# Equity Premiums In a Small Open Economy

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**Preliminary, Comments Welcome**

### Abstract

This paper studies the behaviour of asset prices in relation to consumption and other business cycle variables. While RBC models have been able to successfully explain the dynamics of macroeconomic variables, they fail to replicate similar interesting stylized facts when studying the behavior of asset prices. In an attempt to solve this shortcoming, some progress has been made in models that modify utility in order to account for habit persistence and incorporate capital adjustment costs. We have developed a framework that combines these ingredients by applying the loglinearly reduced form of the general equilibrium model and the asset pricing formula, based on the lognormality of the disturbance distribution for the small open economy case. Our findings indicate that in a small open economy environment this kind of model fails to account for a substantial equity premium.

**Keywords**: Equity premium, habit formation, small open economy

JEL classification: G12, C63, F41

# 1 Introduction

In recent years the frontiers between financial economics and macroeconomics have steadily narrowed. Indeed, many empirical studies focus on the behavior of asset prices in relation to consumption and other business cycle variables. General equilibrium models have been successful in explaining the dynamics of macroeconomic aggregates, but have failed to replicate similar interesting stylized facts when studying the behavior of asset prices. Empirically, financial economic studies have documented important cyclical variations in security returns and risk premia.

To some extent, during the last two decades the principal question in most business cycle studies has been to reconcile data and economic theory, and therefore construct models with endogenous processes able to generate the fluctuations observed in the data. In fact, the data (taken from postwar quarterly data for the industrialized countries) shows for example that consumption is smooth and that the covariance between quarterly real consumption growth and real dividend growth is very weak. Nevertheless, the study of the equity premium can be seen as a prime example of where these models fall apart.

In fact, among the abundant literature treating the relation between economic fluctuations and asset prices, the endowment model of Lucas (1978) was the first to be established as a baseline. A financial asset model based on consumption was later introduced by Hansen and Singleton (1983). In this kind of model the quantity of risk in the financial market is measured by the covariance of the excess stock return with consumption growth, while the risk price is the coefficient of the representative agent's relative risk aversion. Based on the data[1] however the average stock return is very high and the riskless interest rate is low. This means there is high expected excess return on stock (the equity premium), while on the other hand the data reveals low covariance between stock

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[1]For a survey of the stylized facts related to the consumption-asset pricing framework, see Campbell (2001).

returns and consumption. In this case only a very high coefficient of risk aversion can explain the high equity premium, which Mehra and Prescott (1985) have called the 'equity premium puzzle'.[2]

Our purpose here is to introduce a foreign sector to the model studied by Jermann (1998),[3] and in this way permit the representative household to have access to financial credits on the foreign economy. Incorporating the foreign sector will thus provide another opportunity for agents to smooth their consumptions. With this in mind we study the model's business cycle and asset pricing implications, enabling us to determine whether the results obtained by preceding studies[4] will hold once the foreign economy is introduced. This essentially allows us to study equity premium behavior, in the case of a small open economy, with habit persistence in preferences and adjustment costs of capital, the case we intend to assess here.

Kandel and Stambaugh (1991), in response to the equity premium puzzle, argue that risk aversion is much higher than what has been traditionally thought. But, if agents are very risk averse, they have a strong desire to transfer wealth from a 'good' period (with high consumption) to a 'bad' period. Since consumption grows steadily over time, the high risk aversion makes agents want to borrow in order to reduce the discrepancy between present and future consumption. Campbell (2001) shows that to reconcile this with the low observed real interest rate, we must postulate that agents are very patient; thus they have a low or even negative rate of time preference. Weil (1989) calls this the 'risk-free rate puzzle'.

Several studies tried to resolve those enigmas. One method was to introduce a class of utility functions and payout structures that could generate large variability for consumption's marginal

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[2]See also Cochrane and Hansen (1992) and Kocherlakota (1996) for more details on this puzzle. A brief summary of the other enigmas found in literature concerns: 'the riskfree rate puzzle' in Weil (1989), 'the stock market volatility puzzle' in LeRoy and Porter (1981) and Shiller (1981); to quote only the most documented.

[3]The one sector version of the RBC model with adjustment cost of capital and fixed labor.

[4]These include Jermann (1998), Benninga and Protopapadakis (1990), Danthine et al. (1992) and Rouwenhorst (1995).

utility. A model with a representative agent whose utility displays habit formation, introduced by Sundaresan (1989) and Constantinides (1990) produces this variability and can, in this way, resolve the puzzling equity premia problem.

The idea behind the relative success of this utility function (as shown recently by Campbell and Cochrane, 1999), is that specifications for habit formation make agents more risk averse in bad times than in good times, when consumption is high compared to its past history. Thus the equity premium can be explained by the high volatility of market stocks together with a reasonable degree of average level of risk aversion.

Nevertheless, Rouwenhorst (1995) explains that in a closed economy, consumption smoothing must arise through capital accumulation. Thus, it is difficult to explain substantial risk premia because agents smooth their consumption when risk aversion increases. The introduction of capital adjustment costs may however solve this problem and with appropriate specifications for the utility function, equity premiums with several percentage points can be generated (Jermann, 1994). Indeed an extended version of the RBC model including consumption habits may provide a key channel within which risk premiums may be generated, because the agents in this model become more risk averse. Consumption smoothness is desired here but when combined with capital adjustment costs, stock return becomes risky and large equity premiums may result (as shown by Budría, 2002).

Constantinides (1990) demonstrates that in models with trivial production sectors, habit persistence in preferences may potentially account for both risk-free and equity premium asset price puzzles, while implying only a modest level of household risk aversion. In addition to habit formation,[5] another ingredient needed to successfully obtain equity premia consists of those technology features that prevent households from smoothing their consumption. Jermann (1998) shows that capital adjustment costs play a crucial role in this way. The model thus specified will not only

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[5]Given its success in solving the puzzling equity premia in models including production. See Abel (1990) and Constantinides (1990).

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explain the puzzling asset prices but also match a set of salient macroeconomic statistics.

Boldrin et al. (2001) introduce two modifications into the standard real business cycle model: habit persistence preferences and limitations on factor mobility between the two sectors[6] of the economy. The second assumption concerns the fact that the sectorial and aggregate allocations of capital and labor are determined before the achievement of uncertainty in the current period. The resulting model is consistent with observed mean equity premiums and mean risk free rates.

We follow Jermann (1998) through combining the loglinear reduced form along the lines of King, Plosser and Rebelo (1988) and the asset pricing formulae based on the lognormality of the distribution introduced by Hansen and Singleton (1983), and more recently by Campbell (1986 and 1996). The framework presented here extends that of Jermann (1998) by introducing a foreign sector into the model and a 'risk premium' term to remove the model's built-in random walk property as shown below. A standard feature of most current open economy models is a relation implying uncovered interest parity (UIP), despite its prominent empirical weaknesses as shown by McCallum and Nelson (2000).

Our results show that this model is not able to explain equity premium. Indeed the model generates small equity premiums when compared to that observed in the historical data. This failure can be attributable to the fact that, with access to international financial markets, domestic households may again play on the smoothing of their consumption. In this case the substantial addition brought to the standard RBC model by habit formation and capital adjustment costs would be canceled by the opening of the economy to international financial markets. Additionally, domestic agents may reduce fluctuations in consumption by borrowing (or lending) from foreigners in bad (or good) periods. Nevertheless our model is able to match business cycle statistics, and compared to the standard RBC model it is better able to explain equity premia in several basis points.

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[6]Boldrin et al. (1999) assume that consumption and investment are non-homogeneous goods produced in separate sectors.

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This paper is structured as follows. Section 1.2 presents the model setting and discusses its solution, Section 1.3 examines the model predictions and presents the results, and Section 1.4 contains concluding remarks.

