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Productivity shocks and Optimal Monetary Policy in a Unionized Labor Market Economy
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Abstract

This paper presents a New Keynesian model characterized by labor indivisibilities, unemployment and a unionized labor market. The bargaining process between unions and firms introduces real wage rigidity and creates an endogenous trade-off between inflation and output stabilization. Under an optimal discretionary monetary policy a negative productivity shock requires an increase in the nominal interest rate. Moreover, an operational instrument rule will satisfy the Taylor principle, but will also require that the nominal interest rate does not necessarily respond one to one to an increase in the efficient rate of interest. The model calibration studies the response of the unionzed economy to productivity shocks under different monetary policy rules.

1 Introduction

In the last ten years, the Dynamic Stochastic General Equilibrium New Keynesian (DSGE-NK henceforth) model has emerged as an important paradigm in macroeconomics and as a useful framework for the study of monetary policy. Most of the models proposed so far, however, completely ignore the role that trade unions play in determining wages and employment conditions in many countries. If this is probably an acceptable (although very strong) simplification for countries, like the U.S. where, in the year 2002, only about 15% of workers were covered by collective contract agreements,

it becomes instead problematic for other countries such as France, Italy or Sweden where the percentage of workers covered by collective contracts is above 84%.[1] Given that wage bargaining may introduce significant distortions in the functioning of a modern economy and have an impact on its behavior at the aggregate level, the study of unionized labor markets and of the consequences of these markets for monetary policy becomes of crucial importance if one wants to understand the functioning of many important economies around the world.

The purpose of this paper is to propose a model where i) movements of the rate of unemployment are explicitly accounted for; ii) wages are the result of a contractual process between unions and firms. We accomplish the first aim by assuming, as in Hansen [35] and Rogerson and Wright [46], that labor supply is indivisible and that workers face a positive probability of remaining unemployed. We deal with the second problem by assuming that wages are set by unions according to the popular monopoly-union model introduced by Dunlop [19] and Oswald [41].

Unions, in our paper, do not simply maximize the utility of their members, but are institutions that also have "political" objectives in the sense that their objective function takes into account the preferences of workers, the preferences of leaders and market constraints. In this respect we take side on the old and never settled debate initiated by Dunlop [19] and Ross [48] over the appropriate maximand for the unions' utility function, and we assume that the unions' objective function is a Stone-Geary utility function as in Dertouzos and Pencavel [17], Pencavel [42] and, more recently, by De la Croix et al. [16], Raurich and Sorolla [45] and by Chang et al. [10]. This function is extremely flexible and, depending on parameter values, allows for different distribution of power, inside the union, between members and leaders who may have diverging objectives. Moreover, the fact that unions maximize such a function, which includes among its arguments the actual real wage, the reservation wage as well as the level of employment, allows us to introduce real wage rigidity in the model. This distortion is actually the result of the divergence between the union's objective and the households' utility function: a union that simply maximized a utility function would simply set the real wage as a constant mark-up over the competitive wage, in which case the real wage would be fully flexible.

The papers that are closest to this one are the RBC model by Maffezzoli [39] and the New Keynesian model proposed by Zanetti [58].[2] Other models

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[1]More precisely, the number of persons covered by collective agreeements over total employment was 94.5% in France in 2003, 84.1% in Italy in the year 2000 and 85.1% in Sweden in the year 2000. For a complete set of data on union coverage on the various countries see Lawrence and Ishikawa [37].

[2]Both Maffezzoli [39] and Zanetti [58] assume that unions simply maximize a risk neutral utility function, which is a special case of the more general union's objective function we consider in this paper.

characterized by labor market frictions and price staggering, where labor is allowed to move not only along the intensive margin but also along extensive margin, have been proposed also, by Chéron and Langot [11], Walsh [54] [55], Trigari [53], Moyen and Sahuc [40] and Andres et al. [2]. All these models show that search and matching frictions improve the ability of the standard New Keynesian model to replicate the dynamics of inflation and unemployment by explaining, in particular, the persistence of output and the sluggishness of inflation. More recently Christoffel and Linzert [14] and Blanchard and Galì [6] [7] have proposed models characterized by both labor market frictions and real wage rigidities. Blanchard and Galì [6] show that, if real wages are assumed to adjust slowly, what they define as the "divine coincidence" does not hold any more: for a central bank pursuing, as a policy objective, the level of output that would prevail under flexible prices is not equivalent to pursuing the efficient level of output, in which case a trade-off between inflation stabilization and output gap stabilization arises. Blanchard and Galì [7] use a model with search and matching frictions and sluggish real wages, and show that a policy trade-off does not only pertain to the output gap, but also to the rate of unemployment.

Our model differs from previous studies in several respects. First, as in Zanetti [58] we abstract from the search and matching frictions based on the Mortensen-Pissarides [43] model, and we concentrate on the consequences of union behavior, studying in particular the implications of microfounded real wage rigidities. An important aspect of our framework with labor indivisibilities is that wage rigidities apply also to ongoing relationships and not only to new hirings, a problem that is usually found when wage rigidities are introduced in models with search and matching frictions (see for example Thomas [52]). Moreover, differently from Zanetti, we propose a simpler model without human and physical capital accumulation and we are able, therefore, to study analytically the optimal interest rate rule a Central Bank should implement in our unionized economy.[3] Morever, by assuming Rogerson and Wright [46] indivisible labor model, we are able to analyze unemployment in a simple and tractable way which allows us to establish an inverse relationship between unemployment and the output gap. The model is capable of producing a series of interesting results.

First, it shows that productivity shocks and reservation wage shocks give rise to a significant policy trade-off between stabilizing inflation and stabilizing unemployment, and in this respect it provides a way to overcome an important shortcoming of the NK model, i.e. its inability to account for the significant challenges that exogenous changes in technology represent for monetary policy in the real world. According to the "standard" NK

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[3]As we will show in the calibration section, the introduction of phisycal capital does not change the qualitative dynamics of our model. If we calibrate the model using the Taylor-type rule assumed by Zanetti [58] we show that our impulse response functions are very similar to the ones obtained by this author.

model, in fact, an optimal monetary policy that stabilizes output around its flexible price equilibrium also produces zero inflation,[4] so that stabilizing inflation implies automatically an optimal response to a productivity shock. This, however, not only is at odds with the historical accounts[5] and the widespread perception of financial markets, but there is also some recent empirical evidence indicating that, in most countries, central banks have actively responded to technology shocks, increasing or decreasing the nominal interest rate.[6] What is interesting, in our model, is that this result is not the consequence of some kind of exogenous real wage inertia, as in Blanchard and Galì [6], but is simply the consequence of monopolistic unions pursuing a well defined contractual strategy in the labor market. In our economy, in fact, a productivity slowdown, i.e. a negative productivity shock, tends to lower efficient output but, since unions will keep real wages constant, the level of output that would prevail under price flexibility (that we define as "natural" output) decreases even more, so that the difference between efficient output and "natural" output increases. Since in sticky price models inflation depends on marginal costs and, in turn, marginal costs depend on the difference between "natural" output and actual output, then a Phillips curve, correctly defined as depending on the gap between efficient output and actual output, will depend on productivity shocks, and a trade-off between inflation stabilization and output gap stabilization arises.

Second, we show that a policy trade-off for the central bank arises not only in response to technology shocks, but also in response to exogenous wage push shocks. If the unions' reservation wage is subject to exogenous changes, and these changes tend to be persistent over time, then a welfare maximizing central bank must again face the problem of whether to accommodate these shocks with a easier monetary policy. Our model therefore provides a convenient framework to address important normative issues such as, for example, the optimal behavior of central banks in periods characterized by labor market turmoil and wage shocks.

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[4] This is shown quite clearly, for example, by Galì, Lopez Salido and Valles [25].

[5] There is a lot of anectodical evidence that the Fed has spent large efforts in understanding the increase in productivity growth that has characterized the American economy since the mid 1990s. The success of monetary policy in this period has been attributed by important commentators to the ability of the Fed to respond to exogenous technological progress.

[6] Galì et al. [24] found that the Fed, in the post-Volcker period, did not change the nominal interest rate in response to productivity shocks. In a recent paper, Francis, Owyang and Theodoru [22] analyze monetary policy for the G7 countries and found a wide range of variation in the behavior of different countries: while France, Japan, the United Kingdom (after the break in monetary regime) seem to have reacted to a technology shock by increasing nominal interest rates, the U.S. (before and after Vocker), Canada, Germany, the U.K (pre-break) and Italy, seem to have reacted instead by lowering interest rates. A policy of no-reaction to a productivity shock consistent with the prescriptions of the standard DSGE-New Keynesian model, therefore, does not seem to have been widely adopted by the major industrialized countries.

Third, we are able to derive, also in a model with non separable utility and indivisible labor, the objective function of the central bank as a second order Taylor approximation of the expected utility of the representative household, and we show that, when the economy is hit by technology and wage shocks, monetary policy presents some interesting peculiarities relative to the standard case. An optimal discretionary policy requires an increase in the interest rate following a negative productivity shock and an increase in the interest rate following a positive reservation wage shock. An optimal instrument rule that implements such policy can be expressed as an interest rate reacting to the expected rate of inflation and to the efficient rate of interest. In this model monetary policy satisfies the Taylor principle, i.e. the nominal interest rate must be raised more than proportionally with respect to the expected rate of inflation. The response of the nominal interest rate to the "efficient" rate of interest, however, is not one to one like in the standard model: if the persistence of the technological shock is greater than the persistence of the reservation wage shock, the nominal interest rate will increase less than proportionately to an increase in the efficient rate of interest. This is due to the fact, unlike what happens in the standard model, the central bank must react, at the same time, both to a technology shock and to a reservation wage shock.

