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ABSTRACT

This paper examines (real-time) equilibrium interest rates in the Czech Republic in 2001:1-2005:12 estimating various specifications of simple Taylor-type monetary policy rules. First, we estimate it using GMM. Second, we apply structural time-varying coefficient model with endogenous regressors to evaluate fluctuations of equilibrium interest rate over time. The results suggest that there is substantial interest rate smoothing and central bank primarily responds to inflation (forecast) developments. The estimated parameters seem to sustain the equilibrium determinacy. We find that the equilibrium interest rates gradually decreased over sample period to the levels comparable to those of in the euro area reflecting capital accumulation, smaller risk premium and successful disinflation in the Czech economy.

JEL Classification: E43, E52, E58

Keywords: equilibrium interest rates, Taylor rule, augmented Kalman filter

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1. Introduction

Modern central banks set short-term interest rates in the way to achieve their goals, i.e. price stability and/or output close to its potential. Woodford (2003) notes that central banks should track the equilibrium interest rates to stabilize the economy. Taylor (1999) emphasizes that measurement of equilibrium interest rates is one of key issues for countries targeting inflation. In this respect, it is of paramount importance for central banks to identify as precisely as possible, what is the equilibrium (or natural) level of the interest rates that maximizes the central bank objectives. In this regard, the role of equilibrium interest rate for monetary policy conduct is discussed extensively by e.g. Taylor (1993), Woodford (2003) or Amato (2005).

In this paper, we address the issue of equilibrium interest rate estimation in one of EU new member states, the Czech Republic, based on various specifications of simple Taylor-type monetary policy rules. Former transition country provides an interesting case to evaluate nominal as well as real equilibrium interest rates, as one can expect certain pattern in the development of nominal and real equilibrium interest rates over time. Namely, Lipschitz *et al.* (2006) points out that at the outset of transition the capital/labor ratios were much lower than those in Western Europe and therefore marginal product of capital and for that reason real equilibrium interest rate was rather high. Given the capital accumulation over the course of transition, there should be tendency for real equilibrium rates to decrease. From open economy perspective, EU new members exhibited a fall of exchange rate risk premium during their transition process to market economy, which also puts a downward pressure on real equilibrium interest rates (Archibald and Hunter, 2001).

Additionally, the path of nominal equilibrium interest rates should reflect not only the decrease of real equilibrium rates, but also successful disinflation in transition countries (see

Korhonen and Wachtel, 2006). While there are dozens of studies on equilibrium exchange rates in EU new members, there is surprisingly very little evidence on equilibrium interest rates (Brzoza-Brzezina, 2006, is an exception with evidence on Poland). This imbalance is rather striking, as nearly half of EU new members target inflation (Czech Republic, Hungary, Poland and Slovakia), for which the concept as well as measurement of equilibrium interest rates is of primary importance for the conduct of monetary policy.[1] Consequently, this paper tries to bridge this gap.

Concretely, we provide estimation of short-term real equilibrium interest rate as well as policy neutral rate (defined as short-term real equilibrium interest rate plus expected inflation). In order to do so, we first estimate monetary policy rules using GMM. Second, monetary policy rules with time-varying intercept are used to assess the fluctuations of equilibrium interest rates over time. The novelty of our approach is estimation of time-varying equilibrium interest rates by augmented Kalman filter (Kim, 2006). This filter is robust to endogeneity in regressors, which, as we find, is indeed relevant for our exercise. Additional feature of this paper is that in line with Orphanides (2001) we also make use of real-time data (in contrast to ex-post revised data), namely real-time Czech National Bank's (CNB) output gap and inflation forecasts to estimate the monetary policy rules.

The paper is organized as follows. Section 2 discusses various methodologies to measure equilibrium interest rates. Section 3 describes our data and empirical methodology. Section 4 gives the results on the estimation of intercept-invariant monetary policy rules as well as time-varying estimates of equilibrium interest rates. Section 5 concludes. Appendix with additional results follows.

[1] See Coats et al. (2003) and Kotlan and Navratil (2003) on Czech monetary policy.

2. Methods for Equilibrium Interest Rate Estimation

Generally, there are several methods to estimate the equilibrium interest rates (see e.g. Giammarioli and Valla, 2004, for survey and pros and cons of various methods). The simplest is to assume that the equilibrium is captured reasonably well by the trend. The trend can be estimated by some univariate filter such as HP filter. Nevertheless, number of papers document that the estimates based on these filters can be often misleading (Clark and Kozicki, 2005). In general, the limitations of the univariate methods have been also pointed out by many authors (e.g. Canova, 1998).

