$\hat{\nu}_t^r = \rho_{\nu^r} \hat{\nu}_{t-1}^r + \varepsilon_t^{\nu^r}, \quad \varepsilon_t^{\nu^r} \sim \text{iid } \mathcal{N}(0, \sigma_{\nu^r}^2). \quad (102)$$

The innovations driving these exogenous stochastic processes are assumed to be independent, which combined with our distributional assumptions implies multivariate normality. In deterministic steady state equilibrium, $\nu^C = \nu^I = \nu^M = 1$.

2.6.2. Trend Components

The trend components of the prices of output, consumption, investment, government consumption, and imports follow random walks with time varying drift π_t :

$$\ln \bar{P}_t^Y = \pi_t + \ln \bar{P}_{t-1}^Y + \varepsilon_t^{\bar{P}^Y}, \quad \varepsilon_t^{\bar{P}^Y} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{P}^Y}^2), \quad (103)$$

$$\ln \bar{P}_t^C = \pi_t + \ln \bar{P}_{t-1}^C + \varepsilon_t^{\bar{P}^C}, \quad \varepsilon_t^{\bar{P}^C} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{P}^C}^2), \quad (104)$$

$$\ln \bar{P}_t^I = \pi_t + \ln \bar{P}_{t-1}^I + \varepsilon_t^{\bar{P}^I}, \quad \varepsilon_t^{\bar{P}^I} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{P}^I}^2), \quad (105)$$

$$\ln \bar{P}_t^G = \pi_t + \ln \bar{P}_{t-1}^G + \varepsilon_t^{\bar{P}^G}, \quad \varepsilon_t^{\bar{P}^G} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{P}^G}^2), \quad (106)$$

$$\ln \bar{P}_t^M = \pi_t + \ln \bar{P}_{t-1}^M + \varepsilon_t^{\bar{P}^M}, \quad \varepsilon_t^{\bar{P}^M} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{P}^M}^2). \quad (107)$$

It follows that the trend components of the relative prices of consumption, investment, government consumption, and imports follow random walks without drifts. This implies that along a balanced growth path, the levels of these relative prices are constant but state dependent.

The trend components of output, consumption, investment, government consumption, exports, and imports follow random walks with time varying drift $g_t + n_t$:

$$\ln \bar{Y}_t = g_t + n_t + \ln \bar{Y}_{t-1} + \varepsilon_t^{\bar{Y}}, \quad \varepsilon_t^{\bar{Y}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{Y}}^2), \quad (108)$$

$$\ln \bar{C}_t = g_t + n_t + \ln \bar{C}_{t-1} + \varepsilon_t^{\bar{C}}, \quad \varepsilon_t^{\bar{C}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{C}}^2), \quad (109)$$

$$\ln \bar{I}_t = g_t + n_t + \ln \bar{I}_{t-1} + \varepsilon_t^{\bar{I}}, \quad \varepsilon_t^{\bar{I}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{I}}^2), \quad (110)$$

$$\ln \bar{G}_t = g_t + n_t + \ln \bar{G}_{t-1} + \varepsilon_t^{\bar{G}}, \quad \varepsilon_t^{\bar{G}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{G}}^2), \quad (111)$$

$$\ln \bar{X}_t = g_t + n_t + \ln \bar{X}_{t-1} + \varepsilon_t^{\bar{X}}, \quad \varepsilon_t^{\bar{X}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{X}}^2), \quad (112)$$

$$\ln \bar{M}_t = g_t + n_t + \ln \bar{M}_{t-1} + \varepsilon_t^{\bar{M}}, \quad \varepsilon_t^{\bar{M}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{M}}^2). \quad (113)$$

It follows that the trend components of the ratios of consumption, investment, government consumption, exports, and imports to output follow random walks without drifts. This implies that along a balanced growth path, the levels of these great ratios are constant but state dependent.

The trend component of the nominal wage follows a random walk with time varying drift $\pi_t + g_t$, while the trend component of employment follows a random walk with time varying drift n_t :

$$\ln \bar{W}_t = \pi_t + g_t + \ln \bar{W}_{t-1} + \varepsilon_t^{\bar{W}}, \quad \varepsilon_t^{\bar{W}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{W}}^2), \quad (114)$$

$$\ln \bar{L}_t = n_t + \ln \bar{L}_{t-1} + \varepsilon_t^{\bar{L}}, \quad \varepsilon_t^{\bar{L}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{L}}^2). \quad (115)$$

It follows that the trend component of the income share of labour follows a random walk without drift. This implies that along a balanced growth path, the level of the income share of labour is constant but state dependent. The trend component of real marginal cost satisfies $\ln \bar{\Phi}_t = \ln \bar{\Phi}$, while the trend component of the shadow price of capital satisfies $\ln \bar{Q}_t = \ln \bar{P}_t'$. The trend component of the rate of capital utilization satisfies $\ln \bar{u}_t = 0$, while the trend component of the capital stock satisfies $\ln \frac{\bar{K}_{t+1}}{\bar{Y}_t} = \ln \frac{K}{Y}$.

The trend components of the nominal interest rate, tax rate, and nominal exchange rate follow random walks without drifts:

$$\bar{i}_t = \bar{i}_{t-1} + \varepsilon_t^{\bar{i}}, \quad \varepsilon_t^{\bar{i}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{i}}^2), \quad (116)$$

$$\ln \bar{\tau}_t = \ln \bar{\tau}_{t-1} + \varepsilon_t^{\bar{\tau}}, \quad \varepsilon_t^{\bar{\tau}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{\tau}}^2), \quad (117)$$

$$\ln \bar{\mathcal{E}}_t = \ln \bar{\mathcal{E}}_{t-1} + \varepsilon_t^{\bar{\mathcal{E}}}, \quad \varepsilon_t^{\bar{\mathcal{E}}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{\mathcal{E}}}^2). \quad (118)$$

It follows that along a balanced growth path, the levels of the nominal interest rate, tax rate, and nominal exchange rate are constant but state dependent. The trend component of the output

based real interest rate satisfies $\bar{r}_t^Y = \bar{i}_t - E_t \bar{\pi}_{t+1}^Y$, while the trend component of the consumption based real interest rate satisfies $\bar{r}_t^C = \bar{i}_t - E_t \bar{\pi}_{t+1}^C$. The trend component of the real exchange rate satisfies $\ln \bar{Q}_t = \ln \bar{\mathcal{E}}_t + \ln \bar{P}_t^{Y,f} - \ln \bar{P}_t^Y$, while the trend component of the terms of trade satisfies $\ln \bar{T}_t = \ln \bar{P}_t^M - \ln \bar{P}_t^X$, where $\ln \bar{P}_t^X = \ln \bar{P}_t^Y$. The trend component of the net government debt satisfies $\ln \left(-\frac{\bar{B}_{t+1}^G}{\bar{P}_t^Y B} \right) = \ln \left(-\frac{B^G}{PY} \right)$, while the trend component of the net foreign debt satisfies $\ln \left(-\frac{\bar{B}_{t+1}}{\bar{P}_t^Y} \right) = \ln \left(-\frac{B}{PY} \right)$.

Long run balanced growth is driven by three common stochastic trends. Trend inflation, productivity growth, and population growth follow random walks without drifts:

$$\pi_t = \pi_{t-1} + \varepsilon_t^\pi, \quad \varepsilon_t^\pi \sim \text{iid } \mathcal{N}(0, \sigma_\pi^2), \quad (119)$$

$$g_t = g_{t-1} + \varepsilon_t^g, \quad \varepsilon_t^g \sim \text{iid } \mathcal{N}(0, \sigma_g^2), \quad (120)$$

$$n_t = n_{t-1} + \varepsilon_t^n, \quad \varepsilon_t^n \sim \text{iid } \mathcal{N}(0, \sigma_n^2). \quad (121)$$

All innovations driving variation in trend components are assumed to be independent, which combined with our distributional assumptions implies multivariate normality.

3. Estimation, Inference and Forecasting

Unobserved components models feature prominently in the empirical macroeconomics literature, while DSGE models are pervasive in the theoretical macroeconomics literature. The primary contribution of this paper is the joint modeling of cyclical and trend components as unobserved components while imposing theoretical restrictions derived from the approximate multivariate linear rational expectations representation of a DSGE model.

This merging of modeling paradigms drawn from the theoretical and empirical macroeconomics literatures confers a number of important benefits. First, the joint estimation of parameters and trend components ensures their mutual consistency, as estimates of parameters appropriately reflect estimates of trend components, and vice versa. As shown by Nelson and Kang (1981) and Harvey and Jaeger (1993), decomposing integrated observed nonpredetermined endogenous variables into cyclical and trend components with atheoretic deterministic polynomial functions or linear filters may induce spurious cyclical dynamics, invalidating subsequent estimation, inference and forecasting. Second, jointly modeling cyclical and trend components as unobserved components ensures stochastic nonsingularity of the resulting approximate linear state space representation of the DSGE model, as associated with each observed nonpredetermined endogenous variable is at least one exogenous stochastic process.

As discussed in Ruge-Murcia (2003), stochastic nonsingularity requires that the number of observed nonpredetermined endogenous variables employed in full information maximum likelihood estimation of the approximate linear state space representation of a DSGE model not exceed the number of exogenous stochastic processes, with efficiency losses incurred if this constraint binds. Third, and of perhaps greatest practical importance, jointly modeling cyclical and trend components as unobserved components while ensuring the existence of a well defined balanced growth path facilitates the generation of forecasts of the levels of nonpredetermined endogenous variables as opposed to their cyclical components, while ensuring that these forecasts satisfy the stability restrictions associated with balanced growth. These stability restrictions are necessary but not sufficient for full cointegration, as along a balanced growth path, great ratios are constant but state dependent, robustifying forecasts to intermittent structural breaks that occur within sample.

3.1. Estimation

The traditional econometric interpretation of macroeconometric models regards them as representations of the joint probability distribution of the data. Adopting this traditional econometric interpretation, Bayesian full information maximum likelihood estimation of a linear state space representation of an approximate unobserved components representation of this DSGE model of a small open economy, conditional on prior information concerning the values of parameters and trend components, facilitates an empirical evaluation of its impulse response and predictive accuracy properties.

3.1.1. Estimation Procedure

Let \mathbf{x}_t denote a vector stochastic process consisting of the levels of N nonpredetermined endogenous variables, of which M are observed. The cyclical components of this vector stochastic process satisfy second order stochastic linear difference equation

$$\mathbf{A}_0 \hat{\mathbf{x}}_t = \mathbf{A}_1 \hat{\mathbf{x}}_{t-1} + \mathbf{A}_2 \mathbf{E}_t \hat{\mathbf{x}}_{t+1} + \mathbf{A}_3 \hat{\mathbf{v}}_t, \quad (122)$$

where vector stochastic process $\hat{\mathbf{v}}_t$ consists of the cyclical components of K exogenous variables. This vector stochastic process satisfies stationary first order stochastic linear difference equation

$$\hat{\mathbf{v}}_t = \mathbf{B}_1 \hat{\mathbf{v}}_{t-1} + \boldsymbol{\varepsilon}_{1,t}, \quad (123)$$

where $\boldsymbol{\varepsilon}_{1,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_1)$. The trend components of vector stochastic process \mathbf{x}_t satisfy first order stochastic linear difference equation

$$\mathbf{C}_0 \bar{\mathbf{x}}_t = \mathbf{C}_1 + \mathbf{C}_2 \mathbf{u}_t + \mathbf{C}_3 \bar{\mathbf{x}}_{t-1} + \boldsymbol{\varepsilon}_{2,t}, \quad (124)$$

where $\boldsymbol{\varepsilon}_{2,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_2)$. Vector stochastic process \mathbf{u}_t consists of the levels of L common stochastic trends, and satisfies nonstationary first order stochastic linear difference equation

$$\mathbf{u}_t = \mathbf{u}_{t-1} + \boldsymbol{\varepsilon}_{3,t}, \quad (125)$$

where $\boldsymbol{\varepsilon}_{3,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_3)$. Cyclical and trend components are additively separable, that is $\mathbf{x}_t = \hat{\mathbf{x}}_t + \bar{\mathbf{x}}_t$.

If there exists a unique stationary solution to multivariate linear rational expectations model (122), then it may be expressed as:

$$\hat{\mathbf{x}}_t = \mathbf{D}_1 \hat{\mathbf{x}}_{t-1} + \mathbf{D}_2 \hat{\mathbf{v}}_t. \quad (126)$$

Consider the following real generalized Schur decomposition, where stable generalized eigenvalues are ordered first:

$$\begin{bmatrix} \mathbf{Q}_{1,1} & \mathbf{Q}_{1,2} \\ \mathbf{Q}_{2,1} & \mathbf{Q}_{2,2} \end{bmatrix} \begin{bmatrix} \mathbf{I}_N & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{Z}_{1,1} & \mathbf{Z}_{1,2} \\ \mathbf{Z}_{2,1} & \mathbf{Z}_{2,2} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{1,1} & \mathbf{S}_{1,2} \\ \mathbf{0} & \mathbf{S}_{2,2} \end{bmatrix}, \quad (127)$$

$$\begin{bmatrix} \mathbf{Q}_{1,1} & \mathbf{Q}_{1,2} \\ \mathbf{Q}_{2,1} & \mathbf{Q}_{2,2} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{I}_N \\ -\mathbf{A}_1 & \mathbf{A}_0 \end{bmatrix} \begin{bmatrix} \mathbf{Z}_{1,1} & \mathbf{Z}_{1,2} \\ \mathbf{Z}_{2,1} & \mathbf{Z}_{2,2} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{1,1} & \mathbf{T}_{1,2} \\ \mathbf{0} & \mathbf{T}_{2,2} \end{bmatrix}. \quad (128)$$

Following Klein (2000), matrices \mathbf{D}_1 and \mathbf{D}_2 may be expressed in terms of the results of this ordered real generalized Schur decomposition as

$$\mathbf{D}_1 = \mathbf{Z}_{2,1} \mathbf{Z}_{1,1}^{-1}, \quad (129)$$

$$\mathbf{D}_2 = (\mathbf{Z}_{2,2} - \mathbf{Z}_{2,1} \mathbf{Z}_{1,1}^{-1} \mathbf{Z}_{1,2}) \mathbf{R}, \quad (130)$$

where $\text{vec}(\mathbf{R}) = (\mathbf{I}_K \otimes \mathbf{T}_{2,2} - \mathbf{B}_1 \otimes \mathbf{S}_{2,2})^{-1} \text{vec}(\mathbf{Q}_{2,2} \mathbf{A}_3)$. This unique stationary solution exists if the number of unstable generalized eigenvalues equals N .

Let \mathbf{y}_t denote a vector stochastic process consisting of the levels of M observed nonpredetermined endogenous variables. Also, let \mathbf{z}_t denote a vector stochastic process consisting of the levels of $N - M$ unobserved nonpredetermined endogenous variables, the

cyclical components of N nonpredetermined endogenous variables, the trend components of N nonpredetermined endogenous variables, the cyclical components of K exogenous variables, and the levels of L common stochastic trends. Given unique stationary solution (126), these vector stochastic processes have linear state space representation

$$\mathbf{y}_t = \mathbf{F}_1 \mathbf{z}_t, \quad (131)$$

$$\mathbf{z}_t = \mathbf{G}_1 + \mathbf{G}_2 \mathbf{z}_{t-1} + \mathbf{G}_3 \boldsymbol{\varepsilon}_{4,t}, \quad (132)$$

where $\boldsymbol{\varepsilon}_{4,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_4)$ and $\mathbf{z}_0 \sim \mathcal{N}(\mathbf{z}_{0|0}, \mathbf{P}_{0|0})$. Let \mathbf{w}_t denote a vector stochastic process consisting of preliminary estimates of the trend components of M observed nonpredetermined endogenous variables. Suppose that this vector stochastic process satisfies

$$\mathbf{w}_t = \mathbf{H}_1 \mathbf{z}_t + \boldsymbol{\varepsilon}_{5,t}, \quad (133)$$

where $\boldsymbol{\varepsilon}_{5,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_5)$. Conditional on known parameter values, this signal equation defines a set of stochastic restrictions on selected unobserved state variables. The signal and state innovation vectors are assumed to be independent, while the initial state vector is assumed to be independent from the signal and state innovation vectors, which combined with our distributional assumptions implies multivariate normality.

Conditional on the parameters associated with these signal and state equations, estimates of unobserved state vector \mathbf{z}_t and its mean squared error matrix \mathbf{P}_t may be calculated with the filter proposed by Vitek (2006a, 2006b), which adapts the filter due to Kalman (1960) to incorporate prior information. Given initial conditions $\mathbf{z}_{0|0}$ and $\mathbf{P}_{0|0}$, estimates conditional on information available at time $t-1$ satisfy prediction equations:

$$\mathbf{z}_{t|t-1} = \mathbf{G}_1 + \mathbf{G}_2 \mathbf{z}_{t-1|t-1}, \quad (134)$$

$$\mathbf{P}_{t|t-1} = \mathbf{G}_2 \mathbf{P}_{t-1|t-1} \mathbf{G}_2^\top + \mathbf{G}_3 \boldsymbol{\Sigma}_4 \mathbf{G}_3^\top, \quad (135)$$

$$\mathbf{y}_{t|t-1} = \mathbf{F}_1 \mathbf{z}_{t|t-1}, \quad (136)$$

$$\mathbf{Q}_{t|t-1} = \mathbf{F}_1 \mathbf{P}_{t|t-1} \mathbf{F}_1^\top, \quad (137)$$

$$\mathbf{w}_{t|t-1} = \mathbf{H}_1 \mathbf{z}_{t|t-1}, \quad (138)$$

$$\mathbf{R}_{t|t-1} = \mathbf{H}_1 \mathbf{P}_{t|t-1} \mathbf{H}_1^\top + \boldsymbol{\Sigma}_5. \quad (139)$$

Given these predictions, under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, estimates conditional on information available at time t satisfy updating equations

$$\mathbf{z}_{t|t} = \mathbf{z}_{t|t-1} + \mathbf{K}_{y_t} (\mathbf{y}_t - \mathbf{y}_{t|t-1}) + \mathbf{K}_{w_t} (\mathbf{w}_t - \mathbf{w}_{t|t-1}), \quad (140)$$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{K}_{y_t} \mathbf{F}_1 \mathbf{P}_{t|t-1} - \mathbf{K}_{w_t} \mathbf{H}_1 \mathbf{P}_{t|t-1}, \quad (141)$$

where $\mathbf{K}_{y_t} = \mathbf{P}_{t|t-1} \mathbf{F}_1^\top \mathbf{Q}_{t|t-1}^{-1}$ and $\mathbf{K}_{w_t} = \mathbf{P}_{t|t-1} \mathbf{H}_1^\top \mathbf{R}_{t|t-1}^{-1}$. Given terminal conditions $\mathbf{z}_{T|T}$ and $\mathbf{P}_{T|T}$ obtained from the final evaluation of these prediction and updating equations, estimates conditional on information available at time T satisfy smoothing equations

$$\mathbf{z}_{t|T} = \mathbf{z}_{t|t} + \mathbf{J}_t (\mathbf{z}_{t+1|T} - \mathbf{z}_{t+1|t}), \quad (142)$$

$$\mathbf{P}_{t|T} = \mathbf{P}_{t|t} + \mathbf{J}_t (\mathbf{P}_{t+1|T} - \mathbf{P}_{t+1|t}) \mathbf{J}_t^\top, \quad (143)$$

where $\mathbf{J}_t = \mathbf{P}_{t|t} \mathbf{G}_2^\top \mathbf{P}_{t+1|t}^{-1}$. Under our distributional assumptions, these estimators of the unobserved state vector are mean squared error optimal.

Let $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbb{R}^J$ denote a J dimensional vector containing the parameters associated with the signal and state equations of this linear state space model. The Bayesian full information maximum likelihood estimator of this parameter vector has posterior density function

$$f(\boldsymbol{\theta} | \mathcal{I}_T) \propto f(\mathcal{I}_T | \boldsymbol{\theta}) f(\boldsymbol{\theta}), \quad (144)$$

where $\mathcal{I}_t = \{\{\mathbf{y}_s\}_{s=1}^t, \{\mathbf{w}_s\}_{s=1}^t\}$. Under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, conditional density function $f(\mathcal{I}_T | \boldsymbol{\theta})$ satisfies:

$$f(\mathcal{I}_T | \boldsymbol{\theta}) = \prod_{t=1}^T f(\mathbf{y}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta}) \cdot \prod_{t=1}^T f(\mathbf{w}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta}). \quad (145)$$

Under our distributional assumptions, conditional density functions $f(\mathbf{y}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta})$ and $f(\mathbf{w}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta})$ satisfy:

$$f(\mathbf{y}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta}) = (2\pi)^{-\frac{M}{2}} |\mathbf{Q}_{t|t-1}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} (\mathbf{y}_t - \mathbf{y}_{t|t-1})^\top \mathbf{Q}_{t|t-1}^{-1} (\mathbf{y}_t - \mathbf{y}_{t|t-1})\right\}, \quad (146)$$

$$f(\mathbf{w}_t | \mathcal{I}_{t-1}, \boldsymbol{\theta}) = (2\pi)^{-\frac{M}{2}} |\mathbf{R}_{t|t-1}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} (\mathbf{w}_t - \mathbf{w}_{t|t-1})^\top \mathbf{R}_{t|t-1}^{-1} (\mathbf{w}_t - \mathbf{w}_{t|t-1})\right\}. \quad (147)$$

Prior information concerning parameter vector $\boldsymbol{\theta}$ is summarized by a multivariate normal prior distribution having mean vector $\boldsymbol{\theta}_1$ and covariance matrix $\boldsymbol{\Omega}$:

$$f(\boldsymbol{\theta}) = (2\pi)^{-\frac{J}{2}} |\boldsymbol{\Omega}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\theta}_1)^\top \boldsymbol{\Omega}^{-1}(\boldsymbol{\theta} - \boldsymbol{\theta}_1)\right\}. \quad (148)$$

Independent priors are represented by a diagonal covariance matrix, under which diffuse priors are represented by infinite variances.