# 2 The model

## 2.1 Model Setting

We consider the case of a small open economy[7] with a continuum of identical infinitely lived households. The representative agent in both countries (the home country and the rest of the world) maximizes the expected discounted sum of utility. There is a single consumption/investment good in the world, produced by domestic and foreign firms according to constant-returns-to-scale production technology, such that import and local production are perfect substitutes. Each firm finances its investment through retained earnings.

### 2.1.1 Firms

We assume that the representative domestic firm, which is owned by domestic households, has two types of purchasers, who are domestic and foreign customers to whom it may sell its goods. In each period the firm has to decide how much labor to hire and how much to invest. The manager's problem is to maximize the value of the firm to its owners[8]

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} \{Y_{t+j} - W_{t+j} n_{t+j} - I_{t+j}\}$$

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[7]The model is a modification of the one-sector, fixed labor model.

[8]The value of the firm is equal to the present discount value of all current and future expected cash flows (as shown by Jermann, 1998). Here for the financing path we use the Modigliani-Miller theorem.

subject to the constraint given below

$$Y_t = Z_t K_t^\alpha n_t^{1-\alpha}, \ \ 0 < \alpha < 1 \tag{1}$$

where $\beta^j \Lambda_{t+j}/\Lambda_t$ is the marginal rate of substitution of the household and $n_t$ the quantity of labor input. The state of technology evolves according to the AR(1) process

$$Z_t = \rho_z Z_{t-1} + \epsilon_{zt}, \tag{2}$$

where $\epsilon_{zt}$ is a normally distributed white noise with mean 0 and variance $\sigma^2$ for all $t \geqslant 0$.

Prior research – in the case of closed economy – has found that endogenous consumption becomes even smoother as risk aversion is increased. In this way, it is more difficult to explain substantial risk premia (Rouwenhorst, 1995). The intuition behind this is that agents can easily alter their production plans to smooth their consumption. Thus, with this frictionless and instantaneous capital stock adjustment, this problem cannot be solved. Jermann (1994) suggests the introduction of capital adjustment costs in order to overcome this weakness. The specification of the function follows Jermann (1998), that is,

$$\phi(\frac{I_t}{K_t}) = \frac{a_1}{1 - (1/\xi)}(\frac{I_t}{K_t})^{1-(1/\xi)} + a_2$$

where $\phi(.)$ is a positive, concave function.[9] Thus, the resources allocated to investment are not transformed into the next period capital with a rate equal to one. The parameter $\xi$ is the elasticity of investment, $I_t$, with respect to Tobin's $q$ , and $a_1, a_2$ are chosen so as to yield a balanced growth path, for those variables in the model that are invariant to $\xi$ (see Boldrin et al., 2001 for more details).[10]

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[9]The concavity of the cost function captures the idea that changing capital stock rapidly costs more than changing it slowly (see Eisner and Strotz ,1963 and Lucas and Prescott, 1971).

[10]Here, as in Boldrin et al. (2001), we set $a_1$ and $a_2$ to:
$a_1 = (\exp(\bar{x}) - 1 + \delta)^{(1/\xi)}, \ a_2 = \frac{(1/\xi)}{1-(1/\xi)}(1 - \delta - \exp(\bar{x}))$.

The value of $\xi$ strongly affects the concavity of the adjustment cost function. Indeed as shown in Figure 1.1,[11] the function $\phi(.)$ is more concave when $\xi$ is low.

The technology for accumulating capital is as follows

$$K_{t+1} = (1-\delta)K_t + \phi(\frac{I_t}{K_t})K_t, \quad 0 < \delta < 1. \tag{3}$$

where $\delta$ is the depreciation rate of capital.

There are no new shares issued by the firm and the capital stock is financed through retained earning (RE), defined as

$$I_t = RE_t.$$

The domestic household has access to incomplete international financial markets, because the only foreign asset they can hold is a risk-free bond whose rate of return is exogenously determined. In this case the initial conditions, in particular the home country's initial foreign debt position, govern the model's steady state values. As a consequence, a random walk[12] component can prevent the model's dynamic equilibrium from reaching a stable solution. To induce stationarity and remove the model's built-in random walk property, we use an endogenous country-specific risk premium term $\kappa_t$, that reflects departures from uncovered interest parity (UIP).[13] Following Senhadji (1997), Mendoza and Uribe (2000), Schmitt-Grohe and Uribe (2003), and Dib (2003), this risk premium term is given by

$$\kappa_t = \exp(-\frac{\varphi \tilde{B}_t^*}{Y_t}), \tag{4}$$

where $\tilde{B}_t^*$ is the average of aggregate foreign debt and $\varphi$ measures the level of risk premium. The term risk premium implies that the equilibrium is unique and induces stationarity in the model. At

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[11] The figure is taken from Budría, (2002) who uses the same calibration and functional form that we use.

[12] At least one eigenvalue in the model is equal to unity.

[13] That is, the equilibrium steady state is unique and the model is stationary (see Dib 2003).

equilibrium the market clearing condition yields

$$\tilde{B}_t^* = B_t^* \quad \text{for all } t.$$

There are three assets in this economy that are traded in incomplete financial markets. Household can then purchase a perfectly divisible equity share of the representative domestic firm that is a claim to an infinite stream of the firm's dividends ($A_t$); so at time t, this asset delivers a payout (dividends) denoted by $D_t$. This asset can be purchased only by domestic households[14] who must pay $P_t^s$ to obtain it. They can also purchase two types of one-period riskless bonds (domestic and foreign bond). At the end of period t, the firm's dividends to shareholders satisfy the following equation

$$D_t = Y_t - I_t - W_t n_t.$$

### 2.1.2 Households

The representative agent derives utility from consumption of a final good $C_t$. The preferences exhibit a simple form of habit formation, that is a stock of past consumption $X_t$ that affects current utility

$$E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{(C_{t+j} - X_{t+j})^{1-\gamma}}{1 - \gamma} \right], \qquad 0 < \beta < 1 \tag{5}$$

where $\gamma$ is a positive parameters[15] different from 1. The habit stock $X_t$ evolves as follows

$$X_t = bC_{t-1} \tag{6}$$

wherein we define the case where $b > 0$ as the habit persistence preferences case. When $b = 0$, these preferences correspond to those in a standard RBC model with fixed labor.[16]

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[14]It is assumed here that foreigners purchase only those bonds denominated in their own output.

[15]In the special case where $\gamma \to 1$ , the logarithmic function is obtained.

[16]The term $bC_{t-1}$ can be seen as the household's habit stock, thus, b cannot be negative.

In the case of habit persistence in the utility function, the representative agent is concerned with maintaining the same level of consumption period by period. As shown by Constantinides (1990) and Lettau and Uhlig (1997), the coefficient of relative risk aversion, $\gamma$ must not be high, because in this case relative risk aversion becomes more sensible. To show this we can compute elasticity of intertemporal substitution ($ES$) and relative risk aversion ($RRA$). Following Lettau and Uhlig (1997) and Allais et al. (2000), and assuming that the logarithm of consumption follows a random walk with drift yields

$$C_{t+1} = g + C_t + \varepsilon_{t+1}.$$

The inverse of ES is given by[17]

$$\frac{1}{ES} = \left(\frac{\gamma}{1 - b\exp(-g)}\right)\left(\frac{1 + \beta b^2 \exp(-(\gamma+1)g)}{1 - \beta b\exp(-\gamma g)}\right)$$

and the RRA follows

$$RRA = \frac{\gamma}{1 - b\exp(-g)\frac{\exp(-\gamma g) - \beta\gamma}{\exp(-\gamma g) - b\gamma}}.$$

Evidently, with no habit persistence ($b = 0$) the inverse of $ES$ is simply $\gamma$. Moreover, relative risk aversion is strongly related to the habit parameter.[18]

In its portfolio the household has a domestic firm share and can also purchase one type of one period riskless bond $B_t$ (domestic risk-free bond[19]) denominated in consumption units for $P_t^f$. It may also make a period $t$ acquisition of one bond $B_t^*$, denominated on foreign output redeemed

[17] See Lettau and Uhlig (1997) for more details.

