Solution of the Ellsberg paradox by means of the principle of uncertain future

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The principle of uncertain future: the probability of a future event contains an (hidden) uncertainty. The first consequence of the principle: the real values of high probabilities are lower than the preliminarily determined ones; conversely, the real values of low probabilities can be higher than the preliminarily determined ones. The first consequence provides an uniform solution of the underweighting of high and the overweighting of low probabilities, of the Allais paradox, risk aversion, loss aversion, the equity premium puzzle, the "fourfold pattern" paradox, etc. The second consequence: the present probability system of a future event is incomplete. The second consequence provides a solution of the incompleteness of systems of preferences, of ambiguity aversion, of the Ellsberg paradox, etc.

Contents

Introduction

This paper presents a part of the results of Харин (2007) including a solution of the Ellsberg paradox in English.

The final statement of Hey and Orme (1994) was "... we are tempted to conclude by saying that our study indicates that behavior can be reasonably well modeled (to what might be termed a "reasonable approximation") as "expected utility plus noise." Perhaps we should now spend some time thinking about the noise, rather than about even more alternatives to expected utility?"

Harin (2004), Harin (2007), Харин (2007) and this paper renew, generalize and develop this statement.

1. Principle of uncertain future

1.1. Statement and formal proof of the principle

General principle of uncertain future

Future events may be considered as, at least partially, uncertain.

This uncertainty or partial uncertainty may be invisible or imperceptible. It may be crucial. In any case, the overwhelming majority of future events contain, at least a part of uncertainty. In a simple form this principle may sound like:

"A future event contains an uncertainty"

Formal proof of the principle

The principle of uncertain future may be proved, e.g., by means of the Heisenberg's uncertainty principle. The Heisenberg's uncertainty principle states:

One cannot simultaneously measure both impulse and position better than with uncertainty

$$\Delta p \times \Delta x \geq \hbar / 2$$

where

Δp - impulse uncertainty,
Δx - position uncertainty,
$\hbar$ - Planck's constant divided by 2π

Indeed, the Heisenberg's uncertainty principle is true for every object involved in every event, including every future event. Hence, every future event contains an uncertainty. The proof has been completed.

Specific principle of uncertain future

The specific principle of uncertain future emphasizes one of uncertain aspects of future events, namely probability. It states probabilities of future events are, to some extent, uncertain. This extent may be invisible, imperceptible. It may be considerable, even crucial. In any case, the overwhelming majority of future events contain, at least, a degree of uncertainty. In a simple form this principle may sound like:

"The probability of a future event contains an (hidden) uncertainty"

Mathematically, the specific principle may be written in the form of two simplified (in detail see Harin 2007) expressions:
The first

$$\boldsymbol{P_{real} \sim P_{plan} \pm \Delta P}$$

where
ΔP = (plus) $\Delta_{+}P$ and (minus) $\Delta_{-}P$
The second

$$\boldsymbol{P_{mean\ real} = P_{plan} + \delta P}$$

where
$P_{mean\ real}$ - the mean real value of P;
δP - the shift, the bias of the mean real value of real or future P in the comparison with the value of planned or preliminarily determined P (δP may be positive or negative or zero).

1.2. Consequences of the principle

1.2.1. First consequence. The "repulsion" from the "rigid" bounds of the range of probability (from 0% and from 100%)

Suppose we wish to test the probability values, which are very close (but not equal) to 0% or 100%. For example, we choose 1% or 99%.

Suppose the uncertainty value (ΔP) is essentially more than the distance of the probability value from the bound. For example, ΔP = 10%.

Then, evidently, (if we make the test as if there is no uncertainty) the mean distance of the probability value from the bound cannot be as small as 1% (if the uncertainty value is 10%).

Generally, the mean distance of the probability value from the bound cannot be considerably less than the uncertainty value.

