



Munich Personal RePEc Archive

Bank Incentives, Contract Design, and Bank Runs

Andolfatto, David

Simon Fraser University, Federal Reserve Bank of Chicago

June 2007

Online at <https://mpra.ub.uni-muenchen.de/8146/>
MPRA Paper No. 8146, posted 08 Apr 2008 00:30 UTC

Bank Incentives, Contract Design and Bank Runs*

David Andolfatto
Department of Economics
Simon Fraser University
8888 University Drive
Burnaby, B.C.
Canada V5A 1S6

and

Research Centre for Economic Analysis

Ed Nosal*
Federal Reserve Bank of Cleveland
Research Department
P.O. Box 6387
Cleveland, OH 44101-1387

June 2007

*We would like to thank Régis Breton, Ricardo Cavalcanti, Narayana Kocherlakota and the participants at the conference in honor of Neil Wallace. In addition, we received several useful comments from seminar participants at various schools and conferences. We would especially like to thank Neil Wallace for many enjoyable hours of conversation regarding money and banking; conversations that greatly influenced the content of this paper. Andolfatto acknowledges the financial support of SSHRC. This article does not necessarily reflect the views of the Federal Reserve Bank of Cleveland or the Board of Governors. Corresponding author: ed.nosal@clev.frb.org

Abstract

We study the Diamond-Dybvig [3] model as developed in Green and Lin [5] and Peck and Shell [7]. We dispense with the notion of a bank as a coalition of depositors. Instead, our bank is a self-interested agent with a technological advantage in record-keeping. We examine the implications of the resulting agency problem for the design of bank contracts and the possibility of bank-run equilibria. For a special case, we discover that the agency problem may or may not simplify the qualitative structure of bank liabilities. We also find that the uniqueness result in [5] is robust to our form of agency, but that the non-uniqueness result in [7] is not.

JEL codes: D82, G21

Keywords: self-interested banker, private record keeping, bank contracts, bank runs

1 Introduction

A distinguishing characteristic of banks is their propensity to issue demandable liabilities. While the option to redeem debt presumably serves an economic purpose (Diamond and Dybvig [3], Calomaris and Kahn [2]), it is commonly asserted this liability structure opens the door to welfare-reducing bank-runs driven by non-fundamental factors. That is, depositors without any pressing need to redeem their liabilities, may nevertheless choose to do so if they believe—for whatever reason—that others will behave similarly.

Diamond and Dybvig [3] were the first to formalize the concept of an equilibrium bank-run as a coordination failure. It appears, however, that their conclusion relies more on an *ad hoc* restriction on the set of admissible contracts, than on any fundamental property of the environment they study. In particular, Green and Lin [5], building on the work of Wallace [8], demonstrate that when the bank is modeled as an optimal allocation mechanism, the prospect of bank-run equilibria in the Diamond-Dybvig environment disappear entirely. Andolfatto, Nosal, and Wallace [1] demonstrate that this latter result generalizes considerably. On the other hand, Peck and Shell [7] demonstrate—using a preference specification that slightly different than [5]—that it is possible to generate a bank-run equilibrium when one employs a different sort of mechanism.

A common approach adopted in this literature ([1], [5], [7]) is to interpret a bank as a coalition of depositors (or, equivalently, as a benevolent social planner). While this approach has some merit, it abstracts from potentially relevant agency problems that are likely to exist between bankers and depositors. In the present context, we think this may be important for two reasons.

First, the optimal bank contract that emerges in (say) the Green-Lin [7] environment bears little resemblance to any empirical counterpart. In particular, the returns on early redemptions must vary in a complicated manner on the history of depositor-types arriving at the bank. But as individual depositors are not privy to these histories, this outcome relies heavily on the assumption that the bank faithfully conditions allocations on true information as it unfolds over time. In contrast, a self-interested banker might be tempted to fabricate the historical record for personal gain. One question we ask is how the problem of aligning bank incentives with those of depositors restricts the set of incentive-compatible allocations. Does the resulting bank contract, for example, look any less complicated?

Second, it seems natural to explore how bank-run phenomena might be related to any agency problems existing between bankers and depositors. Green and Lin [7] themselves offer a conjecture that the likelihood of a bank-run may increase when depositors must worry about how banks might exploit their private information. They also suggest that these same agency problems may be one explanation for why the

banking contract in their model is not observed.¹ Our paper constitutes an attempt to formalize these related ideas.

To do so, we consider the environment as specified by [5] and modify it by introducing a self-interested banker; we assume that the banker’s objective is to maximize own-wealth, rather than depositor utility. The banker’s comparative advantage lies in the fact that he possesses a superior record-keeping technology. For simplicity, we take an extreme view by assuming that the banker has a perfect memory, while depositors have no memory at all. Any implementable allocation must therefore rely entirely on the banker’s version of recorded history. As a self-interested banker may have an incentive to falsify records, additional incentive-compatibility restrictions must be placed on the allocation—the implications of which constitute the focus of our study.

Even in the context of this relatively simple environment, the analysis becomes increasingly complicated as the number of depositors becomes large. To keep the analysis tractable, we ultimately resort to a special case involving only two depositors. We find that the agency problem studied here may or may not simplify the qualitative structure of the optimal bank contract. Evidently, there is a trade-off between “complicated” contracts, that render relatively good risk-sharing properties, and “simple” contracts, that economize on the inefficient use of resources needed to align bank incentives. Not surprisingly, this trade-off depends on parameters; but we find that simple contracts prevail under a wide range of empirically plausible parameter values. We also find that the agency problem highlighted here in no way increases the likelihood of a bank-run. Indeed, we find this to be the case even when we alter our environment and mechanism in the manner suggested by [7].

The paper is organized as follows. In section 2, we describe the environment. We temporarily depart from the standard assumption that depositors possess private information; a simplification that allows us to focus on the private information problem associated with the banker’s record-keeping advantage. We describe mechanisms in section 3. Section 4 demonstrates that the “first-best” allocation derived in [5] is not implementable in our environment and section 5 characterizes the optimal incentive-feasible allocation. Private information over depositor types is introduced in section 6 and the implications for bank-runs are examined. The paper concludes with section 7.

¹Calomiris and Kahn [2] also stress the role of bank incentives in contract design. Their analysis differs from the standard Diamond-Dybvig [3] set-up along several dimensions. First, demandability is not desired as a form of consumption insurance; rather, it serves as a mechanism to discipline potentially fraudulent behavior. Second, their sequential service constraint emerges endogenously for the same purpose. Finally, a “bank-run” in their model corresponds to bank liquidation based on a set of fundamental shocks (information pertaining to the quality of the bank’s assets). Others have also stressed agency problems in relation to determining a bank’s capital structure (see Diamond [4] and Krasa and Villamil [6], among others) but do not examine the implications for bank-runs.

2 The Environment

The economy is populated by one banker and N depositors, where $N \geq 2$ is a finite integer. There are two dates—date-1 and date-2—and one good per date. All agents have access to a constant returns to scale storage technology. Date-1 goods are invested in the technology at the beginning of date-1; a unit of invested date-1 goods can be converted into a unit of date-1 goods during date-1 and into $R > 1$ units of date-2 goods. Each depositor is endowed with $0 < y < \infty$ units of the date-1 good; the banker has no endowment of goods.

A depositor’s utility is denoted $U(c, c', \omega)$, where c is date-1 consumption, c' is date-2 consumption, and ω is the depositor’s *type*. Assume that $\omega \in \{p, i\} \equiv \Omega$, where p denotes “patient” and i denotes “impatient.” Following [5], preferences are restricted to be:

$$U(c, c', \omega) = \begin{cases} u(c + c') & \text{if } \omega = p; \\ u(c) & \text{if } \omega = i; \end{cases} \quad (1)$$

where $u(x) = (1 - \sigma)^{-1}x^{1-\sigma}$ and $\sigma > 1$. That is, a patient depositor views date-1 and date-2 consumption as perfect substitutes, while an impatient depositor only values date-1 consumption. The banker has linear preferences defined over his date-2 consumption, which we denote as b' .

Depositor types are generated by an exogenous *i.i.d.* process, where realizations occur at date-1 and $0 < \pi < 1$ denotes the probability that any given depositor is patient. Hence, the probability that k patient depositors are patient is $\binom{N}{k}\pi^k(1 - \pi)^{N-k}$, for $k \in \{0, 1, 2, \dots, N\} \equiv \mathbb{N}$. There is a second exogenous stochastic process that determines a depositor’s *place-in-line* $n \in \{1, 2, \dots, N\}$ at date-1. Assume that any given place-in-line is equally likely for all depositors. Together, these processes determine a date-1 queue $\omega^N = (\omega_1, \omega_2, \dots, \omega_N)$, where $\omega_n \in \Omega$ denotes the type of a depositor with place-in-line $n = 1, 2, \dots, N$. We will at times refer to $\omega^N \in \Omega^N$ as a *state*.