[18] Allais et al. (2000) compute the RRA and ES for Canada and argue that the presence of habit forming in preferences is likely to reach the value found in the data, and the model similar to what we present here can better account for price changes for financial assets.

[19] As in Jermann (1998), we suppose that the possibility of bankruptcy is excluded, so that the corporate and riskfree bonds are perfect substitutes.

for one unit of foreign output one period later. The price[20] the household must pay for this bond is $\kappa_t^{-1} P_t^{e*-1}$. Thus, the price households must pay increases the ratio of foreign-debt to output, where the rate of return on $A_t$ is conditional to date $t+1$ state of nature achievement while those on $B_t$ and $B_t^*$ are not. The two riskless bonds pay one unit of the consumption good (for each) at time $t+1$ and expire.[21]

Let $O_t = [B_t, A_t, B_t^*]$ be the asset vector that contains the domestic firm shares and assets described above. Likewise, let $V_t^o$ and $D_t^o$ denote the asset price vectors and current period payouts respectively.

The budget constraint is then given by the following inequality

$$O_{t+1} V_t^o + C_t \leqslant W_t n_t + O_t(V_t^o + D_t^o) \tag{7}$$

where $W_t$ is the wage rate.

Since the representative firm does not issue new shares at date t, the household takes $A_{t-1}, B_{t-1}, B_{t-1}^*, X_{t-1}$ as given and maximizes

$$\max_{\{A_j, B_j, B_j^*, C_j, j \geqslant t\}} E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{(C_{t+j} - X_{t+j})^{1-\gamma}}{1 - \gamma} \right]$$

subject to the habit persistence constraint $X_t = bC$, the budget constraint in (7) and the technology function (1).

The gross domestic product $Y_t$ can either be used for consumption or investment and to pay foreign debt, or be considered as surplus (see appendix A for details).

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[20]McCallum and Nelson (1998) suppose a random "risk-premium" term that reflects temporary but persistent departures from uncovered interest parity, while here instead we assume an endogenous premium term to induce stationarity into our small open economy model.

[21]Domestic riskless bonds are assumed to be in aggregate zero supply.

### 2.1.3 General Equilibrium Model Solution

General equilibrium model solutions usually involve the application of the linearization method developed by King, Plosser and Rebelo (1988). This method implies that expected returns are equal across securities, so that risk premiums cannot be study. Danthine et al. (1992) show that the use of a solution technique with nonlinear functions can yield interesting results.

Following Jermann (1994), we use a combination of the loglinear and lognormal environments.[22] The solution in this case is to solve for the model's approximate dynamics represented by a loglinear state space system of the form $s_t = M s_{t-1} + \epsilon_t$, where $M$ is the square matrix that governs the system's dynamics. This step involves loglinearizing the first order conditions and solving the dynamic system.

The second step involves the lognormal pricing formula used by Hansen and Singleton (1983) and Campbell (1993). In this formula, the random future payout of dividends can be evaluated by the present value relationship (see Jermann 1994 for more details)

$$V_t[D_{t+k}] = \frac{\beta^k E_t[\Lambda_{t+k} D_{t+k}]}{\Lambda_t}$$

where $\Lambda_t$ is the marginal numeraire valuation at period $t$. Furthermore, the relations between the dividend payout and the marginal valuation on the one hand and the state vector on the other hand, pass through the factors $l_d$ and $l_\lambda$ as follows

$$\lambda_t = l_\lambda s_t$$
$$d_t = l_d s_t, \tag{8}$$

where the error terms are assumed to follow a multivariate normal $iid$ process.

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[22]As in Jermann (1994), we assume that the system's variables are stationary.

## 2.2 Numeraire Valuation

In our model the valuation (or marginal utility $h_t$,) is computed as follows

$$h_t = \frac{\partial U_t}{\partial C_t} = (C_t - bC_{t-1})^{-\gamma} - \beta b E_t (C_{t+1} - bC_t)^{-\gamma}. \tag{9}$$

Then through loglinearizing (9) and taking its first order Taylor series approximation around the steady state of consumption (c), it can be shown that

$$
\begin{aligned}
h_t &= \log[((1-b)c)^{-\gamma}(1-\beta b)] - \frac{\gamma(1+\beta b^2)}{(1-b)(1-\beta b)}(\frac{C_t - c}{c}) + \\
&\quad \frac{\gamma b}{(1-b)(1-\beta b)}(\frac{C_{t-1} - c}{c}) + \frac{\gamma \beta b}{(1-b)(1-\beta b)}(\frac{C_{t+1} - c}{c}).
\end{aligned} \tag{10}
$$

Next, we approximate $\frac{C_t - c}{c}$ as the difference between log of time t consumption and the log of steady state consumption, i.e.

$$\frac{C_t - c}{c} \simeq \log(C_t) - \log(c),$$

to finally compute the following expression for $h_t$

$$
\begin{aligned}
h_t &= \log[((1-b)c)^{-\gamma}(1-\beta b)] + \frac{\gamma[b(\beta - b\beta - 1) - 1]}{(1-b)(1-\beta b)} \log(c) \\
&\quad - \frac{\gamma(1+\beta b^2)}{(1-b)(1-\beta b)} c_t + \frac{\gamma b}{(1-b)(1-\beta b)} c_{t-1} + \frac{\gamma \beta b}{(1-b)(1-\beta b)} c_{t+1}.
\end{aligned} \tag{11}
$$

where $c_t$ stands for the log of the time $t$ consumption expenditure.

In this formula we can ignore the constant term when evaluating the relation between consumption expenditure and realized marginal utility $h_t$, and marginal valuation $\Lambda_t$. Thus, we can approximate the marginal utility locally by

$$h_t = \frac{\gamma \beta b}{(1-b)(1-\beta b)} c_{t+1} - \frac{\gamma(1+\beta b^2)}{(1-b)(1-\beta b)} c_t + \frac{\gamma b}{(1-b)(1-\beta b)} c_{t-1}.$$

Now we turn to the risk premium issue.

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## 2.3 Risk Premium Computation

We let $H_{t,t+s}$ be lifetime marginal utility and then we assume the following relation between its log, $(h_{t,t+s})$ and a distributed lead of the state vector's log[23]

$$h_{t,t+s} = \sum_{j=0}^{s} l_h(j)s_{t+j}. \tag{12}$$

Therefore, we can evaluate time $t$ expectations over the lifetime marginal utility within the framework of asset pricing case as follows

$$E_t(H_{t,t+s}) = E_t \exp(h_{t,t+s}).$$

Assuming that $H_{t,t+s}$ is normally distributed, lognormality implies that the right hand side (RHS) of the above equation can be rewritten as

$$E_t(H_{t,t+s}) = \exp(E_t h_{t,t+s}) + \frac{1}{2}var_t(h_{t,t+s}).$$

Consequently, the value of a claim to a potentially random future payout $D_{t+k}$ reduces to

$$V_t(D_{t+k}) = \frac{\beta^k E_t(\exp(h_{t+k,t+s+k})D_{t+k})}{E_t \exp(h_{t,t+s})}. \tag{13}$$

Using the law of iterative expectations, the numerator of (13) can be rewritten as

$$E_t[E_{t+k}(\exp(h_{t+k,t+s+k}))D_{t+k}].$$

Given that $D_{t+k}$ is deterministic at t+k, and under the lognormality assumption, the previous expression becomes

$$E_t[\exp(E_{t+k}(h_{t+k,t+s+k}) + \frac{1}{2}var_{t+k}(h_{t+k,t+s+k}))D_{t+k}].$$

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[23]In what follow we use the presentation of Jermann (1994).