A fourth, important result is that the model is able to account for a well known stylized fact in macroeconomics, i.e. the relatively smooth behavior of wages and the relatively volatile behavior of unemployment over the business cycle. When the level of unemployment that the economy achieves under an optimal discretionary policy is written as a function of the relevant shocks, an exogenous wage shock will in general induce a movement both in the real wage and in the rate of unemployment; a productivity shock, instead, will induce a movement in the rate of unemployment, but not in the real wage. An economy frequently hit by exogenous changes in technology will show, therefore, a strong variability in the rate of unemployment without experiencing, at the same time, significant movements in the real wage.[7]

Finally, the model is calibrated to the Euro area quarterly data. We start by comparing our economy characterized by staggered prices and real wage rigidity with an economy characterized by flexible prices and real wage rigidity under the interest rate rule estimated by Smets and Wouters [57]. This allows us to understand better which is the role of real wage rigidity in the transmission of monetary policy. We then compare the dynamics of the unionized economy with the dynamics of an economy with a walrasian labor market. We find that the response of the relevant variables to a productivity shock is usually larger and more persistent in the unionized economy than

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[7]Also Gertler and Trigari [32] propose a model where wages and unemployment move consistently with the observed data. They achieve this result, however, by introducing exogenous multiperiod wage contracts.

in the one characterized by walrasian labor markets. One important result arising from the calibration exercise is that when the central bank utilizes the Smets and Wouters [57] interest rate rule the model is consistent with the positive correlation between technology shocks and the labor input found in the data. An interest rate rule that replicates the optimal one, moreover, implies a larger response to inflation and a smaller response to the output gap than the one usually implied by standard Taylor-type rules.

The paper is organized as follows. In Section 2 we discuss the standard indivisible labor model (hereafter, IL model). In Section 3 we develop the monopoly union model (hereafter, MU model). In Section 4 we study optimal monetary policy in the MU model. The model is calibrated in section 5.

2 The Indivisible Labor Model

2.1 Households

We consider an economy populated by many identical, infinitely lived worker-households each of measure zero. Households demand a Dixit, Stiglitz [18] composite consumption bundle produced by a continuum of monopolistically competitive firms. In each period households sell labor services to firms. As in Hansen [35], Rogerson [46] and Rogerson and Wright [47], for each household the alternative is between working a fixed number of hours and not working at all. We assume that agents enter employment lotteries, i.e. sign, with a firm, a contract that commits them to work a fixed number of hours, that we normalize to one, with probability N_t. Markets are complete and the contract itself is traded, so a household gets paid whether it works or not. Through this mechanism firms are able to provide complete unemployment insurance to the workers. Since all households are identical, all will choose the same contract, i.e. the same N_t. However, although households are ex-ante identical, they will differ ex-post depending on the outcome of the lottery: a fraction N_t of the continuum of households will work and the rest $1 - N_t$ remains unemployed. The allocation of individuals to work or leisure is determined completely at random by the lottery, and lottery outcomes are independent over time.

Before the lottery draw, the expected intratemporal utility function is:

$$\frac{1}{1-\sigma} N_t \left[C_{0,t} \upsilon(0) \right]^{1-\sigma} + \frac{1}{1-\sigma} (1 - N_t) \left[C_{1,t} \upsilon(1) \right]^{1-\sigma} \tag{1}$$

where $C_{0,t}$ is the consumption level of employed individuals, $C_{1,t}$ is the consumption of unemployed individuals, N_t is the ex-ante probability of being employed and $\upsilon(\cdot)$ is the utility of leisure. Since the utility of leisure of employed individuals $\upsilon(0)$ and the utility of leisure of unemployed individuals $\upsilon(1)$ are positive constants, we assume $\upsilon(0) = \upsilon_0$ and $\upsilon(1) = \upsilon_1$. As in

King and Rebelo [34], we assume $v_0 < v_1$. Under the assumption of complete markets we obtain:

$$C_{0,t}^{-\sigma} v_0^{1-\sigma} = C_{1,t}^{-\sigma} v_1^{1-\sigma} \tag{2}$$

so that the marginal utilities of consumption are equal for employed and unemployed individuals. Defining the average consumption level as: $C_t = N_t C_{0,t} + (1 - N_t) C_{1,t}$ and given (2), equation (1) can be rewritten as:

$$\frac{1}{1-\sigma} C_t^{1-\sigma} \left[N_t v_0^{\frac{1-\sigma}{\sigma}} + (1 - N_t) v_1^{\frac{1-\sigma}{\sigma}} \right]^{\sigma}. \tag{3}$$

This allows us to write the life-time expected intertemporal utility function of a representative household as:

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\sigma} \left[C_t \phi\left(N_t\right) \right]^{1-\sigma}, \ \sigma > 1, \ 0 < \beta < 1 \tag{4}$$

β is the subjective discount rate, while $\phi\left(N_t\right) = \left[N_t v_0^{\frac{1-\sigma}{\sigma}} + (1 - N_t) v_1^{\frac{1-\sigma}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}$ can be interpreted as the disutility of employment for the representative household. The elasticity of $\phi\left(N_t\right)$ with respect to its argument is given by $\xi_\phi = \frac{\phi_N(N_t)}{\phi(N_t)} N < 0$. The flow budget constraint of the representative household is given by:

$$P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} + \Pi_t - T_t \tag{5}$$

where P_t is the corresponding consumption price index (CPI) and W_t is the wage rate. Notice that here a worker is paid according to the probability that it works, not according to the work it does; in other words, the firm is automatically providing full employment insurance to the households. The purchase of consumption goods, C_t, is financed by labor income, profit income Π_t, and a lump-sum transfers T_t from the Government. We denote by B_t the quantity of nominally riskless one period bonds carried over from period $t-1$, and paying one unit of the numéraire in period t.[8] The maximization of (4) subject to (5) gives:

$$1 = \beta R_t E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{\phi\left(N_{t+1}\right)}{\phi\left(N_t\right)} \right)^{1-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{6}$$

$$\frac{W_t}{P_t} = -C_t \frac{\phi_N\left(N_t\right)}{\phi\left(N_t\right)} \tag{7}$$

where equation (6) is the standard consumption Euler equation and (7) gives us the supply of labor of the representative household.

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[8] As is standard in New Keynesian models, government bonds are introduced here as a simple way to allow for the existence of a nominal interest rate in the economy, which will be the policy instrument of the Central Bank.

2.2 The Finished Goods-Producing Sector

The representative finished goods-producing firm uses $Y_t(j)$ units of each intermediate good $j \in [0, 1]$ purchased at a nominal price $P_t(j)$ to produce Y_t units of the finished good with the constant returns to scale technology:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}} \tag{8}$$

where θ is the elasticity of substitution across intermediate goods. Profit maximization yields the following set of demands for intermediate goods: $Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t$, where $P_t = \left[\int_0^1 P_t(j)^{1-\theta} \right]^{\frac{1}{1-\theta}}$ is the aggregate price index.

2.3 The Intermediate Goods-Producing Sector

We abstract from capital accumulation and assume that there is a continuum of intermediate good-producing firms $j \in (0, 1)$ which hire $N_t(j)$ units of labor from the representative household and produce $Y_t(j)$ units of the intermediate good using the following technology:

$$Y_t(j) = A_t N_t(j)^{\alpha} \tag{9}$$

where A_t is an exogenous productivity shock. We assume that $\ln A_t \equiv a_t$ follows the autoregressive process

$$a_t = \rho_a a_{t-1} + \hat{a}_t \tag{10}$$

where $\rho_a < 1$ and $\hat{a}_t$ is a normally distributed serially uncorrelated innovation with zero mean and standard deviation σ_a. Before choosing the price of its goods, a firm chooses the level of $N_t(j)$ which minimizes its total costs, $TC_t = (1 - \tau) W_t N_t(j)$ subject to (9), where τ represents an employment subsidy to the firm[9]. The first order condition with respect to $N_t(j)$ is given by:

$$(1 - \tau) \frac{W_t}{P_t} = MC_t(j) \alpha \frac{Y_t(j)}{N_t(j)}, \tag{11}$$

where $MC_t(j)$ represent firm's j real marginal costs.[10] The aggregate real marginal costs are defined as:

$$MC_t = \frac{(1 - \tau)}{\alpha} \frac{W_t}{P_t} \frac{N_t}{Y_t}, \tag{12}$$

and therefore $MC_t = MC_t(j) \frac{Y_t(j)}{N_t(j)} \frac{N_t}{Y_t}$.

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[9] We assume that the subsidy is covered by a lump sum tax in that the Government runs always a balanced budget.

[10] The assumption of decreasing return to scale technology, which is in line with a non-competitive intermediate good sector, has important implication on the optimal price-settings rule, and then on the derivation of the traditional Phillips curve. See appendix A5.