Another method to derive equilibrium interest rates is based on estimation of simple monetary policy rule of central bank (Taylor, 1993). The reaction function typically associates short-term interest rates to its lagged value, a difference between inflation (forecast) and its target, and output gap. The intercept of the estimated reaction function can be interpreted as the equilibrium interest rate (this is, the interest rate that would prevail when inflation is at the targeted value and output on its potential). This method has been applied to estimate the equilibrium interest rates by e.g. Clarida et al. (1998, 2000) and Orphanides (2001) for the United States and Germany, Adam et al. (2005) for the United Kingdom and Gerdesmeier and Roffia (2004, 2005) for the Euro area. Nevertheless, the assumption of constant equilibrium interest rates is often found too restrictive. It is possible to model the equilibrium interest rate as time-varying within this approach using Kalman filter (see Plantier and Scrimgeour, 2002, and Elkhoury, 2006). Typically, these studies find rationale to model the monetary policy rule as time-varying, given that the equilibrium interest rate sometimes fluctuates considerably over longer time horizons. Generally, the monetary policy rules approach measures the behavior of central bank and assumes that central bank measures equilibrium interest rates

correctly. In case of central bank's systematic mis-measurement of equilibrium rates, it is likely that equilibrium rates retrieved from the estimation of reaction function are mis-measured as well.

Structural time series models represent another common method to measure equilibrium interest rates as well. The primary contribution in this area is Laubach and Williams (2003), who formulate a simple empirical model containing IS curve, Phillips curve and an equation linking equilibrium interest rate to trend growth and model equilibrium interest rates and potential output as unobserved components. Their method has gained popularity recently and has been applied by Manrique and Marques (2004) for the U.S. and Germany, Mesonnier and Renne (2004) for the Euro area and Wintr, Guarda and Rouabah (2005) for the Euro area[2] and Luxembourg as well. In principle, the joint estimation of equilibrium interest rates and output gap is an advantage of this approach; however it also reduces the degrees of freedom, which may be an issue for transition countries with rather short time series.

Equilibrium interest rates can also be estimated within stochastic dynamic general equilibrium models. The advantage of this type of literature is that it fully specifies the structure of economy and allows an identification of variety of shocks hitting the economy. On the other hand, Levin et al. (1999) find that more complex models are less robust to model uncertainty (see also Giammarioli and Valla, 2004). These model outcomes may be highly sensitive to model assumptions. The recent examples of this approach to estimate equilibrium interest rates include Giammarioli and Valla (2003), Neiss and Nelson (2003) and Smets and Wouters (2003).

[2] See Crespo-Cuaresma et al. (2004) on related estimates on Euro area using somewhat different methodology.

The last major stream of literature estimates equilibrium interest rates from the yield curve and asset pricing models. Bomfim (2001) uses inflation linked bonds in order to eliminate the distortions from inflation expectations and retrieves equilibrium interest rates from the realized yields on U.S. Treasury inflation-indexed securities. In this regard, Giammarioli and Valla (2004) discuss equilibrium interest rate estimates in relation to consumption capital asset pricing models. In general, this stream of literature hinges on a notion of liquid financial markets and thus this approach is viable especially for countries with developed financial markets.

3. Data and Empirical Methodology

In this part, we discuss the methodology we employ to evaluate the equilibrium interest rate in the Czech Republic. We estimate a variety of backward or forward looking monetary policy rules with or without time-varying equilibrium interest rate.

3.1 Monetary Policy Rules

We follow Clarida, Gali and Gertler (1998, 2000) assuming that central bank sets nominal interest rate in line with the state of economy. This is reflected in the equation (1):

$$r_t^* = \bar{r} + \alpha\left(E\left\{\pi_{t+i} \middle| \Omega_t\right\} - \pi_{t+i}^*\right) + \beta E\left\{x_t \middle| \Omega_t\right\} \qquad (1)$$

r_t^* denotes the targeted interest rate, $\bar{r}$ is the equilibrium nominal interest rate, π_{t+i} stands for the forecast of yearly inflation rate *i* periods ahead (hereinafter, we set *i* either equal to 12 months to reflect the CNB's actual targeting horizon[3] or to 0, i.e. using current inflation for sensitivity analysis), π_{t+i}^* is the central bank's inflation target. x_t represents a measure of

[3] This in line with the CNB main forecasting model – Quarterly Prediction Model, see Coats *et al.*, 2003. In general, see also Batini and Nelson, 1999, for contributions on optimal targeting horizon.

output gap. As usual, E is the expectation operator and Ω_t is the information set available at the time when interest rates are set. Therefore, the equation (1) links targeted nominal interest rates to a constant (i.e. interest rate that would prevail, when expected inflation is at the target and output gap is null), the deviation of expected inflation from target and the output gap.

Nevertheless, the equation (1) tends to be too restrictive, as it does not account for important empirical regularity - interest rate smoothing of central banks. Typically, central bank adjusts the interest rate sluggishly to the targeted value. This is so for a number of reasons such as the concerns over the stability of financial markets (Goodfriend, 1991) or uncertainty about the effects of interest rate changes on the economy (Sack, 1997). Instead of explicit listing of various factors behind the interest rate smoothing, it is assumed for simplicity that actual policy interest rates are a combination of its lagged value and the targeted policy rate as in the equation (2).

$$r_t = \rho r_{t-1} + (1-\rho) r_t^* + \nu_t \qquad (2).^4$$

In line with Clarida *et al.* (1998), inserting (2) into (1) and eliminating unobserved forecast variables results in the equation (3):

$$r_t = (1-\rho)\left[\bar{r} + \alpha\left(\pi_{t+i} - \pi_{t+i}^*\right) + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t \qquad (3)$$

Note that ε_t is a combination of forecast errors and is thus orthogonal to all information available in time t (Ω_t). $\bar{r}$ can be interpreted as the estimate of nominal equilibrium interest

[4] We have estimated the monetary policy rules including higher lags of interest rates, but failed to find it significant.

rate (short term equilibrium real interest rate plus the expected inflation at $t+i$). We label $\bar{r}$ as the policy neutral rate.