Inference on the parameters is based on an asymptotic normal approximation to the posterior distribution around its mode. Under regularity conditions stated in Geweke (2005), posterior mode $\hat{\boldsymbol{\theta}}_T$ satisfies

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{d} \mathcal{N}(\mathbf{0}, -\mathcal{H}_0^{-1}), \quad (149)$$

where $\boldsymbol{\theta}_0 \in \boldsymbol{\Theta}$ denotes the pseudotrue parameter vector. Following Engle and Watson (1981), Hessian \mathcal{H}_0 may be estimated by

$$\begin{aligned} \hat{\mathcal{H}}_T &= \frac{1}{T} \sum_{t=1}^T \mathbb{E}_{t-1} \left[\nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^\top \ln f(\mathbf{y}_t | \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T) \right] + \frac{1}{T} \sum_{t=1}^T \mathbb{E}_{t-1} \left[\nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^\top \ln f(\mathbf{w}_t | \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T) \right] \\ &\quad + \frac{1}{T} \nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^\top \ln f(\hat{\boldsymbol{\theta}}_T), \end{aligned} \quad (150)$$

where $\mathbb{E}_{t-1} \left[\nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^\top \ln f(\mathbf{y}_t | \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T) \right] = -\nabla_{\boldsymbol{\theta}} \mathbf{y}_{t|t-1}^\top \mathbf{Q}_{t|t-1}^{-1} \nabla_{\boldsymbol{\theta}} \mathbf{y}_{t|t-1} - \frac{1}{2} \nabla_{\boldsymbol{\theta}} \mathbf{Q}_{t|t-1}^\top (\mathbf{Q}_{t|t-1}^{-1} \otimes \mathbf{Q}_{t|t-1}^{-1}) \nabla_{\boldsymbol{\theta}} \mathbf{Q}_{t|t-1}$,
 $\mathbb{E}_{t-1} \left[\nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^\top \ln f(\mathbf{w}_t | \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T) \right] = -\nabla_{\boldsymbol{\theta}} \mathbf{w}_{t|t-1}^\top \mathbf{R}_{t|t-1}^{-1} \nabla_{\boldsymbol{\theta}} \mathbf{w}_{t|t-1} - \frac{1}{2} \nabla_{\boldsymbol{\theta}} \mathbf{R}_{t|t-1}^\top (\mathbf{R}_{t|t-1}^{-1} \otimes \mathbf{R}_{t|t-1}^{-1}) \nabla_{\boldsymbol{\theta}} \mathbf{R}_{t|t-1}$, and
 $\nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^\top \ln f(\hat{\boldsymbol{\theta}}_T) = -\boldsymbol{\Omega}^{-1}$.

3.1.2. Estimation Results

The set of parameters associated with this DSGE model of a small open economy is partitioned into two subsets. The first subset is calibrated to approximately match long run averages of functions of observed nonpredetermined endogenous variables where possible, and estimates derived from existing microeconomic studies where necessary. The second subset is estimated by Bayesian full information maximum likelihood, conditional on prior information concerning the values of parameters and trend components.

Subjective discount factor β is restricted to equal 0.99, implying an annualized deterministic steady state equilibrium real interest rate of approximately 0.04. In deterministic steady state

equilibrium, the output price markup $\frac{\theta^y}{\theta^y-1}$, import price markup $\frac{\theta^M}{\theta^M-1}$, and wage markup $\frac{\theta^L}{\theta^L-1}$ are restricted to equal 1.15. Depreciation rate parameter δ is restricted to equal 0.015, implying an annualized deterministic steady state equilibrium depreciation rate of approximately 0.06. In deterministic steady state equilibrium, the consumption import share $1-\phi^C$, investment import share $1-\phi^I$, and government consumption import share $1-\phi^G$ are restricted to equal 0.30. The deterministic steady state equilibrium ratio of consumption to output $\frac{C}{Y}$ is restricted to equal 0.60, while the deterministic steady state equilibrium ratio of domestic output to foreign output $\frac{Y}{Y^f}$ is restricted to equal 0.11. In deterministic steady state equilibrium, the foreign consumption import share $1-\phi^{C,f}$, foreign investment import share $1-\phi^{I,f}$, and foreign government consumption import share $1-\phi^{G,f}$ are restricted to equal 0.02. The deterministic steady state equilibrium income share of labour $\frac{WL}{PY}$ is restricted to equal 0.50. In deterministic steady state equilibrium, the ratio of government consumption to output $\frac{G}{Y}$ is restricted to equal 0.20, while the tax rate τ is restricted to equal 0.22.

Table 1. Deterministic steady state equilibrium values of great ratios

Ratio	Value	Ratio	Value
C/Y	0.6000	WL/PY	0.5000
I/Y	0.1723	K/Y	2.8710
G/Y	0.2000	B^G/PY	-0.4950
X/Y	0.3194	B/PY	-0.6866
M/Y	0.2917		

Note: Deterministic steady state equilibrium values are reported at an annual frequency based on calibrated parameter values.

Bayesian full information maximum likelihood estimation of the remaining parameters of this DSGE model of a small open economy is based on the levels of twenty six observed nonpredetermined endogenous variables for Canada and the United States described in Appendix A. Those parameters associated with the conditional mean function are estimated subject to cross-economy equality restrictions. Those parameters associated exclusively with the conditional variance function are estimated conditional on diffuse priors. Initial conditions for the cyclical components of exogenous variables are given by their unconditional means and variances, while the initial values of all other state variables are treated as parameters, and are calibrated to match functions of preliminary estimates of trend components calculated with the linear filter described in Hodrick and Prescott (1997). The posterior mode is calculated by numerically maximizing the logarithm of the posterior density kernel with a modified steepest ascent algorithm. Estimation results pertaining to the period 1971Q3 through 2005Q1 are reported in Appendix B. The sufficient condition for the existence of a unique stationary rational expectations equilibrium due to Klein (2000) is satisfied in a neighbourhood around the posterior

mode, while the Hessian is not nearly singular at the posterior mode, suggesting that the approximate linear state space representation of this DSGE model of a small open economy is locally identified.

The prior mean of indexation parameter γ^Y is 0.75, implying considerable output price inflation inertia, while the prior mean of nominal rigidity parameter ω^Y implies an average duration of output price contracts of two years. The prior mean of capital utilization cost parameter κ is 0.10, while the prior mean of elasticity of substitution parameter ϑ is 0.75, implying that utilized capital and effective labour are moderately close complements in production. The prior mean of habit persistence parameter α is 0.95, while the prior mean of intertemporal elasticity of substitution parameter σ is 2.75, implying that consumption exhibits considerable persistence and moderate sensitivity to real interest rate changes. The prior mean of investment adjustment cost parameter χ is 5.75, implying moderate sensitivity of investment to changes in the relative shadow price of capital. The prior mean of indexation parameter γ^M is 0.75, implying moderate import price inflation inertia, while the prior mean of nominal rigidity parameter ω^M implies an average duration of import price contracts of two years. The prior mean of elasticity of substitution parameter ψ is 1.50, implying that domestic and foreign goods are moderately close substitutes in consumption, investment, and government consumption. The prior mean of indexation parameter γ^L is 0.75, implying considerable sensitivity of the real wage to changes in consumption price inflation, while the prior mean of nominal rigidity parameter ω^L implies an average duration of wage contracts of two years. The prior mean of elasticity of substitution parameter η is 2.00, implying considerable insensitivity of the real wage to changes in employment. The prior mean of the consumption price inflation response coefficient ξ^π in the monetary policy rule is 1.50, while the prior mean of the output response coefficient ξ^Y is 0.125, ensuring convergence of the level of consumption price inflation to its target value. The prior mean of the net foreign debt response coefficient ζ^G in the fiscal expenditure rule is -0.10 , while the prior mean of the net government debt response coefficient ζ^r in the fiscal revenue rule is 1.00, ensuring convergence of the levels of the ratios of net foreign debt and net government debt to nominal output to their target values. All autoregressive parameters ρ have prior means of 0.85, implying considerable persistence of shocks driving variation in cyclical components.

The posterior modes of these structural parameters are all close to their prior means, reflecting the imposition of tight independent priors to ensure the existence of a unique stationary rational expectations equilibrium. The estimated variances of shocks driving variation in cyclical components are all well within the range of estimates reported in the existing literature, after accounting for data rescaling. The estimated variances of shocks driving variation in trend

components are relatively high, indicating that the majority of variation in the levels of observed nonpredetermined endogenous variables is accounted for by variation in trend components.

Prior information concerning the values of trend components is generated by fitting third order deterministic polynomial functions to the levels of all observed nonpredetermined endogenous variables by ordinary least squares. Stochastic restrictions on the trend components of all observed nonpredetermined endogenous variables are derived from the fitted values associated with these ordinary least squares regressions, with innovation variances set proportional to estimated prediction variances assuming known parameters. All stochastic restrictions are independent, represented by a diagonal covariance matrix, and are harmonized, represented by a common factor of proportionality. Reflecting moderate confidence in these preliminary trend component estimates, this common factor of proportionality is set equal to one.

Predicted, filtered and smoothed estimates of the cyclical and trend components of observed nonpredetermined endogenous variables are plotted together with confidence intervals in Appendix B. These confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. The predicted estimates are conditional on past information, the filtered estimates are conditional on past and present information, and the smoothed estimates are conditional on past, present and future information. Visual inspection reveals close agreement with the conventional dating of business cycle expansions and recessions.

3.2. Inference

Whether this estimated DSGE model approximately accounts for the empirical evidence concerning the monetary transmission mechanism in a small open economy is determined by comparing its impulse responses to domestic and foreign monetary policy shocks with impulse responses derived from an estimated SVAR model.

3.2.1. Empirical Impulse Response Analysis

Consider the following SVAR model of the monetary transmission mechanism in a small open economy

$$\mathbf{A}_0 \mathbf{y}_t = \boldsymbol{\mu}(t) + \sum_{i=1}^p \mathbf{A}_i \mathbf{y}_{t-i} + \mathbf{B} \boldsymbol{\varepsilon}_t, \quad (151)$$

where $\boldsymbol{\mu}(t)$ denotes a third order deterministic polynomial function and $\boldsymbol{\varepsilon}_t \sim \text{iid } \mathcal{N}(\mathbf{0}, \mathbf{I})$. Vector stochastic process \mathbf{y}_t consists of domestic output price inflation π_t^Y , domestic output $\ln Y_t$, domestic consumption price inflation π_t^C , domestic consumption $\ln C_t$, domestic investment price inflation π_t^I , domestic investment $\ln I_t$, domestic import price inflation π_t^M , domestic exports $\ln X_t$, domestic imports $\ln M_t$, domestic nominal interest rate i_t , nominal exchange rate $\ln \mathcal{E}_t$, foreign output price inflation $\pi_t^{Y,f}$, foreign output $\ln Y_t^f$, foreign consumption $\ln C_t^f$, foreign investment $\ln I_t^f$, and foreign nominal interest rate i_t^f . The diagonal elements of parameter matrix \mathbf{A}_0 are normalized to one, while the off diagonal elements of positive definite parameter matrix \mathbf{B} are restricted to equal zero, thus associating with each equation a unique endogenous variable, and with each endogenous variable a unique structural innovation.

This SVAR model is identified by imposing restrictions on the timing of the effects of monetary policy shocks and on the information sets of the monetary authorities, both within and across the domestic and foreign economies. Within the domestic and foreign economies, prices and quantities are restricted to not respond instantaneously to monetary policy shocks, while the monetary authorities can respond instantaneously to changes in these variables. Across the domestic and foreign economies, the domestic monetary authority is restricted to not respond instantaneously to foreign monetary policy shocks, while foreign variables are restricted to not respond to domestic monetary policy shocks.

This SVAR model of the monetary transmission mechanism in a small open economy is estimated by full information maximum likelihood over the period 1971Q3 through 2005Q1. As discussed in Hamilton (1994), in the absence of model misspecification, this full information maximum likelihood estimator is consistent and asymptotically normal, irrespective of the cointegration rank and validity of the conditional multivariate normality assumption. The lag order is selected to minimize multivariate extensions of the model selection criterion functions of Akaike (1974), Schwarz (1978), and Hannan and Quinn (1979) subject to an upper bound equal to the seasonal frequency. These model selection criterion functions generally prefer a lag order of one.

Table 2. Model selection criterion function values

p	$AIC(p)$	$SC(p)$	$HQ(p)$
1	-110.8778	-102.3386*	-107.4079*
2	-111.2069	-98.2780	-105.9532
3	-111.1651	-93.8465	-104.1276
4	-111.6281*	-89.9197	-102.8068

Note: Minimized values of model selection criterion functions are indicated by *.

Since this SVAR model is estimated to provide empirical evidence concerning the monetary transmission mechanism in a small open economy, it is imperative to examine the empirical validity of its overidentifying restrictions prior to the conduct of impulse response analysis. On the basis of bootstrap likelihood ratio tests, these overidentifying restrictions are not rejected at conventional levels of statistical significance.

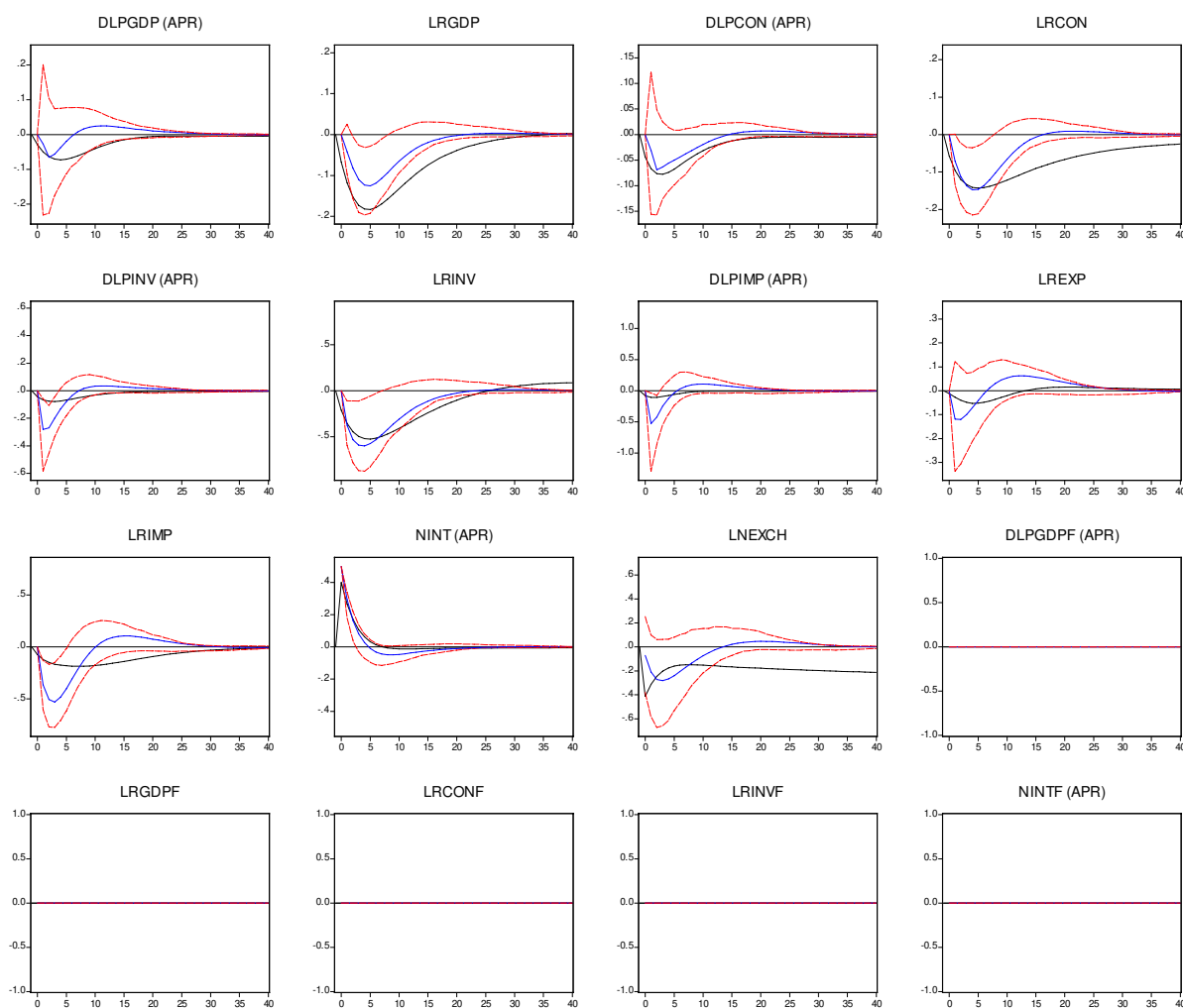
Table 3. Results of tests of overidentifying restrictions

Test Statistic	<i>P</i> Values		
	Asymptotic	Parametric Bootstrap	Nonparametric Bootstrap
278.1389	0.0000	0.9499	0.9990

Note: This likelihood ratio test statistic is asymptotically distributed as χ^2_{65} . Bootstrap distributions are based on 999 replications.

Theoretical impulse responses to a domestic monetary policy shock are plotted versus empirical impulse responses in Figure 1. Following a domestic monetary policy shock, the domestic nominal interest rate exhibits an immediate increase followed by a gradual decline. These nominal interest rate dynamics induce persistent and generally statistically significant hump shaped negative responses of domestic output price inflation, output, consumption price inflation, consumption, investment price inflation, investment, import price inflation, exports and imports, with peak effects realized after approximately one year. The domestic currency appreciates, with the nominal exchange rate exhibiting delayed overshooting. These results are qualitatively consistent with those of SVAR analyses of the monetary transmission mechanism in open economies such as Eichenbaum and Evans (1995), Clarida and Gertler (1997), Kim and Roubini (1995), and Cushman and Zha (1997).

Figure 1. Theoretical versus empirical impulse responses to a domestic monetary policy shock

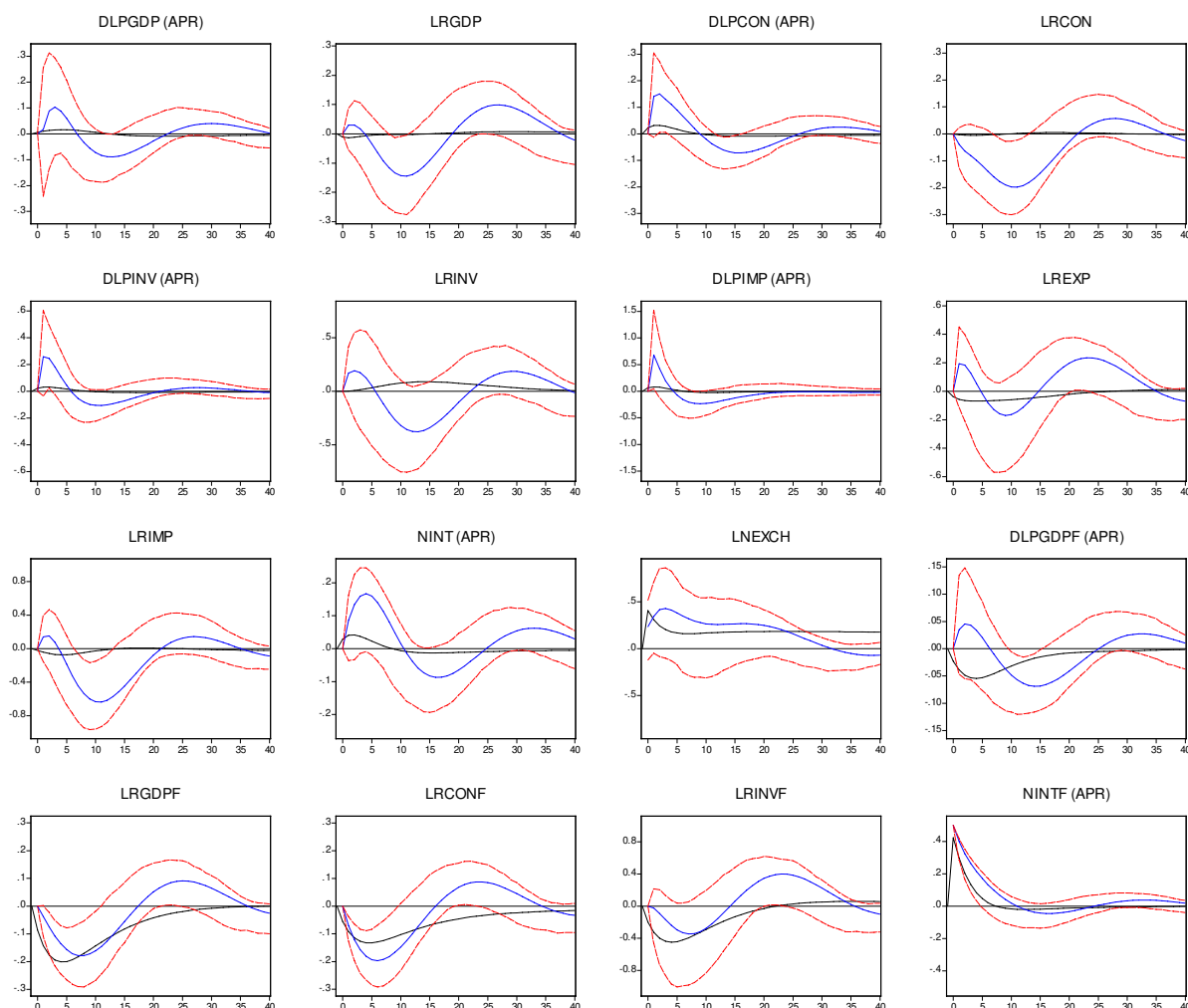


Note: Theoretical impulse responses to a 50 basis point monetary policy shock are represented by black lines, while blue lines depict empirical impulse responses to a 50 basis point monetary policy shock. Asymmetric 95% confidence intervals are calculated with a nonparametric bootstrap simulation with 999 replications.

Theoretical impulse responses to a foreign monetary policy shock are plotted versus empirical impulse responses in Figure 2. Following a foreign monetary policy shock, the foreign nominal interest rate exhibits an immediate increase followed by a gradual decline. In response to these nominal interest rate dynamics, there arise persistent and generally statistically significant hump shaped negative responses of foreign output price inflation, output, consumption and investment, with peak effects realized after approximately one to two years. Although domestic output, consumption, investment and imports decline, domestic consumption price inflation, investment price inflation and import price inflation rise due to domestic currency depreciation. These results are qualitatively consistent with those of SVAR analyses of the monetary transmission mechanism in closed economies such as Sims and Zha (1995), Gordon

and Leeper (1994), Leeper, Sims and Zha (1996), and Christiano, Eichenbaum and Evans (1998, 2005).

