Environmental policy in dynamic models: The impact of the elasticity of substitution if consumers pollute

Barthel, Jens

Humboldt-Universität zu Berlin, Wirtschaftswissenschaftliche Fakultät

23 November 2005

Environmental Policy in Dynamic Models: The Impact of the Elasticity of Substitution if Consumers Pollute

Jens Barthel
Humboldt-Universität zu Berlin,
School of Business and Economics

June 27, 2006

Abstract

This paper discusses first results of a comparative study of different environmental policy instruments. In a model with pollution as a side effect of consumption different environmental policies are studied. In simulations we observe the dynamic behavior of models with utility functions of the Leontief, CES, and Cobb-Douglas type. Environmental policy is modeled as a consumption tax. Tax revenues are used to pay a subsidy for environment-friendly activities, are reimbursed as lump-sum payments or vanish. Furthermore we investigate the implications of errors in the choice of instruments.

1 Introduction

Environmental policy instruments are usually evaluated in a static context. In addition to dimensions such as static efficiency, information intensity, ease of monitoring and enforcement, flexibility and political considerations Bohm and Russell (1985) examine dynamic incentives of policy instruments. Their focus lies on effects on the development of new technologies, the impact on relative factor prices and on consequences for locational decisions. With respect to instruments such as taxes, tradeable rights, and direct regulation they discuss the adaptation to changes of exogenous variables, the incentives to develop new technologies and the effect on market structures.

Dynamic aspects of environmental policy instruments are discussed in studies about the interaction of environmental policy and economic growth. Fundamental studies were published by Bovenberg and de Mooij (1997), Bovenberg and Smulders (1995, 1996), Forster (1973), Gradus and Smulders (1993), Huang and Cai (1994), Lighthart and van der Ploeg (1994) as well as Smulders and Gradus (1996). Hettich (2000) summarizes the above-mentioned literature about the interaction between economic growth and environmental policy. Furthermore he analyzes several aspects in the linear growth model and in three different versions of the Uzawa-Lucas growth model. To study transitional dynamics he uses discrete versions of his models. In these models it is possible to analyze the impact of parameter changes resulting, for example, from increased environmental care caused by better information about the consequences of pollution.

Pittel (2002) investigates different issues of the interrelation between sustainable development and economic growth. Besides an in-depth survey of the theoretical studies she develops models with a focus on recycling, endogenous time preferences and the effects of economic integration on growth and pollution - topics that are hardly mentioned in the theoretical literature, although they are extensively discussed in applied economic studies.

Here we focus on aspects that are also not in the center of theorists' debate. First we generalize previous investigations by analyzing explicitly Cobb-Douglas, CES and Leontief utility functions. Studies published up to now concentrate on Cobb-Douglas utility functions or were made without any specification of the elasticity of substitution. In any case, the impact of the specific form of the utility function has been neglected so far. However, we consider its specification to be very important. One reason is that we can speculate that the elasticity of substitution is not constant in the long run. If degradation is severe, substitution may be impossible. But of course, if environmental quality is high enough, we may have the Cobb-Douglas case. The consequences of such a change should be clarified.

Assuming different types of utility functions has repercussions on the effect of environmental policy instruments. If substitution is easily possible, a rising price of the use of the environment can be expected to have little impact on environmental quality. This results from high elasticities of substitution; investment in environment-friendly activities may not be that necessary. Otherwise, easy substitution possibilities may also call for strong policy measures if welfare gains are high.

In the political process time plays a significant role. Environmental degradation and protection are slow-moving processes; political decision-making - although it seems sometimes sluggish - is comparatively a day-to-day affair and tends to be myopic. The influence of variables - exogenous as well as those determined in the political process - on the speed of convergence of an economic system to the optimal path is therefore an important matter.

One important issue is information. Usually perfect information is assumed. Instead, in a second set of models we look at situations where policy makers choose a too high or too low tax rate due to incomplete information about preferences and technologies. This is far more realistic, since especially the estimation of preferences for environmental quality is a serious, unsolved problem (Mäler, 1985). Furthermore, also information about technologies - and consequently about abatement costs - is private, and it is widely known that there are barely incentives to reveal this information.

Since taxes generate revenues, we can wonder about the impact of the combination of taxes with subsidies on environment-friendly activities (abatement or cleaning). And we can compare the results with lump-sum reimbursements (negative lump-sum taxes). The welfare implications of environmental policy are still an important point in the political discussion and influence the acceptance of green policy measures quite a bit.

The paper is organized as follows: In the next section we introduce the basic model. Section (3) discusses the market solution of the model and the influence of the elasticity of substitution on the equilibrium solution and the speed of convergence. Section (4) introduces environmental policy. To construct a reference point we calculate the solution of the social planner. Examples of different combinations of instruments are examined. Section (5) summarizes the results and gives a brief outlook on possible extensions and variations of the model.

2 The Basic Model

In the following we introduce the basic model and its underlying assumptions. Specifications of the functions are given in the following sections.

2.1 Environment

The quality of the natural environment $N(t)$ depends only on the flow of pollution. There is no accumulation of pollutants. We assume that all pollutants which are not eliminated due to environmental protection vanish in the next moment. This resembles a situation with infinite but somewhat lagged self-regenerating capacity of the environment. Examples of pollutants of this type are traffic noise, malodor from thinners or other chemical substances and - sometimes - food, and last but not least cigarettes and cigars.[1] In summary one can say pollution $P(C)$ is a damaging side effect of consumption C. On the other hand, the burden on the environment depends on effect $E(S)$ of the share of income devoted to clean the environment S. Without any economic activities such as consumption or cleaning, the quality of the natural environment is $\overline{N}$. It follows:

$$N = N\left(E(S), P(C), \overline{N}\right)$$

with:

$$N_E > 0 \qquad N_P < 0$$

2.2 Households and Preferences

We assume n identical households, especially of equal size and small. The representative household exhibits preferences over consumption goods and environmental amenities. Population growth is zero. The rate of time preference is ρ with $\rho > 0$. The elasticity of substitution, $0 \leq \sigma \leq 1$, and the relative weight of environmental amenities in utility, $\phi > 0$, are constant. The utility function of the individual household can be written as:

$$W_i = \int_0^\infty U(c_i, N, \phi) \cdot e^{-\rho \cdot t} dt \tag{1}$$

with the household's consumption being c_i and the public good environmental quality N.

Households supply perfectly inelastic one unit of labor and receive a wage w. Each household holds assets a with a rate of return r. Part of its income can be invested "into the nature" to improve the regenerative capacity of the environment. This is something like trash collection with costs or engagement in environmental activities. The endogenous rate of these investments is $s_{(N)}$. The remaining income will be used for consumption c and saving $\dot{a}$. For the average consumption and investment into the regenerative capacity of the environment follows:

$$C = \sum_{i=1}^{n} c_i \qquad c = \frac{C}{n}$$

$$S_{(N)} = \sum_{i=1}^{n} s_{(N)_i} \qquad s_{(N)} = \frac{S_{(N)}}{n}$$

The flow budget constraint for the household is:

$$w + r \cdot a = \dot{a} + c + s_{(N)} \tag{2}$$

The household's optimization problem is to maximize (1), subject to the budget constraint (2). As derived in Appendix 6.1 the control variables change

[1] We ignore that especially cigars cause stench for days if one cannot open the window.

according to:

$$
\begin{aligned}
g_{(c)} &\equiv \frac{\dot{c}}{c} = \frac{\xi_4 - \xi_2}{\xi_1 \cdot \xi_4 - \xi_3 \cdot \xi_2} \cdot \frac{(\rho - r)}{c} && (3)\\
g_{(s)} &\equiv \frac{\dot{s}}{s} = \frac{\xi_1 - \xi_3}{\xi_1 \cdot \xi_4 - \xi_2 \cdot \xi_3} \cdot \frac{(\rho - r)}{s} && (4)
\end{aligned}
$$

with

$$
\begin{aligned}
\xi_1 &\equiv \frac{U_{cc} + \left[\begin{array}{c} U_N \cdot \left(N_P \cdot P_{CC} + P_C^2 \cdot N_{PP} \right) \\ + U_{NN} \cdot P_C^2 \cdot N_P^2 \end{array} \right] \cdot n + U_{cN} \cdot P_C \cdot N_P \cdot (n+1)}{U_N \cdot N_E \cdot E_S} \\
\xi_2 &\equiv \frac{\left[\left(U_{NN} \cdot N_E \cdot N_P + U_N \cdot N_{EP} \right) \cdot P_C + U_{cN} \cdot N_E \right] \cdot n}{U_N \cdot N_E} \\
\xi_3 &\equiv \frac{\left(U_{NN} \cdot N_E \cdot N_P + U_N \cdot N_{EP} \right) \cdot P_C \cdot n + U_{cN} \cdot N_E}{U_N \cdot N_E} \\
\xi_4 &\equiv \frac{\left(U_N \cdot E_{SS} \cdot N_E + U_{NN} \cdot E_S^2 \cdot N_E^2 + U_N \cdot E_S^2 \cdot N_{EE} \right) \cdot n}{U_N \cdot N_E \cdot E_S}
\end{aligned}
$$

For the change of the quality of nature we can write:

$$
\begin{aligned}
\dot{N} &= N_E \cdot E_S \cdot \dot{S} + N_P \cdot P_C \cdot \dot{C} && (5)\\
&= n \cdot \left(N_E \cdot E_S \cdot \dot{s} + N_P \cdot P_C \cdot \dot{c} \right)
\end{aligned}
$$

2.3 Production

The technology to produce goods in this economy is described by a linear-homogeneous production function with labor L and capital K in efficiency units.

$$
Y = F(K, L) \tag{6}
$$

Since each of the n households supplies one unit of labor and owns the same share of total capital K it follows:

$$
\begin{aligned}
Y &= F(K, n) = n \cdot F\left(\frac{K}{n}, 1 \right) \\
k &\equiv \frac{K}{L} \\
f(k) &\equiv F(k, 1)
\end{aligned}
$$

Output per capita can be expressed by:

$$
y \equiv \frac{Y}{n} = f(k)
$$

The marginal productivities are then given by:

$$\begin{aligned}
\frac{\partial Y}{\partial K} &= n \cdot \frac{\partial f(k)}{\partial k} \cdot \frac{1}{n} \\
&= \frac{\partial f(k)}{\partial k} \\
\frac{\partial Y}{\partial L} &= f(k) + n \cdot \frac{\partial f(k)}{\partial k} \cdot \frac{\partial k}{\partial n} \\
&= f(k) - \frac{K}{n} \cdot \frac{\partial f(k)}{\partial k}
\end{aligned}$$

Output is equal to the sum of the marginal factor productivities multiplied by the quantities:

$$\begin{aligned}
Y &= \frac{\partial Y}{\partial K} \cdot K + \frac{\partial Y}{\partial L} \cdot L \\
&= \frac{\partial f(k)}{\partial \frac{K}{n}} \cdot K + \left[f(k) - \frac{K}{n} \cdot \frac{\partial f(k)}{\partial k} \right] \cdot n \\
&= f(k) \cdot n
\end{aligned}$$

In equilibrium, supply and demand on capital and labor markets are equal. This results in factor returns equal to marginal productivities:

$$\begin{aligned}
r &= \frac{\partial Y}{\partial K} = \frac{\partial f(k)}{\partial k} \\
w &= \frac{\partial Y}{\partial L} = f(k) - k \cdot \frac{\partial f(k)}{\partial k}
\end{aligned}$$

Equilibrium on the capital market ensures that savings are equal to investments. The total amount of capital equals the total amount of assets:

$$a \cdot n = K$$

The interest rate therefore equals the marginal return to investment; the wage rate equals output per capita reduced by capital costs:

$$\begin{aligned}
r &= \frac{\partial f(a)}{\partial a} \\
w &= f(a) - a \cdot \frac{\partial f(a)}{\partial a}
\end{aligned}$$

The household's budget constraint can be written as:

$$\dot{a} + c + s_{(N)} = f(a)$$

3 Solution of the Model

3.1 Steady State

In this model - with no other engine of growth than capital accumulation - a steady state is characterized by constant variables c^*, s^* and a^*. It follows (with the co-state variable for capital denoted as $\theta_{(a)}$):

$$\begin{aligned}
\frac{\dot{\theta}_{(a)}}{\theta_{(a)}} &= \rho - r = 0 \\
\rho &= r && (7) \\
U_c + U_N \cdot N_P \cdot P_C &= U_N \cdot N_E \cdot E_S = \theta_{(a)} \\
\frac{U_c}{U_N} &= N_E \cdot E_S - N_P \cdot P_C && (8) \\
\dot{a} &= 0 \\
c + s_{(N)} &= w + r \cdot a = f(a) && (9)
\end{aligned}$$

For a given parameterization, these equations allow to compute solutions for the steady state values c^*, s^* and a^*. To run numerical simulations, we have to specify particular functional forms for the general equations used so far.

3.2 The Function of the Quality of Nature

The following functional form for the environmental quality is chosen:

$$N = \overline{N} + E(S) - P(C) \tag{10}$$

The relevant derivatives are:

$$\begin{aligned}
N_E &= 1 \\
N_P &= -1 \\
N_{EE} &= N_{PP} = N_{EP} = 0
\end{aligned}$$

We can rewrite condition (8) in the following way:

$$U_c = U_N \cdot (E_S + P_C)$$

It follows:

$$\begin{aligned}
\xi_1 &\equiv \frac{U_{cc} - U_N \cdot P_{CC} \cdot n - U_{cN} \cdot P_C \cdot (n+1) + U_{NN} \cdot P_C^2 \cdot n}{U_N \cdot E_S} \\
\xi_2 &\equiv \frac{(U_{cN} - U_{NN} \cdot P_C) \cdot n}{U_N} \\
\xi_3 &\equiv \frac{U_{cN} - U_{NN} \cdot P_C \cdot n}{U_N} \\
\xi_4 &\equiv \frac{\left(U_N \cdot E_{SS} + U_{NN} \cdot E_S^2\right) \cdot n}{U_N \cdot E_S}
\end{aligned}$$

The path of the system is now determined by the equations:

$$\begin{aligned}
g_{(c)} &\equiv \frac{\dot{c}}{c} = \frac{\xi_4 - \xi_2}{\xi_1 \cdot \xi_4 - \xi_3 \cdot \xi_2} \cdot \frac{\rho - r}{c} && (11) \\
g_{(s)} &\equiv \frac{\dot{s}}{s} = \frac{\xi_1 - \xi_3}{\xi_1 \cdot \xi_4 - \xi_2 \cdot \xi_3} \cdot \frac{\rho - r}{s} && (12) \\
g_{(a)} &\equiv \frac{\dot{a}}{a} = \frac{f(a) - c - s_{(N)}}{a} && (13)
\end{aligned}$$

With respect to the impact of economic activities on environmental quality we assume:

$$\begin{aligned} E(S) &= \tau_{(S)} \cdot S^{\gamma} \\ P(C) &= \tau_{(C)} \cdot C^{\beta} \\ 0 &< \gamma < 1 < \beta \\ 0 &\leq \tau_{(S)}, \tau_{(C)} \end{aligned}$$

The relevant derivatives are:

$$\begin{aligned} E_S &= \tau_{(S)} \cdot \gamma \cdot S^{\gamma-1} > 0 \\ E_{SS} &= \tau_{(S)} \cdot \gamma \cdot (\gamma - 1) \cdot S^{\gamma-2} < 0 \\ P_C &= \tau_{(C)} \cdot \beta \cdot C^{\beta-1} > 0 \\ P_{CC} &= \tau_{(C)} \cdot \beta \cdot (\beta - 1) \cdot C^{\beta-2} > 0 \end{aligned}$$

This implies decreasing marginal effects of investments into environmental quality and increasing marginal environmental damage due to consumption.