2.4 Market clearing

Equilibrium in the goods market of sector j requires that the production of the final good be allocated to expenditure, so that $Y_t(j) = C_t(j)$. In aggregate terms it implies that $Y_t = C_t$, which represents the economy resource constraint. As standard, its log-linearization around the efficient steady state implies,

$$y_t = c_t \tag{13}$$

Since the net supply of bonds, in equilibrium is zero, we have that $B_t = 0$. Labor market clearing implies $N_t = \int_0^1 N_t(j)\,dj$. The aggregate production function is instead given by, $D_t Y_t = A_t N_t^\alpha$, where $D_t = \left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{\theta}{\alpha}} dj\right]^\alpha$ is a measure of price dispersion. Given that in a neighborhood of a symmetric equilibrium and up to a first order approximation $D_t \simeq 1$, the log-linearized aggregate production function can be expressed as,

$$y_t = a_t + \alpha n_t. \tag{14}$$

2.5 The First Best Level of Output

The efficient level of output can be obtained by solving the problem of a benevolent planner that maximizes the intertemporal utility of the representative household, subject to the resource constraint and the production function. This problem is analyzed in the Appendix A1, where we show that the efficient supply of labor, in our economy, is given by:

$$\frac{\phi_N(N_t)}{\phi(N_t)} N_t = -\alpha. \tag{15}$$

Log-linearizing (15), and considering (14), we obtain[11]

$$y_t^{Eff} = a_t. \tag{16}$$

2.6 The Flexible Price Equilibrium and the Natural Output

Equilibrium in the labor market is obtained by equating (7) and (12).

$$-Y_t \frac{\phi_N(N_t)}{\phi(N_t)} = \frac{1}{(1-\tau)} \alpha M C_t \frac{Y_t}{N_t} \tag{17}$$

where we have considered that $C_t = Y_t$. Under flexible prices, all firms set their prices equal to a constant markup over marginal cost. Assuming that firms mark-up, μ_t^P is constant, under the flexible price-equilibrium firms real

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[11]See appendix A2.

marginal costs are constant at their steady state level and therefore given by $MC_t = \frac{1}{1+\mu^P}$. Considering now the log-linearization of (17) we obtain[12],

$$mc_t = \left[1 + \frac{\phi_N(N)}{\phi(N)}N - \frac{\phi_{NN}(N)N^2}{\phi(N)\alpha}\right] n_t \qquad (18)$$

Considering that $mc_t = 0$, then $n_t = 0$ and, from the aggregate production function, we have that under the flexible price equilibrium:

$$y_t^f = a_t \qquad (19)$$

Taking the difference between the log-linearized flexible and efficient output we obtain:

$$y_t^{Eff} - y_t^f = 0 \qquad (20)$$

As in the standard NK model, when labor market is frictionless the difference between the efficient output (its first best) coincides with its flexible price equilibrium level (its second best) that we have defined as the natural level of output. In other words, what Blanchard and Galì [6] call "the divine coincidence" will hold, since any policy that stabilizes output around its natural level, will stabilize it also around its efficient level.

2.7 The Phillips Curve

Firms choose $P_t(j)$ in a staggered price setting à la Calvo-Yun [8]. In the appendix A5 we show that, in our decreasing return to scale economy, the solution of the firm's problem is given by:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_\alpha mc_t \qquad (21)$$

where $\lambda_\alpha = \frac{(1-\psi)(1-\beta\psi)}{\psi}\frac{\alpha}{\alpha+\theta(1-\alpha)}$ and ψ is the probability with which firms reset prices.[13]
Given (14), (18) and (19), marginal costs can be rewritten in terms of the gap between actual and natural output,

$$mc_t = \left(1 + \frac{\phi_N(N)}{\phi(N)}N - \frac{\phi_{NN}(N)N^2}{\phi(N)\alpha}\right)\left(y_t - y_t^f\right) \qquad (22)$$

so that, equation (21) can be rewritten as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \left(1 + \frac{\phi_N(N)}{\phi(N)}N - \frac{\phi_{NN}(N)N^2}{\phi(N)\alpha}\right)x_t \qquad (23)$$

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[12]See appendix A3.

[13]It is worth noticing that because of decreasing returns technology the output gap coefficient, λ_a, of the NK Phillips curve, is lower than the traditional coefficient found with constant return to scale. This means that the elasticity of inflation to output gap is lower than in the case of constant returns, which is consistent with the empirical estimates. In fact, inflation does not seems to respond strongly to the output gap. See for example Galì et al ([31]) and Sbordone ([49]) for a more exaustive explanation.

where

$$x_t = y_t - y_t^f \tag{24}$$

is the output gap with respect to the natural rate of output. As in the standard case there is no trade-off between output stabilization and inflation stabilization, since a central bank that sets inflation to zero will immediately stabilize output.

3 The Monopoly Union Model

As in the previous subsection the individual labor supply is indivisible. Each firm is endowed with a pool of households from which it can hire. In fact, as in Maffezzoli [39] and Zanetti [58], firms hire workers from a pool composed of infinitely many households so that the individual household member is again of measure zero. Since each household supplies its labor to only one firm, which can be clearly identified, workers try to extract some producer surplus by organizing themselves into a firm-specific trade union. The economy is populated by decentralized trade unions, so that each intermediate goods-producing firm negotiates with a single union $i \in (0,1)$ which is too small to influence the outcome of the market. Unions negotiate the wage on behalf of their members.

Once unions are introduced in the analysis, two important issues arise: what is the objective function of the union and what are the variables subject to bargaining. Both these questions have been extensively investigated by the literature, although no conclusive agreement has been reached on the issue.[14] The problem of identifying an appropriate maximand for the union dates back to Dunlop [19] and Ross [48]; since then the debate has revolved over the relative importance of economic considerations (basically how employers respond to wage bargaining) and political considerations in the determination of union wage policy. For political considerations we intend how the preferences of workers, the preference of union leaders and market constraints interact in determining a union's objective. One approach often followed in the literature is the "utilitarian" approach pioneered by Oswald [41] which consists on assuming that all workers are equal and that the union simply maximizes the sum of workers' utility, defined over wages. An alternative approach , initially proposed by Dertouzos and Pencavel [17] and Pencavel [42] and, more recently, reproposed by De la Croix et al. [16] and Raurich and Sorolla [45], is to assume that unions maximize a modified Stone-Geary utility function of the form:

$$V\left(\frac{W_t(i)}{P_t}, N_t(i)\right) = \left(\frac{W_t(i)}{P_t} - \frac{W_t^r(i)}{P_t}\right)^\gamma N_t(i)^\varsigma \tag{25}$$

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[14]For a survey of unions model see Farber [20], and, more recently, Kaufman [33].

The relative value of γ and ς is an indicator of the relative importance of wages and employment in in the union's objective function.[15] The reservation wage $W_t^r(i)$ is the absolute minimum wage the union i can tolerate. This reservation wage has many possible interpretations. One possible interpretation is that W_t^r is the opportunity wage of the workers (Pencavel 1984) since it is unlikely that a union can survive if it negotiates a wage below such level. Another possible interpretation is that W_t^r is what Blanchard and Katz [5] define as an "aspiration wage", i.e. a wage that workers have come to regard as "fair". Unions' reservation wage is generally unobservable and therefore hard to model. As in De la Croix[16] [16], however, we assume that:

$$\frac{W_t^r(i)}{P_t} = \varpi e^{\varepsilon_t^w} \tag{26}$$

where $\varpi > 0$ is a positive constant and with

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \hat{\varepsilon}_t^w \tag{27}$$

where $\rho_w < 1$ and ε_t^w is a normally distributed serially uncorrelated innovation with zero mean and standard deviation σ_w. If the real reservation wage is constant, $\hat{\varepsilon}_t^w = 0$. The fact that the reservation wage is subject to persistent shocks is meant to capture the exogenous wage shocks, often associated with political and social factors that have often characterized industrialized economies, especially in Europe.[17]

The Stone-Geary utility function not only is appealing, both for its ability to approximate the actual behavior of unions and for its flexibility and tractability, but also for its generality. The parameters γ and ς correspond to the elasticities of the union's objective $V(\cdot)$ to the excess wage $\frac{W_t(i)}{P_t} - \frac{W_t^r(i)}{P_t}$ and to the employment level $N_t(i)$ respectively. The larger the difference $\varsigma - \gamma$, the more the union approaches the extreme of a "democratic" (or "populist") union. When $\varsigma = \gamma$, these two parties have an identical discretionary power in formulating policies. If unions are "wage oriented" then $\gamma > \varsigma$, on the other hand if they are "employment oriented" $\gamma < \varsigma$. If we

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[15]Our objective function is closed to the one suggested by the Ross tradition. In fact, for different parameters values, the union's objective function is almost equivalent to the one of a union which maximizes his income or his membership, as for example in Skatun [50] and in Booth [4].

[16]In the model of De la Croix et al. [16] the real reservation wage is a weighted sum of a constant term and of the past real wage. In order not to add further ad-hockery to the model, we chose not to include past real wages. Nevertheless adding these to equation (26) would leave the results unchanged. A technical appendix is avalable upon request.

