Income Distribution, Market Structure, and Individual Welfare

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Abstract

This paper explores how income distribution influences market structure and affects the economic well-being of different groups. It shows that inequality may be good for the poor via a trickle-down effect operating through entry. I consider a general equilibrium model of monopolistic competition with free entry, heterogenous firms and consumers that share identical but non-homothetic preferences. The general model is solved. The case of two types of consumers, rich and poor, is considered in detail. I show that higher income inequality in the economy can benefit the poor. An increase in personal income of the rich raises welfare of the poor, while an increase in the fraction of the rich has an ambiguous impact on the poor: welfare of the poor has an inverted U shape as a function of the fraction of the rich. At the same time, an increase in the personal income of the rich together with a decrease in the fraction of the rich, keeping the aggregate income in the economy fixed, raises the well-being of the poor. I also analyze the effect of changes in market size and entry cost. I show that the rich gain more from an increase in market size and lose more from an increase in the cost of entry than the poor.

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1 Introduction

What are the possible consequences of income redistribution for market structure, consumption allocation, and welfare? As Atkinson and Bourguignon (2000) argue, "it is difficult to think of economic issues without distributive consequences and it is equally difficult to imagine distributive problems without some allocational dimension." There is a large empirical and theoretical literature that relates income distribution and inequality to a number of social and economic outcomes[1]. Alesina and Rodrik (1994) show that an increase in income inequality has a negative impact on economic growth (see also Persson and Tabellini (1994)). Waldmann (1992) argues that the level of inequality is positively correlated with infant mortality. Glaeser, Scheinkman and Shleifer (2003) suggest that high inequality can negatively affect social and economic progress through the subversion of institutions in the economy. This paper develops another insight into the interaction between income distribution and economic outcomes, which has not been explored extensively. I examine how income distribution affects market structure, pricing, and the welfare of heterogenous agents. In particular, I show that higher income inequality in the economy may benefit the poor via a trickle-down effect operating through entry.

I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. In traditional models of monopolistic competition, income distribution plays no role. This rests on two standard preference assumptions. First, if preferences are identical and homothetic, it is well understood that the distribution of income does not affect equilibrium: only aggregate income matters. Second, when preferences are quasi-linear, the presence of a numeraire good eliminates the influence of income distribution on equilibrium outcomes. In this paper, I assume that all consumers share identical but non-homothetic preferences. I introduce income heterogeneity in the model by assuming that consumers differ in the efficiency units of labor they are endowed with. In models with identical homothetic preferences, any price changes have the same impact on all consumers regardless of whether consumers are identical or not. Non-homothetic preferences and income heterogeneity imply that changes in prices may affect different groups differently. In the model, the presence of market power induces variable markups across firms, which are in turn affected by income distribution. Hence, changes in income distribution may have different consequences for different groups of agents.

I adopt the preference structure from Murphy, Shleifer and Vishny (1989) and Matsuyama (2000). The basic idea is that goods are indivisible and potential consumers want to buy only one unit of each good. This implies that given prices, goods can be arranged so that consumers can be seen as moving down some list in choosing what to buy. For instance, in developing countries, consumers first buy food, then clothing, then move up the chain of durables from kerosene stoves to refrigerators, to cars.

[1] See Atkinson and Bourguignon (2000) for more substantial literature review.

Notice that the consumer utility can only be increased by the consumption of a greater number of goods. Moreover, higher income consumers consume the same set of goods as lower income consumers plus some others. This structure of consumer preferences has enough flexibility to be applied as to the whole economy as to a certain industry where goods differ in quality. On the one hand, each good can be interpreted as a distinct good sold in the market. In this case, the structure describes the whole economy. On the other hand, we might think that firms sell not distinct goods but some characteristics of a good produced in a certain industry. For instance, consider a car industry. Each good can be treated as some characteristic of a car. The poor purchase main characteristics associated with a car, while the rich buy the same characteristics as the poor plus some additional luxury characteristics. That is, both groups of consumers buy the same good but of different quality.

Goods differ in terms of the valuations consumers attach to them. By the valuation of a certain good, I mean the utility delivered to consumers from the consumption of one unit of this good. That is, there are goods that are more essential in consumption (necessities) and goods that are less essential (luxuries). There is free entry in the market. To enter the market, ex ante identical firms have to make costly investments that are sunk. Once firms enter, they learn about the valuations attached to their goods. The only source of firm ex-post heterogeneity is the difference in the valuations placed on their goods[2]. Depending on the valuations drawn, firms choose whether to stay or to leave the market. Firms that decide to stay engage in price competition with other firms. This and the preference structure lead to the endogenous distribution of markups, which is influenced not only by market size, but also by the distribution of income in the economy. Hence, the model incorporates two key features: imperfect competition and non-homothetic preferences, which allow analyzing the consequences of changes in the income distribution on pricing in the equilibrium, the market structure and, thereby, welfare of different groups of consumers.

While the general model is established and solved, the heart of the paper focuses on the case of two types of consumers: rich and poor[3]. Depending on the valuations attached to the goods they produce, firms are endogenously divided into three groups in the equilibrium. Firms with high valuations choose to serve all consumers. Firms with medium valuations decide to sell only to the rich. Finally, firms with low valuations leave the market. Therefore, the poor consume on average more essential goods than the rich. This is in line with common intuition that the rich spend higher share of their income on luxury goods, which are less essential in consumption[4].

[2]To simplify the analysis, I assume that the marginal cost of production is identical across all firms.

[3]Recall that in the model, income distribution is exogenous. I deliberately leave out of the scope the situation when income distribution is endogenous, since my main goal is to understand the effects of changes in income distribution not in some other parameters.

[4]Notice that the present model is not a model about quality differences. Agents do not choose between high quality and low quality potatoes. They choose between potatoes, TV's, refrigerators, and so on.

As sources of income inequality, I consider changes in the income and the fraction of the rich consumers. An increase in the income level of the rich has two effects: redistribution of firms across the groups and the higher number of firms entering the market, which results in tougher competition. The former effect is negative for the poor, while the latter one is positive. I show that due to additional entry, the poor gain from an increase in the income of the rich. This is reminiscent of the trickle-down effect in Aghion and Bolton (1997), who show that in the presence of imperfect capital markets, the accumulation of wealth by the rich may be good for the poor. The intuition, which is behind these results, may also work in traditional models with homothetic preferences. In Melitz (2003), higher income of some consumers results in higher entry, tougher competition, and, thereby, higher welfare of all consumers. However, there are some differences. In the short run, when the mass of firms is unchanged, there is only a negative impact on the poor resulting in welfare losses. In traditional models, if we fix the mass of firms then higher income of one part of consumers does not affect welfare of the other part. Moreover, in the present model higher income of the rich raises the markups of firms selling only to the rich and decreases the markups of firms serving all consumers. In traditional models, there is the same or no impact on firms' markups.

Another intriguing issue is to compare welfare of the poor in economies with different fractions of the rich. What is better for the poor: tiny minority or vast majority of the rich? Keeping the same personal incomes and the mass of the consumers, an increase in the fraction of the rich has two opposite implications for the poor. First, some firms that served all consumers choose to sell only to the rich. Second, the larger fraction of the rich results in more firms entering the market. The former effect hurts the poor and the latter one benefits them. I show that if the fraction of the rich is small then the positive effect prevails, while if the fraction of the rich is sufficiently high the opposite happens. Hence, we might expect that welfare of the poor has an inverted U shape as a function of the fraction of the rich. The fact that firms endogenously choose the type of consumers they wish to serve makes the results regarding changes in fraction of the rich different from that ones in traditional models with homothetic preferences. In Melitz (2003), higher fraction of the rich always leads to higher welfare of the poor in the long run and has no impact in the short run. In the present model, we observe an ambiguous impact in the long run and a negative impact in the short run. There is a common feature of both comparative statics considered above. An increase in the personal income of the rich as well as an increase in the fraction of the rich raises the aggregate income in the economy. In the view of policy implications, we need to explore the effects of changes in income distribution keeping aggregate income constant. To capture a pure redistribution effect, I consider an increase in the personal income of the rich together with a decrease in the fraction of the rich keeping aggregate income in the economy fixed. In models with homothetic preferences, these changes in income distribution do not affect entry, prices, and welfare of the poor. In the present model, I show that they result in higher entry in the market. This in turn leads to higher welfare of the poor.

The effects of changes in entry cost or market size on consumer welfare are similar to those in traditional literature. However, consumers do not equally gain or lose from these changes as we might expect in models with homothetic preferences. Who gains more: the rich or the poor? I show that given some plausible assumption about the distribution function of valuation draws, the rich gain more from a rise in market size and lose more from a rise in entry cost than the poor.

The related literature in this area can be divided into three strands. First, there are papers that consider monopolistic competition models with firm heterogeneity assuming homothetic or quasi-linear preferences. Melitz (2003) develops a general equilibrium model with firm heterogeneity and Dixit-Stiglitz preferences, which imply constant markups. Melitz and Ottaviano (2005) examine a similar framework, but incorporate variable markups considering a linear demand system. However, in both these papers, the distribution of income does not play any role. In contrast, the model presented here includes all the key features of the papers mentioned while also establishing a connection between income distribution and the market structure. The second group of papers, for instance Flam and Helpman (1987), Stokey (1991), and Matsuyama (2000), explores the implications of non-homothetic preferences in a perfectly competitive environment for open economies. These papers mainly analyze the interaction between income distribution and trade patterns. There is a set of papers written by Krishna and Yavas[5], in which the role of indivisibilities and market distortions is investigated. However, the impact of income distribution on market structure is not considered in these papers. Finally, the third group of papers deals with both monopolistic competition and non-homothetic preferences. Markusen (1986) extends the Krugman type model of trade with monopolistic competition by adding non-homothetic demand. He examines the role of income per capita in interindustry and intra-industry trade. Mitra and Trindade (2005) also consider a model of monopolistic competition with non-homothetic preferences. However, the way they introduce nonhomotheticity has a shortcoming: the share of consumer income spent on a certain type of goods is exogenous and depends on the income.