Figure 2. Theoretical versus empirical impulse responses to a foreign monetary policy shock



Note: Theoretical impulse responses to a 50 basis point monetary policy shock are represented by black lines, while blue lines depict empirical impulse responses to a 50 basis point monetary policy shock. Asymmetric 95% confidence intervals are calculated with a nonparametric bootstrap simulation with 999 replications.

Visual inspection reveals that the theoretical impulse responses to domestic and foreign monetary policy shocks generally lie within confidence intervals associated with the corresponding empirical impulse responses, suggesting that this estimated DSGE model approximately accounts for the empirical evidence concerning the monetary transmission mechanism in a small open economy. However, these confidence intervals are rather wide, indicating that considerable uncertainty surrounds this empirical evidence.

3.2.2. *Theoretical Impulse Response Analysis*

In an open economy, exchange rate adjustment contributes to both intratemporal and intertemporal equilibration, while business cycles are generated by interactions among a variety of nominal and real shocks originating both domestically and abroad. Theoretical impulse responses and forecast error variance decompositions to domestic and foreign preference, output technology, investment technology, import technology, output price markup, import price markup, wage markup, monetary policy, fiscal expenditure, and fiscal revenue shocks are plotted in Appendix B.

Following a domestic output technology shock, there arise persistent hump shaped positive responses of domestic output, consumption, investment, and government consumption. Domestic output price inflation, consumption price inflation, investment price inflation, and government consumption price inflation exhibit persistent hump shaped declines in response to a reduction in real marginal cost. The domestic nominal and real interest rates exhibit persistent hump shaped declines in response to a reduction in consumption price inflation, mitigated by an increase in output. The domestic currency appreciates in nominal terms and depreciates in real terms, while the terms of trade deteriorate. Since the increase in nominal output exceeds the increase in domestic demand, the trade balance rises, facilitating an intertemporal resource transfer between the domestic and foreign economies.

Following a domestic monetary policy shock, the domestic nominal and real interest rates exhibit immediate increases followed by gradual declines, inducing persistent hump shaped negative responses of domestic output, consumption, investment, and government consumption. The nominal and real exchange rates overshoot, with immediate appreciations followed by gradual depreciations. Domestic output price inflation, consumption price inflation, investment price inflation, and government consumption price inflation exhibit persistent hump shaped declines in response to a reduction in real marginal cost. These declines in domestic consumption price inflation, investment price inflation, and government consumption price inflation are amplified and accelerated by an improvement in the terms of trade. This reduction in the price of imports in terms of exports induces intratemporal expenditure switching, with a decline in the trade balance reflecting a reduction in nominal output relative to domestic demand.

Following a domestic fiscal expenditure shock, there arise immediate positive responses of domestic output and government consumption, together with persistent hump shaped negative responses of domestic consumption and investment. Domestic output price inflation rises in response to an increase in real marginal cost. The domestic nominal and real interest rates exhibit immediate increases followed by gradual declines, causing the domestic currency to

appreciate in nominal and real terms, while the terms of trade improve. Domestic consumption price inflation, investment price inflation, and government consumption price inflation rise in response to an increase in real marginal cost, amplified and accelerated by an improvement in the terms of trade. Since the increase in nominal output is less than the increase in domestic demand, the trade balance declines, facilitating an intertemporal resource transfer between the domestic and foreign economies.

3.3. Forecasting

While it is desirable that forecasts be unbiased and efficient, the practical value of any forecasting model depends on its relative predictive accuracy. In the absence of a well defined mapping between forecast errors and their costs, relative predictive accuracy is generally assessed with mean squared prediction error based measures. As discussed in Clements and Hendry (1998), mean squared prediction error based measures are noninvariant to nonsingular, scale preserving linear transformations, even though linear models are. It follows that mean squared prediction error based comparisons may yield conflicting rankings across models, depending on the variable transformations examined.

To compare the dynamic out of sample forecasting performance of the DSGE and SVAR models, forty quarters of observations are retained to evaluate forecasts one through eight quarters ahead, generated conditional on parameters estimated using information available at the forecast origin. The models are compared on the basis of mean squared prediction errors in levels, ordinary differences, and seasonal differences. The DSGE model is not recursively estimated as the forecast origin rolls forward due to the high computational cost of such a procedure, while the SVAR model is. Presumably, recursively estimating the DSGE model would improve its predictive accuracy.

Mean squared prediction error differentials are plotted together with confidence intervals accounting for contemporaneous and serial correlation of forecast errors in Appendix B. If these mean squared prediction error differentials are negative then the forecasting performance of the DSGE model dominates that of the SVAR model, while if positive then the DSGE model is dominated by the SVAR model in terms of predictive accuracy. The null hypothesis of equal squared prediction errors is rejected by the predictive accuracy test of Diebold and Mariano (1995) if and only if these confidence intervals exclude zero. The asymptotic variance of the average loss differential is estimated by a weighted sum of the autocovariances of the loss differential, employing the weighting function proposed by Newey and West (1987). Visual inspection reveals that these mean squared prediction error differentials are generally negative,

suggesting that the DSGE model dominates the SVAR model in terms of forecasting performance, in spite of a considerable informational disadvantage. However, these mean squared prediction error differentials are rarely statistically significant at conventional levels, indicating that considerable uncertainty surrounds these predictive accuracy comparisons.

Dynamic out of sample forecasts of levels, ordinary differences, and seasonal differences are plotted together with confidence intervals versus realized outcomes in Appendix B. These confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Visual inspection reveals that the realized outcomes generally lie within their associated confidence intervals, suggesting that forecast failure is absent. However, these confidence intervals are rather wide, indicating that considerable uncertainty surrounds the point forecasts.

4. Conclusion

This paper develops and estimates a DSGE model of a small open economy which approximately accounts for the empirical evidence concerning the monetary transmission mechanism, as summarized by impulse response functions derived from an estimated SVAR model, while dominating that SVAR model in terms of predictive accuracy. Cyclical components are modeled by linearizing equilibrium conditions around a stationary deterministic steady state equilibrium which abstracts from long run balanced growth, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. This estimated DSGE model consolidates much existing theoretical and empirical knowledge concerning the monetary transmission mechanism in a small open economy, provides a framework for a progressive research strategy, and suggests partial explanations for its own deficiencies.

Jointly modeling cyclical and trend components as unobserved components while imposing theoretical restrictions derived from the approximate multivariate linear rational expectations representation of a DSGE model confers a number of benefits of particular importance to the conduct of monetary policy. As discussed in Woodford (2003), the levels of the flexible price and wage equilibrium components of various observed and unobserved nonpredetermined endogenous variables are important inputs into the optimal conduct of monetary policy, in particular the measurement of the stance of monetary policy. Jointly modeling cyclical and trend components as unobserved components facilitates estimation of the levels of the flexible price and wage equilibrium components of nonpredetermined endogenous variables while imposing relatively few identifying restrictions on their trend components. The analysis of optimal

monetary policy under an inflation targeting regime and the estimation of the levels of flexible price and wage equilibrium components within the framework of an extended and refined version of this DSGE model of a small open economy remains an objective for future research.

Acknowledgements

The author gratefully acknowledges advice provided by Henry Siu and Paul Beaudry, in addition to comments and suggestions received from seminar participants at the Bank of Canada and the University of British Columbia. The author thanks the Social Sciences and Humanities Research Council of Canada for financial support.

Appendix A. Description of the Data Set

The data set consists of quarterly seasonally adjusted observations on twenty six macroeconomic variables for Canada and the United States over the period 1971Q1 through 2005Q1. All aggregate prices and quantities are expenditure based. Model consistent employment is derived from observed nominal labour income and a nominal wage index, while model consistent tax rates are derived from observed nominal output and disposable income. The nominal interest rate is measured by the three month Treasury bill rate expressed as a period average, while the nominal exchange rate is quoted as an end of period value. National accounts data for Canada was retrieved from the CANSIM database maintained by Statistics Canada, national accounts data for the United States was obtained from the FRED database maintained by the Federal Reserve Bank of Saint Louis, and other data was extracted from the IFS database maintained by the International Monetary Fund.

Appendix B. Tables and Figures

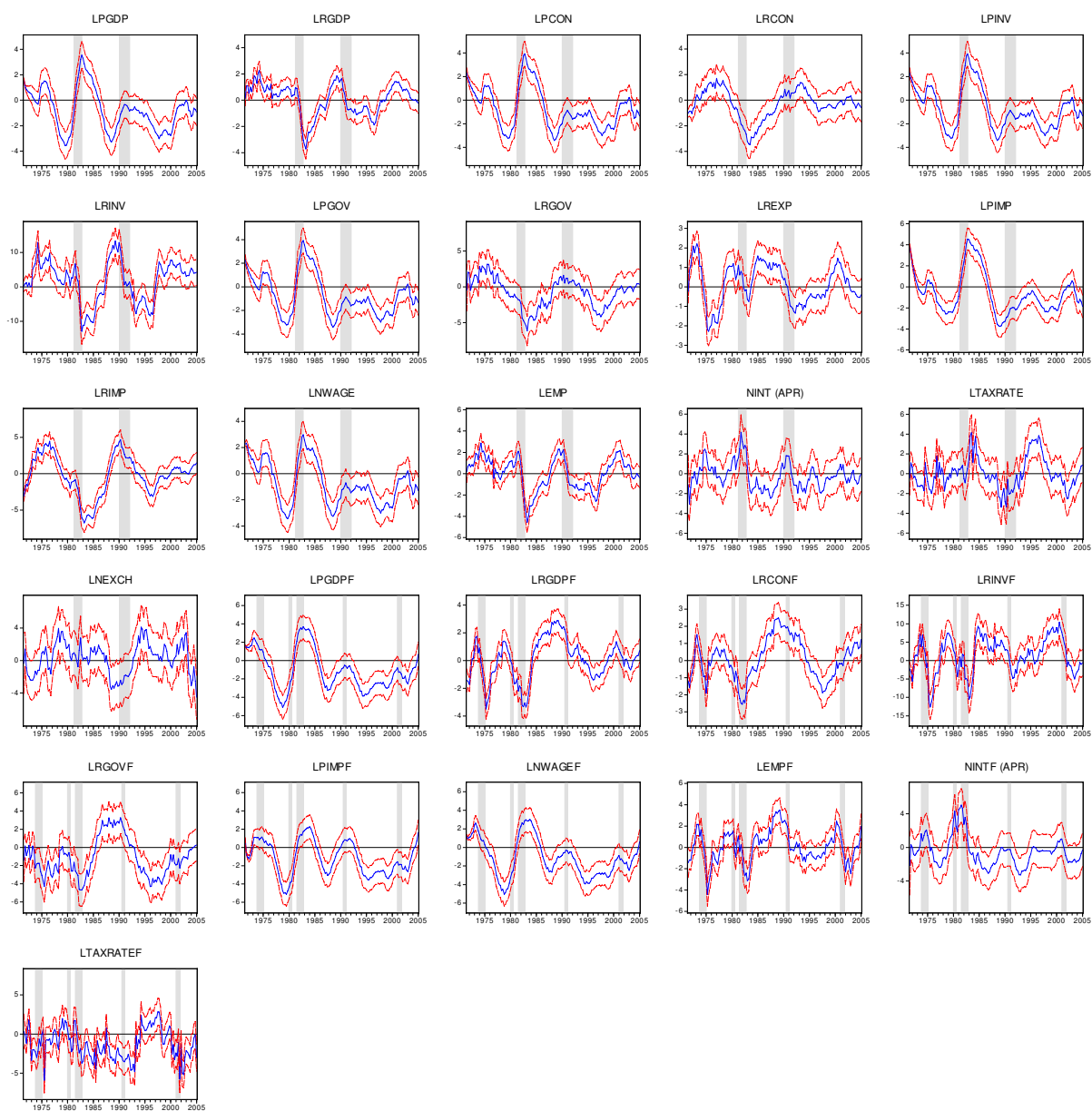
Table 4. Bayesian full information maximum likelihood estimation results

Parameter	Prior Distribution		Posterior Distribution	
	Mean	Standard Error	Mode	Standard Error
α	0.950000	0.000950	0.949020	0.000925
χ	5.750000	0.005750	5.745100	0.005750
η	2.000000	0.002000	2.000100	0.002000
κ	0.100000	0.000100	0.099999	0.000100
ψ	1.500000	0.001500	1.500000	0.001500
σ	2.750000	0.002750	2.751200	0.002750
ϑ	0.750000	0.000750	0.750000	0.000750
γ^Y	0.750000	0.000750	0.749910	0.000750
γ^M	0.750000	0.000750	0.750060	0.000750
γ^L	0.750000	0.000750	0.750020	0.000750
ω^Y	0.875000	0.000875	0.874010	0.000872
ω^M	0.875000	0.000875	0.874270	0.000874
ω^L	0.875000	0.000875	0.874950	0.000874
ξ^π	1.500000	0.001500	1.499600	0.001500
ξ^Y	0.125000	0.000125	0.124950	0.000125
ζ^G	-0.100000	0.000100	-0.099998	0.000100
ζ^τ	1.000000	0.001000	0.999040	0.001000
ρ_{v^c}	0.850000	0.000850	0.849250	0.000850
ρ_Λ	0.850000	0.000850	0.850500	0.000849
ρ_{v^f}	0.850000	0.000850	0.851580	0.000849
ρ_{v^M}	0.850000	0.000850	0.850060	0.000850
ρ_{θ^Y}	0.850000	0.000850	0.850060	0.000850
ρ_{θ^M}	0.850000	0.000850	0.850020	0.000850
ρ_{θ^L}	0.850000	0.000850	0.850030	0.000850
ρ_{v^i}	0.850000	0.000850	0.852660	0.000844
ρ_{v^G}	0.850000	0.000850	0.850300	0.000849
ρ_{v^f}	0.850000	0.000850	0.850150	0.000849
$\sigma_{v^c}^2$	—	∞	0.203690	0.058358
σ_Λ^2	—	∞	0.056415	0.013065
$\sigma_{v^f}^2$	—	∞	0.306120	0.052463
$\sigma_{v^M}^2$	—	∞	0.110820	0.014633
$\sigma_{\theta^Y}^2$	—	∞	0.285680	1.557700
$\sigma_{\theta^M}^2$	—	∞	0.253650	2.182200
$\sigma_{\theta^L}^2$	—	∞	0.290040	3.829100
$\sigma_{v^i}^2$	—	∞	0.015802	0.002923
$\sigma_{v^G}^2$	—	∞	0.441830	0.062389
$\sigma_{v^f}^2$	—	∞	0.242930	0.030785
$\sigma_{v^c, f}^2$	—	∞	0.109320	0.028552
$\sigma_{\Lambda^f}^2$	—	∞	0.102130	0.016336
$\sigma_{v^i, f}^2$	—	∞	0.226170	0.035216
$\sigma_{v^M, f}^2$	—	∞	0.193580	0.502700
$\sigma_{\theta^Y, f}^2$	—	∞	0.243750	1.049700
$\sigma_{\theta^M, f}^2$	—	∞	0.243030	13.116000
$\sigma_{\theta^L, f}^2$	—	∞	0.220930	2.595100
$\sigma_{v^i, f}^2$	—	∞	0.073864	0.009300
$\sigma_{v^G, f}^2$	—	∞	0.239540	0.030234
$\sigma_{v^f, f}^2$	—	∞	0.394560	0.038424

Parameter	Prior Distribution		Posterior Distribution	
	Mean	Standard Error	Mode	Standard Error
$\sigma_{P^i}^2$	—	∞	0.622800	0.078904
σ_Y^2	—	∞	0.062492	0.008983
$\sigma_{P^c}^2$	—	∞	0.577490	0.073005
σ_C^2	—	∞	0.096753	0.011677
$\sigma_{P^i}^2$	—	∞	0.708920	0.088374
σ_I^2	—	∞	0.449850	0.056239
$\sigma_{P^o}^2$	—	∞	0.712530	0.090229
σ_G^2	—	∞	0.123730	0.026314
$\sigma_{P^M}^2$	—	∞	0.362830	0.040564
σ_X^2	—	∞	1.460000	0.185300
σ_M^2	—	∞	0.533020	0.062373
σ_W^2	—	∞	0.783770	0.098795
σ_L^2	—	∞	0.114950	0.020643
σ_i^2	—	∞	0.002181	0.000388
σ_τ^2	—	∞	0.178260	0.027004
σ_ε^2	—	∞	0.448390	0.052147
$\sigma_{P^i,j}^2$	—	∞	0.303080	0.037457
$\sigma_{Y^j}^2$	—	∞	0.021367	0.002802
$\sigma_{C^j}^2$	—	∞	0.026166	0.002760
$\sigma_{I^j}^2$	—	∞	0.497880	0.055539
$\sigma_{G^j}^2$	—	∞	0.086916	0.015808
$\sigma_{P^M,j}^2$	—	∞	2.147700	0.258730
$\sigma_{W^j}^2$	—	∞	0.352380	0.041770
$\sigma_{L^j}^2$	—	∞	0.080368	0.014333
$\sigma_{i^j}^2$	—	∞	0.001500	0.000242
$\sigma_{\tau^j}^2$	—	∞	0.053588	0.011479
σ_π^2	—	∞	0.000136	0.000019
σ_g^2	—	∞	0.000004	0.000005
σ_n^2	—	∞	0.000025	0.000009
$\sigma_{\pi^j}^2$	—	∞	0.000068	0.000012
$\sigma_{g^j}^2$	—	∞	0.000022	0.000011
$\sigma_{n^j}^2$	—	∞	0.000029	0.000012

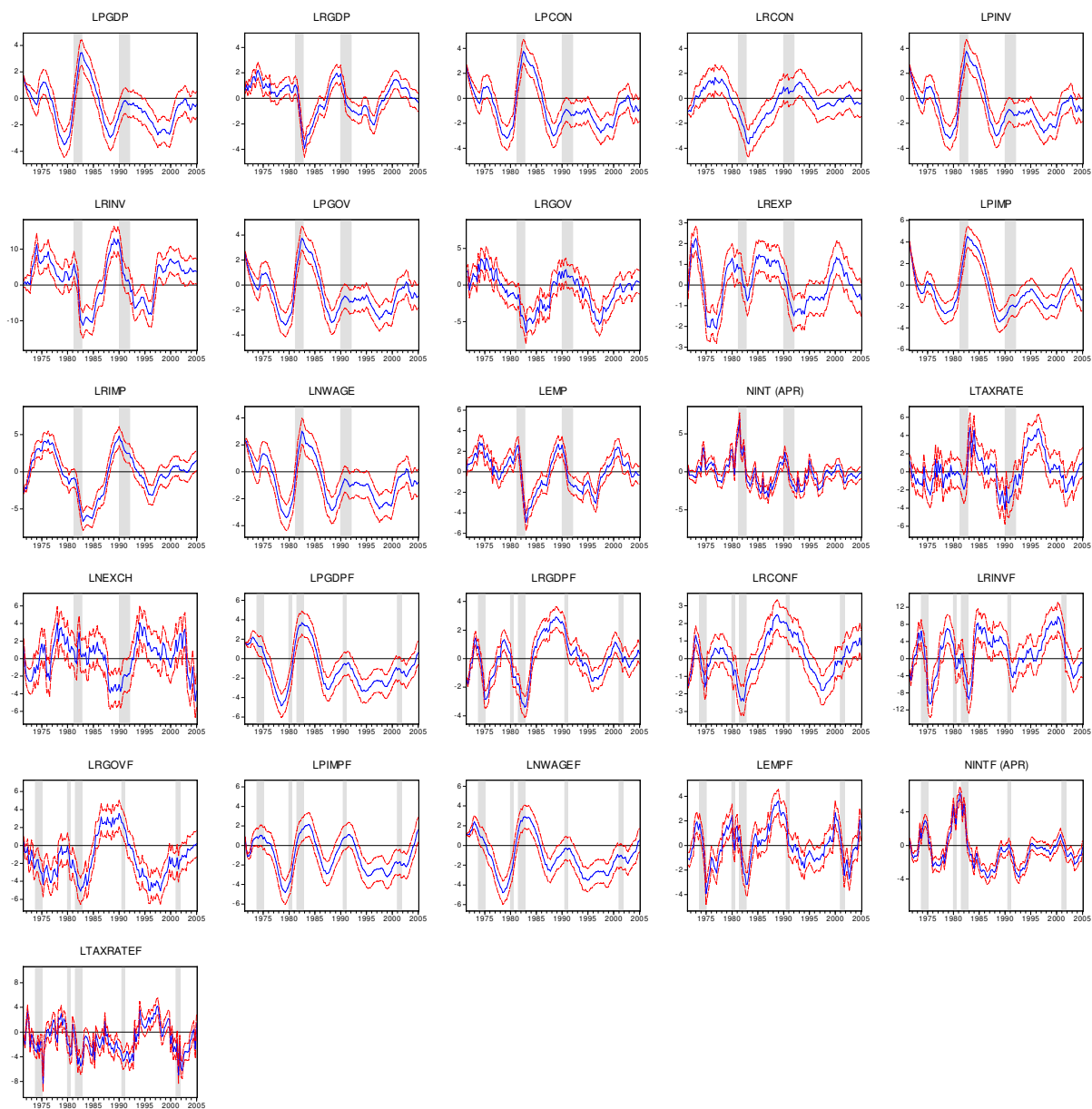
Note: All observed nonpredetermined endogenous variables are rescaled by a factor of 100.