Furthermore, with the variance term which does not depend on the state of the system, (13) then reduces to

$$V_t(D_{t+k}) = \frac{\beta^k E_t(\exp(E_{t+k}(h_{t+k,t+s+k}) + \frac{1}{2}var_{t+k}(h_{t+k,t+s+k}))D_{t+k})}{\exp(E_t h_{t,t+s} + \frac{1}{2}var_t(h_{t,t+s}))}.$$

Moreover, as the two variance terms cancel out, it follows that

$$V_t(D_{t+k}) = \frac{\beta^k E_t(\exp(E_{t+k}(h_{t+k,t+s+k}))D_{t+k})}{\exp(E_t h_{t,t+s})}, \tag{14}$$

so that $\Lambda_t = \exp(E_t h_{t,t+s})$ can be used as the marginal numeraire valuation.

### 2.3.1 Expected Return

Following Jermann (1998), we focus on single-payout assets in order to define a one-period holding return as

$$R_{t,t+1}(D_{t+k}) = \frac{V_{t+1}(D_{t+k})}{V_t(D_{t+k})}. \tag{15}$$

Hence, we first have to evaluate the period $t$ expected value of $V_{t+1}(D_{t+k})$. The lognormality assumption therefore implies that

$$
\begin{aligned}
E_t[V_{t+1}(D_{t+k})] &= \beta^{k-1} E_t \exp[(E_{t+1}(d_{t+k} + h_{t+k} - h_{t+1}) + \frac{1}{2}var_{t+1}(d_{t+k} + h_{t+k})] \\
&= \beta^{k-1} \exp[E_t(d_{t+k} + h_{t+k} - h_{t+1}) + \frac{1}{2}var_t(E_{t+1}(d_{t+k} + h_{t+k} - h_{t+1})) \\
&\quad + \frac{1}{2}var_{t+1}(d_{t+k} + h_{t+k})].
\end{aligned}
$$

The conditional expectation on the holding return can then be calculated as follows

$$
\begin{aligned}
E_t[R_{t,t+1}(D_{t+k})] &= E_t\{\exp[\frac{1}{2}var_{t+1}(d_{t+k} + h_{t+k})] \\
&\quad \frac{\beta^{k-1} \exp[E_t(d_{t+k} + h_{t+k} - h_{t+1}) + \frac{1}{2}var_t(E_{t+1}(d_{t+k} + h_{t+k} - h_{t+1}))]}{\beta^k \exp[E_t(h_{t+k} + d_{t+k} - h_t) + \frac{1}{2}var_t(d_{t+k} + h_{t+k})]}\}.
\end{aligned}
$$

As we assumed early on, the variance term is state-independent so that

$$E_t[R_{t,t+1}(D_{t+k})] = E_t \beta^{-1} \frac{\exp[E_t(d_{t+k} + h_{t+k} - h_{t+1}) + \frac{1}{2}var_t(E_{t+1}(d_{t+k} + h_{t+k} - h_{t+1}))]}{\exp[E_t(h_{t+k} + d_{t+k} - h_t)]}.$$

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As a result,

$$E_t[R_{t,t+1}(D_{t+k})] = E_t\beta^{-1}\exp[E_t(h_t - h_{t+1}) + \frac{1}{2}var_t(E_{t+1}(d_{t+k} + h_{t+k} - h_{t+1}))]\,,$$

which can be shown to reduce to

$$E_t[R_{t,t+1}(D_{t+k})] = R_{t,t+1}(1_{t+1})\exp[\frac{1}{2}var_t(E_{t+1}(d_{t+k} + h_{t+k} - h_{t+1}))]\,,$$

or, alternatively,

$$E_t[R_{t,t+1}(D_{t+k})] = R_{t,t+1}(1_{t+1})\exp[-cov_t(h_{t+1}, E_{t+1}(h_{t+k}-h_{t+1}))-cov_t(h_{t+1}, E_{t+1}(d_{t+k}))]$$

$$(16)$$

which is the conditional expected return.

As shown by Jermann (1994), even though the RHS of this equation can be divided into three components,[24] we are only interested in the first term which represents the risk-free rate

$$R_{t,t+1}[1_{t+1}] = 1/V_t[1_{t+1}] \tag{17}$$

$$= \beta^{-1}\exp(h_t - E_t h_{t+1} - \frac{1}{2}var_t(h_{t+1})).$$

In order to quantify the equity premium, we need to obtain the risk-free rate's conditional expectation. To do so let us first compute the conditional variance.

### 2.3.2 Conditional Variance

With the lognormality assumption it is possible and useful to compute the conditional variance of asset returns. As by definition[25]

$$R_{t,t+1}[D_{t+k}] = E_t(R_{t,t+1}[D_{t+k}])\frac{V_{t+1}[D_{t+k}]}{E_t(V_{t+1}[D_{t+k}])}, \tag{18}$$

---

[24]See Jermann (1998) for more details on this specification. The three components are: riskfree rate, the term uncertainty premium which represents the term premium for a k-period discount bond and the last element, which is the payout uncertainty premium.

[25]Here we use the fact that :

$$\frac{1}{V_t[D_{t+k}]} = \frac{E_t(R_{t,t+1}[D_{t+k}])}{E_t(V_{t+1}[D_{t+k}])}.$$

and focusing on the RHS, it can be shown that

$$
\begin{aligned}
\frac{V_{t+1}[D_{t+k}]}{E_t(V_{t+1}[D_{t+k}])} = \quad & \exp(E_{t+1}(d_{t+k} + h_{t+k} - h_{t+1}) - E_t(d_{t+k} + h_{t+k} - h_{t+1}) \\
& -\frac{1}{2}var_t(E_{t+1}d_{t+k} + h_{t+k} - h_{t+1})).
\end{aligned}
$$

As a result,

$$
var_t(R_{t,t+1}[D_{t+k}]) = E_t(R_{t,t+1}[D_{t+k}])^2(\exp(E_{t+1}(d_{t+k} + h_{t+k} - h_{t+1}))) \tag{19}
$$

which represents the conditional variance of asset returns.

# 3 Model Predictions

## 3.1 Market Equilibrium

The market-clearing condition for the goods market requires that all produced final goods be con-sumed, invested or used in order to pay capital adjustment costs and period asset returns. If we normalize the number of households and firms to one, then the resource constraint holds in an equal manner, and also labor demand equals labor supply. Financial market equilibrium occurs when agents hold all outstanding shares and corporate bonds[26] and all other assets are in zero supply. The sequence of markets equilibrium is defined as usual.

## 3.2 Model Calibration

The values assigned to model parameters are those estimated by Letendre (2003) for the Canadian economy. Certain other values have been chosen from the literature so that the model can reproduce some small open economy features. We consider parameters within the range of values generally considered as being linked to the habit formation (HF) case. Indeed the preference parameter $\gamma$

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[26]The domestic corporate bonds are detained by domestic agents.

is set to 2. As discussed in Jermann (1998) and Budría (2002), in the case of HF preferences this parameter is close to risk aversion. Campbell (1993) estimates this value to be between 5 and 8, but mean reversion in asset prices may increase this value by up to three times (Black, 1990). Boldrin et al. (1995) assume a value of 1 for the HF case. The depreciation rate is set to 0.025, the subjective discount factor is set to 0.96, and the steady state value of $n$ to 0.33. Also the steady state risk premium parameter $\varphi$ is set to 0.0054, which implies an average risk premium of 98 basis points at an annual rate (as in Clinton, 1998, reporting estimates for Canada). The capital share in production is set to 0.32, the parameter of habit persistence $b$ to 0.58. Cochrane and Hansen (1992) use 0.5 and 0.6 for this parameter, while Constantinides (1990) requires a level of 0.8.

The elasticity of investment with respect to Tobin q is estimated in the literature to have values ranging from 0.4 to 1.14. Abel (1980) estimates this parameter to be between 0.27 and 0.52, and Jermann (1998) sets $\xi$ equal to 0.23, which is the high adjustment cost case. We adopt this parametrization and set $\xi$ to 0.23. Finally the productivity shock parameter is set to $0.94436^{27}$ as estimated by Letendre (2003) for Canada, with a standard deviation of 0.00599. See Table 1.1 for a summary of these values.