Closer to this paper is the work of Foellmi and Zweimueller (2004) that develops a general equilibrium model with an exogenous mass of identical firms. In contrast, I consider heterogenous firms and free entry in the market, which in turn implies endogeneity of the mass of potential producers in equilibrium. Moreover, Foellmi and Zweimueller (2004) do not address welfare issues. They show that, depending on the parameters of the model, an increase in income inequality has either no impact on firm markups or increases them. The present paper suggests that this is not necessarily the case; in

[5] See Krishna and Yavas (2001), Krishna and Yavas (2004), and Krishna and Yavas (2005).

fact, an increase in income inequality affects different firms differently. Due to free entry, greater income inequality may raise markups for firms that sell their goods only to the rich and reduce markups for firms that sell their goods to all consumers. Murphy, Shleifer and Vishny (1989) study how income inequality affects the adoption of modern technologies. In their model, prices and markups are exogenous. In fact, Murphy, Shleifer and Vishny (1989) leave the questions of competition, markups, and welfare outside their analysis. In the paper closest to the present work, Foellmi and Zweimueller (2006) examine a dynamic variation of Murphy, Shleifer, and Vishny (1989). Assuming learning by R&D, they focus their analysis on the link between possible growth and inequality. In contrast, I do not consider the learning by R&D spillover and explore the impact of income distribution and inequality on the level of competition, markups, and individual welfare.

The rest of the paper is organized as follows. Section 2 introduces the basic concepts of the general model. Section 3 develops a special case with two types of consumers, rich and poor, and establishes existence and uniqueness of equilibrium for this case. It also derives the implications of the distribution of income on market structure and individual welfare. Section 4 extends the analysis to the general case with N types of consumers, and Section 5 concludes.

2 The Model

I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. The preference structure is adopted from Murphy, Shleifer and Vishny (1989) and Matsuyama (2000).

2.1 Production

The timing of the model is as follows. There is free entry in the market. To enter the market, firms have to make sunk investments f_e. If a firm incurs the cost of entry, it obtains a draw b of the valuation of its good from a common distribution $G(b)$ on $[0, A]$. This is meant to capture the idea that before they enter, firms do not know how well they will end up doing, as they do not know how highly consumers will value their products. I assume that $G'(b) = g(b)$ exists. The valuation b is interpreted as the utility delivered to consumers from the consumption of one unit of the good. Depending on the valuation they draw, firms choose to leave the market or to stay. Firms that decide to stay compete in price with other firms. The only factor of production is labor. I assume that marginal costs of production are the same for all firms and equal to c, i.e., it takes c effective units of labor (which are paid a wage of unity[6]) to produce a unit of any good[7].

Consumers differ in the number of efficiency units of labor they are endowed with[8]. I assume that there are N types of consumers indexed by n. A consumer of type n is endowed with I_n efficiency units of labor. I choose indices so that $I_n > I_{n-1}$. Let α_n be the fraction of type n consumers in the aggregate mass L of consumers. Then, the total labor supply in the economy in efficiency units is $L\sum_{i=1}^{N}\alpha_i I_i$.

2.2 Consumption

All consumers have the same non-homothetic preferences given by utility function

$$U = \int_{\omega\in\Omega} b(\omega)x(\omega)d\omega,$$

where Ω is the set of available goods in the economy, $b(\omega)$ is the valuation of good ω, and $x(\omega) \in \{0, 1\}$ is the consumption of good ω. Each consumer owns a balanced portfolio of shares of all firms. Due to free entry, the total profits of all firms are equal to zero in the equilibrium. This implies that the value of any balanced portfolio is equal to zero. Thus, all consumers have the same wealth, while their incomes vary with their productivity. To simplify the notation, I assume that consumers have equal shares of all firms. Let π be the total profit of all firms in the economy. Given prices of available goods, a type n consumer maximizes

$$\int_{\omega\in\Omega} b(\omega)x(\omega)d\omega$$

subject to the budget constraint

$$\int_{\omega\in\Omega} p(\omega)x(\omega)d\omega \leq I_n + \frac{\pi}{L},$$

where $p(\omega)$ is the price of good ω. The utility maximization merely involves moving down the list of products ordered by their valuation to price ratios, $\frac{b(\omega)}{p(\omega)}$, until all income is exhausted.

The analysis of the general case with N types is rather complicated. Therefore, I focus the analysis on the simpler case when consumers have one of two possible labor productivities. In the next section, I show the existence and uniqueness of the equilibrium and analyze its properties. In section 4, I prove the existence and uniqueness of the equilibrium in the general case and briefly discuss the case when the distribution of labor productivities is continuous.

[6] In the model, wage is taken as numeraire.

[7] The assumption that marginal costs of production are the same across firms simplifies analytical derivations in the model and does not change qualitative results. In general, we can assume that marginal costs are also drawn from some common distribution.

[8] Throughout the paper, I use terms, endowments of efficiency units of labor and labor productivities, interchangeably.

3 A Special Case: Two Types of Consumers

There are two types of consumers: a high income (high productivity) type and a low income type. The productivity of the high income type is defined by I_H, the productivity of the low income type is I_L. Given the preferences, all goods consumed by less productive consumers are also consumed by more productive. Thus, goods in the economy can be divided into three groups: the "common" group includes goods that are consumed by all consumers; the "exclusive" group includes goods that are only consumed by high income consumers; finally, there is the group of goods that are consumed by no one.

A firm that produces a good ω obtains the profit of $(p(\omega) - c)Q(\omega)$, where $Q(\omega)$ is demand for good ω. If all consumers buy the good then the demand is L. If only the rich buy it, the demand is $\alpha_H L$, where α_H is the fraction of a high income type. Hence, $Q(\omega) \in \{L, \alpha_H L, 0\}$. Each firm takes the valuation to price ratio of all other firms as given and maximizes its profit. The following proposition holds.

Proposition 1 *Even though all goods have different valuation to marginal cost ratios, goods from the same group have the same valuation to price ratio in the equilibrium.*

Proof. Suppose not. In this case, there exists some group, in which there are at least two goods with different $\frac{b(\omega)}{p(\omega)}$ ratios. Since both goods belong to the same group, the firm that produces its good with higher $\frac{b(\omega)}{p(\omega)}$ can raise its $p(\omega)$ without affecting the demand. This in turn would increase its profit. ■

Define V_C as the valuation to price ratio of goods from the "common" group and V_E as valuation to price ratio of goods from the "exclusive" group in the equilibrium. Here V_C and V_E are endogenous parameters and V_C is strictly greater than V_E[9]. Thus, if a firm with valuation $b(\omega)$ sells to all consumers then its price is equal to $\frac{b(\omega)}{V_C}$ and its profit is given by

$$(p(\omega) - c)\, L = \left(\frac{b(\omega)}{V_C} - c\right) L,$$

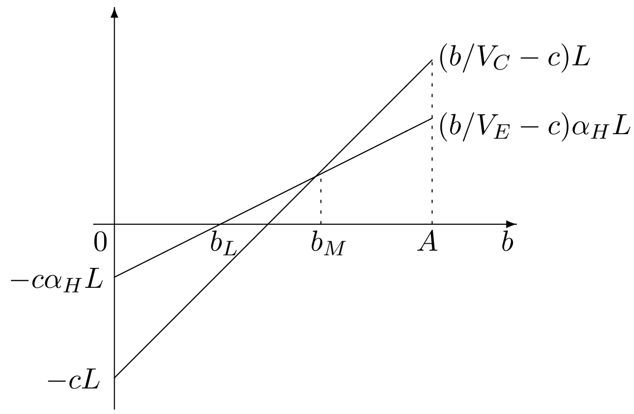
while if the firm sells only to the rich, its profit is given by

$$(p(\omega) - c)\alpha_H L = \left(\frac{b(\omega)}{V_E} - c\right) \alpha_H L.$$

As $V_C > V_E$, the firm chooses between selling to more people at a lower price and selling to fewer of them but at a higher price. Hence, the firm chooses $p(\omega) \in \{\frac{b(\omega)}{V_C}, \frac{b(\omega)}{V_E}\}$ to maximize its profit,

[9] Notice that by definition, $V_C \geq V_E$. If $V_C = V_E$ then all available goods have the same valuation to price ratios. In this case, the equilibrium concept implies that the high income consumers buy all goods, while the poor buy only some part (for instance, this part can be randomly determined). This means that expected demand for a certain good is strictly less than L. Thus, firms can increase their profits by slightly decreasing their prices and acquiring greater demand share. Therefore, if $V_C = V_E$, equilibrium does not exist.

Figure 1: The Profit Function



taking V_C and V_E as given. In the equilibrium, the price of good ω only depends on $b(\omega)$. Therefore, hereafter I omit the notation of ω and consider prices as a function of b.

Let b_M be the unique solution of the equation

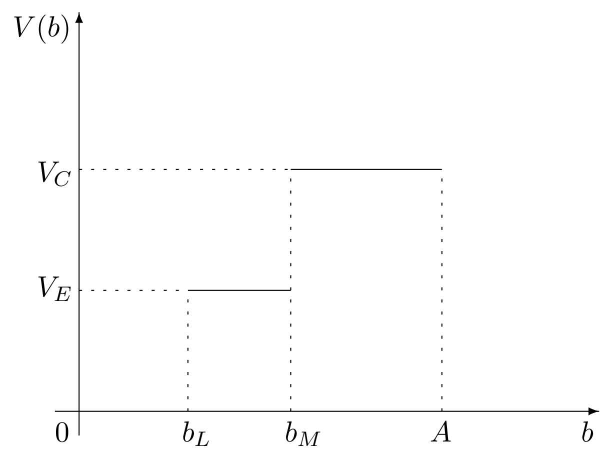
$$\left(\frac{b}{V_C} - c\right) L = \left(\frac{b}{V_E} - c\right) \alpha_H L. \tag{1}$$

In the equilibrium, the condition $\frac{\alpha_H}{V_E} < \frac{1}{V_C}$ is satisfied. Otherwise, for any $b \geq 0$, $\left(\frac{b}{V_E} - c\right) \alpha_H L > \left(\frac{b}{V_C} - c\right) L$ and all firms would choose to sell only to high income consumers. However, this is impossible in the equilibrium. This condition guarantees that

$$\begin{aligned} \left(\frac{b}{V_C} - c\right) L &\geq \left(\frac{b}{V_E} - c\right) \alpha_H L \quad \text{if } b \geq b_M, \\ \left(\frac{b}{V_C} - c\right) L &< \left(\frac{b}{V_E} - c\right) \alpha_H L \quad \text{otherwise.} \end{aligned}$$

This means that if a firm draws $b \geq b_M$ then in the equilibrium, it sells to both types of consumers, otherwise it sells only to the rich or exits. A firm with valuation b_M of its good is indifferent between selling to all consumers and selling only to the rich (see *Figure 1*). Hence, even in the presence of market power, products have a natural hierarchy: consumers first buy goods with higher b, i.e., goods that are more essential in consumption. This result is supportive of the common intuition that the poor mostly spend their incomes on necessities, which are more essential in consumption, while the rich can afford to buy not only necessities but also luxuries. Without loss of generality, I assume that a firm with valuation b_M sells to both types of consumers. Let a function $V(b)$ be defined by $\frac{b}{p(b)}$. In the equilibrium, $V(b)$ looks as in *Figure 2*, where $b_L \geq 0$ is a cutoff level such that firms drawn $b < b_L$ exit.

