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Abstract. This paper investigates the impact of externalities on economic growth in an AK model. The paper considers finitely-lived agents along the continuous time, overlapping generations literature. New results, *not* holding for infinitely-lived agent economies, emerge. Negative consumption externalities generally imply overconsumption, and savings (growth) rates are lower than optimal. The inefficiency occurs even when labor supply is exogenous and there is no concurrent production externality. Transition paths are considered. The model employed encompasses the infinitely-lived agent economy as a special case, thus helps understanding the differences in results between finite-horizon overlapping-generations and infinitely-lived agents economies.

Keywords and Phrases: AK growth, externality, finite lifetime, overlapping generations, optimum.

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1 Introduction

This paper analyzes the (distortionary) impact of consumption and production externalities on economic growth. In contrast to the existing literature, this paper considers finitely-lived agents.

Individual demand is conventionally considered to be independent of others' consumption demands. The rate at which the sales-share of sports utility vehicles (purchased by non-adventurous people) in total car sales increases, however, casts doubt on this independence. It rather is anecdotal evidence for the possibility of a negative consumption externality.

There is significant empirical evidence of consumption and production externalities. Brekke and Howarth (2002), Frank (1999), Johansson-Stenman et al. (2002, 2006), Luttmer (2005), and Solnick and Hemenway (1998, 2005) present impressive evidence for negative consumption externalities ("keeping (catching) up with the Jonses" preferences).[1] Their consideration in economic analysis is important as they allow us to explain data that cannot be otherwise explained. For example, various papers demonstrate that consideration of consumption externalities help resolve many economic puzzles, including the equity premium, excess sensitivity, and excess smoothness puzzles.

Externalities also have an impact on the growth dynamics. In addition to investigating the effects of externalities for consumption growth and capital accumulation, it is of crucial importance to know whether this impact

[1] Regarding consumption externalities, further important references include Alpizar et al. (2005), Carlsson et al. (2003), Easterlin (1995), Ferrer-i-Carbonell (2005), McBride (2001), Neumark et al. (1998). For production externalities, see, e.g., Caballero and Lyons (1990, 1992), and Lindstrom (2000).

is distortionary or not. For example, if a negative consumption externality imposed a distortion, a shift in the tax base towards consumption would possibly raise efficiency. If the consumption externality had no distortionary impact, the same tax reform would reduce efficiency. In this paper, we show that *only* in a framework with *finite* horizons, a negative consumption externality *always* leads to "overconsumption" (consumption levels higher than optimal), and to savings rates (growth rates) lower than optimal. This finding helps explaining the very low savings rates seen in many industrialized countries. As argued below, in the framework of infinitely-lived agent (ILA) models, consumption externalities either do not introduce a distortion at all, or they result in "underconsumption," thereby implying savings and growth rates *higher* than optimal.

More specifically, this paper offers a systematic analysis of the effects of externalities, in the context of a continuous-time overlapping generations (C-OLG) model with AK technology. The model contains the case of infinite lifetime as a special case. Therefore, it is possible to relate the results obtained from the C-OLG economy, with a finite horizon, to those of the ILA economy, with an infinite horizon. The analysis displays three important results.

First, with a negative consumption externality, the steady state growth rate in a decentralized economy always falls short from the one in a command optimum, contrary to the infinitely-lived agent case (where the consumption externality does not introduce a distortion in the steady state). The reason is seen to come from overconsumption. Households consume more and save less than optimal. Consequently consumption growth is lower than optimal.

Second, growth rates along the transition paths always differ between a decentralized economy and the ones in a command optimum. Whether this difference is positive or negative depends on the nature of the consumption externality. In particular, we demonstrate that the consumption growth rate can be larger than optimal for some time, along a transition path, and it can later become smaller than optimal, close to and in a steady state equilibrium. This reversal is never implied by an ILA model. These results hold – even when there is no production externality, and labor supply is exogenous. Third, in the ILA model, consumption is equal across agents in equilibrium. One might ask, whether a consumption externality (such as keeping up with the Joneses preferences) can appropriately be treated in such a symmetrical context, as the externality stems from *different* consumption levels. The C-OLG model introduces the required heterogeneity. As wealth differs among cohorts, so do individual consumption levels *in equilibrium*. Therefore, the C-OLG framework seems to be a natural framework to consider consumption externalities.

This work relates to several papers in the literature. Brekke and Howarth (2002), and Johansson-Stenman et al. (2002, 2006) discuss the impact of consumption externalities on consumption choice and growth in some detail. Liu and Turnovsky (2005) consider consumption and production externalities in an economy, populated by infinitely-lived households. They show that if labor supply is inelastic, without a production externality, consumption externalities do *not* introduce a distortion with respect to both the steady state equilibrium and the transitional dynamics, when the marginal rate of substitution between individual and aggregate consumption is constant through time. Carroll et al. (1997) analyze the effects of consumption externalities in an endogenous growth model populated with infinitely-lived dynasties. In their analysis, they show that the steady state growth rates of consumption, capital and output increase in the consumption externality, that is shown to raise the long-horizon intertemporal elasticity of substitution in consumption. Abel (2005) extents these analyses in that he considers the impact of a consumption externality within the framework of an overlapping generations economy, where each generation is alive for two periods. He focuses on optimal fiscal policy and demonstrates that in addition to a lump-sum tax/transfer scheme (that ensures optimal savings), an additional instrument is needed for internalizing the consumption externality. In contrast to Abel (2005), the consumption externality introduces a distortion in the C-OLG model, even when the social planner's discount rate equals the market one.[2]

This paper systematically studies the distortionary impact of externalities on economic growth in the context of a C-OLG model where households are finitely-lived. In contrast to the existing ILA literature, the present study shows that in the finite-horizon case, consumption externalities (alone) are always distortionary, that is, they introduce an inefficiency. Many studies show significant evidence for *negative* consumption externalities (see above). As one consequence, this paper's results support arguments in favor of a shift in the tax base towards consumption, on grounds of efficiency.

The paper is structured in the following way. Section 2 presents the economy's structure of both the market economy framework and a command optimum. Section 3 first compares the steady state effects of the externalities

[2]In Abel (2005), benchmarks introduce no distortion, when private and social discount rates are equal, and benchmarks are equally weighted averages of all consumers.

between both economies, and then considers the effects of the externalities on transitional dynamics. Section 4 concludes the paper. The appendix contains major derivations and proofs, and is found at the end of this paper.

2 The Economy's Structure

In this section, we extend the continuous time overlapping generations model by consideration of consumption and production externalities. In light of the evidence referred to in the introduction, we focus on a *negative* consumption externality ("catching up with the Joneses" preferences), and a *positive* production externality, as in Romer (1986).

Population. An individual born at time v ("vintage") is uncertain about the length of his or her life. Following the usual perpetual-youth assumption (Blanchard, 1985), we employ two assumptions. First, the instantaneous probability of death (the "death rate"), β, is constant and independent of age. Second, the death rate equals the birth rate. At each instant of time, a *large* new cohort of size $\beta\, P(t)$ is born, where $P(t)$ is the size of population in t. Clearly, as the population size is "large," the number of births and deaths coincide, and we can normalize population size to unity: $P(t) \equiv 1$. Under this simple population structure, at time t, the size of a vintage-v cohort is $\beta\, e^{-\beta\,(t-v)}$. Moreover, the expected remaining lifetime of any agent is β^{-1}. As a special case, the representative-agent model emerges from the continuous time overlapping generations model as β approaches zero.

Households. Time-t utility of a vintage-v household is a function $u(.)$ of consumption $c(v,t)$. The first argument in $c(.)$ refers to the birth date, and the second argument refers to time. At time t, an individual household, however, not only cares about its *own* consumption, but also about how own consumption compares to some reference consumption level $X(t)$. Therefore, instantaneous utility is given by $u(c(v,t), X(t))$.

In this paper, we consider the standard case of a CRRA utility function. We follow Galí (1994) and Carroll et al. (1997) in specifying the instantaneous utility function as:

$$\begin{aligned} u(c(v,t), X(t)) &= \frac{c(v,\, t)^{(1-\sigma)}\, X(t)^{-\gamma(1-\sigma)}}{1-\sigma} \\ &= \frac{1}{(1-\sigma)} \left[c(v,\, t)^{(1-\gamma)} \left(\frac{c(v,\, t)}{X(t)} \right)^{\gamma} \right]^{1-\sigma} , \quad \sigma > 1\,. \end{aligned} \tag{1}$$

Parameter $\gamma \in [0,1)$ measures the extent of the consumption externality, i.e., the importance of the reference stock. Suppose, $\gamma = 0$, then the model reduces to the usual model with interpersonally separable CRRA utility. Parameter σ governs the intertemporal elasticity of substitution. If $\gamma = 0$, the intertemporal elasticity of substitution is given by σ^{-1}. If, however, $\gamma > 0$, both, own consumption and the reference stock determine the elasticity of substitution between consumption at any two points in time. As is overwhelmingly suggested by the empirical literature, σ is considered to be larger than unity.