The timing of events and the structure of information is as follows. Depositors do not know their type *ex ante*; at this stage, they may choose to enter a risk-sharing arrangement by giving their date-1 endowment to the banker, who invests it in the storage technology. Nature then selects a state $\omega^N \in \Omega^N$ according to the stochastic processes described above. Depositors then interact with the banker at date-1 sequentially according to their realized place-in-line. To highlight the role played by bank incentives, we begin by assuming that depositor types are not private information; this assumption is subsequently relaxed. At meeting n , the banker knows $\omega^{n-1} = \{\omega_1, \omega_2, \dots, \omega_{n-1}\} \in \Omega^{n-1}$ and the depositor knows $\omega_n \in \Omega$; note that the depositor does not know n . Each depositor may communicate with the bank, but depositors cannot communicate with each other at date-1. The date-1 payouts to depositors are subject to a *sequential service constraint*; i.e., the date-1 payout to depositor ω_n can only depend upon the realizations ω^{n-1} and cannot depend on

subsequent type realizations ω_j for $j > n$. After these N sequential meetings, any remaining investment endowment is augmented by a factor of R in date-2.

The banker then interacts with all depositors at date-2, with terminal payouts made at this date. At date-2, the banker knows the true state ω^N , but depositors are assumed to know less than this. There are several ways one might choose to model the asymmetry in information between depositors and the banker. One might, for example, assume that depositors cannot communicate with each other at date-2, but can recall their private communications with the banker. Alternatively, one might restrict the memory of depositors regarding their private date-1 communications with the banker but allow for date-2 communication among depositors (this implies that the depositors will know the number of patient and impatient depositors as depositor type is publicly observable). For either of these information structures, the banker could tell all depositors at date-1 that they are the last. Whether or not the banker can actually get away making such reports depends upon how announcements and payoffs are resolved in date-2, and this depends on the details of the information structure. In what follows, we choose the latter approach for two reasons: first, it is simpler to work with; and second, the qualitative aspects of our main results remain robust across these two information structures.² To this end, we assume that depositors do not have a record keeping device and, as a result, cannot “remember” anything, i.e., they cannot remember their date-1 communications with the bank or their date-1 consumption. However, as a by-product of their date-2 communication with each other, they can observe $k(\omega^N) \equiv \{\#(p) \in \omega^N\}$, i.e., the number of patient depositors realized in state ω^N .

3 Mechanisms

We consider mechanisms in which the banker’s strategy is to make a sequence of N reports at date-1—one for each depositor—and one report at date-2. At date-1, associated with each depositor $n > 1$ there is a true history of types $\omega^{n-1} \in \Omega^{n-1}$. For depositor $n = 1$, it will be convenient to denote the “null” history as $\omega^0 \equiv \emptyset \in \Omega^0$.

The banker sends a report of this history for each depositor. Since the banker is the only agent in the economy that has access to a record keeping device and as this history constitutes private information for the banker, the banker’s report associated with depositor n may be an element of any conceivable history $\bar{\Omega} \equiv \cup_{j=0}^{N-1} \Omega^j$. The banker also makes a date-2 report, which will be described shortly.

The mechanism requires that the banker’s date-1 report associated with depositor n be made before the banker observes the depositor’s type. The report is made after depositor $n - 1$ departs and before depositor n arrives.³ Thus, a date-1 strategy for

²We discuss the implications of the former information structure in appendix B.

³If the mechanism has the banker making his date-1 report *after* he observes the depositor’s type,

the banker is a set of functions:

$$m_n : \Omega^{n-1} \rightarrow \bar{\Omega} \text{ for } n = 1, 2, \dots, N.$$

A date-1 allocation or outcome function for depositors is a recommendation $C_n(\cdot)$ made by the mechanism. The allocation is made contingent on the banker's date-1 reports and each depositor's type; i.e.,

$$C_n : \bar{\Omega} \times \Omega_n \rightarrow \mathcal{R}_+ \text{ for } n = 1, 2, \dots, N,$$

where $\Omega_n = \{i, p\}$. Note that the mechanism's date-1 recommendation is made contingent on the depositor's true type, which is observable by the mechanism, together with the *banker's* version of the historical record.

The banker's date-2 reporting strategy is a function $m' : \Omega^N \rightarrow \Omega^N$, where the domain represents the set of true histories. At date-2, all depositors reconvene. Since depositors have no record-keeping device, they are unable to communicate the banker's date-1 report, m_n , or the amount that they consumed at date-1. Depositors can, however, report any pertinent contemporaneous information at this stage. This information is summarized by the function $k : \Omega^N \rightarrow \mathbb{N}$, where $k(\omega^N)$ reveals the number of patient depositors at date-2. (Recall that depositors' types are observable.) A date-2 allocation or outcome function for depositors is a recommendation $C'_n(\cdot)$ made contingent on the banker's date-2 report and $k(\omega^N)$; i.e.,

$$C'_n : \Omega^N \times \mathbb{N} \rightarrow \mathcal{R}_+ \text{ for all } n = 1, 2, \dots, N.$$

Let $\mathbf{C} \equiv \{C_n, C'_n\}_{n=1}^N$ represent an allocation (for depositors) and let $\mathbf{m} \equiv \{m_n, m'\}_{n=1}^N$ represent a strategy profile for the banker.

In what follows, we restrict the banker's date-2 report to be consistent with what the mechanism and depositors can observe at date-2, i.e., $k(\omega^N)$; since, otherwise, the banker would be making a report that would be known to be false.

Definition 1 The date-2 strategy m' is said to be *consistent* if $k(m'(\omega^N)) = k(\omega^N)$ for all $\omega^N \in \Omega^N$.

Note that consistency does not imply truth-telling. If consistency is imposed, then we can reduce notation by making the date-2 allocation solely a function of the banker's consistent date-2 report. Through a slight abuse of notation, we now let $C'_n : \Omega^N \rightarrow \mathcal{R}_+$ for all $n = 1, 2, \dots, N$, where the domain of this function is now understood to be the set of *consistent* date-2 reports.

then it can be easily shown that risk-sharing possibilities are destroyed and autarky will be the only outcome. Such a mechanism cannot be optimal.

Now fix an allocation \mathbf{C} and a consistent strategy \mathbf{m} . Then, conditional on a realization $\omega^N \in \Omega^N$, the *ex post* payoff or outcome function for the banker is given by:

$$B'(\mathbf{C}, \mathbf{m}) \equiv R \left[Ny - \sum_{n=1}^N C_n(m_n(\omega^{n-1}), \omega_n) \right] - \sum_{n=1}^N C'_n(m'(\omega^N)). \quad (2)$$

A mechanism $(\mathbf{\Omega}, \mathbf{C})$ is a collection of strategy sets, $\mathbf{\Omega}$, and an outcome function, \mathbf{C} . The collection of strategy sets is $\mathbf{\Omega} = (\bar{\Omega}, \Omega^N, \Omega_1, \dots, \Omega_N)$, where $\bar{\Omega}$ and Ω^N represents the banker's date-1 and date-2 strategy sets, respectively; and Ω_n , $n = 1, \dots, N$ represent the depositors' type set.

Definition 2 The strategy profile \mathbf{m} for mechanism $(\mathbf{\Omega}, \mathbf{C})$ is said to be *feasible* if:

$$B'(\mathbf{C}, \mathbf{m}) \geq 0 \quad (3)$$

for all $\omega^N \in \Omega^N$.

Let $\mathcal{M}(\mathbf{\Omega}, \mathbf{C})$ denote the set of feasible and consistent strategy profiles that are available to the banker for mechanism $(\mathbf{\Omega}, \mathbf{C})$. We are effectively imposing a form of commitment on the banker by requiring him to choose his strategy profile $\mathbf{m} \in \mathcal{M}(\mathbf{\Omega}, \mathbf{C})$. Since the horizon is finite, some degree of commitment is required; otherwise, there is nothing that prevents the banker from, for example, giving out zero payoffs in date-1 and consuming RNy at date-2. We assume that the banker can commit to make payouts consistent with his announcements; this implies that $\mathbf{m} \in \mathcal{M}(\mathbf{\Omega}, \mathbf{C})$.