Figure 2: The Valuation to Price Function: A Special Case



3.1 The Equilibrium

Let M_e be the mass of firms that enter the market. One can think of M_e as that there are $M_e g(b)$ different firms with a particular valuation b. In the equilibrium, several conditions should be satisfied. First, as there is free entry in the market, the ex ante expected profits of firms have to be equal to zero. Second, the goods market clears. Since the poor consume only goods from the "common" group, the aggregate cost of the bundle of goods from the "common" group should be equal to the income of a poor consumer. Similarly, the aggregate cost of the bundle of all available goods in the economy should be equal to the income of a rich consumer.

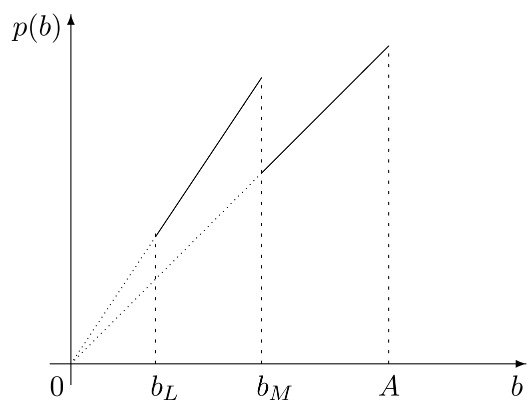
Definition 1 *The equilibrium of the model is defined by the price function $p(b)$ on $b \geq b_L$, the cutoff level $b_L \geq 0$, b_M, M_e, and the valuation to price ratios V_C and V_E such that*

1) The ex ante expected profits of firms are equal to zero.

2) The goods market clears.

Further, I derive equations that satisfy the conditions mentioned above and prove that equilibrium in the model always exists and is unique. Let $\pi(b)$ be the variable profit of a firm with valuation b. To find the equilibrium, I express $\pi(b)$ and $p(b)$ as functions of b, b_L, b_M and exogenous parameters. As b_L is the cutoff level, firms with valuation b_L have zero profits. This implies that $\left(\frac{b_L}{V_E} - c\right)\alpha_H L = 0$ or $V_E = \frac{b_L}{c}$. From (1), we can express V_C as a function of b_L and b_M. As a result, the following lemma holds.

Figure 3: The Price Function



Lemma 1 *In the equilibrium,*

$$p(b) = \begin{cases} \frac{b}{V_C} = cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) & \textit{if} \;\; b \geq b_M, \\ \frac{b}{V_E} = cb\frac{1}{b_L} & \textit{if} \;\; b \in [b_L, b_M), \end{cases}$$

$$\pi(b) = \begin{cases} \left(cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - c \;\right) L & \textit{if} \;\; b \geq b_M, \\ \left(cb\frac{1}{b_L} - c\right)\alpha_H L & \textit{if} \;\; b \in [b_L, b_M). \end{cases}$$

Since firms with valuation b_M have the same profits from selling to all consumers as from selling only to the rich, the price function has a jump at b_M; i.e., to compensate for lower demand, firms raise their prices (see *Figure 3*). This results in the nonmonotonicity of the price function.

Due to free entry in the market, the ex ante expected profits of firms are equal to zero in the equilibrium. Using the results from *Lemma 1* and taking into account that firms with $b < b_L$ exit, I obtain

$$\begin{aligned} f_e &= (G(b_M) - G(b_L))E(\pi(b)|b_L \leq b < b_M) + (1 - G(b_M))E(\pi(b)|b \geq b_M) \iff \\ \frac{f_e}{cL} + 1 &= \alpha_H H(b_L) + (1 - \alpha_H)H(b_M), \end{aligned} \tag{2}$$

where $H(x) = G(x) + \frac{\int_x^A x dG(x)}{x}$. The goods market clearing condition implies that

$$\begin{cases} I_L = M_e \int_{b_M}^A p(t)dG(t) \\ I_H = M_e \int_{b_L}^A p(t)dG(t) \end{cases}. \tag{3}$$

The aggregate cost of the bundle of goods from the "common" group is equal to the income of a poor consumer, while the aggregate cost of the bundle of all available goods in the economy is equal to the income of a rich consumer. Dividing the second line in (3) by the first one and using *Lemma 1*, I obtain

$$\frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{A} tdG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right).$$

Hence, given the exogenous parameters I_H, I_L, α_H, f_e, c, L, and the distribution of draws $G(\cdot)$, we can find endogenous b_M and b_L from the system of equations, which is given by

$$\begin{cases} \frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{A} tdG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right) \\ \frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1-\alpha_H)H(b_M) \end{cases}. \quad (4)$$

The following lemma states the existence and uniqueness of the equilibrium.

Lemma 2 *The system of equations* (4) *has a unique solution.*

Proof. In the Appendix. ∎

Once b_M and b_L are found, V_C and V_E can be derived using the results in *Lemma 1.* Finally, the mass of firms can be found from (3).

3.2 Income Inequality and Welfare

Before analyzing the effects of income inequality on market structure and welfare, I examine how consumer welfare and income inequality are determined in the model.

3.2.1 Welfare

Given the preference structure, welfare of a certain consumer is equal to the sum of valuations of goods she consumes. In this way, welfare of a poor consumer is equal to $M_e \int_{b_M}^{A} tdG(t)$. From (3), $M_e = \frac{I_L}{\int_{b_M}^{A} p(t)dG(t)}$. This implies that

$$W_p = I_L V_C.$$

Welfare of a poor consumer naturally rises with an increase in either her income or the valuation to price ratio of goods she consumes. Similarly, welfare of a rich consumer is given by

$$W_r = I_L V_C + (I_H - I_L)V_E.$$

As the rich consume the same bundle of goods as the poor plus some others, welfare of the rich is equal to welfare of the poor plus additional welfare from the consumption of "exclusive" goods, which is in turn equal to income spent on these goods multiplied by their valuation to price ratio.

Notice that all changes in individual welfare are divided into two components: an income effect and a price effect. The price effect is determined by changes in V_C and V_E, which implicitly depend on the incomes (I_H and I_L) and the level of competition inside the groups of goods. The income effect is explicitly determined by changes in exogenous I_L and I_H.

3.2.2 Income Inequality

As income inequality in the economy, I consider the variance of the income distribution[10], which is given by

$$VAR = \alpha_H(1-\alpha_H)\,(I_H - I_L)^2\,. \tag{5}$$

Income inequality is increasing in the income difference $I_H - I_L$ and has an inverted U shape as a function of α_H. Since one of the comparative statics I analyze deals with fixed aggregate income per capita, I express the variance in terms of aggregate income per capita, the fraction of the rich, and the income of the poor. Aggregate income per capita in the economy is given by

$$AG = \alpha_H I_H + (1-\alpha_H) I_L.$$

Then, the variance can be rewritten as follows

$$VAR = (\frac{1}{\alpha_H} - 1)\,(AG - I_L)^2\,. \tag{6}$$

The expression in (6) implies that keeping AG fixed, an increase in I_H together with a decrease in α_H raise income inequality in the economy.

In the next sections, I examine the impact of income inequality on the market structure and individual welfare. Since one of my main goals is to consider the effects on welfare of the poor consumers, in the subsequent analysis, I keep the income of the poor fixed and only consider changes in α_H and I_H. Recall that while changes in α_H affect consumer welfare only through the price effect, changes in I_H affect welfare through both the price and the income effects.

3.2.3 Changes in the Income of the Rich

If the rich get even richer, do the poor gain or lose? What is the impact on prices? In this section, I consider an increase in the income of the rich I_H. Higher I_H has an impact on the poor only through

[10] Another possible way to describe income inequality in the model is to use the Gini coefficient. However, in the case of the income distribution considered in the paper, the Gini coefficient is highly correlated with the variance. Changes in the parameters of the distribution, which increase the Gini coefficient, usually increase the variance. The exception is changes in α_H. In some cases, higher α_H decreases the Gini coefficient but increases the variance. As my main goal is to analyze the qualitative implications of changes in income distribution, without loss of generality, I consider the variance of the distribution as the measure of income inequality.

changes in the prices of the "common" goods. Two opposite effects influence these prices. First, since I_H increases, some firms that used to sell their goods to all consumers find it more profitable to sell only to the rich. This reduces competition among firms serving all consumers and, therefore, raises the prices of the "common" goods. Second, higher income of the rich results in higher expected profits of firms, this in turn implies that more firms enter the market inducing tougher competition and reducing the prices. I show that the latter effect prevails over the former one. As a result, higher I_H positively affects V_C increasing welfare of the poor. The following proposition summarizes the results above.

Proposition 2 *An increase in income of the rich reduces the prices of the "common" goods increasing welfare of the poor.*

Proof. In the Appendix. ■

In contrast, an increase in I_H affects the rich through both the price and the income effects. Higher income of the rich allows firms that sell only to the rich to raise their prices. In spite of higher entry in the market, the prices of the "exclusive" goods rise and as a result, V_E falls. However, the income effect is stronger than the effect of changes in prices of the "exclusive" goods and the rich gain from higher I_H. The following proposition holds[11].

Proposition 3 *An increase in income of the rich raises the prices of the "exclusive" goods and increases welfare of the rich.*

Proof. In the Appendix. ■

The intuition, which is behind the results above, may also work in traditional models with homothetic preferences. In Melitz (2003), higher income of some consumers results in higher entry, tougher competition, and, thereby, higher welfare of all consumers. However, there are some differences. In the present model, higher income of the rich raises the markups of firms selling only to the rich and decreases the markups of firms serving all consumers. In traditional models, there is the same or no impact on firms' markups. Moreover, assume for a moment that the mass of firms does not change in the model[12]. In this case, higher income of the rich raises prices of all goods and the poor are worse off. In traditional models, if we fix the mass of firms then higher income of one part of consumers does not affect welfare of the other part.