At time t, expected lifetime utility of a household born in v is given by

$$E[U(v,t)] = \int_t^{\infty} u(c(v,\tau), X(\tau))\, e^{-(\rho+\beta)\,(\tau-t)}\, d\tau\,, \tag{2}$$

where ρ is the household's pure rate of time preference. As in Yaari's (1965) analysis, the possibility of death leads to a subjective discount rate $(\rho + \beta)$

higher than the pure rate of time preference.[3]

Reference Consumption. Aggregate consumption is defined by $C(t) \equiv \beta \int_{-\infty}^{t} e^{-\beta(t-v)} c(v,t)\, dv$. As the population size is normalized to unity, $C(t)$ equals average consumption. The reference consumption stock develops according to:

$$\dot{X}(t) = \varphi\, (C(t) - X(t))\ , \quad X(0) = X_0\,, \tag{3}$$

with X_0 exogenously given.[4] The reference level $X(t)$ is a weighted average of past *average* consumptions. The parameter $\varphi \geq 0$ determines the weight of average consumption at different times. The larger is φ, the more important is average consumption in the *recent* past. As $\varphi \to \infty$, $X(t) \to C(t)$.

Sign Restrictions. With $\gamma \in [0,1)$, and $\sigma > 1$, marginal utility rises in the reference stock: $u_{cX}(.) > 0$. In the taxonomy of Dupor and Liu (2003), preferences exhibit the "keeping (catching) up with the Jonses" property, for these sign restrictions. Moreover, $\gamma < 1$ ensures positivity of marginal utility of a proportionate increase in both individual consumption and the reference stock: $u_c(.) + u_X(.) > 0$. To keep the economy from explosive growth, we need to impose the following sign restriction:

$$\sigma - \gamma\,(\sigma - 1) > 0\,, \tag{4}$$

which ensures: $u_{cc}(.) + u_{cX}(.) < 0$. That is, a proportionate increase of both individual consumption and reference stock has diminishing marginal utility.[5]

[3] Observe that the subjective (effective) discount rate, $\rho + \beta$, is essentially independent of the specification of the utility function.

[4] Different specifications could potentially be adopted. The proposed specification, however, is rather general, and there is no empirical evidence supporting an alternative specification.

[5] For a CRRA utility function, sign restriction (4) is implied by $\gamma < 1$.

Production. There is a large number of perfectly competitive, identical firms producing a homogeneous output, $Y(t)$, according to

$$Y(t) = Z(t)\, K(t)^{\alpha}\, L(t)^{1-\alpha}\,, \quad 0 < \alpha \leq 1\,, \tag{5}$$

where $K(t)$ is capital, and $L(t)$ are labor services. Total factor productivity, $Z(t)$, is considered given by individual firms. Firms maximize profits and hire factors from households on competitive factor markets:

$$\alpha\, Z(t)\, L(t)^{1-\alpha}\, K(t)^{-1+\alpha} = r(t) + \delta\,, \quad (1-\alpha)\, Z(t)\, L(t)^{-\alpha}\, K(t)^{\alpha} = w(t)\,, \tag{6}$$

where $r(t)$ is the rate of interest, $w(t)$ is the wage rate, and δ is the rate of depreciation of capital.

Factor productivity, $Z(t)$, is given by:

$$Z(t) = A\, K(t)^{1-\alpha}\,, \quad \alpha < 1\,. \tag{7}$$

As in Romer (1986), the aggregate capital stock serves as a proxy for knowledge, which gives rise to a (potentially) positive production externality. The magnitude of the production externality is determined by factor $(1-\alpha)$. As long as α is strictly smaller than unity, there is a positive externality of capital accumulation.[6]

We conclude this description with two important remarks. First, on an aggregate level, considering (7) together with the labor market clearing condition, $L(t) = P(t) = 1$, production is given by: $Y(t) = A\, K(t)$, and the market interest and wage rates are obtained by:

$$r(t) = r = \alpha\, A - \delta\,, \quad w(t) = (1-\alpha)\, A\, K(t)\,. \tag{8}$$

[6]Certainly, the analysis can be extended to *negative* production externalities, that is, $\alpha > 1$. This case, however is in contrast with the evidence, given in the introduction.

That is, the production externality has a direct impact on the rate of interest. The larger the externality, the lower is the market interest rate as compared to the socially optimal rate of interest: $A - \delta$. Second, the aggregate stock of capital evolves according to:

$$\dot{K}(t) = Y(t) - C(t) - \delta\, K(t)\,. \tag{9}$$

2.1 The Market Economy

Every household inelastically supplies labor services and chooses consumption at all $t \geq v$ such as to maximize expected lifetime utility (2) subject to its intertemporal budget constraint:

$$a(v,t) + h(t) - \int_t^{\infty} c(v,\tau)\, e^{-R^A(t,\tau)}\, d\tau = 0\,, \tag{10}$$

where $a(v,t)$ stands for time-t assets of a vintage-v household, and $h(t) \equiv \int_t^{\infty} w(\tau)\, e^{-R^A(t,\tau)} d\tau$ is the discounted integral of future wage payments. In the market framework, a household does *not* consider the impact of its individual consumption on average consumption (on the consumption reference level, X).

The market framework is one in which individuals may buy or sell actuarially fair notes, for which they pay or receive an interest rate $r^A(t)$. The notes are canceled upon death of an individual. Actuarial fairness requires $r^A(t) = r(t) + \beta$. In this framework, the relevant discount factor is: $R^A(t,\,\tau) \equiv \int_t^{\tau} [r(s) + \beta]\, d\,s$.[7]

[7] The transversality condition required to prevent households from running Ponzi schemes is: $\lim_{\tau\to\infty} e^{-R^A(t,\tau)}\, a(v,\tau) = 0$. Clearly, budget constraint (10) follows from combining the budget identity, $\dot{a}(v,t) = r^A(t)\, a(v,t) + w(t) - c(v,t)$, with the transversality condition.

Define $g_x(t) \equiv \dot{X}(t)/X(t)$, and $\Gamma(t,\tau) = \int_t^\tau g_x(s)\,ds$.[8] Then, the first order conditions are:

$$c(v,t)\,e^{\Gamma(t,\tau)\,\gamma\,\frac{(\sigma-1)}{\sigma}+\frac{(\rho+\beta)}{\sigma}(t-\tau)-R^A(t,\tau)\frac{(\sigma-1)}{\sigma}} = c(v,\tau)\,e^{-R^A(t,\tau)}\,, \quad \tau \geq t\,. \tag{11}$$

Define $\zeta(t) \equiv \sigma^{-1}\rho + (1-\sigma^{-1})r(t) + \beta$. Integration of both sides of (11) yields:

$$c(v,t) = \zeta(t)\,[a(v,t)+h(t)] - \gamma\,(1-\sigma^{-1})\,g_x(t)\,[a(v,t)+h(t)]\,. \tag{12}$$

As we do not consider a bequests motive, we have $a(t,t) = 0$, hence,

$$c(t,t) = \zeta(t)\,h(t) - \gamma\,(1-\sigma^{-1})\,g_x(t)\,h(t)\,. \tag{12a}$$

Notice that consumption levels are *not* equal across cohorts. Consumption levels are proportional to wealth, with a factor of proportionality given by: $[\zeta(t) - \gamma\,(1-\sigma^{-1})\,g_x(t)]$. As $a(t,t) = 0$, we know that $a(t,t)$ is smaller than average wealth. Thus $c(t,t)$ is smaller than average consumption.

Differentiating (11) with respect to τ yields:

$$\frac{\dot{c}(v,t)}{c(v,t)} = \frac{r(t)-\rho}{\sigma} + \gamma\,\frac{\sigma-1}{\sigma}\,\frac{\dot{X}(t)}{X(t)}\,. \tag{13}$$

The growth rate of individual consumption not only depends on the rate of interest and the pure rate of time preference but it also depends *positively* on the growth rate of the consumption reference stock. It does *not* directly depend on β.

Aggregation. Aggregate wealth, $\Omega(t)$, is given by $\Omega(t) \equiv \beta \int_{-\infty}^t e^{-\beta\,(t-v)}\,a(v,t)\,dv$.

[8] Throughout the paper, g_z denotes the growth rate of some variable z.