Definition 3 The strategy profile $\mathbf{m}^* \in \mathcal{M}(\mathbf{\Omega}, \mathbf{C})$ constitutes an *equilibrium* if:

$$E_{\omega^N \in \Omega^N} [B'(\mathbf{C}, \mathbf{m}^*)] \geq E_{\omega^N \in \Omega^N} [B'(\mathbf{C}, \mathbf{m})] \quad (4)$$

for all $\mathbf{m} \in \mathcal{M}(\mathbf{\Omega}, \mathbf{C})$.

Note that our definition of equilibrium satisfies the notion of sequential rationality for the banker even though the banker's expected payoff in (4) is calculated in an *ex ante* sense. To see this, suppose the proposed equilibrium strategy profile for the banker \mathbf{m}^* satisfies (4). Now, consider some history $\hat{\omega}^n \in \Omega^n$ and suppose that the banker can increase his expected payoff relative to the proposed equilibrium by playing the feasible and consistent continuation strategy $\hat{\mathbf{m}}^n$ for the remainder of the game. Define a new strategy \mathbf{m} that is identical to \mathbf{m}^* for all histories, except following history $\hat{\omega}^n$, where the continuation strategy $\hat{\mathbf{m}}^n$ is played instead. Clearly then, \mathbf{m}^* cannot be an equilibrium strategy since the constructed strategy $\mathbf{m} \in \mathcal{M}(\mathbf{\Omega}, \mathbf{C})$ violates inequality (4).

If strategy $\mathbf{m} \in \mathcal{M}(\Omega, \mathbf{C})$ is an equilibrium, then associated with each state $\omega^N \in \Omega^N$ are payoff functions to depositors $\mathbf{c} = \{c_n, c'_n\}_{n=1}^N$, where $c_n(\omega^n) \equiv C_n(m_n(\omega^{n-1}), \omega_n)$ and $c'_n(\omega^N) = C'_n(m'_n(\omega^N))$, together with a banker payoff function $b'(\mathbf{c}, \omega^N)$, where $b'(\mathbf{c}, \omega^N) \equiv B'(\mathbf{C}, \mathbf{m}(\omega^N))$. Hence, we can construct an alternative mechanism (Ω, \mathbf{c}) such that if $\mathbf{m} \in \mathcal{M}(\Omega, \mathbf{C})$ is an equilibrium for mechanism (Ω, \mathbf{C}) , then $\mathbf{t} \in \mathcal{M}(\Omega, \mathbf{c})$ is an equilibrium for mechanism (Ω, \mathbf{c}) , where \mathbf{t} is defined as the *truth-telling strategy* $t_n(\omega^{n-1}) \equiv \omega^{n-1}$ for all $n = 1, 2, \dots, N$ and $t'(\omega^N) \equiv \omega^N$.

Definition 4 An allocation \mathbf{c} is said to be *truthfully implementable* as an equilibrium for mechanism (Ω, \mathbf{c}) if $\mathbf{t} \in \mathcal{M}(\Omega, \mathbf{c})$ is a truth-telling strategy and

$$E_{\omega^N \in \Omega^N} [b'(\mathbf{c}, \omega^N)] \geq E_{\omega^N \in \Omega^N} [b'(\mathbf{c}, \mathbf{m}(\omega^N))] \quad (5)$$

for all $\mathbf{m} \in \mathcal{M}(\Omega, \mathbf{C})$.

In what follows, we can, without loss of generality, restrict attention to allocations that are truthfully implementable.

Under a truth-telling strategy \mathbf{t} for mechanism (Ω, \mathbf{c}) , the *ex ante* utility payoff for depositors is given by:

$$W(\mathbf{c}) = \sum_{\omega^N \in \Omega^N} \Pr(\omega^N) \left[\sum_{n=1}^N U(c_n(\omega^{n-1}, \omega_n), c'_n(\omega^N), \omega_n) \right]. \quad (6)$$

Definition 5 An *optimal allocation* \mathbf{c} maximizes $W(\mathbf{c})$ subject to: [1] $b'(\mathbf{c}, \omega^N) \geq 0$ for all $\omega^N \in \Omega^N$ [feasibility]; and [2] $E_{\omega^N \in \Omega^N} [b'(\mathbf{c}, \omega^N)] \geq E_{\omega^N \in \Omega^N} [b'(\mathbf{c}, \mathbf{m}(\omega^N))]$ for all $\mathbf{m} \in \mathcal{M}(\Omega, \mathbf{C})$ [equilibrium].

4 Is the First-Best Allocation Implementable?

In this section, we consider the benchmark allocation that would result if depositors have access to a public record-keeping device. The resulting allocation, which we refer to as the *first-best allocation*, corresponds to that derived in [5]. In what follows, we ask whether the first-best allocation can be implemented under our assumed information structure; i.e., when the banker has a “monopoly on memory.”

When depositors have access to a record-keeping technology, the banker is redundant. Hence, the first-best allocation maximizes $W(\mathbf{c})$ subject to feasibility, $b'(\mathbf{c}, \omega^N) \geq 0$ for all $\omega^N \in \Omega^N$.

Owing to the specification of a depositor’s utility function (1), a number of properties associated with the first-best allocation immediately emerge. First, since impatient depositors do not value date-2 consumption, they do not receive any date-2

consumption in the first-best allocation, i.e., $c'_n(\omega^N) = 0$ if $\omega_n = i$ for all $\omega^N \in \Omega^N$. Second, since patient depositors view date-1 and date-2 consumption as perfect substitutes, but one unit of date-1 consumption can be converted into $R > 1$ units of date-2 consumption, patient depositors do not receive any date-1 consumption in the first-best contract, i.e., $c_n(\omega^{n-1}, \omega_n) = 0$ if $\omega_n = p$ for all $\omega^{n-1} \in \Omega^{n-1}$, $n = 1, \dots, N$. And finally, since depositor types are observable and since they are risk-averse, optimal risk-sharing implies that patient depositors receive the same levels of date-2 consumption, independent of their place in line, i.e., $c'_n(\omega^N) = c'_j(\omega^N) > 0$ for all n, j whenever $\omega_n = \omega_j = p$. To better understand other aspects associated with risk-sharing, we solve the above maximization problem explicitly. For this purpose, it will be sufficient to consider the case where $N = 2$.

Feasibility implies $c'_2(i, p) = R[2y - c_1(i)]$, $c'_1(p, i) = R[2y - c_2(p, i)]$ and $c_2(i, i) = 2y - c_1(i)$. Substituting these conditions into (6) results in the following maximization problem for $N = 2$,

$$\begin{aligned} \max_{c_1(i), c_2(p, i)} \quad & \pi^2 2u(Ry) + (1 - \pi) \pi \{u(c_1(i)) + u(R[2y - c_1(i)])\} \\ & + \pi(1 - \pi) \{u(c_2(p, i)) + u(R[2y - c_2(p, i)])\} + \\ & (1 - \pi)^2 \{u(c_1(i)) + u(2y - c_1(i))\}. \end{aligned}$$

It is straightforward to establish that the solution to this problem has the following properties:

$$\begin{aligned} 0 &< c_2(i, i) < y < c_1(i) < c_2(p, i) \\ y &< c'_1(p, i) < c'_2(i, p) < Ry = c'_1(p, p) = c'_2(p, p). \end{aligned}$$

The inequalities above describe the nature of optimal risk-sharing in this environment. In particular, note that in the event that there is only one impatient depositor, this agent receives a date-1 consumption that exceeds the autarkic level y ; this is evidence of risk-sharing between impatient and patient depositors. Note further that an impatient depositor receives a larger payoff if he is second in line and follows a patient depositor; i.e., $c_2(p, i) > c_1(i)$. Intuitively, if the first depositor is patient, the planner is better able to share risks with a subsequent impatient depositor and the patient depositor, than if the first depositor is impatient, i.e., in the former case, no payment is made, leaving the banker with greater resources. Although the first impatient depositor receives an amount in excess of y , if the second depositor turns out to be impatient, he will receive a date-1 payoff that falls below the autarkic level.

We now demonstrate that the first-best allocation cannot be implemented as a truth-telling equilibrium when memory resides solely with the banker.