### 3.3 Model Solution

As shown by Boldrin et al. (1999) and Jermann (1988), in the RBC model the equity premium is low. Intuitively, this result is due to the fact that in RBC models the Sharpe ratio for equity (SR) and the standard deviation for real return to equity, $\sigma_{r^e}$, are low. Hence, for a production economy defining the equity premium as $E(r_{t+1}^e - r_t^f) = SR.\sigma_{r^e}$ naturally leads to a low value for the premium. Indeed, the equity premium remains at zero and the result is invariant to the introduction of habit persistence in the utility function.[28] As discussed before the introduction of adjustment cost

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[27]See Prescott (1986) for a discussion of Solow residual estimates.

[28]In our model the change in $b$ let the equity with no significant change.

of capital in a model with habit preferences and fixed worked hours increases $\sigma_{r^e}$ to a large value and this yields a substantial equity premium.

In the case of the lognormal pricing model, we assume that the dividend ($D_t$) and the marginal valuation ($\Lambda_t$) are lognormal. The joint distribution is given by the vector process for state variables. In this case, the equity premium as defined is usually computed as the difference between the unconditional mean equity return and the unconditional mean risk-free rate, thus we need to apply the lognormal pricing formulae to the two rates (on the equity and risk-free rates), whereas the one period holding return is defined as

$$R_{t,t+1}^e = \frac{V_{t+1}(D_{t+k})}{V_t(D_{t+k})}$$

for k period asset holding . In the risk-free rate case the expression becomes

$$R_{t,t+1}^f = \frac{1}{V_t(1_{t+k})}.$$

Following Jermann (1998), the risk-free rate can thus be rewritten as

$$R_{t,t+1}^f = \beta^{-1} \exp[\lambda_t - E_t\lambda_{t+1} - \frac{1}{2}var_t(\lambda_{t+1})] \tag{20}$$

where $\lambda_t$ is the logarithm of the valuation, $\Lambda_t$. The unconditional expectation corresponding to (20) is then given by

$$E(R_{t,t+1}^f) = \beta^{-1} \exp[\frac{1}{2}(var(E_t\lambda_{t+1} - \lambda_t) - var(\lambda_{t+1} - E_t\lambda_{t+1}))],$$

which is trivially computed from the model solution. However, although return to the firm's equity can be written as

$$R_{t,t+1}^e = \frac{V_{t+1}^D + D_{t+1}}{V_t^D},$$

it is still difficult to get an analytical closed form solution for the unconditional expectation of this return. As shown by Jermann (1998) we can overcome this shortcoming by applying numerical simulation.

### 3.4 Summary of Numerical Results

The models' predictions for mean returns and business cycle statistics are shown in Tables 1.2 and 1.3. The results obtained show that the equity premium computed using the lognormal formulae is about 0.025% (2.5 basis points annually), which is extremely low when compared to the premium obtained using historical data. For example Allais et al. (2000) report an equity premium of 3.47% for Canada. See Table 1.2 for a summary of results, with Canadian data used for comparison purposes.

This means that even with habit persistence in preferences and capital adjustment costs, the model fails to account for a substantial equity premia when a foreign sector is introduced. This failure can be explained by the smoothing of household consumption when confronted with fluctuations in their consumption plans. In this case they can borrow from or lend to a foreign country to obtain the same level of consumption. As explained before with habit persistence, economic agents are not only concerned by the actual level of consumption, but they are also intended to maintain the same consumption level period by period.

Despite this failure, the model is able to provide high equity premiums compared to those obtained by the standard RBC model (which gives no equity), and is also able to match selected business cycle statistics. For example, the standard deviation of output is 1.78, compared to about 1.72 for the Canadian data. The relative deviation between consumption and output is about 0.179 while the data gives 0.54, which can be explained by the model's smoothness of consumption. In fact, consumption's standard deviation at 0.32 is three times less volatile compared to the value of 0.93 obtained in the data[29]. These results show that the RBC model augmented with habit formation and adjustment costs fails to account for asset pricing statistics when a new element, the foreign

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[29]The consumption volatility has dropped to 0.32. Basically, the agents in this economy use the near linear technology to smooth out consumption, counteracting the effect on risk premia that habit formation has on the preference side.

sector, is introduced into the model. As discussed in Abel (1991), interest rate volatility is too high, representing a problem with habit persistence in that it makes this rate too volatile. Likewise, habit formation preferences display a strong aversion to intertemporal substitution which in turn leads to high variations in interest rates (Jermann, 1998). Furthermore, the model overpredicts riskless rate volatility in part because a high value of the parameter that governs habit persistence is needed to generate a sizeable equity premium. In models of this class, if one is willing to increase risk-aversion, less habit persistence is required to match mean asset returns, which simultaneously leads to lower volatility of the risk-free rate. However, Boldrin et al. (2001) point out that higher risk aversion also has adverse implications for employment dynamics in such models.

## 3.5  Impulse Response Functions

To get insight into the workings of the model, the second part of our analysis concerns a set of impulse responses to a unit positive productivity shock. The impulse responses for this version of the model, under the baseline calibration in Table 1.1 are shown in Figure 1.2 and 1.3. In this kind of model the output and investment responses to unit positive productivity impulses are standard. The dividend responses show that the dividends are procyclical, as they are in the model's closed economy version. Indeed Jermann (1998) found that, even with and without habit, dividends are more procyclical with respect to capital adjustment costs. The marginal utility response is also in line with what can be found in a closed economy version of the model, and it is negatively serially correlated with a hump-shaped response. Consumption also displays a hump-shaped response because, under habit formation, households smooth both the level and the change in consumption. The peak of the consumption and marginal utility responses takes place after 10 quarters. Responses of the other variables to unit technology shocks correspond to literature standards.

# 4   Concluding Remarks

Prior research on endowment model following Lucas (1978) and Campbell (1986) has focussed on various modifications of a standard RBC model in an effort to resolve its puzzling pricing implications. In fact, the version of the RBC model containing habit persistence and capital adjustment costs properly accounts for equity premiums and other asset pricing components.

The same model however when augmented by a foreign sector fails to generate substantial equity premiums nor explain equity generated by historical data in small open economy cases.

Using the lognormal-loglinear model solution, in this paper we evaluate asset prices in small open economy cases and highlight some shortcomings. First, as discussed above, the model generates low risk premia and its second shortcoming is that, consistent with the findings of Heaton (1995) and Boldrin et al. (1999) the volatility of the risk-free (and risky) rate is too high, This is a typical problem for those utility functions that display habit formation. Habit persistence makes marginal utility very volatile, even for smooth consumption profiles (Budría, 2002). This creates large fluctuations of the expected marginal utility at successive dates, and also involves large movements in the risk-free rate.

In summary, this model does well when compared to selected business cycle statistics, but fails to improve the performances obtained using the closed economy version of the model with respect to asset pricing. Obvious directions for future work include estimating the parameters used to generate the results, and finding features of technology that can prevent households from smoothing their consumption in an open economy model. Indeed, if we can limit the access to foreign debt (assets) the model obtained can allows one to solve for asset returns. These areas are a few of the many areas where further work might be focused in order to address the issue of best way to resolve the puzzle equity premium.