[17]We consider both these two alternative in order to show that the our results on the endogenous inflation unemployment (output) trade-off is not qualitatively influenced by the fact that the reservation wage shock is an exogenous shock. Moreover, to our knowledge this is the first attempt to study how the optimal interest rate rule should react in response to more than one supply shock.

set $\gamma = 1$, $\varsigma = 1$ and $\hat{\varepsilon}_t^w = 0$, maximizing (25) is equivalent to maximize the unions' objective function assumed by Maffezzoli [39] and Zanetti [58] in their recent papers. The bargaining process we consider here is in the tradition of the "right to manage" models. In particular, we follow the popular "monopoly union" model first proposed by [19] and Oswald [41], where the employment rate and the wage rate are determined in a non-cooperative dynamic game between unions and firms. We restrict the attention to Markov strategies, so that in each period unions and firms solve a sequence of independent static games. Each union behaves as a Stackelberg leader and each firm as a Stackelberg follower. Once the wage has been chosen, each firm decides the employment rate along its labor demand function. Even if unions are large at the firm level, they are small at the economy level, and therefore they take the aggregate wage as given. The ex-ante probability of being employed is equal to the aggregate employment rate and the allocation of union members to work or leisure is completely random and independent over time. Finally, as in the previous IL economy, we assume that workers are able to perfectly insure themselves against the possibility of being unemployed. This result can either be obtained through the lottery mechanism previously described or by assuming, as in Maffezzoli [39] and Zanetti [58] that, in order to impede workers from leaving the Union, the Union pursues a redistributive goal, acting as a substitute for competitive insurance market. Insurance is supplied under zero-profit condition and is therefore actuarially fair. The problem of the firm is the same as in the IL model.

From the first order conditions of the union's maximization problem with respect to $W_t(i)$, given that in this model the labor demand elasticity with respect to the real wage $\frac{1}{1-\alpha}$ is constant, after imposing the symmetric equilibrium we obtain:

$$\frac{W_t}{P_t} = \frac{\varsigma}{\varsigma - \gamma(1-\alpha)} \frac{W_t^r}{P_t}.$$ (28)

with $\varsigma > \gamma(1-\alpha)$. The technology shock has no effect on the real wage rate chosen by the monopoly union. Since $\frac{\varsigma}{\varsigma - \gamma(1-\alpha)} > 1$, we see that the real wage rate is always set above the reservation wage.

It is interesting to compare, at this point, equation (28) with the real wage equation we would obtain if the union simply maximized agents' utility (4) subject to a firm's labor demand (11). In this case the real wage would be given by

$$\frac{W_t}{P_t} = -\frac{1}{\alpha} C_t \frac{\phi_N(N_t)}{\phi(N_t)}$$ (29)

where $\frac{1}{\alpha} > 1$ is the mark-up over the competitive real wage (7) a monopoly union would be able to capture. Notice that when unions maximize the objective function (25), real wages are always set above the reservation wage

and vary only in response to changes in the reservation wage so that we have real wage rigidity. When unions simply maximize workers'utility real wages are instead fully flexible.

3.1 Households

If the union offers actuarially fair insurance, household will again perfectly share the risk to be unemployed. The model is quite similar to the IL model except for the fact that now households, in solving their problem, take N_t as given, since their wage schedule is determined by the maximization problem of the monopoly union. The maximization of utility function (4) subject to budget constraint (5) gives the same Euler equation as in the Walrasian model, which is given by equation (6).

3.2 The Flexible Price Equilibrium and the Natural Level of Output

Given that both intermediate goods and finished goods producing firm problem are the same as in the previous problem, the aggregate labor demand function is again given by equation (12). Labor market equilibrium therefore implies,

$$\frac{1}{\alpha}\frac{W_t^r}{P_t} = \frac{1}{(1-\tau)}\alpha MC_t \frac{Y_t}{N_t},\tag{30}$$

also in this case τ guarantees that the MU model steady state is equal to the pareto-efficient one. Considering now the log-linearization of (30), together with $n_t = y_t - a_t$ we obtain the following expression for real marginal costs,

$$mc_t = \varepsilon_t^w + \frac{1-\alpha}{\alpha}y_t - \frac{1}{\alpha}a_t.\tag{31}$$

Considering that $mc_t = 0$, substituting in (31) and solving for y_t we find an expression for the flexible-price level of output, which we define as the natural rate of output for our unionized economy:

$$y_t^f = \frac{1}{1-\alpha}a_t - \frac{\alpha}{1-\alpha}\varepsilon_t^w\tag{32}$$

Recalling now that the efficient level of output, for our economy with indivisible labor, is given by equation (16) we immediately see that the difference between natural output and efficient output of the unionized economy is given by

$$y_t^{Eff} - y_t^f = -\frac{\alpha}{1-\alpha}a_t + \frac{\alpha}{1-\alpha}\varepsilon_t^w.\tag{33}$$

Unlike what happens in the walrasian model, this difference is not constant, but is a function of the relevant shocks that hit the economy. In this model therefore, stabilizing the output gap - the difference between actual and

natural output - is not equivalent to stabilizing the *welfare relevant* output gap - the gap between actual and efficient output. In other words, what Blanchard and Galì call "the divine coincidence" will not hold, since any policy that brings the economy to its natural level is not necessarily an optimal policy.[18]

3.3 The IS-Curve

To obtain an IS curve we log-linearize the Euler equation (6) as:

$$c_t = E_t \{c_{t+1}\} - \frac{1-\sigma}{\sigma} \frac{\phi_N(N)N}{\phi(N)} E_t \{\Delta n_{t+1}\} - \frac{1}{\sigma} (\hat{r}_t - E_t \{\pi_{t+1}\}) \quad (34)$$

with $\hat{r}_t = r_t - \varrho$, where $r_t = \ln R_t$ and $\varrho = -\ln\beta$ which is the steady state interest rate all the variables without a subscript are taken at their steady state levels. Given that optimal subsidy setting implies $\frac{\phi_N(N)N}{\phi(N)} = -\alpha$, we rewrite equation (34) as,

$$c_t = E_t \{c_{t+1}\} + \frac{\alpha(1-\sigma)}{\sigma} E_t \{\Delta n_{t+1}\} - \frac{1}{\sigma} (\hat{r}_t - E_t \{\pi_{t+1}\}). \quad (35)$$

Given the economy resource constraint (13), the production function (14), and the definition of the output gap $x_t = y_t - y_t^{Eff}$, the Euler equation (35) becomes:

$$x_t = E_t x_{t+1} + \sigma E_t \{\Delta a_{t+1}\} - (\hat{r}_t - E_t \{\pi_{t+1}\}).$$

The efficient rate of interest, instead, can be expressed as, $\hat{r}_t^e = -\sigma(1-\rho_a)a_t$. Therefore, the IS relation can be rewritten as:

$$x_t = E_t \{x_{t+1}\} - (\hat{r}_t - E_t \{\pi_{t+1}\} - r_t^e). \quad (36)$$

Note that (36) relates the output gap rate to current and anticipated deviations of the real interest rate from its efficient counterpart.

3.4 The Phillips Curve

As in the walrasian case, firms choose $P_t(j)$ in a staggered price setting à la Calvo-Yun [8] and the Phillips curve is again given by (21). Given (31) and (32), marginal costs can be rewritten in terms of the gap between actual output and its natural level,

$$mc_t = \frac{1-\alpha}{\alpha} \left(y_t - y_t^f\right) \quad (37)$$

so that, equation (21) can now be rewritten as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \frac{1-\alpha}{\alpha} \left(y_t - y_t^f\right) \quad (38)$$

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[18]It is worth noticing that even if the real reservation wage is constant, the flexible price equilibrium output is different from the efficient one.

Given the relationship between efficient and natural output, (see eq. (33)), equation (38) can finally be expressed as:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \frac{1-\alpha}{\alpha} x_t - \lambda_a a_t + \lambda_a \varepsilon_t^w \qquad (39)$$

We can now state:

Result 1. *In a unionized labor market economy the "divine coincidence" does not hold, i.e., stabilizing inflation is not equivalent to stabilizing the output gap defined as the deviation of output from the efficient output. A positive (negative) productivity shock has a negative (positive) effect on inflation, while a cost push shock has an effect of the same size but with the opposite sign on inflation.*

This result depends on the existence of a real distortion in the economy, beside the one induced by monopolistic competition, and the nominal distortion caused by firms' staggered price setting. When a productivity shock hits the economy, efficient output, given by equation (16), increases by the same amount. Natural output instead (i.e., the level of output that would prevail in a flexible price equilibrium) increases more than proportionally so that the difference between efficient output and natural output decreases. This is due to the fact that in a unionized economy, following a productivity shock, real wages remain constant and therefore do not offset the effects of the shock on real marginal cost (see equation (37)). Because of staggered price adjustment we know that inflation is proportional to real marginal costs which, in turn, because of monopolistic competition (see equation (38)) are proportional to the difference between actual and natural output. As we will see in the following paragraphs, a Central Bank pursuing an optimal monetary policy will decide to stabilize the distance between output and its efficient level. If the difference between efficient and flexible output were constant, as in the standard model with Walrasian labor markets, stabilizing the gap between actual and natural output would be equivalent to stabilizing the gap between actual and efficient output. In this case, stabilizing the output gap with respect to the natural output would be sufficient to stabilize inflation. In our unionized economy, instead, the natural level of output differs from the efficient level because of productivity and cost-push shocks. As it is evident from equation (39), if the Central Bank stabilizes output around the efficient level, inflation will be completely vulnerable to productivity and cost-push shocks; in other words the output gap is no longer a sufficient statistics for the effect of real activity on inflation. Note that if the union maximized households' utility and real wages were given by equation (29), the log-linearized flexible price equilibrium output would coincide with (19), i.e. the flexible price equilibrium output obtained in the IL model with walrasian labor markets. Therefore, the divine coincidence would still hold.

