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Abstract:

This paper analyses the conditions under which savings should receive special tax treatment. A two–class overlapping generation model is presented, and a simple condition for the taxation or subsidisation of savings is derived and interpreted.

1   Introduction

There is a longstanding debate among economists on the choice of the personal tax base.[1] The two main contenders for the tax base are consumption and income. The primary distinction between the two is that under the former savings are exempt from taxation. Meanwhile, some argue that savings are taxed twice under the latter (when income is first earned and again when it bears interest). Double taxation *per se* is not a sufficient reason to justify the special treatment (or exemption) of savings from the tax base. Yet, it has been argued that a tax on savings leads to an inefficiently low amount of savings and capital formation. Moreover, the consumption tax base has several administrative advantages, arising mainly from the difficulty in measuring and taxing some forms of asset income. Still, there remains the possibility that taxes on savings may be of some use in counterbalancing the distortions imposed by labour income taxation. It is also reasonable to expect that, under some circumstances, an exclusion of asset income from the tax base may make the tax system more inequitable.

Any attempt to shed light on the tax base issue must be framed inside a dynamic model of the economy, if only because the saving decision is intertemporal. This note outlines one of the simplest models in which this question can be addressed. It is an attempt to embed the standard model of personal income taxation, in the Mirrlees (1971) tradition, into an overlapping generations model. The distortionary effects of personal income taxation are modelled as arising out of information asymmetry between the taxation authority and individuals. The population of workers is divided into two classes, differing in productivity. Following the standard set of assumptions,

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[1]See Kesselman (1994) and Boadway and Wildasin (1994) for surveys of the issues involved.

the taxation authority is assumed to observe only market earnings, which is a mixture of innate ability and hours of work. The dynamic structure is equally simple, deriving in a straightforward way from a commonly used deterministic overlapping generations model.

Whilst this type of analysis has been carried out before,[2] the current framework is relatively simple. The two–class model has proven to be flexible enough to formulate extensions of the static nonlinear income tax model that incorporate such items as public expenditure (Boadway and Keen (1993), Boadway and Marchand (1995)) and income maintenance (Besley and Coate (1992)). It is hoped that the current model could be used in a similar way.[3]

The remainder of this note is organised as follows. The next section provides a description of the model, paying careful attention to the information assumptions contained therein. Section 3 derives some qualitative features of optimal taxation in this environment, and derives a condition under which it is optimal to tax (or subsidise) savings. Some concluding remarks are then offered.

2   The Model

There are two consumers born each period. During the first period of their lives, they supply labour elastically and they consume. In the second period of life, each individual retires. Within a generation, individuals differ in productivity. Denote the productivity of a person of type i by a_i, $i = 1, 2$, $a_1 < a_2$. Thus, if a person supplies l_i units of labour, her effective labour is $y_i := a_i l_i$. At any date (apart from the start–up period), t, the following

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[2]See Myles (1995, pp. 509–514) for a textbook treatment of the best–known of these analyses — that of Ordover and Phelps (1979).

[3]Indeed, since the first version of this note was written, Pirttilä and Tuomala (1998) have provided an extension to the case of the provision of a public good.

individuals are alive:

- two young individuals, one of type a_1, the other with productivity a_2;

- two retired individuals, born at time $t-1$, living off the proceeds of their savings.

Individuals derive utility from consumption when young (c) and consumption during retirement (x). Moreover, they are assumed to have a disutility of labour. The utility of a person of type i born at time t is given by[4]

$$u_i^t := U(c_i^t, x_i^t, l_i^t) = U(c_i^t, x_i^t, \frac{y_i^t}{a_i}). \tag{1}$$

Total consumption at time t is made up of consumption by the young born at that date and the spending–in–retirement of those born at date $t-1$. $y^t := y_1^t + y_2^t$ denotes the total effective labour supplied by the generation born at time t.

Total output at any date t is a function of the capital stock, k^t, and total effective labour. Let $F(k^t, y^t)$ be the production function, assumed to exhibit constant returns to scale. The prices of inputs are determined by the profit–maximisation conditions:

$$r^t = F_k(k^t, y^t); \quad w^t = F_y(k^t, y^t), \tag{2}$$

where w^t is the price of effective labour. Thus the before–tax income of an individual is given by

$$z_i^t := w^t a_i l_i^t = w^t y_i^t. \tag{3}$$

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[4]Throughout this analysis, subscripts are used to denote the type of an individual, whilst superscripts denote the date of birth of an individual. Quantities denoted without subscripts are within–period aggregates.