Thus, the mean value of probability ($P_{real\ mean} \equiv P_{real}$) cannot be as high as 99%. It may be (*see the second consequence below) more than 1% also. Or

$$\boldsymbol{P_{high\ real} < P_{high\ plan}}$$
$$\boldsymbol{{}^{*}P_{low\ real} > P_{low\ plan}}$$

In other words

"High probability will be lower"
"Low probability can be higher"

1.2.2. Second consequence. Incompleteness of the present probability system of a future event

The probability of an event, which is not forbidden by objective laws is more than zero (in the microcosm virtual events can occur that infringe the laws of conservation). Hence, an unforeseen event with the probability more than zero will occur in any forecast or plan. Or

"The present probability system of future events is incomplete"

Or

$$\sum \boldsymbol{P_{unforeseen}} > 0\%$$
$$\sum \boldsymbol{P_{foreseeable}} < 100\%$$

where

$\sum P_{foreseeable}$ - real sum of probabilities of all foreseeable events

$\sum P_{unforeseen}$ - real sum of probabilities of all unforeseen events

2. General applications of the principle

The principle of uncertain future, due to its general nature, may be used and is successfully used in a variety of fields.

2.1. Logic

2.1.1. Possible infringement of the law of the excluded middle

Suppose in present there are a class of events and the negation of this class. The second consequence of the principle allows in future an event occur which do not exactly belong neither to this class nor to its negation.

So, the application of the law of the excluded middle to future events is questionable. Note, already Aristotel proposed similar opinion.

2.2. Probability theory

2.2.1. The need of creation of a section of the probability theory which will consider real noises and uncertainties

There is the need of creation of a section of the probability theory which will consider real noises and uncertainties, errors in measurements, etc.

2.2.2. Splitting of the continuous range of probability near 0% and 100%

Noises and uncertainties can lead to splitting of the continuous range of probability near 0% and 100%. Similar phenomena take place in the physics of elementary particles as the violation of symmetries.

2.3. Forecasting and planning

2.3.1. Creation of a basic equation of forecasting

The principle of uncertain future and sectionally continuous transformation provide creation of a basic equation of forecasting. This equation is supposed in Харин (2008).

3. First consequence. Solution of problems of economics

3.1. Uniform solution of the underweighting of high and the overweighting of low probabilities, the Allais paradox, risk aversion, loss aversion, the equity premium puzzle, the "fourfold pattern" paradox, etc

Solution of the Allais paradox (Allais 1953), risk aversion, loss aversion, overweighting of low probabilities, uniform explanation of choices for both gains and losses, the equity premium puzzle, etc has been reported in Harin (2007).

A simplified solution of the "fourfold pattern" paradox

The "fourfold pattern" paradox (see, e.g., Tversky and Wakker 1995, Fehr-Duda et al 2006) is one of the strongest qualitative tests for utility theories. The well-determined facts are: For positive (gains) risky prospects, people typically overweight low probabilities but underweight high probabilities. For negative (losses) risky prospects, people typically underweight low probabilities but overweight high probabilities. This may be written as

1) For gains at high probabilities people choose guarantee

Risk < Guarantee

2) For gains at low probabilities people choose risk

Risk > Guarantee

3) For losses at high probabilities people choose risk

Risk > Guarantee

4) For losses at low probabilities people choose guarantee

Risk < Guarantee

From the first consequence of the principle of uncertain future

$\boldsymbol{P_{high\ real}} < \boldsymbol{P_{high\ plan}}$

$*\boldsymbol{P_{low\ real}} > \boldsymbol{P_{low\ plan}}$

making evident conclusion the possibility of $*\boldsymbol{P_{low\ real}} < \boldsymbol{P_{low\ plan}}$ at the equilibrium is enough to people to choose the corresponding choice and to write $\boldsymbol{P_{low\ real}} < \boldsymbol{P_{low\ plan}}$,
denoting the value of the risky gain as $\boldsymbol{G}$ and the value of the risky loss as $\boldsymbol{-G}$,
and remembering ***planned Risk = planned Guarantee*** we obtain:

1) Risk = $\boldsymbol{G} \times \boldsymbol{P_{high\ real}} < \boldsymbol{G} \times \boldsymbol{P_{high\ plan}}$ = Guarantee

Risk < Guarantee

2) Risk = $\boldsymbol{G} \times \boldsymbol{P_{low\ real}} > \boldsymbol{G} \times \boldsymbol{P_{low\ plan}}$ = Guarantee

Risk > Guarantee

3) Risk = $\boldsymbol{-G} \times \boldsymbol{P_{high\ real}} > \boldsymbol{-G} \times \boldsymbol{P_{high\ plan}}$ = Guarantee

Risk > Guarantee

4) Risk = $\boldsymbol{-G} \times \boldsymbol{P_{low\ real}} < \boldsymbol{-G} \times \boldsymbol{P_{low\ plan}}$ = Guarantee

Risk < Guarantee

4. Second consequence. Solution of problems of economics

4.1. Solution of the incompleteness of systems of preferences, ambiguity aversion, the Ellsberg paradox, etc

The Ellsberg paradox (Ellsberg 1961) (simplified and modified): The urn U1 contains red and black balls with unknown proportion. The urn U2 contains red and black balls with certain proportion 1:1. You will win $100 if you draw a ball of the determined color from the urns U1 or U2. Most people stated that they prefer U1 to U2 for both red and black ball.