Analogously, one can derive short-term equilibrium real interest rate, $\bar{rr}$. Rewriting the equation (1) in the real terms and eliminating unobserved forecast variables results in the equation (4):

$$r_t = (1-\rho)\left[\bar{rr} + \alpha\pi_{t+i} + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t \qquad (4)$$

It is common in literature (see for example Gerdesmeier and Roffia, 2004) to include additional economic variables, trying to capture the state of economy more precisely, in the equations 3-4.[5] We follow this stream of literature merely to address the sensitivity of our results and examine if the CNB reacts systematically to the exchange rate and monetary developments as well. Our prior is that inflation targeting central bank pays rather little attention to money. However, in case of exchange rate fluctuations, the picture is less clear-cut. While the CNB has stated several times that it does not directly react to exchange rate fluctuations, it acknowledged that exchange rate plays important role for inflation developments in small, open economies. As a result, CNB may indirectly react to exchange rate fluctuations, if they jeopardize the inflation developments (Kotlan and Navratil, 2005).

Originally, the design of Taylor rule lacked forward-looking element characteristic for the modern monetary policy conduct. Additional way to address the sensitivity of our results is

[5] This is typically done in rather *ad hoc* manner. Although Kirsonova *et al.* (2006) show that under some conditions exchange rate should enter the central bank reaction function along with inflation and output gap.

estimation of both backward- and forward-looking Taylor-type rules. Another important point is about timeliness of information in the monetary policy conduct (Orphanides, 2001). Output data are typically revised at later stage, but monetary policy is conducted based on information available at the time. Therefore, we collect real-time based CNB output gap estimates (note that inflation is not revised at later stage). In addition, we also use one year ahead CNB's real-time inflation forecasts in estimating monetary policy rule.

There are further modeling issues stemming from the fact that policy interest rates are not changed in a continuous fashion. For instance, the CNB Bank Board meets on a monthly basis to discuss the policy interest rate settings. Besides, the policy rate change itself is not continuous. Typically, if the rates are changed, the respective magnitude is 0.25 percentage points (or eventually multiple of 0.25), despite the change that maximizes economic stability according to model-based forecast might be of different magnitude. In consequence, policy rate is censored. In this case, Podpiera (2006) claims that linear estimators overstate monetary policy inertia. This is so, as the estimates capture not only the true extent of interest rate smoothing, but also the aforementioned censoring.

Given the inherent censoring of policy interest rates, many authors such as Clarida et al. (1998, 2000) or Adam *et al.* (2005) rely on using short-term interbank rates as the approximation of the censored policy rate. If the central bank monetary policy actions were largely predictable, short-term interbank rate is likely to serve as a useful approximation of the policy rate. In the case of the CNB, Kotlan and Navratil (2005) find that about 75% of monetary policy decisions have been priced in by the market participants in 2000-2004. Similarly, Perez-Quiros and Sicilia (2002) argue that market has predicted the monetary policy decisions of the European Central Bank and the Federal Reserve relatively well. In

consequence, we opt for using short-term interbank rate in estimation of the monetary policy rules.

Next, we also estimate time-varying short-term real equilibrium interest rate and neutral policy rate. We apply structural time-varying coefficient model with endogenous regressors. Kim (2006) shows that standard time-varying coefficient model delivers inconsistent estimates, when explanatory variables are correlated with the error term (as we find in our case). Then he derives a consistent estimator and labels it as augmented Kalman filter. In line with Kim (2006), we estimate the following empirical model:

$$r_t = (1-\rho)\left[\bar{r}_t + \alpha\left(\pi_{t+i} - \pi^*_{t+i}\right) + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t \qquad (7)$$

$$\bar{r}_t = \bar{r}_{t-1} + \vartheta_t \qquad (8)$$

$$\pi_{t+i} = Z'_{t-j}\xi + \sigma_\varphi \varphi_t \ , \quad \varphi_t \sim i.i.d.N(0,1) \qquad (9)$$

$$x_t = Z'_{t-j}\psi + \sigma_v v_t \ , \ v_t \sim i.i.d.N(0,1) \qquad (10)$$

The measurement equation (7) is Taylor rule for policy neutral rate as outlined above (analogously for short-term real equilibrium interest rate). However, we relax here the assumption of constant policy neutral rate and let it vary over time, as specified in the transition equation (8). We assume that $\bar{r}_t$ follows random walk without drift.[6] The "first-stage" equations (9) and (10) lay out the relationship between endogenous regressors (π_{t+i} and x_t) and its instruments, Z_t. The list of instruments, Z_{t-j}, is as follows: π_{t-1}, π_{t-2}, x_{t-1},

[6] We also experimented with AR(1) structure in the equation (8), but this only reduced the likelihood and the estimated AR parameter has been close to one, anyway.