Figure 3. Predicted cyclical components of observed nonpredetermined endogenous variables



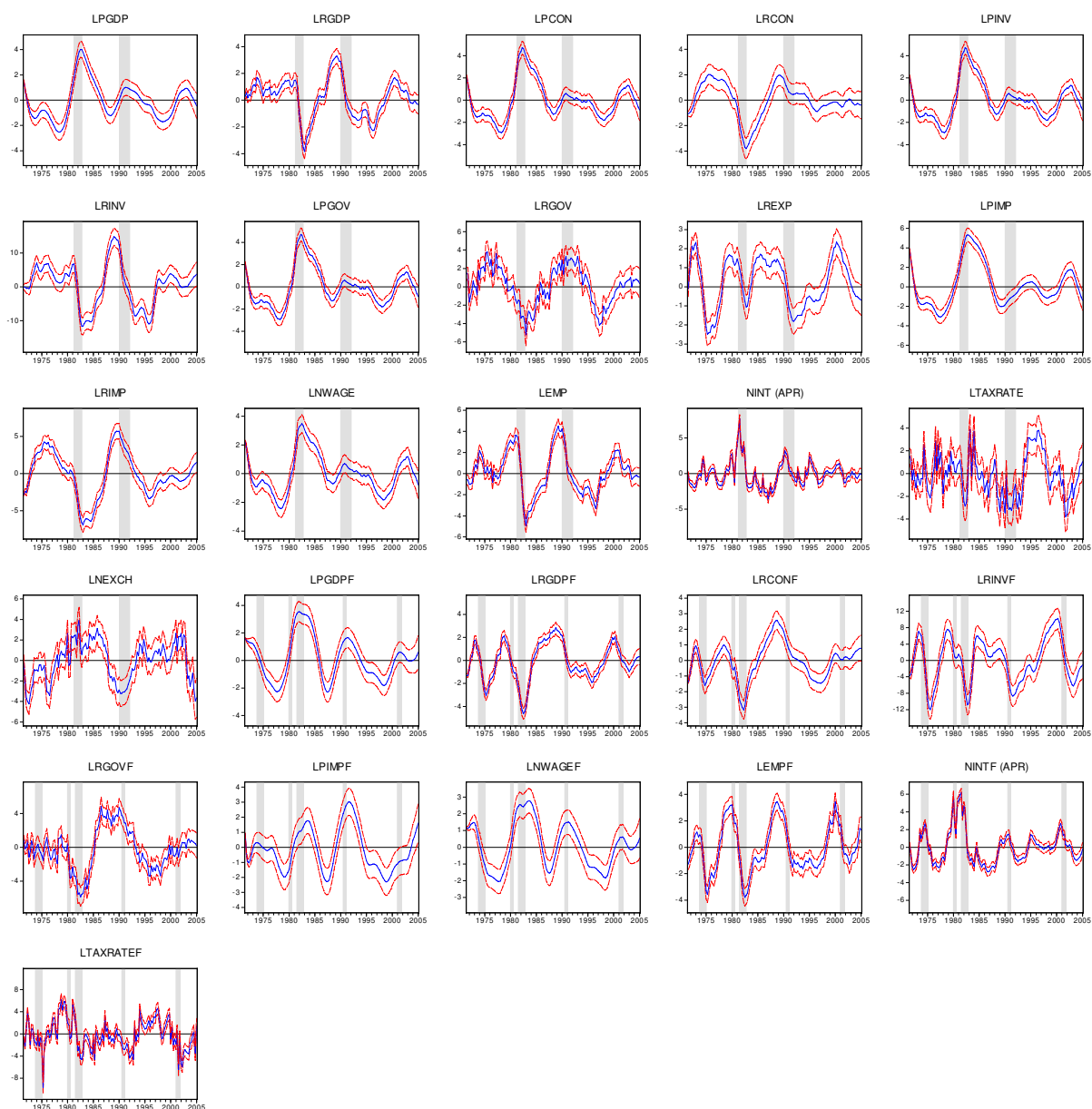
Note: Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 4. Filtered cyclical components of observed nonpredetermined endogenous variables



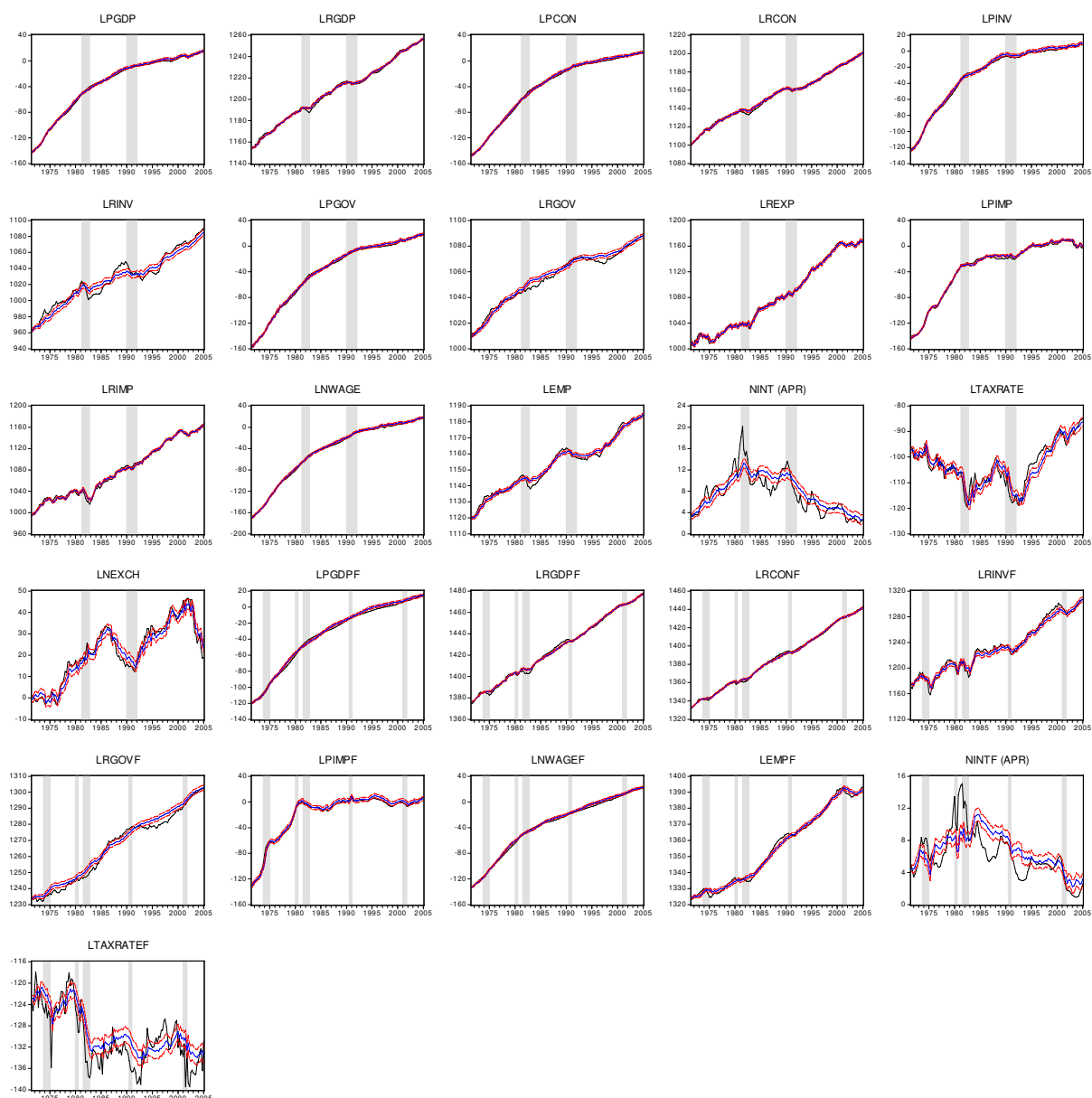
Note: Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 5. Smoothed cyclical components of observed nonpredetermined endogenous variables



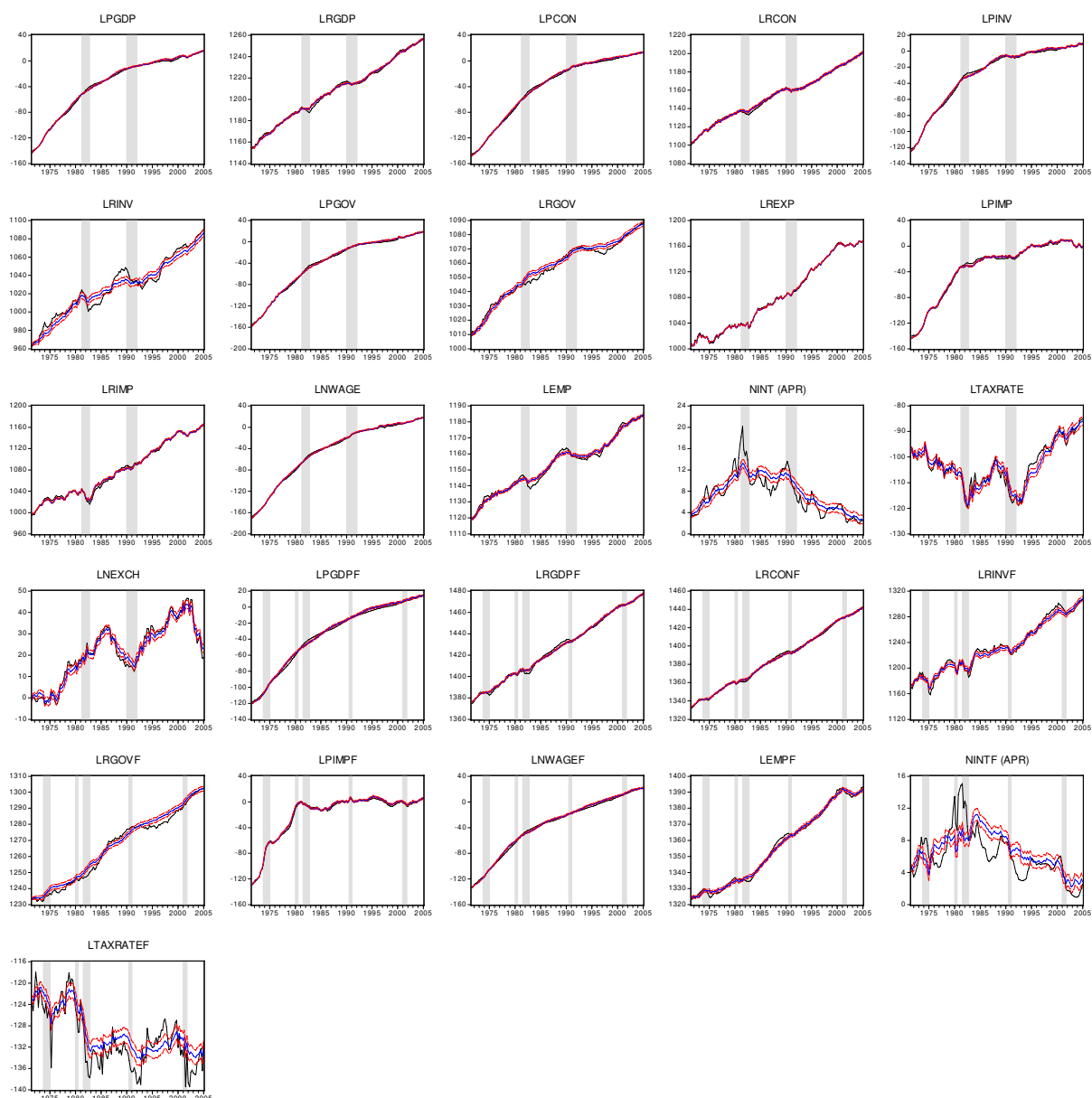
Note: Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 6. Predicted trend components of observed nonpredetermined endogenous variables



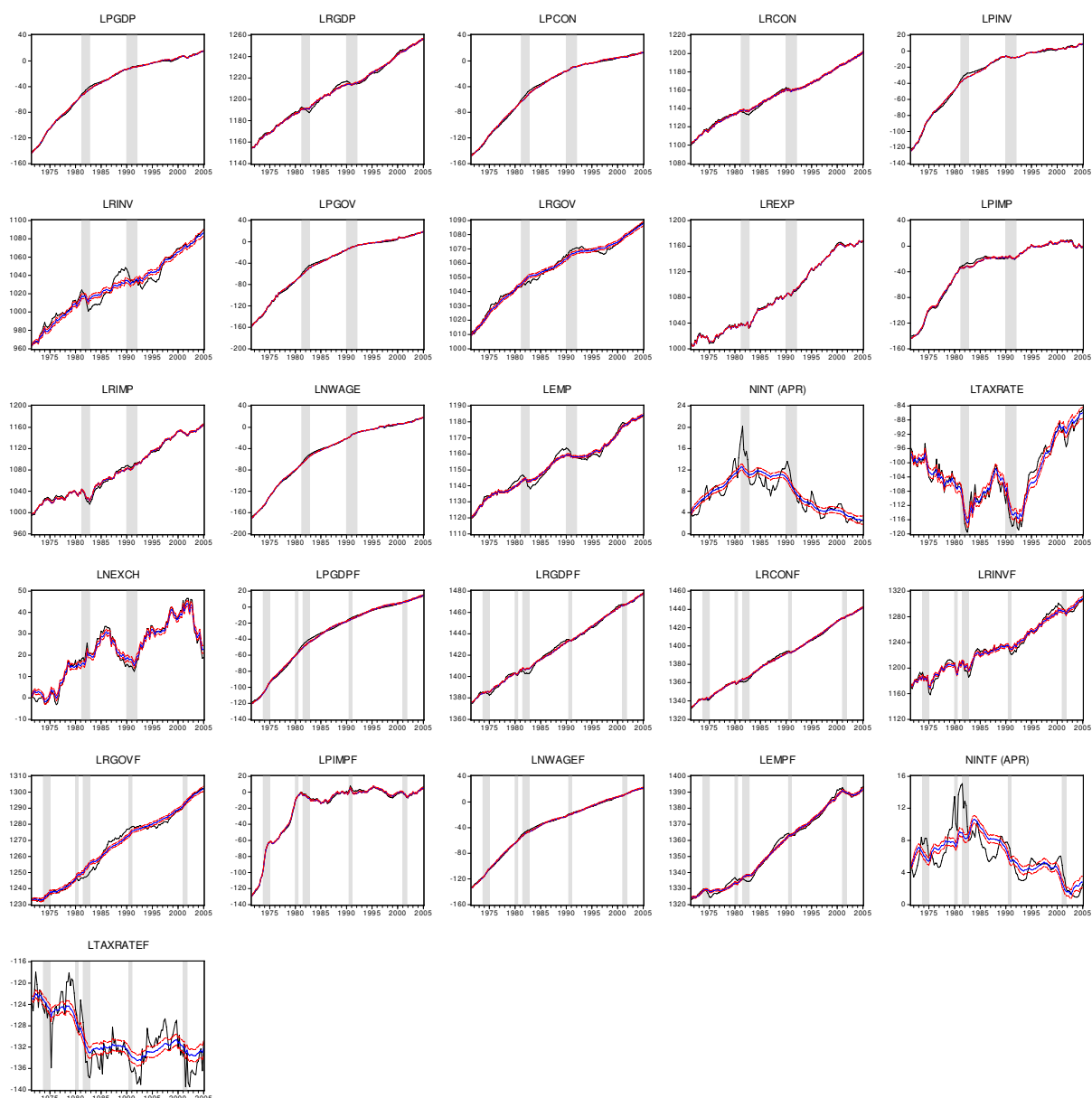
Note: Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 7. Filtered trend components of observed nonpredetermined endogenous variables



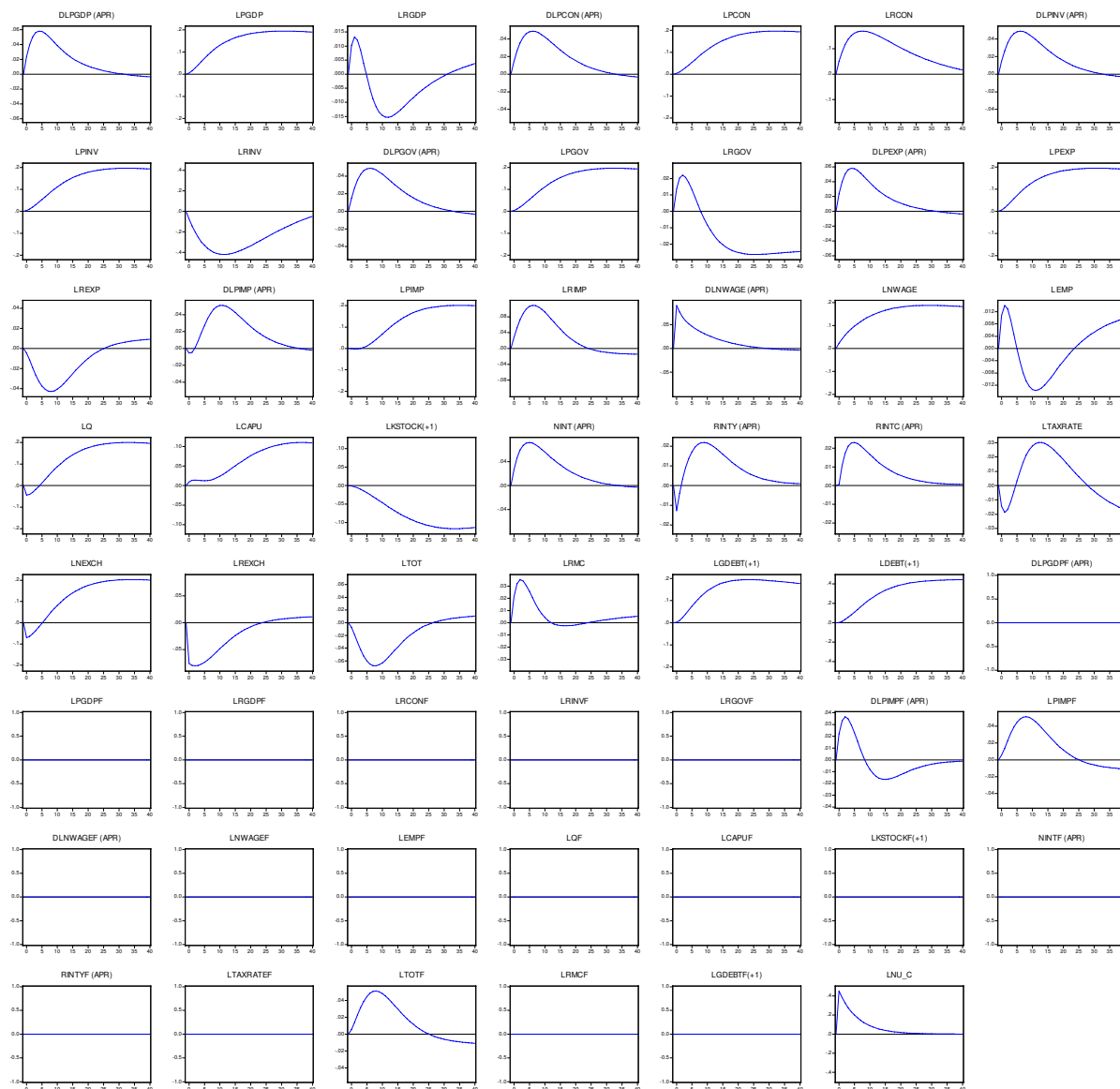
Note: Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 8. Smoothed trend components of observed nonpredetermined endogenous variables



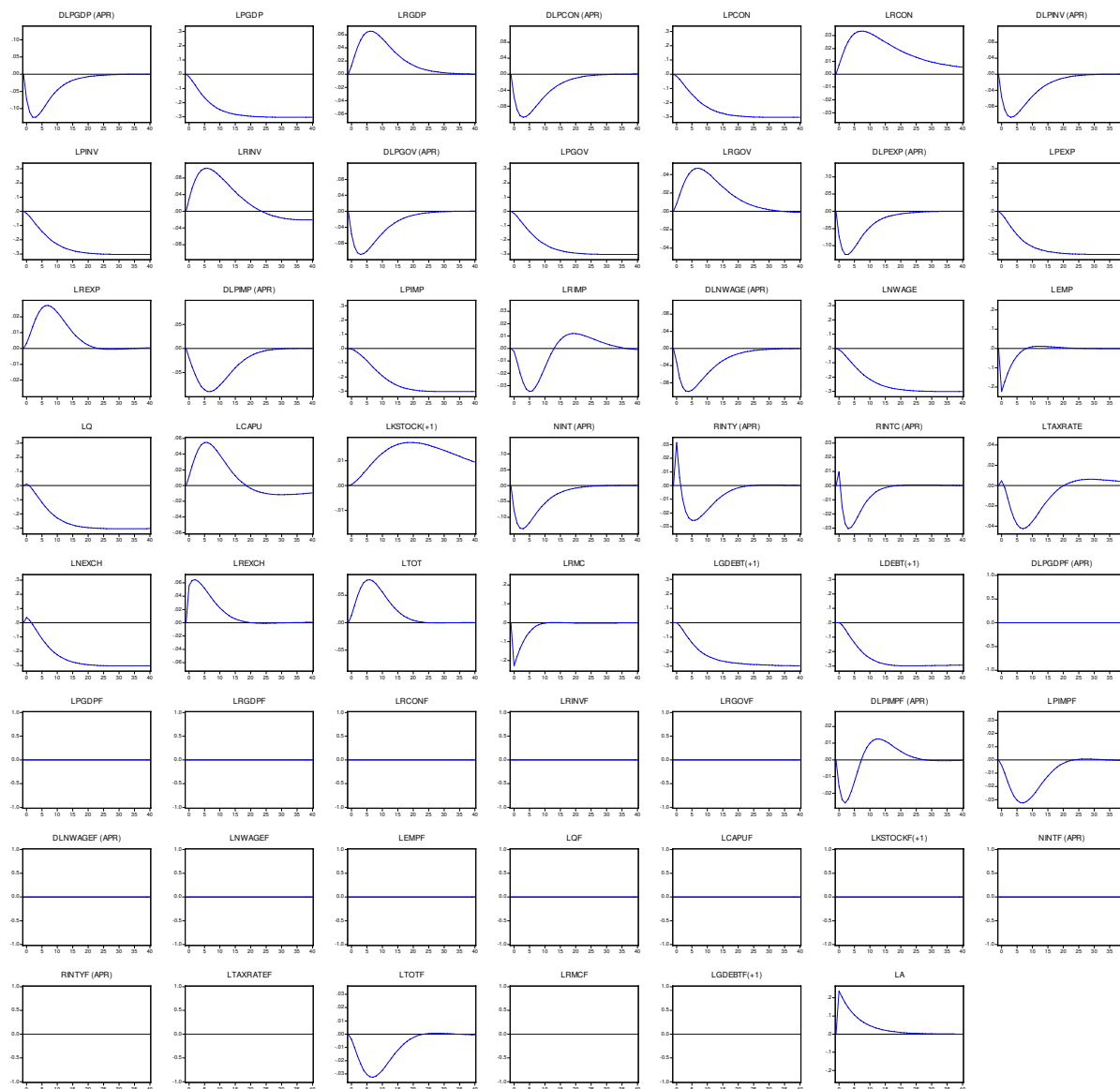
Note: Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the Economic Cycle Research Institute reference cycle.

