Wage Differences, Bonus and Team Performances: A parametric non-linear integer programming model

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Abstract

We formulate a non-linear integer programming model and use plausible parameters to examine: (i) the effects of wage differences between Super- and Normal- players in the performance of four teams which participate in the UEFA CL group matches; (ii) whether the expected qualification bonus received by UEFA and paid to the players of the non-qualified teams, enhances effort and the teams manage to qualify. When performance is measured by points' maximization, higher wage equality seems to improve the performance of three teams, irrespectively if the elasticity of substitution between Super- and Normal- players is high or low, while the most efficient team of the tournament is not affected by the wage structure. The U-formed performance for that team is not excluded. When performance is measured by profits' maximization, the performance depends on both the "production" technology and on wage differences. When all teams operate under increasing returns and all pay the same, but varying relative wages, or when they operate under decreasing returns and pay the marginal value product of their players, the most "balanced" team performs better. The most "unbalanced" team performs best under increasing returns to scale and egalitarian wages. In the last case, the non-qualified teams did not manage to improve their performance and qualify, even if their players should receive the expected qualification bonus that UEFA pays.

Keywords: Players, Teams, Wages, Bonus, Performance, Tournament

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1. Introduction

Over the last decades different empirical studies have been conducted to determine whether compressed wages among a team's players have a stronger impact on the performance of the team, than the dispersed wages. Since players are heterogeneous, Lazear & Rosen (1981) argued that a team's performance increases if the best talents are paid higher wages than the normal players. Milgrom (1988) and Lazear (1989) on the other hand, stressed the possibility of bad field cooperation between players, and consequently the performance could be inferior, if the under-paid players feel discriminated. Levine (1991) took an extreme position and supported the egalitarian wages if a team wants to improve its performance. Fehr and Schmidt (1999) tried to balance these two effects and argued that, as a whole, the team losses more if wages are more unequal than equal.

Franck & Nüesch (2007) have recently reviewed the empirical studies from sport teams, in baseball, hockey, basket and football. Some of them seem to support the compressed wages hypothesis. They argue that these findings can partly be explained because most studies assume a linear relationship between wage dispersion and team performance. In their own study, based on 5281 individual salary proxies from German soccer players between 1995 and 2007, they allowed for squares of the Gini coefficient and the coefficient of variation of the wage distribution at the beginning of the season. They found a U-formed sportive success, i.e. teams do better by either a rather egalitarian pay structure or a steep one. In addition, they found evidence that teams with dispersed wages entertain better since the number of seasonal dribblings and runs increases significantly! Pokorny (2004) on the other hand, found an inverted U-performance. Torgler et al (2008), using also data from the German Bundesliga (and the NBA), found that players care more about the salary distribution within the team and not just about their own salary. In general, they prefer a reduced inequality and in that case their performance improves. In addition, a detailed investigation of the basketball data shows also that when a player moves from a relative income advantage to a relative disadvantage, his performance decreases in a statistically significant way. On the other hand, moving from relative income disadvantage to relative advantage has no effects.

The impact of a win bonus has recently been analyzed by Kesenne (2007), assuming a Cournot-Nash model. Using various parameters, he found that if the small team introduces a 10% win bonus, it hires fewer talents, but they are more efficient, leading to higher winning percentage and profits. In that case, even the demand for talents from the large team increases. The effects on winning percentage are rather robust, irrespectively if the fixed wages remain unchanged, or are reduced. Profits might decrease though, if the expected increase in effort that the bonus is assumed to lead, does not realize. A problem with the Kesenne model is the "normalization" and the interpretation of his results. For instance, while talents are defined as "units", the optimal solution is a non-integer one, such as 4/9. Moreover, by "normalizing" 4/9 to 44 as Kesenne does, is not consistent with the selected parameters. In addition, the frequently used winning percentages exclude draws and would require a large number of games to interpret them properly. Finally, predictions from duopoly models might be erroneous in sport tournaments when the number of teams is higher. It is therefore important to develop a consistent model with more teams that participate in a tournament, with integer players, victories and draws.

In this paper we formulate and solve a non-linear integer programming model to examine mainly two effects: (i) the effects of wage differences between Super (S)- and Normal (N)- players in the performance of teams, measured by (a) points collected from a tournament like the UEFA Champions League (CL) twelve group matches, in which the four participating teams play, and (b) profits; (ii) since only two teams qualify, the non-qualified teams have an incentive to qualify, by paying their players the qualification bonus they will receive from UEFA, in case of qualification. Which teams perform better, those with compressed wages or those with dispersed ones? Does the qualification bonus enhance effort of players and lead to qualification?

A key feature of this model is the different "team production" functions. For instance, in points' maximization, the S- and N-players of these teams are considered as being from almost complements (or non-substitutes) to almost substitutes. Also, in profits' maximization, teams operate under both increasing and decreasing returns to scale. In section two we formulate and solve the points' maximization model and in section three we formulate and solve the profits' maximization model.

2. The model

There are four teams, 1, 2, 3, and 4 which play home and away matches in a tournament like the group matches in the UEFA CL. As a whole there are 12 matches and the maximum number of points is 36. The two teams which collect most points are qualified for the next round and the other two are eliminated[1].

We follow Kesenne and also assume that all teams have a certain number of S-talented and N-players. All S-players are equally "Super" and all N-players are equally "Normal". Obviously, there are players who, objectively, belong to one category or the other, like AC Milan's world player of the year 2007, Kaká, Barcelona's star Messi or Manchester United's, Ronaldo. Moreover, most players are neither that excellent to be classified in the first category, or that "Normal" to be classified in the second category. In the football world, it is very common that the supporters of a team tend to overvalue their own players and undervalue the competitor's players. This is an empirical issue, left to supporters, to managers, to journalists or even to those who do research in efficiency analysis. Teams use their S- and N- players in order to (a) "produce" points, or (b) maximize profits. The supply of S- and N-players is unlimited. The wages the S- and N-players receive are exogenous to all teams or implicitly determined, based on the value of the marginal product of players. We assume that teams have no other fixed costs (like managers or other facilities); they have only variable costs, the wages they pay to their players. In reality, these teams participate in other home leagues or championships as well

[1] The third team continues in the UEFA-Cup.

and pay wages to their players according to the contracts signed. We simplify the model and concentrate only on these six matches each team play. Finally we simplify and assume that all teams receive similar revenues, either directly from UEFA, and/or from their public and sponsors.

We will formulate and solve two "Cournot" type problems: (a) all teams simultaneously maximize points, subject to non-negative profits and some other "technical" or "structural" constraints; (b) all teams maximize profits, either simultaneously, or individually. In both models we will mainly examine how teams perform when their own initial wage difference between the S- and N-players increases or decreases, or if the competitors' wage difference changes. We will also investigate whether the tournament remains balanced or turns to imbalanced, and whether teams who use more S-players collect more points than teams who use less S-players.