3.3 Production Function

We use a Cobb-Douglas production function:

$$Y = F(K, L) = A \cdot K^{\delta} \cdot L^{1-\delta} \tag{14}$$

with A being the level of technology. In this case, the per-capita output and the interest rate are given by:

$$\begin{aligned} y &= A \cdot k^{\delta} \\ r &= \delta \cdot A \cdot k^{\delta-1} \end{aligned}$$

It follows, that the unique equilibrium is determined by exogenous parameter values. The equilibrium capital stock is given by:

$$k^* = \left(\frac{\delta \cdot A}{\rho} \right)^{\frac{1}{1-\delta}}$$

The labor supply is one unit per capita. Since there are n households in the economy, this implies:

$$k = a$$

3.4 CES Utility Function

To analyze the influence of different elasticities of substitution we use a CES utility function. This allows easily to cover the range $0 < \sigma \leq 1$. The Leontief case ($\sigma = 0$) will be calculated separately. The CES utility function is given by:

$$U(c, N) = \left(\alpha \cdot c^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) \cdot (\phi \cdot N)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{15}$$

The marginal utilities for the general CES utility function are given by:

$$
\begin{aligned}
U_c &= \alpha \cdot c^{-\frac{1}{\sigma}} \cdot \left(\alpha \cdot c^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \cdot (\phi \cdot N)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \\
U_{cc} &= \frac{(\alpha-1) \cdot \alpha \cdot c^{\frac{1-\sigma}{\sigma}} \cdot (\phi \cdot N)^{\frac{1+\sigma}{\sigma}}}{\sigma \cdot \left((\alpha-1) \cdot \phi \cdot c^{\frac{1}{\sigma}} \cdot N - \alpha \cdot c \cdot (\phi \cdot N)^{\frac{1}{\sigma}}\right)^2} \cdot U
\end{aligned}
$$

$$
\begin{aligned}
U_N &= (1-\alpha) \cdot \phi \cdot (\phi \cdot N)^{-\frac{1}{\sigma}} \cdot \left(\alpha \cdot c^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \cdot (\phi \cdot N)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \\
U_{NN} &= \frac{(\alpha-1) \cdot \alpha \cdot \phi \cdot c^{\frac{1+\sigma}{\sigma}} \cdot (\phi \cdot N)^{\frac{1}{\sigma}}}{\sigma \cdot N \cdot \left((\alpha-1) \cdot \phi \cdot c^{\frac{1}{\sigma}} \cdot N - \alpha \cdot c \cdot (\phi \cdot N)^{\frac{1}{\sigma}}\right)^2} \cdot U
\end{aligned}
$$

$$
\begin{aligned}
U_{cN} &= \frac{(1-\alpha) \cdot \alpha \cdot \phi \cdot c^{\frac{1}{\sigma}} \cdot (\phi \cdot N)^{\frac{1}{\sigma}}}{\sigma \cdot \left((\alpha-1) \cdot \phi \cdot c^{\frac{1}{\sigma}} \cdot N - \alpha \cdot c \cdot (\phi \cdot N)^{\frac{1}{\sigma}}\right)^2} \cdot U \\
\frac{U_c}{U_N} &= \frac{\alpha}{(1-\alpha) \cdot \phi} \cdot \left(\frac{\phi \cdot N}{c}\right)^{\frac{1}{\sigma}}
\end{aligned}
$$

Since we know from (8) that $U_c = U_N \cdot (E_S + P_C)$ on the optimal path we have:

$$
\begin{aligned}
\frac{N}{c} &= \frac{1}{\phi} \cdot \left[\frac{(1-\alpha) \cdot \phi}{\alpha} \cdot (E_S + P_C)\right]^{\sigma} \\
N &= \frac{1}{\phi} \cdot \left[\frac{(1-\alpha) \cdot \phi}{\alpha} \cdot \left(\tau_{(S)} \cdot \gamma \cdot S^{\gamma-1} + \tau_{(C)} \cdot \beta \cdot C^{\beta-1}\right)\right]^{\sigma} \cdot c
\end{aligned}
$$

3.5 Numerical Results with Different Elasticities of Substitution

The following parameter values are used: $A = 5$, $n = 1000$, $\alpha = 0.75$, $\beta = 1.1$, $\gamma = 0.9$, $\delta = 0.5$, $\rho = 0.05$, $\phi = 0.5$, $\overline{N} = 1000$, $\tau_{(S)} = 5$ and $\tau_{(C)} = 0.05$.[2] To analyze the dynamics of the model we want to show the stable branch of the model for various starting values. It is possible to use various methods to determine the correct initial value of the control variables.[3] However, we use the method of backward integration as described by Brunner and Strulik (2002). The following figures illustrate the trajectories of the variables over 30 periods of time. The solid line is calculated under the assumption that in $t = 30$ all variables take their equilibrium values. The other lines are calculated under the assumption that assets deviate after 30 periods by -0.01% (two dots, dash), -0.005% (dot, dash), $+0.005\%$ (short dashes), and $+0.01\%$ (long dashes) from

[2] All numerical calculations and plots were made with Mathematica 4.0.
[3] For an overview see Barro and Sala-i-Martin (1995, 471-491).

their equilibrium value. Furthermore, it is assumed that savings - and therefore the growth of the capital stock - in $t = 30$ amount to the same percentage of output.[4]

3.5.1 Low Elasticity of Substitution

The first simulation is calculated with $\sigma = 0.1$. Here, equilibrium values are $c^* = 229.0372$, $s^* = 20.9627$, $N^* = 407.286$ and $U^* = 219.442$.

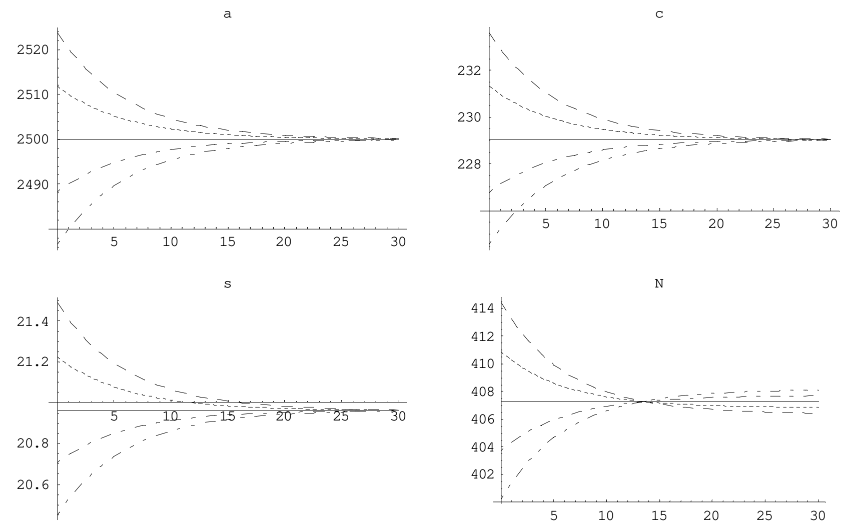

Figure 3.1: Market solution: assets (a), consumption (c), environmental expenditures (s) and quality of nature (N) with $\sigma = 0.1$

[4] Note that this is the reason for the intersection of time paths in some figures. If the savings for the given starting values of assets are relatively low (high), environmental quality and utility can be higher (lower) than those on the equilibrium path. To keep all models comparable, we had to accept this overshooting effect.

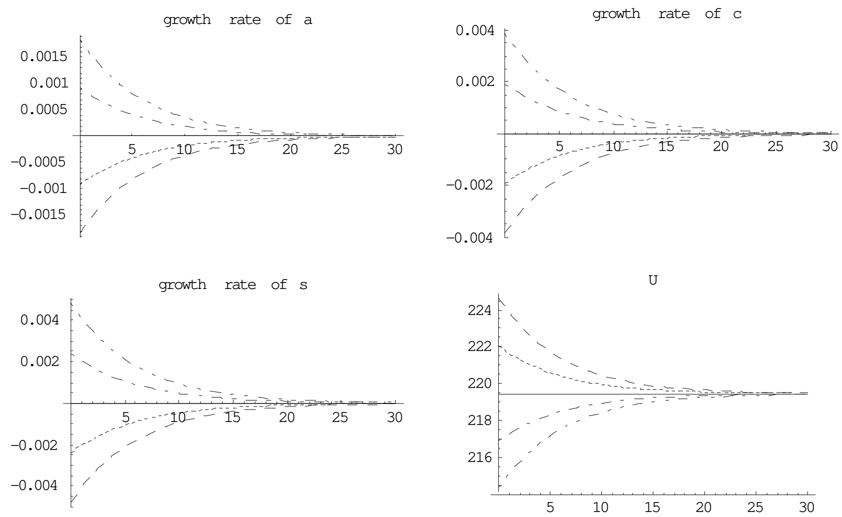

Figure 3.2: Market solution: growth rates of assets, consumption and environmental expenditures, and utility level (U) with $\sigma = 0.1$

3.5.2 High Elasticity of Substitution

In the next set of figures, the elasticity of substitution is assumed to be higher ($\sigma = 0.9$). Equilibrium values are here $c^* = 229.1710$, $s^* = 20.8289$, $N^* = 159.254$ and $U^* = 173.873$.

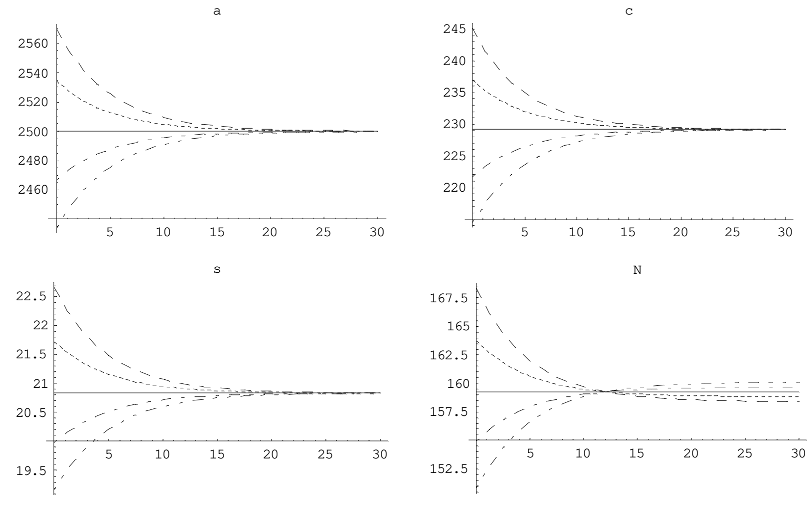

Figure 3.3: Market solution: assets (a), consumption (c), environmental expenditures (s) and quality of nature (N) with $\sigma = 0.9$

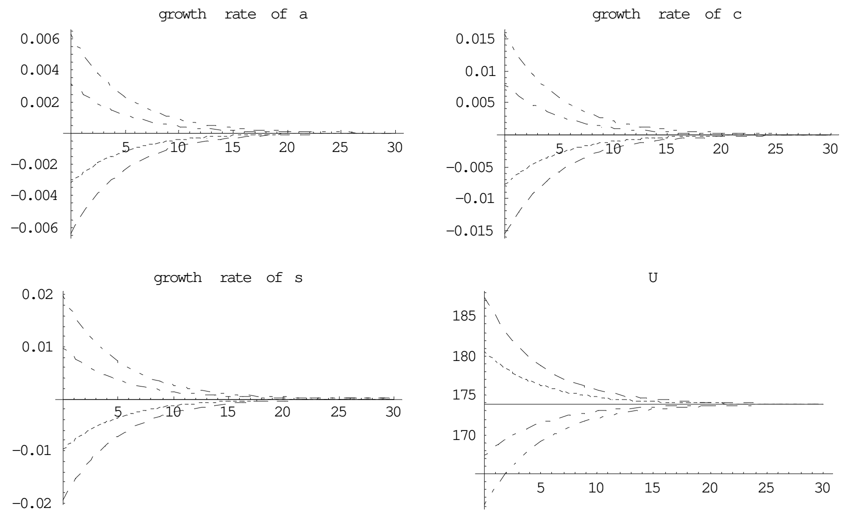

Figure 3.4: Market solution: growth rates of assets, consumption and environmental expenditures, and utility level (U) with $\sigma = 0.9$

Although consumption is only slightly higher and investments in environmental quality are a little bit lower than in the case of a low elasticity of substitution, environmental quality differs a lot. On the other hand, the speed of convergence is much higher. If the elasticity of substitution is higher, the economy will close a given gap between an initial and equilibrium values determined by exogenous variables much faster.

3.5.3 Cobb-Douglas Utility Function

To analyze the case of a elasticity of substitution equal to one ($\sigma = 1$), we have to replace the general CES utility function by a Cobb-Douglas utility function:

$$U = c^{\alpha} \cdot (\phi \cdot N)^{1-\alpha} \tag{16}$$

Equilibrium values are now $c^* = 229.1805$, $s^* = 20.8194$, $N^* = 141.617$ and $U^* = 170.866$.

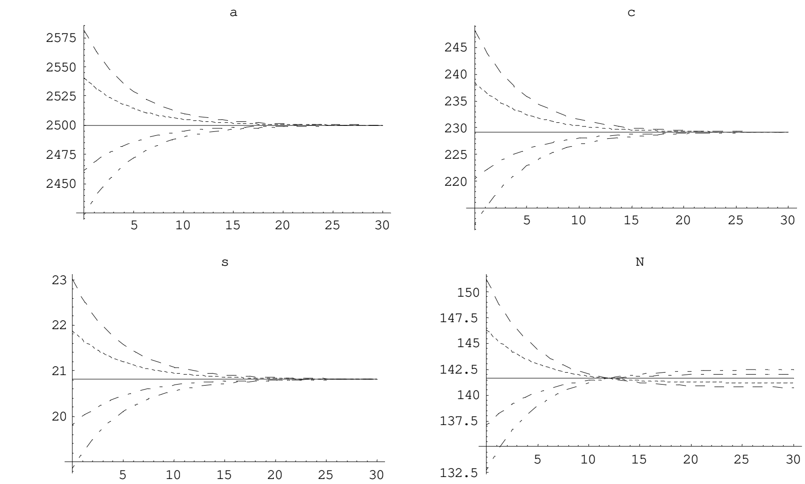

Figure 3.5: Market solution: assets (a), consumption (c), environmental expenditures (s) and quality of nature (N) with $\sigma = 1$

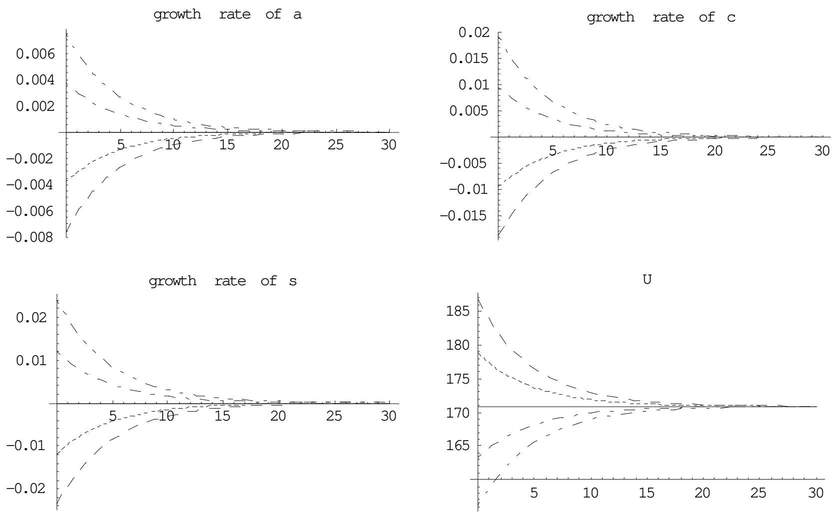

Figure 3.6: Market solution: growth rates of assets, consumption and environmental expentitures, and utility level (U) with $\sigma = 1$

Here, the effect of a higher elasticity of substititution is - compared with the first scenario - even stronger. The equilibrium value of consumption is higher, investments in environmental quality and - as a consequence - environmental quality itself are lower. Again, the speed of convergence is higher than in the case with a high elasticity of substitution and obviously much higher than in the case of a low elasticity of substitution. If we compare the time necessary to close the gap between a deviation from the equilibrium value of assets per capita of -5% to a deviation of -0.01%, the economy needs 39.29 time units in the case of a low elasticity of substitution ($\sigma = 0.1$), 32.44 time units in the case of a high elasticity of substitution ($\sigma = 0.9$), and 31.65 time units in the Cobb-Douglas case. The growth rate of capital in $t = 0$ is with 0.00747 in the Cobb-Douglas case more than four times as high as in the case with a low elasticity of substitution (0.00184).