Figure 9. Theoretical impulse responses to a domestic preference shock



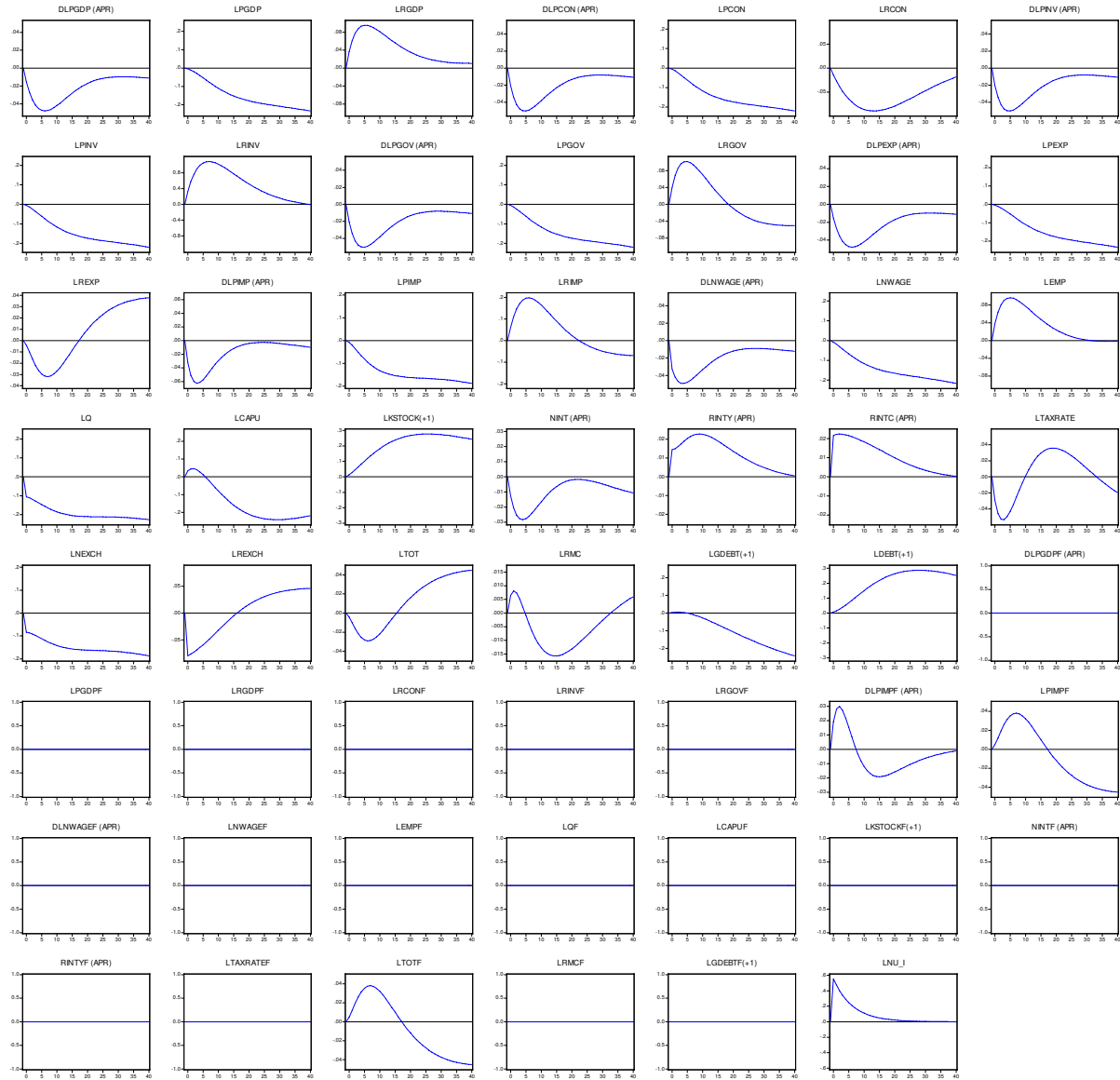
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 10. Theoretical impulse responses to a domestic output technology shock



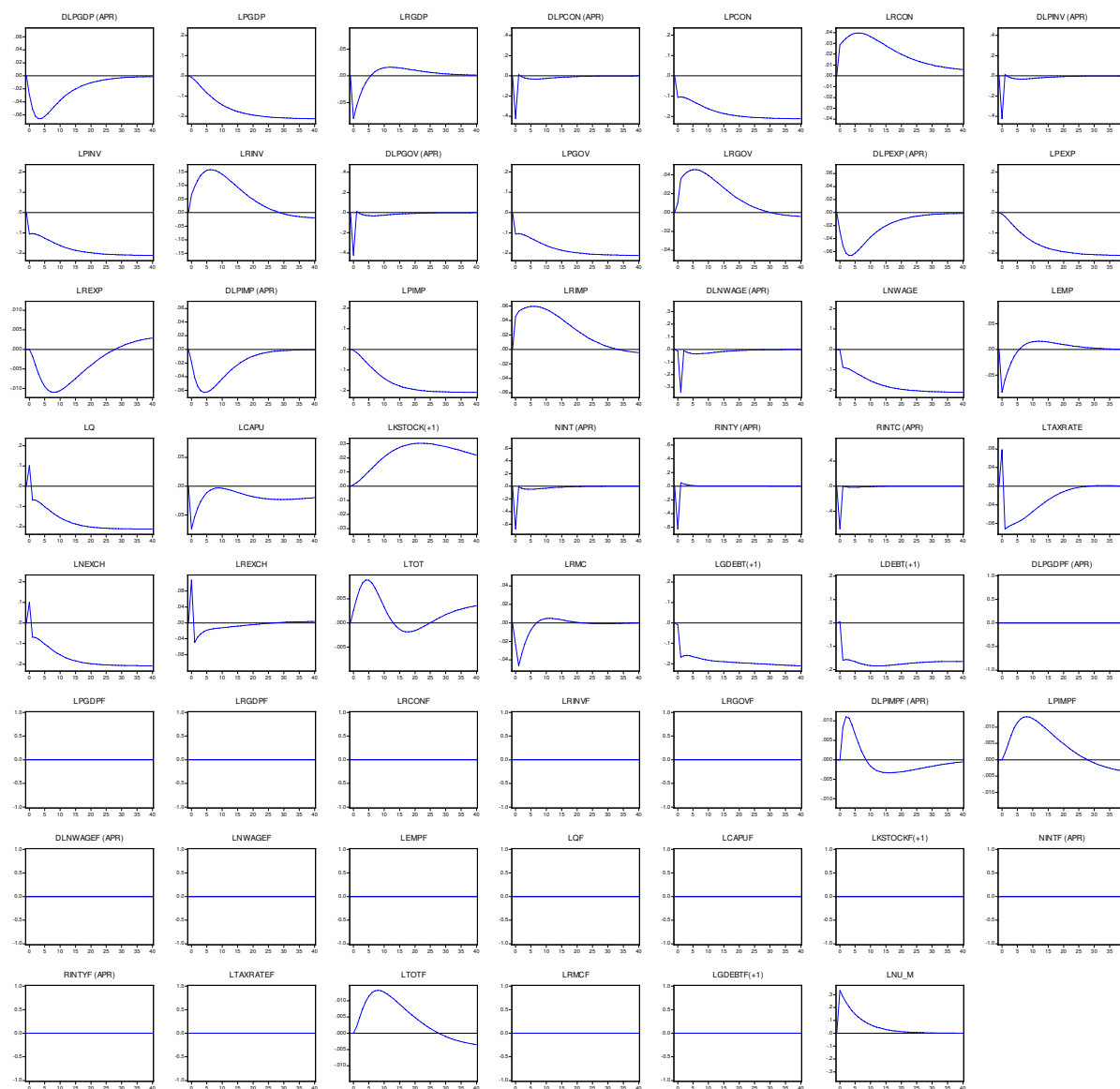
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 11. Theoretical impulse responses to a domestic investment technology shock



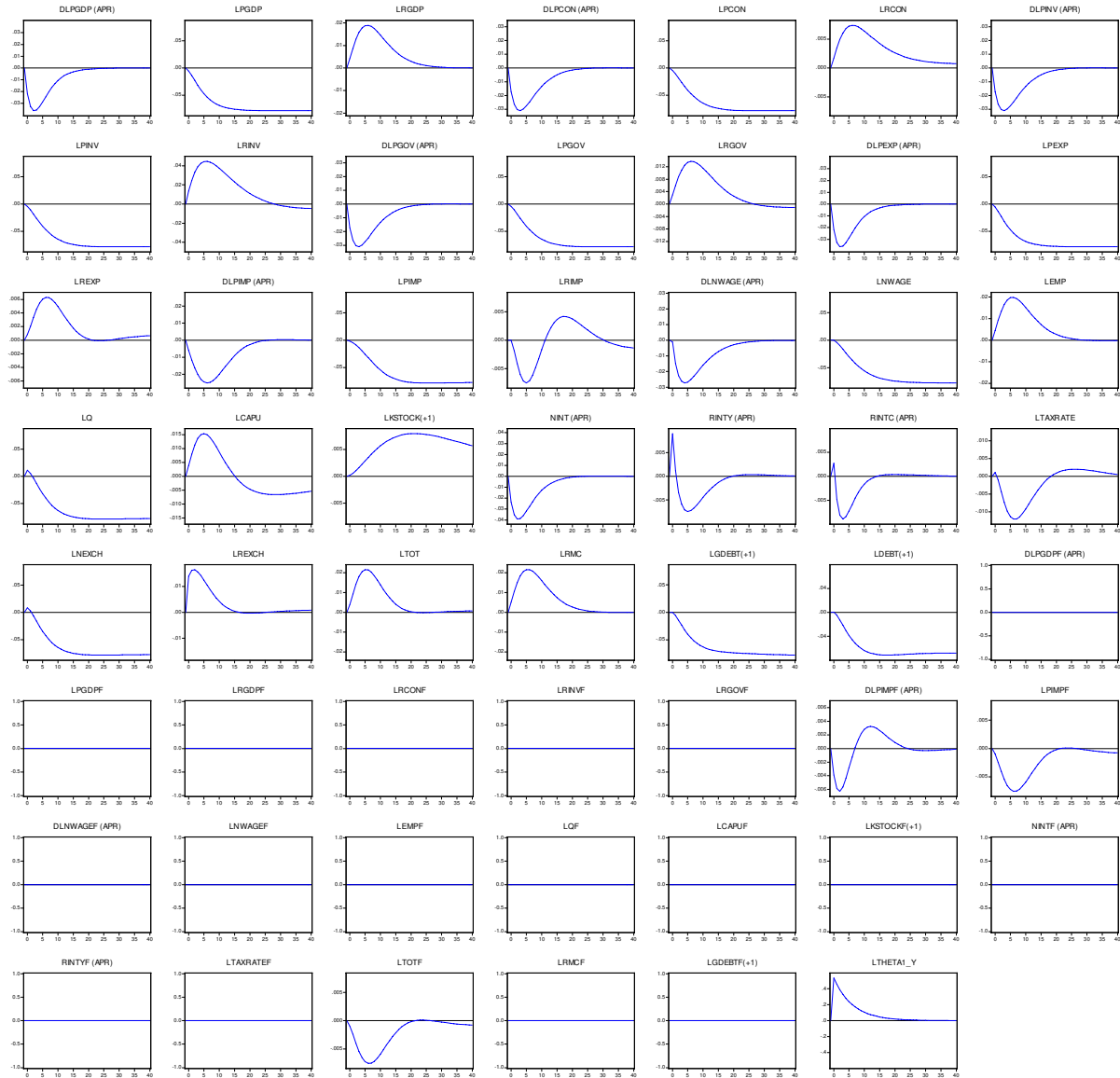
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 12. Theoretical impulse responses to a domestic import technology shock



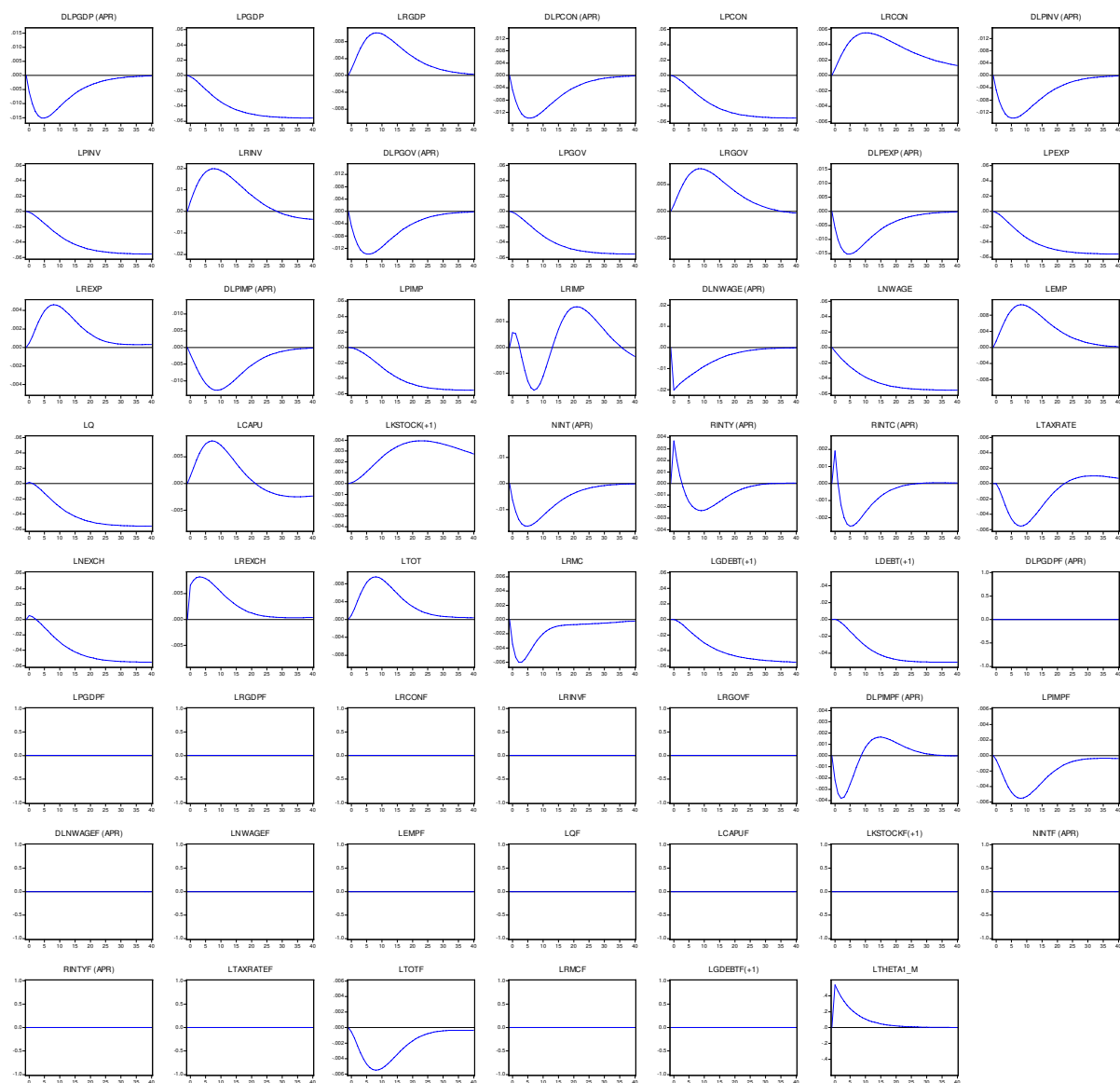
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 13. Theoretical impulse responses to a domestic output price markup shock



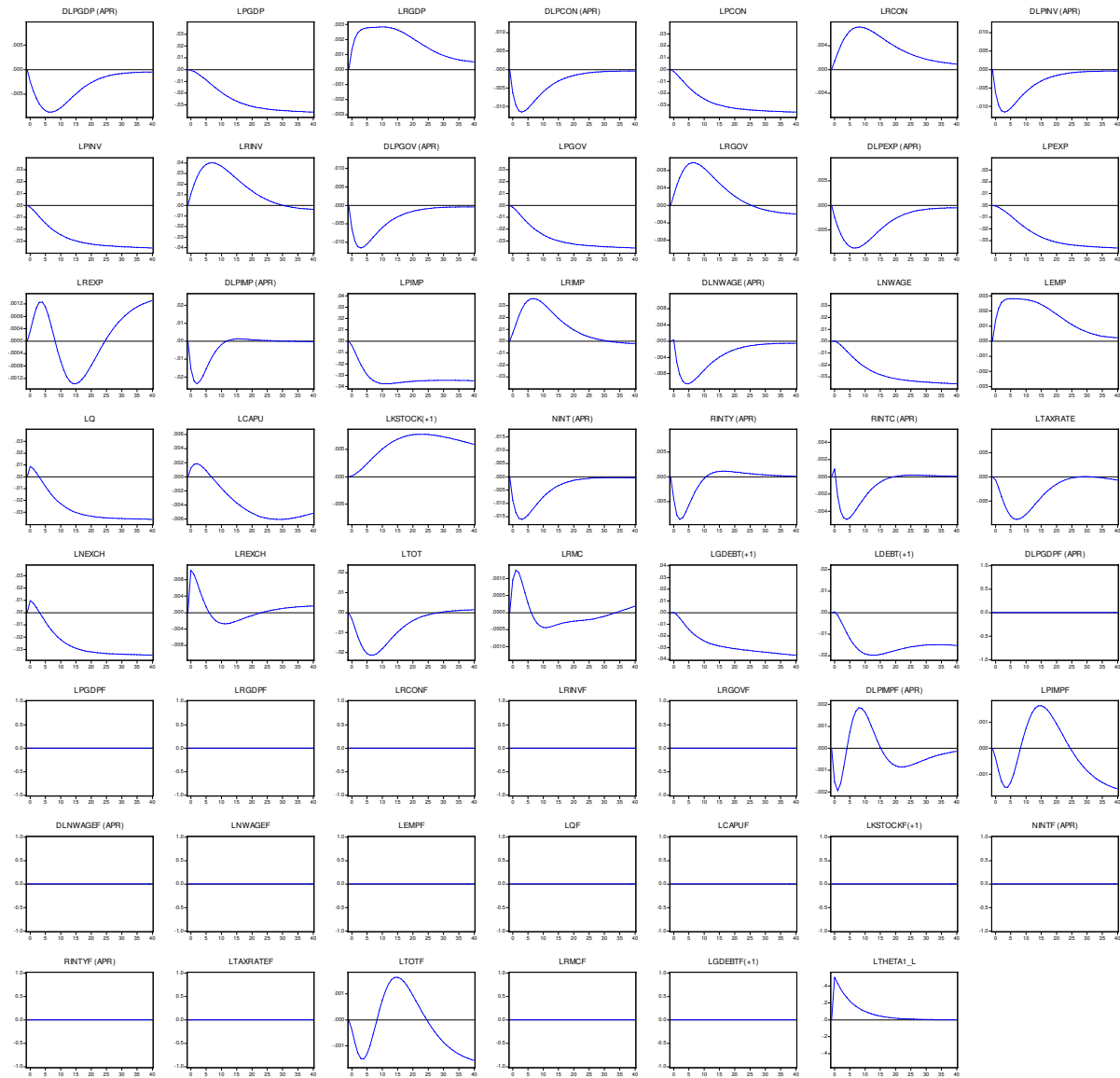
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 14. Theoretical impulse responses to a domestic import price markup shock



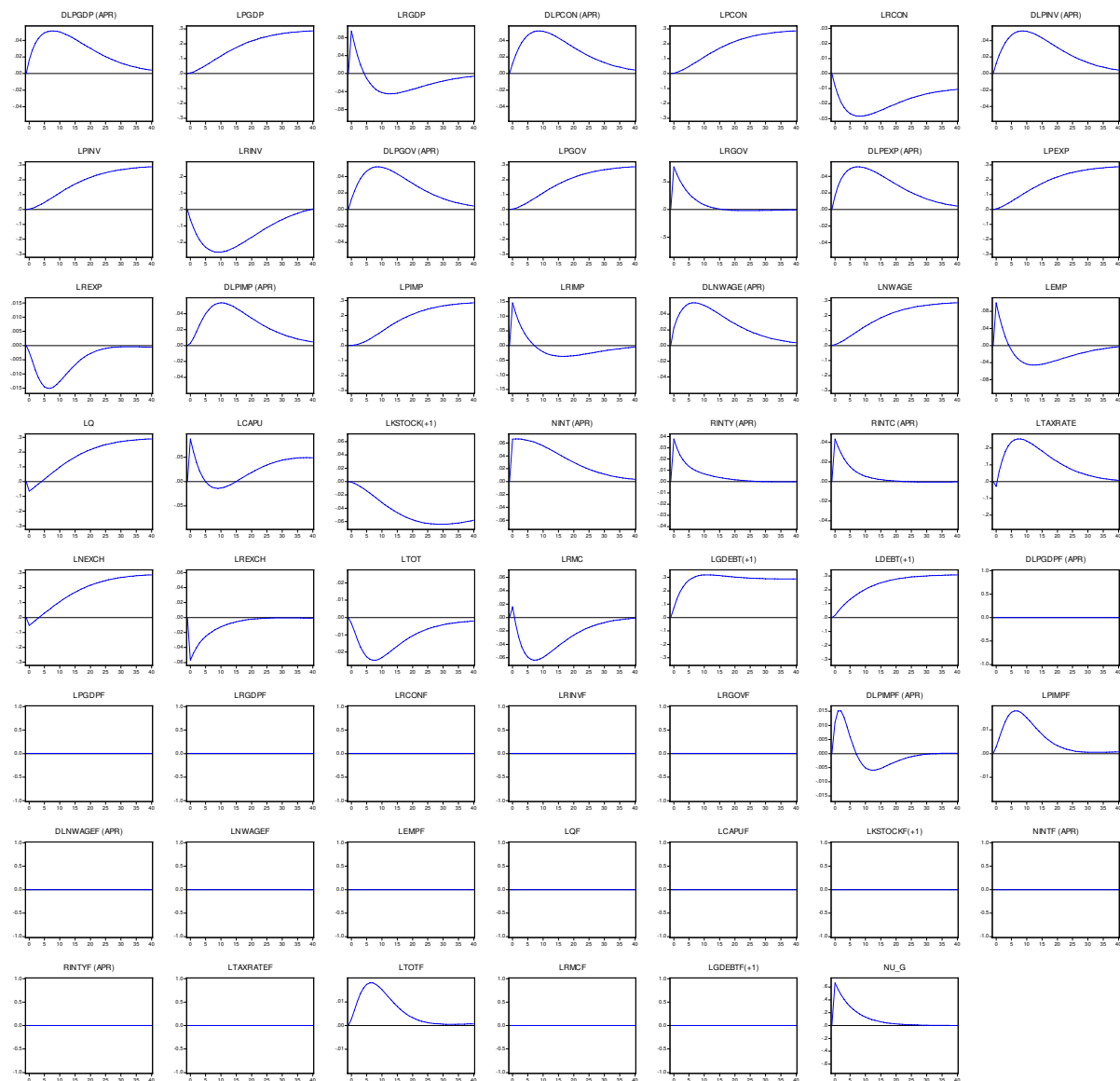
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 15. Theoretical impulse responses to a domestic wage markup shock