2.1 Teams maximize points

Let P_1, P_2, P_3, P_4, be the points collected by the teams; let S_1, S_2, S_3, S_4, be the S-players, and N_1, N_2, N_3, N_4 the N-players the teams use in these matches. Let w_1, w_2, w_3, w_4, be the victories of the teams, d_1, d_2, d_3, d_4, the draw matches and l_1, l_2, l_3, l_4, the losses (defeats) of the teams. All these 24 variables are positive integers.

Since all firms aim at maximizing points, the tournament's objective function is:

$$Max \sum_{i}^{4} P_i \qquad (1)$$

There are various kinds of "technical" or "structural" constraints valid to all teams. The key constraints that differentiate teams are the teams' "production" functions constraints. We assume that all functions are of Constant Elasticity of Substitution (CES) type of degree one, but with different elasticity. Team 1 has almost Leontief "production" function, i.e. very low elasticity of substitution between its S- and N-players[2]. Team 4 is the other extreme, with almost Cobb-Douglas "production" function, i.e. almost excellent elasticity of substitution. Team 2 is close to team 1, and team 3 close to team 4.

The demand for S- and N-players for each team will be endogenously determined from the optimal solution. When the composition of the team has been determined, the same team will be used for all six matches. Teams of course change the composition of their players for tactical reasons, for instance if they play away against a stronger team or if they play at home against a weaker team, or because some of their players might be injured or punished and are not available for a particular match. The model neglects such possibilities. Moreover, since the roster of teams consists of more than 11 players, teams might change their starting eleven, if some key parameters, like the "price" per point collected, or the wage costs of players, change. Notice that, due to the integer constraint of players, the composition of the team can remain unchanged, even if the players' wages change. The four teams "production" functions are:

$$\begin{aligned}
P_1 &= t_1\left(\alpha_1 S_1^{-100} + (1-\alpha_1) N_1^{-100}\right)^{-\frac{1}{100}} \\
P_2 &= t_2\left(\alpha_2 S_2^{-10} + (1-\alpha_2) N_2^{-10}\right)^{-\frac{1}{10}} \\
P_3 &= t_3\left(\alpha_3 S_3^{-0.5} + (1-\alpha_3) N_3^{-0.5}\right)^{-\frac{1}{0.5}} \\
P_4 &= t_4\left(\alpha_4 S_4^{-.1} + (1-\alpha_4) N_4^{-.1}\right)^{-\frac{1}{.1}}
\end{aligned} \qquad (2) - (5)$$

t_i is the efficiency parameter and a_i is the distribution parameter, with i =1, 2, 3, 4, assuming the following bounds: $t_i > 0$ and $0 < a_i < 1$.

The next constraint is the number of players each team uses in its matches. Thus, given some key parameters, they might use 9 N- and 2 S-players, while for other

[2] In reality, an S-player can substitute an N-player more easily than the reverse. We assume that the elasticity of substitution is unchanged, irrespectively if the S- substitutes the N-player, or if the N-substitutes the S-player.

parameters they might use 7 N- and 4 S-players. Since the only costs are the players' wages, the wage bill to non-used players is zero.

$S_i + N_i = 11$ (6) – (9)

In reality, even for top and rich teams, the number of S- seems to lower than the number of N-players. Despite the fact, that such condition is not necessary, we state it explicitly in order to speed up the solution of this complex model.

$S_i \leq N_i$ (10) - (13)

As is well known, victories are worth 3 points draws are worth only 1 point and defeats, zero points. Thus, the constraints are:

$P_i = 3w_i + d_i$ (14) – (17)

Each one of the 12 matches in the group ends either with a home team victory (*wh*), with an away team victory (*wa*), or with a home team draw (*dh*), and in the last case with the away team draw (*da*) as well. But, in order to identify the correct pair of teams which play draw at home (*dh*) and/or draw away (*da*), we can separate the home team draw from the away team draw, i.e. we need 24 additional constraints. The first 12 constraints below relate (*wh*) and (*wa*) to (*dh*) and the remaining 12 relate (*wh*) and (*wa*) to (*da*). For instance, $wh12 = 1$, means that team 1 wins at home against team 2, and $dh13 = 1$, means that team 1 plays draw at home against team 3.

If $dh12 = 1$ (and consequently $wh12 = wa21 = 0$), from constraints (18) and (30) we are ensured that $da21 = 1$ as well. If that match ends with a home or away victory, it implies that $dh12 = da21 = 0$. Moreover, these constraints do not exclude impossible (non-binary) match results, such as $dh12 = da21 = 0.5$ and $wh12 = wa21 = 0.25$. Therefore we require that all these variables are binary, as given in the following constraints.

$1 = (wh12+wa21)+ (dh12)$

$1 = (wh21+wa12)+ (dh21)$

$1 = (wh13+wa31)+ (dh13)$

$1 = (wh31+wa13)+ (dh31)$

$1 = (wh14+wa41)+ (dh14)$

$1 = (wh41+wa14)+ (dh41)$ (18) – (29)

$1 = (wh23+wa32)+ (dh23)$

$1 = (wh32+wa23)+ (dh32)$

$1 = (wh24+wa42)+ (dh24)$

$1 = (wh42+wa24)+ (dh42)$

$1 = (wh34+wa43)+ (dh34)$

$1 = (wh43+wa34)+ (dh43)$

$1 = (wh12+wa21)+ (da21)$

$1 = (wh21+wa12)+ (da12)$

$1 = (wh13+wa31)+ (da31)$

$1 = (wh31+wa13)+ (da13)$

$1 = (wh14+wa41)+ (da41)$

$1 = (wh41+wa14)+ (da14)$ (30)– (41)

$1 = (wh23+wa32)+ (da32)$

$1 = (wh32+wa23)+ (da23)$

$1 = (wh24+wa42)+ (da42)$

$1 = (wh42+wa24)+ (da24)$

$1 = (wh34+wa43)+ (da43)$

$1 = (wh43+wa34)+ (da34)$

Obviously, the draws (and the victories) for each firm is the sum of all possible draws (and victories) against all other teams. Constraints (18)-(41) above, together with constraints (42)-(45) below ensure that the number of draws in constraints is always an integer (even) number, or zero. For instance, when team 1 plays only one match draw, it is $d_1=1$. If the draw match was a home match against team 2, it must be $dh12 = 1$, and $da21 = 1$, as well. In that case, if team 2 does not play another match draw, it is $d_2=1$ as well. Of course, if team 2 plays three draw matches, $d_2=3$, implying that it must have played draw against other teams.