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One interesting aspect of this model is that we are able to express the Phillips curve in its more traditional form, i.e. in terms of unemployment. From equations (14), (16) and (24) we find that $n_t = \frac{x_t}{\alpha}$. Expressing the rate of unemployment as $U_t = 1 - N_t$ and log linearizing around the steady state we obtain $u_t = -\frac{\eta}{\alpha}x_t$, where $\eta = \frac{N}{1-N}$. We can therefore rewrite the Philllips curve as:

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\lambda_a(1-\alpha)}{\eta}u_t - \lambda_a a_t + \lambda_a \varepsilon_t^w. \tag{40}$$

The relationship between unemployment and the output gap that we find in this model, therefore, allows us to consider, indifferently, the output gap and the unemployment rate as policy objectives for the central bank.

4   Optimal Monetary Policy

In Appendix A6 we show that also for the non-separable preferences assumed in our framework, consumers' utility can be approximated up to the second order by a quadratic equation of the kind:

$$W_t = E_t \sum_{t=0}^{\infty} \beta^t \tilde{U}_{t+k} = -\frac{U_{Y,t}}{2} E_t \sum_{t=0}^{\infty} \beta^t \left[\pi_{H,t+k}^2 + \frac{\lambda_a}{\theta\sigma}x_{t+k}^2 \right] + \bigcirc\left(\|\alpha\|^3 \right) \tag{41}$$

where $\tilde{U}_{t+k} = U_{t+k} - \bar{U}_{t+k}$ is the deviation of consumers' utility from the level achievable in the frictionless equilibrium, and θ is the elasticity of substitution between intermediate goods, which are used as input in the final good sector. Notice that, the relative weights assigned to inflation and to the output gap are linked to the structural parameters reflecting preferences and technology. We will consider optimal monetary policy under discretion, i.e. when the Central Bank cannot credibly commit in advance to a future policy action or a sequence of future policy actions.[19] Policy makers choose in each period the value to assign to the policy instrument, i.e., the short-term nominal interest rate $\hat{r}_t$. In order to do so, the Central Bank maximizes the welfare-based loss function (41), subject to the Phillips curve (39), and (36). The first order conditions imply:

$$x_t = -\frac{1-\alpha}{\alpha}\theta\sigma\pi_t. \tag{42}$$

Substituting into (39) and iterating forward,

$$\pi_t = -\frac{\lambda_a}{\Omega} E_t \sum_{i=0}^{\infty} \left(\frac{\beta}{\Omega} \right)^i \left(a_{t+i} - \varepsilon_{t+i}^w \right) \tag{43}$$

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[19]The case of constrained commitment, i.e. when the Central Bank is committed to follow a well specified policy rule, is analyzed in a separate appendix avalaible upon request.

where $\Omega = 1 + \lambda \left(\frac{1-\alpha}{\alpha}\right)^2 \theta\sigma$. The interest rate rule can be obtained by substituting (42), (43) and (43) one period ahead, into the IS curve (36), we obtain:

$$\hat{r}_t^* = -\left[\left[1 + \left(\frac{1-\rho_a}{\rho_a}\right)\left(\frac{1-\alpha}{\alpha}\right)\theta\sigma\right]\left(\frac{\lambda_a \rho_a}{\Omega - \beta\rho_a}\right) + \sigma\left(1-\rho_a\right)\right] a_t +$$

$$+ \left[1 + \left(\frac{1-\rho_w}{\rho_w}\right)\left(\frac{1-\alpha}{\alpha}\right)\theta\sigma\right]\left(\frac{\lambda_a \rho_w}{\Omega - \beta\rho_w}\right)\varepsilon_t^w \tag{44}$$

We can therefore state

Result 2. *Under discretion an optimal monetary policy requires a decrease in the nominal interest rate following a positive productivity shock and an increase in the nominal interest rate following a positive reservation wage shock.*

An interest rate rule that implements such optimal policy, can be found using (42), iterating forward (43) and remembering that $E_t\{a_{t+i+1}\} = \rho_a^i a_t$ and that $E_t\{\varepsilon_{t+i+1}^w\} = \rho_w^i \varepsilon_t^w$ we obtain:

$$\hat{r}_t^* = \left[1 + \left(\frac{1-\rho_w}{\rho_w}\right)\frac{1-\alpha}{\alpha}\theta\sigma\right]E_t\pi_{t+1} + \left[1 + \frac{(\rho_w - \rho_a)}{\rho_w(1-\rho_a)}\frac{\lambda_a\theta}{\Omega - \beta\rho_a}\frac{1-\alpha}{\alpha}\right]\hat{r}_t^e. \tag{45}$$

Assuming, as a particular case $\rho_a = \rho_w = \rho$, equation (45) becomes

$$\hat{r}_t^* = \left[1 + \left(\frac{1-\rho}{\rho}\right)\frac{1-\alpha}{\alpha}\theta\sigma\right]E_t\pi_{t+1} + r_t^e \tag{46}$$

which is equivalent on assuming that the real reservation wage $\hat{\varepsilon}_t^w = 0$. We can now state:

Result 3. *Optimal monetary policy under discretion requires a more than proportional increase in the nominal interest rate following an increase in the expected rate of inflation. However, an increase in the efficient rate of interest implies a proportional increase in the nominal interest rate if and only if $\rho_a = \rho_w = 1$. Otherwise an increase in the efficient rate implies a more than proportional increase in the nominal interest rate if $\rho_w > \rho_a$ and a less than proportional increase if $\rho_w < \rho_a$.*

As in the standard NK model, optimality requires that the Central Bank responds to increasing inflationary expectations by raising more than proportionally nominal interest rates. In other words, also for our unionized economy, the Taylor principle applies. The optimal response of the nominal interest rate to an increase in the efficient rate of interest, instead, is different from the one that is usually obtained in the "standard" NK model.

Notice that (39) together with $u_t = -\frac{\eta}{\alpha} x_t$ imply,

$$u_t = -\frac{\eta(1-\alpha)\theta\sigma\lambda_a}{\alpha^2(\Omega - \beta\rho_a)} a_t + \frac{\eta(1-\alpha)\theta\sigma\lambda_a}{\alpha^2(\Omega - \beta\rho_w)} \varepsilon_t^w. \qquad (47)$$

Given the log-linearization of equation (28), we can now state

Result 4. *Under an optimal discretionary monetary policy a productivity shock will induce a change in the rate of unemployment without affecting the real wage rate.*

This result is quite important since it is consistent with a well known fact in macroeconomics, i.e. the relatively smooth behavior of wages along the business cycle together with the relatively volatile behavior of unemployment. In this simple model, wages move only when there is a shock in the reservation wages of households. Productivity shocks imply some degree of volatility in unemployment while real wages remain constant. Wages, in the simple set up we consider in this paper, are probably too rigid, as we assume that all markets are unionized. Nevertheless, the model makes an interesting point, i.e. that the behavior of monopoly unions, in itself, is able to generate a dynamics of wages and unemployment that is roughly consistent with the one typically observed in the real economy.

5 Calibration

Our calibration exercise is aimed at illustrating the qualitative properties of our model. We start by comparing our economy characterized by staggered prices and real wage rigidity with an economy characterized by flexible prices and real wage rigidity under the interest rate rule estimated by Smets and Wouters [57] for the Euro area. The purpose of this exercise is to obtain a better understanding of the role of real wage rigidity in the transmission of monetary policy. Using the same interest rule, in a second step, we compare the dynamics of the unionized economy with the dynamics of an economy with a walrasian labor market. A similar exercise can be found also in Zanetti [58]. Differently from Zanetti's model, ours does not allow for human and physical capital accumulation and this allows us to study optimal monetary policy and provide, therefore, a useful benchmark to evaluate actual monetary policy. In a third step, we calibrate the model under the optimal monetary policy rule and we look for a simple rule that can actually replicate such policy. As in Zanetti [58], the variables of the model are calibrated using data from the Euro area.

The model is calibrated on quarterly frequencies. For the parameters describing preferences, we set the elasticity of intertemporal substitution at $\sigma = 2$. The output elasticity of labor, $\alpha = 0.72$, is based on the estimate of

Christoffel et al. [15]. We set $\beta = 0.99$, $\varphi = 0.75$, which implies an average price duration of one year, and finally $\theta = 6$, which is consistent with a 10 percent markup in the steady state. The persistence of the technology shock is set to $\rho_a = 0.93$ as in Amato and Laubach [3]. The persistence of the wage shock is assumed to be equal to $\rho_w = 0.7$, as the persistence of a cost-push shock estimated by Ireland [36]. As discussed in Zanetti [58] $N = 0.61$.

In the first and in the second exercise, the Central Bank is assumed to follow the following Taylor-type monetary policy rule:

$$\hat{r}_t = \phi_r \hat{r}_{t-1} + (1 - \phi_r) \left[\phi_\pi \pi_{t-1} + \phi_x x_{t-1} \right] \tag{48}$$

As advocated by Carlstrom and Fuerst [9], we employ lagged values for output and inflation because it can be considered consistent with the information set of the Central Bank at time t. The parameters of the Taylor rule follow the estimates of Smets and Wouters [57] for the Euro area. In particular, the degree of interest rate smoothing is set at $\phi_r = 0.9$, the nominal interest rate response to inflation at $\phi_\pi = 1.658$ and the response to output at $\phi_x = 0.148$.