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Depreciation is assumed away, so that the capital stock evolves according to the equation:

$$k^{t+1} = F(k^t, y^t) + k^t - c^t - x^{t-1}. \tag{4}$$

That is, capital next period equals current output plus current capital less total consumption of those currently alive.

It is assumed that the government can observe both z and w, but that it cannot observe l. This accords with the standard assumptions of nonlinear tax theory. It is equivalent to say that the planner can observe y. Implicitly, then, the planner can also observe k. Because l is unobserved, the planner must resort to distortionary taxation. Exactly which tax instruments are available to the planner depend on the further assumptions one makes about the use of non–income information. It is assumed that the planner knows the age of each individual, so that the young cannot pretend to be old, nor can the old pretend to be young. The old do not work, so there is no direct interaction between them and the income tax schedule. Thus, the only concern is that the young may have incentive to misrepresent their ability.

Given that information about type is revealed when young, the planner can distinguish between retirees of the same generation. Thus, without loss of generality, it is assumed that the tax on consumption of the old is pre–paid at the end of the first period of life. Because retirees simply consume their after–tax savings, one need not worry about the "ratchet effect". No economically meaningful decision made in the second period of life.

The taxation authority is assumed to select a tax system that specifies an amount of tax to be paid on labour income, along with a levy on the amount of savings. Equivalently, it can be modelled as choosing the consumption levels and effective labour time for each type of worker at each date in time, subject to incentive compatibility constraints. I analyse only the case in

which a person of high ability may wish to misrepresent its type. That is, at each date, only one form of self–selection constraints is considered, namely

$$U(c_2^t, x_2^t, \frac{y_2^t}{a_2}) \geq U(c_1^t, x_1^t, \frac{y_1^t}{a_2}) \quad t = 1, 2, \ldots \tag{5}$$

This is the case most commonly analysed in the literature. A more general analysis can be easily carried out, but would not shed a great deal more light on the issues addressed in this note.

3   Optimal Taxation Rules

The planner is assumed to be Paretian both across and within generations. If we ignore the generation that is born old (generation 0), we may write the planner's objective function as

$$W(u_1^1, u_2^1, \ldots, u_1^t, u_2^t, \ldots).$$

For this specification of objectives, the problem of designing an optimal taxation mechanism may be stated in mathematical terms as:

$$\max_{\mathbf{c}_1, \mathbf{c}_2, \mathbf{x}_1, \mathbf{x}_2, \mathbf{y}_1, \mathbf{y}_2, \mathbf{k}} W\left(U(c_1^1, x_1^1, \frac{y_1^1}{a_1}), U(c_2^1, x_2^1, \frac{y_2^1}{a_2}), \ldots\right) \tag{6}$$

subject to

$$F(k^t, y^t) + k^t - c^t - x^{t-1} \geq k^{t+1} \quad t = 1, 2, \ldots \tag{7}$$

$$U(c_2^t, x_2^t, \frac{y_2^t}{a_2}) \geq U(c_1^t, x_1^t, \frac{y_1^t}{a_2}) \quad t = 1, 2, \ldots \tag{8}$$

where $\mathbf{z}_i := (z_i^t)_{t=1}^{\infty}$, for any variable z. It is helpful to consider the (infinite) Lagrangean for this problem, namely:

$$\mathcal{L} := W\left(U(c_1^1, x_1^1, \frac{y_1^1}{a_1}), U(c_2^1, x_2^1, \frac{y_2^1}{a_2}), \ldots\right)$$

$$+ \sum_{t=1}^{\infty} \rho^t \left[F(k^t, y^t) + k^t - c^t - x^{t-1} - k^{t+1}\right] \tag{9}$$

$$+ \sum_{t=1}^{\infty} \mu^t \left[U(c_2^t, x_2^t, \frac{y_2^t}{a_2}) - U(c_1^t, x_1^t, \frac{y_1^t}{a_2})\right].$$

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Ordover and Phelps (1979) analysed a planner's problem very similar to the one above, but with a continuum of types. They found that the traditional 'no distortion at the top' result to hold for both labour taxation and savings taxation. The current model gives rise to the same conclusion. For completeness, it is presented in the following result.