Capital market clearing requires $\Omega(t) = K(t)$. Similarly, aggregate consumption is defined by $C(t) \equiv \beta \int_{-\infty}^{t} e^{-\beta\,(t-v)}\, c(v,t)\, dv$. Differentiation of aggregate consumption with respect to time, and using (13) yields:

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t)-\rho}{\sigma} - \beta\,\zeta(t)\,\frac{\Omega(t)}{C(t)} + \gamma\,(1-\sigma^{-1})\,\frac{\dot{X}(t)}{X(t)}\,\left(1+\beta\,\frac{\Omega(t)}{C(t)}\right)\,.$$

We are now ready to represent aggregate behavior by a series of differential equations in the variables $\kappa(t) \equiv K(t)/X(t)$, and $\chi(t) \equiv C(t)/X(t)$:[9]

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t)-\rho}{\sigma} + \gamma\,(1-\sigma^{-1})\,\frac{\dot{X}(t)}{X(t)} - \beta\,\left[\zeta(t) - \gamma\,(1-\sigma^{-1})\,\frac{\dot{X}(t)}{X(t)}\right]\,\frac{\kappa(t)}{\chi(t)}\,, \tag{14}$$

$$\frac{\dot{X}(t)}{X(t)} = \varphi(\chi(t)-1)\,, \tag{3'}$$

$$\frac{\dot{K}(t)}{K(t)} = A - \delta - \frac{\chi(t)}{\kappa(t)}\,. \tag{15}$$

Equation (14) deserves four remarks. First, aggregate consumption growth depends on the difference between the rate of interest and the pure rate of time preference. If $\beta = \gamma = 0$, equation (14) represents the usual Euler equation. Second, if, in addition, consumption causes a negative externality, the consumption growth rate is raised by the growth of the reference stock. This is intuitive. From the viewpoint of an individual household, if all other households raise their rates of consumption growth (and thereby the reference stock), the best response of the individual household will be to raise his or her rate of consumption growth as well. As a consequence, aggregate consumption growth increases in the negative consumption externality. Third, if the length of lifetime is finite, $\beta > 0$, consumption growth not only depends

[9] Notice that the dynamics of the model is two-dimensional: $\dot{\chi} = \dot{\chi}(\chi,\kappa)$, $\dot{\kappa} = \dot{\kappa}(\chi,\kappa)$. The dimension of the stable manifold is one, and the system is saddle point stable.

on the reference stock but also on the capital stock.[10] As the term in square brackets is positive[11], a rise in β lowers the aggregate consumption growth rate. This effect is due to the "intergenerational turnover effect," which is discussed in Lemma 1, below. Forth, a rise in the production externality (a decline in α) lowers the consumption growth rate. A decline in α lowers the rate of interest according to (8), and we expect the rate of consumption growth to decline: $-\partial\,(\dot{C}/C)/\partial\, r = -(1-\beta(\sigma-1))/\sigma < 0$.

The second equation, above, displays the dynamics of the reference stock. It is a restatement of equation (3). Capital accumulation (15) follows from (9).

Before analyzing the impact of finite horizons in more detail, we note an important relationship between individual and aggregate consumption growth, which is obtained by differentiating the definition of $C(t)$ with respect to time: $\dot{C}(t) = \beta\, c(t,t) - \beta\, C(t) + \beta \int_{-\infty}^{t} [\dot{c}(v,t)/c(v,t)]\; c(v,t)\, e^{-\beta\,(t-v)}\, dv$. Using (13) and (12a) yields:

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{c}(v,t)}{c(v,t)} - \beta\; \frac{C(t) - c(t,t)}{C(t)} = \frac{\dot{c}(v,t)}{c(v,t)} - \beta\; \frac{\Omega(t)}{\Omega(t) + h(t)}\,. \tag{16}$$

If $\beta > 0$, the rate of aggregate consumption growth is smaller than the rate of individual consumption growth. The difference in individual and aggregate growth rates is caused by the intergenerational turnover effect.

Intergenerational Turnover Effect. According to (16), aggregate and individual rates of consumption growth differ if and only if $\beta > 0$ and $c(t,t) \neq C(t)$.

[10]If $\beta = 0$ (ILA case) the capital stock has no effect on consumption growth, as long as the rate of interest is independent of K, as is the case here. This was already shown by Carroll et al. (1997).

[11]The term in square brackets in (14) is positive by the fact that $c(v,t) > 0$, as can easily be verified by (12).

The following Lemma 1 is particularly useful for explaining the differences between *optimal* growth paths and those obtained in a decentralized, competitive equilibrium.

LEMMA 1 (Intergenerational Turnover Effect) *If $\beta > 0$, $c(v,t)$ has the following properties:*
(i) $c(v,t)$ is proportional to wealth $[a(v,t)+h(t)]$, where the factor of proportionality is positive and invariant with respect to the vintage index v;
(ii) $c(t,t) - C(t) < 0$;
(iii) $\dot{c}(v,t)/c(v,t) = \dot{c}(v',t)/c(v',t)$, for all $v, v' \leq t$;
(iv) the difference in individual and aggregate consumption growth rates, $[\dot{c}(v,t)/c(v,t) - \dot{C}(t)/C(t)]$, rises in β;
(v) the difference in individual and aggregate consumption levels, $|c(t,t) - C(t)|$, rises in the level of capital..

The lemma shows an important property of the C-OLG model. While the growth rate of individual consumption is *independent* of wealth, the growth rate of *aggregate* consumption depends on wealth when $\beta > 0$.

By (13), individual consumption growth rates are independent of wealth (κ). If $c(v,t)$ were equal to $c(v',t)$ for all $v \leq t$, then $c(t,t)$ would be equal to $C(t)$, and the aggregate consumption growth rate would be equal to the individual one. However, individual consumption levels rise in wealth, and $c(t,t) < C(t)$. Therefore, $(\dot{c}(v,t)/c(v,t)) > (\dot{C}(t)/C(t))$. Intuitively, existing (wealthy) individuals are replaced by newborn individuals without wealth ("intergenerational turnover"), which lowers the average (aggregate) consumption growth rate.

2.2 The Planner's Objective

Following Calvo and Obstfeld (1988), our planner's objective, at time t, is the sum of two components. First, the integral of the lifetime expected utilities of *representative* agents from each of the generations to be born, as measured from the time of birth. Second, the integral of the lifetime expected utilities over the remainder of their lifetimes of representative agents from all cohorts alive at time t, measured from the perspective of his and her birthdate.[12] The planner's discount rate needs not equal the private pure time-preference rate. In the following, however, we assume both discount rates to coincide.[13]

Welfare at time t is

$$\begin{aligned} W(t) = & \int_t^\infty \left\{ \int_v^\infty u(c(v,\tau), X(\tau)) e^{\rho(v-\tau)}\, \beta\, e^{-\beta(\tau-v)}\, d\tau \right\} e^{-\rho(v-t)}\, d\, v \\ & + \int_{-\infty}^t \left\{ \int_t^\infty u(c(v,\tau), X(\tau)) e^{\rho(v-\tau)}\, \beta\, e^{-\beta(\tau-v)}\, d\tau \right\} e^{-\rho(v-t)}\, d\, v\, . \end{aligned}$$

Define age as $y \equiv t - v$ (number of "$y$ears"), and change the order of integration. Then,

$$W(t) = \int_t^\infty \left\{ \int_0^\infty u(c(\tau - y, \tau), X(\tau)) \beta\, e^{(\rho-(\rho+\beta))\, y}\, d\, y \right\} e^{-\rho(\tau-t)}\, d\, \tau\, . \quad (17)$$

The planner's problem of maximizing (17) subject to

(i) $C(\tau) = \beta \int_0^\infty e^{-\beta\, y}\, c(\tau - y, \tau)\, d\, y\,,$

(ii) $\dot{X}(\tau) = \varphi(C(\tau) - X(\tau))\,,$

(iii) $\dot{K}(\tau) = (A - \delta)\, K(\tau) - C(\tau)\,,$

can be clarified by decomposing it into two stages. First, given levels of

[12] As shown by Calvo and Obstfeld (1988), discounting back to birthdates ensures time-consistency of optimal fiscal policy.

[13] The more general case case where the planners' discount rate potentially is different from ρ is available as a working paper from the author. This case, however, does not add substantive results to the analysis.

aggregate consumption $C(\tau)$ and $X(\tau)$, the planner must allocate $C(\tau)$ in order to maximize time τ welfare: $\int_0^\infty u(c(\tau-y,\tau),X(\tau))\beta\, e^{(\rho-(\rho+\beta))\,y}\,d\,y$:

$$V[C(\tau),X(\tau)] = \max_{\{c(\tau-y,\tau\}_{y=0}^{\infty}} \int_0^\infty u(c(\tau-y,\tau),X(\tau))\beta\, e^{(\rho-(\rho+\beta))\,y}\,d\,y\,, \quad (18)$$

subject to constraint (i). At the second stage, the planner's problem becomes the optimal control problem of choosing $C(\tau)$, and $X(\tau)$, such as to maximize

$$W(t) = \int_t^\infty V[C(\tau),X(\tau)]\, e^{-\rho(\tau-t)}\,d\,\tau\,, \quad (19)$$

subject to constraints (ii) and (iii). As can easily be shown,

$$V[C(\tau),X(\tau)] = \frac{[C(\tau)\,X(\tau)^{-\gamma}]^{1-\sigma}}{1-\sigma}\,. \quad (20)$$

Next, we characterize both individual and aggregate consumption. Denote the Lagrange multiplier associated with the constraint of stage 1 above by λ:

$$\lambda(\tau) = c(\tau-y,\tau)^{-\sigma}\,X(\tau)^{-\gamma(1-\sigma)}\,. \quad (21)$$

It follows that $c_y(\tau-y,\tau)=0$. This is very intuitive. An allocation of aggregate consumption is optimal, if there is no incentive to shift consumption between cohorts at any time τ. As private and social discount rates are equal, it is optimal to implement an egalitarian plan under which all cohorts receive the *same* consumption level at any point in time.