Proposition 1 *The first-best allocation is not implementable when the banker has a monopoly on memory.*

Proof Observe that the truth-telling strategy $(t_1, t_2) = (\emptyset, \omega_1)$ and $t' = \omega^2$ delivers a zero payoff to the banker under every realization ω^2 . Consider the following deviation: the banker tells the truth everywhere except if $\omega_1 = p$, in which case he reports $m_2 = \emptyset$ (a lie) to the second depositor and $m' = t'$ (the truth) at date 2. If the state turns out to be $\omega^2 = (p, p)$, then the date-1 payoff to the second depositor associated with the report $m_2 = \emptyset$ is $c_1(p) = c_2(p, p) = 0$. Consistency requires that the banker report $m' = (p, p)$ at date 2; the date 2 payoffs $c'_1(p, p) = c'_2(p, p) = Ry$ are feasible since the banker did not make any payments at date-1. If, however, the state turns out to be $\omega^2 = (p, i)$, then the payoff to the second depositor associated the report $m_2 = \emptyset$ is $c_1(i) < c_2(p, i)$. Now, consistency requires that $m' \in \{(p, i), (i, p)\}$. If $m' = (p, i)$, then after paying $c'_2(p, i)$ to the patient depositor, there will be $[c_2(p, i) - c_1(i)]R > 0$ resources left over, which the banker can consume. Since the banker's deviant strategy is feasible, consistent and provides a higher expected payoff than the truth-telling strategy, the first-best allocation cannot be an equilibrium outcome.

It is perhaps instructive to point out how the banker's ability to get away with a profitable lie in the proof above relies heavily on the first depositor's inability to remember the past. Note that the banker actually reports $m' = (p, i)$ at date-2, which is the truth. If the second depositor had access to a record-keeping device, he would recall that the banker told him that he was the first in line, whereas the date-2 report reveals that he was, in fact, the second in line.

5 The Optimal Incentive-Feasible Allocation

The formal derivations involved in characterizing an optimal incentive-feasible allocation are rather involved and so we relegate them to an appendix. In this section, we present the main results and offer some explanation for the logic that underlies them.

Our first result is that it is possible to implement allocations with the following properties: $c_2(p, i) = c_1(i) > c_2(i, i)$ or $c_2(p, i) > c_1(i) > c_2(i, i)$. We call the former allocation a *partial risk-sharing* allocation and the latter a *full risk-sharing* allocation. We describe the partial risk-sharing allocation as "simple" because the date-1 payment to impatient depositors depends only on the number of impatient depositors who have made withdrawals, and not on the history of patient depositors who may have arrived earlier. Our second result is that either the partial or full risk-sharing allocation may be optimal, depending on model parameters.

To develop some intuition, recall that the proof to proposition 1 shows that if $c_2(p, i) > c_1(i)$, then the banker has an incentive to lie to the second depositor if

$\omega^1 = p$; in state $\omega^2 = (p, i)$ the banker can obtain a payoff of $R[c_2(p, i) - c_1(i)] > 0$ if he tells the second depositor that he is the first in line. One obvious way to prevent the banker from behaving in this manner is to restrict the allocation so that $c_2(p, i) = c_1(i)$. Of course, while this has the benefit of better aligning bank incentives, it comes at the cost of reducing the risk-sharing properties of the allocation.

The restriction $c_2(p, i) = c_1(i)$, however, only prevents lying along one dimension. When date-1 payoffs to patient depositors are zero, if $c_1(i) > c_2(i, i)$, then the banker can get away with a profitable lie by reporting $m_1 = i$ to the first depositor and telling the truth everywhere else. To see this, suppose that the actual state turns out to be $\omega^2 \in \{(p, p), (p, i)\}$. As the first depositor is patient, the bank pays out $c_2(i, p) = 0$. Since no resources are withdrawn and since the banker subsequently tells the truth, the banker has enough resources to make good on his date-1 and date-2 payouts. On the other hand, suppose that the state turns out to be $\omega^2 \in \{(i, p), (i, i)\}$. In this case, the first depositor receives a payment $c_2(i, i) < c_1(i)$ and the bank makes a profit of $R[c_1(i) - c_2(i, i)] > 0$, while at the same time being able to make his date-2 payouts.

One way to prevent the banker from behaving in the manner described above is to restrict the allocation further so that $c_1(i) = c_2(i, i)$. Unfortunately, this would have the effect of eliminating all risk-sharing; the result would be autarky. There is, however, another way to elicit truthful reporting when $c_1(i) = c_2(p, i)$: by setting $c_2(i, p)$ to an arbitrarily small but positive number. The effect of this restriction is to render the lie $m_1 = i$ *infeasible*. That is, forcing the bank to pay out something to the patient depositor leaves the bank with insufficient resources—by the amount $Rc_2(i, p)$ —to make good on its date-2 payouts in state (i, p) .⁴

Under the partial risk-sharing allocation, it turns out that the banker makes zero profits in every state except for $\omega^2 = (p, i)$, where he earns a return equal to $Rc_2(i, p)$. But as $c_2(i, p)$ can be made arbitrarily small, this inefficient date-1 payment to the patient depositor and the inefficient payment to the banker can be made arbitrarily small. Hence, the first-order welfare cost associated with the partial risk-sharing allocation stems solely from the loss of risk-sharing associated with setting $c_2(p, i) = c_1(i)$.

The logic described above extends to the case of the full risk-sharing allocation. That is, since $c_1(i) > c_2(i, i)$, the allocation must have the property $c_2(i, p) > 0$ in order to prevent the lie $m_1 = i$. The banker, however, may have an incentive to misreport the truth since $c_2(p, i) > c_1(i)$. In particular, the banker will have an incentive to report the lie $m_2 = \emptyset$ when the first depositor is patient (as described in the proof to proposition 1). Recall from the proof, if $\omega^1 = p$ and the banker reports $m_2 = \emptyset$ in state $\omega^2 = (p, p)$, the banker pays out $c_1(p) = 0$ to the second depositor,

⁴If the banker does not make good on its date-2 payouts—either by choice or because he has insufficient resources—the contract can specify that the banker can consume any of the remaining deposits and receive a payoff of $-Z$, where Z can be made arbitrarily large.

leaving the him with sufficient resources to meet his future payouts. Hence, in state $\omega^2 = (p, i)$, the banker pays out $c_1(i)$ instead of $c_2(p, i)$ to the second depositor and captures a positive payoff. One way to elicit truth-telling here is by setting $c_1(p)$ to an arbitrarily small but positive number. Again, the effect of this inefficient payoff is to render the lie $m_2 = \emptyset$ infeasible in state (p, i) .

Under the full risk-sharing allocation, it turns out that the banker makes zero profits in every state. But to induce truth-telling, the allocation must be restricted in the following manner

$$c_2(i, p) - c_1(p) = c_2(p, i) - c_1(i), \quad (7)$$

where $c_1(p)$ can be made arbitrarily small.⁵ Hence, equation (7) tells us that in the full risk-sharing allocation (with $c_1(p)$ made arbitrarily small), the magnitude of $c_2(i, p)$ will depend positively on desired the degree of risk-sharing $c_2(p, i) - c_1(i)$.

Here then, we see the trade-offs that are involved. A full risk-sharing allocation can be implemented, but at the expense of allocating at a nontrivial date-1 payment to patient depositors, $c_2(p, i)$. A partial risk-sharing allocation effectively eliminates this nontrivial inefficient payment, but only at the expense of reducing the risk-sharing properties of the allocation.

Which of these two scenarios is optimal turns out to depend on model parameters. Numerical examples suggest that a full risk-sharing allocation becomes more desirable as $\pi \rightarrow 0$ and $\sigma \rightarrow \infty$. The intuition for this is straightforward. In particular, as the probability of patient depositors decreases, the probability of making inefficient date-1 payments to patient depositors falls as well, so that risk-sharing objectives can be met more cheaply. Likewise, as depositors become more risk-averse, they are more willing to bear the cost of inefficient payments that render full risk-sharing incentive-compatible. We summarize the discussion in this section in the following two propositions, with the proofs relegated to appendix A.

Proposition 2 *If the optimal allocation is characterized by full risk-sharing— $c_2(p, i) > c_1(i) > c_2(i, i)$ —then truth-telling on the part of the banker will require date-1 payments to patient depositors satisfy $c_2(i, p) > c_1(p) = \eta > 0$. As in the first-best allocation, date-2 payments to impatient depositors are zero and $c_2(p, p) = 0$. The banker makes zero profit in each state of the world.*

Proposition 3 *If the optimal allocation is characterized by partial risk sharing— $c_2(p, i) = c_1(i) > c_2(i, i)$ —then truth-telling on the part of the banker will require*

⁵Since the optimal allocation delivers zero payoffs to the banker in states (i, p) and (p, i) , $c_1(i) + c_2(i, p) + c'_2(i, p) = c_1(p) + c_2(p, i) + c'_1(p, i)$. As well, the date-2 payoffs to patient depositors must be the same in states (i, p) and (p, i) ; otherwise, the banker will announce at date-2 that state which gives the depositor the smallest payoff. Since $c'_2(i, p) = c'_1(p, i)$, the above equation can be rearranged to (7).

date-1 payments to patient depositors satisfy $c_2(i, p) = \xi > 0$. As in the first-best allocation, date-2 payments to impatient depositors are zero and $c_1(p) = c_2(p, p) = 0$. The banker makes zero profit in all states of the world, except state $\omega^2 = (p, i)$, where he makes an arbitrarily small profit, equal to $R\xi$.