x_{t-2} and r_{t-1}.[7] We assume that the parameters in the equations (9) and (10) are time-invariant. It is also assumed that the correlation between the standardized residuals φ_t and ν_t with ε_t is constant at $\kappa_{\varphi,\varepsilon}$ and $\kappa_{\nu,\varepsilon}$, respectively (note that σ_φ and σ_ν are standard errors of φ_t and ν_t, respectively). The consistent estimates of coefficients in the equation (7) are then obtained in two steps. First, we estimate the equations (9) and (10) and save the standardized residuals φ_t and ν_t. Second, we estimate the equation (11) along with the equation (8) using maximum likelihood via the Kalman filter. Note that (11) now includes additional terms, (standardized) residuals from (9) and (10), to address the aforementioned endogeneity of regressors. Consequently, the estimated parameters are consistent.

$$r_t = (1-\rho)\left[\bar{r}_t + \alpha\left(\pi_{t+i} - \pi^*_{t+i}\right) + \beta x_t\right] + \rho r_{t-1} + \kappa_{\nu,\varepsilon}\sigma_{\varepsilon,t}\nu_t + \kappa_{\varphi,\varepsilon}\sigma_{\varepsilon,t}\varphi_t + \iota_t \quad (11)$$

3.2 Data

Our sample contains monthly data over the period 2001:1-2005:12 on yearly CPI inflation, yearly net inflation, output gap (difference between actual and potential GDP growth), short-term interbank rate (3M PRIBOR), end of month CZK/EUR exchange rate, nominal effective exchange rate, and monetary aggregate M2. We also use the real-time CNB internal forecasts of CPI and net inflation and output gap. We have three different estimates of output gap: a) estimate using HP filter, b) ex-post revised output gap from CNB's QPM (main forecasting model) and c) real-time based output gap from CNB's QPM. All these variables are available on the monthly basis, except the output gap. Following Adam *et al.* (2005), we linearly

[7] Note that for the GMM estimation the list of instruments includes the instruments as for the time-varying coefficient model and additionally, also higher lags of explanatory variables, when found significant in the first-stage regressions.

interpolate quarterly estimates of output gap to monthly values. We use the mid-points of CNB inflation target. The choice of 2001-2005 period is motivated to have as long sample period as possible, while not rejecting stationarity of all variables at 5% significance level (using KPSS test). As a robustness check, we also estimate the monetary policy rules with net inflation (regulated prices are excluded from the consumer basket) instead of CPI inflation.

4. Results

This section contains the estimates of monetary policy rules and time-varying equilibrium interest rates. We first report the GMM estimates of monetary policy rules. Estimation of monetary policy rules gives us the estimate of equilibrium interest rates. Depending on the specification of rules, we receive either the estimate of short-term real equilibrium interest rate, or policy neutral rate. Next, we relax the restriction of constant equilibrium interest rates and present the estimates of time-varying equilibrium interest rates. This is done using time-varying parameter models estimated by augmented Kalman filter. Augmented Kalman filter is used in order to address the endogeneity of explanatory variables (Kim, 2006). Appendix presents some additional results and charts.

4.1 GMM Results

As a prelude to the GMM estimation, we test the stationarity of the variables and do not reject the stationarity of the variables at 5% level using KPSS test (see Table A.1 in the Appendix). In this context, we also find OLS estimates inconsistent based on the results of Hausman test.

The GMM estimates of backward looking monetary policy rules based on various specifications are reported in Table 1. The results suggest that 3 months real equilibrium interest rate is around 1%, while policy neutral rate lies somewhere between 2-3%. Notably,

there is evidence for substantial interest rate smoothing (note that we use monthly data, the value of smoothing parameter corresponding to the quarterly frequency is around 0.75). As expected, greater inflation tends to increase the nominal interest rates. We use three alternative estimates of output gap: HP filter, real-time and ex-post gap from the CNB's QPM (see Chart B.1 in the Appendix), but fail to find them significant. Interestingly, the parameters are more precisely estimated and monetary policy inertia is somewhat smaller, when using real-time data.

Table 1 – Backward looking Policy Rules

$$r_t = (1-\rho)\left[\bar{rr} + \alpha\pi_t + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t$$

$\bar{rr}$	ρ	α	β	Sargan test	Notes:
1.1**	0.92***	0.59**	0.48	0.12	GDP HP-filtered gap, CPI
(0.48)	(0.02)	(0.21)	(0.45)	(0.73)	inflation
1.22	0.90***	0.35	-0.24	2.28	CNB GDP gap (ex-post), CPI
(1.12)	(0.02)	(0.28)	(0.48)	(0.52)	inflation
			Real-time estimates		
0.42	0.87***	0.63***	-0.59*	2.04	CNB GDP gap (real-time), CPI
(0.62)	(0.02)	(0.14)	(0.3)	(0.56)	inflation

$$r_t = (1-\rho)\left[\bar{r} + \alpha(\pi_t - \pi_t^*) + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t$$

$\bar{r}$	ρ	α	β	Sargan test	Notes:
3.27***	0.95***	0.69**	0.88	0.3	GDP HP-filtered gap, CPI
(0.67)	(0.02)	(0.33)	(0.73)	(0.58)	inflation
2.4***	0.91***	0.32	-0.22	2.04	CNB GDP gap (ex-post), CPI
(0.58)	(0.02)	(0.32)	(0.6)	(0.56)	inflation
			Real-time estimates		
2.02***	0.89***	0.65*	-1.03	2.63	CNB GDP gap (real-time), CPI
(0.53)	(0.03)	(0.34)	(0.68)	(0.11)	inflation

Note: $\bar{rr}$ stands for the short-term equilibrium real interest rate, $\bar{r}$ for the policy neutral rate. Inflation is not revised ex-post. Therefore, inflation series is identical for all specifications in this Table. HAC robust standard errors in brackets. Sargan test is test for overidentifying restrictions (p-value in the brackets). ***, **, and * - denotes significance at 1 percent, 5 percent, and 10 percent, respectively.