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# 5 Appendix

**Model setting**

**The household maximizes**

$$E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{(C_{t+j} - X_{t+j})^{1-\gamma_1}}{1-\gamma_1} \right], \tag{21}$$

**subject to the following constraint**

$$X_t = bC_{t-1} \tag{22}$$

and the budget constraint:

$$P_t^f B_{t+1} + P_{t+1}^e A_{t+1} + (1+\kappa_t)^{-1} P_t^{f*} B_{t+1}^* + C_t \leqslant W_t n + B_t + B_t^* + (P_t^e + D_t) A_t. \quad (\Lambda_t) \tag{23}$$

where $P_t^e$, $P_t^f$ and $P_t^{f*}$ denote the prices of the risky asset, the home and foreign riskless bond respectively. At time t, the equity pays a dividend payout $D_t$ and each bond pays one unit of the consumption good at time t+1 and expires. Here that $\kappa_t$ is expressed as:

$$\kappa_t = \exp(-\frac{\varphi \tilde{B}_t^*}{Y_t}),$$

We can introduce the first equation directly in the utility function. Thus the controls in this economy are: $C_t$, $B_{t+1}$, $B_{t+1}^*$, $A_{t+1}$.

**The firm's problem** The manager of the firm maximize the value of the firm to its owners:

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} \{ Z_{t+j} K_{t+j}^{\alpha} (X_{t+j} n_{t+j})^{1-\alpha} - W_{t+j} n_{t+j} - I_{t+j} \} \tag{24}$$

subject to the following constraints:

$$K_{t+1} = (1-\delta)K_t + (\frac{a_1}{1-\xi}(\frac{I_t}{K_t})^{1-\xi} + a_2)K_t, \; 0 < \delta < 1.$$

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$$Y_t = Z_t K_t^\alpha n_{t+j}^{1-\alpha}, \tag{25}$$

with $\beta^j \frac{\Lambda_{t+j}}{\Lambda_t}$ is the marginal rate of substitution of the household and $n_t$ the quantity of labor input. The state of technology evolves according to the AR(1) process:

$$Z_t = \exp(Z_{t-1}) \text{ or in log: } z_t = \rho_z z_{t-1} + \epsilon_{zt}, \tag{26}$$

with $z_{-1}$ given and $\epsilon_{zt}$ is a normally distributed white noise with mean $0$ and variance $\sigma_z^2$ for all $t \geqslant 0$. The controls here are: $I_t, n_t$.

**Other equations of the model**   The dividends equation:

$$D_t = Y_t - W_t n_t - I_t \tag{27}$$

**The resources constraint equation**   To compute the resources constraint equation we can set: $B_t = 0$ and we normalize $A_t = 1$;

The budget constraint can be written as:

$$P_{t+1}^e + \kappa_t^{-1} P_t^{f*} B_{t+1}^* + C_t \leqslant W_t n + B_t^* + (P_t^e + D_t)$$

and we know that:

$$D_t + W_t n_t = Y_t - I_t,$$

at equilibrium the strict equality holds, that is the resource constraint can be written as:

$$B_{t+1}^* - B_t^* = Y_t - C_t - I_t - \Delta P_{t+1}^e - ((1+\kappa_t)^{-1} P_t^{f*} - 1) B_{t+1}^* \tag{28}$$

The production function:

$$Y_t = Z_t K_t^\alpha n_{t+j}^{1-\alpha} \tag{29}$$

From the law of motion of capital one can isolate the term of investment:

$$I_t = [(\frac{K_{t+1}}{K_t} - 1 + \delta - a_2)\frac{(1-\zeta)}{a_1}]^{\frac{1}{1-\zeta}} K_t \tag{30}$$

## 5.1 The First Order Conditions

The dynamic of this economy is captured by the following equations from the household's problem (FOC):

$\partial/\partial c_t$ :

$$(C_t - bC_{t-1})^{-\gamma_1} - \beta b E_t (C_{t+1} - bC_t)^{-\gamma_1} = \Lambda_t, \tag{31}$$

$\partial/\partial B_{t+1}$ :

$$P_t^f = \beta E_t [\frac{\Lambda_{t+1}}{\Lambda_t}] \quad \text{Euler equation 1}, \tag{32}$$

$\partial/\partial B_{t+1}^*$ :

$$P_t^{f*} = \beta E_t [\frac{\Lambda_{t+1}}{\Lambda_t}](1 + \kappa_t) \quad \text{Euler equation 2}, \tag{33}$$

$\partial/\partial A_{t+1}$ :

$$P_t^e = \beta E_t [\frac{\Lambda_{t+1}}{\Lambda_t}(P_{t+1}^e + D_{t+1})] \quad \text{Euler equation 3}, \tag{34}$$

The first order conditions for the firm's problem are:

$\partial/\partial K_{t+1}$ :

$$
\begin{aligned}
0 = \quad & -\frac{1}{a_1}(\frac{k_{t+1}}{k_t} - 1 + \delta - a_2)^{\frac{\zeta}{1-\zeta}} + E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t}\{\alpha Z_{t+1} K_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} - [(\frac{k_{t+2}}{k_{t+1}} - 1 + \delta - a_2) \\
& (\frac{1-\zeta}{a_1})]^{\frac{1}{1-\zeta}} + \frac{1}{a_1}\frac{k_{t+2}}{k_{t+1}}[(\frac{k_{t+2}}{k_{t+1}} - 1 + \delta - a_2)(\frac{1-\zeta}{a_1})]^{\frac{\zeta}{1-\zeta}}]\}
\end{aligned}
$$

$\partial/\partial n_t$ :

$$(1 - \alpha)Z_t K_t^\alpha n_t^{-\alpha} = W_t \tag{35}$$

As shown by Jermann (1994), the leisure does not enter utility function, so Agents will allocate their entire time endowment to productive work (Fixed Labor Economy). In this framework n is fixed to it's steady state value (n=0.33).

## 5.2 Steady state

The equations of the model can be expressed at steady state in this manner:

$$K = \{(\frac{1}{\beta \alpha a_1 Z n^{1-\alpha}}[\bar{V}^\zeta + \beta a_1 \bar{V} - \beta(1 - \delta + \frac{a_1}{1 - \zeta}\bar{V} + a_2)\bar{V}^\zeta\}^{\frac{1}{\alpha-1}}$$

where

$$\bar{V} = [(\delta - a_2)\frac{(1 - \zeta)}{a_1}]^{\frac{1}{1-\zeta}}$$

$$W = (1 - \alpha)ZK^\alpha n^{-\alpha}$$

$$Y = ZK^\alpha n^{1-\alpha}$$

$$D = Y - Wn - I$$

$$I = [(\delta - a_2)\frac{(1 - \zeta)}{a_1}]^{\frac{1}{1-\zeta}} K]$$

$$\kappa = \exp(-\frac{\varphi B^*}{Y})$$

$$P^f = \beta$$

$$P^{f*} = \beta.\kappa$$

$$C = Y - I - \beta$$

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$$\Lambda = ((1-b)C)^{-\gamma_1}(1-\beta b)$$

$$P^e = \frac{\beta}{1-\beta}D \ .$$

## 5.3 The log-linearization of the model

We define the variables without $t$ subscripts as steady-state values and the variables with circumflex as a percentage deviation of the variables from their steady states. For example, $\hat{C}_t = (C_t - c)/c$ denotes the percentage deviation of consumption from its steady state (c).