Aggregate consumption growth follows from the first order conditions of the Hamiltonian (see appendix) as well as from the canonical equations:

$$\frac{\dot{C}(\tau)}{C(\tau)} = \frac{\tilde{r}-\rho}{\sigma} + \gamma\,\frac{\sigma-1}{\sigma}\frac{\dot{X}(\tau)}{X(\tau)} - \frac{\gamma}{\sigma}\,\dot{\chi}(\tau)\,(1-\gamma\,\chi(\tau))^{-1}\,, \quad (22)$$

where $\tilde{r}\equiv A-\delta$ defines the *social rate of interest*. As for any given time, all individual consumption levels are identical in the command optimum, individual and aggregate consumption growth rates coincide:

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{c}(v,t)}{c(v,t)} = \frac{\dot{c}(t,t)}{c(t,t)}\,. \tag{23}$$

We end this section by observing three important properties of the command optimum. First, optimal consumption is independent of β. In the market economy, this is true on an individual level only. Aggregate consumption growth, however, depends on β in the market economy. Second, if $\gamma = 0$, we are back to the standard Cass-Koopmans-Ramsey representative agent planning model. Moreover, as in the decentralized economy, individual consumption growth rates are independent of age. Third, in contrast to the market economy, individual consumption *levels* are independent of age in the command optimum.

3 Overlapping Generations, Consumption and Production Externalities

In the previous literature[14], it was shown that in ILA economies with exogenous labor supply, a consumption externality has an impact on the steady state only in the presence of a production externality. Moreover, a consumption externality alone has no impact on transitional paths when the reference level is given by current average consumption. In the following, we complement the above findings and show that consumption externalities generally do have an impact, both along transitional paths and in steady state, if agents live for a finite time only ($\beta > 0$). We also briefly address the impact

[14] See Liu and Turnovsky (2005), and Turnovsky and Monteiro (2007)

of β on the speed of convergence at the end of this section.[15]

3.1 Comparison of Steady State Equilibria

In a steady state, the growth rates of aggregate consumption, capital, and the reference stock are equal.[16] Denote this growth rate as g^* for the market economy (the "market growth rate"), and as $\tilde{g}$ for the planner's solution (the "optimal growth rate"). It follows that $\kappa(t)$ and $\chi(t)$ are constant in a steady state. Define $\tilde{\sigma} \equiv \sigma - \gamma(\sigma - 1) > 1$. We first consider the impact of β on the steady state growth rate, and on κ and χ. We then consider the distortionary effects of the externalities.

LEMMA 2 *Consider an economy without a production externality* $(\alpha = 1)$*. Then, the optimal steady state growth rate exceeds the market growth rate if and only if* $\beta > 0$*:*

$$\tilde{g} = \frac{\tilde{r} - \rho}{\tilde{\sigma}}, \quad g^* = \frac{r - \rho - \beta\,\sigma}{\tilde{\sigma}}. \tag{24}$$

Proof. Calculate g^* by equating (14) with (3) and (15), and $\tilde{g}$ by equating (22) with (3) and (15). Clearly, as $\alpha = 1$ we have $\tilde{r} = r$, and the lemma follows. ‖

Lemma 2 shows that the aggregate optimal and market growth rates do not

[15] All of the following results hold regardless of whether the planner and market discount rates are equal or not. Differences of the discount rates as a source of distortion are emphasized by Abel (2005). Such differences are *one* source of distortion here as well. However, to sharpen the main results we assume that the planner and the market discount the future at the *same* rate.

[16] The necessary and sufficient condition for a positive steady state to exist, in a market economy, is: $(r - \rho - \beta\,\sigma)(\tilde{r} + \beta) + (\tilde{r} - r)\,\beta\,\sigma > 0$. That is, a sufficient condition is: $(r - \rho - \beta\,\sigma) > 0$.

coincide when $\beta > 0$. This is true regardless of whether there is a consumption externality or not. Even if $\gamma = 0$ — thereby $\tilde{\sigma} = \sigma$ — Lemma 2 shows that $\tilde{g} > g^*$ whenever $\beta > 0$. The market growth rate of aggregate consumption is smaller than the optimal growth rate because of the intergenerational turnover effect, indicated in Lemma 1 (property iv). This difference in growth rates, when $\gamma = 0$, does *not* indicate any distortion. However, we show below that Lemma 2 proves important to demonstrate the existence of a distortion in case $\gamma > 0$.

Figure 1 illustrates the impact of a rise in β on κ and χ in the steady state. The Figure shows, in (χ, κ) space, all loci for which $\dot{\chi} = 0$, and all loci for which $\dot{\kappa} = 0$. The former loci are denoted by $\chi\chi$, the latter by $\kappa\kappa$. Socially optimal loci and values are marked with a tilde.

The $\kappa\kappa$-curve follows from $\dot{K}/K = \dot{X}/X$. It is the same for both a social optimum and a market economy, and it is independent of both externalities — that is, the $\kappa\kappa$-curve is independent of the parameters α, β, γ. Its slope is clearly positive[17], and increasing in (χ, κ) space. Intuitively, the propensity to consume (χ/κ) declines along the convex $\kappa\kappa$-curve, as κ rises. This leads to a rise in savings and capital growth. Constancy of κ then requires a higher growth rate of the reference stock, which is implied by a higher χ.

The $\chi\chi$-curve follows from $\dot{X}/X = \dot{C}/C$. Only if $\beta = 0$, it is the same for both a social optimum and a market economy, in which case it is independent of κ, according to (14). If $\beta > 0$, two additional properties characterize consumption growth. First, wealth of young cohorts is smaller than wealth

[17] Observe that $g_X = g_X(\underset{(+)}{\chi})$, and $g_K = g_K(\underset{(-)}{\chi}, \underset{(+)}{\kappa})$. For a given κ, a rise in χ raises g_X and lowers g_K, and $\dot{\kappa} < 0$. By increasing κ, $\dot{\kappa}$ rises and becomes zero again, as required along the $\kappa\kappa$ locus.

of elder cohorts, and the aggregate consumption growth rate declines, as shown by Lemma 1 (iv). Second, consumption growth depends on wealth, and thereby on κ. Thus, for a given κ, a rise in β requires a decline in χ, to ensure that $\dot{\chi} = 0$. In other words, as a consequence of a rise in β, the $\chi\chi$ locus shifts downward in (χ, κ) space.

[Figure 1 about here.]

Figure 1 shows that χ declines in β. It is largest at the social optimum (point A)[18], and becomes lower, as β rises (points B and C). From (3'), g_X declines in χ. In a steady state, $g = g_X = g_K = g_C$. Therefore, the steady state growth rate declines in β, as stated in the discussion above.

Distortionary Impact of a Consumption Externality. To show the distortionary impact of externalities, we need to consider *individual* consumption growth rates rather than aggregate ones. If $\alpha = 1$ and $\gamma = 0$, equations (13) and (22) show that individual market and optimal consumption growth rates are equal, regardless of whether β is zero or not.

PROPOSITION 1 *Consider an economy with inelastic labor supply and without a production externality* ($\alpha = 1$). *Then, the optimal individual consumption growth rate, in steady state, exceeds the market growth rate if and only if* $\beta > 0$ and $\gamma > 0$. *That is, in the C-OLG economy, the consumption externality introduces a distortion (an inefficiency) even without a concurrent production externality.*

[18] If $\beta = 0$, $\tilde{\chi} = \chi$ in a steady state.

Proof. If $\gamma = 0$, individual optimal and market growth rates are equal, even if $\beta > 0$: $\tilde{g}_c|_{\gamma=0} = g_c^*|_{\gamma=0} = (r-\rho)/\sigma$. However, if $\gamma > 0$, the individual consumption growth rates become:

$$\tilde{g}_c = \frac{r-\rho}{\sigma} + \gamma \frac{\sigma-1}{\sigma} \tilde{g} = \tilde{g} = \frac{r-\rho}{\tilde{\sigma}},$$
$$g_c^* = \frac{r-\rho}{\sigma} + \gamma \frac{\sigma-1}{\sigma} g^* = \frac{r-\rho-\beta\,\gamma(\sigma-1)}{\tilde{\sigma}}. \qquad \|$$

Clearly, if $\beta > 0$, the negative consumption externality introduces a distortion (an inefficiency): $\tilde{g}_c > g_c^*$. Proposition 1 essentially follows from an "overconsumption" ("undersaving" or "undergrowth") result. Households consume more than optimal, and save less than optimal. Therefore, the rate of consumption growth is lower than optimal. The economic intuition is as follows.

A rise in γ raises the desired individual consumption growth rates in both the market economy and the command optimum. The reason is seen to come from a rise in the (steady state) intertemporal elasticity of substitution in individual consumption, which amounts to $\tilde{\sigma}^{-1}$. The higher γ the more a household is willing to substitute intertemporally, as the reference stock diminishes the utility gain derived by a given increase in consumption.

In the market economy, the negative consumption externality gives rise to *overconsumption.* Households consume more and save less than optimal. The level of savings, however, determines the consumption growth rate. An increase in γ strengthens the extent of overconsumption, and thereby raises the optimal consumption growth rate by more than the market growth rate.

If and only if $\beta > 0$, the consumption externality introduces a distortion (an inefficiency). In the competitive equilibrium, a household does *not* consider the impact of its individual consumption on the consumption reference level, X. Considering the definition of aggregate consumption, this impact is given by:

$$\frac{\partial \dot{X}(t)}{\partial c(v,t)} = \varphi \, \beta \, e^{-\beta(t-v)} \geq 0 \, . \tag{25}$$

Equation (25) shows that the impact of individual consumption on the consumption reference level is strictly positive, when $\beta > 0$. As the consumption reference level enters all other individuals' utilities — and individual households do not consider this impact —, the resulting competitive equilibrium is not Pareto efficient.[19] Individual households overconsume. Therefore, the competitive equilibrium is not optimal, and the consumption externality is distortionary.[20]

In the context of the ILA framework ($\beta = 0$), equation (25) shows that the impact of individual consumption on the consumption reference level is zero. Thus, the consumption externality does not introduce a distortion in the infinite horizon case, while the externality does introduce a distortion in the finite horizon case.