Next, we evaluate the equilibrium determinacy ("equilibrium with stable prices") of the estimated monetary policy rules. This is important in order to assure that economy is not subject to sunspot equilibria, when there is infinite number of paths for the real variables leading back to equilibrium. In this context, Woodford (2003) in a basic model for analysis of monetary policy effects shows that in case of backward-looking Taylor rule with interest rate smooting it is necessary for the determinacy of equilibrium that $\alpha \succ 1-\rho-(1-\psi/q)\beta$, where α, β, ρ are defined as above, ψ is discount factor and q is a parameter linking output gap to inflation in the standard New Keynesian Phillips curve. Note that it is assumed that $\alpha, \beta, \rho \geq 0$. The results in Table 1 indicate that α is around 0.6 and ρ is around 0.9. Assuming realistically that ψ is close to 1 and $q = 0.5$ (see Coats *et al.*, 2003 for a calibration of Phillips curve for the Czech Republic), the inequality above is fulfilled, when $\beta \prec 0.5$, which is typically the case in Table 1. Although the results must be interpreted with caution, CNB monetary policy does not seem to lead to equilibrium indeterminacy, where fluctuations of output and inflation would result from self-fulfilling expectations.

Table 2 reports the forward looking estimates of monetary policy rules. The results suggest that the real equilibrium interest rate, $\bar{rr}$, is slightly above 1% albeit the standard errors of the estimates are rather large. Policy neutral rate, $\bar{r}$, is estimated to be around 2.5%. Generally, there are greater margins of uncertainty in the estimates presented in Table 2, as compared to those backward-looking presented in Table 1.

Table 2 – Forward looking Policy Rules

$r_t = (1-\rho)\left[\bar{rr} + \alpha\pi_{t+12} + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t$					
$\bar{rr}$	ρ	α	β	Sargan test	Notes:
1.29	0.94***	0.21	0.99	3.42	GDP HP-filtered gap, CPI
(1.57)	(0.03)	(0.5)	(1.16)	(0.84)	inflation forecast (real-time)
1.19	0.92	0.37	0.21	2.22	CNB GDP gap (ex-post), CPI
(1.42)	(0.03)	(0.38)	(0.31)	(0.53)	inflation forecast (real-time)
			Real-time estimates		
1.2	0.9***	0.45	0.31	3.84*	CNB GDP gap (real-time), CNB
(1.29)	(0.02)	(0.35)	(0.29)	(0.05)	CPI inflation forecast (real-time)
$r_t = (1-\rho)\left[\bar{r} + \alpha\left(\pi_{t+12} - \pi^*_{t+12}\right) + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t$					
$\bar{r}$	ρ	α	β	Sargan test	Notes:
2.54***	0.92***	0.85	0.00	4.8*	CNB GDP HP-filtered gap, CPI
(0.45)	(0.03)	(0.61)	(0.66)	(0.09)	inflation forecast (real-time)
2.43***	0.92***	0.58	0.08	2.16	CNB GDP gap (ex-post), CPI
(0.7)	(0.03)	(0.72)	(0.39)	(0.54)	inflation forecast (real-time)
			Real-time estimates		
2.43***	0.93***	0.62*	0.05	2.1	CNB GDP gap (real-time), CPI
(0.77)	(0.03)	(0.21)	(0.41)	(0.35)	inflation forecast (real-time)

Note: See Table 1 for mnemonics.

Similarly to aforementioned backward-looking Taylor rule, Woodford (2003) demonstrates that for forward-looking Taylor rule with interest rate smoothing it is necessary for the determinacy of equilibrium that both $\alpha \prec 1 + \rho + ([1+\psi]/q)(\beta + 8\sigma^{-1}(1+\rho))$ and $\alpha \succ 1 - \rho - \beta(1-\psi)/q$ are satisfied. $\alpha, \beta, \rho, \psi, q$ are defined as above, and σ is intertemporal elasticity of substitution of aggregate expenditures. Regarding the former inequality note that $([1+\psi]/q)(\beta + 8\sigma^{-1}(1+\rho))$ is expected to be positive (if $\beta \geq 0$, it is always positive) and thus it is enough, if $\alpha \prec 1 + \rho$, which is the case for all estimated specifications reported in Table 2. Analogously for the latter inequality, it is vital to note that

if $\beta \geq 0$, then the equilibrium is determinate, if $\alpha \succ 1-\rho$, which is again the case for all the specifications reported in Table 2.[8]

4.2 Time-varying equilibrium interest rates

In this section we estimate time-varying short-term equilibrium interest rate and the policy neutral rate. This is done using time-varying parameter models estimated by the augmented Kalman filter.