$d_1=dh12+da12+dh13+da13+dh14+da14$

$d_2=dh21+dh23+dh24+da21+da23+da24$

$d_3=dh31+dh32+dh34+da31+da32+da34$ (42) – (45)

$d_4=dh41+dh42+dh43+da41+da42+da43$

A similar interpretation is for the victory constraints (46)-(49).

$w_1=wh12+wa12+wh13+wa13+wh14+wa14$

$w_2=wh21+wh23+wh24+wa21+wa23+wa24$

$w_3=wh31+wh32+wh34+wa31+wa32+wa34$ (46) – (49)

$w_4=wh41+wh42+wh43+wa41+wa42+wa43$

Since each team play six matches, there are 6 possible results from its games.

$$w_i+d_i+l_i=6 \qquad (50) – (53)$$

No team can collect more than 18 points. That is simply:

$$P_i \leq 18 \qquad (54) – (57)$$

In addition, the maximum number of points from all matches is 36, composed only with 12 victories (or 12 defeats), without any draws.

$$\sum_{i}^{4} P_i \leq 36 \qquad (58)$$

Finally, we need the non-negative profits constraints. One can assume a simple (linear) or a non-linear revenue function. We assume a quadratic revenue function in the points collected, such as $R_i = 12P_i - 0.5P_i^2$. This function has revenue maximum at $P = 12$, which is high enough collected points for qualification to the next round.

As we mentioned above, the only costs are the players' wages. We assume that all teams pay the same, higher wages to their S-players, and the same, lower wages to their N-players. Initially, we assume the following wages: $w^i_S = 8$ and $w^i_N = 4.8$, i.e. the N-players receive 60% of the S-players' wages. Thereafter, we increase and decrease the w^i_N to investigate the wage dispersion on the own and other teams' performance and on own and other teams' starting team (demand for S- and N-players). According to the Italian Gazzeta, (http://www.gazzeta.it, calcio/11 September 2007), three of the participating teams in the UEFA CL, AC Milan, Inter and Roma paid the following annual wages: AC Milan, an average of €2.86 million, varying from €6 (Kaká) to €1 million; Inter, an average of €2.7 million, varying from €5 (Ibrahimovic, Adriano, Vieira) to €0.3 million; and Roma, an average of €1.27 million, varying from €5.46 (Totti) to €0.15 million.

Given the revenue and costs parameters, the non-negative profits constraints for all teams are satisfied if each team collects at least 6 points. With less than 6 points there is no integer value of N- and S-players to ensure non-negative profits. Thus, the non-negative profits constraints are:

$$\pi_i = 12P_i - 0.5P_i^2 - 8w_S^i - 4.8w_N^i \geq 0 \qquad (59) – (62)$$

As mentioned earlier, all P_i, S_i, N_i, d_i and w_i are positive integers; all 12 dh_{ij}, da_{ij}, wh_{ij} and wa_{ij} are binaries, and finally all $t_i > 0$ and $0 < a_i < 1$.

The model is now complete. We solved the problem in Lingo, using Global Solver.

2.1.1 The Solution

The initial solution, with $w^i_S = 8$ and $w^i_N = 4.8$, shows that the tournament is completely balanced, because each team wins its three home matches and collects 9 points. Despite the differences in their "production" functions, all four teams have a similar team composition, consisting of 1 S-player and 10 N-players and each team makes a profit of 11.5.

We solved the model 124 times, i.e. using 31 different N-wages per team, assuming N-wages starting from 3.4 and increasing up to 6.4, at a range of 0.1. When each team at a time changed its N-wages, we assumed that all other teams keep their own S- and N-wages unchanged. Notice that N-wages above 6.4 lead to non-feasible solution, because the non-negative profits constraint(s) are violated.

In Table 1, we summarize the regression estimates of both a linear model, $P = \alpha + \beta(8 - w_N)$ and a quadratic one, $P = \alpha + \beta_1(8 - w_N) + \beta_2(8 - w_N)^2$ for every team, to investigate if a U-type or an inverse U-type performance exists. The right hand side variable is the wage difference between the S-wages (assumed to be fixed at 8) and the various N-wages, while the dependant variable is points collected. The estimates for every team are based on 31 observations, obtained from the 31 corresponding optimal solutions when the team under consideration changes its N-wages. Negative (positive) β-estimates imply that the performance of teams improves if the wage equality (inequality) between the S- and N-players increases. Similarly, negative β_1-estimates and positive β_2-estimates imply a U-type performance, i.e. either highly unequal or highly equal wages improve performance. In addition we show the average number of points and its variance the teams have collected from these 31 optimal solutions.

The estimates show that the two non-qualified teams, 2 and 4, and also the second qualified team 1 improve their performances when the wage equality increases. On the other hand, the first qualified team 3 is very robust in its collected points, irrespectively if it pays more or less egalitarian wages. Not only is its $\overline{R}^2$ almost zero, its β-estimate is not statistically significant from zero either, implying that its wage differences have no impact on its performance. Team 3 collects the highest average number of points (11.42), which is strongly different from zero, precisely as its constant α estimate (10.71) is.

Table 1: OLS estimates: $P = \alpha + \beta(8 - w_N)$ & $P = \alpha + \beta_1(8 - w_N) + \beta_2(8 - w_N)^2$

	Team 1 $\overline{P} = 8.84$ $\sigma^2 = 5.21$	**Team 2** $\overline{P} = 8.26$ $\sigma^2 = 5.87$	**Team 3** $\overline{P} = 11.42$ $\sigma^2 = 1.45$	**Team 4** $\overline{P} = 8.16$ $\sigma^2 = 4.81$
α	13.5**	15.23**	10.71**	15.04**
	(11.22)	(17.84)	(13.7)	(26.05)
β	-1.50**	-2.25**	0.23	-2.22**
	(-4.32)	(8.50)	(0.95)	(-12.62)
$\overline{R}^2$	0.337	0.704	0.00	0.841
F	16.25	72.26	0.90	159.2
α	20.95**	18.52**	16.20**	20.22**
	(5.09)	(6.12)	(6.18)	(11.3)
β_1	-6.743*	-4.56*	-3.64*	-5.87**
	(-2.41)	(-2.21)	(-2.04)	(-4.83)
β_2	0.845	0.373	0.624*	0.59**
	(1.89)	(1.13)	(2.19)	(3.03)
$\overline{R}^2$	0.391	0.707	0.112	0.876
F	10.63	37.12	2.90	106.7

** significant at 0.01 level; * significant at 0.05 level; t-statistics is in parentheses

Apart from team 3, the U-type performance for all other teams is rejected, either because the quadratic function overestimate the constant α, and/or the β_2 estimates for teams 1 and 2 are not statistically different from zero. On the other hand, despite the fact that the explanatory power of the quadratic function for team 3 is very low, the existence of a U-type performance is not rejected. Team 3 minimizes its performance if its N-wages are approximately 5.1, (i.e. if its N-players receive 63.75% of the S-wages), reaching almost 11 points (with three victories, two draws and one defeat). Moreover, that team can collect more than 12 points (i.e. one more draw instead of a defeat, or four victories and two defeats), if its N-players were paid either less than 44%, or more than 81% of what its S-players were paid.