[11] Similar intuition works if we consider changes in I_L. An increase in I_L raises the prices of the "common" goods and decreases the prices of "exclusive" goods. The poor and the rich are better off (see details in the Appendix).

[12] In some sense, this case can be interpreted as a short run version of the model.

3.2.4 Changes in the Fraction of the Rich

In the previous section, I have established that higher income of the rich always benefits the poor. What is the impact of an increase in the fraction of the rich? What is better for the poor: tiny minority or vast majority of the rich? In this section, I analyze how changes in α_H affect the poor consumers. As above, a rise in α_H affects the poor through the price effect. Because of higher α_H, profits from selling to the rich become higher. This implies that some firms switch from serving all consumers to serving only the rich. This reduces competition among firms selling the "common" goods and, consequently, raises the prices of the "common" goods. At the same time, higher fraction of the rich results in higher ex ante expected profits and this in turn increases entry in the market inducing tougher competition and lower prices of all goods. In the previous section, the negative effect on the prices of "common" goods always dominates the positive one. In this case, it is not necessarily true. I show that in a neighborhood around $\alpha_H = 0$, a rise in α_H increases welfare of the poor. While in a neighborhood around $\alpha_H = 1$, higher fraction of the rich decreases welfare of the poor consumers. The following proposition holds.

Proposition 4 *If α_H is close to zero (close to one), a rise in α_H decreases (increases) the prices of the "common" goods increasing (decreasing) welfare of the poor.*

Proof. In the Appendix. ■

The last proposition suggests that welfare of the poor has an inverted U shape as a function of α_H. Because of mathematical difficulties arising in the analysis, I cannot strictly prove this conjecture. Instead, I make a number of numerical exercises where I consider welfare of the poor as a function of α_H. The results are supportive of the claim that welfare of the poor has an inverted U shape as a function of α_H[13].

The fact that firms endogenously choose the type of consumers they wish to serve makes the results regarding changes in α_H different from that ones in traditional models with homothetic preferences. In Melitz (2003), higher fraction of the rich always leads to higher welfare of the poor in the long run and has no impact in the short run (when the mass of firms is fixed). In this model, we observe an

[13] It appears that the sign of $(W_p)'_{\alpha_H}$ is the same as the sign of $K(\alpha_H) - \frac{\alpha_H}{1-\alpha_H}$, where $K(\alpha_H) > 0$ for any $\alpha_H \in [0,1]$ and $K(1) < \infty$. This implies that in the neighborhood of $\alpha_H = 0$ ($\alpha_H = 1$), $(W_p)'_{\alpha_H} > 0$ ($(W_p)'_{\alpha_H} < 0$). If $K(\alpha_H)$ is well behaved, i.e., the equation $K(\alpha_H) = \frac{\alpha_H}{1-\alpha_H}$ has a unique solution, then W_p has an inverted U as a function of α_H. Unfortunately, the analysis of the behavior of $K(\alpha_H)$ on $[0,1]$ is rather complicated. We cannot exclude the possibility that the equation $K(\alpha_H) = \frac{\alpha_H}{1-\alpha_H}$ has multiple solutions (see details in the Appendix). In the numerical examples I consider, I take the power distribution $G(b) = \left(\frac{b}{A}\right)^k$ with $k > 0$ as the distribution of draws. For a number of different sets of the exogenous parameters, I find the solution of $K(\alpha_H) = \frac{\alpha_H}{1-\alpha_H}$. In all cases, the solution is unique.

ambiguous impact of α_H on the poor in the long run and a negative impact in the short run[14].

3.2.5 Changes in the Income and the Fraction of the Rich Keeping Aggregate Income Fixed

There is a common feature for both comparative statics mentioned above. An increase in I_H as well as an increase in α_H raises aggregate income in the economy. To capture a pure redistribution effect, I consider an increase in the personal income of the rich together with a decrease in the fraction of the rich keeping aggregate income in the economy fixed[15]. In models with homothetic preferences, these changes in income distribution do not affect entry, prices, and welfare of the poor. In the present model, I show that they result in higher entry in the market and, therefore, higher welfare of the poor.

For better understanding of the intuition behind, I first consider the short run implications of the changes in income distribution. From the previous sections we know that in the short run, higher I_H decreases welfare of the poor, while lower α_H increases it. Thus, two effects work in opposite directions. However, it appears that the impact of I_H is always stronger than that of α_H. Here the assumption that aggregate income is unchanged plays a key role[16]. This implies that in the short run, the poor are worse off from the changes in the income distribution considered.

What is about the long run? On the one hand, higher income of the rich allows firms to impose higher prices of their goods and, consequently, leads to higher entry in the market. On the other hand, lower fraction of the rich reduces the demand for the "exclusive" goods making ex ante expected profits

[14] While a rise in α_H has an ambiguous impact on the poor, the rich always benefit from it. Higher α_H raises competition among firms selling the "exclusive" goods and, thereby, reduces the prices of these goods. Recall that welfare of the rich is equal to $W_r = I_L V_C + (I_H - I_L)V_E$. Even though in some cases V_E falls, due to a rise in V_C, W_r increases. As a result, the rich gain from higher α_H. See details in the Appendix.

[15] Another comparative static that holds aggregate income constant is changes in incomes of the rich and the poor keeping the fraction of the rich fixed. Since the main goal of this paper is to explore the effects on welfare of the poor, I do not pay a lot of attention on this comparative static. If we consider a rise in I_H and a decrease in I_L holding AG constant then we might expect that the income effect prevails over the price effect. That is, the rich gain and the poor lose. Notice that the poor consume on average more valuable goods than the rich. At the same time, the changes in the incomes substitute the consumption of more valuable goods for the consumption of less valuable goods. Therefore, given that $bg(b)$ is increasing in b, total welfare in the economy may decrease.

[16] Recall that welfare of a poor consumer is given by $W_p = M_e \int_{b_M}^{A} t dG(t)$. Since in the short run M_e is fixed, we only need to examine the effects on b_M. Notice that b_M solves $\left(\frac{b}{V_C} - c\right) L = \left(\frac{b}{V_E} - c\right) \alpha_H L$. This implies that $b_M \left(\frac{1}{V_C} - \frac{\alpha_H}{V_E}\right) = c(1-\alpha_H)$. From the goods market clearing condition, $I_L = \frac{M_e}{V_C} \int_{b_M}^{A} t dG(t)$ and $I_H - I_L = \frac{M_e}{V_E} \int_{b_L}^{b_M} t dG(t)$. This results in $\frac{b_M}{M_e} \left(\frac{I_L}{\int_{b_M}^{A} t dG(t)} - \frac{\alpha_H (I_H - I_L)}{\int_{b_L}^{b_M} t dG(t)} \right) = c(1-\alpha_H)$. Notice that in the long run, both an increase in I_H and a decrease in a_H drive the prices of the "exclusive" goods up (see the previous sections). This implies that in the long run, $b_L = cV_E$ falls. In the short run, only firms that were active before may operate in the market. Therefore, in the short run, b_L is unchanged. That is, firms that produced before the changes in I_H and α_H find it profitable to produce after. Moreover, as aggregate income in the economy is unchanged, $\alpha_H (I_H - I_L)$ does not change too. This implies that only b_M changes in $\frac{b_M}{M_e} \left(\frac{I_L}{\int_{b_M}^{A} t dG(t)} - \frac{\alpha_H (I_H - I_L)}{\int_{b_L}^{b_M} t dG(t)} \right)$. As a result, an increase in $c(1-\alpha_H)$ leads to a rise in b_M.

lower. This results in lower entry in the market. As in the previous section, I focus the analysis on two extreme cases: $\alpha_H \approx 0$ and $\alpha_H \approx 1$. I show that in these cases, the impact of I_H prevails over that of α_H leading to higher entry in the market. Moreover, I show that the positive effect on welfare of the poor from higher entry is stronger than the negative short run effect. These results are derived for neighborhoods of $\alpha_H = 0$ and $\alpha_H = 1$ and an arbitrary distribution function $G(b)$. However, if we limit the analysis to the cases when $bg(b)$ is increasing in b then the results hold for any $\alpha_H \in [0, 1]$[17]. The next proposition summarizes these findings.

Proposition 5 *If α_H is in neighborhoods of $\alpha_H = 0$ and $\alpha_H = 1$ or $G(b)$ is such that $bg(b)$ is increasing in b, an increase in I_H together with a decrease α_H keeping aggregate income fixed raise welfare of the poor and the number of firms entering the market.*

Proof. In the Appendix. ■

The assumption that $bg(b)$ is increasing in b has a strong economic interpretation. It implies that $g(b)$ does not decrease too fast; i.e., the probability of getting higher values of b does not decrease too fast with b. Moreover, in some sense, utility from the consumption of all goods with a particular valuation b is equal to $M_e bg(b)$. Hence, this assumption also guarantees that this utility increases with b.

3.3 Entry Cost, Market Size, and Welfare

The impact of higher entry cost and market size on consumer welfare is the same as in traditional models. However, the present model implies that changes in market size or the cost of entry have different impacts on different types of consumers. In this section, I briefly describe the effects of changes in f_e and L on individual welfare and focus my analysis on the effects on the relative welfare of the rich with respect to the poor.

An increase in the cost of entry f_e reduces the ex ante expected profits of firms. This in turn decreases the number of firms entering the market and reduces competitive pressure. As a result, prices of goods from both groups rise and welfare of all consumers falls. An increase in L results in higher ex ante expected profits of firms. This leads to the higher number of firms entering the market and tougher competition. Prices of goods from both groups fall and consumers of both types are better off. Finally, any changes in f_e and L such that the ratio $\frac{f_e}{L}$ remains the same do not change prices and individual welfare. Two opposite effects completely compensate each other (see (4)). The following proposition holds.

[17] For instance, the set of power distributions satisfies this condition.

Proposition 6 *Larger countries and countries with lower entry cost have higher individual welfare: a rise in* $\frac{f_e}{L}$ *reduces welfare of all individuals.*

Proof. In the appendix. ■

In the next section, I examine the effect of $\frac{f_e}{L}$ on relative welfare of the rich with respect to the poor.