Linearizing the first-order conditions yields:

$$\frac{\gamma_1 \beta b}{(1-b)(1-\beta b)}E_t\hat{C}_{t+1} = \frac{\gamma_1(\beta b^2 + 1)}{(1-b)(1-\beta b)}\hat{C}_t - \frac{\gamma_1 b}{(1-b)(1-\beta b)}\hat{C}_{t-1} + \hat{\Lambda}_t, \qquad (36)$$

$$\beta\hat{\lambda}_t + P^f\hat{P}_t^f = \beta E_t\hat{\Lambda}_{t+1}, \qquad (37)$$

$$P^e\hat{P}_t^e + \beta DP^e\hat{\Lambda}_t = \beta DP^e E_t\hat{\Lambda}_{t+1} + \beta DP^e E_t\hat{D}_{t+1} + \beta DP^e E_t\hat{P}_{t+1}^e \qquad (38)$$

$$P^{f*}\hat{P}_t^{f*} + \beta(1+\kappa)\hat{\Lambda}_t - \beta\kappa\hat{\kappa}_t = \beta(1+\kappa)E_t\hat{\Lambda}_{t+1} \qquad (39)$$

$$\zeta(\frac{I}{k})^{\zeta}\hat{k}_t - \zeta(\frac{I}{k})^{\zeta}\hat{I}_t + \beta\alpha a_1 n^{1-\alpha}k^{\alpha-1}\hat{Z}_{t+1} = [\beta\zeta(1-\delta+\frac{a_1}{1-\zeta}(\frac{I}{k})^{1-\zeta}+a_2)(\frac{I}{k})^{\zeta}-\beta\alpha a_1(\alpha-1)$$

$$Zk^{\alpha-1}n^{1-\alpha}]\hat{K}_{t+1}-\beta\zeta[1-\delta+\frac{a_1}{1-\zeta}(\frac{I}{k})^{1-\zeta}+a_2)(\frac{I}{k})^{\zeta}]\hat{I}_{t+1}$$

$$+\beta[\alpha a_1 ZK^{\alpha-1}n^{1-\alpha}-a_1\frac{I}{K}+(1-\delta+\frac{a_1}{1-\zeta}(\frac{I}{k})^{1-\zeta}+a_2)$$

$$(\frac{I}{k})^{\zeta}]E_t\hat{\Lambda}_{t+1}-\beta[\alpha a_1 ZK^{\alpha-1}n^{1-\alpha}-a_1\frac{I}{K}+(1-\delta$$

$$+\frac{a_1}{1-\zeta}(\frac{I}{k})^{1-\zeta}+a_2)(\frac{I}{k})^{\zeta}]\hat{\Lambda}_t \tag{40}$$

$$\alpha\hat{K}_t + \hat{Z}_t = \hat{W}_t, \tag{41}$$

And finally, the linearization of the identities:

$$\hat{K}_{t+1} = [1-\delta-a_1\frac{\zeta}{1-\zeta}(\frac{I}{K})^{1-\zeta}+a_2]\hat{K}_t + a_1(\frac{I}{K})^{1-\zeta}\hat{I}_t$$

$$\hat{\kappa}_t = -\frac{\varphi B^*}{Y}B_t^* + \frac{\varphi B^*}{Y}\hat{Y}_t, \tag{42}$$

$$\hat{Y}_t = \alpha\hat{K}_t + \hat{Z}_t \tag{43}$$

$$D.\hat{D}_t = Y.\hat{Y}_t - nW.\hat{W}_t - I.\hat{I}_t \tag{44}$$

$$Y.\hat{Y}_t = C.\hat{C}_t + I.\hat{I}_t \tag{45}$$

The tech. shock process:

$$z_t = \rho z_{t-1} + d\varepsilon_t, \tag{46}$$

The controls in this economy:

$Y_t, C_t, I_t, W_t, P_t^e, P_t^f, B_t^*, D_t, \kappa_t.$

The state and co-state variables:

$K_t, C_{t-1}, \Lambda_t.$

We assume that the foreign exogenous variable $P_t^{f*}$ is constant for all t. So the exogenous variable are:

$Z_t, P_t^{f*}.$

**Table 1:** Model Calibration

| Parameter | value assigned |
| --- | --- |
| $\rho_z$ | 0.94436 |
| $\sigma_z$ | 0.00599 |
| $\beta$ | 0.96 |
| $\bar{x}$ | 0.0040 |
| $\alpha$ | 0.32 |
| $b$ | 0.58 |
| $\delta$ | 0.025 |
| $\gamma$ | 2.0 |
| $\zeta$ | 1/0.23 |
| $\varphi$ | 0.006 |

**Table 2:** Equity Premium Statistics

| Statistics | Model | Data* |
|:---:|:---:|:---:|
| $E(r^e - r^f)$ | 0.0247 | 3.47 |
| $\sigma(r^e - r^f)$ | 79.252 | 15.53 |
| $\sigma(r^e)$ | 78.864 | na |
| $\sigma(r^f)$ | 7.058 | na |
| $\sigma(\Delta \ln(C))$ | 0.6822 | 2.04 |
| $\rho(r^e - r^f, \Delta \ln(C))$ | 0.004782 | 0.33 |
| $\mathrm{cov}(r^e - r^f, \Delta \ln(C))$ | 0.2584 | 10.58 |

\* Data statistics are from Allais et al.(2000)(we report the case of Canada).

$Note$ : The first column represents the average excess return. The second and fifth

column are the standard-errors of the excess return and the consumption growth.

In the last two column the covariance and correlation coefficients between the

excess return and the consumption growth are represented.

Moments are averages of 100 replications of length 500.

**Table 3:** Business cycle Statistics

| Statistics | Model | Data* |
|:---:|:---:|:---:|
| $\sigma_Y$ | 1.78003 | 1.72 |
| $\sigma_C$ | 0.31971 | 0.93 |
| $\sigma_I$ | 2.18099 | 5.13 |
| $\sigma_{Pe}$ | 1.50586 | na |
| $\sigma_D$ | 1.52692 | na |
| $\sigma_C/\sigma_Y$ | 0.17961 | 0.54 |
| $\sigma_I/\sigma_Y$ | 1.22526 | 2.98 |
| $\rho(Y,C)$ | 0.88255 | 0.80 |
| $\rho(Y,I)$ | 0.99038 | 0.77 |

* Data statistics are taken from Letendre (2003).

$Note$ : This study uses quarterly Canadian Data (from 1981Q1 to 2001Q4)