3.5.4 Leontief Utility Function

Finally, we consider the case in which the utility function of the individual household is of Leontief type. Again, the representative household exhibits preferences over consumption goods and environmental amenities. Population growth is zero. The rate of time preference is ρ. The relative weight of environmental amenities in utility, ϕ, is constant. The utility function can be written as:

$$U = \min\left[c^{\lambda}, (\phi \cdot N)^{\lambda}\right] \qquad 0 < \lambda \leq 1 \tag{17}$$

with the household's consumption being c_i and environmental quality N. Efficient points are characterized by:

$$\begin{aligned} c^{\lambda} &= (\phi \cdot N)^{\lambda} \\ c &= \phi \cdot N \end{aligned}$$

which in fact reduces the utility function to:

$$U = (\phi \cdot N)^{\lambda} = c^{\lambda}$$

and the marginal utilities to:

$$\begin{aligned} U_N &= \lambda \cdot \phi \cdot (\phi \cdot N)^{\lambda - 1} \\ U_{NN} &= \lambda \cdot (\lambda - 1) \cdot \phi^2 \cdot (\phi \cdot N)^{\lambda - 2} \\ U_c &= \lambda \cdot c^{\lambda - 1} \\ U_{cc} &= \lambda \cdot (\lambda - 1) \cdot c^{\lambda - 2} \end{aligned}$$

The flow budget constraint for the household is:

$$w + r \cdot a = \dot{a} + c + s_{(N)} = \dot{a} + \phi \cdot N + s_{(N)} \tag{18a}$$

The system is fully described by:[5]

$$g_{(c)} = \frac{U_c \cdot (\rho - r)}{U_{cc} \cdot c} \tag{19}$$

$$g_{(a)} = \frac{f(a) - c - s_{(N)}}{a} \tag{20}$$

[5] See Appendix 6.2.

Again we use the simplest form to model - with the same specifications as before - the impact of economic activities on the environmental quality:

$$N = \overline{N} + E(S) - P(N)$$

Then we can calculate the level and the associated growth rate of investments in environmental quality necessary to fulfill the optimality condition $c = \phi \cdot N$:

$$\begin{aligned} s &= \frac{1}{n} \cdot \left(\frac{c}{\phi \cdot \tau_{(S)}} + \frac{\tau_{(C)}}{\tau_{(S)}} \cdot (n \cdot c)^{\beta} - \frac{\overline{N}}{\tau_{(S)}} \right)^{\frac{1}{\gamma}} \\ g_{(s)} &= \frac{1}{\gamma \cdot n} \cdot \left(\frac{c}{\phi \cdot \tau_{(S)}} + \frac{\tau_{(C)} \cdot (n \cdot c)^{\beta}}{\tau_{(S)}} - \frac{\overline{N}}{\tau_{(S)}} \right)^{\frac{1}{\gamma}-1} \\ &\quad \cdot \left(\frac{1}{\phi \cdot \tau_{(S)}} + \frac{\tau_{(C)} \cdot \beta \cdot (n \cdot c)^{\beta-1}}{\tau_{(S)}} \right) \cdot g_{(c)} \cdot c \end{aligned}$$

The steady state is again given by:

$$\rho = r \tag{21}$$
$$\dot{a} = 0 \tag{22}$$
$$c + s_{(N)} = w + r \cdot a = f(a) \tag{23}$$

The specifications of all other functions remain unchanged. Equilibrium values are now $c^* = 229.0098$, $s^* = 20.99017$, $N^* = 458.02$ and $U^* = 58.8695$.

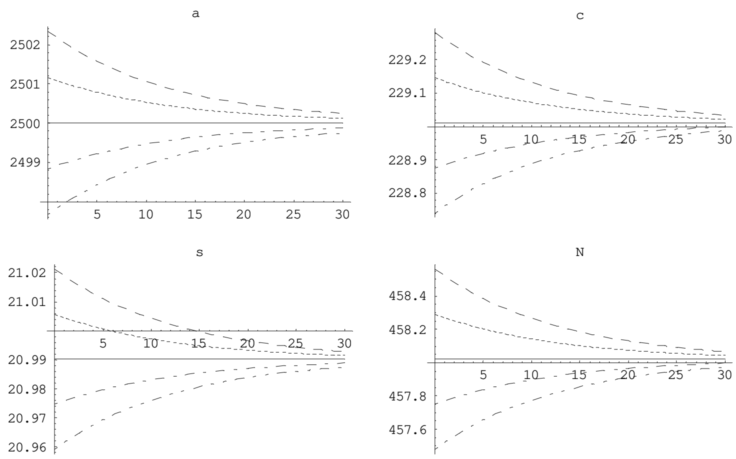

Figure 3.7: Market solution: assets (a), consumption (c), environmental expenditures (s) and quality of nature (N) with $\sigma = 0$

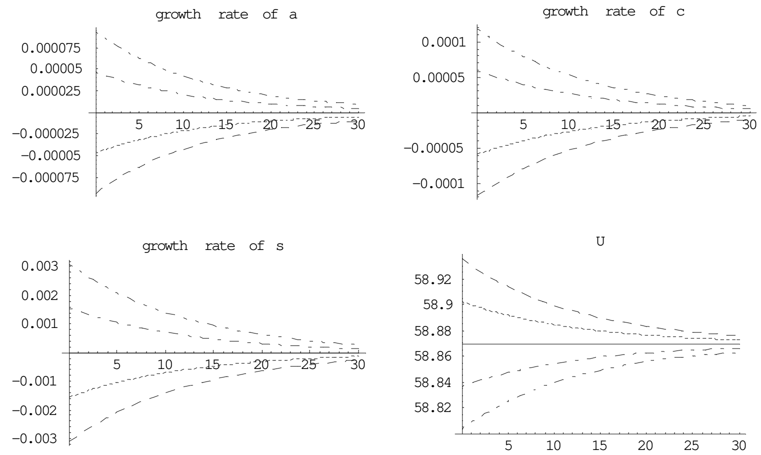

Figure 3.8: Market solution: growth rates of assets, consumption and environmental expenditures and utility level (U) with $\sigma = 0$

Although the equilibrium value of consumption is only about 1% lower and the investment in environmental quality a bit more than 1% higher than in the Cobb-Douglas case, the environmental quality is more than three times as high (323%). The speed of convergence is very low. To close the gap between a deviation from the equilibrium value of assets per capita of -5% to a a deviation of -0.01%, the economy needs 77.57 time units (compared with 39.29 time units in the case of a low elasticity of substitution ($\sigma = 0.1$), 32.44 time units in the case of a high elasticity of substitution ($\sigma = 0.9$), and 31.65 time units in the Cobb-Douglas case).

3.6 The Influence of the Elasticity of Substitution

We have seen that paths approaching from "above" are mirror images of paths converging from lower values to the equilibrium values. Therefore, the influence of the elasticity of substitution can be illustrated with the case of a deviation of -0.01% at the point of time $t = 30$, compared to the equilibrium value. Trajectories calculated with a deviation of $+0.01\%$ at the point of time $t = 30$ are in principle simple upside down copies of the trajectories as can be seen in the following. The solid line is calculated under the assumption of a Cobb-Douglas utility function, the other lines with elasticities of substitution of 0.1 (two dots, dash), 0.9 (dot, dash), and for the Leontief case $\sigma = 0$ (short dashes).

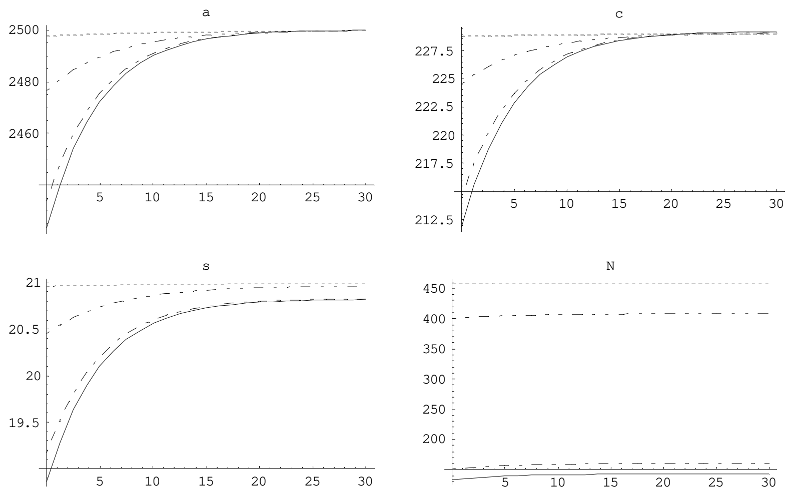

Figure 3.9: Market solution: assets (a), consumption (c), environmental expenditures (s) and quality of nature (N) with varying elasticity of substitution

Table 3.1 shows the values in $t = 0$ of the variables assets per capita, consumption, investment in environmental quality, and environmental quality for different elasticities of substitution.

Table 3.1: Variable values in $t = 0$

	a	c	s	N
$\sigma = 0$	2497.05	228.67	20.95	457.34
$\sigma = 0.1$	2472.05	223.70	20.35	397.90
$\sigma = 0.9$	2422.32	211.96	18.89	148.36
$\sigma = 1$	2410.16	208.92	18.54	130.33

Table 3.2 shows the equilibrium values of the variables assets per capita, consumption, investment in environmental quality, and environmental quality for different elasticities of substitution.

Table 3.2: Equilibrium values of variables[6]

σ	a	c	s	N	U
0	2500.00	229.01	20.99	458.02	58.87
0.1	2500.00	229.04	20.96	407.29	219.44
0.9	2500.00	229.17	20.83	159.25	173.87
1	2500.00	229.18	20.82	141.62	170.87

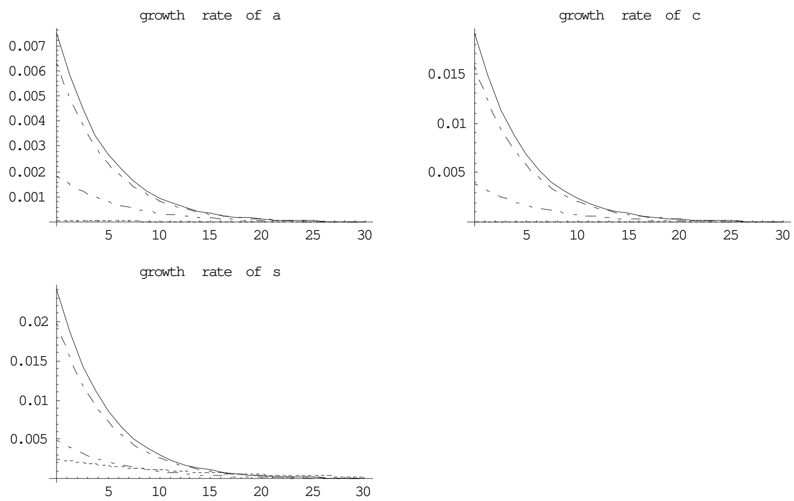

Figure 3.10: Market solution: growth rates of assets, consumption and environmental expenditures with varying elasticity of substitution

Table 3.3 shows the values in $t = 0$ of the growth rates of the variables assets per capita, consumption, and investment in environmental quality for different elasticities of substitution. Note that the equilibrium values of these variables equal zero.

[6] Note that a comparison of utility levels calculated with different utility functions is meaningless. The column with equilibrium values of U serves as reference for comparisons with different environmental policy instruments.

Table 3.3: Values of growth variables in $t = 0$

σ	$g_{(a)}$	$g_{(c)}$	$g_{(s)}$
0	0.00009	0.0001	0.0031
0.1	0.00183	0.0039	0.0048
0.9	0.00629	0.0157	0.0197
1	0.00747	0.0191	0.0240

Table 3.4: Speed of convergence[7]

	Δa		
σ	$-5\% \to -0.01\%$	$-1\% \to -0.01\%$	$-0.1\% \to -0.01\%$
0	77.570	57.052	27.908
0.1	39.294	29.314	15.203
0.9	32.440	24.270	12.746
1	31.655	23.691	12.460

The lower the elasticity of substitution is, the longer it takes to reach the equilibrium value. A high speed of convergence (a small time interval necessary to close a gap between an initial and a "target" level of the variable assets per capita) corresponds obviously to high growth rates. The equilibrium value of environmental quality is higher if the elasticity of substitution is low. Note that a comparison of utilitiy levels evaluated with different utility functions is meaningless as long as we have no cardinal utility measures.

4 Implementing Environmental Policy

4.1 Social Optimum

In order to construct a reference point we derive the social optimum. We assume that the social planner maximizes the sum over the households' welfare. Since all households are equal, distributional aspects can be ignored. Therefore, the optimization problem can be reduced to maximizing the utility of the representative household. If we use the same specifications for all functions, the system can be described by:[8]

$$g^P_{(a)} \equiv \frac{\dot{a}}{a} = \frac{f(a) - c - s_{(N)}}{a} \tag{24}$$

$$g^P_{(c)} \equiv \frac{\dot{c}}{c} = \frac{\xi_2 - \xi_3}{\xi_2^2 - \xi_1 \cdot \xi_3} \cdot \frac{\rho - f_a}{c} \tag{25}$$

$$g^P_{(s)} \equiv \frac{\dot{s}}{s} = \frac{\xi_2 - \xi_1}{\xi_2^2 - \xi_1 \cdot \xi_3} \cdot \frac{\rho - f_a}{s} \tag{26}$$

[7] The speed of convergence is measured by the time necessary to close the gap Δa between a certain amount of assets and the target level in $t = 30$. We calculated the time the economy needs to increase the amount of assets from 5%, 1% and 0.1% below the equilibrium and the "target" level of 0.01% below the equilibrium.

[8] See Appendix 6.3.

with:

$$\begin{aligned}
\xi_1 &\equiv \frac{U_{cc}}{U_N \cdot E_S \cdot n} - \frac{P_{CC} \cdot n}{E_S} + \frac{U_{NN} \cdot P_C^2 \cdot n - 2 \cdot U_{cN} \cdot P_C}{U_N \cdot E_S} \\
\xi_2 &\equiv \frac{U_{cN} - U_{NN} \cdot P_C \cdot n}{U_N} \\
\xi_3 &\equiv \frac{U_{NN} \cdot E_S \cdot n}{U_N} + \frac{E_{SS} \cdot n}{E_S}
\end{aligned}$$

The steady state of a planned economy is now characterized by:

$$\rho = f_a \tag{27}$$

$$\frac{U_c}{U_N} = n \cdot (E_S + P_C) \tag{28}$$

$$f(a) = c + s_{(N)} \tag{29}$$

With the given parameters, we can now find the planner's path from a certain initial point to the equilibrium and compare this path with the trajectories in the market equilibrium - at least theoretically. It turns out that boundary value problems of this type can not be solved in every case. Even with very simple functional forms this is usually a complicated problem which has to be solved by trial and error.[9]

Therefore, we focus on another aspect. We introduce different environmental policies and analyze their effects in a decentralized economy. We address the following questions: How can the environmental quality be influenced by policy instruments? Is it possible to influence the speed of convergence? Can a society characterized by rather high elasticities of substitution attain a comparatively high level of environmental quality?

4.2 The Tax Regime

Various tax regimes are possible: constant tax rates, tax rates dependent on the actual environmental quality or time-dependent tax rates.[10] Constant tax rates are similar to the existing energy taxes or taxes on fossil fuels.[11] The Pigou tax which internalizes the external effects is an ideal type of such a tax. Tax rates dependent on the environmental quality influence the behavior of households or firms over time. It is possible to start with high tax rates to induce an initial jump in the control variables. Starting with low rates would maybe imply a smooth behavior of the system. Time-dependent tax rates may allow to adapt to changing variables more easily. Welfare losses due to jumps - which are a serious problem in real-world scenarios - can be diminished by such a regime. On the other hand, changing tax rates are a source of uncertainty and may cause welfare losses resulting from the increase in transaction costs.

[9] Mathematica usually produces error messages.

[10] Here, we understand constant tax rates as rates that do not change automatically with time or environmental quality. But of course, the rates are subject to changes due to legislative activities of governments.

[11] For European countries, an overview can be found in European Commission (2004).