Figure 1 shows a comparison between the IRFs to a negative productivity shock in an economy with real wage rigidity and flexible prices (dashed line) and in our economy characterized by real wage rigidity and staggered prices (solid line). In both cases the presence of real wage rigidity introduces a trade-off between inflation and output stabilization: inflation goes up and output decreases, the nominal interests rate increases more than proportionately with respect to the inflation rate. It is worth noticing, however, that if prices are fully flexible all variables are much more volatile. Moreover, in the staggered price economy unemployment decreases and the output gap increases, whereas, in the flexible price economy we have the opposite result: unemployment increases and the output gap decreases. The intuition is the following. In both cases, unless a reservation wage shock hits the economy, unions tend to keep real wages constant. This means that the negative effect on marginal costs caused by a decrease in productivity is not partially offset by a reduction in real wages. Nominal marginal costs increase. Given that in the flexible price economy prices are always set as a constant markup over nominal marginal costs, firms increase prices immediately by the same amount of the increase in nominal marginal costs. Consequently, inflation increases and, given the type of Taylor rule we have assumed, the nominal interest rate goes up more than proportionally. Then, consumption decreases and aggregate demand goes down. Given that firms cannot adjust real wages they are forced to reduce their demand for labor. Therefore, unemployment increases. As shown in figure 1 unemployment increases by about 6 percentage points.

On the contrary, when prices are staggered, only a proportion of firms adjust their prices immediately, while the others leave their prices unchanged.

This means that inflation increases less than in the flexible price economy. Therefore the nominal interest rate increases less. Consequently, the reduction of aggregate demand is lower than the reduction in productivity. This means that, in order to satisfy the aggregate demand firms need to hire new workers. In this case unemployment decreases, instead of increasing. In fact, as shown in figure 1 unemployment decreases by almost 2% points.

In the second exercise we evaluate the dynamics of the walrasian model and the dynamics of the unionized model in response to a negative productivity shock. In figure (2) we plot the response of the interest rate, inflation, output, the output gap and unemployment to a one unit standard deviation negative productivity shock in the unionized economy (solid line) and in the walrasian labor market economy (dashed line). Our model, although simpler, produces results that are very similar to the ones found by Zanetti [58]. This suggests that adding physical and human capital accumulation does not sensibly change the dynamics of the model.

A negative productivity shock, in both the walrasian and the unionized economies, causes the output gap to rise, total output to decline, the nominal interest rate and inflation to rise and unemployment (employment) to decline (rise). The difference between the responses of the walrasian and the unionized economies lies not on the sign of the effect, but on the size of the effect: in the unionized economy the response of the main variables to a productivity shock is amplified and all the variables in the unionized economy are characterized by a higher degree of persistence.

The fact that unemployment experiences a large decline after a negative productivity shock, as shown in figure (2), is extremely interesting. A negative comovement between productivity shocks and various measures of the labor input has been recently found in the empirical literature, among others, by Galì [23], [27], [29], and by Francis et al. [22] [21].[20]. This empirical result has given rise to an important debate[21], since the standard RBC model is not able to replicate it, and has been used to cast serious doubts on the relevance of productivity shocks in explaining business fluctuations. In contrast, our model seems to suggest that technology shocks might still be a driving force of the business cycle if the economy is characterized by labor indivisibilities and sticky prices and the central bank follows a Taylor-type rule.

This formulation of our model provides a direct comparison with the Blanchard Galì [7] model, characterized by search frictions and real wage rigidity. In that model, when monetary policy follows the simple rule proposed by Taylor [51], which is in turn used to approximate the optimal interest rate rule, the correlation between productivity and unemployment

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[20]See Galì and Rabanal [26] for a survey.

[21]For alternative estimations of the effect of productivity shocks on the labor input see Christiano et al. [13].

is negative. Differently from ours, the Blanchard Galì [7] model is not able to replicate the empirical findings of Galì [23], [27], [29] and others.

In figure (3) we describe the response of the main variables of the unionized model under the optimal discretionary rule (45). In this case a one standard deviation negative productivity shock requires a 0.3% initial increase in the nominal interest rate of 0.3% and an increase in inflation of almost 0.13%. Initially output gap falls by 1% and the rate of unemployment has an initial increase of about 2 percentage points, while output decrease by 2%. An optimal monetary policy, therefore, will take into account the trade-off that exists between inflation stabilization and output stabilization and will require some degree of accommodation: as a response to a productivity shock output will decrease and inflation will increase.

In order to evaluate to what extent Taylor rules found in the empirical literature on monetary policy compare relatively to the optimal monetary policy analyzed in this paper, we report, in figure (4), the results of an exercise aimed at replicating the optimal policy through a simple Taylor rule. We found that a rule that approximates quite well the optimal monetary policy (i.e. that achieves a response of the major variables quite close to the one achieved by our economy under the optimal discretional monetary policy) is given by,

$$i_t = 2.5\pi_t + 0.05x_t \tag{49}$$

Notice that this rule implies a stronger response to inflation and a weaker response to the output gap than the ones found in the literature. It is also worth noticing that, differently from what happens under the Taylor rules considered above, under the optimal policy rule negative productivity shocks have a negative effect on unemployment. This result, together with the one described in figures (2), (3) and (4), suggests that the negative correlation between productivity and employment found in the data by Galì and others might be the result of a monetary policy too accommodating with respect to inflation, rather than consequence of some "structural" characteristic of the economy.

In figures (5) and (6) we show the responses of the interest rate, output and unemployment to a one standard deviation shock to the reservation wage under the optimal rule and under the rule which mimic the optimal one. The responses are similar to the ones found for the productivity shocks, although, in the case of wage push shocks, the correlation between output and unemployment is always negative.

6   Conclusions

We have considered a DSGE-NK model where labor is indivisible and where wages are set by monopoly unions. We found that, with respect to the

standard NK framework, our model gives a more satisfactory description of the reality of modern industrialized economies, especially of those where collective bargaining dominates the labor market. In a unionized economy, significant trade-offs between stabilizing inflation and stabilizing unemployment arise in response to technology and exogenous wage shocks. Because of real wage rigidity, an optimizing central bank must respond to negative technology shocks by increasing the interest rate and, similarly, must respond to exogenous increases in unions' reservation wage with an interest rate increase. Interestingly, if we consider an optimal instrument rule, an optimizing central bank not only will increase the interest rate more than proportionately in response to an increase in future expected inflation, but will also react to increases in the natural rate that are not necessarily one to one. The model is also capable of accounting for the greater volatility of unemployment relative to the wage volatility that is usually found in the data. Moreover, once calibrated on Euro-area parameters, with the addition of an exogenous interest rate rule, our model is consistent with the positive correlation between technology shocks and the labor input found in the data. This correlation, however, becomes negative once the optimal discretionary rule is included in the model, suggesting that this correlation might be explained by the nature of monetary policy.

Even though, for the sake of simplicity, many other market imperfections, like search and matching frictions and firing costs are absent from our model, and therefore only some of the characteristics of European labor markets are taken into account, the model provides a coherent framework for the analysis of monetary policy in countries where unions play an important role. Obviously, there are many possible extensions to this model that could provide a deeper understanding of the relationship between monetary policy and different institutional settings in the labor market; we leave however these challenges to future research.

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A   Technical Appendix

A.1   The Ramsey Problem

We consider a social planner which maximizes the representative household utility subject to the economy resource constraint and production function as follows:

$$\max_{N_t} U\left(C_t, N_t\right) = \frac{1}{1-\sigma} C_t^{1-\sigma} \phi\left(N_t\right)^{1-\sigma} \tag{A1}$$

subject to the economy resource constraint and to the aggregate production function. The first order condition requires

$$\left(A_t N_t^{\alpha}\right)^{-\sigma} \alpha \frac{Y_t}{N_t} \phi\left(N_t\right)^{1-\sigma} = -\left(A_t N_t^{\alpha}\right)^{1-\sigma} \phi\left(N_t\right)^{-\sigma} \phi_N\left(N_t\right) \tag{A2}$$

simplifying

$$Y_t \frac{\phi_N\left(N_t\right)}{\phi\left(N_t\right)} = -\alpha \frac{Y_t}{N_t} \tag{A3}$$

Multiplying both sides of equation for $\frac{N_t}{Y_t}$ we find equation (15) in the text.

A.2 Derivation of the Efficient Output

We consider the Ramsey solution (A4)

$$\phi_N(N_t) N_t = -\alpha \phi(N_t) \qquad (A4)$$

in order to find an equation for the efficient output we first log-linearizing equation (A5) around the steady state, which implies

$$[\phi_N(N) + \phi_{NN}(N) N n_t] N (1 + n_t) = -\alpha (\phi(N) + \phi_N(N) N n_t). \qquad (A5)$$

Considering the steady state equation, i.e. that $\phi_N(N) N = -\alpha \phi(N)$ and collecting terms in n_t we obtain:

$$\left(1 + \frac{\phi_N(N) N_t}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\phi(N)} \left(\frac{\phi_N(N) N_t}{\phi(N)} \right)^{-1} \right) n_t = 0 \qquad (A6)$$

given that $1 + \frac{\phi_N(N) N_t}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\alpha \phi(N)} \left(\frac{\phi_N(N) N_t}{\phi(N)} \right)^{-1} \neq 0$ we require $n_t = 0$, and then from the aggregate production function we obtain equation (16) in the text.