Positive Production Externality. In a market economy, consumption growth (savings) is governed by the private (rather than the social) rate of return. Therefore, a *decline* in α (an increase in the positive production externality) lowers the consumption growth rate – and thereby the steady state growth rates. The production externality does not have an impact on the social op-

[19]In contrast, the social planner considers the impact of consumption on the reference level.

[20]Certainly, the consumption externality introduces a distortion not only for the employed social welfare function (17) but for other social welfare functions as well. If $\gamma > 0$, the equilibrium is not Pareto efficient. Thus, there is no social welfare function, for which such an equilibrium is optimal.

timum. While the production externality always introduces a distortion, the consumption externality introduces a distortion only when $\beta > 0$ or there is a concurrent production externality.

PROPOSITION 2 *In an overlapping generations economy with inelastic labor supply and with $\gamma > 0$, a rise in the consumption externality exacerbates the distortion between market growth rates and optimal growth rates the more the larger is β.*

Proof. It can easily be verified that $\partial\,(\tilde{g}_c - g_c^*)/\partial\,\gamma = \beta\,\sigma\,(\sigma - 1)/\tilde{\sigma}^2 > 0$. Also, $\partial^{\,2}(\tilde{g}_c - g_c^*)/(\partial\,\gamma\,\partial\,\beta) = \sigma\,(\sigma - 1)/\tilde{\sigma}^2 > 0$. As $\sigma > 1$, both derivatives are positive. ‖

Without a consumption externality, the market allocation is efficient. Proposition 1 shows that the consumption externality is a source of inefficiency, when $\beta > 0$. This inefficiency rises in γ, and it rises the more the larger the β.

Even if there is no production externality, the consumption externality raises the (positive) difference between the socially optimal and the market consumption growth rates. This result is specific to the finite lifetimes of households in the overlapping generations framework. For the infinitely-lived agent economy, the consumption externality alone, without capital externality, has no impact on the steady state growth rates.

To shed light on Proposition 2, notice that the higher the β, the lower is the aggregate growth rate g^*, as shown in Lemma 2.[21] The individual market consumption growth rate depends positively on the aggregate growth rate (of the reference level). Thus, the higher the β the lower the rise in aggregate growth, thereby the lower the rise in the individual consumption growth rate, upon a rise in γ. As the optimal growth rate is independent of β, the distortionary impact of the consumption externality rises in β.

Impact of the Externality on Levels. We next briefly investigate the relationships between socially optimal and market consumption and capital *levels*.

In an economy with inelastic labor supply and without a production externality, the stronger is the consumption externality the higher are χ, κ, and κ/χ, and the stronger is the distortion with respect to χ and κ. This result is proven in the appendix. Intuitively, a rise in γ increases the intertemporal elasticity of substitution in individual consumption. Therefore agents prefer a higher rate of consumption growth, which is accomplished by lowering the steady state propensity to consume out of wealth: χ/κ.

Initially, upon the rise in γ, consumption is lowered, as households raise the rate of consumption growth. Due to the negative consumption externality, households lower the consumption levels by *less* than optimal. Consequently, the consumption growth effect of the externality is lower in a market economy as compared to the command optimum.

[Figure 2 about here.]

[21] The higher is β, the smaller is the fraction of older, wealthy people with a high consumption *level*, and therefore, the smaller is the consumption growth effect of γ on aggregate consumption growth, according to property (iv) of Lemma 1.

Figure 2 illustrates these results. As of a rise in γ, the $\chi\chi$ locus shifts down and outwards to the right. It intersects the original $\chi\chi$ locus at $\chi = 1 \Leftrightarrow g^* = 0$, and is located above for $\chi > 1$. As the positively sloped $\kappa\kappa$ line is independent of γ, the steady state levels of κ, χ, and g^* increase.

At the same time, a rise in γ induces the vertical $\widetilde{\chi\chi}$ line to shift parallel to the right. The $\kappa\kappa$ locus is the same for both the market economy and the command optimum. The dots indicate the old and new optimal steady state equilibria.

There is an important lesson to be learnt from the previous discussion. In the C-OLG framework, consumption levels are higher than optimal, and the savings rate $(1 - \chi/\kappa)$ is lower than optimal. Figure 2 shows that the market savings rate falls the shorter of the optimal one the higher is γ. This finding helps explaining the low savings rates observed in many industrialized countries.

We end this subsection by considering the impact of a production externality, that is, $(1 - \alpha) > 0$, on steady state allocations. Certainly, α has no impact on the command optimum, as the planner takes the production externality into account. However, individual households take $Z(t)$ as given, and $A - \delta = \tilde{r} \geq r = \alpha\, A - \delta$. The steady state market growth rate becomes:

$$g^* = \frac{r - \rho - \beta\,\sigma}{2\tilde{\sigma}} + \frac{\beta + \tilde{r}}{2} - \frac{1}{2}\sqrt{\left(-\frac{r - \rho - \beta\,\sigma}{\tilde{\sigma}} + (\beta + \tilde{r})\right)^2 + \frac{4\,\beta\,\sigma}{\tilde{\sigma}}(r - \tilde{r})}\,. \tag{26}$$

Notice that the production externality is *decreasing* in α. Thus, for evaluating a rise in the production externality, we consider $-\partial\,(.)/(\partial\,\alpha)$ in the following proposition.

PROPOSITION 3 *Consider an overlapping generations economy. The production externality raises the difference between the socially optimal growth rate and the market growth rate:*

$$-\frac{\partial\,(\tilde{g}-g^*)}{\partial\,\alpha} > 0\,.$$

The production externality exacerbates the distortionary impact of the consumption externality.

Proposition 3 passes on two results. First, the production externality causes optimal and market growth rates to differ from each other, which does not come as a surprise. When α declines — that is, the production externality increases — the market rate of interest also declines. As a consequence, the individual consumption growth rates fall, and so does the aggregate one. As the socially optimal growth rate is not affected by α, a decline in α strengthens the distortion $\tilde{g}_c - g_c^*$. Second, the consumption and production externalities exacerbate each other. The higher is γ the more are households willing to substitute intertemporally. That is, the impact of a given decline in α (thereby in r) on the individual consumption growth rate is the stronger, the higher is the consumption externality.

3.2 Dynamics

Externalities not only have an impact on steady state paths but also on transitional dynamics. Here, we characterize transition paths, and compare those obtained from the market economy framework with *optimal* transition paths. We will end this subsection by a note on the impact of β on (asymptotic) convergence speeds in market economies.

In the following, in order to clarify the discussion of dynamics, we present some results in terms of the general utility functions $u(.)$ and $V(.)$ rather than the corresponding specified CRRA functions. As before, an asterisk refers to variables of the market economy, while a tilde refers to variables of the social optimum.

The market economy framework implies the following growth rate of individual consumption:

$$\frac{\dot{c}(v,t)}{c(v,t)} = (r-\rho)\,\theta^{*}(v,t)\,, \tag{27}$$

where $\theta^{*}(v,t)$ is the transitional intertemporal elasticity of substitution in consumption of the market economy.[22] In the social optimum, consumption grows according to:

$$\frac{\dot{c}(v,t)}{c(v,t)} = (\tilde{r}-\rho)\,\tilde{\theta}(v,t)\,, \tag{28}$$

where $\tilde{\theta}(v,t)$ is the intertemporal elasticity of substitution in consumption in the social optimum.[23] Clearly, the growth rates of *individual* consumption in the two economies coincide if and only if $(r-\rho)\,\theta^{*}(v,t) = (\tilde{r}-\rho)\,\tilde{\theta}(v,t)$. To shed light on the dynamics, we will now discuss transitional dynamics for a number of important cases. In particular, we demonstrate that a consumption externality introduces a distortion with respect to the optimal transition paths in the cases listed in Table 1.

[22]Partial derivatives are denoted by subindexes. E.g., $u_c(v,t)$ stands for $\partial\, u(c(v,t), X(t))/\partial\, c(v,t)$.

[23]$\theta^{*}(v,t) \equiv -\frac{u_c(v,t)}{\left(u_{cc}(v,t)+u_{cX}(v,t)\,\frac{\dot{X}(t)}{\dot{c}(v,t)}\right)\,c(v,t)}$,
$\tilde{\theta}(v,t) \equiv -\frac{V_C(t)+V_X(t)}{\left(V_{CC}(t)+V_{CX}(t)+\frac{\dot{X}(t)}{\dot{C}(t)}(V_{CX}(t)+V_{XX}(t))\right)\,C(t)}$.

TABLE 1.

DISTORTIONARY IMPACT OF CONSUMPTION EXTERNALITY

		ILA ($\beta = 0$)	C-OLG ($\beta > 0$)
$\varphi \to \infty$:	$X(t) \equiv C(t)$	no	yes ($\tilde{g}_c > g_c^*$)
$\varphi < \infty$:	$\dot{X}(t) = \varphi\ (C(t) - X(t))$	yes ($\tilde{g}_c < g_c^*$)	yes ($\tilde{g}_c \gtrless g_c^*$)

The indicated signs holds for $\dot{\chi} > 0$, and even for $\alpha = 1$. To sharpen our results, in all cases, we assume that there is no production externality ($\alpha = 1$).