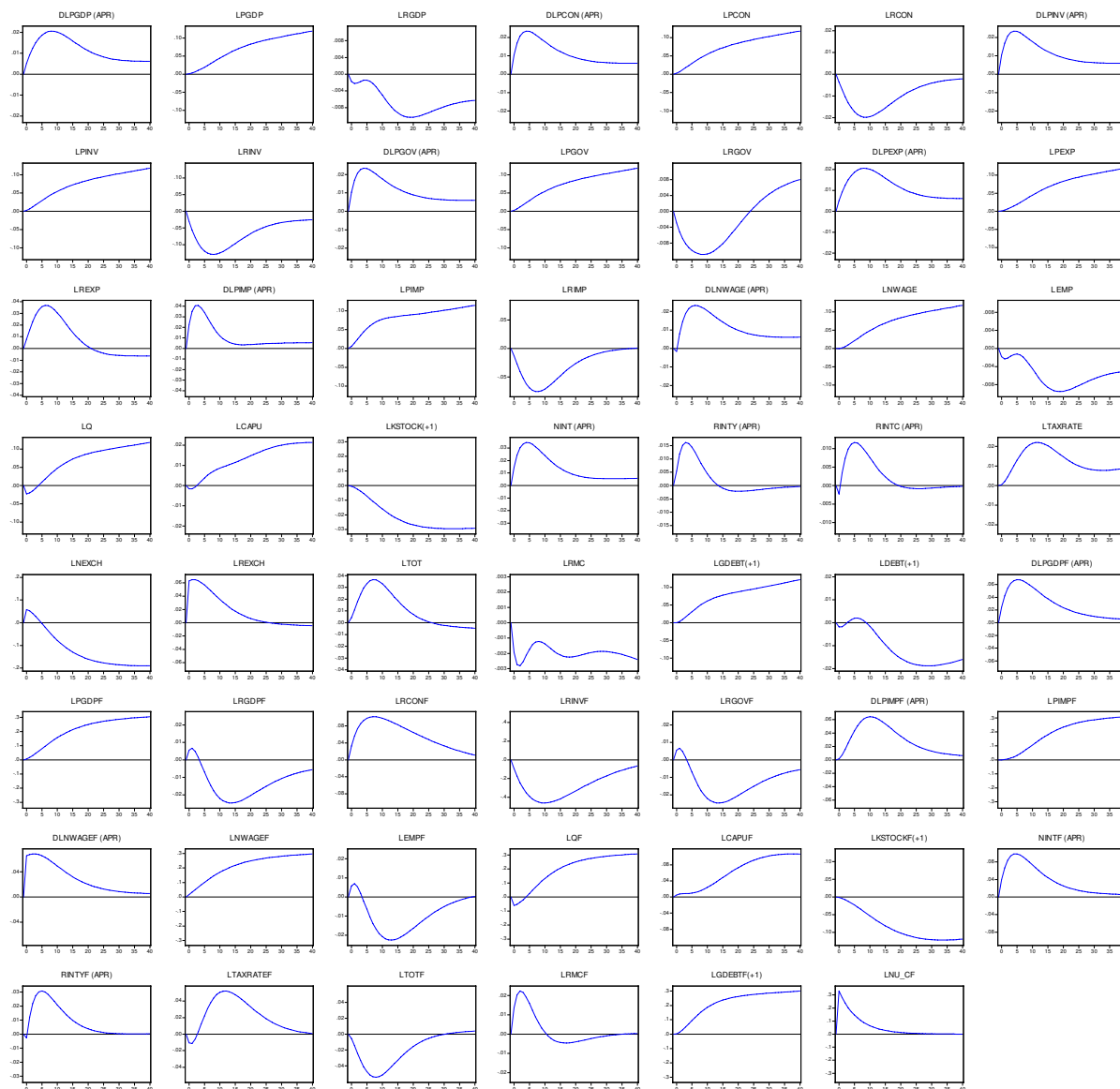
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 17. Theoretical impulse responses to a domestic fiscal expenditure shock



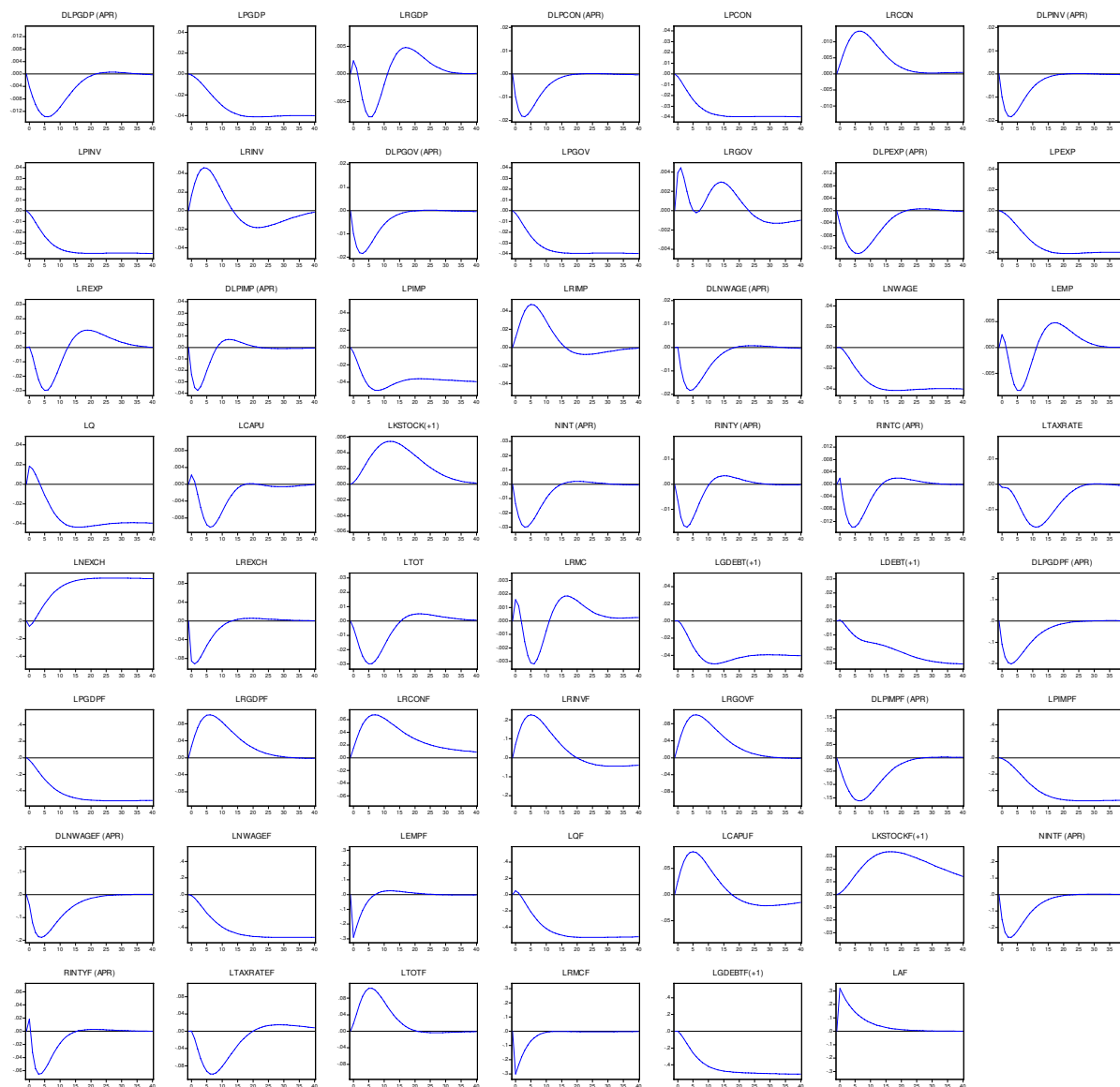
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 19. Theoretical impulse responses to a foreign preference shock



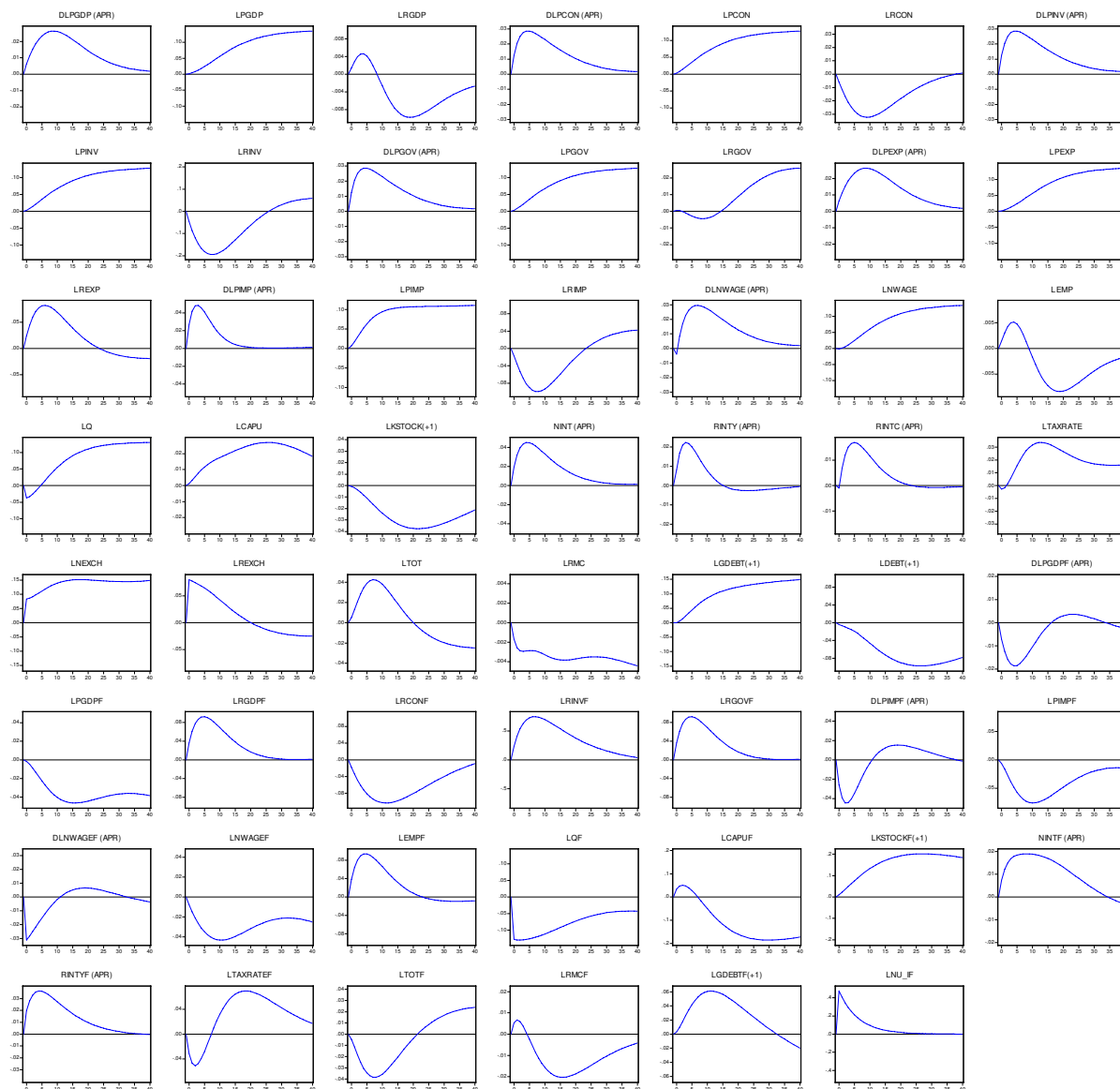
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 20. Theoretical impulse responses to a foreign output technology shock



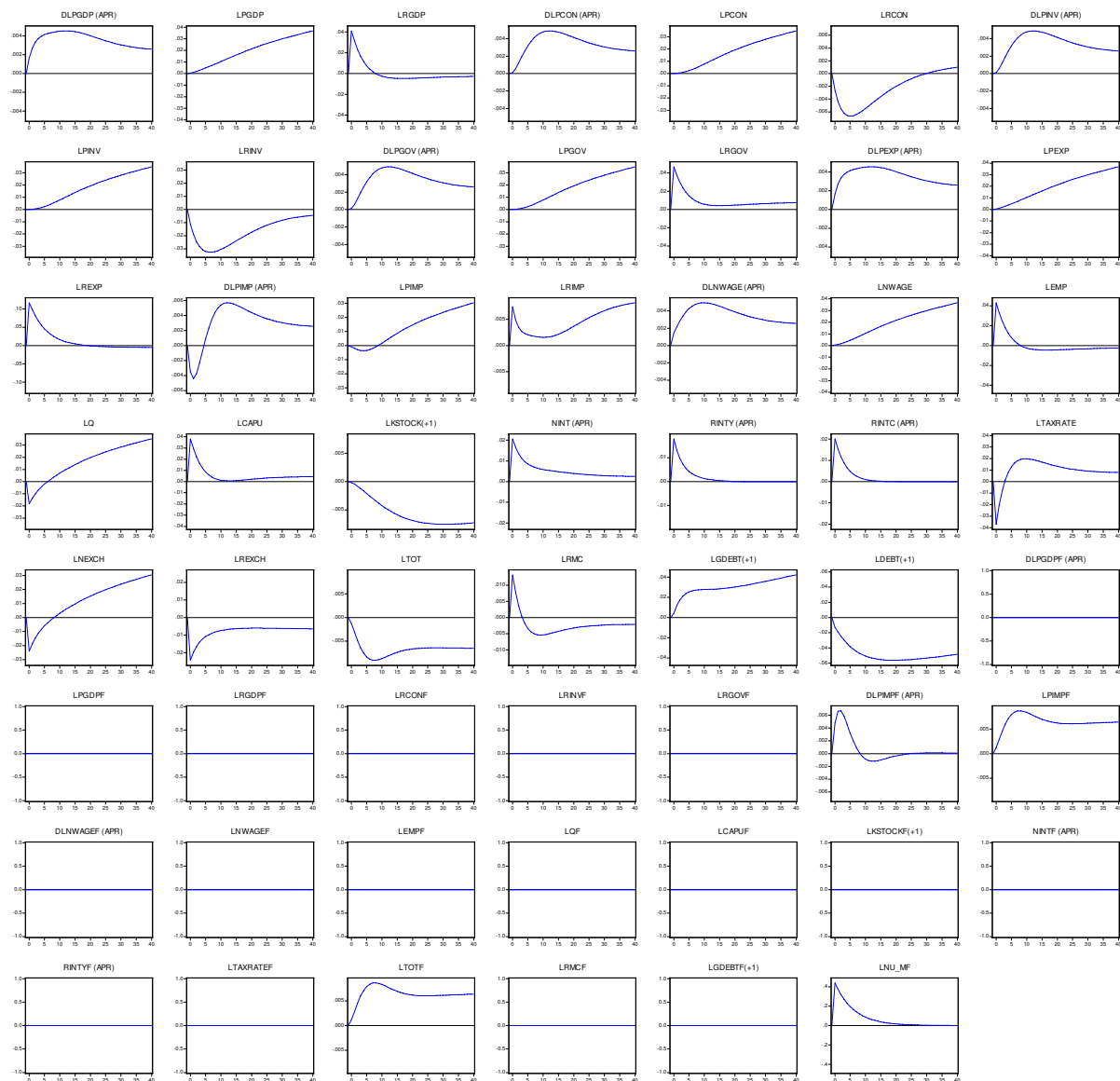
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 21. Theoretical impulse responses to a foreign investment technology shock



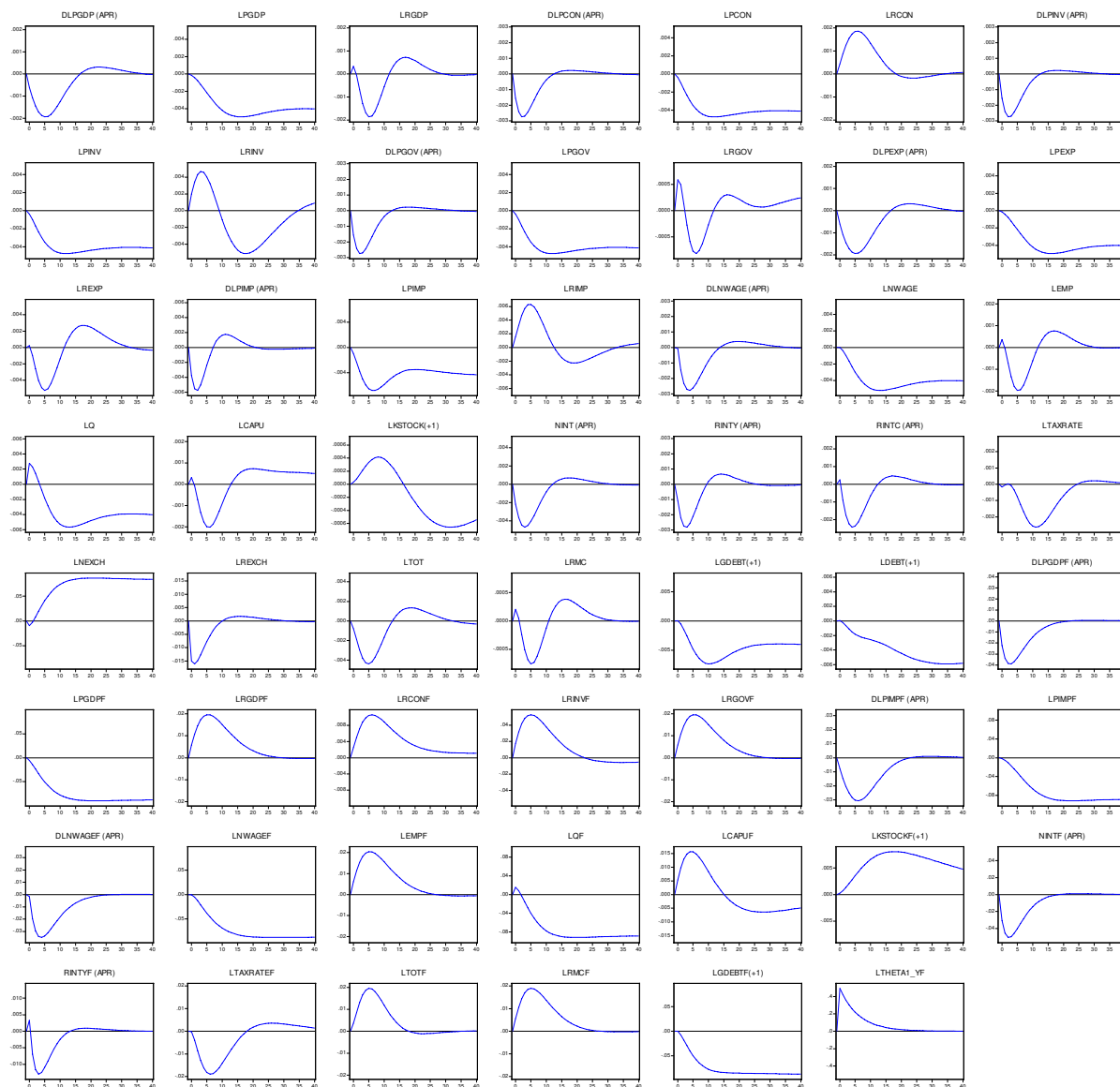
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 22. Theoretical impulse responses to a foreign import technology shock



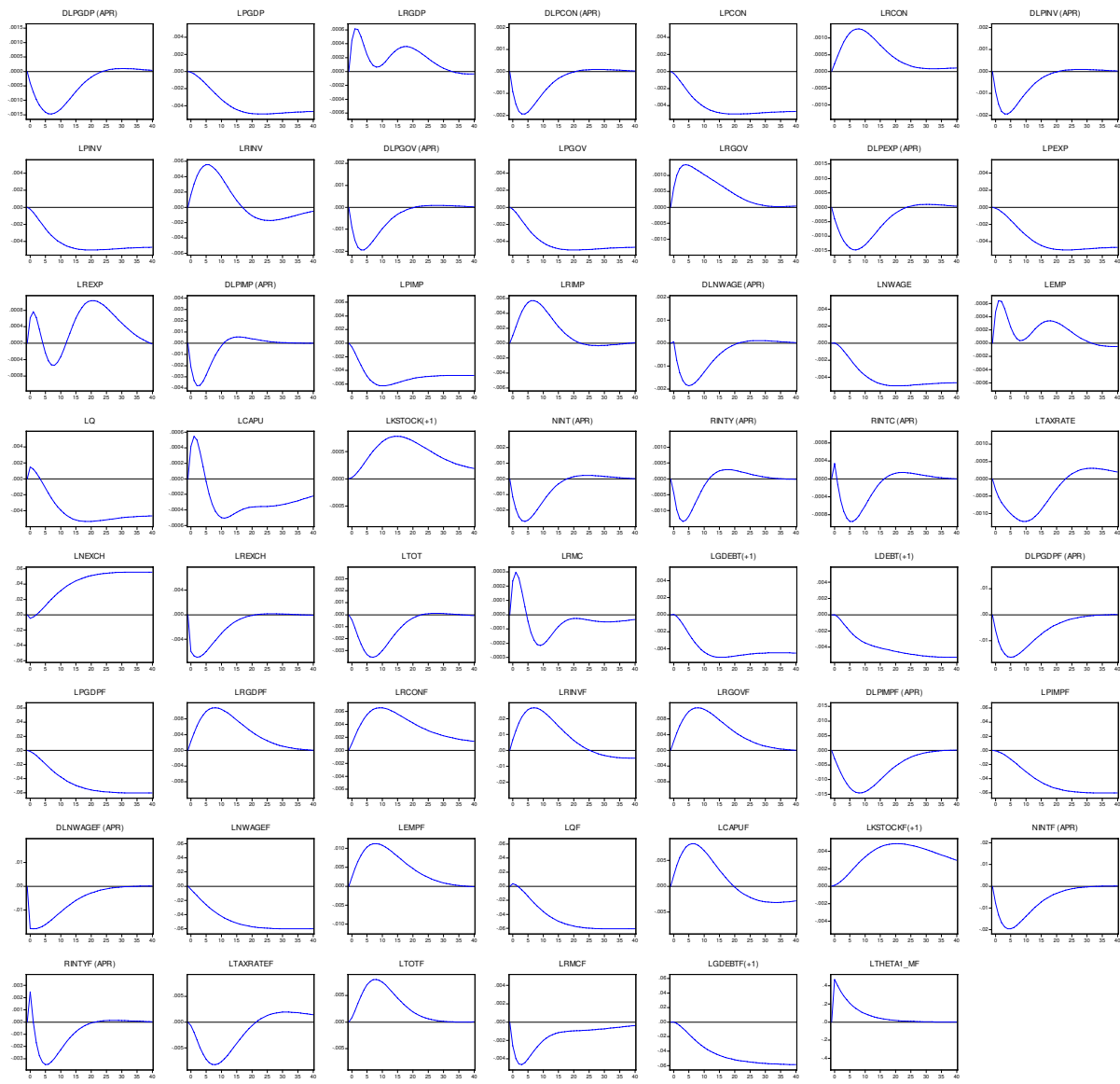
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 23. Theoretical impulse responses to a foreign output price markup shock