In Table 2 we show both the own effects on performance and team composition, based on just four N-wages decreases and six N-wages increases (in bald in diagonal), and also the effects to other teams (off-diagonal). As a whole, for each team we show only 40/124 possible team compositions, points collected and profits. Some points from the Table 2 are worth mentioned.

1. As expected, team 1, with almost Leontief technology, never changes its team composition. It always uses 1 S- and 10 N-players. On the other hand, team 4, with almost Cobb-Douglas technology, does not change it more frequent than team 2 does, which is close to team 1.
2. There are both positive and negative cross effects to the performances to other teams, if another team changes its own N-wages. (i) If team 1 decreases its N-wages, teams 2 and 3 seem to be favoured, but not team 4. On the other hand, if it increases its N-wages, the effects on other teams' performance are unchanged. (ii) If team 2 reduces its N-wages, teams 3 and 4 are favoured, while team 1 is always hurt. (iii) If team 3 changes its N-wages, it has no effect on the performance of team 4, but it often deteriorates the performance of teams 1 and 2. (iv)If team 4 changes its N-wages, it improves the performance of team 3 and has no effect on teams 1 and 2. Thus, the first qualified team 3 is always benefited!

Table 2: Points maximization, subject to non-negative profits for selected N-wages; initial parameters: $w^i_N = 4.8$, $w^i_S = 8$

W_N	S^1	N^1	P^1	π^1	S^2	N^2	P^2	π^2	S^3	N^3	P^3	π^3	S^4	N^4	P^4	π^4
$W_N^1 = 4.0$	**1**	**10**	**12**	**24**	1	10	9	11	2	9	7	0.3	1	10	7	3.5
$W_N^1 = 4.5$	**1**	**10**	**6**	**1**	1	10	12	16	1	10	9	11	1	10	9	11
$W_N^1 = 4.6$	**1**	**10**	**6**	**0**	2	9	9	8.3	5	6	12	3.2	1	10	9	11.5
$W_N^1 = 4.7$	**1**	**10**	**7**	**4.5**	4	7	9	1.9	1	10	12	16	1	10	7	3.5
$W_N^1 = 4.9$	**1**	**10**	**9**	**10.5**	2	9	9	8.3	2	9	9	8.3	1	10	9	11.5
$W_N^1 = 5.0$	**1**	**10**	**9**	**9.5**	1	10	9	11.5	1	10	9	11.5	1	10	9	11.5
$W_N^1 = 5.1$	**1**	**10**	**9**	**8.5**	1	10	9	11.5	1	10	9	11.5	1	10	9	11.5
$W_N^1 = 5.2$	**1**	**10**	**9**	**7.5**	1	10	9	11.5	1	10	9	11.5	2	9	9	8.3
$W_N^1 = 5.3$	**1**	**10**	**12**	**11**	1	10	9	11.5	1	10	7	3.5	1	10	7	3.5
$W_N^1 = 5.4$	**1**	**10**	**9**	**5.5**	1	10	9	11.5	1	10	9	11.5	1	10	9	11.5
$W_N^2 = 4.0$	1	10	7	3.5	**1**	**10**	**6**	**6**	1	10	15	11.5	1	10	7	3.5
$W_N^2 = 4.5$	1	10	7	3.5	**1**	**10**	**6**	**1**	1	10	15	11.5	1	10	7	3.5
$W_N^2 = 4.6$	1	10	7	3.5	**1**	**10**	**6**	**0**	1	10	7	3.5	1	10	15	11.5
$W_N^2 = 4.7$	1	10	7	3.5	**1**	**10**	**7**	**4.5**	1	10	12	16	1	10	9	11.5
$W_N^2 = 4.9$	1	10	7	3.5	**1**	**10**	**9**	**10.5**	2	9	12	12.8	1	10	7	3.5
$W_N^2 = 5.0$	1	10	7	3.5	**1**	**10**	**7**	**1.5**	1	10	12	16	1	10	9	11.5
$W_N^2 = 5.1$	1	10	7	3.5	**1**	**10**	**7**	**0.5**	1	10	12	16	1	10	9	11.5
$W_N^2 = 5.2$	1	10	7	3.5	**1**	**10**	**12**	**12**	2	9	9	8.3	1	10	7	3.5
$W_N^2 = 5.3$	1	10	7	3.5	**1**	**10**	**12**	**11**	2	9	9	8.3	1	10	7	3.5
$W_N^2 = 5.4$	1	10	7	3.5	**1**	**10**	**12**	**10**	1	10	9	11.5	1	10	7	3.5
$W_N^3 = 4.0$	1	10	7	3.5	1	10	7	3.5	**1**	**10**	**12**	**24**	1	10	9	11.5
$W_N^3 = 4.5$	1	10	7	3.5	1	10	7	3.5	**1**	**10**	**12**	**19**	1	10	9	11.5
$W_N^3 = 4.6$	1	10	7	3.5	1	10	7	3.5	**1**	**10**	**12**	**18**	1	10	9	11.5
$W_N^3 = 4.7$	1	10	9	11.5	2	9	9	8.3	**1**	**10**	**9**	**12.5**	1	10	9	11.5
$W_N^3 = 4.9$	1	10	7	3.5	1	10	7	3.5	**1**	**10**	**12**	**15**	1	10	9	11.5
$W_N^3 = 5.0$	1	10	7	3.5	1	10	7	3.5	**1**	**10**	**12**	**14**	1	10	9	11.5
$W_N^3 = 5.1$	1	10	9	11.5	1	10	9	11.5	**1**	**10**	**9**	**8.5**	2	9	9	8.3
$W_N^3 = 5.2$	1	10	7	3.5	2	9	7	3.5	**1**	**10**	**12**	**12**	1	10	9	11.5
$W_N^3 = 5.3$	1	10	7	3.5	1	10	7	3.5	**1**	**10**	**12**	**11**	1	10	9	11.5
$W_N^3 = 5.4$	1	10	9	11.5	3	8	9	5.1	**1**	**10**	**9**	**5.5**	3	8	9	5.1
$W_N^4 = 4.0$	1	10	9	11.5	1	10	9	11.5	1	10	12	16	**1**	**10**	**6**	**6**
$W_N^4 = 4.5$	1	10	9	11.5	1	10	9	11.5	1	10	12	16	**1**	**10**	**6**	**1**
$W_N^4 = 4.6$	1	10	9	11.5	1	10	9	11.5	1	10	12	16	**1**	**10**	**6**	**0**
$W_N^4 = 4.7$	1	10	7	3.5	1	10	9	11.5	1	10	12	16	**1**	**10**	**7**	**4.5**
$W_N^4 = 4.9$	1	10	7	3.5	1	10	7	3.5	1	10	12	16	**1**	**10**	**9**	**10.5**
$W_N^4 = 5.0$	1	10	9	11.5	3	8	9	5.1	2	9	9	8.3	**4**	**7**	**9**	**0.5**
$W_N^4 = 5.1$	1	10	7	3.5	2	9	7	0.3	1	10	12	16	**2**	**9**	**9**	**5.6**
$W_N^4 = 5.2$	1	10	7	3.5	2	9	7	0.3	1	10	12	16	**2**	**9**	**9**	**4.7**
$W_N^4 = 5.3$	1	10	9	11.5	1	10	9	11.5	1	10	9	11.5	**1**	**10**	**9**	**6.5**
$W_N^4 = 5.4$	1	10	9	11.5	1	10	9	11.5	1	10	9	11.5	**1**	**10**	**9**	**5.5**