Generally, we find that the equilibrium interest rates decreases over time. This is in line with Lipschitz *et al*. (2006) noting that real equilibrium interest rate should be initially at high levels in transition countries, as capital accumulation was typically low and thus marginal product of capital high. Consequently, as capital accumulates over the course of transition, real equilibrium interest rates should decrease. In addition, a decrease in risk premium inevitably provides a downward pressure on the real equilibrium rates as well (Archibald and Hunter, 2001). This corresponds to Beneš and N'Diaye (2004), who find that risk premium decreased during transition of Czech economy. The respective decrease of policy neutral rate also shows successful disinflation of Czech economy and well-anchored inflation expectations (see chart B.2 in the Appendix on inflation developments in 2001-2005). This is confirmed by variety of specifications presented in Charts 1-4. Generally, the results suggest that short-term real equilibrium interest rates gradually decreased from some 3-4% to the

[8] It is worth emphasizing that we also estimate the monetary policy rules including CZK/EUR exchange rate, nominal effective exchange rate and monetary aggregate M2. Money is never significant in any specifications, while exchange rate and effective exchange rate is significant in some specifications. The results using net inflation are generally quite close to those of CPI inflation. This is not surprising, as the correlation of net and CPI inflation is 0.94 in our sample. These results are available on request.

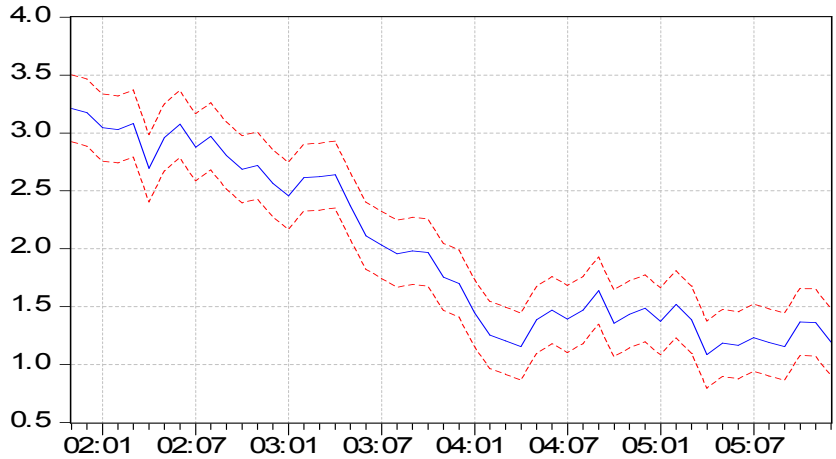
values slightly above 1%. These numbers are thus quite close to those estimated for the euro area. Messonier and Renne (2004) estimate euro area real equilibrium interest rates around 1% at the end of their sample (i.e. year 2002) and Wintr *et al.* (2005) find it a bit below 1% in 2004. Policy neutral rate is estimated to be around 5% in 2001 converging to some 2.5% at the end of 2005.

Interestingly, the difference between nominal and real equilibrium interest rates, which can be interpreted as the measure of inflation expectations is around 2% suggesting that the expectations were typically below CNB's inflation target (3% y-o-y inflation rate) in our sample period. We can also see that equilibrium interest rates fluctuate substantially. This should not come as surprise, as we focus on equilibrium interest rates at very short-term horizon.

The results support the usefulness of applying augmented version of Kalman filter. The auxiliary variables, as in (11), are typically significant and the log likelihood increases after their inclusion. Comparing the estimated equilibrium interest rates based on augmented Kalman filter with standard Kalman filter, we find that the resulting difference varies and is generally between 0.1-0.2 p.p. However, for certain observations in our sample the difference reaches values as high as 0.6-0.8 p.p.

Chart 1 –Time-Varying Short-Term Real Equilibrium Interest Rate, Backward Looking

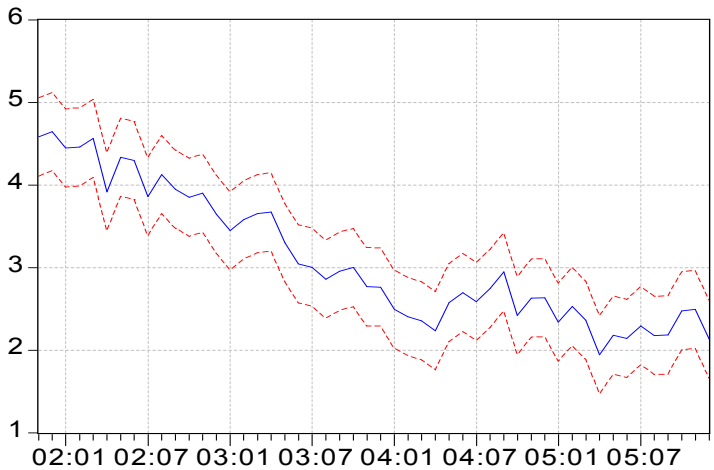
$$r_t = (1-\rho)\left[\bar{rr}_t + \alpha\pi_t + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t,\ \bar{rr}_t = \bar{rr}_{t-1} + v_t$$



Note: Smoothed parameter estimate based on augmented Kalman filter and ±2 s.e. Output gap estimate, x_t, based on HP filter.