4.3 The Decentralized Economy with a Green Tax

Here, we will model one simple policy: a tax on consumption d with or without returning the tax revenue as a lump-sum payment or as subsidies for environmental quality. Without paying reimbursements to the households the money has to be burnt.[12] Complete repayments make a balanced budget possible. For the cases of no repayments at all (30), lump-sum payments (31) and subsidies (32) the budget constraints of the households are then given by:

$$\begin{aligned} w + r \cdot a &= \dot{a} + (1+d) \cdot c + s_{(N)} & (30) \\ w + r \cdot a + D &= \dot{a} + (1+d) \cdot c + s_{(N)} & (31) \\ w + r \cdot a &= \dot{a} + (1+d) \cdot c + (1-p) \cdot s_{(N)} & (32) \end{aligned}$$

Another point is the information problem: Is it known that at a certain point of time a tax will be introduced? Or does this step come unanticipated? In the first case the household will choose an optimal path which takes this step into account. In the second case the household follows an optimal path which depends on parameters and initial conditions. When the tax is introduced, the households calculate a new path in consideration of the new parameters. In the present study, we assume an existing tax regime with complete or incomplete knowledge about the optimal tax rate.

In general the policy makers can use the tax to influence the system with respect to two aspects: First they may try to improve environmental quality permanently by shifting expenditures from consumption to abatement. Second, they may try to influence the adjustment speed.

4.3.1 No or Lump-sum Repayment of Tax Revenues

In the following, we assume that there is either a lump-sum or no repayment of tax revenues, i.e. we apply the budget constraints (30) and (31). Since households are assumed to be "small", they can not influence the tax regime and see no connection between their consumption pattern and the additional income from the lump-sum transfers. The system can be described by the following three equations:[13]

$$\begin{aligned} g_{(a)} &\equiv \frac{\dot{a}}{a} = \frac{r \cdot a + w + D - (1+d) \cdot c - s_{(N)}}{a} & (33) \\ g_{(c)} &\equiv \frac{\dot{c}}{c} = \frac{(\xi_5 - \xi_2) \cdot (\rho - r) - \xi_3 \cdot \xi_5 \cdot \dot{d}}{(\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4) \cdot c} & (34) \\ g_{(s)} &\equiv \frac{\dot{s}}{s} = \frac{(\xi_1 - \xi_4) \cdot (\rho - r) + \xi_3 \cdot \xi_4 \cdot \dot{d}}{(\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4) \cdot s} & (35) \end{aligned}$$

[12] Maybe not in a physical sense, but from the households' point of view. The use of the tax revenue does not necessarily produce income and substitution effects for the households.

[13] See Appendix 6.4.

with:

$$\begin{aligned}
\xi_1 &\equiv \frac{U_{cc} + \left(U_{NN} \cdot P_C^2 - U_N \cdot P_{CC}\right) \cdot n - U_{cN} \cdot P_C \cdot (n+1)}{(1+d) \cdot U_N \cdot E_S} \\
\xi_2 &\equiv \frac{(U_{cN} - U_{NN} \cdot P_C) \cdot n}{(1+d) \cdot U_N} \\
\xi_3 &\equiv -\frac{1}{1+d} \\
\xi_4 &\equiv \frac{U_{cN} - U_{NN} \cdot P_C \cdot n}{U_N} \\
\xi_5 &\equiv \frac{\left(U_N \cdot E_{SS} + U_{NN} \cdot E_S^2\right) \cdot n}{U_N \cdot E_S}
\end{aligned}$$

4.3.2 Numerical Simulations: No Repayments

If we assume a constant tax rate without repayments it follows that $\dot{d} = 0$ and $D = 0$. Consequently:[14]

$$\begin{aligned}
g_{(a)} &= \frac{r \cdot a + w - (1+d) \cdot c - s_{(N)}}{a} \\
g_{(c)} &= \frac{(\xi_5 - \xi_2) \cdot (\rho - r)}{(\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4) \cdot c} \\
g_{(s)} &= \frac{(\xi_1 - \xi_4) \cdot (\rho - r)}{(\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4) \cdot s}
\end{aligned}$$

This assumption implies that the use of the tax revenue does not influence a household's utility at all. As mentioned before, it is used to "build pyramids" or other things which are unimportant for the individual's utility level. A second implication is that the household's utility level will fall compared to the situation without the tax or with reimbursement of tax revenues. With full lump-sum repayments of the tax revenue it is in principle possible to stay on the same utility level and simply adjust to the changed price relation between consumption and quality of nature. This adjustment can lead to the social optimal values for consumption and expenditures for environmental quality. Hence, without repayments the social optimal point cannot be reached. Note that basic functions remain unchanged. As an example, we assume a tax rate of $d = 0.1$ and a CES utility function with a high elasticity of substitution of $\sigma = 0.9$.

[14] Note that ξ_3 vanished from the equations of motion completely.

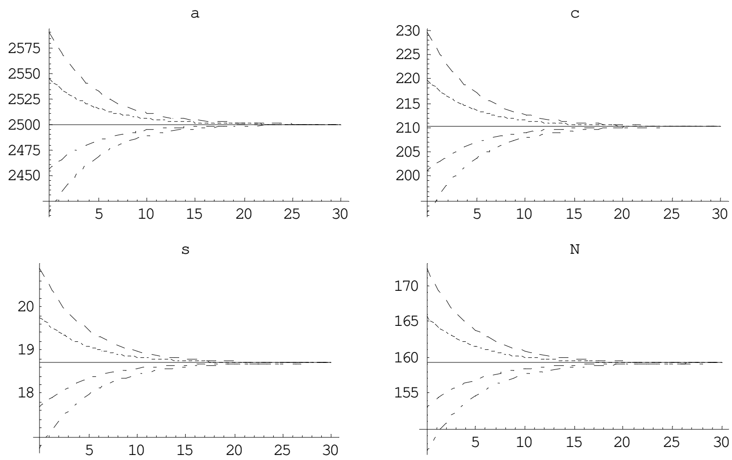

Figure 4.1: Green consumption tax without repayment of the tax revenue: assets (a), consumption (c), environmental expenditures (s) and quality of nature (N) with $\sigma = 0.9$

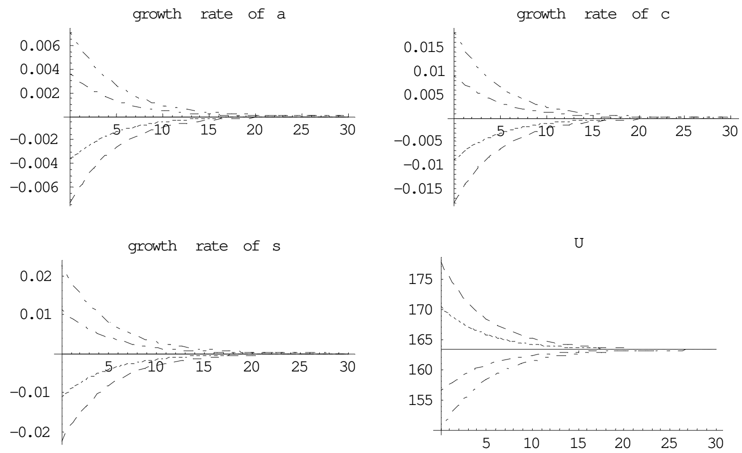

Figure 4.2: Green consumption tax without repayment of the tax revenue: growth rates of assets, consumption and environmental expentitures, and utility level (U) with $\sigma = 0.9$

Similar figures can be generated with higher tax rates. In the following we want to compare selected parameter values for different tax rates and elasticities of substitution. We concentrate on the path that starts in $t = 30$ with a deviation of 0.01% below the equilibrium value of assets.[15]

[15] See Appendix 6.6.

Higher taxes decrease the equilibrium levels of utility. This result is intuitive since the households' income is reduced. The decrease in environmental quality due to taxation in the case of CES utility functions is not that blatant. With the Cobb-Douglas function this effect does not appear, but this is caused by decreasing pollution due to a lower consumption level, the high speed of convergence and the overshooting effect. The reason is that the income effect dominates the substitution effect. Note that the effect vanishes off equilibrium. Easy substitution possibilities between goods result in a relative decrease of the environmental quality: Consumption is nearly unaffected by the elasticity of substitution, but environmental quality is much higher for lower elasticities of substitution. Consumption decreases with the tax rate, and overshooting cannot be observed. The equilibrium asset level is given by the rate of time preference and technological parameters and therefore independent of the elasticity of substitution. But the speed of convergence increases with the elasticity of substitution and with the tax rate. With higher taxes expenditures for environmental quality are lower. This results in decreasing environmental quality in the CES case. For higher tax rates and higher elasticities of substitution the growth rates are higher. The growth rate of consumption is lower than the growth rate of expenditures for environmental quality. The households try to approach the equilibrium level of consumption before the optimal environmental quality is reached.

Additionally let us have a look at the consequences for the government's budget. First, the tax revenues for a given point of time are calculated; second, we compute the accumulated tax revenues for all 30 time periods, for the last 20, the last 10, the last 5 and the last, beginning in $t = 29$ and ending in $t = 30$. In all cases tax revenues per period rise over time and are nearly independent of the elasticity of substitution. It is obvious that they depend on the tax rate. The overall tax revenue is higher for low elasticities of substitution.

4.3.3 Numerical Simulations: Lump-sum Repayments

Now we assume a constant tax rate and lump-sum repayments of the tax revenue. It follows:[16]

$$\begin{aligned} g_{(a)} &= \frac{r \cdot a + w + D - (1+d) \cdot c - s_{(N)}}{a} \\ g_{(c)} &= \frac{(\xi_5 - \xi_2) \cdot (\rho - r)}{(\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4) \cdot c} \\ g_{(s)} &= \frac{(\xi_1 - \xi_4) \cdot (\rho - r)}{(\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4) \cdot s} \end{aligned}$$

The case in which the lump-sum transfers exceed the tax revenue is excluded. Therefore, we can restrict the share of revenues that is paid back to:

$$\widehat{d} = \frac{D}{d \cdot c}, \qquad 0 < \widehat{d} \leq 1$$

[16] Note that $\xi_3 =$ vanished from the expressions for the growth rates completely.

If the tax is a constant and we assume a balanced government budget, the tax revenue equals the lump-sum payments. It follows:

$$D = d \cdot c$$

Consequently, in this special case the budget constraint of the household is given by:

$$\dot{a} = r \cdot a + w - c - s_{(N)}$$

and the growth rate of assets is:

$$g_{(a)} = \frac{r \cdot a + w - c - s_{(N)}}{a}$$

All other functions remain unchanged.

Here, an optimal (Pigou) tax leads to the socially optimal equilibrium with $c^* = 228.819$, $s^* = 21.181$, $N^* = 811.808$ and $U^* = 236.201$:

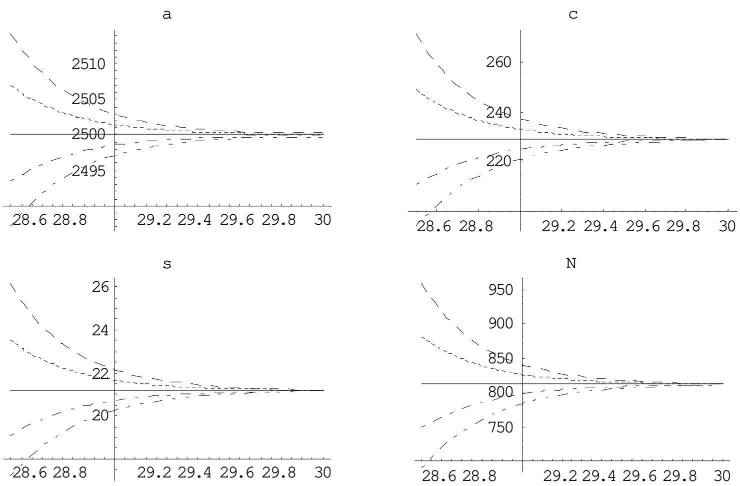

Figure 4.3: Green consumption tax with lump-sum repayment of the tax revenue: assets, consumption, environmental expenditures and quality of nature with $\sigma = 0.1$ and an optimal tax rate ($d = 1112.796$)

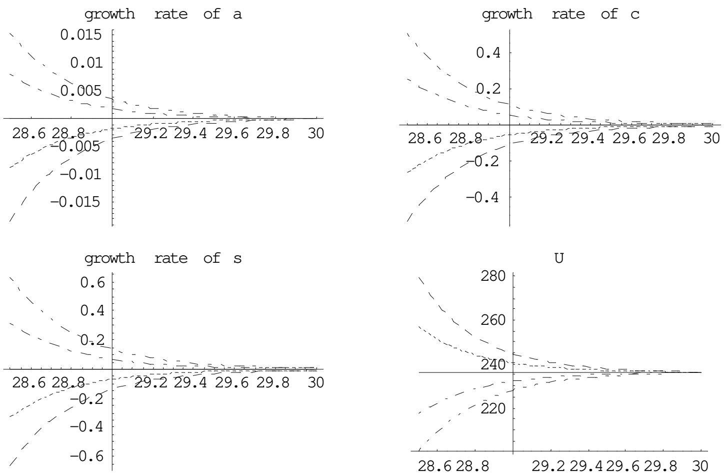

Figure 4.4: Green consumption tax with lump-sum repayment of the tax revenue: growth rates of assets, consumption and environmental expenditures, and utility level (U) with $\sigma = 0.1$ and an optimal tax rate ($d = 1112.796$)

Consumption is slightly lower, investment in environmental quality slightly higher than in the market solution. Environmental quality is nearly twice as high. The attained utility level is higher than in the market (236.201 compared to 219.442, see Table 3.2). Note that the tax rate is unrealistically high because of the lump-sum transfer back to the households. The regulatory effect of the tax is very low since the decrease of income due to the tax is nearly compensated by the lump-sum transfer. As a result, very high rates of convergence for low elasticities of substitution are a striking feature of all models with lump-sum transfers.

Table 4.1: Convergence of models if revenues from an optimal tax are reimbursed as lump-sum payments

	Δa	
σ	$-1\% \rightarrow -0.01\%$	$-0.1\% \rightarrow -0.01\%$
0.1	1.730	0.957
0.9	14.014	7.562
1	17.363	9.284

With high elasticities of substitution, the speed of convergence is roughly comparable to the market solution. With low elasticities of substitution, the equilibrium will be reached almost immediately. The picture does not change qualitatively if we vary the tax rates. In a second example with a low elasticity of substitution we assume a tax rate below the optimal value ($d = 100$). The equilibrium value are $c^* = 228.912$, $s^* = 21.088$, $N^* = 638.919$ and $U^* = 235.915$.

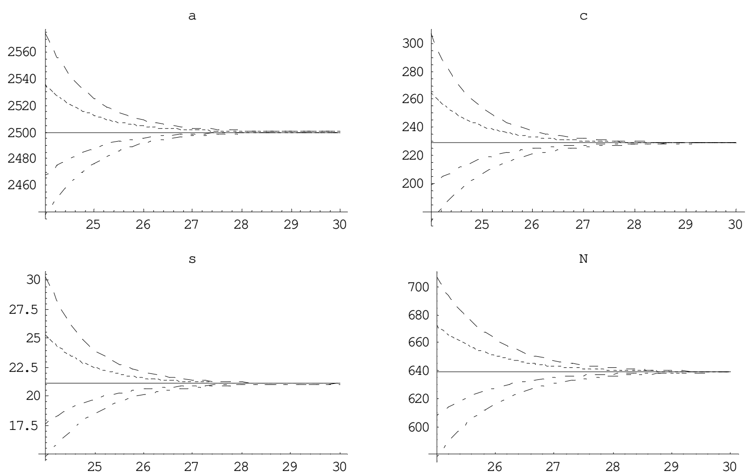

Figure 4.5: Green consumption tax with lump-sum repayment of the tax revenue: assets, consumption, environmental expenditures and quality of nature with $\sigma = 0.1$ and a too low tax rate ($d = 100$)

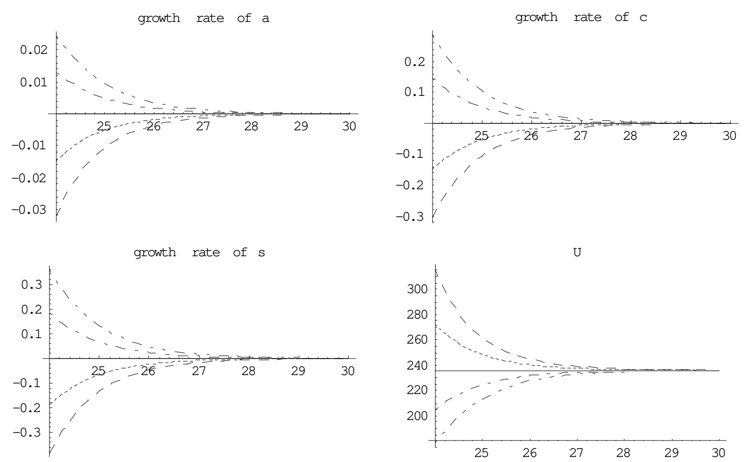

Figure 4.6: Green consumption tax with lump-sum repayment of the tax revenue: growth rates of assets, consumption and environmental expenditures, and utility level (U) with $\sigma = 0.1$ and too low tax rate ($d = 100$)

The next picture captures the case of an optimal tax with a higher elasticity of substitution. The equilibrium values are: $c^* = 193.884$, $s^* = 56.116$, $N^* = 62250.2$ and $U^* = 541.271$.[17] Here, the difference to the market solution is more

[17] See Table 3.2 for a comparison.

obvious. All variables differ remarkably from the values of the market solution, the utility level is more than three times as high, the environmental quality is nearly 400 times higher.