6 Depositor Private Information and Bank-Runs

To address the issue of bank-run equilibria, we need to extend the model so depositor types are private information. The basic structure of the environment requires only some minor modifications to accommodate the private information assumption regarding depositor types.

As in section 3, the banker makes a date-1 report to the mechanism m_n after depositor $n - 1$ departs and before depositor n arrives for each $n = 1, 2, \dots, N$. In addition, we now require that depositor n make a report a_n to the mechanism. Here, we follow [7] by assuming that

$$a_n : \Omega_n \rightarrow \Omega_n \text{ for } n = 1, 2, \dots, N,$$

where the domain represents the depositor's true type. This specification implies that the mechanism withholds information from the depositor (in particular, the bank report m_n). An important implication of this restriction is that the depositor does not know his place-in-line when making his report. [7] stresses that this property of the mechanism is essential for admitting the possibility of a bank-run equilibrium.

Since that banker is the only agent in the model with a record-keeping device, it is optimal for the mechanism to inform the banker of a_n , $n = 1, \dots, N$. The date-1 outcome function for depositor n , $C_n(\cdot)$, is contingent on the banker's date-1 reports and depositor reports; i.e.,

$$C_n : \bar{\Omega} \times \Omega_n \rightarrow \mathcal{R}_+ \text{ for } n = 1, 2, \dots, N.$$

At date-2, the banker and all of the depositors (simultaneously) make a report to the mechanism. As in section 3, the banker's date-2 reporting strategy is the function $m' : \Omega^N \rightarrow \Omega^N$ and each depositor reports

$$a'_n : \Omega_n \rightarrow \Omega_n \text{ for } n = 1, 2, \dots, N.$$

Here we assume that the depositor is able to remember the announcement that he made at date-1. Let $k(a'_1, \dots, a'_N) \in \mathbb{N}$ denote the number of patient reports contained in the vector (a'_1, \dots, a'_N) . Again, we stress that the mechanism only knows $k(\cdot)$ and not the specific configuration of (a'_1, \dots, a'_N) . Hence, a date-2 allocation or outcome function for depositors is a recommendation $C'_n(\cdot)$ made contingent on the banker's date-2 report and $k(\cdot)$; i.e.,

$$C'_n : \Omega^N \times \mathbb{N} \rightarrow \mathcal{R}_+ \text{ for all } n = 1, 2, \dots, N.$$

Let $\mathbf{a} = (a_n, a'_n)_{n=1}^N$ represent a strategy profile for the N depositors.⁶

We restrict attention to allocations that are truthfully implementable. In keeping with our earlier analysis, we focus on the case where $N = 2$. ([7] is able to generate a bank run equilibrium for $N = 2$.) Note that when depositors reports their types truthfully, all of the analysis—and the associated implications, i.e., propositions 2 and 3—in section 5 remain valid here.

Assuming that the banker reveals depositors' reports truthfully, the truth-telling condition for an *impatient* depositor is,

$$\begin{aligned} & \frac{(1-\pi)}{2} \{u(c_1(i)) + u(c_2(i,i))\} + \frac{\pi}{2} \{u(c_1(i)) + c_2(p,i)\} \\ \geq & \frac{(1-\pi)}{2} \{u(c_1(p)) + u(c_2(i,p))\} + \frac{\pi}{2} (1-\pi) u(c_1(p)). \end{aligned}$$

This condition is *always* satisfied. First, note that $c_1(p)$ is an arbitrarily small number so that for our preferences $u(c_1(p)) \rightarrow -\infty$. Second, note that $u(c_1(i)) + u(c_2(i,i)) > u(y) > u(c_2(i,p))$ since $c_1(i) + c_2(i,i) = y$ and $c_2(i,p) < y$.

The truth-telling condition for a patient depositor is,⁷

$$\begin{aligned} & \pi u \left(Ry - \frac{R+1}{2} c_1(p) \right) + \\ & \frac{(1-\pi)}{2} \{u(c_2(i,p) + R(2y - c_2(i,p) - c_1(i))) + u(c_1(p) + R(2y - c_2(p,i)))\} \\ \geq & \frac{\pi}{2} \{u(c_1(i)) + u(c_2(p,i))\} + \frac{(1-\pi)}{2} \{u(c_1(i)) + u(c_2(i,i))\}. \end{aligned} \tag{8}$$

Allocation \mathbf{c} for the mechanism (Ω, \mathbf{c}) is said to be implementable as a truth-telling equilibrium if conditions (5) and (8) hold.

Suppose that when depositor type is observable, the optimal allocation is characterized by partial risk-sharing; i.e., $c_2(p,i) = c_1(i) > c_2(i,i)$. In this case, it is straightforward to demonstrate that condition (8) is always satisfied with a strict inequality for the preferences considered here. If, instead, the optimal allocation is characterized by full risk-sharing; i.e., $c_2(p,i) > c_1(i) > c_2(i,i)$, then we can demonstrate (numerically) that condition (8) holds with strict inequality for a wide range

⁶What if the reports of the depositors are not consistent with the banker's report? In this situation, suppose that $c_n(a'_n) = 0$ for all n and $b' = R(Ny - \sum_n c_n) - Z$, where Z is made arbitrarily large. That is, if the reports are inconsistent, then the depositors receive a zero date-2 payoff and the banker gets to consume all of its deposits; and, in addition, receives a large, negative payoff from the mechanism. This payoff scheme will ensure that all reports will be consistent.

⁷Since depositors do not know their place in line, it is optimal to deliver to depositors the same utility payoff in state (p,p) .

of parameter values.⁸ This implies that the optimal allocation with observable depositor types can be implemented as a truth-telling equilibrium when depositor types are private information. Hence, for the preferences considered here, having depositor types private information in no way affects our earlier analysis.

We now move to investigate the possibility of bank-run equilibria. To this end, let \mathbf{c}^* represent the optimal allocation that can be implemented as a truth-telling equilibrium. Given the mechanism (Ω, \mathbf{c}^*) , does there exist another equilibrium in which patient depositors misreport their type? In particular, does there exist an equilibrium where the strategy of all depositors is to announce i and the banker reveals the history of depositor announcements truthfully?

It is straightforward to demonstrate that there does not exist a bank-run equilibrium of this form. To see this, suppose that all depositors play the bank-run strategy. Then, the banker will announce $m_n = i$ for $n = 1, 2$ and $m' = (i, i)$. The banker will announce $m_1 = i$, instead of the truth $m_1 = \emptyset$, because this results in a payment to the first depositor of $c_1(i, i) < c_1(i)$ leaving the banker with payoff $R[c_1(i) - c_2(i, i)] > 0$, which is higher than the proposed equilibrium payoff of zero. Hence, when depositors play bank-run strategies in the mechanism (Ω, \mathbf{c}^*) , the banker will depart from proposed equilibrium play. We conclude that the uniqueness result reported in [5] appears robust to our form of agency.

Note that, as in [5], the depositor truth-telling constraint (8) does not bind. Interestingly, [7] finds no evidence of bank-runs even under the mechanism they consider when the depositor truth-telling constraint is slack. They are, however, able to generate a bank-run equilibrium when they modify preferences so that the depositor truth-telling constraint (8) binds. In particular, it turns out that (8) will bind for preferences:

$$U(c, c', \omega) = \begin{cases} u(c + c') & \text{if } \omega = p; \\ Au(c) & \text{if } \omega = i; \end{cases} \quad (9)$$

where $u(x) = (1 - \sigma)^{-1}x^{1-\sigma}$, $\sigma > 1$ and $A > 0$ sufficiently large.

Suppose that we too modify preferences in the manner suggested in [7] and assume that (8) binds. Since this is important for their result, we continue to follow [7] in assuming that the mechanism does not provide depositors with any information regarding previous announcements or their place in line. Denote the optimal allocation that can be implemented as a truth-telling equilibrium when condition (8) binds as $\bar{\mathbf{c}}$. As demonstrated in [7]—via an example for $N = 2$ —when the depositor truth-telling constraint binds, it is possible to generate a bank-run equilibrium for allocation $\bar{\mathbf{c}}$; that is, there exists an equilibrium where all depositors, independent of type, report i at date-1.