3.3.1 Relative Welfare

Relative welfare of the rich with respect to the poor is given by

$$\frac{W_r}{W_p} = 1 + \frac{I_H - I_L}{I_L}\frac{V_E}{V_C}.$$

Notice that welfare inequality is divided into two components: income inequality and consumption inequality. The income inequality is determined by the ratio $\frac{I_H - I_L}{I_L}$, while the consumption inequality $\frac{V_E}{V_C}$ depends on relative prices of the "exclusive" goods with respect to the "common" goods. Changes in the exogenous parameters of the model may affect either type of inequality or both. For instance, higher income of the rich raises income inequality but decreases consumption inequality.

The relative welfare can be rewritten as follows

$$\frac{W_r}{W_p} = 1 + \frac{I_H - I_L}{I_L}\left(\alpha_H + (1-\alpha_H)\frac{b_L}{b_M}\right). \tag{7}$$

From (7), changes in $\frac{f_e}{L}$ affect $\frac{W_r}{W_p}$ only through the ratio $\frac{b_L}{b_M}$. Moreover, changes in $\frac{f_e}{L}$ have no direct impact on the goods market equilibrium condition

$$\frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{A} tdG(t)} - \frac{I_H - I_L}{I_L}\left(\alpha_H + (1-\alpha_H)\frac{b_L}{b_M}\right) = 0. \tag{8}$$

From (8), we can find an implicit dependence of b_M on b_L: $b_M = b_M(b_L)$. Notice that $\frac{f_e}{L}$ is negatively correlated with the cutoff $b_L = cV_E$. Hence, exploring the impact of $\frac{f_e}{L}$ on relative welfare, we need to analyze the sign of $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$. In the Appendix, I show that to determine the sign of $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$, we need to know the sign of $\left(\frac{b^2 g(b)}{\int_b^A tdG(t)}\right)'_b$. If $\left(\frac{b^2 g(b)}{\int_b^A tdG(t)}\right)'_b$ is always greater than zero then $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$ is always positive. Otherwise, depending on the exogenous parameters of the model, the sign of $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$ might be either. The following proposition formalizes the findings above.

Proposition 7 *If* $\left(\frac{b^2 g(b)}{\int_b^A tdG(t)}\right)'_b > 0$ *for any* $b \in [0, A]$, *then the rich gain more from an increase in market size and lose more from an increase in the cost of entry than the poor.*

Proof. In the Appendix. ■

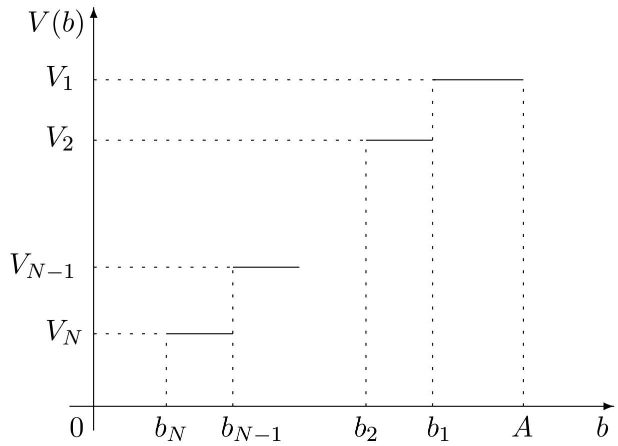
Limiting the analysis to the cases when $\left(\frac{b^2 g(b)}{\int_b^A t dG(t)}\right)'_b$ is always positive, we derive that the rich lose more from an increase in $\frac{f_e}{L}$ than the poor. To understand the intuition, I separately consider two markets. The first market is the market for goods from the "common" group, while the second one is the market for the "exclusive" goods. I divide the effect of higher $\frac{f_e}{L}$ into two steps. First, given an increase in $\frac{f_e}{L}$, fewer firms enter the both markets decreasing b_L and b_M. Second, due to less competitive pressure, some firms that sold their goods only to the rich switch to selling to all consumers. This effect decreases b_M even more and in turn reduces competition in the second market allowing firms with low valuations to survive. As a result, the cutoff b_L falls. Since firms that switched from the second market to the first one have relatively high valuations of their goods compared with firms that "survived", the prices of these goods were relatively high. This implies that b_L has to fall by more than b_M to compensate for the difference in the prices.

4 A General Model

To complete the model, I consider the general case with N types of consumers. I show the existence and uniqueness of the equilibrium and discuss some issues related to the case when the distribution of efficiency units of labor among consumers is continuous.

In the general case, consumers differ in the number of efficiency units of labor they are endowed with. A consumer of type n is endowed with I_n efficiency units of labor. I choose indices so that $I_n > I_{n-1}$. Here α_n is the fraction of consumers of type n in the aggregate mass L of consumers. The equilibrium in the general model is similar to the equilibrium in the simple case considered before. All goods that are consumed by a certain type of consumers are also consumed by more productive consumers. Thus, goods in the economy are divided into $N+1$ groups. Goods belong to group $k = 1..N$ if they are only consumed by consumers whose type is greater or equal to k. Goods belong to group $N+1$ if they are consumed by nobody. In the equilibrium, goods from the same group have the same valuation to price ratio. Let V_k be the valuation to price ratio of goods from group k. Then, in the equilibrium, $V(b)$ looks as in *Figure 4,* where b_k is such that firms with b_k are indifferent between selling to consumers with types greater or equal to k and selling to consumers with types greater or equal to $k+1$. For instance, firms with b_1 are indifferent between selling to all consumers and selling to everyone except the poorest. Firms with $b < b_N$ leave the market. Without loss of generality, I assume that firms with b_k choose to sell to consumers with types greater or equal to k. As before, let M_e be the mass of firms that enter the market and draw valuation of their goods.

Figure 4: The Valuation to Price Function: A General Model



Definition 2 *The equilibrium of the model is defined by the price function $p(b)$ on $b \geq b_N$, M_e, the sequences $\{V_k\}_{k=1..N}$ and $\{b_k\}_{k=1..N}$ such that*

1) The ex ante expected profits of firms are equal to zero.

2) The goods market clears.

Let $\pi_k(b)$ and $p_k(b)$ be the profit and the price of a firm with valuation $b \in [b_k, b_{k-1})$, respectively[18]. Then, the following lemma holds.

Lemma 3 *In the equilibrium,*

$$p_k(b) = \frac{b}{V_k} = bc\frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i},$$

$$\pi_k(b) = cL\sum_{i=k}^{N}\frac{\alpha_i(b-b_i)}{b_i}.$$

Proof. In the appendix. ■

In the equilibrium, the expected profits of firms are equal to zero. This implies that

$$\begin{aligned} f_e &= \sum_{k=1}^{N}(G(b_{k-1}) - G(b_k))E(\pi_k(b)|b \in [b_k, b_{k-1})) \iff \\ \frac{f_e}{cL} + 1 &= \sum_{k=1}^{N}\alpha_k H(b_k). \end{aligned}$$

In addition, the goods market clearing condition should be satisfied. This implies that the aggregate cost of the bundle of goods from group k should be equal to income of a consumer of type k. In this

[18] $b_0 = A$.

way, I obtain

$$I_k = M_e \int_{b_k}^{A} p(t)dG(t) \quad k = 1..N.$$

Hence, there is the system of $N+1$ equations

$$\begin{cases} I_k = M_e \int_{b_k}^{A} p(t)dG(t) \quad k = 1..N \\ \frac{f_e}{cL} + 1 = \sum_{k=1}^{N} \alpha_k H(b_k) \end{cases} \tag{9}$$

with $N+1$ unknowns: $\{b_k\}_{k=1..N}$ and M_e.

Proposition 8 *The equilibrium in the general model always exists and is unique.*

Proof. The proof is based on the fact that the system of equations (9) has a unique solution. Details are in the Appendix. ■

Assume that the distribution of consumer productivities is continuous. Notice that any continuous distribution can be approximated by the sequence of discrete distributions. Therefore, we can interpret equilibrium in the continuous model as the limit of equilibria in the discrete models. In this case, the function $V(b)$ is continuous, increasing on $[b_L^c, b_M^c)$, and flat on $[b_M^c, A]$, where $0 \geq b_L^c > b_M^c \geq A$. The parameter b_L^c represents the cutoff level: firms with $b < b_L^c$ leave the market. While b_M^c is determined by the support of the productivity distribution. Namely, goods with $b \in [b_M^c, A]$ are consumed by everybody in the equilibrium. This implies that $b_M^c < A$ if and only if the lower bound of the distribution support is strictly greater than zero; i.e., the minimum income in the economy is greater than zero.

Due to mathematical difficulties, it is hard to solve the continuous model for an arbitrary distribution of productivities[19]. To solve the problem explicitly, I need to make a simplifying assumption about the distribution of efficiency units of labor. I assume that this distribution has a constant hazard rate. That is, I consider the set of exponential distributions on $[s, \infty)$, where $s \geq 0$ is the minimum endowment of efficiency units of labor. Since the upper bound of the support is infinity, the maximum income in the economy is also equal to infinity. This implies that the cutoff b_L^c equals to zero in the equilibrium. I show that in a neighborhood $b = 0$, the price function $p(b)$ is decreasing in b and $p(0) = \infty$. Hence, this model gives us a simple straightforward explanation of why some luxury goods with relatively low valuation (or quality) to price ratios are so expensive: the rich are ready to pay such high prices for these goods.

[19] See details in the Appendix.

5 Conclusion

In this paper, I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. The model incorporates two key features: imperfect competition and non-homothetic preferences, which allow us to analyze the consequences of changes in income distribution on pricing, market structure and, thereby, welfare of different groups of consumers in equilibrium. The general model is constructed and solved. Due to technical difficulties in exploring comparative statics in the general case, I focus on the case of two types of consumers: rich and poor.

This framework leads to interesting theoretical results that help to understand the impact of income inequality on individual well-being. In particular, I analyze how income inequality influences welfare of the poor. I show that higher income inequality in the economy may benefit the poor via a trickle-down effect operating through entry. This model also allows us to analyze the effects of changes in market size and entry cost. An increase in market size leads to tougher competition. Therefore, markups of all firms fall and welfare of all consumers rises. Similarly, an increase in entry cost induces lower competition, raises markups, and, thereby, decreases welfare of all consumers. Moreover, I show that the rich may gain more from an increase in market size and lose more from an increase in entry cost compare to the poor.