3. The most striking result is that more S-players do not necessary increase the performance of teams! For instance, in four cases (three for team 2 and one for

team 3) teams with 2 S- and 9 N-players collect only seven points and these teams are eliminated. Also, in two other cases (one for team 2 and one for team 4), teams with 4 S- and 7 N-players collect just nine points.

4. Regarding the effect of wage differences on the balance of tournament, there is no clear relationship. The tournament remains balanced in 11/40 cases, mainly if team 1 increases its N-wages up to 5.3, or if teams 3 and 4 increase their N-wages to some specific values. When team 2 changes its N-wages, the tournament is always unbalanced.
5. Zero profits appear only in very few cases.
6. In many cases, the total number of points is 35, indicating that one match ends in draw, and that the tournament is often unbalanced.

Thus, the non-negativity profits constraints, together with the integer conditions, on players, victories and draws, can lead to, positive, negative or zero effects on points (or wins and draws), and on own and other teams' demands for N- and S-players, when teams change their N-wages. The model seems to favour the performance of team 3, which has a "production" function close to team 4. Moreover, team 4, with the highest elasticity of substitution between the S- and N-players, performs worse, while team 1 with the lowest elasticity of substitution performs better.

Additional simulations are therefore required to find out if can identify the clear "winners" or "losers". For instance, a possible way to obtain higher differences in performances is to increase the "price" parameter from 12 to 15 or 18, so that the maximum performance will be even higher, when the non-negative profits constraints will be satisfied with less than 6 points. Another interesting path to follow is to allow different teams compositions in different matches, or combine the "production" functions per match, to find out if the "Leontief team" beats or is defeated by the "Cobb-Douglas team", at home or away. Finally, since our CES functions do not let teams use various tactical dispositions of players, it would be desirable to stress the field positions of the S- and N-players. That is similar in spirit to Hirotsu & Wright (2006), who applied a Nash-Cournot game to figure out the win probabilities of the 4-4-2 strategy over the 4-5-1 one.

3. Teams maximize profits

Let us now assume that teams' objective function is to maximize profits. One can choose that either all teams maximize profits simultaneously, or individually, subject to non-negative profits of all other teams.

We simplified the revenue function to be linear in the points collected. On the other hand, with regard to costs, we assumed that S- and N-players will receive the value of their marginal products[3]. We maximized profits for all teams simultaneously and for each one separately, subject to non-negative profits for the other teams. Keeping our players as integers, we failed to obtain feasible solutions. Feasible solutions exist only if players are non-integers.

In order to obtain feasible solutions with integer players, we had to simplify the profit function and/or the production and cost functions as well. Below we will present four cases. In the first three we assumed simultaneous profit maximization with different parameters in revenues and costs, while in the fourth case we assumed individual profit maximization.

3.1 Simultaneous profit maximization under increasing returns (case 1)

The objective function is linear in revenues and costs, assuming equal wages to both S- and N-players, for all teams:

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[3] The marginal products of the S- and N-player are in a CES function are: $\dfrac{\alpha t}{\left(\frac{1-\alpha}{N^{\rho}}+\frac{\alpha}{S^{\rho}}\right)^{\frac{1+\rho}{\rho}} S^{1+\rho}}$, $\dfrac{(1-\alpha)t}{\left(\frac{1-\alpha}{N^{\rho}}+\frac{\alpha}{S^{\rho}}\right)^{\frac{1+\rho}{\rho}} N^{1+\rho}}$, where, $\rho = \frac{1-\sigma}{\sigma}$, and σ is the elasticity of substitution.

$$Max = 12\sum_{1}^{4} P_i - 6\sum_{1}^{4}\left(S_i + N_i\right) \qquad (1)'$$

All "production" functions constraints are of Cobb-Douglas type with increasing returns to scale. The sum of S- and N-players' elasticity is 1.75 for all teams and the elasticity of the S-players is higher than of the N-players. The parameters are selected to ensure both integer players (around 3 to 5 S- and 8 to 6 N-players) and a balanced tournament, with approximately 9 points per team. For the same P, S and N, the S-players of the teams 1 and 2 have higher marginal products compared to S-players of teams 3 and 4, and the N-players of teams 3 and 4 have higher marginal products compared to N-players of teams 1 and 2. We define team 1 as the most "unbalanced" team, with extremely good S-players and rather poor N-players. Team 3 on the other hand is the most "balanced" team[4], because the marginal products of its S-players are not much higher than the respective ones of its N-players. Notice that positive profits are ensured, if teams collect at least 6 points, because the "price" per point is 12 and the ad-hoc unit cost per player is 6.

The "production" functions are given in (2)´-(5)´.

$$\begin{aligned} P_1 &= t_1\left(S_1^{1.25}N_1^{0.5}\right) \\ P_2 &= t_2\left(S_2^{1.15}N_2^{0.6}\right) \\ P_3 &= t_3\left(S_3^{1.05}N_3^{0.7}\right) \\ P_4 &= t_4\left(S_4^{1.1}N_4^{0.65}\right) \end{aligned} \qquad (2)' - (5)'$$

As before, t_i is the positive efficiency parameter of the teams.