PROPOSITION 4 *Consider an economy with $\beta = 0$. The consumption externality does not introduce a distortion with respect to the optimal transition paths if and only if:*

$$\dot{\left(\frac{u_X}{u_c}\right)}|_{c=C} = 0\,. \tag{29}$$

The proof of Proposition 4 is given in the appendix. Proposition 4 encompasses two important cases.

Case 1. $\varphi \to \infty : X(t) \equiv C(t)$. For this case, the consumption externality is represented by *current aggregate (average) consumption*, and condition (29) becomes:[24]

$$\dot{\left(\frac{u_C}{u_c}\right)}|_{c=C} = 0\,.$$

Condition (29) clearly holds for our CRRA specification of the utility function (1). It says that the marginal rate of substitution of $C(t)$ for $c(v,t)$ is constant through time. A proportionate increase in both $c(v,t)$ and $C(t)$ does not

[24]When $\varphi \to \infty$, X is to be replaced by C. As $\beta = 0$, individual consumption cannot be differentiated according to birth date: $c(v,t) = c(t)$. In equilibrium, $c(t) = C(t)$.

alter this marginal rate of substitution. For a given consumption level at some time t, $\theta^*(v,t) = \tilde{\theta}(v,t)$, and all variables in the market economy grow at socially optimal rates along the transition paths.

Proposition 4 does *not* imply that the consumption externality does not have any impact on transition paths. It shows that the consumption externality does not introduce a *distortion*, that is, the externality has the same impact in both frameworks, the market economy and the social optimum.

Suppose, there is a production externality $(1-\alpha > 0)$ in addition to the consumption externality. Then $\tilde{r} > r$, $\dot{\tilde{C}}(t) > \dot{C}^*(t)$, and $\tilde{C}(t+\Delta) > C^*(t+\Delta)$, in which case, condition (29) does not longer hold. With a production externality, Proposition 4 shows that transition paths deviate. Consumption growth is larger in a social optimum than in the market economy, and the consumption externality *here* influences the distortion introduced by the production externality.

Case 2: $\varphi < \infty$: $\dot{X}(t) = \varphi\,(C(t) - X(t))$. As can easily be verified, condition (29) does *not* hold for our CRRA utility function, when $\gamma > 0$. Over time, the marginal rate of substitution changes according to: $\dot{(u_X/u_c)} = -\gamma\,\dot{[c(v,t)/X(t)]} = -\gamma\,[\dot{\chi(t)}]$. The consumption externality introduces a distortion with respect to transition paths, even in the absence of a production externality (and even with exogenous labor supply). Three remarks are in order.

First, the nature of the distortion (sign) depends on whether the optimal $\chi(t)$ is smaller than or larger than its steady state value. If $\chi(t)$ is below its steady state value, $\dot{\chi}(t) > 0$ along the transition path. In this case, $\tilde{g}_c < g_c^*$. That is, the optimal consumption growth rate is lower than the market growth rate.

The social planner internalizes the negative consumption externality, while individual households do not. Therefore, the social planner "penalizes" $\dot{\chi} > 0$. If $\dot{\chi} > 0$, it follows that $g_C > g_X$, and $\dot{X} > 0$. That is, the reference level, which lowers utility of all households, increases. The consumption externality raises the (transitional) optimal intertemporal elasticity of substitution, $\tilde{\theta}(v,t)$, by less than the decentralized one, $\theta^*(v,t)$, and $\tilde{g}_c < g_c^*$.

Second, condition (29) fails to hold for a wide class of utility functions and specifications of consumption externalities. In particular, along a transition path, (29) does not hold for the class of CRRA utility functions, including the externality specifications proposed by Dupor and Liu (2003).[25]

Third, only in a steady state, χ does not change over time. Therefore, the consumption externality does *not* introduce a distortion in the steady state, when $\beta = 0$. Along the transition path, however, the consumption externality always introduces a distortion.

PROPOSITION 5 *Consider an overlapping generations economy ($\beta > 0$). If lifetimes are finite, the consumption externality always introduces a distortion (an inefficiency) with respect to the optimal transition paths.*

As for the ILA framework ($\beta = 0$) above, we consider two cases: $\varphi \to \infty$, and $\varphi < \infty$.

[25] They propose a utility function that includes, as special cases, many standard specifications of consumption externalities used in the literature:

$$v(c,X) = \frac{1}{1-\sigma}\left[\left(\frac{c^\mu - \gamma X^\mu}{1-\gamma}\right)^{1/\mu}\right]^{1-\sigma},$$

where the utility function we employ follows from the limit as μ approaches zero. Clearly, $(v_X/v_c) = -\gamma\,(c/X)^{1-\mu}$, which is constant over time in a steady state only.

Case 1. $\varphi \to \infty$: $X(t) \equiv C(t)$. In an ILA economy, the consumption externality did not cause a distortion. Proposition 5 shows that for a C-OLG economy, the consumption externality creates a distortion in transitional paths (even in the absence of a production externality). With $\beta > 0$, the market's aggregate consumption growth rate is smaller than the individual consumption growth rate, due to the intergenerational turnover effect. In contrast to the optimal growth rate, in a market economy, the consumption reference level — aggregate consumption — grows by less than individual consumption. As individual growth rates positively depend on the aggregate growth rate, $\tilde{g}_c > g_c^*$.[26]

Case 2: $\varphi < \infty$: $\dot{X}(t) = \varphi\,(C(t) - X(t))$. In an ILA economy, we showed that the consumption externality caused the optimal individual consumption growth rate to fall short of the market growth rate. The reason was seen to be the "internalization effect" (the social planner internalizes the consumption externality, while households do not). With $\beta > 0$, a second, opposing effect enters the story: the intergenerational turnover effect, described above for Case 1. It cannot generally be determined whether the internalization effect dominates the intergenerational turnover effect or not. If, however, β is "very small," the internalization effect dominates, and $\tilde{g}_c < g_c^*$. If $\dot{\chi}$ is positive and "very small," the intergenerational turnover effect dominates, and $\tilde{g}_c > g_c^*$. In particular, close enough to the steady state, as $\dot{\chi}$ is close to zero, the intergenerational turnover effect always dominates. That is, the overconsumption result ($\tilde{g}_c > g_c^*$) not only holds in the steady state but also

[26] The distortion becomes immediate when considering: $\tilde{g}_c = (r-\rho)/\sigma + \gamma\,(\sigma-1)/\sigma\,\tilde{g}_C$, and $g_c^* = (r-\rho)/\sigma + \gamma\,(\sigma-1)/\sigma\,g_C^*$, where $g_C^* < \tilde{g}_C$ when $\beta > 0$.

along the transition path, close to the steady state.

There are two lessons to be drawn from this discussion. First, along the transition path, it is possible to observe a switch from $\tilde{g}_c < g_c^*$ to $\tilde{g}_c > g_c^*$. Second, close to the steady state, $\tilde{g}_c > g_c^*$. This finding reverses the result given by the ILA framework, where $\tilde{g}_c < g_c^*$ along the transition path.

Speed of Convergence.

Finally, we briefly consider the impact of longevity on the asymptotic speed of convergence in the C-OLG model (a detailed account is available from the author upon request). In the previous section we already showed that a rise in β has a dampening effect on steady state growth in a market economy. The same holds along the stable trajectory as well. The higher the β the stronger is the fraction of older, wealthy consumers with a high consumption level, and the smaller is *aggregate* consumption growth. This intergenerational turnover effect is the stronger, the larger is κ. Consequently, the rise in κ along the stable arm has a dampening effect on growth, when $\beta > 0$. This dampening effect is absent in the ILA model, where $\beta = 0$.

In numerical terms, with $\beta > 0$ *and* $\gamma > 0$, and for reasonable calibrations, our model implies asymptotic convergence speeds below 3% a year. This result accords with empirical evidence (Barro and Sala-i-Martin 1992, and Mankiw et al. 1992). This property of the C-OLG model is important, as the typical one-sector ILA growth model implies convergence speeds that are too high and not consistent with empirical evidence.[27]

[27] It goes without saying that finiteness of lifetime is just one mechanism lowering the implied convergence speed of the growth model. The introduction of a second (sluggish) capital stock will result in a similar decline of the convergence speed, as does the introduction of adjustment costs of investment.

4 Conclusions

The impact of externalities on economic growth is usually considered in economies populated with *infinitely*-lived households. In this paper, we consider the question of how the finiteness of lifetimes (a "finite horizon") affects the effects of externalities on economic growth. We employ an overlapping generations model in continuous time (C-OLG model) for our analysis.

This endeavor is motivated by two facts. First, we argue that a consumption externality cannot be treated most appropriately in a symmetrical context, where the individual consumption levels of all households are the same, as the externality stems from *different* consumption levels. The C-OLG model introduces the required heterogeneity. As wealth differs among cohorts, so do individual consumption levels. Therefore, the C-OLG framework is a natural framework to consider consumption externalities. Second, the introduction of finite lifetimes (a finite horizon) has a significant impact on the answer to the research question, whether or not externalities introduce an inefficiency (a distortion).