There are a number of plausible extensions of this model. For instance, it would be interesting to consider an open economy version of the model. In this case, the paper can be modified in two ways. First, one can explore a model of trade between two countries with different income distributions and examine how this difference affects the trade pattern. Second, it would be interesting to consider the case when income distribution is endogenous and, for instance, affected by the level of openness. I leave these issues for future work.

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Appendix

Proof of Lemma 2

Consider

$$\frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{A} tdG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right). \tag{10}$$

Let $b_M = F_1(b_L)$ be an implicit solution of (10). $F_1(b_L)$ is strictly increasing in b_L and $A \geq F_1(b_L) \geq b_L$. This implies that $F_1(A) = A$. Now, consider

$$\frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1-\alpha_H)H(b_M). \tag{11}$$

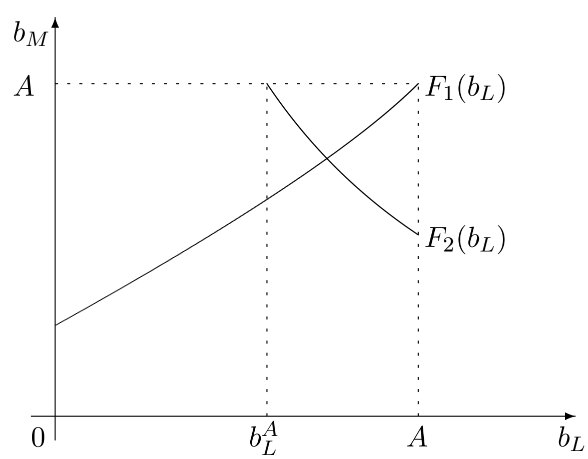
By analogy, let $b_M = F_2(b_L)$ be an implicit solution of (11). As $H(\cdot)$ is strictly decreasing, $F_2(b_L)$ is also strictly decreasing in b_L. Since $H(A) = 1$, $H(F_2(A)) = \frac{f_e}{cL(1-\alpha_H)} + 1 > 1$. This implies that $F_2(A) < A$. Let b_L^A be such that $F_2(b_L^A) = A$. Then, $H(b_L^A) = \frac{f_e}{cL\alpha_H} + 1 > 1$, i.e., $b_L^A < A$. Hence, the solution of (4) exists and is unique (see *Figure 5*).

Proof of Lemma 3

Demand for goods from group k is equal to $L\sum_{i=k}^{N}\alpha_i$. From the definition of the sequence $\{b_k\}_{k=1..N}$, $\left(\frac{b_k}{V_k} - c\right)\sum_{i=k}^{N}\alpha_i = \left(\frac{b_k}{V_{k+1}} - c\right)\sum_{i=k+1}^{N}\alpha_i$. By induction,

$$\frac{\sum_{i=k}^{N}\alpha_i}{V_k} = \frac{1}{V_1} - c\sum_{i=1}^{k-1}\frac{\alpha_i}{b_i}. \tag{12}$$

Figure 5: The Equilibrium



From (12), $\pi_N(b) = \left(\frac{b}{V_N} - c\right)\alpha_N L = \frac{bL}{V_1} - cbL\sum_{i=1}^{N-1}\frac{\alpha_i}{b_i} - c\alpha_N L$. Recall that $\pi_N(b_N) = 0$. This implies that $\frac{1}{V_1} = c\sum_{i=1}^{N}\frac{\alpha_i}{b_i}$. From (12), $\frac{1}{V_k} = \frac{c\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}$ $k = 1..N$. Therefore,

$$\begin{aligned} p_k(b) &= bc\frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}, \\ \pi_k(b) &= cL\sum_{i=k}^{N}\frac{\alpha_i(b-b_i)}{b_i}. \end{aligned}$$

Proof of Proposition 8

Using *Lemma 3*, the system of equations (9) can be rewritten as follows[20]

$$\left\{\begin{array}{c} \frac{f_e}{cL} + 1 = \sum_{k=1}^{N}\alpha_k H(b_k), \\ \frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}\int_{b_k}^{b_{k-1}} tdG(t) \quad k = 1..N \end{array}\right. . \tag{13}$$

Consider $k = N$. Then,

$$\frac{I_N - I_{N-1}}{cM_e} = \frac{1}{b_N}\int_{b_N}^{b_{N-1}} tdG(t). \tag{14}$$

Given M_e and b_{N-1}, there exists a unique solution $b_N(b_{N-1}, M_e)$ of the equation (14). The function $b_N(b_{N-1}, M_e)$ is strictly increasing in M_e and b_{N-1}. Given b_{N-1}, $\frac{M_e}{b_N(b_{N-1},M_e)} = \frac{I_N - I_{N-1}}{c\int_{b_N}^{b_{N-1}} tdG(t)}$ is strictly increasing in M_e.

Consider $k = N-1$. Then,

$$\frac{I_{N-1} - I_{N-2}}{cM_e} = \frac{\frac{\alpha_N}{b_N} + \frac{\alpha_{N-1}}{b_{N-1}}}{\alpha_N + \alpha_{N-1}}\int_{b_{N-1}}^{b_{N-2}} tdG(t). \tag{15}$$

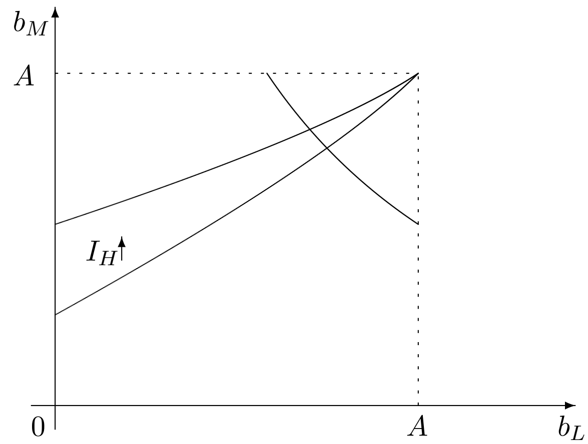
Given M_e and b_{N-2}, there exists a unique solution $b_{N-1}(b_{N-2}, M_e)$ of the equation (15). The function $b_{N-1}(b_{N-2}, M_e)$ is strictly increasing in b_{N-2}. Since $\frac{M_e}{b_N(b_{N-1},M_e)}$ is strictly increasing in M_e, $b_{N-1}(b_{N-2}, M_e)$ is also strictly increasing in M_e. Finally, $\frac{\left(\frac{\alpha_N}{b_N} + \frac{\alpha_{N-1}}{b_{N-1}}\right)M_e}{\alpha_N + \alpha_{N-1}} = \frac{I_{N-1} - I_{N-2}}{c\int_{b_{N-1}}^{b_{N-2}} tdG(t)}$ is strictly increasing in M_e.

Using the backward induction, it can be proved that for any $k = 1..N$, there exists a unique solution $b_k(b_{k-1}, M_e)$ of the equation $\frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}\int_{b_k}^{b_{k-1}} tdG(t)$ such that $b_k(b_{k-1}, M_e)$ is strictly increasing in b_{k-1} and M_e. This implies that for any M_e, there exists a unique solution $\{b_k(M_e)\}_{k=1..N}$ of the system of equations $\frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}\int_{b_k}^{b_{k-1}} tdG(t)$ $k = 1..N$. And for any $k = 1..N$, $b_k(M_e)$ is strictly increasing in M_e. Hence, (13) is equivalent to

$$\left\{\begin{array}{ll} \frac{f_e}{cL} + 1 = \sum_{k=1}^{N}\alpha_k H(b_k(M_e)), & \\ b_k = b_k(M_e) & k = 1..N. \end{array}\right. \tag{16}$$

[20] $I_0 = 0$.

Figure 6: An Increase in I_H



Consider $D(M_e) = \sum_{k=1}^{N} \alpha_k H(b_k(M_e))$. As $H(x)$ is a strictly decreasing function, $D(M_e)$ is strictly decreasing in M_e. If M_e is close to zero then $b_N(M_e)$ is close to zero and, thereby, $D(M_e)$ is high enough[21]. If M_e is sufficiently high then for any $k = 1..N$, $b_k(M_e)$ is close to A and $D(M_e) \approx \sum_{k=1}^{N} \alpha_k H(A) = 1 < \frac{f_e}{cL} + 1$. This implies that there exists a unique solution M_e of (16). Therefore, there exists a unique solution of (9).

Comparative Statics

In this section, I use a simplifying notation: $\int_x^y$ means $\int_x^y tdG(t)$.

Proof of Proposition 2

An increase in I_H shifts the curve $F_1(b_L)$ up, while the curve $F_2(b_L)$ is unchanged. As a result, b_L falls and b_M rises (see *Figure 6*). The impact on welfare of the poor is not so straightforward. Rewrite (10) and (11) as follows

$$\begin{cases} J_1 \equiv (1-\alpha_H)cLH(b_M) + \alpha_H cLH(b_L) - f_e - cL = 0 \\ J_2 \equiv I_L \int_{b_L}^{b_M} - (I_H - I_L)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^{A} = 0 \end{cases}. \tag{17}$$

[21] Recall that $H(0) = \infty$.

Notice that equilibrium values of b_L and b_M solve (17). Using implicit differentiation, I obtain

$$\frac{\partial b_M}{\partial I_H} = \frac{\frac{\partial J_2}{\partial I_H}\frac{\partial J_1}{\partial b_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} > 0 \tag{18}$$

$$\frac{\partial b_L}{\partial I_H} = \frac{-\frac{\partial J_2}{\partial I_H}\frac{\partial J_1}{\partial b_M}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} < 0. \tag{19}$$

Consider $\frac{1}{V_C} = \frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}$. $\left(\frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}\right)'_{I_H}$ is equal to $\frac{-\alpha_H c}{(b_L)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{(b_M)^2}\frac{\partial b_M}{\partial I_H}$. From (18) and (19),

$$\frac{-\alpha_H c}{(b_L)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{(b_M)^2}\frac{\partial b_M}{\partial I_H} = \frac{c^2 L\alpha_H(1-\alpha_H)\frac{\partial J_2}{\partial I_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\left(\frac{H'(b_M)}{(b_L)^2} - \frac{H'(b_L)}{(b_M)^2}\right).$$

Recall that $H'(x) = -\frac{\int_x^A tdG(t)}{x^2} < 0$. Then,

$$\frac{-\alpha_H c}{(b_L)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{(b_M)^2}\frac{\partial b_M}{\partial I_H} = \frac{c^2 L\alpha_H(1-\alpha_H)\frac{\partial J_2}{\partial I_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\frac{\int_{b_L}^A - \int_{b_M}^A}{(b_L)^2(b_M)^2}.$$

Since $\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L} > 0$ and $\frac{\partial J_2}{\partial I_H} < 0$, $\left(\frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}\right)'_{I_H} < 0$. Therefore, $(V_C)'_{I_H} > 0$. This implies that an increase in I_H leads to the lower prices of the "common" goods and higher welfare of the poor, which is equal to $I_L V_C$.

