A general equilibrium theory of North-South trade

Chichilnisky, Graciela

Cambridge University

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CHAPTER 1

A general equilibrium theory of North–South trade

Graciela Chichilnisky

This chapter presents an application of competitive general equilibrium theory of markets in the spirit of Walras, formalized in the 1950s by K. Arrow and by G. Debreu. In using general equilibrium theory to generate insights into current policy issues, it follows a tradition established by Arrow in his work on welfare economics of medical care (1963), on the organization of economic activity (1969), on the evaluation of public investment (Arrow and Lind, 1970), and in urban economic development (1970).

The intention is to use formalized general equilibrium theory to derive general statements about the economic behavior and interrelations between two groups of countries: industrial and developing countries. The first group is represented by a cluster of competitive market economies called the *North,* and the second by a similar cluster of competitive economies called the *South:* thus the name North–South trade. The goal is to obtain simple and general results, and for this purpose we consider a stylized model with the minimum of characteristics needed for the task: two regions, two produced goods, and two factors of production. Within this simple model, we explore issues of current import, such as export-led policies and the transmission of economic activity between regions. The underlying theme is that general equilibrium analysis is indeed useful for disclosing patterns of economic behavior and for suggesting policies, a point of view that guided classical international economics.

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1 Policy issues and main results

The classical trade models developed by Heckscher, Ohlin, Lerner, and Samuelson were concerned with gains from trade between countries having similar preferences and technologies, but with different endowments of factors of production. These models explained why trade takes place between similar countries. In the earliest part of this century, trade among similar countries, indeed industrial countries, was the most important segment of international trade. However, North–South trade, which now accounts for 40 percent by value of OECD trade, takes place between countries of very different characteristics. They differ not only in factor endowments but also in preferences and technologies. To understand this important and growing component of world trade, we need a framework that can incorporate explicitly the diversity of technologies and demand patterns as well as the more traditional differences in factor endowments and that can relate this diversity to the welfare effects of trade.

This chapter develops a rigorous general equilibrium analysis of trade between two competitive market economies with significant differences both in technologies and in endowments. Within this framework, we use general equilibrium comparative statistics analysis to study the welfare effects of changes in the volume of trade across free trade equilibria. We study changes in the market equilibrium in response to changes in parameters that are exogenous to the model. We also examine the comparative statics effects that an expansion in the North has on the South. These are two current topics: Export policies of the developing countries are at the forefront of discussions on the international debt, and the issue of whether or not an economic expansion in the industrial countries is transmitted to the developing countries underlies many policy prescriptions. Many oil-exporting countries and exporters of other raw materials show disappointing records after a decade of concentration on production for exports.

The aim of this essay is to explain why, in the words of Arthur Lewis, the international market works at times to concentrate rather than to diffuse the gains from trade (Lewis 1983). I hope to explore in some detail how the international market transmits economic activity from one region to the other. The explanations that I seek are in terms of the primitive structural characteristics of the domestic economies of the trading regions, such as technologies factor supplies and demand structures, and not in terms of the derived parameters of international markets, such as elasticities of international demand at a market equilibrium.

For many developing countries, the degree of involvement in the international economy is a major policy decision, with export-oriented domestic production having been strongly recommended by international agencies for many years. However, as Samuelson has noted, even under the assumptions of classical trade theory we cannot, in general, claim that a country will benefit from orienting its production toward international trade.[1] The policy issue facing most countries is not one of choosing between free trade and autarky. It is one of choosing between policies that would result in more or less emphasis on an international sector of the economy. The classical theorems on the welfare effects of trade provide little information about such choices. This is the issue we study here.

Initial results on these problems were obtained in Chichilnisky (1981), who dealt with trade between an industrial country and a labor-abundant developing country and showed how the structural differences between the countries (and also between sectors within a country) play an important role in determining the welfare impact of changes in the volume of trade and in the transmission of economic activity through international markets.

A feature of the 1981 results that seemed counterintuitive and attracted attention is that an increase in the exports of a labor-intensive product could lower real wages and terms of trade in the labor-abundant South. Labor abundance was described by the responsiveness of labor supply to real wages, and it was proved that even in cases where the labor supply is abundant, an expansion of labor-intensive exports could have these negative effects. Of course, these same effects occur when labor is *not* abundant, and in this chapter we give necessary and sufficient conditions for such results. We consider here cases where the South's demand for basics derives both from capital and from wage income.

Another feature of the results that attracted attention was that an expansion in the North parameterized by an increase in its demand for industrial goods could lead to an expansion in the South's exports and simultaneously could lower terms of trade and real wages in the South. This result led to several comments that centered on the question: How is it possible that in a stable market economy such as that of the North–South model, one can have simultaneously an increase in the volume of exports from the South demanded by the North and a drop in their market prices? This chapter deals with this question. It shows how under conditions similar to those discussed above, a move toward an equilibrium with a higher industrial demand in the North leads indeed to a higher volume of exports of labor-intensive goods from the South but to lower terms of trade and lower real wages in the South, within a stable market economy. The results are traced to simultaneous supply and demand responses, which are not readily perceived within a partial equilibrium framework but appear quite naturally in general equilibrium models. Further results are obtained here showing that in such cases the North may actually consume more of both goods and the South less, following the industrial expansion in the North and the increase in exports from the South.