We show that significant results discussed in the literature only hold at the limit, when agents are infinitely-lived. However, they cease to hold in finite-horizon economies. E.g., with a finite horizon, a consumption externality always introduces a distortion (an inefficiency) with respect to steady state growth rates, even if there is no production externality (and labor supply is exogenous). Likewise, a consumption externality introduces a distortion with respect to transitional paths, even if the externality stems from current average consumption. These results do not hold when agents are infinitely-lived.

Another result concerns the nature of the distortion caused by a consumption externality that stems from a slowly adjusting reference level. In an economy with infinitely-lived agents, *optimal* consumption growth is smaller than the growth rate observed in a market economy along the transition path. In contrast, in the C-OLG model, optimal consumption growth exceeds the market growth rate along the transition path, close to the steady state. That is, in a decentralized economy, consumption levels are higher than optimal, and the savings rate is lower than optimal. This finding helps explaining the low savings rates observed in many industrialized countries.

Next, the finiteness of lifetime reduces the speed of convergence to a magnitude that accords with empirical evidence. This finding is important, as the typical one-sector ILA growth model implies convergence speeds that are too high and not consistent with empirical evidence.

The reason for the distortionary effect (inefficiency) introduced by a consumption externalities is seen to come from overconsumption. The impact of individual consumption on the consumption reference level is strictly positive, when lifetime is finite, that is, $\beta > 0$. As the consumption reference level enters (negatively) all other individuals' utilities, and individual households do not consider this impact, the resulting competitive equilibrium is not Pareto efficient. Therefore, the consumption externality is distortionary in the C-OLG model. In the ILA framework, in contrast, this impact of individual consumption on the reference level is shown to be nil. Thus, the consumption externality does not introduce a distortion in the infinite horizon case, while it introduces a distortion in the finite horizon case.

These results have important (optimal) fiscal policy implications. First, the results support arguments in favor of a shift in the tax base towards consumption, as a negative consumption externality implies overconsumption in the C-OLG framework (which never is the case in the ILA framework). The distortion always occurs, regardless of the presence of production externalities. Second, regarding the design of optimal policy, in the ILA case, the only instrument needed is a capital income tax to correct for the production externality. With finitely-lived agents, however, a consumption tax is needed in addition. Along the transition path, in the ILA case, two instruments are required: a capital income tax along with a consumption tax. With finitely-lived agents, the optimal consumption tax needs not only vary in time, but it needs to be age-conditioned, in addition. An optimum can be obtained only, once a sufficiently rich set of policy instruments is available.

I hope this study helps to clarify the differences in effects of externalities between the ILA and C-OLG frameworks, and it will contribute to the future debate on economic growth, cross-country income dynamics, and externalities.

Appendix

A.1 Proof of Lemma 1. Property (i) is shown in (12). The factor of proportionality is given by $[\zeta(t) - \gamma\,(1-\sigma^{-1})\,g_x(t)]$, which is positive by the fact that $c(v,t) > 0$. |

Property (ii) follows from the fact that at any given point in time $0 = a(t,t) < \Omega(t)$. From (i) it follows that $c(t,t)$ falls short of average (aggregate) consumption, which is given by $C(t)$. Formally, $C(t) - c(t,t) = [\zeta(t) - \gamma\,(1-\sigma^{-1})\,g_x(t)]\,\Omega(t)$. |

Property (iii) is shown in (13): individual consumption growth rates are equal across cohorts. |

Property (iv): Consider that β represents the ("intergenerational turnover") rate at which existing, wealthy cohorts are replaced by newborn individuals without wealth. For given individual and aggregate consumption levels, a rise in β increases the share of young households who hold no wealth at all. While consumption growth rates are equal across generations (by iii), a rise in β amounts to a rise in the share of consumers with a below-average consumption level (by ii). Therefore, the *average* consumption growth rate declines in β. Formally, $\beta\,[C(t) - c(t,t)]/C(t) = \beta\,\Omega/(h+\Omega)$. Thus, $\partial\,[\beta\,[C(t) - c(t,t)]/C(t)]/\partial\,\beta = \Omega/(h+\Omega) > 0$. A rise in β increases the difference between aggregate and individual consumption growth rates. |

Property (v): $\partial\,[C(t) - c(t,t)]/\partial\,\Omega = [\zeta(t) - \gamma\,(1-\sigma^{-1})\,g_x(t)] > 0$. This property reflects the fact that $0 = a(t,t)$ falls the more short of average wealth the higher is aggregate (average) wealth $\Omega(t)$. As consumption is proportional to wealth, $c(t,t)$ falls the more short of average consumption the higher is aggregate (average) wealth. ||

A.2 Equation (22) *and Consumption Growth.* The current value Hamiltonian is given by:

$H_c = \frac{C(\tau)^{1-\sigma}X(\tau)^{-\gamma(1-\sigma)}}{1-\sigma}\,e^{\rho\,t} + \mu_K(\tilde{r}\,K(\tau) - C(\tau)) + \mu_X(\varphi\,(C(\tau) - X(\tau)))$.

Equation (22) follows from:

$$\partial\,H_c/\partial\,C(\tau) = 0\,, \quad \partial\,H_c/\partial\,X(\tau) = 0\,,$$
$$-\,\partial\,H_c/\partial\,K(\tau) = \dot{\mu}_K(\tau) - \rho\,\mu_K(\tau)\,, \quad -\partial\,H_c/\partial\,X(\tau) = \dot{\mu}_X(\tau) - \rho\,\mu_X(\tau)\,.$$

Furthermore, differentiating (21) with respect to time implies:

$$\frac{\dot c(\tau - y, \tau)}{c(\tau - y, \tau)} = -\frac{\dot\lambda(\tau)}{\lambda(\tau)\,\sigma} + \gamma\,\frac{\sigma - 1}{\sigma}\frac{\dot X(\tau)}{X(\tau)}\,.$$

Considering the Lagrangian from (18), we observe that $\lambda(\tau) = V_C[C(\tau), X(\tau)]$, thus,

$$-\frac{\dot\lambda(\tau)}{\lambda(\tau)\,\sigma} = \frac{\dot C(\tau)}{C(\tau)} - \gamma\,\frac{\sigma - 1}{\sigma}\frac{\dot X(\tau)}{X(\tau)}\,.$$

Individual consumption growth is then given by:

$$\frac{\dot c(\tau - y, \tau)}{c(\tau - y, \tau)} = \frac{\tilde r - \rho}{\sigma} + \gamma\,\frac{\sigma - 1}{\sigma}\frac{\dot X(\tau)}{X(\tau)} - \frac{\gamma}{\sigma}\,\dot\chi(\tau)\,(1 - \gamma\,\chi(\tau))^{-1}\,,$$

which coincides with the rate of aggregate consumption growth. $\|$

A.3 Summary of the Social Optimum.[28] We represent aggregate behavior by a series of differential equations in the variables $\kappa(t)$, and $\chi(t)$:[29]

$$\begin{aligned}
\frac{\dot C(t)}{C(t)} &= \frac{\tilde r - \rho}{\sigma} + \gamma\,\frac{\sigma - 1}{\sigma}\frac{\dot X(t)}{X(t)} - \frac{\gamma}{\sigma}\,\dot\chi(t)\;(1 - \gamma\,\chi(t))^{-1}\,,\\
\frac{\dot X(t)}{X(t)} &= \varphi(\chi(t) - 1)\,,\\
\frac{\dot K(t)}{K(t)} &= \tilde r - \frac{\chi(t)}{\kappa(t)}\,.
\end{aligned}$$

A.4 Impact of the Externality on Levels. Claim. In an economy with inelastic labor supply and without a production externality, the stronger is the consumption externality the higher are χ and κ, the lower is χ/κ, and the

[28] The appropriate transversality conditions are: $\lim_{\tau\to\infty} e^{-\rho\,\tau}\,\mu_K(\tau)\,K(\tau) = 0$, $\lim_{\tau\to\infty} e^{-\rho\,\tau}\,\mu_X(\tau)\,X(\tau) = 0$, and $K(t), X(t)$ given.

[29] As in the decentralized economy, the dynamics of the model is two-dimensional: $\dot\chi = \dot\chi(\chi)$, $\dot\kappa = \dot\kappa(\chi, \kappa)$. The dimension of the stable manifold is one, and the system is saddle point stable.

stronger is the distortion with respect to χ and κ.

Proof. Clearly, for both the market and the planner's economies $\chi = 1+g/\varphi$. Also, for both economies, $\kappa = (g+\varphi)/(\varphi(\tilde{r}-g))$. Finally, $\chi/\kappa = \tilde{r}-g$. $\widetilde{\chi/\kappa} < \chi/\kappa \Leftrightarrow \tilde{g} > g^*$. The last part of the Lemma follows from the fact that $\partial\,(\tilde{g}-g^*)/\partial\,\gamma > 0$. $\|$

A.5 Proof of Proposition 3.

Step 1. By a first order Taylor series approximation,

$$g^*(\alpha) \approx g^*(1) + \frac{\partial\, g^*(1)}{\partial\, r}\,\frac{\partial\, r}{\partial\, \alpha}\,(\alpha - 1)\,.$$

For $0 < \alpha \leq 1$, $g^*(\alpha) - g^*(1) \approx A\,(\alpha-1)/\tilde{\sigma} \leq 0$. Thus, a decrease in α lowers g^* while it leaves $\tilde{g}$ unaffected.