5.0.2 Proof of Proposition 3

From the previous proof, we know that higher I_H results in lower b_L. That is, the prices of the "exclusive" goods rise. As $W_p = M_e\int_{b_M}^A$ and b_M increases, an increase in I_H raises M_e and, therefore, $W_r = M_e\int_{b_L}^A$.

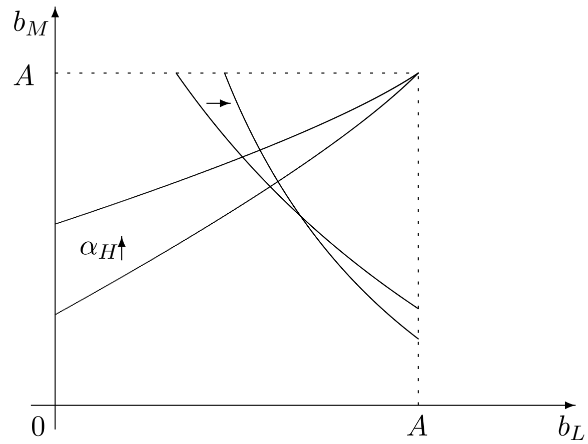
5.0.3 Changes in I_L

Similarly, an increase in I_L shifts the curve $F_1(b_L)$ down, while the curve $F_2(b_L)$ is unchanged. Hence, b_L rises and b_M falls. To analyze the impact on consumer welfare, I use the same technique as in the previous proofs. As $W_r = M_e\int_{b_L}^A$ and $I_H = M_e\int_{b_L}^A p(t)dG(t)$, $W_r = \frac{I_H\int_{b_L}^A}{\int_{b_L}^A p(t)dG(t)}$. The sign of $(W_r)'_{I_L}$ is the same as the sign of $\left(\int_{b_L}^A\right)'_{I_L}\int_{b_L}^A p(t)dG(t) - \left(\int_{b_L}^A p(t)dG(t)\right)'_{I_L}\int_{b_L}^A$. Algebra shows that

$$\left(\int_{b_L}^A\right)'_{I_L}\int_{b_L}^A p(t)dG(t) - \left(\int_{b_L}^A p(t)dG(t)\right)'_{I_L}\int_{b_L}^A =$$

$$= \frac{c^2 L(1-\alpha_H)^2\frac{\partial J_2}{\partial I_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\frac{\int_{b_L}^{b_M}\int_{b_L}^A\int_{b_M}^A}{(b_L)^2(b_M)^2} + c(1-\alpha_H)(b_M - b_L)\left\{\frac{\partial b_L}{\partial I_L}\frac{g(b_L)}{b_M}\int_{b_M}^A - \frac{\partial b_M}{\partial I_L}\frac{g(b_M)}{b_L}\int_{b_L}^A\right\}.$$

Figure 7: An Increase in α_H



From (17), $\frac{\partial b_L}{\partial I_L} > 0$, $\frac{\partial b_M}{\partial I_L} < 0$, and $\frac{\partial J_2}{\partial I_L} > 0$. This implies that $(W_r)'_{I_L} > 0$. As $M_e = \frac{W_r}{\int_{b_L}^A}$, an increase in I_L raises M_e and, thereby, $W_p = M_e \int_{b_M}^A$.

Proof of Proposition 4

An increase in α_H shifts the curve $F_1(b_L)$ up and the curve $F_2(b_L)$ to the right around 45° degree line (see *Figure 7*). In this case, b_M rises. The impact on b_L is not so straightforward. There are two opposite effects. The upward shift of $F_1(b_L)$ decreases b_L, while the shift of the $F_2(b_L)$ increases b_L. I show that $\frac{\partial b_L}{\partial \alpha_H} > 0$.

From (17),

$$\begin{aligned}
\frac{\partial b_M}{\partial \alpha_H} &= \frac{-\frac{\partial J_1}{\partial \alpha_H}\frac{\partial J_2}{\partial b_L} + \frac{\partial J_2}{\partial \alpha_H}\frac{\partial J_1}{\partial b_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} > 0 \\
\frac{\partial b_L}{\partial \alpha_H} &= \frac{-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H} + \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}.
\end{aligned}$$

To determine the sign of $\frac{\partial b_L}{\partial \alpha_H}$, I examine

$$-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H} + \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H} = cL\left(\left(H\left(b_L\right) - H\left(b_M\right)\right)\frac{\partial J_2}{\partial b_M} - \frac{(1-\alpha_H)}{\left(b_M\right)^2}\left(I_H - I_L\right)\left(1 - \frac{b_L}{b_M}\right)\left(\int_{b_M}^A\right)^2\right).$$

The partial derivative of J_2 with respect to b_M can be written as follows

$$\frac{\partial J_2}{\partial b_M} = \left(I_H - I_L\right)\left(\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right) b_M g\left(b_M\right)\frac{\int_{b_L}^A}{\int_{b_L}^{b_M}} + \frac{b_L(1-\alpha_H)}{\left(b_M\right)^2}\int_{b_M}^A\right). \tag{20}$$

Then,

$$\frac{-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H}+\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H}}{cL\left(I_H-I_L\right)} = \left(H\left(b_L\right)-H\left(b_M\right)\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)b_M g\left(b_M\right)\frac{\int_{b_L}^{A}}{\int_{b_L}^{b_M}}+$$
$$+\frac{(1-\alpha_H)}{\left(b_M\right)^2}\int_{b_M}^{A}\left(G(b_L)b_L-G(b_M)b_L+\int_{b_L}^{A}-\int_{b_M}^{A}\right)>0,$$

as $b_M>b_L$ and $G(b_L)b_L-G(b_M)b_L+\int_{b_L}^{A}-\int_{b_M}^{A}$ is increasing in b_M and equal to zero when $b_M=b_L$.

Welfare of the poor is given by $W_p=\frac{I_L}{c\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)}$. To determine the sign of $(W_p)'_{\alpha_H}$, we need to examine the sign of $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}=\frac{1}{b_L}-\frac{1}{b_M}-\left(\frac{\alpha_H}{(b_L)^2}\frac{\partial b_L}{\partial\alpha_H}+\frac{(1-\alpha_H)}{(b_M)^2}\frac{\partial b_M}{\partial\alpha_H}\right)$. The derivative of J_2 with respect to b_L can be expressed as

$$\frac{\partial J_2}{\partial b_L}=-\left(I_H-I_L\right)\left(\frac{b_L g\left(b_L\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^{A}}{\int_{b_L}^{b_M}}+\frac{(1-\alpha_H)}{b_M}\int_{b_M}^{A}\right). \tag{21}$$

Using the expressions (20) and (21), I show that

$$\frac{\alpha_H}{(b_L)^2}\frac{\partial b_L}{\partial\alpha_H}+\frac{(1-\alpha_H)}{(b_M)^2}\frac{\partial b_M}{\partial\alpha_H}=\frac{cL\left(I_H-I_L\right)\left(\left(H\left(b_L\right)-H\left(b_M\right)\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)P_1+\left(1-\frac{b_L}{b_M}\right)P_2\right)}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}},$$

where $P_1=\left(\frac{\alpha_H b_M g(b_M)\int_{b_L}^{A}}{(b_L)^2\int_{b_L}^{b_M}}+\frac{(1-\alpha_H)b_L g(b_L)\int_{b_M}^{A}}{(b_M)^2\int_{b_L}^{b_M}}+\frac{(1-\alpha_H)}{b_L(b_M)^2}\int_{b_M}^{A}\right)$ and $P_2=\frac{(1-\alpha_H)\alpha_H}{(b_M)^2(b_L)^2}\int_{b_L}^{b_M}\int_{b_M}^{A}$. In addition,

$$\frac{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}{cL\left(I_H-I_L\right)}=\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3+\frac{(1-\alpha_H)\int_{b_M}^{A}tdG(t)}{\left(b_M\right)^2}P_4,$$

where $P_3=\frac{(1-\alpha_H)b_L g(b_L)\left(\int_{b_M}^{A}\right)^2}{(b_M)^2}+\frac{\alpha_H b_M g(b_M)\left(\int_{b_L}^{A}\right)^2}{(b_L)^2}$ and $P_4=\frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M}+\frac{\alpha_H\int_{b_L}^{A}}{b_L}$. Therefore,

$$\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}=\frac{1}{b_L}-\frac{1}{b_M}-\frac{\left(H\left(b_L\right)-H\left(b_M\right)\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)P_1+\left(1-\frac{b_L}{b_M}\right)P_2}{\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3+\frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}P_4}.$$

After some simplifications,

$$\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}=\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3+\frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}P_4}P_5,$$

where

$$P_5 = \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}\left(\frac{1}{b_L}+\frac{b_L g(b_L)}{\int_{b_L}^{b_M}}\right)\left(G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_L}\right)$$
$$+\frac{\alpha_H\int_{b_L}^{A}}{(b_L)^2}\frac{b_M g(b_M)}{\int_{b_L}^{b_M}}\left(G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_M}\right).$$

Hence, the sign of $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$ is the same as the sign of P_5. As $b_M > b_L$, $G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_L} < 0$ and $G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_M} > 0$. Hence, if α_H is close enough to zero then $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} < 0$; that is, $(W_p)'_{\alpha_H} > 0$. However, if α_H is close enough to one then $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} > 0$. This implies that $(W_p)'_{\alpha_H} < 0$. It is much more complicated to determine the sign of P_5 for all values of $\alpha_H \in [0,1]$.