Result 1 *For all generations, agents of type 2 face a zero marginal rate of taxation on both labour income and savings.*

Proof: Let $MU_{c,i}^t$ be the marginal utility of first–period consumption for a person of type i born at time t, and let

$$\alpha_i^t := \frac{\partial W(\cdot)}{\partial u_i^t} \tag{10}$$

be the marginal social valuation of an increase in the utility of an individual of type i born at date t. Denote by $\widehat{MU}_{c,2}^t$ the marginal utility of consumption for a person of type 2 born at time t, evaluated at the bundle designed for the person of type 1 born at time t. That is, we follow the Boadway and Keen (1993) convention of using "hats" to denote the potential mimicker. $MU_{x,i}^t$ and similar expressions have analogous meaning.

The first order condition for the planner's maximisation problem include:

$$k_t : \rho^t \left[F_k(k_t, y_t) + 1 \right] - \rho^{t-1} = 0; \tag{11}$$

$$c_1^t : \alpha_1^t MU_{c,1}^t - \mu^t \widehat{MU}_{c,2}^t - \rho^t = 0; \tag{12}$$

$$c_2^t : (\alpha_2^t + \mu^t) MU_{c,2}^t - \rho^t = 0; \tag{13}$$

$$x_1^t : \alpha_1^t MU_{x,1}^t - \mu^t \widehat{MU}_{x,2}^t - \rho^{t+1} = 0; \tag{14}$$

$$x_2^t : (\alpha_2^t + \mu^t) MU_{x,2}^t - \rho^{t+1} = 0; \tag{15}$$

$$y_1^t : \alpha_1^t MU_{l,1}^t \frac{1}{a_1} - \mu^t \widehat{MU}_{l,2}^t \frac{1}{a_2} + \rho^t F_y(k_t, y_t) = 0; \tag{16}$$

$$y_2^t : (\alpha_2^t + \mu^t) MU_{l,2}^t \frac{1}{a_2} + \rho^t F_y(k_t, y_t) = 0. \tag{17}$$

Leading (11) by one period yields

$$\rho^t = \rho^{t+1}\left[F_k(k_{t+1}, y_{t+1}) + 1\right] \Rightarrow \rho^{t+1} = \frac{\rho^t}{1 + F_k(k_{t+1}, y_{t+1})}. \tag{18}$$

Using (18), the conditions $r^{t+1} = F_k(k^{t+1}, y^{t+1})$ and $w^t = F_y(k^t, y^t)$, and taking ratios of (13), (15) and (17) yields:

$$\frac{MU_{l,2}^t}{MU_{c,2}^t} = -a_2 w_t; \tag{19}$$

$$\frac{MU_{c,2}^t}{MU_{x,2}^t} = 1 + r_{t+1}. \tag{20}$$

Relation (19) implies that the marginal rate of substituion between labour and consumption equals the wage rate for agent of type 2 born at time t. Relation (20) says that the marginal rate of substitution between present and future consumption for high–ability agents equals the gross rate of interest. Hence, it is never optimal to distort the savings decision of high–ability agents. $\qquad\square$

The analysis of Ordover and Phelps also uncovered a set of conditions under which it is optimal to not tax the savings of any individuals at the marginal; that is, when labour is separable from consumption in utility. A similar finding holds in the present model, and is a direct consequence of the following, more general, result.

Result 2 *The following statements hold for each generation of agents of type 1;*

1. *The implicit marginal rate of income tax is positive.*

2. *There is an implicit positive (negative) marginal tax rate on savings if and only if*

$$\frac{U_c(c_1^1, x_1^1, \frac{y_1^1}{a_1})}{U_x(c_1^1, x_1^1, \frac{y_1^1}{a_1})} > (<) \frac{U_c(c_1^1, x_1^1, \frac{y_1^1}{a_2})}{U_x(c_1^1, x_1^1, \frac{y_1^1}{a_2})}. \tag{21}$$