All other constraints, (6) – (58), integer, binary and bounds apply.

[4] By "unbalanced" ("balanced") teams we mean high (low) differences in the marginal products (or elasticity in our Cobb-Douglas functions) of the S- and N-players and not by very unequal (almost equal) number of S- and N-players in the team composition.

The solution to these teams is: 15, 9, 6, and 6 points respectively, implying that the tournament is slightly unbalanced. All teams use 5 S- and 6 N-players, their respective efficiency estimates are 0.8190, 0.4825, 0.3159 and 0.3188, and their profits are 114, 42, 6 and 6.

In theory, the estimates are as expected. For instance, since all teams pay exactly the same wage to both types of players, all of them will use their maximum number of S-players who are more productive. As a consequence, the most "unbalanced" team 1, who has the most productive S-players, performs better. That team wins 5 out of 6 matches and obtains the highest profit as well. Thus, if the elasticity parameter of the S-players remains very high, even if they receive exactly the same wages as their N-colleagues, the team performs best. In reality though, there is a risk that the ex-ante (or expected) high elasticity of the S-players will decrease, if wages are equal. On the other hand, the S- and N-players of the "balanced" team 3, with a lower and higher elasticity respectively, are not able to help their team, even if such "egalitarian" wages are considered more fair to them than to team 1.

3.1.1 Pay the qualification bonus to players

As is known, UEFA pays to qualified teams, a qualification bonus of CHF 2.5 million, in addition to CHF 0.5 million per victory. Since teams 3 and 4 do not qualify, they have an incentive to pay a qualification bonus to enhance the effort of their players. On the other hand, the qualified teams 1 and 2 do not need to pay that bonus to their players. We will therefore formulate the qualification bonus for teams 3 and 4, separately.

Under normal conditions, if team collects about 10 points is qualified[5] and will receive from the UEFA a qualification bonus of CHF 2.5 and victory premium of at least CHF 1.5 from its victories and draws.

[5] Depending on how much balanced or unbalanced the group is, a team can qualify with 7 points (like Rangers in 2005/06, with Inter 13, Artmedia 6 and Porto 5 points, the other teams in the group) and

Let team 3 pay its players, at most, the UEFA qualification bonus ε_3 (not the victory premium) it will receive. Let q_3 be a binary variable with the following interpretation: if the team qualifies its sum of bonus and victory premium (i.e. 2.5 + 1.5 = 4) is included in its revenue part of the objective function, i.e. $q_3 = 1$. In that case, the non-qualified team 3 must pay at most $2.5/11 > \varepsilon_3$ to its 11 players. If on the other hand, the sum of bonus and victory premium is not sufficient to increase the revenues of the team and the team does not qualify, $q_3 = 0$.

Thus, the objective function changes now into:

$$Max = 12\sum_{1}^{4} P_i - 6\sum_{1}^{4}\left(S_i + N_i\right) + 4q_3 - q_3\varepsilon_3(S_3 + N_3) \qquad \text{(1a)´}$$

Notice that, if $q_3 = 0$, the product $q_3\varepsilon_3(S_3 + N_3)$ is zero, irrespectively if ε_3 is positive.

We need also the following constraints which ensure that the bonus will be paid if and only if the team is qualified. If the team does not qualify, it will collect again 6 points, as before. Therefore, we use a new binary variable μ_3, with the following meaning: if the team does not qualify, $\mu_3 = 1$, while if it qualifies, $\mu_3 = 0$. And when the team qualifies, it must collect at least 10 points. These conditions are simply formulated as:

$$\begin{aligned} &\varepsilon_3 P_3 \le 6\mu_3 \\ &\varepsilon_3 P_3 \ge 10 q_3 \\ &\mu_3 + q_3 = 1 \\ &2.5/11 \succ \varepsilon_3 \ge 0 \end{aligned} \qquad (a_1)$$

might not qualify with 10 points (like Olympiacos in 2004/05, with Monaco 11, Liverpool 11 and Deportivo 1 point). In theory, a team B can qualify with 6 points (such as 1 victory against C and 3 draws, in both matches against D and 1 against C), if team A wins all its matches and collects 18 points, the third team C collects 5 points (1 victory against D and 2 draws, against B and C) and the fourth team D collects 3 points (3 draws, in both matches against B and 1 against C).

We solved the model with these modifications. Team 3 remains unqualified, but the solution changed to 12, 6, 6 and 12 points, i.e. team 4 qualifies now (!) at the cost of team 2 previously.

We repeated this model for the other non-qualified team 4. Precisely as team 3, team 4 does not qualify either. The solution changed now to 9, 12, 9 and 6, indicating that team 2 qualifies as in the initial case. Team 3, might qualify now, since it ends second together with team 1 with 9 points.

Thus, we conclude that if all teams maximize profits simultaneously, the two non-qualified teams can not improve their position by paying the qualification bonus to its players to enhance performance. Instead they improve the position of the other non-qualified team!

3.2 Simultaneous profit maximization under increasing returns (case 2)

In this case we made two changes compared to case 1. First, we differentiated the wages among the S- and N-players, assuming $w^i_S = 8$ and $w^i_N = (1, 7)$, increased at a range of 0.5. Second, we changed the revenue function to a quadratic one. The objective function is now:

$$Max = 12\sum_1^4 P_i - 0.5\sum_1^4 P_i^2 - 8\sum_1^4 S_i - (1,..,8)\sum_1^4 N_i \qquad (1)''$$

All other constraints, (2)´-(5)´, (6) – (58), integer, binary and bounds apply.

All 13 wage parameters gave the same answer, 6, 6, 18 and 6 points respectively. In 12 solutions, all teams use 1 S- and 10 N-players. In one case only, when $w^i_S = 8$ and $w^i_N = 3.5$, the winner, team 3, uses 3 S- and 8 N-players. These results are also expected. Obviously, when all teams must pay lower (ad hoc) wages to their N-players, all teams use more N-players and the most "balanced" team 3, in which the N-players have the highest elasticity, performs best.