A.3 Derivation of the Flexible Price Equilibrium Output in the IL Model

Let us rewrite equation (17) as:

$$\phi_N(N_t) N_t = -\frac{\alpha}{(1 - \tau)} MC_t \phi(N_t), \qquad (A7)$$

then log-linearizing,

$$[\phi_N(N) + \phi_{NN}(N) N n_t] N (1 + n_t) = -\frac{\alpha MC}{(1 - \tau)} (1 + mc_t) [\phi(N) + \phi_N(N) N n_t] \qquad (A8)$$

considering the steady state of equation (A7) we have,

$$mc_t = \left(1 + \frac{\phi_N(N) N}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\phi(N) \alpha} \right) n_t \qquad (A9)$$

given that in the flexible price equilibrium it must hold that $mc_t = 0$, as in the previous problem it requires that $n_t = 0$. Then, considering the aggregate production function we obtain equation (19) in the text.

A.4 Derivation of the Phillips Curve

Following Calvo [8] we assume that each firm may reset its price with probability $1 - \varphi$ each period, independently from the time elapsed since the last adjustment. This means that each period a measure $1 - \varphi$ of firms reset their price, while a fraction φ of them keep their price unchanged.

The law of motion of the aggregate price is given by: $\ln P_t = \varphi \ln P_{t-1} + (1-\varphi) \ln P_t^*$, which implies

$$\pi_t = (1-\varphi) \ln \left(\frac{p_t^*}{P_{t-1}} \right) \tag{A10}$$

where $\ln P_t^*$ denotes the (log) price set by a firm i adjusting its price in period t. Under Calvo [8] price-setting structure $p_{t+k}(i) = p_t^*$ with probability φ^k for $k = 0, 1, 2, ...$, hence firms have to be forward-looking.

Given that the individual firm technology is characterized by decreasing return to scale, the optimal price setting rule should take into account that marginal cost is no longer common across firms. In particular, in the neighborhood of the zero inflation steady state, we have the following price-setting rule:

$$\ln P_t^*(i) = \mu^P + (1 - \beta\varphi) \sum_{k=0}^{\infty} (\beta\varphi)^k E_t \left\{ mc_{t,t+k}^n \right\} \tag{A11}$$

where $mc_{t,t+k}^n$ is the log-linearized nominal marginal cost in period $t + k$ of a firm which last set its price in period t. Considering the equation of real marginal cost and the one of the aggregate production function,

$$MC_{t,t+k} = (1 - \tau) \frac{(W_{t+k}/P_{t+k})}{\alpha (Y_{t,t+k}/N_{t,t+k})} = MC_{t+k} \left(\frac{P_t^*}{P_{t+k}} \right)^{\theta \frac{1-\alpha}{\alpha}} \tag{A12}$$

taking the logs

$$\ln MC_{t,t+k} = \ln MC_{t+k} - \theta \frac{1-\alpha}{\alpha} \ln \left(\frac{P_t^*}{P_{t+k}} \right) \tag{A13}$$

and considering that all firms resetting prices in period t will choose the same price P_t^* we can rewrite equation (A13) as,

$$\ln \left(\frac{P_t^*(i)}{P_{t-1}} \right) = \mu^P + (1 - \beta\varphi) \sum_{k=0}^{\infty} (\beta\varphi)^k E_t \left\{ \ln MC_{t,t+k}^n - \ln P_{t-1} \right\} \tag{A14}$$

which can be rewritten as

$$\begin{aligned} \ln \left(\frac{P_t^*(i)}{P_{t-1}} \right) &= \mu^P + (1 - \beta\varphi) \sum_{k=0}^{\infty} (\beta\varphi)^k E_t \left\{ \ln MC_{t,t+k}^n - \theta \frac{1-\alpha}{\alpha} \ln \left(\frac{P_t^*}{P_{t+k}} \right) \right\} \\ &\quad + \sum_{k=0}^{\infty} (\beta\varphi)^k \left\{ \pi_{t+k} \right\} \end{aligned} \tag{A15}$$

then

$$\ln P_t^*(i) - \ln P_{t-1} = \mu^P + \beta\varphi E_t \left\{ \ln P_{t+1}^* - \ln P_t \right\} + (1 - \beta\varphi) \ln MC_t, \tag{A16}$$

which allows to obtain equation (21) in the text.

A.5   The Welfare-Based Loss Function

A second-order Taylor expansion of the period utility around the efficient equilibrium yields,

$$U_t = \bar{U}_t + \bar{U}_{\bar{C},t}\bar{C}_t\tilde{C}_t + \frac{1}{2}\bar{U}_{\bar{C}\bar{C},t}\bar{C}_t^2\tilde{C}_t^2 + \bar{U}_{\bar{N},t}\bar{N}_t\tilde{N}_t + \frac{1}{2}\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2\tilde{N}_t^2 +$$
$$+ \bar{U}_{\bar{C}\bar{N},t}\bar{C}_t\bar{N}_t\tilde{C}_t\tilde{N}_t + \bigcirc\left(\|\alpha\|^3\right) \tag{A17}$$

where the generic $\tilde{X} = \ln\left(X/\bar{X}_t\right)$ denotes log-deviations from the efficient equilibrium and $\bar{X}_t$ denotes the value of the variable under efficient equilibrium. Moreover, we denote as $\bar{x}_t = \ln\left(\frac{\bar{X}_t}{\bar{X}}\right)$. Considering the flexible prices economy resource constraint,

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t\tilde{Y}_t + \frac{1}{2}\bar{U}_{\bar{Y}\bar{Y},t}\bar{Y}_t^2\tilde{Y}_t^2 + \bar{U}_{\bar{N},t}\bar{N}_t\tilde{N}_t + \frac{1}{2}\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2\tilde{N}_t^2 +$$
$$+ \bar{U}_{\bar{Y}\bar{N},t}\bar{Y}_t\bar{N}_t\tilde{Y}_t\tilde{N}_t + \bigcirc\left(\|\alpha\|^3\right). \tag{A18}$$

Collecting terms yields

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \left[\begin{array}{c} \tilde{Y}_t + \frac{\bar{U}_{\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}\bar{Y}_t}\tilde{N}_t + \frac{1}{2}\frac{\bar{U}_{\bar{Y}\bar{Y},t}}{\bar{U}_{\bar{Y},t}}\bar{Y}_t\tilde{Y}_t^2 + \\ + \frac{1}{2}\frac{\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2}{\bar{U}_{\bar{Y},t}\bar{Y}_t}\tilde{N}_t^2 + \frac{\bar{U}_{\bar{Y}\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}}\tilde{Y}_t\tilde{N}_t \end{array} \right] + \bigcirc\left(\|\alpha\|^3\right) \tag{A19}$$

Considering that, $\frac{\bar{U}_{\bar{Y}\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}} = \frac{\phi_{\bar{N},t}\left(\bar{N}_t\right)\bar{N}_t}{\phi\left(\bar{N}_t\right)} = -\left(1-\sigma\right)\alpha$, we have,

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \left[\begin{array}{c} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 + (1-\sigma)\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\bar{N}_t\tilde{Y}_t\tilde{N}_t \\ + \frac{1}{2}\left[\frac{\phi_{NN}\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)} - \sigma\left(\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\right)^2\right]\bar{N}_t^2\tilde{N}_t^2 \end{array} \right] + \bigcirc\left(\|\alpha\|^3\right) \tag{A20}$$

It can be shown that $\frac{\phi_{NN,t}\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)} = \frac{2\sigma-1}{\sigma}\left(\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\right)^2$, hence

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \left[\begin{array}{c} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 + (1-\sigma)\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\bar{N}_t\tilde{Y}_t\tilde{N}_t \\ + \frac{1}{2}\left[\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\left(\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\right)^2\right]\bar{N}_t^2\tilde{N}_t^2 \end{array} \right] + \bigcirc\left(\|\alpha\|^3\right) \tag{A21}$$

and therefore,

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \left[\begin{array}{c} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - (1-\sigma)\alpha\tilde{Y}_t\tilde{N}_t + \\ + \frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\alpha^2\tilde{N}_t^2 \end{array} \right] + \bigcirc\left(\|\alpha\|^3\right). \tag{A22}$$

We now take a first-order expansion of the term $\bar{U}_{\bar{Y},t}\bar{Y}_t$ around the steady state.

$$\bar{U}_{\bar{Y},t}\bar{Y}_t = U_Y\left(1 + (1-\sigma)\,\bar{y}_t + (1-\sigma)\frac{\phi_N(N)}{\phi(N)}N\bar{n}_t\right) + \bigcirc\left(\|\alpha\|^2\right)$$

$$= U_Y\left(1 + (1-\sigma)\,\bar{y}_t - (1-\sigma)\,\alpha\bar{n}_t\right) + \bigcirc\left(\|\alpha\|^2\right) \qquad \text{(A23)}$$

with

$$\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\bar{N}_t = \frac{\phi_N(N)}{\phi(N)}N + \Gamma_n\bar{n}_t + + \bigcirc\left(\|\alpha\|^2\right) \qquad \text{(A24)}$$

where $\Gamma_n = \left(\frac{\phi_N(N)N}{\phi(N)} + \frac{\phi_{NN}(N)N^2}{\phi(N)} - \frac{\phi_N(N)^2N^2}{\phi(N)^2}\right)$, and

$$\left(\frac{\phi_N(\bar{N}_t)\bar{N}_t}{\phi(\bar{N}_t)}\right)^2 = \left(\frac{\phi_N(N)N^2}{\phi(N)}\right)^2 + \Lambda_n\bar{n}_t + \bigcirc\left(\|\alpha\|^2\right) \qquad \text{(A25)}$$

where $\Lambda_n = 2\left(\frac{\phi_N(N)\phi_{NN}(N)N}{\phi(N)^2} + \left(\frac{\phi_N(N)N}{\phi(N)}\right)^2 - \left(\frac{\phi_N(N)}{\phi(N)}\right)^3 N\right)$

given that $\bar{n}_t = 0$, and that $\frac{\phi_N(N)N}{\phi(N)} = -\alpha$, substituting (A24) and (A25) into the welfare function,

$$U_t = \bar{U}_t + U_Y\left(1 + (1-\sigma)\,\bar{y}_t\right)\begin{bmatrix} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - \alpha(1-\sigma)\tilde{Y}_t\tilde{N}_t \\ + \frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\alpha^2\tilde{N}_t^2 \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \qquad \text{(A26)}$$