It is, however, straightforward to demonstrate that this result does not extend to our environment. The reason for this is lies, as before, in the fact that conditional

⁸In fact, we could not find parameter values where the condition is violated.

on all depositors announcing that they are impatient, a self-interested banker is able to profitably lie by reporting that every depositor is the last in line and that all previous depositors reported i . Hence, we conclude here that the non-uniqueness result reported in [7] appears *not* to be robust to our form of agency.

It is interesting to note that [7] is only able to generate a bank-run equilibrium by assuming that their banker/mechanism faithfully conditions the allocation on the *true* history of reports (even though the mechanism does not reveal any information to depositors). In contrast, our self-interested banker has a strong motive to depart from truth-telling if it is in his interest to so—as would be the case when depositors take a “run” at the bank. Hence, while both [5] and [7] allude to the possibility that bank incentive problems may expand the possibility of bank-run equilibria, our analysis seems to indicate—at least, for the manner in which we have modeled the banker agency problem—that this need not be the case.

7 Conclusion

Much of the recent literature on bank-runs, notably [5] and [7], adopts the view of a bank as a coalition of depositors. The standard environment is in the tradition of [3], extended to incorporate aggregate uncertainty and an explicit sequential service constraint. In both [5] and [7], the optimal bank contract is an elaborate design. [5] demonstrates that when depositor incentive-compatibility constraints do not bind, the first-best allocation is uniquely implementable, i.e., there is no bank-run equilibrium. [7] demonstrates that when depositor incentive-compatibility constraints do bind—so that the resulting allocation cannot be first-best—then a mechanism that withholds some information from depositors admits the possibility of a bank-run equilibrium.

In contrast to [5] and [7], we chose to view a bank as a self-interested agent with some technological advantage in record-keeping. We found first that the need to align bank and depositor incentives precludes implementation of the first-best allocation even when depositor incentive-compatibility constraints are slack. Second, we found that the agency problem studied here may or may not simplify the structure of the optimal bank contract. Indeed, in some cases, the resulting bank contract becomes even more complicated than in [5] and [7], as positive date-1 payouts to patient depositors are needed to implement allocations with a high degree of risk-sharing. Third, we found that the agency problem studied here in no way affects the conclusion in [5] concerning the possibility of bank-runs. On the other hand, there appears to be an interesting implication for the conclusion in [7]. In particular, the bank-run equilibrium discovered in [7] appears to disappear when the bank is modeled as a self-interested agent.

8 References

- [1] D. Andolfatto, E. Nosal and N. Wallace, The role of independence in the Green-Lin Diamond-Dybvig Model. Forthcoming, *J. Econ. Theory*.
- [2] C. Calomiris and C. Kahn, The role of demandable debt in structuring optimal banking arrangements. *Amer. Econ. Review* 81 (1991), pp. 497-513.
- [3] D. Diamond and P. Dybvig, Bank runs, deposit insurance, and liquidity. *J. Polit. Econ.* 91 (1983), pp. 401-419.
- [4] D. Diamond, Financial intermediation and delegated monitoring, *Rev. Econ. Studies* 51 (1984), pp. 393-414.
- [5] E. Green and P. Lin, Implementing efficient allocations in a model of financial intermediation, *J. Econ. Theory* 109 (2003), pp. 1-23.
- [6] S. Krasa and A.P. Villamil, Monitoring the monitor: An incentive structure for a financial intermediary. *J. Econ. Theory* 57 (1992), pp. 197-221.
- [7] J. Peck and K. Shell, Equilibrium Bank Runs, *J. Polit. Econ.* 111 (2003), pp. 103-123.
- [8] N. Wallace, Another attempt to explain an illiquid banking: the Diamond–Dybvig model with sequential service taken seriously. *Fed. Reserve Bank Minneapolis Quart. Rev.* 12 (1988), pp. 3–16.

9 Appendix A: Proof to Propositions 2 and 3

Here we sketch out the proof to the optimal allocation, as summarized in propositions 2 and 3. In order to determine the nature of the optimal allocation, we start the analysis at date-2.

9.1 Date-2

At date-2, the true history $\omega^2 \in \Omega^2$ is known to the banker. Regardless of what happened historically, the banker at this point will submit a final report m' that minimizes depositor payouts. This final report must respect consistency. Hence, if either $\omega^2 = (p, p)$ or $\omega^2 = (i, i)$, the banker is constrained to tell the truth. However, if the state of the world is either $\omega^2 = (i, p)$ or $\omega^2 = (p, i)$, consistency requires only that the banker's final report satisfy $k(i, p) = k(p, i) = 1$; i.e., the banker's report must contain one i and one p (in either order). A truthful report in this case requires that the date-2 payout to depositors is the same for either of these states; i.e.,

$$c'_1(p, i) + c'_2(p, i) = c'_1(i, p) + c'_2(i, p). \quad (10)$$

One can without loss of generality impose $c'_1(i, i) = c'_2(i, i) = c'_1(i, p) = c'_2(p, i) = 0$, as making date-2 payments to impatient depositors does nothing to align bank incentives, serving only to waste resources. Consequently, condition (10) can be reduced to:

$$c'_1(p, i) = c'_2(i, p) \equiv X. \quad (11)$$

Let $C \equiv c'_1(p, p) + c'_2(p, p)$. Then given condition (11), the banker's *ex post* payoffs under truth-telling (assuming a feasible allocation \mathbf{c}) are given by:

$$\begin{aligned} b'(p, p) &= R[2y - c_1(p) - c_2(p, p)] - C \geq 0; \\ b'(p, i) &= R[2y - c_1(p) - c_2(p, i)] - X \geq 0; \\ b'(i, p) &= R[2y - c_1(i) - c_2(i, p)] - X \geq 0; \\ b'(i, i) &= R[2y - c_1(i) - c_2(i, i)] \geq 0. \end{aligned} \quad (12)$$

In what follows, we conjecture that the optimal allocation can be made to deliver:

$$b'(p, p) = b'(i, i) = 0. \quad (13)$$

Note that zero bank profits can be achieved in any state of the world is not a foregone conclusion when the banker has private information. Hence, this conjecture must subsequently be checked for its validity.

9.2 Date 1 (Depositor 2)

Assume that the true history is $\omega^1 = i$ and that the banker told the truth in the past; i.e., $t_1 = \emptyset$. A truthful report to the second depositors entails $t_2 = i$; there are two possible deviations to consider, $m_2 \in \{\emptyset, p\}$. We assume throughout that date-1 payoffs to impatient depositors follow $c_2(p, i) \geq c_1(i) > c_2(i, i)$.

Lemma 1 *If $\omega^1 = i$, then the banker report $m_2 \in \{\emptyset, p\}$ is infeasible.*

Proof Suppose that the true state turns out to be $\omega^2 = (i, i)$. Then the lie $m_2 = p$ generates an *ex post* profit $R[2y - c_1(i) - c_2(p, i)]$, which is strictly negative if $b'(i, i) = 0$, since $c_2(p, i) > c_2(i, i)$. Likewise, the lie $m_2 = \emptyset$ generates an *ex post* profit $R[2y - c_1(i) - c_1(i)]$, which is strictly negative if $b'(i, i) = 0$, since $c_1(i) > c_2(i, i)$.

Lemma 1 tells us that if the first depositor is impatient, then the banker has no incentive to misreport the truth to the second depositor—and in date 2—when $c_2(p, i) \geq c_1(i) > c_2(i, i)$ (and $c'_1(p, i) = c'_2(i, p)$).

Assume now that the true history is $\omega^1 = p$ and that the banker told the truth in the past; i.e., $t_1 = \emptyset$. A truthful report to the second depositor is $t_2 = p$; there are two possible deviations to consider, $m_2 \in \{\emptyset, i\}$.

Lemma 2 *If $\omega^1 = p$ and $c_2(i, p) = c_2(p, p) = 0$, then the banker report $m_2 = i$ represents a profitable deviation from truthtelling.*

Proof If $\omega^2 = (p, p)$, then the banker's ex post payoffs are the same whether $m_2 = i$ or $m_2 = t_2 = p$, i.e., $R[2y - c_1(p) - c_2(i, p)] - C = R[2y - c_1(p) - c_2(p, p)] - C$. If, however, $\omega^2 = (p, i)$, then the banker's ex post payoff associated with reporting $m_2 = i$ exceeds that of telling the truth, i.e., $R[2y - c_1(p) - c_2(i, i)] - X > R[2y - c_1(p) - c_2(p, i)] - X$ since $c_2(p, i) > c_2(i, i)$.