3.3 Simultaneous profit maximization under decreasing returns (case 3)

In this case we made the following changes. First, the "production" functions are now characterized by decreasing returns to scale with the sum of S- and N-players' elasticity equal to 0.90. With decreasing returns and the necessary points collected, the efficiency parameters t_i would be very high. Thus, we multiplied all "production" functions with the number of matches (6) they play. Second, instead of using "ad hoc" wages to the players, we assumed that all teams pay the value of the marginal product of their players, determined endogenously in the model. Because the mobility of players during the tournament is not permitted, any differences in absolute and relative wages of players will persist. Third, to keep the complex objective function as "simple" as possible, we assumed a linear revenue function, and the "price" per point equal to 15. Thus, given the "production" functions below, (2)´´´- (5)´´´, the objective function is now:

$$Max = 15\begin{pmatrix} \sum_{1}^{4} P_i - (4.8t_1N_1^{0.1}S_1^{-0.2})S_1 - (0.6t_1N_1^{-0.9}S_1^{0.8})N_1 - \\ (4.2t_2N_2^{0.2}S_2^{-0.3})S_2 - (1.2t_2N_2^{-0.8}S_2^{0.7})N_2 - \\ (3.6t_3N_3^{0.3}S_3^{-0.4})S_3 - (1.8t_3N_3^{-0.7}S_3^{0.6})N_3 - \\ (3.0t_4N_4^{0.4}S_4^{-0.5})S_4 - (2.4t_4N_4^{-0.6}S_4^{0.5})N_4 \end{pmatrix} \qquad (1)´´´$$

The parentheses, multiplied by the unit price of points, 15, are the respective values of the marginal player, i.e. $w^i{}_S$ and $w^i{}_N$.

The new "production" functions are:

$$P_1 = 6t_1\left(S_1^{0.8}N_1^{0.1}\right)$$
$$P_2 = 6t_2\left(S_2^{0.7}N_2^{0.2}\right)$$
$$P_3 = 6t_3\left(S_3^{0.6}N_3^{0.3}\right) \qquad (2)^{\prime\prime\prime}\text{- }(5)^{\prime\prime\prime}$$
$$P_4 = 6t_4\left(S_4^{0.5}N_4^{0.4}\right)$$

In this case, team 1 is again the most "unbalanced" team, while team 4 is the most "balanced" one, since the S- and N-players have almost the same marginal product (elasticity).

All other constraints (6) – (58), integer, binary and bounds apply.

Unfortunately, there is no feasible (integer) solution. Feasible solutions exist, only if players are allowed to be non-integer (continuous). In that case, all teams use 5.5 N and 5.5 S, and collect 12, 9, 6 and 9 points respectively.

3.4 Individual profit maximization under decreasing returns (case 4)

In this case we maximized profits separately for every team, given the non-negative profits of all other teams as constraints, and paying wages according to the value of the marginal product of their players, as in case 3. The revenue function is again linear and the "price" per point is 15. Thus the objective function for team *i* is:

$$Max = 15\left(P_i - (MPS_i)S_i - (MPN_i)N_i\right) \qquad (1)^{\prime\prime\prime\prime}$$

The non-negative profits constraints for $j \neq i$ are:

$$15\left(P_j - (MPS_j)S_j - (MPN_j)N_j\right) \geq 0$$

All other constraints, $(2)^{\prime\prime\prime}$-$(5)^{\prime\prime\prime}$and (6) – (58), integer, binary and bounds apply.

No feasible (integer) solutions exist in this case either.

We disregarded the integer condition on players, removed constraints (10) – (13) and assumed instead lower and upper bounds of S_i, equal to 1 and 5 respectively. The solutions are summarized in Table 3[6]. The bald values (in diagonal) show the optimal solution, for the team that maximizes its profits. All other values (off diagonal) show the respective implied solutions for the other teams whose non-negative profits constraints are satisfied.

It is clear that the profit maximizing team wins all its matches and apart from team 3, all other teams use 5 N- and 6 S-players. The most "balanced" team 4 seems to perform better, since it is qualified when teams 1 and 3 maximize their profits and ends second, together with teams 1 and 3, when team 2 maximizes its profits. While teams 1 and 3 need to pay extremely unequal wages when they maximize their profits (9.6 and 8.52 times higher to their S-players respectively), team 4 achieves the same profit, and wins all its matches too, by paying just 1.5 times higher. Finally team 2 is an intermediate case and needs to pay 4.2 times higher wages to its S-players in order to maximize its profits and win all its matches. These ratios are not far away to some published statistics. For instance, the maximum wage range for AC Milan is 6 times, for Inter 16 times and for Roma 34 times. Thus, if these solutions are global, we find again that the most "balanced" team performs better, precisely as we found when we assumed "production" functions characterized by increasing returns to scale (case 2).

Neither compressed nor unequal wages can explain the performances of teams, based on all sixteen observations from Table 3. If we look at every team individually (with just four observations), anything is possible. If team 1 maximizes, it seems that a U-type performance exists. On the other hand, if team 2 maximizes, the performance seems to be of a reverse U-type. If team 3 maximizes, it is unclear if

[6] We first tried to solve the problem for the first team. Since the Global algorithm in Lingo iterated for hours, we set a maximum limit of 1 hour of runtime for all teams (which is equivalent to more than 20 millions of iterations). As a consequence, all reported solutions are the best found, within 1 hour of elapsed runtime and might not be globally optimal.

wage differences matter, and if team 4 maximizes, it seems that higher inequality reduces performance.

Table 3: Individual profits maximization, subject to non-negative profits for the other teams (the variable players, is continuous)

The maximizing team	Team 1	Team 2	Team 3	Team 4
Team 1	**$S^1 = 5$** **$N^1 = 6$** **$t^1 = 0.6920$** **$P^1 = 18$** **$\pi^1 = 27$** **$w_N^1 = 4.5$** **$w_S^1 = 43.2$**	$S^2 = 5$ $N^2 = 6$ $t^2 = 0.1132$ $P^2 = 3$ $\pi^2 = 4.5$ $w_N^2 = 1.5$ $w_S^2 = 6.3$	$S^3 = 5$ $N^3 = 6$ $t^3 = 0.1112$ $P^3 = 3$ $\pi^3 = 4.5$ $w_N^3 = 2.25$ $w_S^3 = 5.4$	$S^4 = 5$ $N^4 = 6$ $t^4 = 0.2912$ $P^4 = 8$ $\pi^4 = 12$ $w_N^4 = 8$ $w_S^4 = 12$
Team 2	$S^1 = 5$ $N^1 = 6$ $t^1 = 0.1538$ $P^1 = 4$ $\pi^1 = 6$ $w_N^1 = 1$ $w_S^1 = 9.6$	**$S^2 = 5$** **$N^2 = 6$** **$t^2 = 0.6795$** **$P^2 = 18$** **$\pi^2 = 27$** **$w_N^2 = 9$** **$w_S^2 = 37.8$**	$S^3 = 5$ $N^3 = 6$ $t^3 = 0.1482$ $P^3 = 4$ $\pi^3 = 6$ $w_N^3 = 3$ $w_S^3 = 7.2$	$S^4 = 4.7$ $N^4 = 6.3$ $t^4 = 0.1472$ $P^4 = 4$ $\pi^4 = 6$ $w_N^4 = 3.81$ $w_S^4 = 6.37$
Team 3	$S^1 = 1$ $N^1 = 10$ $t^1 = 0.2648$ $P^1 = 2$ $\pi^1 = 3$ $w_N^1 = 0.3$ $w_S^1 = 24$	$S^2 = 5$ $N^2 = 6$ $t^2 = 0.0755$ $P^2 = 2$ $\pi^2 = 3$ $w_N^2 = 1$ $w_S^2 = 4.2$	**$S^3 = 2.1$** **$N^3 = 8.9$** **$t^3 = 1$ (limit)** **$P^3 = 18$** **$\pi^3 = 27$** **$w_N^3 = 9.09$** **$w_S^3 = 77.49$**	$S^4 = 5$ $N^4 = 6$ $t^4 = 0.4368$ $P^4 = 12$ $\pi^4 = 18$ $w_N^4 = 12$ $w_S^4 = 18$
Team 4	$S^1 = 5$ $N^1 = 6$ $t^1 = 0.1153$ $P^1 = 3$ $\pi^1 = 4.5$ $w_N^1 = 0.75$ $w_S^1 = 7.20$	$S^2 = 5$ $N^2 = 6$ $t^2 = 0.2265$ $P^2 = 6$ $\pi^2 = 9$ $w_N^2 = 3$ $w_S^2 = 12.6$	$S^3 = 5$ $N^3 = 6$ $t^3 = 0.1482$ $P^3 = 4$ $\pi^3 = 6$ $w_N^3 = 3$ $w_S^3 = 7.2$	**$S^4 = 5$** **$N^4 = 6$** **$t^4 = 0.6552$** **$P^4 = 18$** **$\pi^4 = 27$** **$w_N^4 = 18$** **$w_S^4 = 27$**