Of course, precisely the opposite effects can also happen. This essay also examines sufficient conditions for *positive* outcomes of an export expansion: As exports increase, terms of trade and real wages improve. It also examines conditions for a positive transmission of an expansion in the North to the South: As the North increases its industrial demand, its imports increase, leading to better terms of trade and real wages in the South. The purpose of our general equilibrium analysis is to provide a rigorous framework to analyze which case is likely to occur and under which specifications of the economies. We also discuss some empirical aspects on the basis of recent econometric implementations of the model for the case of trade between Sri Lanka and the United Kingdom in Chichilnisky, Heal, and Podivinsky (1983) and for the case of Argentina and the United States in Chichilnisky and McLeod (1984).

2 The North–South model and its solutions

This section summarizes the general equilibrium model in Chichilnisky (1981, 1984a, b). A version of this model is a special case of an Arrow–Debreu general equilibrium model: This is shown in Section f of the appendix.

There are two regions, North and South. The North represents the industrial countries, the South the developing countries. Each region produces and consumes two goods: basics (B) and industrial goods (I). There are two inputs to production: capital (K) and labor (L). The two regions trade with each other.

Consider first the economy of the South. It produces basics and industrial goods using labor and capital, as described by the Leontief production functions

$$B^S = \min(L^B/a_1, K^B/c_1),$$

$$I^S = \min(L^I/a_2, K^I/c_2),$$

where the superscripts B and I denote the sector in which inputs are used, and the superscript S denotes supply. Basics are labor-intensive and in-

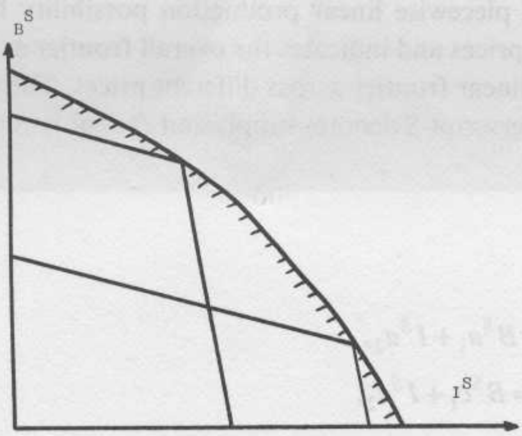


Figure 1. The overall production possibility frontier across equilibria when $D \neq 0$. For each set of prices, the production possibility frontier is piecewise linear; as prices change, endowments of factors vary and a new piecewise linear production set arises. The overall frontier is smooth.

dustrial goods capital-intensive, that is, $D = a_1c_2 - a_2c_1 > 0$. These production functions were chosen for the sake of analytic tractability. More general production functions can be utilized with no significant changes in the results (see, e.g., Benhabib and Chichilnisky 1984). In any case, across equilibria, this economy exhibits substitution between the total amount of labor and capital employed; this is discussed below.

We can now write the equations that specify equilibrium of the model. Competitive behavior on the part of the firms ensures zero profits, so that

$$p_B = a_1 w + c_1 r, \tag{2.1}$$

$$p_I = a_2 w + c_2 r, \tag{2.2}$$

where p_B and p_I are the prices of B and I; w and r are the wages and the rate of return on capital.[2]

Labor and capital supplied are increasing functions of their rewards:

$$L^S = \alpha(w/p_B) + \bar{L} \quad (\alpha > 0), \tag{2.3}$$

$$K^S = \beta r + \bar{K} \qquad (\beta > 0). \tag{2.4}$$

Since factor supplies vary with factor prices by equations (2.3) and (2.4), the model exhibits substitution in the total use of capital and labor across equilibria when $D = a_1c_2 - a_2c_1 \neq 0$. Figure 1 illustrates the production possibility frontier: Across equilibria, commodity prices change and factor prices change so that factor endowments change too, by (2.3) and (2.4).

The diagram exhibits a piecewise linear production possibility frontier for each price vector of prices and indicates the overall frontier as the envelope of the piecewise linear frontier across different prices. The market clearing conditions (superscript S denotes supply and D denotes demand) are

$$L^S = L^D, \tag{2.5}$$

$$K^S = K^D, \tag{2.6}$$

$$L^D = L^B + L^I = B^S a_1 + I^S a_2, \tag{2.7}$$

$$K^D = K^B + K^I = B^S c_1 + I^S c_2, \tag{2.8}$$

$$B^S = B^D + X_B^S, \tag{2.9}$$

where X_B^S denotes exports of B,

$$I^D = X_I^D + I^S, \tag{2.10}$$

where X_I^D denotes imports of I, and

$$p_B X_B^S = p_I X_I^D, \tag{2.11}$$

that is, the value of exports equals the value of imports.[3]

The North is specified by a set of equations similar to (2.1)–(2.11), with possibly different technology and factor supply parameters. In a world equilibrium, the prices of traded goods are equal across regions (factors K and L are not traded) and exports match imports:

$$p_I(\mathrm{S}) = p_I(\mathrm{N}), \tag{2.12}$$

$$p_B(\mathrm{S}) = p_B(\mathrm{N}), \tag{2.13}$$

$$X_B^S(\mathrm{S}) = X_B^D(\mathrm{N}), \tag{2.14}$$

$$X_I^S(\mathrm{N}) = X_I^D(\mathrm{S}), \tag{2.15}$$

where (S) and (N) denote South and North, respectively.