Chart 2 – Real-Time Time-Varying Policy Neutral Rate, Backward Looking

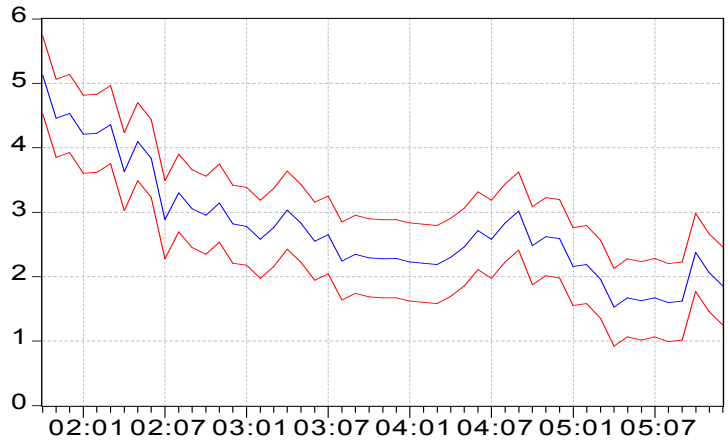
$$r_t = (1-\rho)\left[\bar{r}_t + \alpha\pi_t + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t,\ \bar{r}_t = \bar{r}_{t-1} + v_t$$



Note: Smoothed parameter estimate based on augmented Kalman filter and ±2 s.e. Output gap estimate, x_t, based on HP filter.

Chart 3 – Real-Time Time-Varying Short-Term Real Equilibrium Interest Rate, Forward Looking

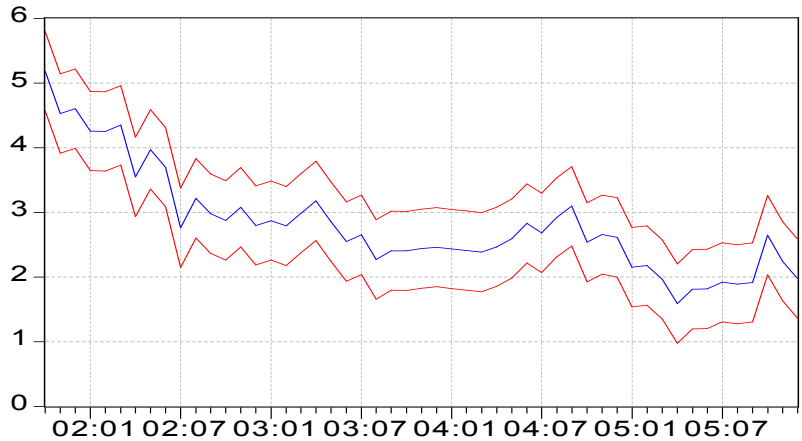
$$r_t = (1-\rho)\left[\bar{rr}_t + \alpha\pi_{t+12} + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t\,,\ \bar{rr}_t = \bar{rr}_{t-1} + \nu_t$$



Note: Smoothed parameter estimate based on augmented Kalman filter and ±2 s.e. Output gap estimate, x_t, is real-time CNB's output gap.

Chart 4 – Real-Time Time-Varying Policy Neutral Rate, Forward Looking

$$r_t = (1-\rho)\left[\bar{r}_t + \alpha\pi_{t+12} + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t\,,\ \bar{r}_t = \bar{r}_{t-1} + \nu_t$$



Note: Smoothed parameter estimate based on augmented Kalman filter and ±2 s.e. Output gap estimate, x_t, is real-time CNB's output gap.

5. Conclusions

This paper analyzes the equilibrium interest rates in the Czech Republic. In order to do so, we estimate various specifications of simple monetary policy rules at the monthly frequency from 2001 to 2005. The specifications differ based on whether we include real-time or ex-post revised data, employ backward or forward-looking versions of monetary policy rules or vary the measures of output gap. Generalized method of moments is first used to retrieve the estimates of short-term equilibrium real interest rate and policy neutral rate (i.e. nominal interest rate, when inflation is at the target and output at its potential). We find that Czech National Bank primarily responds to inflation (forecast) developments and interest rate setting process is relatively inertial. Parameters are more precisely estimated and monetary policy inertia is somewhat smaller, when using real-time data as compared to revised data. The estimated parameters seem to support equilibrium determinacy. Subject to various sensitivity tests, the results suggest that short-term real equilibrium interest rate and policy neutral rate lie around 1% and 2.5-3%, respectively. These numbers are quite close to the levels estimated for the euro area. Interestingly, the difference between nominal and real equilibrium interest rates, which can be interpreted as the measure of inflation expectations is around 2% suggesting that the expectations were lower that CNB's inflation target (3% y-o-y inflation rate) in our sample period.

Next, we estimate time-varying real equilibrium interest rate from the simple monetary policy rules. For this purpose, we use augmented Kalman filter developed by Kim (2006), which is robust to endogeneity of regressors. The results suggest that shot-term real equilibrium interest rates as well as policy neutral rate decrease gradually during sample period. Decrease of real equilibrium interest rate may reflect capital accumulation as well as a decrease in risk

premium, while the respective decrease of policy neutral rate also shows successful disinflation of Czech economy and well-anchored inflation expectations.