Given the aggregate production function and that the log-deviations of the price dispersion index $-d_t = \tilde{Y}_t - \alpha\tilde{N}_t$ are of second-order, and that: $\tilde{Y}_t^2 = \alpha^2\tilde{N}_t^2$ $\quad n_t\alpha\tilde{N}_t = n_t\tilde{Y}_t \quad y_t\alpha\tilde{N}_t = y_t\tilde{Y}_t \quad \tilde{Y}_t\alpha\tilde{N}_t = \tilde{Y}_t^2$. Considering only terms up to the second-order we have:

$$U_t = \bar{U}_t + U_Y\begin{bmatrix} \tilde{Y}_t - \tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - (1-\sigma)\tilde{Y}_t^2 \\ + \frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\tilde{Y}_t^2 \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \qquad \text{(A27)}$$

or

$$\tilde{U}_t \equiv U_t - \bar{U}_t = -U_Y\left\{d_t - \frac{1}{2\sigma}\tilde{Y}_t^2\right\} + \bigcirc\left(\|\alpha\|^3\right) \qquad \text{(A28)}$$

As proven by Galì and Monacelli [28], the log-index of the relative-price distortion is of second-order and proportional to the variance of prices across firms, which implies that[22]: $d_t = \ln\left(\int_0^1\left(\frac{P_t(i)}{P_t}\right)di\right) = \frac{\theta}{2}var_i\left\{p_t(i) + \bigcirc\left(\|\alpha\|^3\right)\right\}$. As shown in Woodford [56], this means that: $\sum_{t=0}^{\infty}\beta^t var_i\{p_t(i)\} = \frac{1}{\lambda_a}\sum_{t=0}^{\infty}\beta^t\pi_t^2$, where $\lambda = (1-\psi)(1-\psi\beta)/\psi$. Finally, denoting the output gap $\tilde{Y}_t$ as in the standard way x_t, the Welfare-Based loss-function can be written as equation (41) in the text.
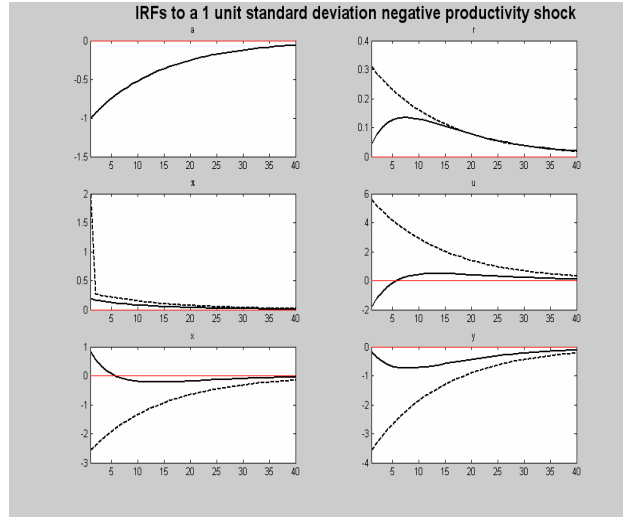
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[22] Proof Galì and Monacelli [28].

Figure 1: Impulse response functions to a 1 unit sd. negative productivity shock under the Taylor rule esitimated by Smets and Wouters (2003). Comparison betweens staggered prices (solid line) and flexible prices (dashed line) economy.
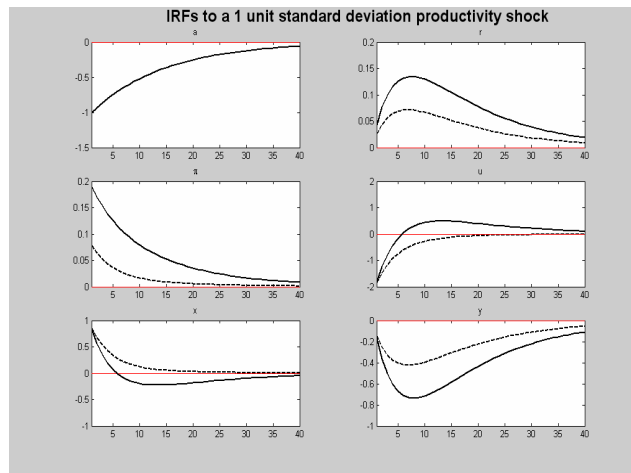

Figure 2: Impulse response functions to a 1 unit sd. negative productivity shock. Comparison between the walrasian labor market economy (dashed line) and the unionized labor market economy (solid line).
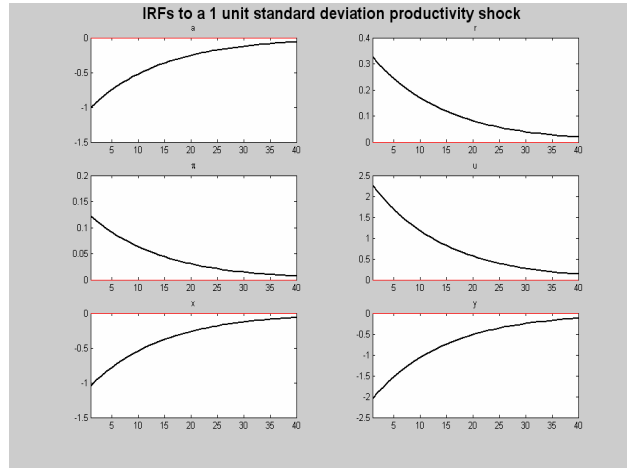
Figure 3: Impulse response functions to a 1 sd. negative productivity shock under the optimal interest rate rule.
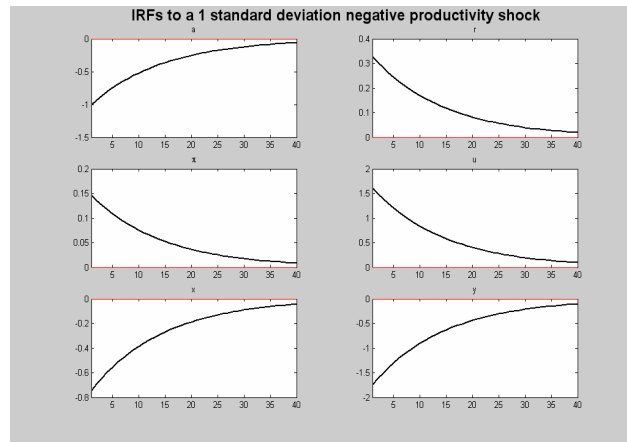

Figure 4: Impulse response functions to a 1 unit sd. negative productivity shock under the simple rule that replicates the optimal one.
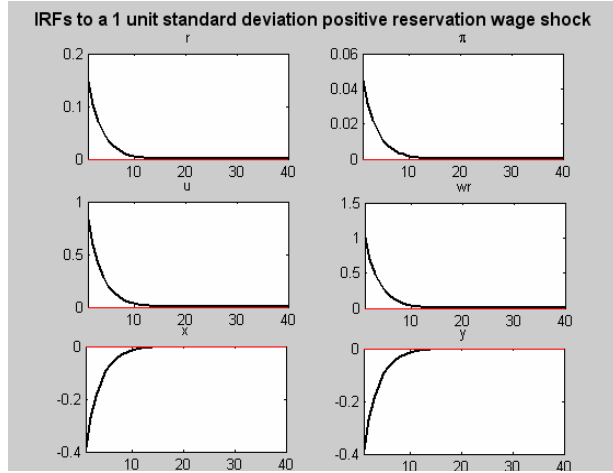
Figure 5: Impulse response functions to a 1 unit sd. positive reservation wage shock under the optimal interest rate rule
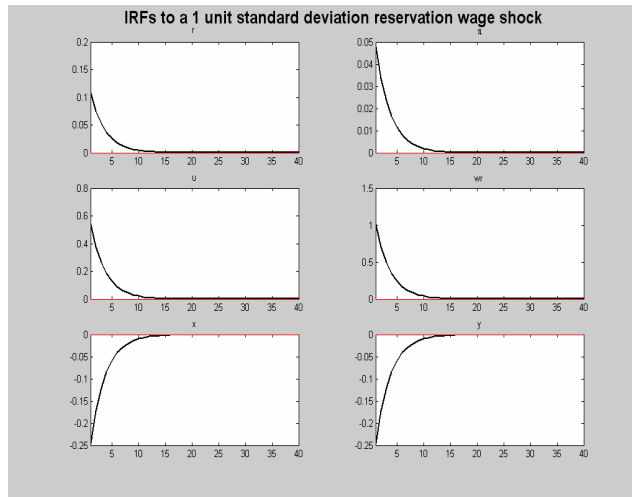

Figure 6: Impulse response functions to a 1 unit sd. positive reservation wage shock under the simple rule that replicates the optimal one.