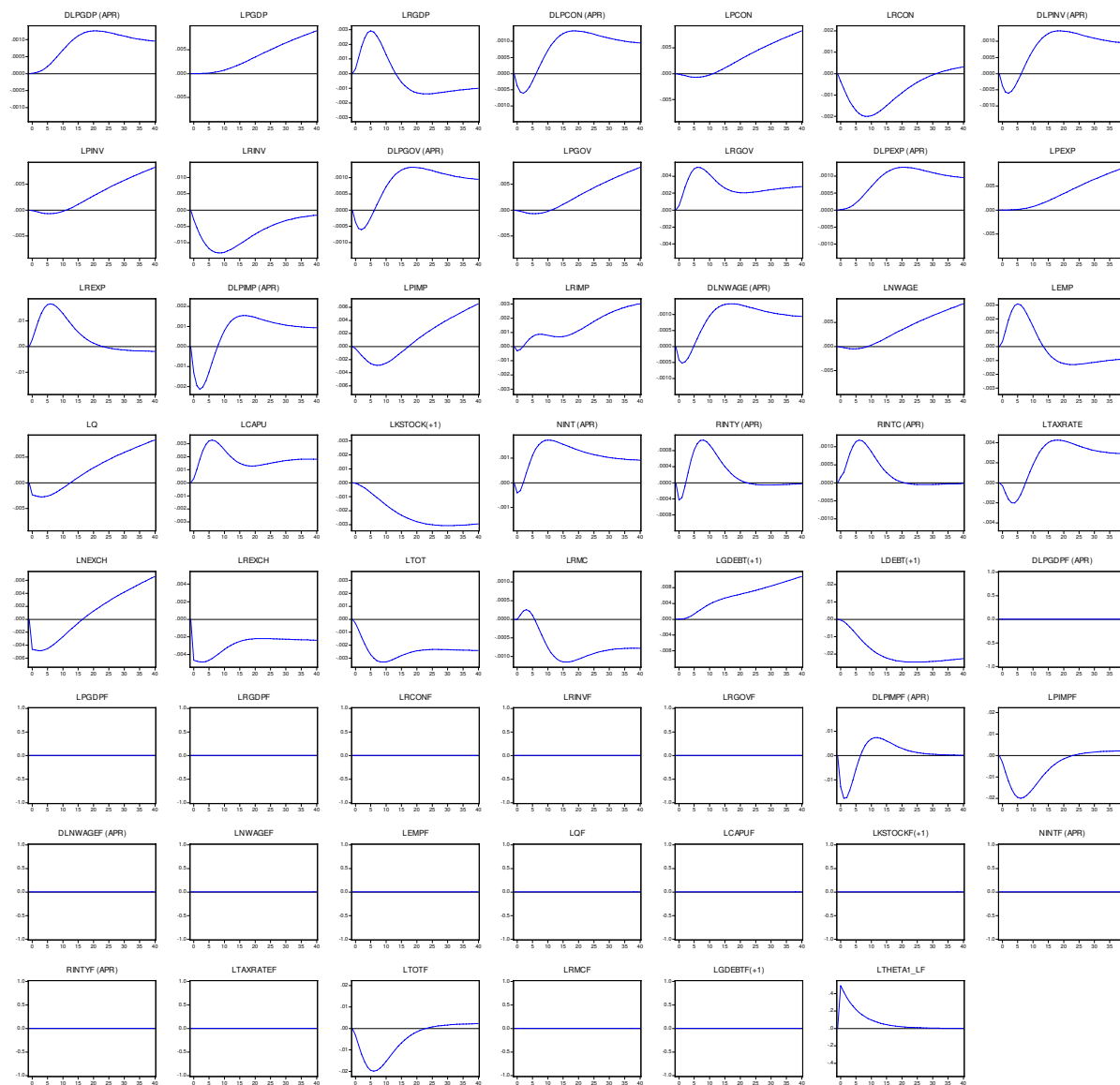
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 24. Theoretical impulse responses to a foreign import price markup shock



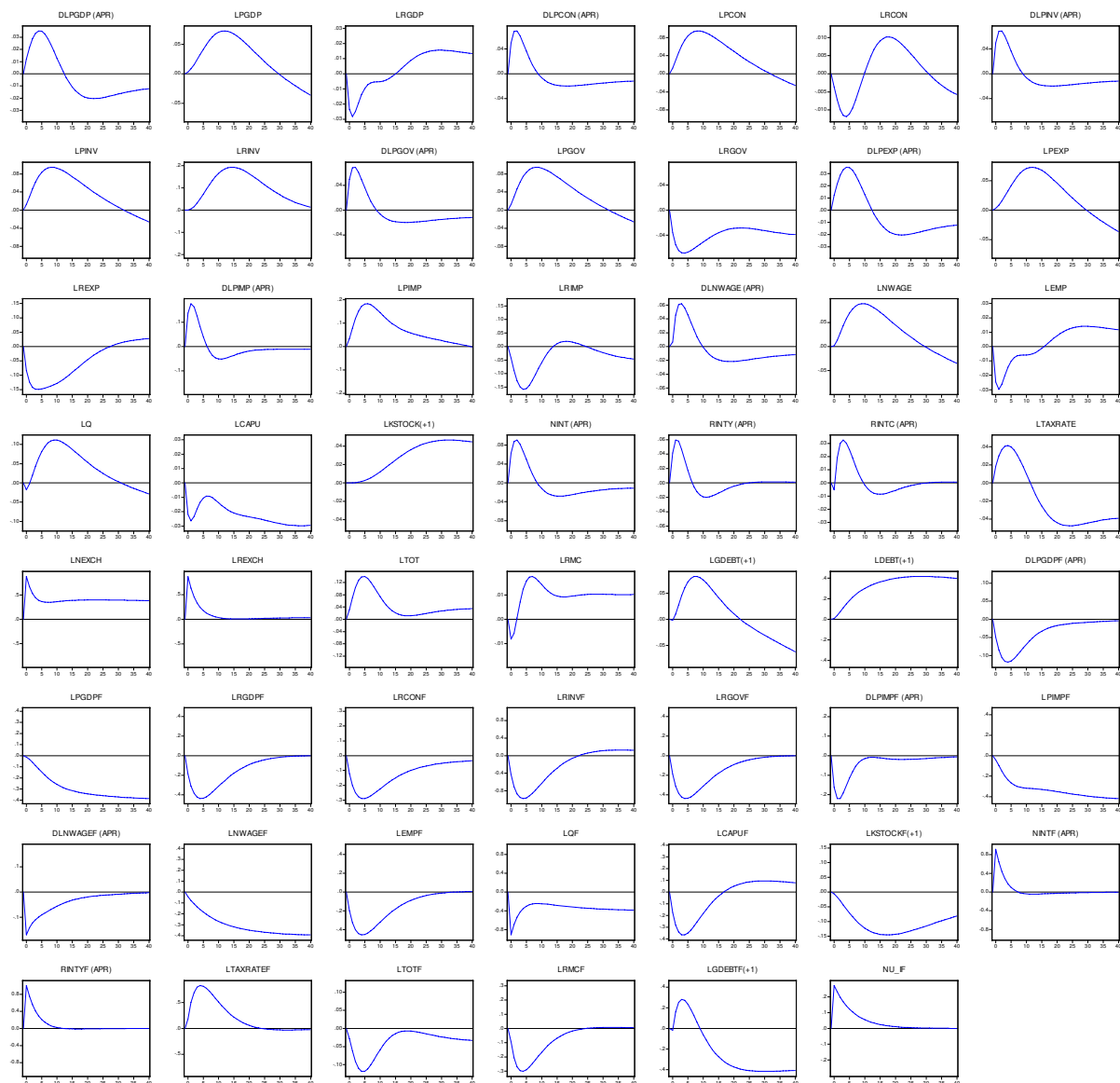
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 25. Theoretical impulse responses to a foreign wage markup shock



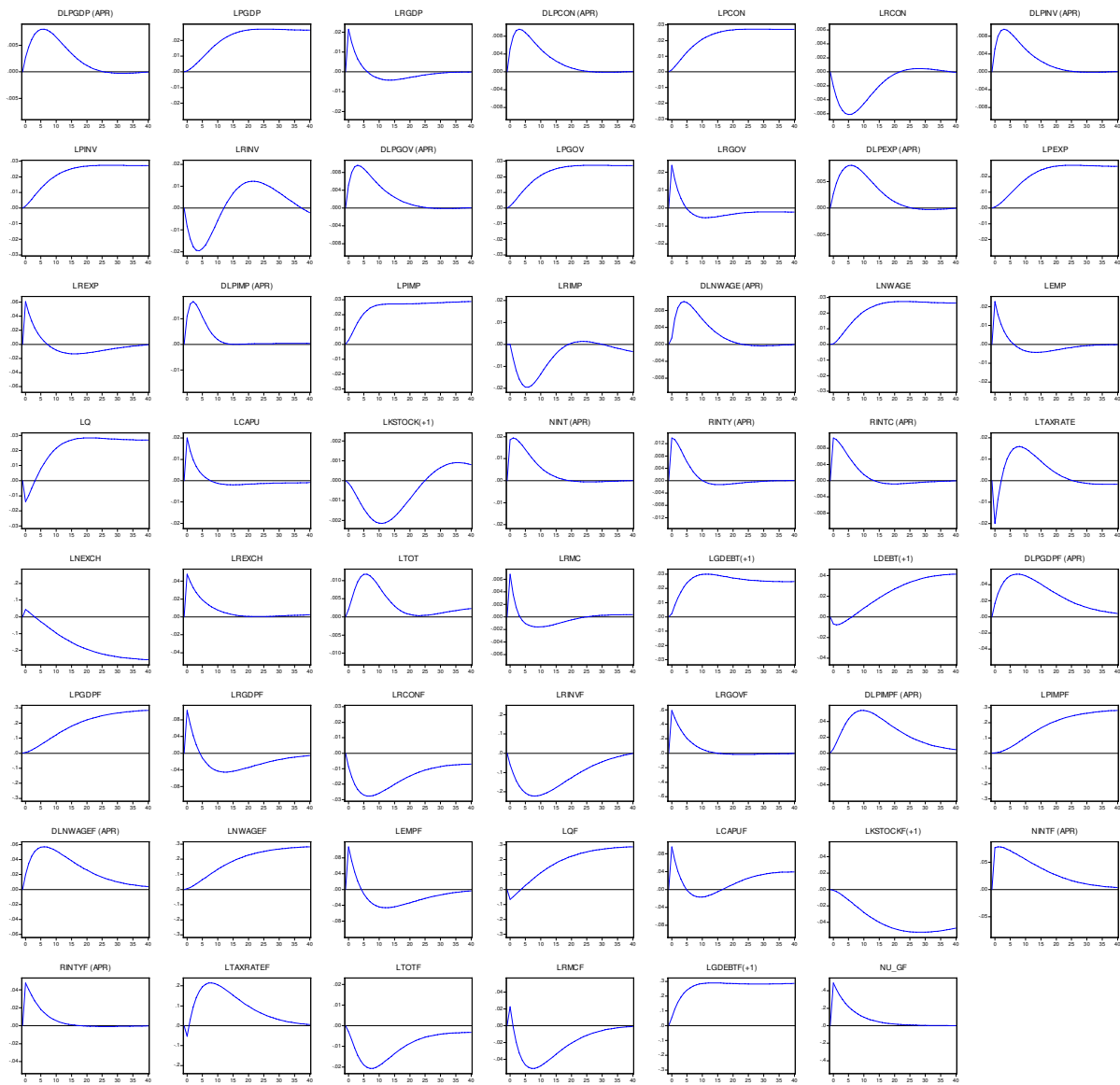
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 26. Theoretical impulse responses to a foreign monetary policy shock



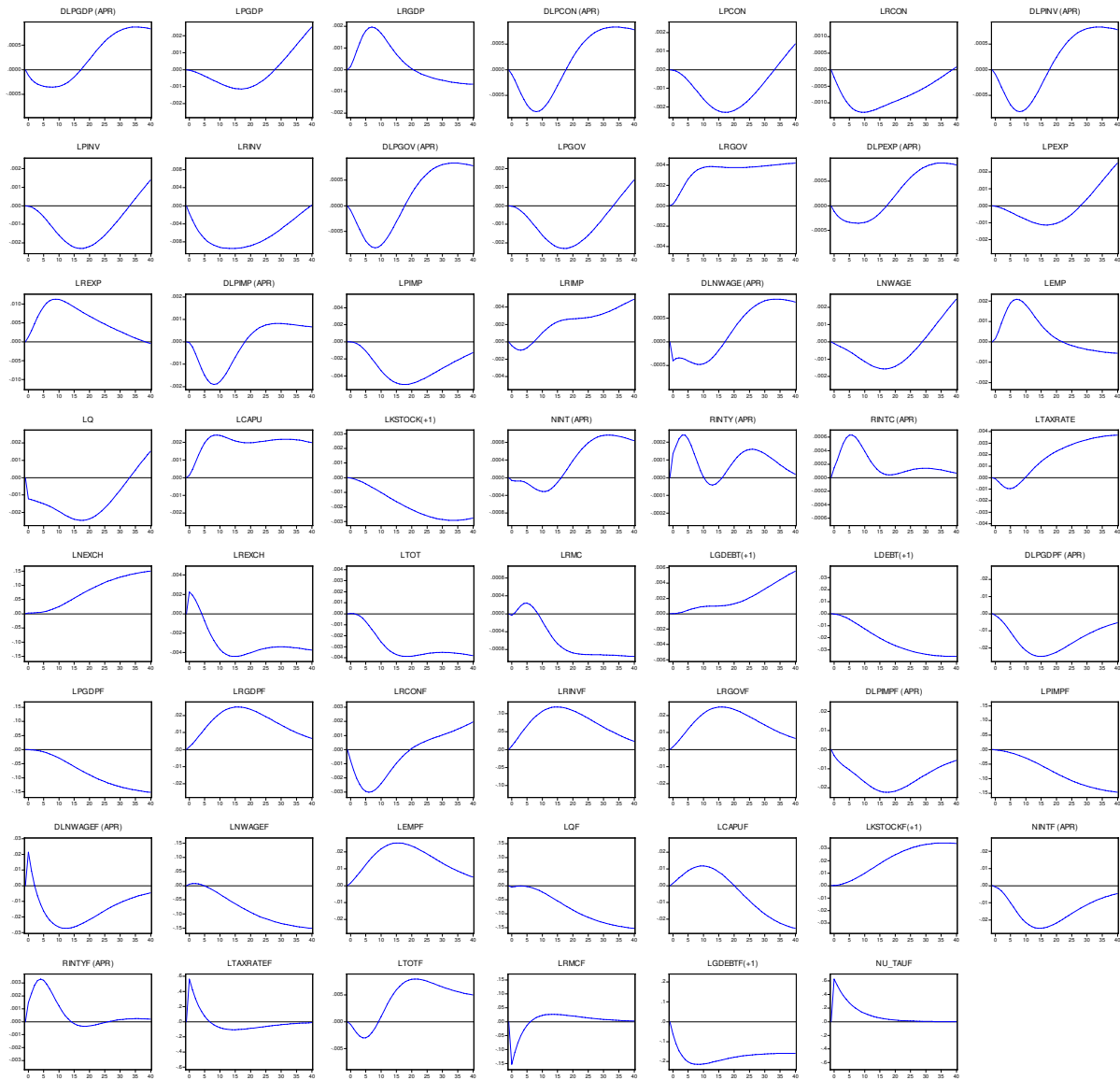
Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 27. Theoretical impulse responses to a foreign fiscal expenditure shock



Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 28. Theoretical impulse responses to a foreign fiscal revenue shock



Note: Theoretical impulse responses to a unit standard deviation innovation are represented by blue lines.