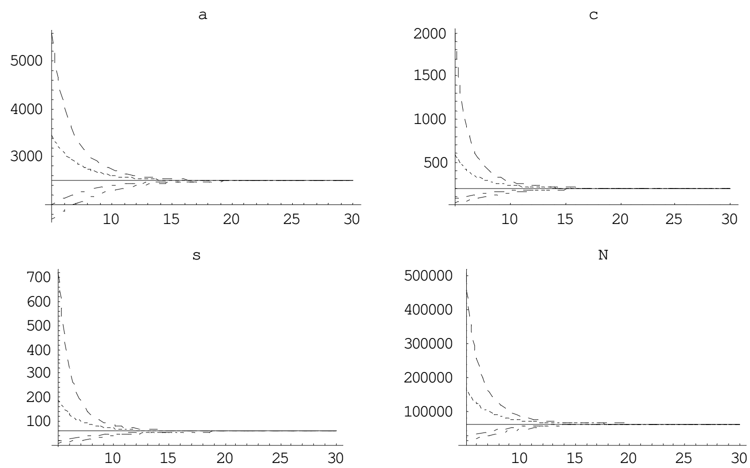

Figure 4.7: Green consumption tax with lump-sum repayment of the tax revenue: assets, consumption, environmental expenditures and quality of nature with $\sigma = 0.9$ and optimal tax rate ($d = 1122.134$)

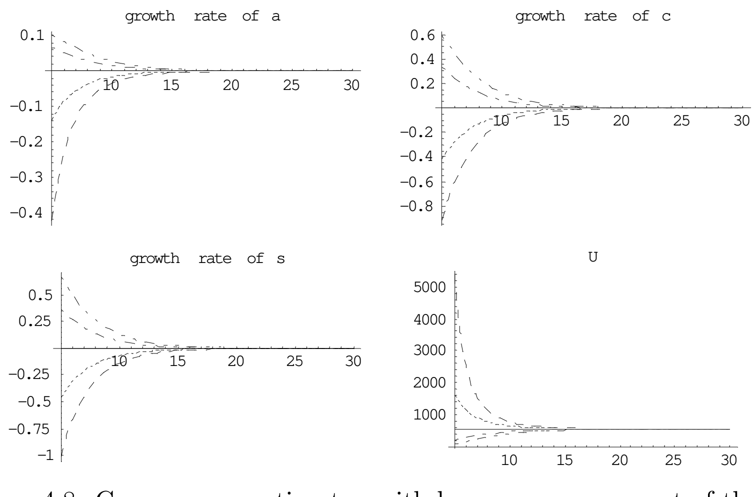

Figure 4.8: Green consumption tax with lump-sum repayment of the tax revenue: growth rates of assets, consumption and environmental expenditures, and utility level (U) with $\sigma = 0.9$ and optimal tax rate ($d = 1122.134$)

In a last example with complete reimbursement of tax revenues, we want to look at the effects of a too high tax rate of $d = 2000$ in the model with a high elasticity of substitution ($\sigma = 0.9$). The equilibrium values are now given by $c^* = 175.759$, $s^* = 74.241$, $N^* = 92531.988$ and $U^* = 529.956$.

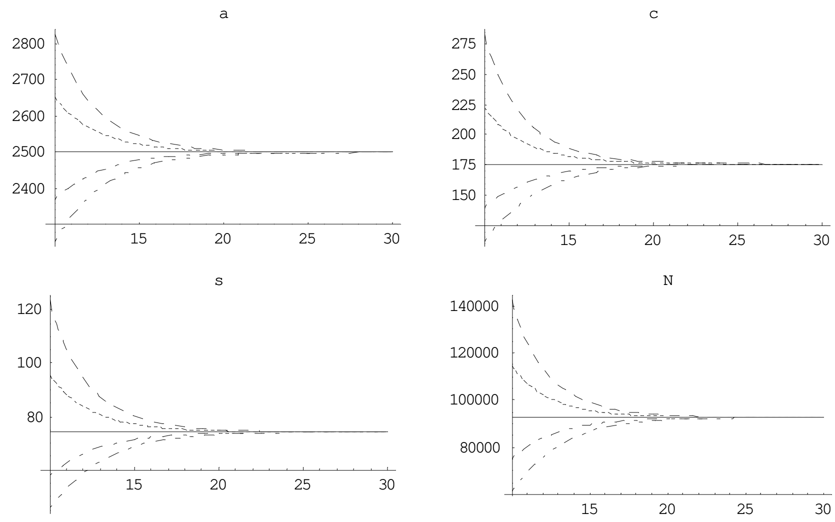

Figure 4.9: Green consumption tax with lump-sum repayment of the tax revenue: assets, consumption, environmental expenditures and quality of nature with $\sigma = 0.9$ and a too high tax rate ($d = 2000$)

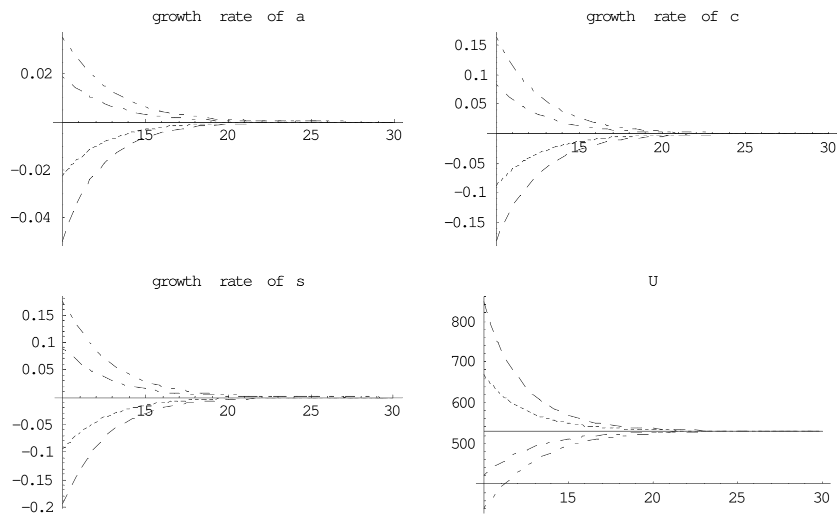

Figure 4.10: Green consumption tax with lump-sum repayment of the tax revenue: growth rates of assets, consumption and environmental expenditures, and utility level (U) with $\sigma = 0.9$ and too high tax rate ($d = 2000$)

In nearly all cases there are - compared with the situation without repayments - slightly changed consumption levels, but dramatically improved environmental qualities.

To complete the picture, we now take a look at the results with a partly repayment of tax revenues. As an example, a model with a Cobb-Douglas utility function, a tax rate of $d = 0.3$ and a repayment of 50% of the tax revenues is chosen. The tax rate is far below the optimal tax rate. But in real life, tax rates exceeding 1 can rarely be observed.[18] To take this fact into account, we calculate in the following models with lower, more realistic tax rates. The equilibrium values are: $c^* = 201.930$, $s^* = 17.780$, $N^* = 160.552$, and $U^* = 160.342$.

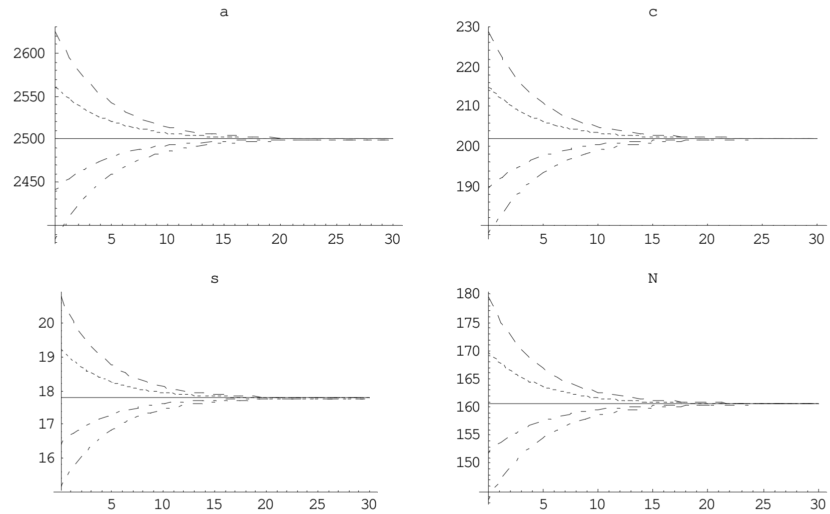

Figure 4.11: Green consumption tax with lump-sum repayment of 50% of the tax revenue: assets, consumption, environmental expenditures and quality of nature with $\sigma = 1$ and a tax rate of $d = 0.3$

[18] An example for rather high tax rates is the tax on gasoline. Today, the consumption tax on gasoline is € 654.50 per 1.000 litres plus value-added tax. An overview can be found at Bundesministerium der Finanzen (ed.) (2005).

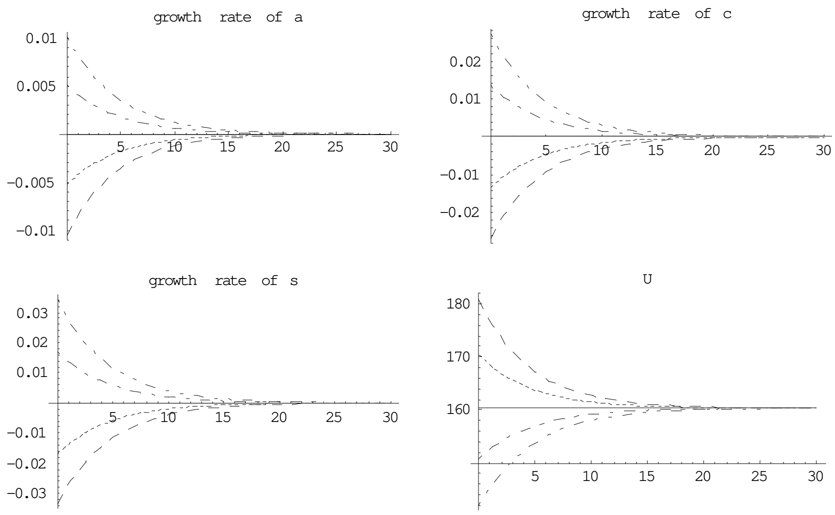

Figure 4.12: Green consumption tax with lump-sum repayment of 50% of the tax revenue: growth rates of assets, consumption and environmental expenditures, and utility level (U) with $\sigma = 1$ and a tax rate of $d = 0.3$

Table 4.2 compares the variables with equilibrium values without or with complete reimbursement of tax revenues, Table 4.3 the speed of convergence in these models.[19]

Table 4.2: Equilibium values of variables if tax revenues are not, partly or fully reimbursed as lump-sum payments

reimbursement	c	s	N	U
0%	180.44	15.429	145.23	143.72
50%	201.93	17.780	160.55	160.34
100%	229.16	20.840	179.74	181.35

Table 4.3: Convergence of models if tax revenues are not, partly or fully reimbursed as lump-sum payments

	Δa		
reimbursement	$-5\% \to -0.01\%$	$-1\% \to -0.01\%$	$-0.1\% \to -0.01\%$
0%	30.334	22.712	11.974
50%	30.355	22.728	11.983
100%	30.383	22.750	11.993

While there is nearly no difference in the speed of convergence, all other variables differ. The decreasing income leads to lower consumption, lower

[19] Note that there is a difference between equilibrium values in Table 4.2 and values for $t = 30$ in Appendix 6.6.

quality of environment as well as a sunken utility level. Similar results were obtained for different elasticities of substitution ($\sigma = \{0.1, 0.9, 1\}$), tax rates ($d = \{0.1, 0.2, 0.3\}$), and rates of reimbursement ($\frac{D}{d \cdot c} = \{0, 0.1, 0.5, 1\}$). The effect of increasing tax rates is always the same: consumption, investments in environmental quality and environmental quality itself as well as utility decrease. A higher rate of reimbursement works in the opposite direction. A higher elasticity of substitution increases consumption and decreases investments in environmental quality slightly with the effect of a remarkable decrease of environmental quality and a moderate decrease of utility. Table 4.4 illustrates the latter effect.

Table 4.4: Equilibrium values of variables if tax revenues are partly (50%) reimbursed as lump-sum payments for a tax rate d = 0.3

σ	c	s	N	U
0.1	201.83	17.892	368.07	195.64
0.9	201.92	17.789	176.05	162.88
1	201.93	17.780	160.55	160.34

4.3.4 Repayment of Tax Revenues as Subsidy

Similarly, we can analyze the third possible budget constraint given in (32), meaning that tax revenues are repaid in the form of subsidies for investments in environmental quality. This yields:[20]

$$
\begin{aligned}
g_{(a)} &\equiv \frac{\dot{a}}{a} = \frac{r \cdot a + w - (1+d) \cdot c - (1-p) \cdot s_{(N)}}{a} \\
g_{(c)} &\equiv \frac{\dot{c}}{c} = \frac{(\xi_5 - \xi_2) \cdot (\rho - r) - \xi_3 \cdot \xi_5 \cdot \dot{d} + \xi_2 \cdot \xi_6 \cdot \dot{p}}{(\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4) \cdot c} \\
g_{(s)} &\equiv \frac{\dot{s}}{s} = \frac{(\xi_1 - \xi_4) \cdot (\rho - r) + \xi_3 \cdot \xi_4 \cdot \dot{d} - \xi_1 \cdot \xi_6 \cdot \dot{p}}{(\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4) \cdot s}
\end{aligned}
$$

with

$$
\begin{aligned}
\xi_1 &\equiv \frac{U_{cc} + \left(U_{NN} \cdot P_C^2 - U_N \cdot P_{CC}\right) \cdot n - U_{cN} \cdot P_C \cdot (n+1)}{\frac{1+d}{1-p} \cdot U_N \cdot E_S} \\
\xi_2 &\equiv \frac{(U_{cN} - U_{NN} \cdot P_C) \cdot n}{\frac{1+d}{1-p} \cdot U_N} \\
\xi_3 &\equiv -\frac{1}{1+d} \\
\xi_4 &\equiv \frac{U_{cN} - U_{NN} \cdot P_C \cdot n}{U_N} \\
\xi_5 &\equiv \frac{\left(U_N \cdot E_{SS} + U_{NN} \cdot E_S^2\right) \cdot n}{U_N \cdot E_S} \\
\xi_6 &\equiv \frac{1}{1-p}
\end{aligned}
$$

[20] See Appendix 6.5.

A balanced government's budget requires that tax revenue equals subsidy payments:

$$c \cdot d = s \cdot p$$

4.3.5 Numerical Simulations: A Subsidy on Investments in Environmental Quality

In the following we consider the government's option to use part or all of the tax revenue to finance a subsidy on investments in environmental quality. Again, several combinations of changing and/or constant subsidy rates - depending on time, environmental parameters or other variables - are possible. A special case is a changing subsidy rate that ensures a balanced budget. We restrict our analysis to the cases of constant tax and subsidy rates and to constant tax rates and a subsidy which keeps the government budget in equilibrium balanced. Constant tax and subsidy rates yield:[21]

$$\begin{aligned} g_{(a)} &= \frac{r \cdot a + w - (1+d) \cdot c - (1-p) \cdot s_{(N)}}{a} \\ g_{(c)} &= \frac{(\xi_5 - \xi_2) \cdot (\rho - r)}{(\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4) \cdot c} \\ g_{(s)} &= \frac{(\xi_1 - \xi_4) \cdot (\rho - r)}{(\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4) \cdot s} \end{aligned}$$

The following table shows the equilibrium values of the variables for varying elasticities of substitution.