The situation seems as if

$$P_{red\ uncertain} + P_{black\ uncertain} < 100\%$$

or, more exactly,

$$P_{red\ uncertain} + P_{black\ uncertain} < P_{red\ certain} + P_{black\ certain}$$

Consider the Ellsberg paradox from the point of view of the second consequence of the principle of uncertain future. The second consequence states the present probability system of a future event is incomplete.

$$\sum P_{unforeseen} > 0\%$$
$$\sum P_{foreseeable} < 100\%$$

And the more present, initial uncertainty - the more the real sum of probabilities of all unforeseen events and the less the real sum of probabilities of all foreseeable events. If the initial uncertainty Unc1 is more than the initial uncertainty Unc2, then

$$\sum P_{unforeseen\ Unc1} > \sum P_{unforeseen\ Unc2}$$
$$\sum P_{foreseeable\ Unc1} < \sum P_{foreseeable\ Unc2}$$

The initial uncertainty of the unknown proportion is evidently more than initial uncertainty of the certain proportion. Hence, the real sum of probabilities of all foreseeable events (red or black) for the unknown proportion is or (due to the experience of tested people) seems less than that of the certain proportion.

$$^*\sum P_{unforeseen\ uncertain} > \sum P_{unforeseen\ certain}$$
$$^*\sum P_{foreseeable\ uncertain} < \sum P_{foreseeable\ certain}$$

*(is or seems)

and

$$^*P_{red\ uncertain} + P_{black\ uncertain} < P_{red\ certain} + P_{black\ certain}$$

*(is or seems)

So, in the light of the second consequence of the principle of uncertain future, the Ellsberg paradox is quite natural.

The incompleteness of systems of preferences and ambiguity aversion are quite natural also.

Conclusions

In a simplified form, the conclusions of this paper may be drawn as follows:

The principle of uncertain future

The principle of uncertain future:

"The probability of a future event contains an (hidden) uncertainty"

The first consequence of the principle:

"High probability will be lower"

"Low probability can be higher"

or

$$\boldsymbol{P_{high\ real} < P_{high\ plan}}$$
$$\boldsymbol{{}^*P_{low\ real} > P_{low\ plan}}$$

(*can be)

The second consequence of the principle:

"The present probability system of future events is incomplete"

or

$$\sum \boldsymbol{P_{unforeseen} > 0\%}$$
$$\sum \boldsymbol{P_{foreseeable} < 100\%}$$

The principle of uncertain future, due to its general nature, may be used and is successfully used in a variety of fields, e.g., in logic, in probability theory, in forecasting and planning.

In economics the first consequence of the principle provides an uniform solution of the underweighting of high and the overweighting of low probabilities, of the Allais paradox, risk aversion, loss aversion, the equity premium puzzle, the "fourfold pattern" paradox, etc. More detailed application of the principle will provide an explanation of the full shape of the probability weighting function.

Solution of the Ellsberg paradox

The Ellsberg paradox seems as if

$$P_{red\ uncertain} + P_{black\ uncertain} < 100\%$$

or, more exactly,

$$P_{red\ uncertain} + P_{black\ uncertain} < P_{red\ certain} + P_{black\ certain}$$

The second consequence of the principle provides a solution of the Ellsberg paradox. The second consequence gives

$${}^*\sum P_{foreseeable\ uncertain} < \sum P_{foreseeable\ certain}$$

*(is or seems)

and

$${}^*P_{red\ uncertain} + P_{black\ uncertain} < P_{red\ certain} + P_{black\ certain}$$

*(is or seems)

So, from the point of view of the second consequence of the principle of uncertain future, the Ellsberg paradox is quite natural.

The incompleteness of systems of preferences and ambiguity aversion are quite natural also.

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