Lemma 3 *If $\omega^1 = p$ and $c_2(i, p) > c_2(p, p) \geq 0$, then the lie $m_2 = i$ is infeasible.*

Proof If $\omega^2 = (p, p)$, then the banker's report $m_2 = i$ generates an *ex post* profit $R[2y - c_1(p) - c_2(i, p)] - C$, which is strictly negative if $c_2(i, p) > c_2(p, p)$ and $b'(p, p) = 0$.

Lemmas 2 and 3 imply that whether the allocation is characterized by partial or full risk sharing, implementation requires that inefficient payments be made to patient depositors; in particular $c_2(i, p) > c_2(p, p) \geq 0$.

Lemma 4 *If $\omega^1 = p$ and the allocation is characterized by full risk sharing— $c_2(p, i) > c_1(i)$ —and $c_1(p) = c_2(p, p) = 0$, then the banker report $m_2 = \emptyset$ represents a profitable deviation from truthtelling.*

Proof If $\omega^2 = (p, p)$, then the banker's ex post payoffs are the same whether $m_2 = \emptyset$ or $m_2 = t_2 = p$, i.e., $R[2y - c_1(p) - c_2(p)] - C = R[2y - c_1(p) - c_2(p, p)] - C$. If, however, $\omega^2 = (p, i)$, then the banker's ex post payoff associated with reporting $m_2 = \emptyset$ exceeds that of telling the truth, i.e., $R[2y - c_1(p) - c_1(i)] - X > R[2y - c_1(p) - c_2(p, i)] - X$ since $c_2(p, i) > c_1(i)$.

Lemma 5 *If $\omega^1 = p$ and $c_1(p) > c_2(p, p) \geq 0$, then the lie $m_2 = \emptyset$ is infeasible.*

Proof Suppose that the true state turns out to be $\omega^2 = (p, p)$. Then the lie $m_2 = \emptyset$ generates an *ex post* profit $R[2y - c_1(p) - c_1(p)] - C$, which is strictly negative if $c_1(p) > c_2(p, p)$ and $b'(p, p) = 0$.

Lemmas 4 and 5 imply that if the allocation is characterized by complete risk sharing then additional inefficient payments must be made; in particular $c_1(p) > c_2(p, p) \geq 0$. Note that if the allocation is characterized by partial risk sharing— $c_2(p, i) = c_1(i)$ —there is *not* a profitable deviation for the banker when $c_1(p) = c_2(p, p) = 0$ and $\omega^1 = p$.

9.3 Date 1 (Depositor 1)

At this stage, the true history is $\omega^0 = \emptyset$. A truthful report now entails $t_1 = \emptyset$ and there are two possible deviations to consider, $m_1 \in \{i, p\}$. In what follows, we assume that the banker continues to tell the truth following any ‘one-shot’ deviation (it can be shown that this represents optimal behavior for the banker.) The first question we ask is there a relationship between $c_2(i, p)$ and $c_1(p)$?

Lemma 6 *Suppose that $c_2(i, p) = c_1(p)$. Then $m_1 = i$ is a profitable deviation.*

Proof If $\omega^2 = (p, p)$ or $\omega^2 = (p, i)$ then the banker's ex post payoffs are the same whether $m_1 = i$ or $m_1 = t_1 = \emptyset$, i.e., for $\omega^2 = (p, p)$ $R[2y - c_1(i, p) - c_2(p, p)] - C = R[2y - c_1(p) - c_2(p, p)] - C$; and similarly for $\omega^2 = (p, i)$. If, however, $\omega^2 = (i, i)$ or $\omega^2 = (i, p)$, then the banker's ex post payoff associated with reporting $m_1 = i$ exceeds that of telling the truth, i.e., $R[2y - c_1(i, i) - c_1(i, i)] - X > R[2y - c_1(i) - c_2(i, i)] - X$ since $c_2(i) > c_1(i, i)$; and similarly for $\omega^2 = (i, p)$.

Lemma 7 *If $c_2(i, p) > c_1(p)$, then the lie $m_1 = i$ is infeasible.*

Proof Suppose that the true state turns out to be $\omega^2 = (p, p)$. Then the lie $m_1 = i$ generates an *ex post* profit $R[2y - c_2(i, p) - c_2(p, p)] - C$, which is strictly negative when $c_2(i, p) > c_1(p)$ and $b'(p, p) = 0$.

Let us now focus on the full risk sharing allocation. Lemmas 6 and 7 imply that any implementable allocation must be characterized by $c_2(i, p) > c_1(p)$. Note

also, that lemmas 2-5, together with lemmas 6 and 7, imply that if the allocation is characterized by full risk sharing, implementation requires that

$$c_2(i, p) > c_1(p) > c_2(p, p) \geq 0. \quad (14)$$

When condition (14) holds, then the deviation $m_1 = p$ is not feasible when the allocation is characterized by full risk sharing. To see this, suppose that the true state turns out to be $\omega^2 = (i, i)$. Then the lie $m_1 = p$ generates an *ex post* profit $R[2y - c_2(p, i) - c_2(i, i)]$, which is strictly negative as $c_2(p, i) > c_1(i)$ and $b'(i, i) = 0$.

As the goal is to maximize depositor welfare, optimality requires that the allocation minimizes inefficient payments to depositors and bank profits, subject to maintaining incentive-compatibility. To begin, note that from (14), we can set $c_2(p, p) = 0$ without altering bank incentives. This implies that we can set $c_1(p)$ to some arbitrarily small (but positive) number $\eta > 0$. *Ex post* bank profits are then given by:

$$\begin{aligned} b'(p, p) &= R[2y - \eta] - C; \\ b'(p, i) &= R[2y - \eta - c_2(p, i)] - X; \\ b'(i, p) &= R[2y - c_1(i) - c_2(i, p)] - X; \\ b'(i, i) &= R[2y - c_1(i) - c_2(i, i)]; \end{aligned} \quad (15)$$

Observe that it remains incentive-feasible to restrict the allocation such that bank profits are driven to zero in every state of the world (this validates our previous conjecture (13)). In this case, $b'(p, i) = b'(i, p)$ implies that:

$$c_2(i, p) - \eta = c_2(p, i) - c_1(i).$$

The condition shows clearly how greater risk-sharing (a larger $c_2(p, i) - c_1(i)$) can only come at the expense of a larger (inefficient) payment $c_2(i, p)$. We can summarize the implications for an optimal full risk sharing allocation by

Proposition 2 *If the optimal allocation is characterized by full risk-sharing— $c_2(p, i) > c_1(i) > c_2(i, i)$ —then truthtelling on the part of the banker will require date-1 payments to patient depositors satisfy $c_2(i, p) > c_1(p) = \eta > 0$. As in the first-best allocation, date-2 payments to impatient depositors are zero and $c_2(p, p) = 0$. The banker makes zero profit in each state of the world.*

Now let's consider the partial risk-sharing allocation. Lemmas 2, 3, 6 and 7 imply that implementation requires that

$$c_2(i, p) > c_1(p) = c_2(p, p) \geq 0 \quad (16)$$

When condition (16) holds, then it is straightforward to show that the deviation $m_1 = p$ is not profitable. Note that from (16), we can set $c_1(p) = c_2(p, p) = 0$ without altering bank incentives. This, along with truthtelling conditions (16), implies that

we are free to set $c_2(i, p)$ to some arbitrarily small (but positive) number $\xi > 0$. *Ex post* bank profits are then given by:

$$\begin{aligned}
b'(p, p) &= R[2y] - C; \\
b'(p, i) &= R[2y - c_1(i)] - X; \\
b'(i, p) &= R[2y - c_1(i) - \xi] - X; \\
b'(i, i) &= R[2y - c_1(i) - c_2(i, i)].
\end{aligned} \tag{17}$$

Observe that it remains incentive-feasible to restrict the allocation such that bank profits are driven to zero in states $\omega^2 \in \{(p, p), (i, i)\}$ (this validates our previous conjecture (13)). It is also possible to set $b'(i, p) = 0$. However, note that it is not possible to do likewise with $b'(p, i)$. Evidently, the bank must make a strictly positive profit in state $\omega^2 = (p, i)$; i.e.,

$$b'(p, i) = R\xi > 0.$$

On the other hand, note that this profit becomes arbitrarily small as $\xi \rightarrow 0$. We can summarize the implications for the optimal partial risk-sharing allocation by