Finally, an interesting difference in this case, compared to all other three cases in profit maximization, is that some matches end in draw and consequently the total number of points which is less than thirty-six.

4. Conclusions

The purpose of this paper was to develop a consistent and a “general equilibrium” kind of model to investigate if teams perform better or worse when they pay rather compressed or more dispersed wages to their different quality players. Working with four teams, with different “production” technologies (of CES type), which participate in a tournament like the UEFA CL group matches, we formulated a non-linear integer programming and used “plausible” parameters to examine the effects of wage differences and of qualification bonus on the performance of the teams.

When we measured performance by their effort to qualify in the next round (maximization of points), our simulations show that higher wage equality seems to improve the performance of three teams. One of these teams has the lowest elasticity of substitution between S- and N- players (almost Leontief “production” function) and the other has the highest elasticity of substitution (almost Cobb-Douglas “production” function). The fourth team, which is characterized by a rather high elasticity of substitution, is the most efficient team of the tournament, and its performance is not affected by the wage structure. The U-formed performance for that team, i.e. both lower and higher wage equality, is not excluded though.

In most cases, the starting team is composed by 1 S- and 10 N-players. And, as it happens in football, in some cases in which teams field more S- players, they do not necessarily perform better than teams with just 1 S-player! Because our model is a kind of “general equilibrium”, the performance of the teams is not only affected by their own team composition, derived from the own differences between the N- and S-wages, but also by the other teams’ differences between their N- and S-wages. As a consequence, the cross effects to some teams are positive and to other teams negative.

When performance is measured by profits’ maximization, the problem becomes complex. We changed the CES “production” functions to “Cobb-Douglas” ones under both increasing and decreasing returns to scale, assuming a sum of elasticity equal to 1.75 respective 0.90. We select one team to be very “balanced” in terms of N- and S-players elasticity, another very “unbalanced” and the other teams to be close to one or the other. Teams can maximize their profits simultaneously or separately, given the non-negative profits constraints for the other three teams. We worked with equal (ad hoc) wages to all players, with (ad hoc) wage differences among S- and N-players and with implicitly optimal wages to their players, reflecting the value of their marginal products. Similarly we worked with linear and quadratic revenue functions.

Under increasing returns to scale, when teams pay equal wages, the team that performs best is the most “unbalanced” one. In that case, the non-qualified teams, the most and the second “balanced” teams, did not manage to improve their performance and qualify, even if their players should receive the qualification bonus that UEFA would pay. On the other hand, under increasing returns to scale and ad-hoc wage differences, the most “balanced” team performs always best and all others equally bad.

Finally, under decreasing returns to scale, we allowed the optimal wages to be determined endogenously. Unfortunately, our solutions violated the non-integer condition on players. However, if we accept our non-integer players’ solution, we can not support that wage differences matter. Some teams need to pay extremely unequal wages, while others can perform exactly equally well with more compressed wages. What matters, is if the team is “balanced” or not. A “balanced” team does it better!

References

Arrow, K., et al (1961), Capital-labor substitution and economic efficiency, *Review of Economics & Statistics, 43, pp. 228-232.*

Fehr, E. & K. M. Schmidt (1999), A theory of fairness, competition, and cooperation, *The Quarterly Journal of Economics, 114, pp. 817-868.*

Franck, E. & S. Nüesch (2007), Wage dispersion and team performance – An empirical panel analysis, *Working Paper No. 73, Institute for Strategy and Business Economics, University of Zurich.*

Hirotsu, N. & M. B. Wright (2006), Modeling tactical changes of formation in association football as a zero-sum game, *2 (2), article 4, Journal of Quantitative Analysis in Sports, The Berkeley Electronic Press.*

Kesenne, S., (2007), Does a win bonus help to increase profit or wins in professional team sports? *Working Paper, Economics Department, University of Antwerp.*

Lazear, E. P. (1989), Pay equality and industrial politics, *Journal of Political Economy, 97, pp. 561-580.*

Lazear, E. P. & S. Rosen (1981), Rank-order tournaments as optimum labor contracts, *Journal of Political Economy, 89, pp. 841-964.*

Levine, D. I. (1991), Cohesiveness, productivity, and wage dispersion, *Journal of Economic Behavior and Organization, 15, pp. 237-255.*

Milgrom, P. R. (1988), Employment contracts. Influence activities, and efficient organization designs, *Journal of Political Economy, 96, pp. 42-60.*

Papahristodoulou, C. (2007), An analysis of UEFA Champions League match statistics, (submitted for publication in the *International Journal of Applied Sports Science*)

Pokorny, K. (2004), Pay – but don't pay too much: An experimental study on the impact of incentives, *Working Paper,* (available at http://ssrn.com/abstract=614742)

Torgler, B., et al (2008), Looking awkward when winning and foolish when losing: Inequality aversion and performance in the field, *Working Paper No 2008-11, Center for Research in Economics, Management and the Arts, Basel.*

http://www.gazzeta.it, 2007-09-11