Step 2. For the second part of the Proposition, consider $\alpha = 1$. Then $\partial\,(\tilde{g}-g^*)/\partial\,\gamma = (\sigma-1)(\tilde{g}-g^*)/\tilde{\sigma} > 0$. This distortionary effect of γ is strengthened by an increase in the production externality, that is, by a decrease in α: $-\partial\,/\partial\,\alpha\,[\partial\,(\tilde{g}-g^*)/\partial\,\gamma] = (\sigma-1)A/\tilde{\sigma}^2 > 0$. $\|$

A.6 Proof of Proposition 4. In the proof, we use the following two relations. $u(c,X)|_{c=C} = V(C,X)$, and as $\beta = 0$, $c(v,t) = c(v',t) = C(t)$.

The optimal and market consumption growth rates are equal if and only if $\tilde{\theta}(v,t) = \theta^*(v,t)$, for all t along the transitional paths, where:

$$\theta^*(v,t) \equiv -\frac{u_c(v,t)}{\left(u_{cc}(v,t) + u_{cX}(v,t)\,\frac{\dot{X}(t)}{\dot{c}(v,t)}\right)\,c(v,t)}\,,$$

$$\tilde{\theta}(v,t) \equiv -\frac{V_C(t) + V_X(t)}{\left(V_{CC}(t) + V_{CX}(t) + \frac{\dot{X}(t)}{\dot{C}(t)}(V_{CX}(t) + V_{XX}(t))\right)\,C(t)}\,.$$

Using $u(c,X)|_{c=C} = V(C,X)$, $c(v,t) = C(t)$, and suppressing time indexes, equality of individual consumption growth rates requires:

$$\frac{V_C}{\left(V_{CC} + V_{CX}\,\frac{\dot{X}}{\dot{C}}\right)} = \frac{V_C + V_X}{\left(V_{CC} + V_{CX} + \frac{\dot{X}}{\dot{C}}(V_{CX} + V_{XX})\right)}\,. \tag{A.1}$$

Condition (29) is equivalent to: $(V_C\,\dot{V}_X - V_X\,\dot{V}_C) = 0$. Clearly, condition (29) implies (A.1). If $\varphi \to \infty$, $X = C$, condition (29) holds, and so does (A.1). If $\varphi < \infty$, condition (29) does *not* hold for our CRRA utility function (as long as $\gamma > 0$). Therefore, (A.1) does not hold as well. $\|$

A.7 Proof of Proposition 5. The individual consumption growth rates are defined by:

$$\tilde{g}_c = \frac{r-\rho}{\sigma} + \gamma\,\frac{\sigma-1}{\sigma}\,\tilde{g}_X - \frac{\gamma}{\sigma}\,\dot{\chi}\,(1-\gamma\,\chi)^{-1}\,, \tag{A.2}$$

$$g_c^* = \frac{r-\rho}{\sigma} + \gamma\,\frac{\sigma-1}{\sigma}\,g_X^*\,. \tag{A.3}$$

If $\varphi \to \infty$, $X = C$, and $\dot{\chi} = 0$. In this case, the consumption externality stems from g_C rather than g_X. Regardless of β, $\tilde{g}_C = \tilde{g}_c$. If $\beta = 0$, $g_c^* = g_C^*$. In this case, $\tilde{g}_c = \tilde{g}_C = g_C^* = g_c^*$, and the consumption externality is non-distortionary. If, however, $\beta > 0$, $g_C^* < g_c^*$ by the intergenerational turnover effect. Suppose that at $t = 0$, $\tilde{g}_c(0) = g_c^*(0)$. Then $\tilde{g}_C(0) = \tilde{g}_c(0) = g_c^*(0) > g_C^*(0)$. At $t = 0 + \Delta = \Delta$, for some small $\Delta > 0$, $\tilde{g}_c(\Delta) > g_c^*(\Delta)$. The individual growth rates start to deviate, as $\tilde{g}_C(\Delta) > g_C^*(\Delta)$ due to the intergenerational turnover effect.

If $\varphi < \infty$, the same effect, depending on the magnitude of β, holds (with X in place of C). In addition, $\dot{\chi} \neq 0$. If $\dot{\chi} > 0$, $\tilde{g}_c$ is lowered (internalization effect). Equations (A.2) and (A.2) show that both the magnitude of $\dot{\chi}$ and that of β determine the sign of $[\tilde{g}_c(t) - g_c^*(t)]$. $\|$

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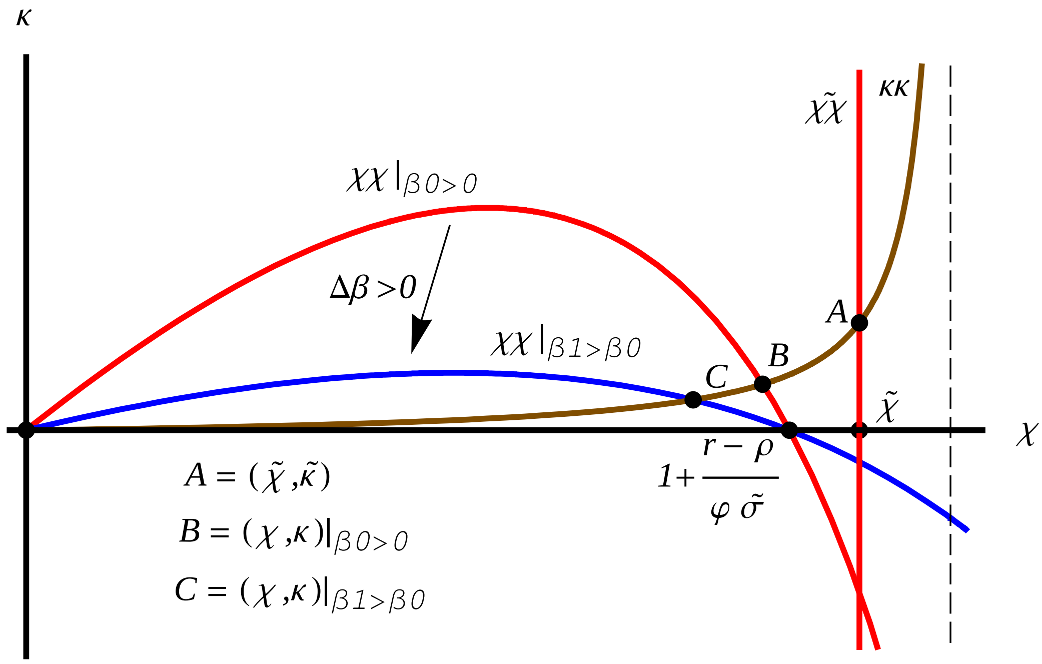

Figure 1: Finite lifetimes – a rise in β

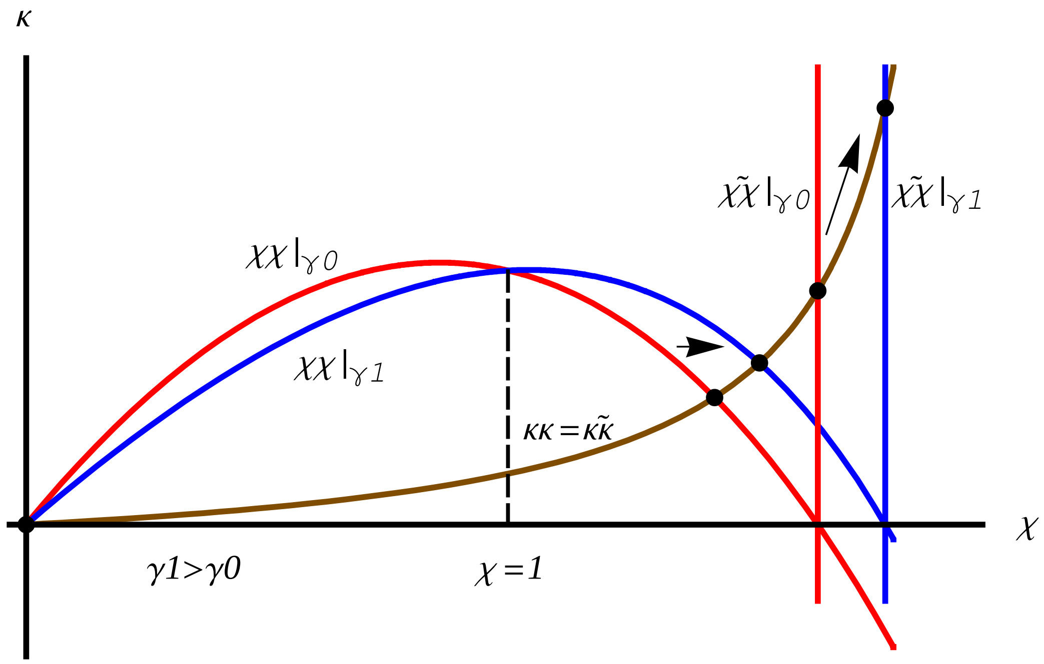

Figure 2: Finite lifetimes – a rise in γ