Proposition 3 *If the optimal allocation is characterized by partial risk sharing— $c_2(p, i) = c_1(i) > c_2(i, i)$ —then truthtelling on the part of the banker will require date-1 payments to patient depositors satisfy $c_2(i, p) = \xi > 0$. As in the first-best allocation, date-2 payments to impatient depositors are zero and $c_1(p) = c_2(p, p) = 0$. The banker makes zero profit in all states of the world, except state $\omega^2 = (p, i)$, where he makes an arbitrarily small profit, equal to $R\xi$.*

9.4 Optimal Incentive-Feasible Allocations

Let $W \equiv \max\{W^F, W^P\}$, where W^j denotes the indirect utility function associated with the $j = F, P$ (full or partial) risk-sharing allocation. For the full risk-sharing allocation, substitute the restrictions embedded in (15) into the objective function; so that the problem can be stated as:

$$\begin{aligned}
W^F &= \max_{c_1(i), c_2(p, i), c'_1(p, p)} \pi^2 \{u(c'_1(p, p)) + u(R[2y - \eta] - c'_1(p, p))\} \\
&+ (1 - \pi) \pi \{u(c_1(i)) + u(c_2(p, i) + \eta - c_1(i) + R[2y - c_2(p, i)])\} \\
&\quad + \pi(1 - \pi) \{u(c_2(p, i)) + u(\eta + R[2y - c_2(p, i)])\} \\
&\quad + (1 - \pi)^2 \{u(c_1(i)) + u(2y - c_1(i))\}.
\end{aligned} \tag{18}$$

Keep in mind that the solution to the program above has economic content only over ranges in which $c_2(p, i) > c_1(i)$.

For the partial risk-sharing allocation, substitute the restrictions embedded in (17) into the objective function; so that the problem can be stated as:

$$\begin{aligned}
W^P = \max_{c_1(i), c'_1(p,p)} & \pi^2 \{u(c'_1(p,p)) + u(2Ry - c'_1(p,p))\} + & (19) \\
& (1 - \pi) \pi \{u(c_1(i)) + u(\xi + R[2y - c_1(i) - \xi])\} + \\
& \pi(1 - \pi) \{u(c_1(i)) + u(R[2y - c_1(i) - \xi])\} + \\
& (1 - \pi)^2 \{u(c_1(i)) + u(2y - c_1(i))\}.
\end{aligned}$$

Again, as reported in the text, which of these two scenarios is optimal turns out to depend on parameters. Through the use of numerical examples, one can demonstrate that $W^F > W^P$ for low values of π and high values of σ ; and conversely, $W^F < W^P$ for high values of π and low values of σ .

10 Appendix B: Alternative Information Structure

One might reasonably wonder to what extent the results above are driven by our particular information structure, i.e., depositors cannot recall anything that happened at date-1 but can communicate with one another at date-2. An interesting alternative information structure, suggested by a referee, is one where depositors can recall the outcomes of their date-1 interactions with the banker—i.e., depositor n can remember m_n and $c_n(m_n, \omega_n)$ —but cannot communicate with one another at date-2.⁹ Under this information structure, the banker deals with depositors on a one-on-one basis in date-2, just as in date-1. This information structure has some appeal because the banker can now be viewed as the “sole aggregator of information”; i.e., depositors can communicate with the bank, but cannot communicate with each other.

In this formulation, one would require that the banker’s date-2 messages to depositor n be consistent with (m_n, ω_n) for all n . However, note that since the banker deals with depositors one-on-one at date-2, he can potentially send *different* date-2 reports to different depositors; this was precluded under our original information structure. Since different (consistent) reports can be sent to different depositors at date-2, the analysis and characterization of the optimal contract turns out to be relatively complicated

In this appendix we show that it is not possible to implement the first-best allocation with this alternative information structure and provide some additional remarks on the nature of implementable contracts.

Assume that each depositor receives a document that embeds the report m_n sent by the banker at date-1. The document cannot be counterfeited nor altered in any way. The banker’s informational advantage over the depositors is formally modeled by assuming that in date-2, as in date-1, depositors meet sequentially and privately with the banker. This implies that depositors are unable to observe each other’s document or bankers reports in either date. As in the information structure in the main text, we continue to assume that the mechanism has no record keeping device; it’s role is to suggest payoffs associated with reports provided by the banker and documentation provided by depositors, as well as their types, at both dates. For simplicity, assume that depositor type is observable.

The date-1 allocation or outcome function is a recommendation $C_n(\cdot)$ made contingent on the banker’s date-1 report and each depositor’s type, i.e.,

$$C_n : \bar{\Omega} \times \Omega_n \rightarrow \mathcal{R}_+ \text{ for } n = 1, 2, \dots, N.$$

(This is identical to the previous analysis.) At date-2, depositors meet sequentially with the banker. The banker reports $m'_n : \Omega^N \rightarrow \Omega^N$ in his meeting with depositor

⁹If depositor n can remember (m_n, ω_n) and depositors can communicate with one another at date-2, then the first-best allocation can be implemented.

n , where the domain represents the set of true histories. Depositor n reports $a'_n : \bar{\Omega} \times \Omega_n \rightarrow \bar{\Omega} \times \Omega_n$, where the domain represents the documented date-1 banker announcement and the depositor's type. The date-2 allocation or outcome function is a recommendation $C'_n(\cdot)$ made contingent on the banker's report and depositor's document and type, i.e.,

$$C'_n : \Omega^N \times \bar{\Omega} \times \Omega_n \rightarrow \mathcal{R}_+ \text{ for } n = 1, 2, \dots, N.$$

As in the main text, we will restrict the banker's date-2 report to be consistent; but now consistency relates to the depositor's type *and* documentation.

Definition 1 *The date-2 strategy m' is said to be consistent if $m'_n = (m_n, \omega_n, \tilde{m}'_n) \in \Omega^N$, where $m_n \in \bar{\Omega}$ is the banker's date-1 message to depositor n , ω_n is depositor n 's type and \tilde{m}'_n is the banker's date-2 report of depositor types that followed depositor n .*

Here, consistency requires that the banker's date 2 announcement to depositor n embed his date-1 announcement, as well as depositor n 's type, i.e., for each depositor, the relevant part of the banker's date-2 announcement must “agree” with his date-1 announcement. As in the previous information structure, consistency does not imply truth-telling.

It is straightforward to demonstrate that with the information structure described above, the first-best allocation cannot be implemented.

Proposition 2 *The first-best allocation cannot be implemented.*

Proof. Recall that the first-best allocation is characterized, in part, by $c'_1(p, i) < c'_1(p, p) = Ry$. Suppose that the banker makes truthful reports in date-1 and likewise in date-2 except in state (p, p) . In state (p, p) , suppose that the banker reports $m'_1 = (p, i)$ and $m'_2(p, p)$ to the first and second depositors, respectively, both of which are consistent announcements. The second depositor will receive $c'_2(p, p) = Ry$, while the first depositor will receive $c'_1(i, p) < c'_1(p, p)$. Since $Ry - c'_2(p, p) - c'_1(i, p) > 0$, the banker will generate a positive payoff for himself at date-2, which is a profitable deviation from truth-telling. Therefore, the first-best allocation is not incentive feasible. ■

The profitable deviation described in the proof to the above proposition gives a hint as to some of the (date-2) incentive compatibility constraints that are required. In particular, it must be the case that $c'_1(p, p) = c'_1(p, i)$; if his were not so, then the banker would be able to generate a higher payoff than the truth-telling payoff by, for example, reporting $m'_1(p, p)$ in state (p, i) if $c'_1(p, p) < c'_1(p, i)$ or by reporting $m'_1(p, i)$ in state (p, p) if $c'_1(p, p) > c'_1(p, i)$.

Just as in the information structure studied in the main text, it can be shown that if the contract is characterized by $c_2(p, i) > c_1(i)$, then a non-trivial (inefficient)

payment must be made to a patient depositor in date-1. If the inefficient payment is not made, then incentive compatibility will require that $c_2(p, i) = c_1(i)$.

For the information structure used in the text, (in)feasibility of banker payments played a major role in disciplining bank reports. In the alternative information structure, since the banker can give different date-2 reports to different depositors, it turns out that the one cannot rely solely on feasibility arguments to ensure truthful bank reports and, instead, explicit truthtelling constraints must be satisfied. Hence, the actual characterization of optimal allocations will be complicated by the required satisfaction of a number of truthtelling constraints (that do not appear—because they are not required) in the maximization problems described in appendix A.

Under this alternative information structure, we are able to demonstrate that the first-best is not implementable. The qualitative structure of the optimal incentive-feasible allocation—whether partial or full risk-sharing—depends on parameters. We further suspect, although we have not demonstrated formally, that there are no bank-run equilibria under the mechanisms studied in [5] and [7].