Table 4.5: Equilibrium values of variables if tax revenues are reimbursed as subsidies on environmental expeditures

σ	c	s	N	U
0.1	228.82	21.81	811.8	236.20
0.9	193.88	56.12	62250.2	541.27
1	173.97	76.03	95476.0	708.07

As a first example, the path with a Cobb-Douglas utility function and an optimal constant tax rate ($d = 0.43645$) and subsidy ($p = 0.99872$) is shown:

[21] Note that ξ_3 and ξ_3 vanish.

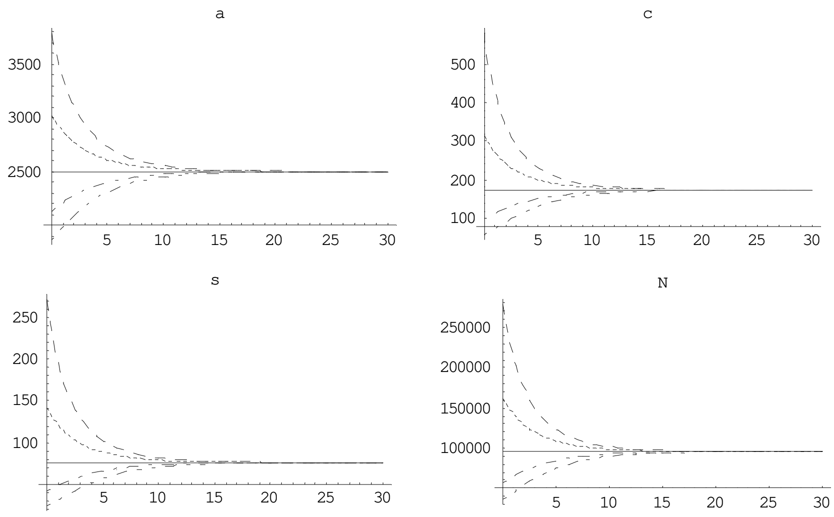

Figure 4.13: Green consumption tax with repayment of the tax as constant subsidy: assets, consumption, environmental expenditures and quality of environment with $\sigma = 1$ and an optimal tax and subsidy rate

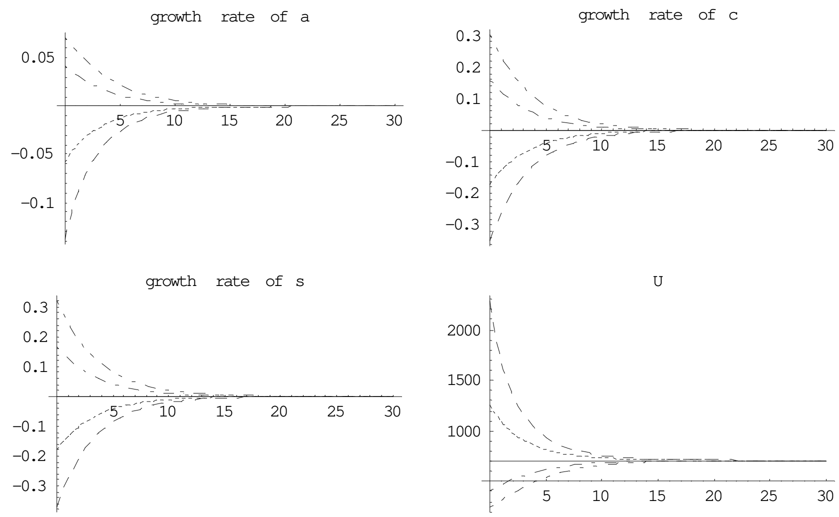

Figure 4.14: Green consumption tax with repayment of the tax as constant subsidy: growth rate of assets, consumption and environmental expenditures and utility with $\sigma = 1$ and an optimal tax and subsidy rate

Again the adjustment speed is much higher with low elasticities of substitution. Table 4.6 illustrates the decreasing speed of convergence if the elasticity of subsititution increases. The calculated numbers are similar to the model with lump-sum repayments of tax revenues (see Table 4.1).

Table 4.6: Convergence of models if tax revenues are used to finance a constant subsidy on investments for environmental quality

σ	Δa $-5\% \rightarrow -0.01\%$	$-1\% \rightarrow -0.01\%$	$-0.1\% \rightarrow -0.01\%$
0.1	2.437	1.748	0.967
0.9	18.847	14.172	7.652
1	23.350	17.532	9.380

The question now is if the government's budget is balanced or not. It is self-evident that long running deficits cause serious problems. The following figure shows that in the long run, the budget is balanced, and even in the short run the size of deficit or surplus does not exceed reasonable limits.

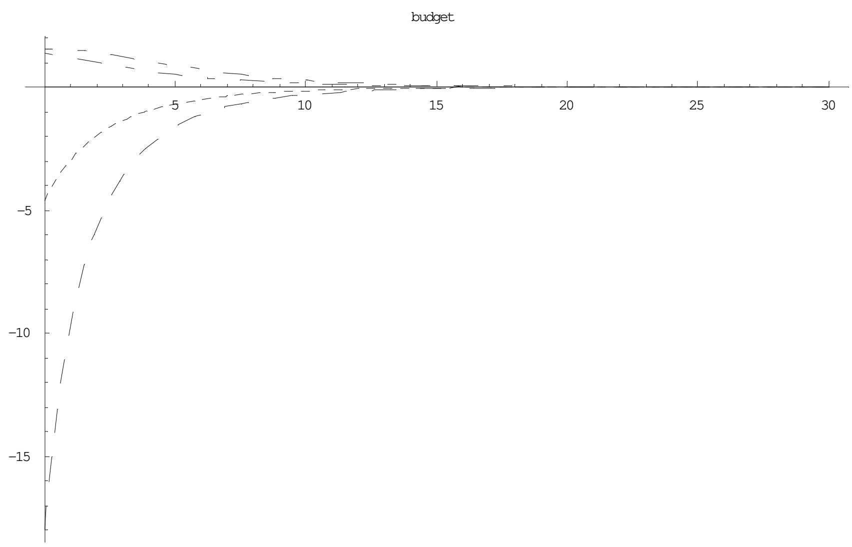

Figure 4.15: Green consumption tax with repayment of the tax as constant subsidy: the government's budget with $\sigma = 1$ and an optimal tax and subsidy rate

Integrated over 30 time periods, the tax revenues exceed (fall short of) expenditures in the case of a deviation from equilibrium of -0.01% ($+0.01\%$) by 10.094 (35.115). Similar results can be obtained for CES utility functions with elasticities of substitution of 0.1 and 0.9, although in earlier periods deficit and surplus are higher for lower elasticities of substitution. These findings correspond with higher speeds of convergence (see Table 4.6).

Again, we look at consequences of errors in the determination of tax and subsidy rates. Various causes of fault are conceivable. Similar to previous models we can assume that the tax revenue is low due to low tax rates or that the subsidy exceeds tax revenues. It is also possible that part of the tax revenues will not be reimbursed or tax rates exceed the optimal level. The first cases end with a permanent deficit, the latter with a growing surplus. Combinations of these errors are possible. Since in our model the only aim of the government is to steer the economy to the optimal point it is questionable that such errors are persistent. But, if the government is interested in a surplus for various reasons, such errors could become permanent conditions.

Here we will concentrate on two other types of errors. First we look at too high (low) tax rates. The tax revenue will be used to finance a subsidy. In equilibrium, the government's budget will be balanced by choosing an appropriate subsidy rate. Second we assume too high (low) tax rates, but now we choose the subsidy rate appropriate to reach equilibrium values of our variables. A long running surplus or deficit of the government is accepted. This model refers to a situation where the government has made commitments with respect to tax rates and the quality of environment - e.g. by signing multilateral environmental agreements - and is therefore forced to finance the budget deficit otherwise.

In the following, we use a Cobb-Douglas utility function. Table 4.7 shows equilibrium values of our variables. A tax rate 10% below (above) the equilibrium level changes consumption +3.1% (−2.9%), investments in environmental quality −7.2 (+6.7) and environmental quality −9.4 (+8.8). Utility is nearly unaffected. Also the speed of convergence does not change very much. It is slightly higher for higher tax rates (see Table 4.7).

Table 4.7: Equilibrium values of variables if tax rates are too high or too low

σ	c	s	N	U
90%	179.42	70.578	86473	706.92
99%	174.50	75.497	94603	708.06
100%	173.97	76.027	95476	708.07
101%	173.44	76.554	96343	708.06
110%	168.84	81.156	103893	707.14

Table 4.8: Convergence of models if tax rates are too high or too low

	Δa		
t	$-5\% \rightarrow -0.01\%$	$-1\% \rightarrow -0.01\%$	$-0.1\% \rightarrow -0.01\%$
90%	23.417	17.582	9.406
99%	23.356	17.537	9.382
100%	23.350	17.532	9.380
101%	23.344	17.528	9.378
110%	23.294	17.491	9.358

With a high elasticity of substitution, CES utility functions result in a slightly higher speed of convergence. The error in determining the tax rate has - as in the models with Cobb-Douglas utility functions - nearly no impact. This changes if the elasticity of substitution is rather low. An error in the determination of tax rates accelerates (slows down) the system considerably (see Table 4.9). On the other hand, equilibrium values are nearly unaffected.

Table 4.9: Convergence of models if tax rates are too high or too low

		Δa		
σ	t	$-5\% \rightarrow -0.01\%$	$-1\% \rightarrow -0.01\%$	$-0.1\% \rightarrow -0.01\%$
0.1	99%	5.637	4.209	2.331
	100%	2.437	1.748	0.967
	101%	0.442[22]	0.410	0.213
0.9	99%	18.867	14.186	7.659
	100%	18.847	14.172	7.652
	101%	18.828	14.158	7.644

In all cases, the budget is balanced in the long run and does not exceed a reasonable deficit or surplus in the short run.

Using again a Cobb-Douglas utility function we will now look at a situation with an error in the determination of the tax rate and a subsidy rate which ensures equilibrium values of consumption and investments in quality of nature. Compared with an optimal tax and subsidy, there is nearly no change in the speed of convergence. But the government's budget is not balanced anymore. The following table shows the budget in $t = 30$ and the integral of the budget over time from 0 to 30 for the equilibrium time path.

Table 4.10: Government's budget if tax rates are too high or too low

Δd	budget in $t = 30$	budget over time from 0 to 30
-50%	-37.980	-1139.39
-10%	-7.596	-227.88
$+10\%$	$+7.596$	$+227.88$
$+50\%$	$+37.980$	$+1139.39$

Here, the budget deficit or surplus adds up to 3% (15.2%) of output per period, if the error is as high as 10% (50%) of the tax rate.

5 Summary and Outlook

We have presented some results from a study of different environmental instruments. Besides the choice of instruments - we have used various combinations of taxes and subsidies -, the exogenous parameter elasticity of substitution has a great influence on the dynamic behavior and the equilibrium levels of variables.

[22] Calculated by interpolation.

Furthermore we have investigated the impact of the precision of determination of tax and subsidy rates.

In our model pollution is a side effect of consumption. Without environmental policy, the equilibrium levels of the variables consumption, investments in environmental quality, environmental quality and utility depend on the elasticity of substitution. For higher elasticities of substitution we observe higher equilibrium levels of consumption and lower equilibrium levels of investments in environmental quality, environmental quality and utility. Since the equilibrium amount of assets depends only on the rate of time preference and technological parameters, it does not change if the elasticity of substitution changes. With a Cobb-Douglas utility function the speed of convergence is high, in case of a Leontief utility function it is low.

We have introduced a simple environmental instrument: a tax on household's consumption. Without repayments of tax revenues in the form of lump-sum transfers (a negative lump-sum tax), the socially optimal values cannot be reached. This follows immidiately from the budget cuts. Environmental quality is lower if the elasticity of substitution is high. The tax rate has little influence on environmental quality, especially if the elasticity of substitution is high. This is the result of the income effect of the tax. This income effect lowers the expenditures for environmental quality and dominates the substitution effect caused by a change of relative prices. The level of consumption is nearly independent of the rate of substitution, but decreases with the tax rate. The growth rates of assets are lower than the growth rates of consumption and environmental expenditures, indicating that households first increase assets and then consumption. Environmental expenditures fall behind and therefore have to grow with the highest rates.

If we introduce a lump-sum repayment of the tax revenue the economy can reach the socially optimal level for all variables, but only if the tax rate is ridiculously high. The repayment of tax revenues cancels the regulatory impact of the tax out. Whereas in the market solution the speed of convergence was higher for high elasticities of substitution we have here a very high speed of convergence for low elasticities of substitution. If we restrict the repayment to part of the tax revenues we get qualitatively the same results for reasonable tax rates. As a matter of course, here again equilibrium values of the variables cannot be reached, i.e. the values for quality of the environment and utility level are lower. The share of reimbursed tax revenues has virtually no effect on the speed of convergence. In all models, the budget of the government is balanced or closes with a surplus.

With a repayment of tax revenues as a subsidy on environmental expenditures, the required tax rate is reasonable. We observe a high adjustment speed and a high optimal level of natural quality. The system converges faster if the elasticity of substitution is low. Optimal consumption decreases, optimal investment in environmental quality and environmental quality increase with elasticity of substitution. The government's budget tends to be balanced.

If taxes and subsidies are not appropriate to approach the optimal values of variables, the budget of the government can still be balanced if a deviating tax rate is combined with an adequate deviation of the subsidy rate. For moderate deviations (10%), the equilibrium utility level remains nearly unchanged, although we can observe considerable changes of environmental quality (˜9%) due to changes of the consumption level and investments in environmental quality. If the elasticity of substitution is low, a rising tax rate results in an acceleration of the system, measured by the speed of convergence. If the government is forced to achieve a certain level of environmental quality, an error in the determination of tax rates may imply a permanent surplus or deficit. The speed of convergence is unaffected.

Future research should focus on a modified model with pollution as a side effect of production. In this case, a tax on capital can be used to achieve the optimal capital stock. The tax revenues can again be used to finance a lump-sum transfer to the households or to pay a subsidy on investments in environmental quality. Furthermore, we will analyze the impact of shocks and of lags in the politician's response. Shocks may result from changing exogenous variables, e.g. productivity, preferences or a destruction of capital due to social or natural unpredictable incidences.

It would be very interesting to conduct empirical studies, especially to learn more about the elasticity of substitution between environmental amenities and consumption. But this goes beyond the scope of the present study.