5.0.4 The Effect of Higher α_H on the Rich

From the previous section, we know that $\frac{\partial b_L}{\partial \alpha_H} > 0$. This means that higher α_H decreases the prices of the "exclusive" goods. Welfare of the rich is given by $\frac{1}{c}\left(\frac{I_L}{\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)}+(I_H-I_L)\,b_L\right)$. This implies

$$c\,(W_r)'_{\alpha_H} = \frac{(I_H-I_L)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}}{\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2}.$$

To determine the sign of $(W_r)'_{\alpha_H}$, we need to examine the sign of $(I_H-I_L)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$. Using the previous results,

$$(I_H-I_L)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} =$$
$$= (I_H-I_L)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3+\frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}P_4}P_5.$$

After some simplifications, it appears that to prove that $(W_r)'_{\alpha_H} > 0$, it is enough to prove that

$$(I_H-I_L)\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)(H(b_L)-H(b_M)) - \frac{I_L}{b_L}\left(G(b_M)-G(b_L)-\frac{\int_{b_L}^{b_M}}{b_M}\right) > 0 \iff$$
$$\frac{I_L\int_{b_L}^{A}}{b_L}\left(\frac{H(b_L)-H(b_M)}{\int_{b_M}^{A}}-\frac{1}{b_L}+\frac{1}{b_M}\right) > 0.$$

For any $b_L < b_M$, $\frac{H(b_L)-H(b_M)}{\int_{b_M}^{A}} - \frac{1}{b_L} + \frac{1}{b_M} > 0$ resulting in that $(W_r)'_{\alpha_H}$ is always greater than zero. Since $(W_r)'_{\alpha_H} > 0$, $(b_L)'_{\alpha_H} > 0$ and $W_r = M_e \int_{b_L}^{A}$; the mass of firms entering the market rises, i.e., $(M_e)'_{\alpha_H} > 0$.

Proof of Proposition 5

Aggregate income per capita AG is given by $\alpha_H I_H + (1-\alpha_H) I_L$. This implies that $\alpha_H (I_H - I_L) = AG - I_L$. In this way, I rewrite (17) as follows

$$\begin{cases} J_1 \equiv (1-\alpha_H) cLH(b_M) + \alpha_H cLH(b_L) - f_e - cL = 0 \\ J_2 \equiv I_L \int_{b_L}^{b_M} - (AG - I_L)\left(1 + \frac{b_L(1-\alpha_H)}{\alpha_H b_M}\right) \int_{b_M}^{A} = 0 \end{cases}. \tag{22}$$

Hence, it is necessary to explore the impact of a decrease in α_H on welfare of the poor given new equilibrium equations (22). Using the same technique as in the proof of *Proposition 4*, I obtain

$$\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \frac{\left(\frac{\alpha_H}{b_L} + \frac{b_L(1-\alpha_H)}{b_M}\right)\left(\frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M^2}\left(G(b_M) - G(b_L)\right) + P_6\right)}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M} t dG(t)} P_3 + \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2} P_4},$$

where $P_6 = \frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M^2} \frac{b_L^2 g(b_L)}{\int_{b_L}^{b_M}}\left(G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_L}\right) + \frac{\alpha_H \int_{b_L}^{A}}{b_L} \frac{b_M g(b_M)}{\int_{b_L}^{b_M}}\left(G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_M}\right)$.

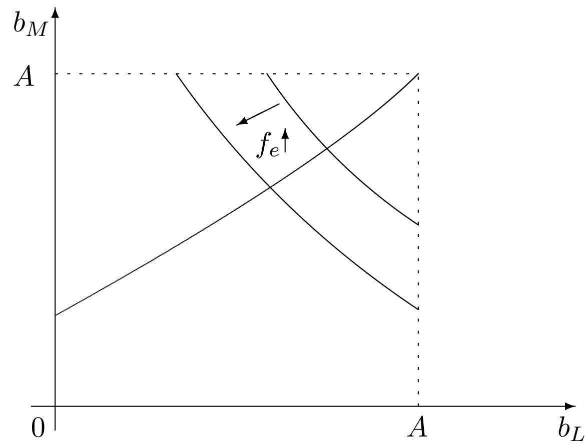
If α_H is close to one then $P_6 > 0$ and $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} > 0$. That is, welfare of the poor rises with a decrease in α_H. This result is also supported by the fact that given sufficiently high α_H, both an increase in I_H and a decrease in α_H have a positive impact on welfare of the poor.

Consider $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$ when α_H is close to zero. From (22), $\lim_{\alpha_H \to 0} b_L(\alpha_H) = 0$ and $\lim_{\alpha_H \to 0} \frac{b_L(\alpha_H)}{\alpha_H}$ is a positive constant. As for any density function $g(\cdot)$, $\lim_{x \to 0} xg(x) = 0$; $\lim_{\alpha_H \to 0} P_6 > 0$. This implies that $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H = 0} > 0$. Finally, it can be shown that if $bg(b)$ is increasing in b, $\frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M^2}\left(G(b_M) - G(b_L)\right) + P_6 > 0$ for any $\alpha_H \in [0, 1]$.

Proof of Proposition 6

A rise in f_e shifts the curve $F_2(b_L)$ to the left, while the curve $F_1(b_L)$ is unchanged. As a result, b_L and b_M fall (see *Figure 8).* Since $W_p = \frac{I_L}{\frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}}$ and $W_r = W_p + \frac{I_H - I_L}{c} b_L$, W_p and W_r decrease. M_e, which is equal to $\frac{W_p}{\int_{b_M}^{A}}$, falls too. In the same way, a rise in L raises M_e, W_p, and W_r. Finally, any changes in f_e and L such that $\frac{f_e}{L}$ remains unchanged do not affect $F_2(b_L)$ and $F_1(b_L)$.

Figure 8: An Increase in f_e



Proof of Proposition 7

I need to show that given $\left(\frac{b^2 g(b)}{\int_b^A}\right)'_b > 0$, $\left(\frac{b_L}{b_M(b_L)}\right)_{b_L} > 0$ where $b_M(b_L)$ is an implicit solution of

$$\int_{b_L}^{b_M} - \left(\frac{I_H}{I_L}-1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^{A} = 0.$$

Notice that the sign of$\left(\frac{b_L}{b_M(b_L)}\right)_{b_L}$ is the same as the sign of $b_M - \frac{\partial b_M}{\partial b_L} b_L$. Algebra shows that

$$b_M - \frac{\partial b_M}{\partial b_L} b_L \quad > \quad 0 \iff$$

$$\frac{b_L g(b_L) + \frac{\int_{b_L}^{b_M}}{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}\frac{(1-\alpha_H)}{b_M}}{\frac{\int_{b_L}^A}{\int_{b_M}^A} b_M g(b_M) + \frac{b_L(1-\alpha_H)}{(b_M)^2}\frac{\int_{b_L}^{b_M}}{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}}\frac{b_L}{b_M} \quad < \quad 1 \iff \frac{(b_L)^2 g(b_L)}{\int_{b_L}^A} < \frac{(b_M)^2 g(b_M)}{\int_{b_M}^A}.$$

The Continuous Distribution of Efficiency Units of Labor

I assume that there is a distribution $F(\cdot)$ on $[s, S]$ (with a density function $f(\cdot)$) of efficiency units of labor. That is, given the mass L of consumers, there are $F(x)L$ consumers with income less or equal to x. Define $V(b) = \frac{b}{p(b)}$. From the main body of the paper, $V(b)$ is increasing on $[b_L^c, b_M^c)$ and flat on $[b_M^c, A]$ (see *Section 4*). I assume that $V(b)$ is differentiable on $[b_L^c, b_M^c)$. To simplify the notation, I also assume that $L = 1$.

Consider a particular firm with valuation b. If $b \in [b_M^c, A]$ then demand for this good is equal to one and $p(b) = \frac{b}{V(b_M^c)}$. Suppose $b \in [b_L^c, b_M^c)$ and the firm imposes price p of its good. Then, given $V(b)$

in the equilibrium, $s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)$ is total spending on goods, which are bought before the good considered: goods that have higher valuation to price ratios. This implies that demand for this good is equal to $1-F\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)$. Hence, in the equilibrium, firms with $b\in[b_L^c,b_M^c)$ solve the following maximization problem

$$\max_p (p-c)\left(1-F\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)\right).$$

The first order condition implies that

$$\frac{1-F\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)}{f\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)}=(p-c)\frac{bM_e p\left(V^{-1}\left(\frac{b}{p}\right)\right)g\left(V^{-1}\left(\frac{b}{p}\right)\right)}{p^2V'\left(V^{-1}\left(\frac{b}{p}\right)\right)}.$$

This equation should be satisfied for any $b\in[b_L^c,b_M^c)$. That is, the price function $p(b)$ on $[b_L^c,b_M^c)$ solves the following differential equation

$$\frac{1-F\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)}{f\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)}=(p(b)-c)\frac{bM_e g(b)}{p(b)V'(b)} \tag{23}$$

where $V(b)=\frac{b}{p(b)}$. Using the solution of (23), free entry condition, and the goods market equilibrium, we can find b_L^c, b_M^c, and M_e.

In general, it is rather complicated to find the solution of (23). To simplify the problem, I assume that $F(x)=1-e^{-\alpha(x-s)}$ on $[s,\infty)$. This implies that $\frac{1-F\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)}{f\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)}=\frac{1}{\alpha}$.Thus, (23) is equivalent to

$$V'(b)=\alpha M_e(b-cV(b))g(b). \tag{24}$$

As the maximum endowment of efficiency unit of labor is infinity, there is no exit and $b_L^c=0$. Using the initial condition $V(0)=0$ and (24), we have

$$\begin{aligned}V(b) &= \frac{1}{c}\left(b-e^{-\alpha M_e cG(b)}\int_0^b e^{\alpha M_e cG(t)}dt\right)\\ p(b) &= \frac{cb}{b-e^{-\alpha M_e cG(b)}\int_0^b e^{\alpha M_e cG(t)}dt}.\end{aligned}$$

From the goods market clearing condition, we obtain that $s=\frac{M_e}{V(b_M^c)}\int_{b_M^c}^A tdG(t)$. Using this equation and free entry condition, we can find M_e and b_M^c[22]. Notice that $\lim_{b\to 0}p(b)=\infty$. This means that goods with the lowest valuations have the highest prices.

[22] In the simplest case when $s=0$, $b_M^c=A$ and M_e can be found from $f_e=\int_0^A(p(b)-c)\,e^{-\alpha M_e\int_b^A p(t)dG(t)}dG(b)$.