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APPENDIX

A. Stationarity tests

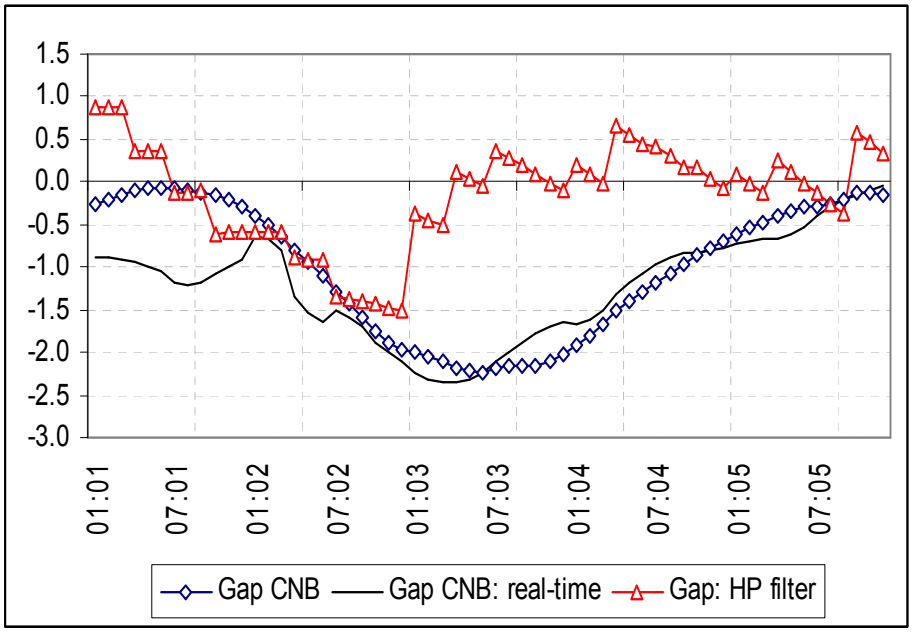
Table A.1 – KPSS Test

Series	Test statistic
PRIBOR 3M	0.359*
CPI Inflation	0.189
CPI Inflation forecast t+12	0.203
Net Inflation	0.149
Output gap – HP filtered	0.156
Output gap – Real-time based from the CNB	0.136
Output gap – Ex post from the CNB	0.136
M2 growth	0.159
Exchange rate changes	0.114

Given the size of our time series, we opt for KPSS test (Kwiatkowski et al. 1992). The null hypothesis is that the series is level stationary. Critical values for the null hypothesis: 10% - 0.347, 5% - 0.463, 1% - 0.739. Sample period: 2001:1-2005:12. * denotes significance at the 10 percent level.

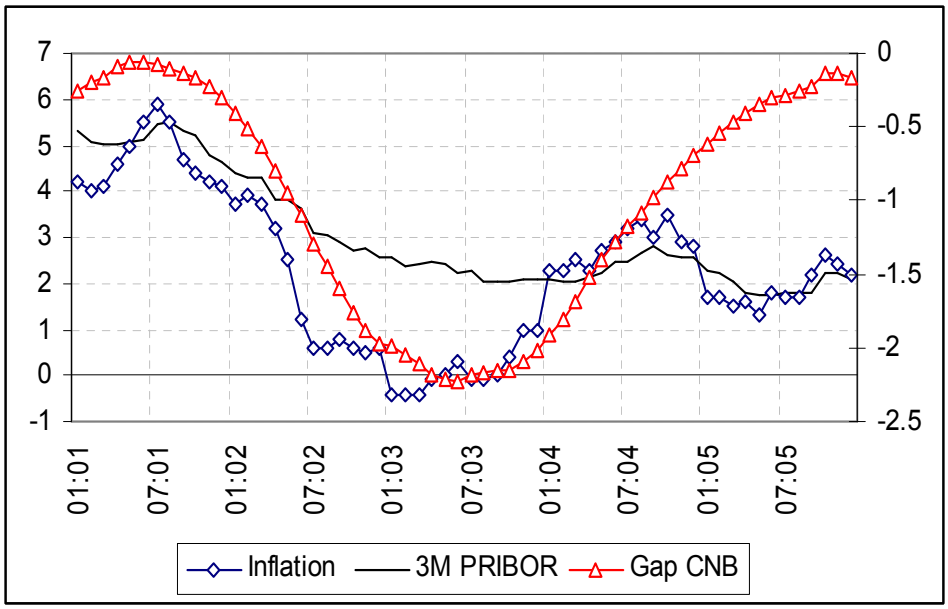
B. Additional Charts

Chart B.1 – Comparison of Output Gap Estimates



Note: This chart presents three measures of output gap used in the paper: Output gap calculated by the CNB as of their April 2006 forecast round (Gap CNB), Real-time based output gap calculated by the CNB (Gap CNB: real-time) and output gap calculated using HP filter as the estimate of potential output

Chart B.2 – Policy Rate, Output Gap and Inflation



Note: This chart presents current inflation, short-tem interbank rate (3M PRIBOR) and CNB output gap as of their April 2006 forecast round