Figure 29. Theoretical forecast error variance decompositions

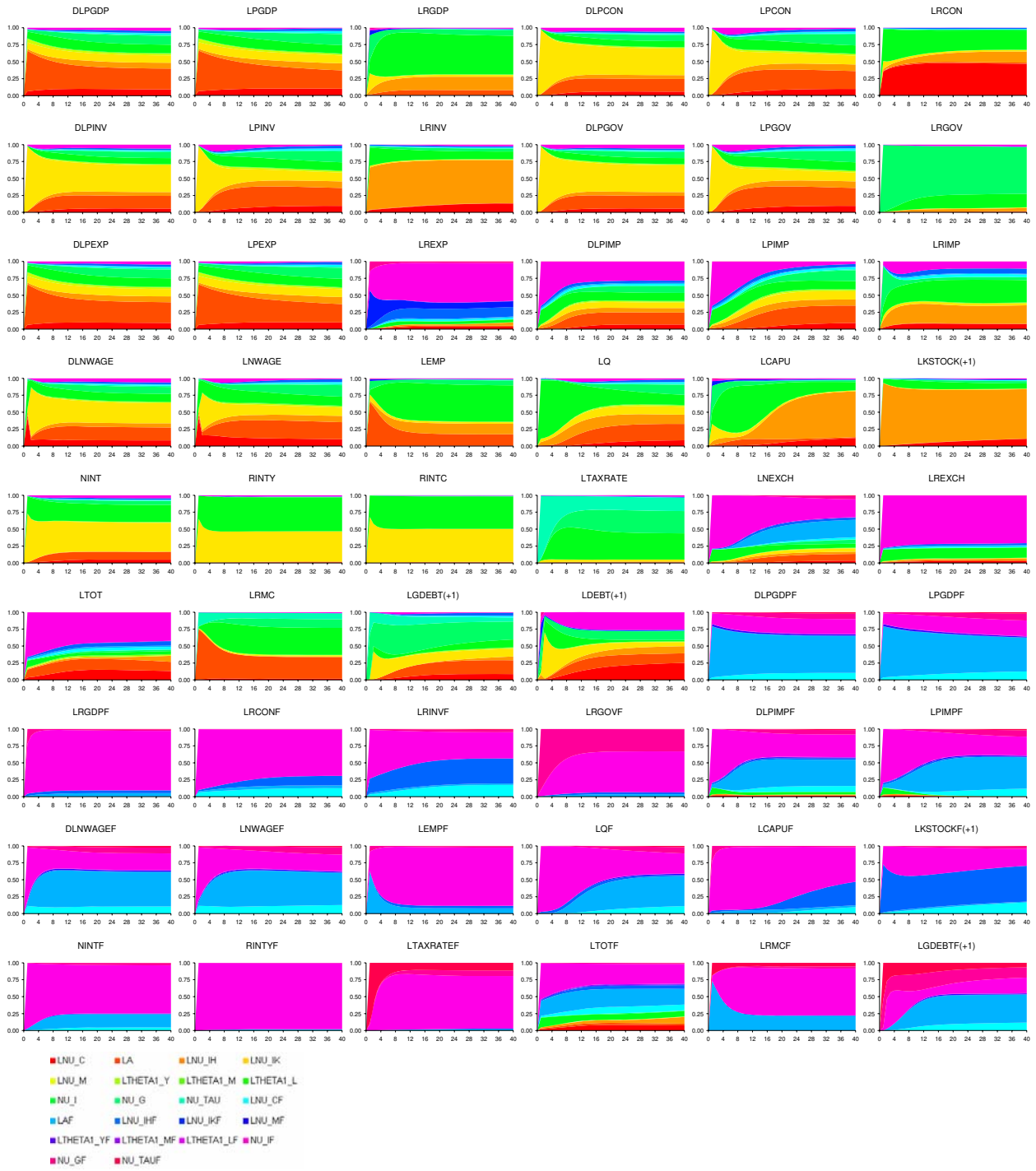
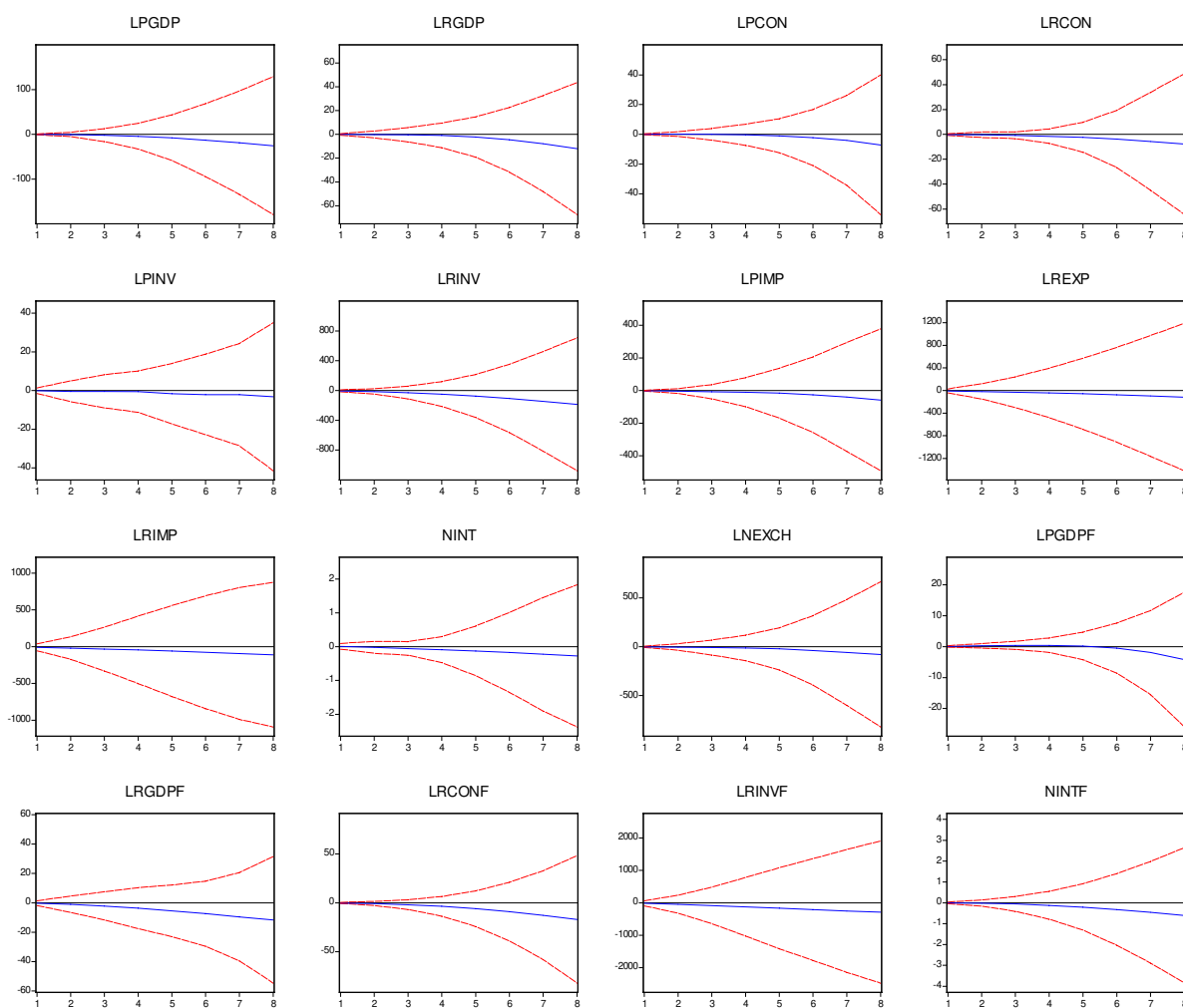
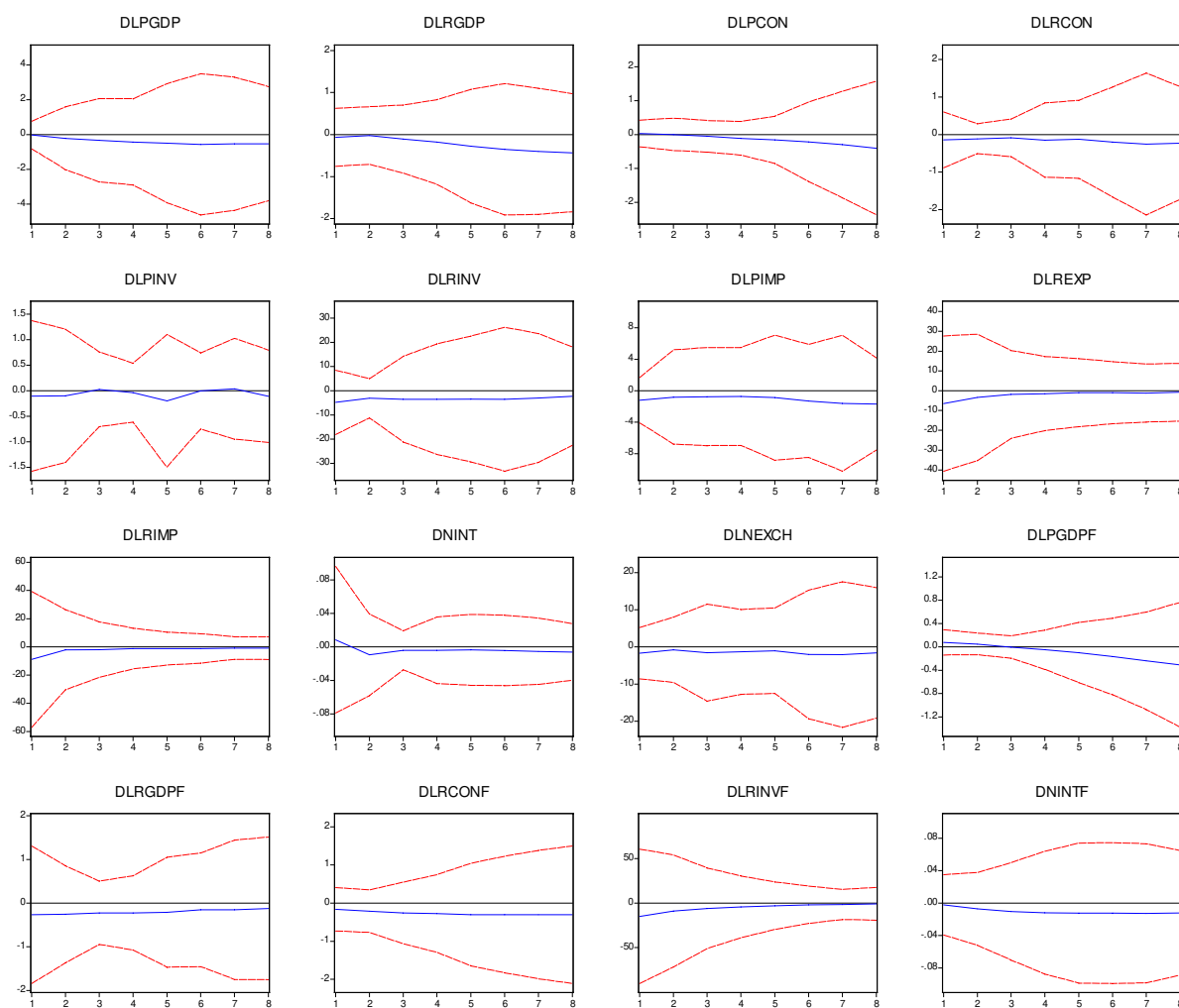


Figure 30. Mean squared prediction error differentials for levels



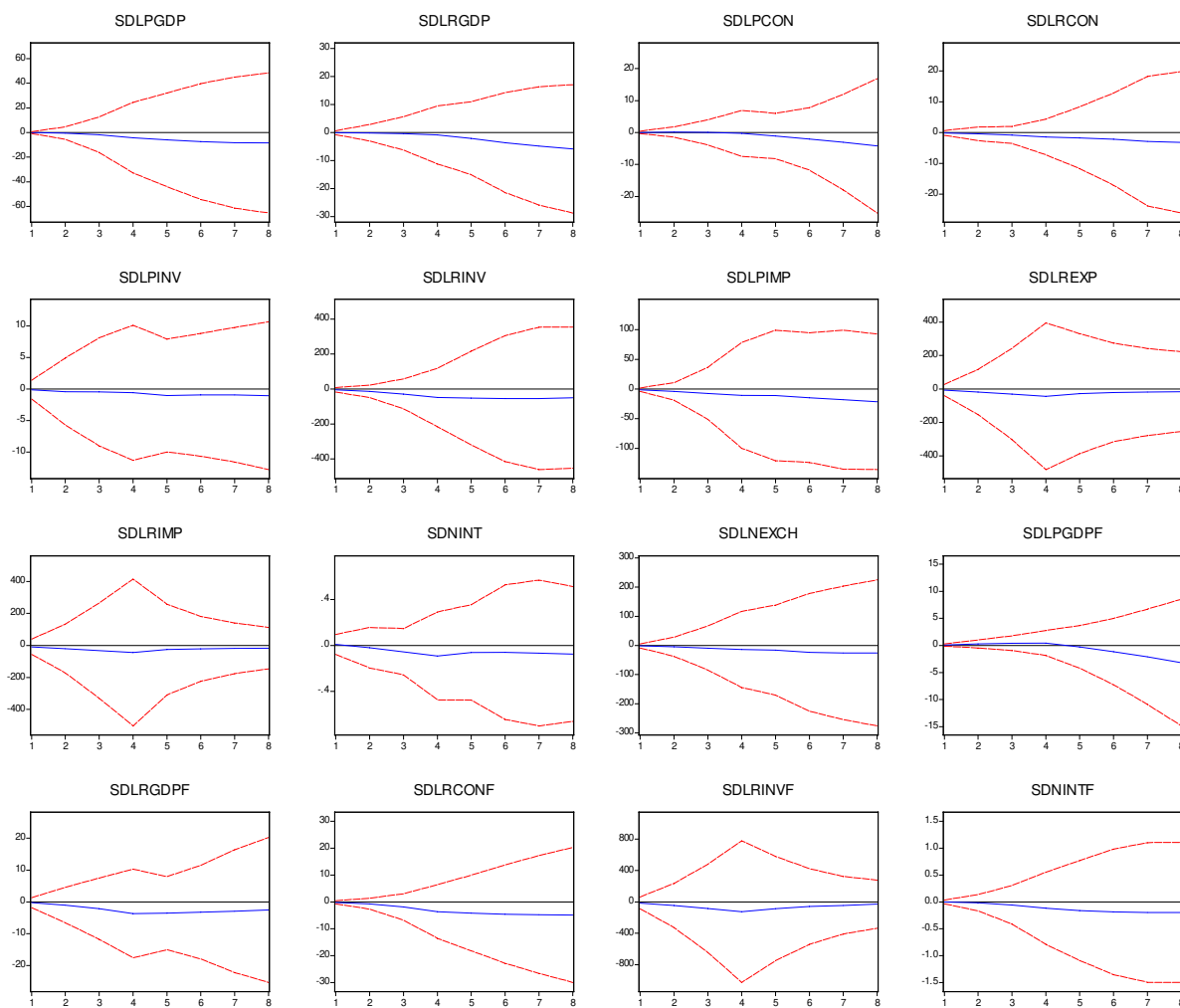
Note: Mean squared prediction error differentials are defined as the mean squared prediction error for the DSGE model less that for the SVAR model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

Figure 31. Mean squared prediction error differentials for ordinary differences



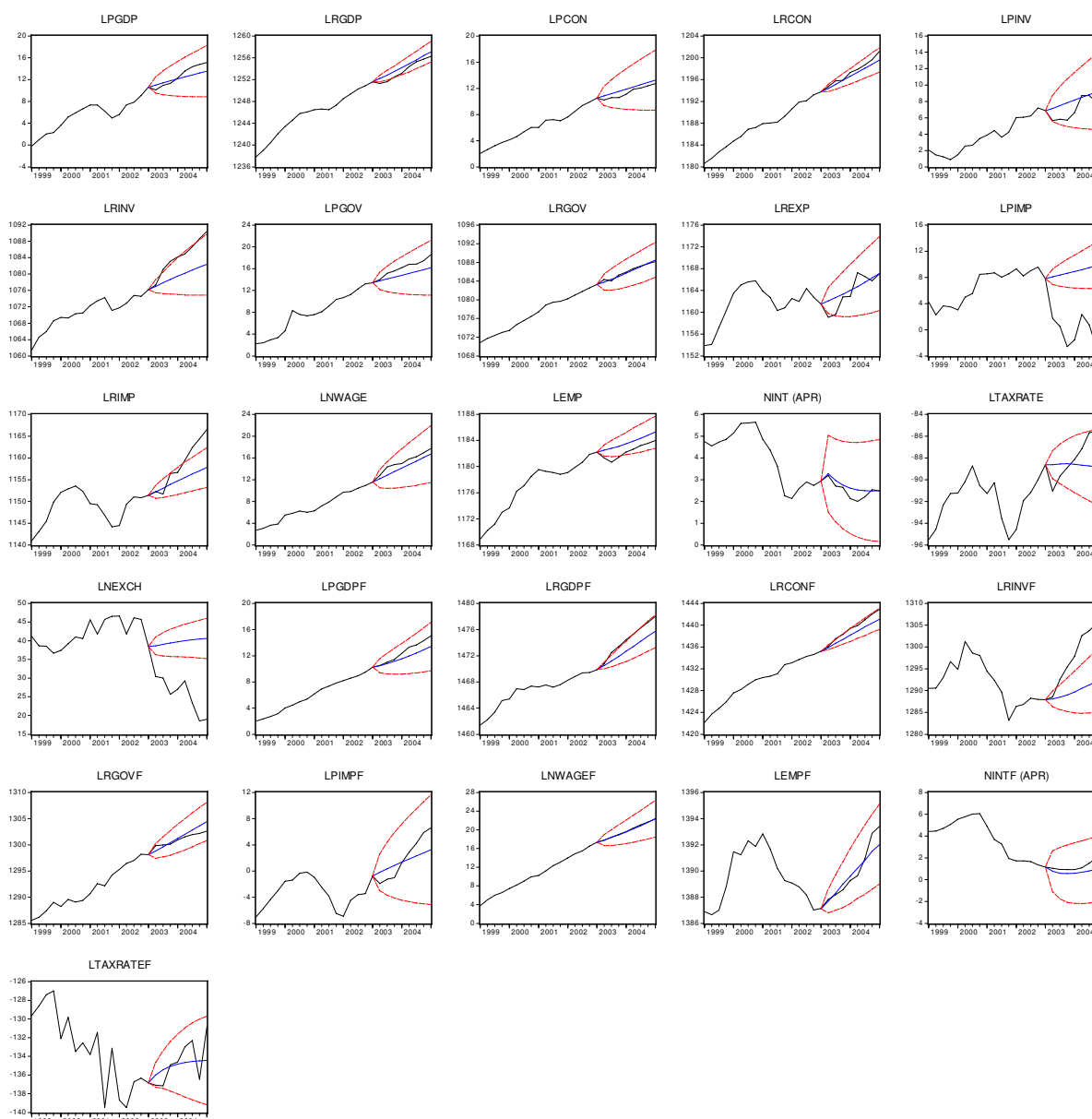
Note: Mean squared prediction error differentials are defined as the mean squared prediction error for the DSGE model less that for the SVAR model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

Figure 32. Mean squared prediction error differentials for seasonal differences



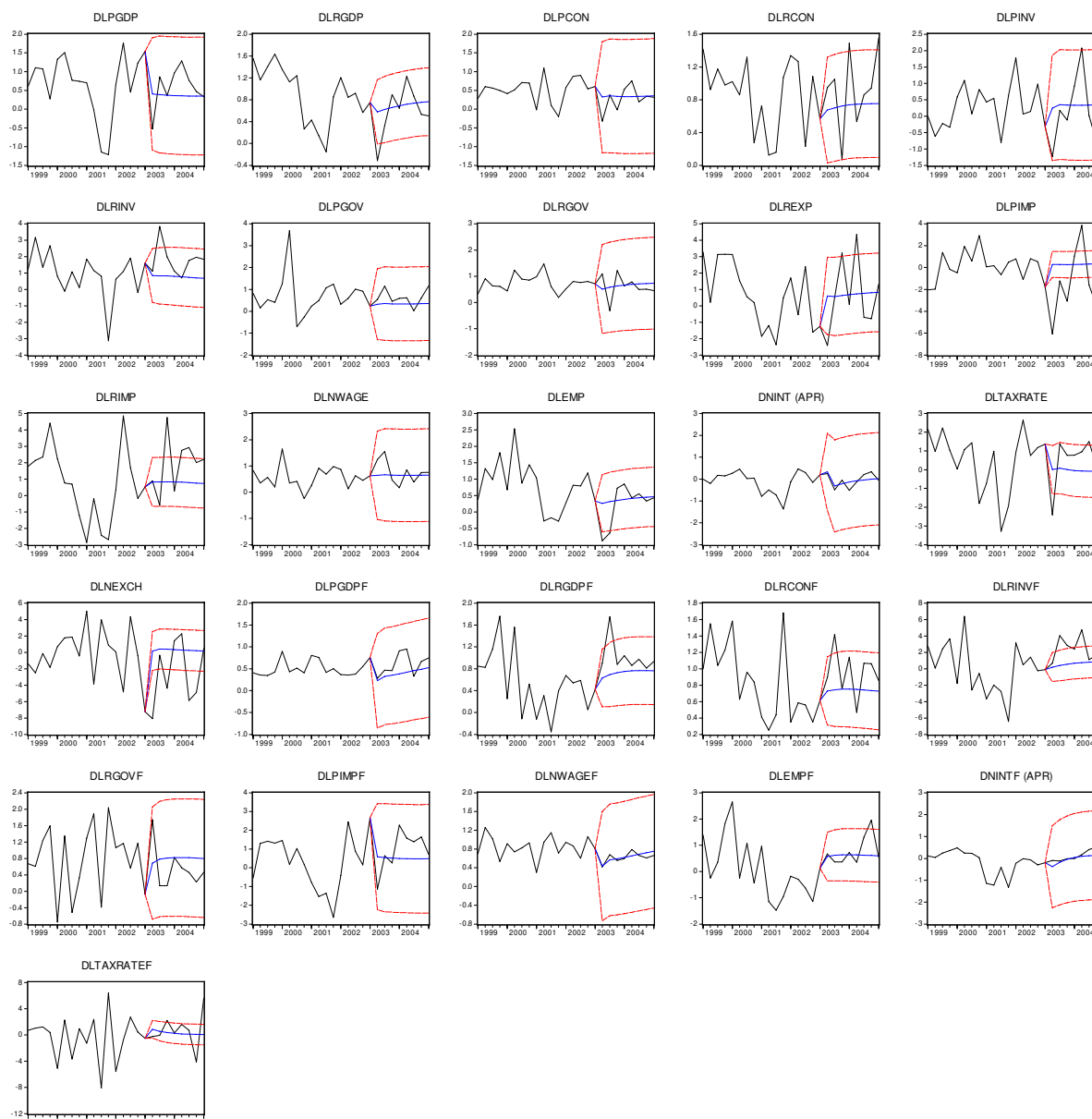
Note: Mean squared prediction error differentials are defined as the mean squared prediction error for the DSGE model less that for the SVAR model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

Figure 33. Dynamic forecasts of levels of observed nonpredetermined endogenous variables



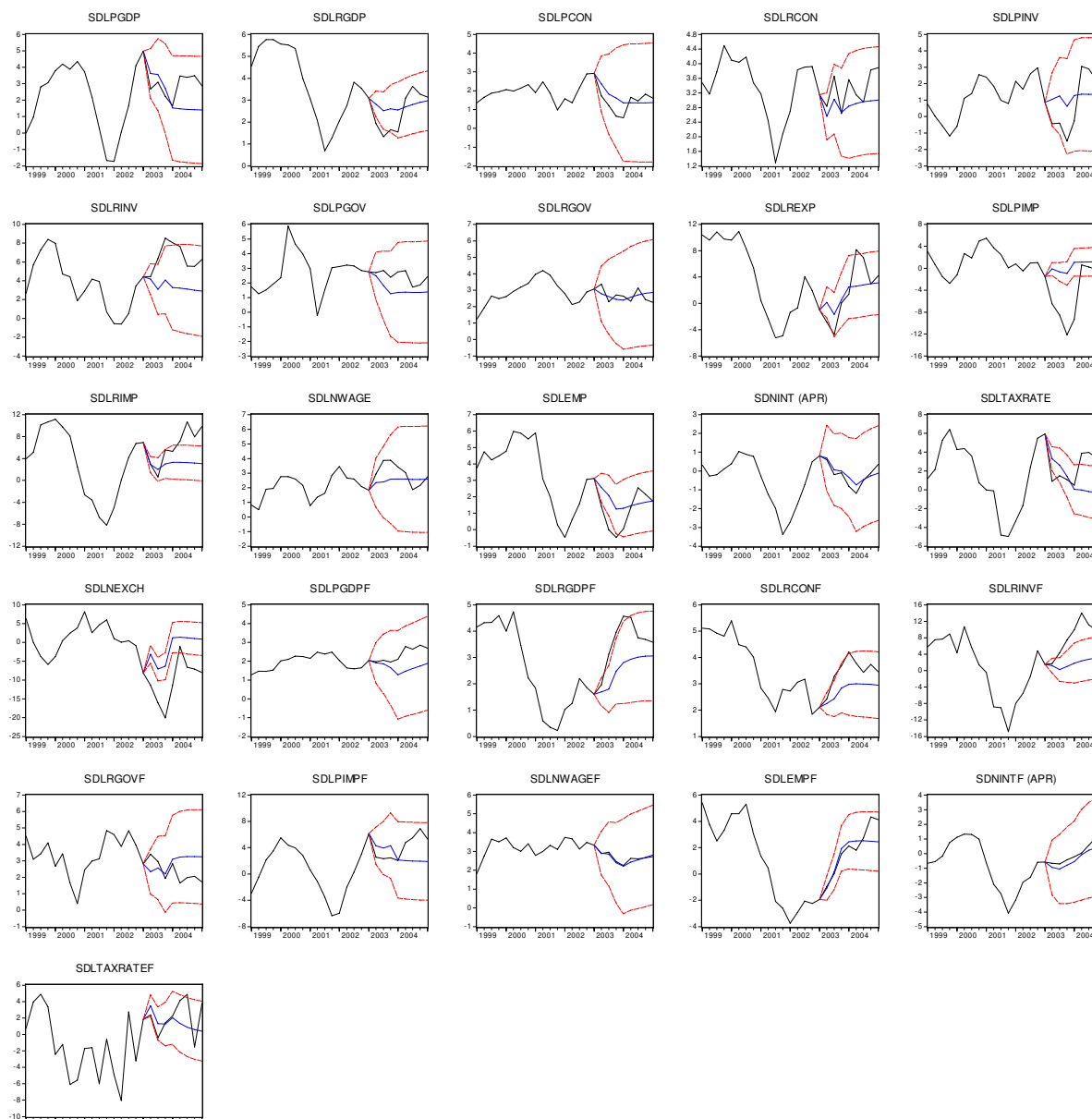
Note: Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters.

Figure 34. Dynamic forecasts of ordinary differences of observed nonpredetermined endogenous variables



Note: Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters.

Figure 35. Dynamic forecasts of seasonal differences of observed nonpredetermined endogenous variables



Note: Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed and independent signal and state innovation vectors and known parameters.

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