6 Appendix

6.1 Solution of the Household's Optimization Problem in the Basic Model

The Hamiltonian for the household i is:

$$J = U\left(c_i, N\left(E\left(S\right), P\left(C\right), \overline{N}\right), \phi\right) + \theta_{(a)} \cdot \left(r \cdot a + w - c_i - s_{(N)_i}\right) \tag{36}$$

The first-order conditions are:

1. $\frac{\partial J}{\partial c_i} = 0$
$$U_c + U_N \cdot N_P \cdot P_C = \theta_{(a)} \tag{37}$$

2. $\frac{\partial J}{\partial s_{(N)_i}} = 0$
$$U_N \cdot N_E \cdot E_S = \theta_{(a)} \tag{38}$$

3. $\frac{\partial J}{\partial a} = \rho \cdot \theta_{(a)} - \dot{\theta}_{(a)}$
$$\rho \cdot \theta_{(a)} - \dot{\theta}_{(a)} = \theta_{(a)} \cdot r \tag{39}$$

The transversality condition[23] is given by:

$$\lim_{t \to \infty} \left[\theta_{(a)} \cdot a\right] = 0$$

[23] See Barro and Sala-i-Martin (1995, 503-508). This condition holds for all model specifications.

which is equivalent to:

$$\lim_{t\to\infty}\left[e^{-\rho\cdot t}\cdot a\right]=0$$

Conditions (37) and (38) yield:

$$U_c = U_N\cdot(N_E\cdot E_S - N_P\cdot P_C)$$

Derivation of conditions (37) and (38) with respect to time yield:

$$\begin{aligned}\frac{\dot\theta_{(a)}}{\theta_{(a)}} &= \xi_1\cdot\dot c+\xi_2\cdot\dot s\\ &= \xi_3\cdot\dot c+\xi_4\cdot\dot s\end{aligned}$$

where:

$$\begin{aligned}
\xi_1 &\equiv \frac{U_{cc}+\left[\begin{array}{c}U_N\cdot\left(N_P\cdot P_{CC}+P_C^2\cdot N_{PP}\right)\\ +U_{NN}\cdot P_C^2\cdot N_P^2\end{array}\right]\cdot n+U_{cN}\cdot P_C\cdot N_P\cdot(n+1)}{U_N\cdot N_E\cdot E_S}\\
\xi_2 &\equiv \frac{((U_{NN}\cdot N_E\cdot N_P+U_N\cdot N_{EP})\cdot P_C+U_{cN}\cdot N_E)\cdot n}{U_N\cdot N_E}\\
\xi_3 &\equiv \frac{(U_{NN}\cdot N_E\cdot N_P+U_N\cdot N_{EP})\cdot P_C\cdot n+U_{cN}\cdot N_E}{U_N\cdot N_E}\\
\xi_4 &\equiv \frac{\left(U_N\cdot E_{SS}\cdot N_E+U_{NN}\cdot E_S^2\cdot N_E^2+U_N\cdot E_S^2\cdot N_{EE}\right)\cdot n}{U_N\cdot N_E\cdot E_S}
\end{aligned}$$

The control variables change therefore according to:

$$\dot c = \frac{\xi_4-\xi_2}{\xi_1\cdot\xi_4-\xi_3\cdot\xi_2}\cdot(\rho-r) \tag{40}$$

$$\dot s = \frac{\xi_1-\xi_3}{\xi_1\cdot\xi_4-\xi_2\cdot\xi_3}\cdot(\rho-r) \tag{41}$$

6.2 Solution of the Household's Optimization Problem with a Leontief Utility Function

The Hamiltonian is given by:

$$J_H = U(c)+\theta_{(a)}\cdot\left(w+r\cdot a-c_i-s_{(N)}\right) \tag{42}$$

The first order conditions are:

1. $\frac{\partial J}{\partial c_i}=0$

$$U_c-\theta_{(a)}=0 \tag{43}$$

2. $\frac{\partial J}{\partial a}=\rho\cdot\theta_{(a)}-\dot\theta_{(a)}$

$$\rho\cdot\theta_{(a)}-\dot\theta_{(a)}=\theta_{(a)}\cdot r \tag{44}$$

The derivative of the condition (43) with respect to time is:

$$\dot{\theta}_{(a)} = U_{cc} \cdot \dot{c}$$

Using condition (44) yields:

$$\begin{aligned} \frac{\dot{\theta}_{(a)}}{\theta_{(a)}} &= \frac{U_{cc}}{U_c} \cdot \dot{c} \\ &= \rho - r \end{aligned}$$

It follows:

$$\dot{c} = \frac{U_c \cdot (\rho - r)}{U_{cc}}$$

6.3 Solution of the Planners's Optimization Problem

The Hamiltonian can be written as:

$$J_P = U\left(c, N\left(E\left(S\right), P\left(C\right), \overline{N}\right), \phi\right) + \theta_{(a)} \cdot \left(f\left(a\right) - c - s_{(N)}\right) \tag{45}$$

The first order conditions are:

1. $\frac{\partial J}{\partial c} = 0$
$$U_c + U_N \cdot N_P \cdot P_C \cdot n = \theta_{(a)} \tag{46}$$

2. $\frac{\partial J}{\partial s_{(N)}} = 0$
$$U_N \cdot N_E \cdot E_S \cdot n = \theta_{(a)} \tag{47}$$

3. $\frac{\partial J}{\partial a} = \rho \cdot \theta_{(a)} - \dot{\theta}_{(a)}$
$$\rho \cdot \theta_{(a)} - \dot{\theta}_{(a)} = \theta_{(a)} \cdot f_a \tag{48}$$

From equations (46) and (47) it follows:

$$U_c = U_N \cdot n \cdot (N_E \cdot E_S - N_P \cdot P_C)$$

Derivation of conditions (46) and (47) with respect to time yields:

$$\begin{aligned} \frac{\dot{\theta}_{(a)}}{\theta_{(a)}} &= \xi_1 \cdot \dot{c} + \xi_2 \cdot \dot{s} \\ &= \xi_2 \cdot \dot{c} + \xi_3 \cdot \dot{s} \end{aligned}$$

where:

$$\begin{aligned} \xi_1 &\equiv \frac{U_{cc} + \left[\begin{array}{c} U_N \cdot \left(N_P \cdot P_{CC} + P_C^2 \cdot N_{PP}\right) \\ +U_{NN} \cdot N_P^2 \cdot P_C^2 \end{array} \right] \cdot n^2 + 2 \cdot U_{cN} \cdot P_C \cdot N_P \cdot n}{U_N \cdot N_E \cdot E_S \cdot n} \\ \xi_2 &\equiv \frac{U_{cN} + U_{NN} \cdot N_P \cdot P_C \cdot n}{U_N} + \frac{N_{EP} \cdot P_C \cdot n}{N_E} \\ \xi_3 &\equiv \frac{U_{NN} \cdot N_E \cdot E_S \cdot n}{U_N} + \frac{N_{EE} \cdot E_S \cdot n}{N_E} + \frac{E_{SS} \cdot n}{E_S} \end{aligned}$$

It follows

$$\begin{aligned}\rho - f_a &= \xi_1 \cdot \dot{c} + \xi_2 \cdot \dot{s} \\ &= \xi_2 \cdot \dot{c} + \xi_3 \cdot \dot{s}\end{aligned}$$

Here, the growth rates are given by:

$$\dot{a} = f(a) - c - s_{(N)} \tag{49}$$

$$\dot{c} = \frac{\xi_2 - \xi_3}{\xi_2^2 - \xi_1 \cdot \xi_3} \cdot (\rho - f_a) \tag{50}$$

$$\dot{s} = \frac{\xi_2 - \xi_1}{\xi_2^2 - \xi_1 \cdot \xi_3} \cdot (\rho - f_a) \tag{51}$$

6.4 Solution of the Household's Optimization Problem without or with Lump-sum Repayment of Tax Revenues

The budget constraint is given by:

$$w + r \cdot a + D = \dot{a} + (1 + d) \cdot c + s_{(N)}$$

with the tax rate being d and the repayment of the tax revenue $D \geq 0$. The Hamiltonian for the household i is:

$$\begin{aligned}J_H &= U\left(c_i, N\left(E(S), P(C), \overline{N}\right), \phi\right) \\ &\quad + \theta_{(a)} \cdot \left(r \cdot a_i + w_i + D_i - (1 + d) \cdot c_i - s_{(N)_i}\right)\end{aligned} \tag{52}$$

The first-order conditions are:

1. $\frac{\partial J}{\partial c_i} = 0$
$$U_c + U_N \cdot N_P \cdot P_C = \theta_{(a)} \cdot (1 + d) \tag{53}$$

2. $\frac{\partial J}{\partial s_{(N)_i}} = 0$
$$U_N \cdot N_E \cdot E_S = \theta_{(a)} \tag{54}$$

3. $\frac{\partial J}{\partial a} = \rho \cdot \theta_{(a)} - \dot{\theta}_{(a)}$
$$\rho \cdot \theta_{(a)} - \dot{\theta}_{(a)} = \theta_{(a)} \cdot r \tag{55}$$

Again, we arrive at:

$$U_c = U_N \cdot [(1 + d) \cdot N_E \cdot E_S - N_P \cdot P_C]$$

From the derivation of the first and second condition with respect to time we obtain:

$$\begin{aligned}\frac{\dot{\theta}_{(a)}}{\theta_{(a)}} &= \xi_1 \cdot \dot{c} + \xi_2 \cdot \dot{s} + \xi_3 \cdot \dot{d} \\ &= \xi_4 \cdot \dot{c} + \xi_5 \cdot \dot{s}\end{aligned}$$

where:

$$\xi_1 \equiv \frac{U_{cc} + \left[\begin{array}{c} U_N \cdot \left(N_P \cdot P_{CC} + P_C^2 \cdot N_{PP}\right) \\ +U_{NN} \cdot P_C^2 \cdot N_P^2 \end{array}\right] \cdot n + U_{cN} \cdot P_C \cdot N_P \cdot (n+1)}{(1+d) \cdot U_N \cdot N_E \cdot E_S}$$

$$\xi_2 \equiv \frac{((U_{NN} \cdot N_E \cdot N_P + U_N \cdot N_{EP}) \cdot P_C + U_{cN} \cdot N_E) \cdot n}{(1+d) \cdot U_N \cdot N_E}$$

$$\xi_3 \equiv -\frac{1}{1+d}$$

$$\xi_4 \equiv \frac{(U_{NN} \cdot N_E \cdot N_P + U_N \cdot N_{EP}) \cdot P_C \cdot n + U_{cN} \cdot N_E}{U_N \cdot N_E}$$

$$\xi_5 \equiv \frac{\left(U_N \cdot E_{SS} \cdot N_E + U_{NN} \cdot E_S^2 \cdot N_E^2 + U_N \cdot E_S^2 \cdot N_{EE}\right) \cdot n}{U_N \cdot N_E \cdot E_S}$$

If we use the specification of the function of quality of nature given in (10), we can simplify to equations in the following way:

$$\xi_1 \equiv \frac{U_{cc} + \left(U_{NN} \cdot P_C^2 - U_N \cdot P_{CC}\right) \cdot n - U_{cN} \cdot P_C \cdot (n+1)}{(1+d) \cdot U_N \cdot E_S}$$

$$\xi_2 \equiv \frac{(U_{cN} - U_{NN} \cdot P_C) \cdot n}{(1+d) \cdot U_N}$$

$$\xi_3 \equiv -\frac{1}{1+d}$$

$$\xi_4 \equiv \frac{U_{cN} - U_{NN} \cdot P_C \cdot n}{U_N}$$

$$\xi_5 \equiv \frac{\left(U_N \cdot E_{SS} + U_{NN} \cdot E_S^2\right) \cdot n}{U_N \cdot E_S}$$

It follows for the control variables:

$$\dot{c} = \frac{(\xi_5 - \xi_2) \cdot (\rho - r) - \xi_3 \cdot \xi_5 \cdot \dot{d}}{\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4}$$

$$\dot{s} = \frac{(\xi_1 - \xi_4) \cdot (\rho - r) + \xi_3 \cdot \xi_4 \cdot \dot{d}}{\xi_1 \cdot \xi_5 - \xi_2 \cdot \xi_4}$$

6.5 Solution of the Household's Optimization Problem with Repayment of Tax Revenues as a Subsidy

The Hamiltonian for the household i is:

$$\begin{aligned} J_H = {} & U\left(c_i, N\left(E(S), P(C), \overline{N}\right), \phi\right) \\ & +\theta_{(a)} \cdot \left(r \cdot a_i + w_i - (1+d) \cdot c_i - (1-p) \cdot s_{(N)_i}\right) \end{aligned} \tag{56}$$

The first-order conditions are:

1. $\frac{\partial J}{\partial c_i} = 0$
$$U_c + U_N \cdot N_P \cdot P_C = \theta_{(a)} \cdot (1+d) \tag{57}$$

2. $\frac{\partial J}{\partial s_{(N)_i}} = 0$
$$U_N \cdot N_E \cdot E_S = \theta_{(a)} \cdot (1-p) \tag{58}$$

3. $\frac{\partial J}{\partial a} = \rho \cdot \theta_{(a)} - \dot{\theta}_{(a)}$
$$\rho \cdot \theta_{(a)} - \dot{\theta}_{(a)} = \theta_{(a)} \cdot r \tag{59}$$

Again, this yields:

$$U_c = U_N \cdot \left(\frac{1+d}{1-p} \cdot N_E \cdot E_S - N_P \cdot P_C \right)$$

The derivation of conditions (57) and (58) with respect to time yields:

$$\begin{aligned}
\dot{\theta}_{(a)} &= \frac{U_{cc} + \left[\begin{array}{c} U_N \cdot \left(N_P \cdot P_{CC} + N_{PP} \cdot P_C^2\right) \\ +U_{NN} \cdot P_C^2 \cdot N_P^2 \end{array} \right] \cdot n + U_{cN} \cdot N_P \cdot P_C \cdot (n+1)}{1+d} \cdot \dot{c} \\
&\quad + \frac{((U_N \cdot N_{EP} + U_{NN} \cdot N_E \cdot N_P) \cdot P_C + U_{cN} \cdot N_E) \cdot E_S \cdot n}{1+d} \cdot \dot{s} \\
&\quad - \frac{\theta_{(a)}}{1+d} \cdot \dot{d} \\
&= \frac{((U_N \cdot N_{EP} + U_{NN} \cdot N_E \cdot N_P) \cdot P_C \cdot n + U_{cN} \cdot N_E) \cdot E_S}{1-p} \cdot \dot{c} \\
&\quad + \frac{\left(U_N \cdot N_E \cdot E_{SS} + \left(U_{NN} \cdot N_E^2 + U_N \cdot N_{EE}\right) \cdot E_S^2\right) \cdot n}{1-p} \cdot \dot{s} \\
&\quad + \frac{\theta_{(a)}}{1-p} \cdot \dot{p}
\end{aligned}$$

or

$$\begin{aligned}
\frac{\dot{\theta}_{(a)}}{\theta_{(a)}} &= \xi_1 \cdot \dot{c} + \xi_2 \cdot \dot{s} + \xi_3 \cdot \dot{d} \\
&= \xi_4 \cdot \dot{c} + \xi_5 \cdot \dot{s} + \xi_6 \cdot \dot{p}
\end{aligned}$$

with:

$$
\begin{aligned}
\xi_1 &\equiv \frac{U_{cc} + \left[\begin{array}{c} U_N \cdot \left(N_P \cdot P_{CC} + N_{PP} \cdot P_C^2\right) \\ +U_{NN} \cdot P_C^2 \cdot N_P^2 \end{array} \right] \cdot n + U_{cN} \cdot N_P \cdot P_C \cdot (n+1)}{\frac{1+d}{1-p} \cdot U_N \cdot N_E \cdot E_S} \\
\xi_2 &\equiv \frac{((U_N \cdot N_{EP} + U_{NN} \cdot N_E \cdot N_P) \cdot P_C + U_{cN} \cdot N_E) \cdot n}{\frac{1+d}{1-p} \cdot U_N \cdot N_E} \\
\xi_3 &\equiv -\frac{1}{1+d} \\
\xi_4 &\equiv \frac{(U_N \cdot N_{EP} + U_{NN} \cdot N_E \cdot N_P) \cdot P_C \cdot n + U_{cN} \cdot N_E}{U_N \cdot N_E} \\
\xi_5 &\equiv \frac{\left(U_N \cdot N_E \cdot E_{SS} + U_{NN} \cdot N_E^2 \cdot E_S^2 + U_N \cdot N_{EE} \cdot E_S^2\right) \cdot n}{U_N \cdot N_E \cdot E_S} \\
\xi_6 &\equiv \frac{1}{1-p}
\end{aligned}
$$

We can use the specification of the function of the quality of the environment to simplify the equations:

$$
\begin{aligned}
\xi_1 &\equiv \frac{U_{cc} + \left(U_{NN} \cdot P_C^2 - U_N \cdot P_{CC}\right) \cdot n - U_{cN} \cdot P_C \cdot (n+1)}{\frac{1+d}{1-p} \cdot U_N \cdot E_S} \\
\xi_2 &\equiv \frac{(U_{cN} - U_{NN} \cdot P_C) \cdot n}{\frac{1+d}{1-p} \cdot U_N} \\
\xi_3 &\equiv -\frac{1}{1+d} \\
\xi_4 &\equiv \frac{U_{cN} - U_{NN} \cdot P_C \cdot n}{U_N} \\
\xi_5 &\equiv \frac{\left(U_N \cdot E_{SS} + U_{NN} \cdot E_S^2\right) \cdot n}{U_N \cdot E_S} \\
\xi_6 &\equiv \frac{1}{1-p}
\end{aligned}
$$

The equations of motion of the control variables are then given by:

$$
\begin{aligned}
\dot{c} &= \frac{(\xi_5 - \xi_2) \cdot (\rho - r) - \xi_3 \cdot \xi_5 \cdot \dot{d} + \xi_2 \cdot \xi_6 \cdot \dot{p}}{\xi_1 \cdot \xi_5 - \xi_4 \cdot \xi_2} \\
\dot{s} &= \frac{(\xi_1 - \xi_4) \cdot (\rho - r) + \xi_3 \cdot \xi_4 \cdot \dot{d} - \xi_1 \cdot \xi_6 \cdot \dot{p}}{\xi_1 \cdot \xi_5 - \xi_4 \cdot \xi_2}
\end{aligned}
$$

6.6 Results of Simulations with a Consumption Tax without Repayment of Tax Revenues

Table 6.1: Utility level over time if the tax revenue is not repaid.