Proof: Taking ratios of the first order conditions (12), (14) and (16) yields:

$$\frac{\alpha_1^t MU_{l,1}^t \frac{1}{a_1} - \mu^t \widehat{MU}_{l,2}^t \frac{1}{a_2}}{\alpha_1^t MU_{c,1}^t - \mu^t \widehat{MU}_{c,2}^t} = -w_t \tag{22}$$

$$\frac{\alpha_1^t MU_{c,1}^t - \mu^t \widehat{MU}_{c,2}^t}{\alpha_1^t MU_{x,1}^t - \mu^t \widehat{MU}_{x,2}^t} = 1 + r_{t+1}. \tag{23}$$

Equation (22) implies

$$\frac{MU_{l,1}^t}{MU_{c,1}^t} < -a_1 w_t, \tag{24}$$

establishing Statement i.).

Savings are taxed at a non–negative marginal rate if and only if

$$MRS_{c_1,x_1}^t = \frac{MU_{c,1}^t}{MU_{x,1}^t} \leq 1 + r_{t+1}. \tag{25}$$

Upon substitution of (23) into (25), one may deduce the following chain of equivalent statements.

$$\frac{MU_{c,1}^t}{MU_{x,1}^t} \leq \frac{\alpha_1^t MU_{c,1}^t - \mu^t \widehat{MU}_{c,2}^t}{\alpha_1^t MU_{x,1}^t - \mu^t \widehat{MU}_{x,2}^t} \tag{26}$$

$$MU_{c,1}^t \left[\alpha_1^t MU_{x,1}^t - \mu^t \widehat{MU}_{x,2}^t \right] \leq MU_{x,1}^t \left[\alpha_1^t MU_{c,1}^t - \mu^t \widehat{MU}_{c,2}^t \right] \tag{27}$$

$$-\mu^t MU_{c,1}^t \widehat{MU}_{x,2}^t \leq -\mu^t MU_{x,1}^t \widehat{MU}_{c,2}^t \tag{28}$$

$$\frac{MU_{c,1}^t}{MU_{x,1}^t} \geq \frac{\widehat{MU}_{c,2}^t}{\widehat{MU}_{x,2}^t}. \tag{29}$$

Note: the multiplication carried out in the first step does not change the sense of the inequality, because the terms in square brackets are all positive. This can be checked by looking at the first–order conditions and noting that the multipliers are all positive. Statement ii.) follows. □

Statement i.) is a restatement of the standard result of a positive marginal rate of taxation for agents of low type (cf. Guesnerie and Seade (1982)). It

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holds in the current model because there is only one potential mimicker of the individual of type 1 born at time t, the type–2 individual born at the same time. Preventing this form of mimicking requires a downward distortion of the labour supply of type–1 agents.

Relation (21) is easily interpreted in light of the work by Boadway and Keen (1993). It says that it is optimal to tax savings if and only if mimickers value future consumption more than individuals of type 1 do. To see why this is the case, consider an initial situation in which savings are untaxed for all agents of some generation t. Now consider increasing c_1^t by one unit, decreasing the future consumption of that same individual by $MRS_{c_1,x_1}^t = 1+r_{t+1}$ (because saving is untaxed at the margin). This change is production feasible, and leaves the affected agent equally well off as before. However, when (21) is satisfied, the young agent of type 2 at time t finds mimicking less attractive. Thus, the self–selection constraint is slackened, and a welfare improvement can be effected.

4 Concluding Remarks

This note has outlined a framework for analysing the optimality of savings taxation in an overlapping generations setting. It was found that saving should be taxed at the margin if and only if high ability agents have, in some sense, a greater preference for future consumption. Taxing future consumption then becomes a means to enhance the effectiveness of redistribution. This intuitive result is presented as an illustration of simplification afforded by treating the two–type model.

Of course, this note presents only a preliminary step on the road to understanding nonlinear taxation in dynamic settings. Importantly, there is an assumption that agents can be induced to reveal their type when young. This

assumption may not be too bad in the current context, given that old individuals supply no labour. However, when labour is supplied in many periods of life, the link between simple revelation mechanisms and tax schedules is lost.[5] The consequences for (modelling) optimal tax schedules remain largely unexplored.

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[5]See Dillén and Lundholm (1996) for an exposition of this issue in a two–period model.

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