filtered with HP (here we use the same filter for moment computations).
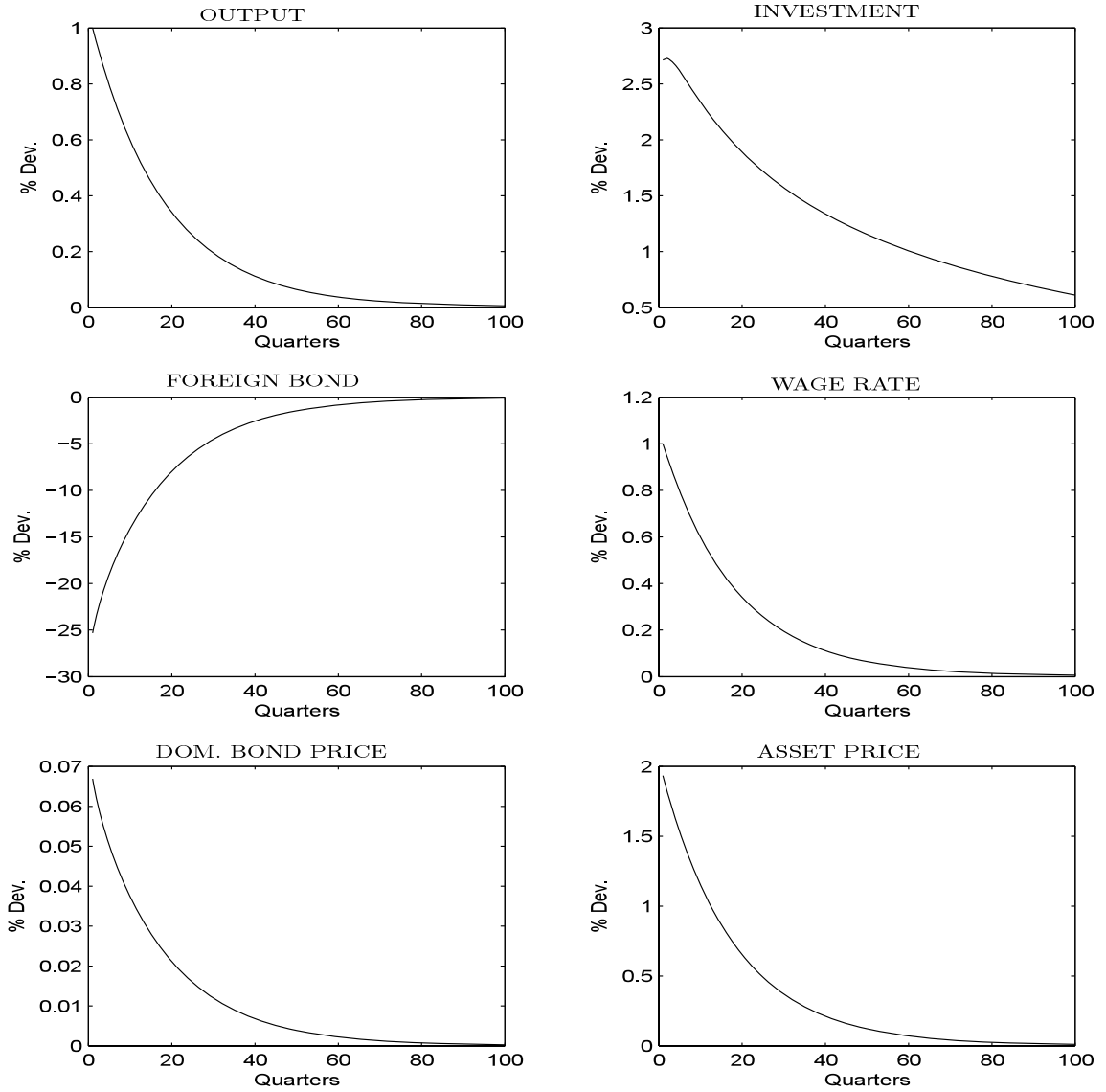
**Figure 1:** The Cost of Adjustment Function
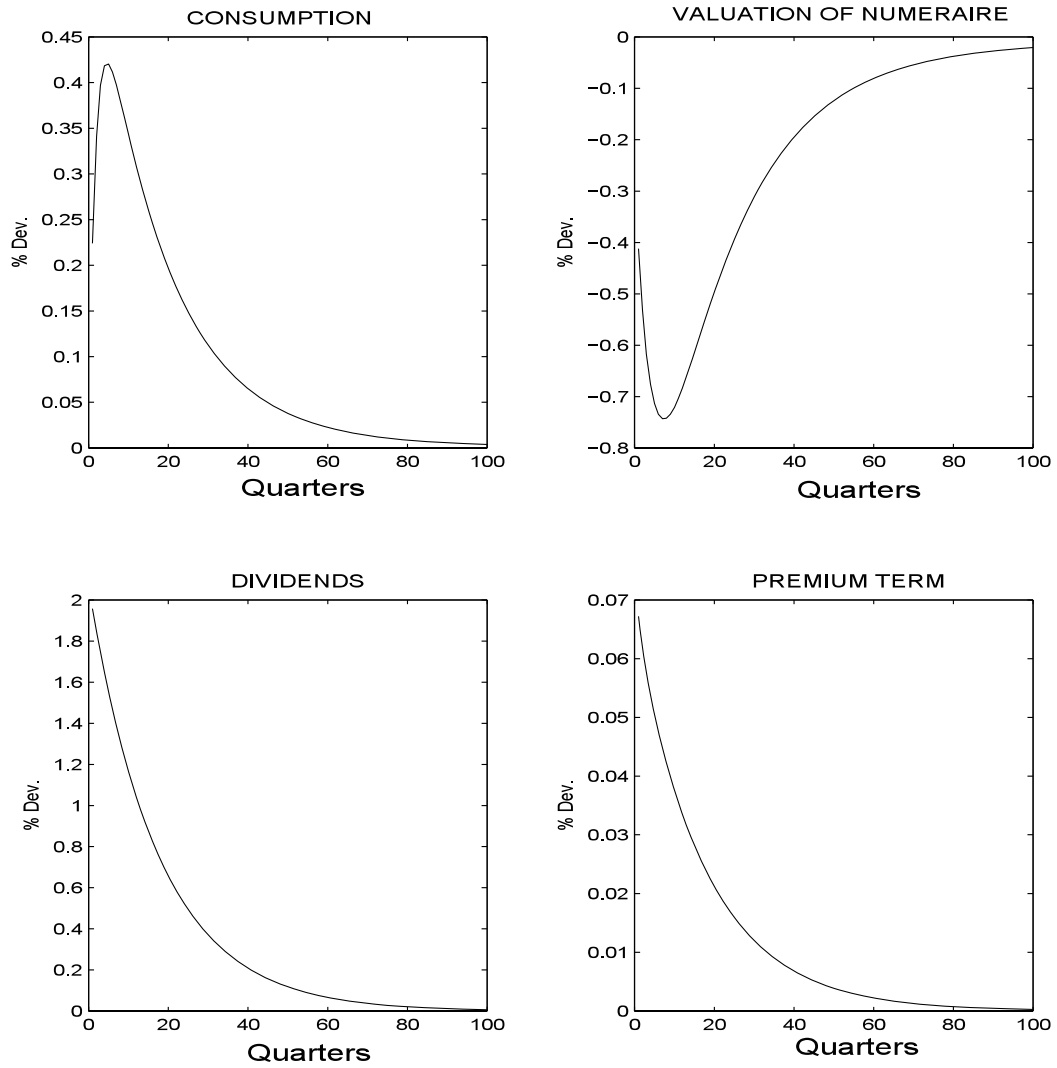
(Source: Budría, 2002)
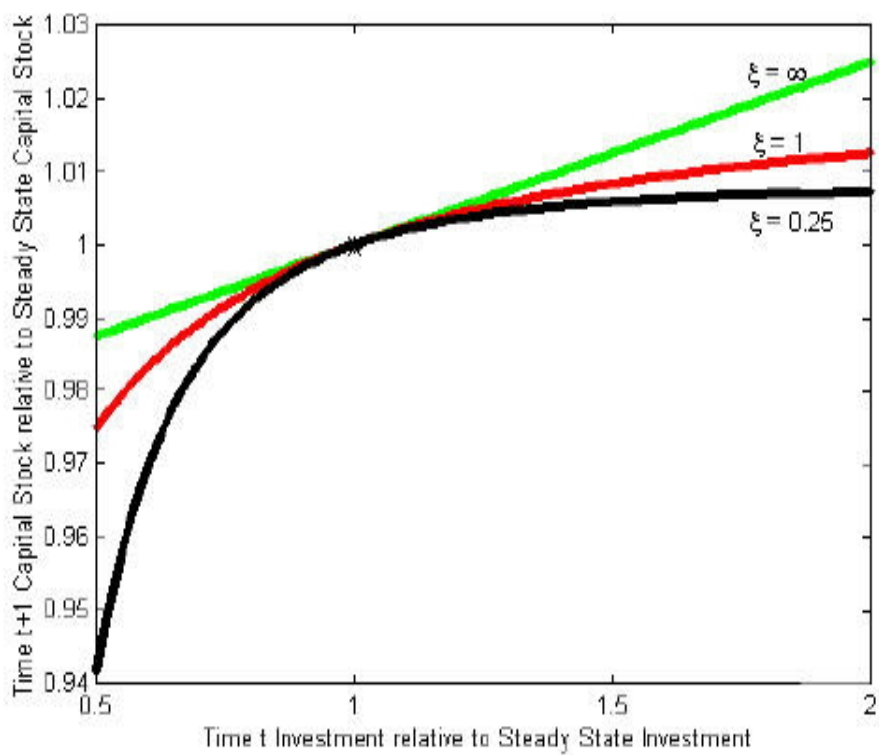


Note: This graphic is based on calibrations similar to what we use in our model.

**Figure 2:** Impulse Response Functions to a Unit Technology Shock



Note: The impulse is a unit positive productivity shock, the responses are in percent deviations from steady state values.

**Figure 3:** Impulse Response Functions to a Unit Technology Shock



Note : The valuation of numeraire is equivalent here to the marginal utility.