σ	d	$t=$ 0	10	20	25	29	30
0.1	0.1	196.92	201.26	202.09	202.20	202.24	202.25
0.1	0.2	181.96	186.58	187.42	187.53	187.57	187.58
0.1	0.3	168.95	173.88	174.74	174.85	174.89	174.90
0.9	0.1	149.85	161.47	163.04	163.20	163.25	163.26
0.9	0.2	140.04	152.23	153.84	153.99	154.04	154.05
0.9	0.3	131.43	144.12	145.75	145.90	145.95	145.96
1	0.1	145.04	158.55	160.29	160.45	160.51	160.52
1	0.2	135.46	149.56	151.33	151.49	151.55	151.56
1	0.3	127.10	141.68	143.48	143.64	143.69	143.69

Table 6.2: Quality of nature over time if the tax revenue is not repaid.

σ	d	$t=$ 0	10	20	25	29	30
0.1	0.1	367.36	375.41	376.94	377.14	377.22	377.23
0.1	0.2	341.07	349.68	351.25	351.45	351.52	351.54
0.1	0.3	318.15	327.36	328.98	329.18	329.26	329.27
0.9	0.1	146.96	157.52	158.94	159.08	159.13	159.14
0.9	0.2	145.61	157.34	158.87	159.02	159.06	159.07
0.9	0.3	144.32	157.18	158.83	158.98	159.03	159.04
1	0.1	130.16	141.25	142.68	142.81	142.85	142.86
1	0.2	129.96	142.32	143.87	144.01	144.06	144.06
1	0.3	129.77	143.34	145.01	145.15	145.20	145.21

Table 6.3: Assets level over time if the tax revenue is not repaid.

σ	d	$t=$ 0	10	20	25	29	30
0.1	0.1	2468.74	2494.12	2498.89	2499.51	2499.72	2499.75
0.1	0.2	2465.10	2493.68	2498.86	2499.50	2499.72	2499.75
0.1	0.3	2461.13	2493.22	2498.82	2499.50	2499.72	2499.75
0.9	0.1	2413.99	2488.59	2498.51	2499.45	2499.72	2499.75
0.9	0.2	2405.90	2487.88	2498.47	2499.44	2499.72	2499.75
0.9	0.3	2398.08	2487.22	2498.43	2499.44	2499.72	2499.75
1	0.1	2400.73	2487.44	2498.44	2499.44	2499.72	2499.75
1	0.2	2391.78	2486.70	2498.40	2499.43	2499.72	2499.75
1	0.3	2383.36	2486.01	2498.36	2499.43	2499.72	2499.75

Table 6.4: Consumption over time if the tax revenue is not repaid

σ	d	$t=$ 0	10	20	25	29	30
0.1	0.1	204.58	209.11	209.96	210.08	210.12	210.13
0.1	0.2	188.27	193.07	193.94	194.06	194.10	194.11
0.1	0.3	174.18	179.28	180.18	180.29	180.33	180.34
0.9	0.1	192.54	207.88	209.96	210.16	210.23	210.24
0.9	0.2	176.08	191.85	193.92	194.12	194.18	194.19
0.9	0.3	161.99	178.08	180.14	180.33	180.40	180.41
1	0.1	189.46	207.60	209.95	210.16	210.24	210.25
1	0.2	173.04	191.57	193.91	194.12	194.19	194.20
1	0.3	159.02	177.81	180.13	180.34	180.40	180.41

Table 6.5: Expenditures for environmental quality over time if the tax revenue is not repaid

σ	d	$t=$ 0	10	20	25	29	30
0.1	0.1	18.20	18.70	18.80	18.81	18.82	18.82
0.1	0.2	16.39	16.92	17.02	17.03	17.03	17.03
0.1	0.3	14.86	15.41	15.51	15.52	15.52	15.52
0.9	0.1	16.74	18.44	18.67	18.69	18.70	18.70
0.9	0.2	14.96	16.67	16.90	16.92	16.93	16.93
0.9	0.3	13.47	15.18	15.40	15.43	15.43	15.43
1	0.1	16.40	18.40	18.66	18.68	18.69	18.69
1	0.2	14.63	16.63	16.89	16.91	16.92	16.92
1	0.3	13.15	15.15	15.40	15.42	15.42	15.43

Table 6.6: Growth rate of assets over time if the tax revenue is not repaid

σ	d	$t=$ 0	10	20	25	29	30
0.1	0.1	0.0021	$3.948 \cdot 10^{-4}$	$7.345 \cdot 10^{-5}$	$3.067 \cdot 10^{-5}$	$1.327 \cdot 10^{-5}$	$1.000 \cdot 10^{-5}$
0.1	0.2	0.0024	$4.340 \cdot 10^{-4}$	$7.762 \cdot 10^{-5}$	$3.177 \cdot 10^{-5}$	$1.343 \cdot 10^{-5}$	$1.000 \cdot 10^{-5}$
0.1	0.3	0.0027	$4.760 \cdot 10^{-4}$	$8.192 \cdot 10^{-5}$	$3.288 \cdot 10^{-5}$	$1.359 \cdot 10^{-5}$	$1.000 \cdot 10^{-5}$
0.9	0.1	0.0070	$9.328 \cdot 10^{-4}$	$1.209 \cdot 10^{-4}$	$4.214 \cdot 10^{-5}$	$1.487 \cdot 10^{-5}$	$1.000 \cdot 10^{-5}$
0.9	0.2	0.0079	0.0010	$1.263 \cdot 10^{-4}$	$4.332 \cdot 10^{-5}$	$1.502 \cdot 10^{-5}$	$1.000 \cdot 10^{-5}$
0.9	0.3	0.0087	0.0011	$1.313 \cdot 10^{-4}$	$4.440 \cdot 10^{-5}$	$1.516 \cdot 10^{-5}$	$1.000 \cdot 10^{-5}$
1	0.1	0.0084	0.0010	$1.296 \cdot 10^{-4}$	$4.403 \cdot 10^{-5}$	$1.512 \cdot 10^{-5}$	$1.000 \cdot 10^{-5}$
1	0.2	0.0093	0.0011	$1.352 \cdot 10^{-4}$	$4.522 \cdot 10^{-5}$	$1.527 \cdot 10^{-5}$	$1.000 \cdot 10^{-5}$
1	0.3	0.0102	0.0012	$1.402 \cdot 10^{-4}$	$4.627 \cdot 10^{-5}$	$1.541 \cdot 10^{-5}$	$1.000 \cdot 10^{-5}$

Table 6.7: Growth rate of consumption over time if the tax revenue is not repaid

		$t =$					
σ	d	0	10	20	25	29	30
0.1	0.1	0.0045	$8.440 \cdot 10^{-4}$	$1.585 \cdot 10^{-4}$	$7.036 \cdot 10^{-5}$	$3.995 \cdot 10^{-5}$	$3.580 \cdot 10^{-5}$
0.1	0.2	0.0053	$9.462 \cdot 10^{-4}$	$1.708 \cdot 10^{-4}$	$7.433 \cdot 10^{-5}$	$4.174 \cdot 10^{-5}$	$3.737 \cdot 10^{-5}$
0.1	0.3	0.0061	0.0010	$1.836 \cdot 10^{-4}$	$7.838 \cdot 10^{-5}$	$4.352 \cdot 10^{-5}$	$3.894 \cdot 10^{-5}$
0.9	0.1	0.0180	0.0023	$3.036 \cdot 10^{-4}$	$1.123 \cdot 10^{-4}$	$5.719 \cdot 10^{-5}$	$5.088 \cdot 10^{-5}$
0.9	0.2	0.0204	0.0026	$3.216 \cdot 10^{-4}$	$1.171 \cdot 10^{-4}$	$5.898 \cdot 10^{-5}$	$5.244 \cdot 10^{-5}$
0.9	0.3	0.0227	0.0028	$3.384 \cdot 10^{-4}$	$1.214 \cdot 10^{-4}$	$6.060 \cdot 10^{-5}$	$5.384 \cdot 10^{-5}$
1	0.1	0.0219	0.0027	$3.319 \cdot 10^{-4}$	$1.196 \cdot 10^{-4}$	$5.991 \cdot 10^{-5}$	$5.324 \cdot 10^{-5}$
1	0.2	0.0246	0.0029	$3.508 \cdot 10^{-4}$	$1.244 \cdot 10^{-4}$	$6.168 \cdot 10^{-5}$	$5.478 \cdot 10^{-5}$
1	0.3	0.0272	0.0032	$3.680 \cdot 10^{-4}$	$1.287 \cdot 10^{-4}$	$6.325 \cdot 10^{-5}$	$5.614 \cdot 10^{-5}$

Table 6.8: Growth rate of expenditures for environmental quality over time if the tax revenue is not repaid

		$t =$					
σ	d	0	10	20	25	29	30
0.1	0.1	0.0057	0.0010	$1.990 \cdot 10^{-4}$	$8.834 \cdot 10^{-5}$	$5.017 \cdot 10^{-5}$	$4.495 \cdot 10^{-5}$
0.1	0.2	0.0066	0.0012	$2.150 \cdot 10^{-4}$	$9.358 \cdot 10^{-5}$	$5.254 \cdot 10^{-5}$	$4.704 \cdot 10^{-5}$
0.1	0.3	0.0078	0.0013	$2.317 \cdot 10^{-4}$	$9.893 \cdot 10^{-5}$	$5.493 \cdot 10^{-5}$	$4.914 \cdot 10^{-5}$
0.9	0.1	0.0227	0.0029	$3.814 \cdot 10^{-4}$	$1.411 \cdot 10^{-4}$	$7.184 \cdot 10^{-5}$	$6.392 \cdot 10^{-5}$
0.9	0.2	0.0258	0.0032	$4.051 \cdot 10^{-4}$	$1.474 \cdot 10^{-4}$	$7.428 \cdot 10^{-5}$	$6.604 \cdot 10^{-5}$
0.9	0.3	0.0288	0.0035	$4.273 \cdot 10^{-4}$	$1.533 \cdot 10^{-4}$	$7.651 \cdot 10^{-5}$	$6.798 \cdot 10^{-5}$
1	0.1	0.0276	0.0034	$4.170 \cdot 10^{-4}$	$1.503 \cdot 10^{-4}$	$7.526 \cdot 10^{-5}$	$6.669 \cdot 10^{-5}$
1	0.2	0.0311	0.0037	$4.418 \cdot 10^{-4}$	$1.567 \cdot 10^{-4}$	$7.769 \cdot 10^{-5}$	$6.899 \cdot 10^{-5}$
1	0.3	0.0346	0.0040	$4.646 \cdot 10^{-4}$	$1.625 \cdot 10^{-4}$	$7.987 \cdot 10^{-5}$	$7.088 \cdot 10^{-5}$

Table 6.9: Speed of convergence of assets per head

		Δa		
σ	d	5%	1%	0.1%
0.1	0.1	38.40	28.66	14.89
0.1	0.2	37.56	28.04	14.59
0.1	0.3	36.78	27.46	14.31
0.9	0.1	31.89	23.86	12.54
0.9	0.2	31.41	23.51	12.37
0.9	0.3	31.00	23.21	12.22
1	0.1	31.14	23.31	12.27
1	0.2	30.70	22.98	12.11
1	0.3	30.33	22.71	11.97

Table 6.10: Tax revenue over time if the tax revenue is not repaid

		$t =$					
σ	d	0	10	20	25	29	30
0.1	0.1	20.458	20.911	20.996	21.008	21.012	21.013
0.1	0.2	37.653	38.614	38.789	38.811	38.820	38.822
0.1	0.3	52.254	53.784	54.053	54.086	54.098	54.101
0.9	0.1	19.254	20.788	20.996	21.016	21.023	21.024
0.9	0.2	35.217	38.370	38.785	38.824	38.837	38.839
0.9	0.3	48.598	53.423	54.043	54.100	54.119	54.122
1	0.1	18.946	20.760	20.995	21.016	21.024	21.025
1	0.2	34.607	38.314	38.782	38.824	38.838	38.840
1	0.3	47.706	53.343	54.040	54.101	54.120	54.124

Table 6.11: Accumulated tax revenue over time if the tax revenue is not repaid

		$\Delta t =$				
σ	d	30	20	10	5	1
0.1	0.1	627.16	419.72	201.07	105.05	21.01
0.1	0.2	1157.97	775.34	388.09	194.08	38.82
0.1	0.3	1612.67	1080.38	540.83	270.47	54.10
0.9	0.1	621.96	419.39	210.14	105.10	21.02
0.9	0.2	1147.47	774.64	388.20	194.16	38.84
0.9	0.3	1596.96	1079.29	540.95	270.56	54.12
1	0.1	620.66	419.29	210.14	105.10	21.02
1	0.2	1144.91	774.45	388.20	194.17	38.84
1	0.3	1593.23	1079.02	540.95	270.57	54.12

7 References

Barro, Robert, and Xavier Sala-i-Martin (1995): *Economic Growth*, McGraw Hill, New York et al.

Bohm, Peter, and Clifford S. Russell (1985): Comparative Analysis of Alternative Policy Instruments, In: Allan Kneese and James L. Sweeney (eds.), *Handbook of Natural Resource and Energy Economics*, North-Holland, 395-460.

Bovenberg, A. Lans, and Ruud A. de Mooij (1997): Environmental Tax Reform and Endogenous Growth, *Journal of Public Economics*, **63**, 207-237.

Bovenberg, A. Lans, and Sjak A. Smulders (1995): Environmental Quality and Pollution-augmenting Technical Change in a Two-sector Endogenous Growth Model, *Journal of Public Economics*, **57**, 369-391.

Bovenberg, A. Lans, and Sjak A. Smulders (1996): Transitional Impacts of Environmental Policy in an Endogenous Growth Model, *International Economic Review*, **37**, 861-891.

Brunner, Martin, and Holger Strulik (2002): Solution of Perfect Foresight Saddlepoint Problems: A Simple Method and Applications, *Journal of Economic Dynamics and Control*, **26**, 737-753.

Bundesministerium der Finanzen (ed.) (2005): Zoll Online - Verbrauchsteuersätze in Euro seit 01.09.2005. Available from: http://www.zoll-d.de/b0_zoll_und_steuern/b0_verbrauchsteuern/a0_allgemeines/f0_vstsaetze_in_euro/index.html.

European Commission, Directorate-General for Energy and Transport (ed.) (2004): *Energy & Transport in Figures 2004*. Available from: http://europa.eu.int/comm/dgs/energy_transport/figures/pocketbook/index_en.htm.

Forster, Bruce A. (1973): Optimal Capital Accumulation in a Polluted Environment, *Southern Economic Journal*, **39**, 544-547.

Gradus, Raymond, and Sjak A. Smulders (1993): The Trade-off Between Environmental Care and Long-term Growth - Pollution in Three Prototype Growth Models, *Journal of Economics / Zeitschrift für Nationalökonomie*, **58**, 25-51.

Hettich, Frank (2000): *Economic Growth and Environmental Policy*, Edward Elgar, Cheltenham (UK) and Northampton (MA, US).

Huang, Chung-Huang, and Deqin Cai (1994): Constant Returns Endogenous Growth with Pollution Control, *Environmental and Resource Economics*, **4**, 383-400.

Lighthart, Jenny E., and Frederick van der Ploeg (1994): Pollution, the Cost of Public Funds and Endogenous Growth, *Economics Letters*, **46**, 351-361.

Mäler, Karl Göran (1985): Welfare Economics and the Environment. In: Allan Kneese and James L. Sweeney (eds.), *Handbook of Natural Resource and Energy Economics*, North-Holland, 1-60.

Pittel, Karen (2002): *Sustainability and Economic Growth*, Edward Elgar, Cheltenham (UK) and Northampton (MA, US).

Smulders, Sjak A., and Raymond Gradus (1996): Pollution Abatement and Long-term Growth, *European Journal of Political Economy*, **12**, 505-532.