In each region, there are eight exogenous parameters: a_1, a_2, c_1, c_2, α, $\bar{L}$, β, and $\bar{K}$, making a total of *sixteen exogenous parameters* for the North–South model. When we add the price-normalizing condition,[4]

$$p_I = 1, \tag{2.16}$$

we have a *total of twenty-six independent equations:* (2.1)–(2.11) for North; (2.1)–(2.11) for South, (2.12)–(2.14), and (2.16).[5] There are in total twenty-eight *endogenous variables,* fourteen for each region: p_B, p_I, w, r, L^S, L^D, K^S, K^D, B^S, B^D, X_B^S, I^S, I^D, and X_I^D. Therefore, the system is undetermined so far up to two variables.[6] Thus, we now specify two more variables exogenously, industrial demand in the South, $I^D(\mathrm{S})$, and in the North, $I^D(\mathrm{N})$, adding two more equations:

$$I^D(\mathrm{N}) = \bar{I}^D(\mathrm{N}), \tag{2.17}$$

$$I^D(\mathrm{S}) = \bar{I}^D(\mathrm{S}). \tag{2.18}$$

Obviously, we could have solved the model by specifying exogenously other variables or else by postulating demand equations; this will be done in the following sections. The demand specifications of the model are chosen to meet two criteria: analytical tractability and empirical plausibility.

The North–South model is, therefore, a system of twenty-eight equations in twenty-eight variables, depending on eighteen exogenous parameters: a_1, a_2, c_1, c_2, α, $\bar{L}$, β, $\bar{K}$, and $\bar{I}$ for each region.

The economies of the North and of the South are identical except possibly for the values of their exogenous parameters. Differences in the structural characteristics of the two regions are described by differences in their exogenous parameters. For instance, in the North the two sectors (B and I) use approximately the same technology, that is, the economy is technologically homogeneous. This means that $a_1/c_1 \sim a_2/c_2$ so that the determinant $D(\mathrm{N})$ of the matrix of technical coefficients

$$\begin{pmatrix} a_1 & c_1 \\ a_2 & c_2 \end{pmatrix}$$

is close to zero in the North. In the South, instead, technologies are dualistic: The two sectors use factors very differently, and $D(\mathrm{S})$ is therefore large. In both regions, $D(\mathrm{N})$ and $D(\mathrm{S})$ are positive, which indicates that the B sector uses labor more intensively than the I sector. Another difference arises in factor markets. In the North, labor is relatively more scarce, that is, less responsive to increases in the real wage w/p_B. This means $\alpha(\mathrm{N})$ is small. In the South the opposite is true, $\alpha(\mathrm{S})$ is large. The reciprocal relations hold in capital markets: $\beta(\mathrm{N})$ is large and $\beta(\mathrm{S})$ is small. These parameter specifications can be presented so as to be independent of the units of measurements.

It is worth noting that whereas most equations are linear in the variables, some are not [e.g., (2.3) is nonlinear]. The solutions also display nonlinearities, as we shall see in the following.

Proposition 1. The North–South model has at most one equilibrium.[7] This equilibrium can be computed explicitly by solving one equation that depends on all exogenous parameters of the model.

Proof: From
$$X_I^D(\mathrm{S}) = X_I^S(\mathrm{N}),$$
we have
$$\bar{I}^D(\mathrm{S}) - I^S(\mathrm{S}) = I^S(\mathrm{N}) - \bar{I}^D(\mathrm{N}). \tag{2.19}$$
Inverting (2.7) and (2.8), we obtain
$$B^S = \frac{c_2 L - a_2 K}{D}, \qquad I^S = \frac{a_1 K - c_1 L}{D}, \tag{2.20}$$
and inverting (2.1) and (2.2),
$$w = \frac{p_B c_2 - c_1}{D}, \qquad r = \frac{a_1 - p_B a_2}{D}. \tag{2.21}$$
Using (2.3), (2.4), (2.20), and (2.21), we can rewrite equation (2.19) as a function of one variable only, p_B (which is the *terms of trade* of the South, since $p_I = 1$) and obtain
$$p_B^2(A + A(\mathrm{N})) + p_B[C + C(\mathrm{N}) + \bar{I}^D(\mathrm{S}) + \bar{I}^D(\mathrm{N})] - (V + V(\mathrm{N})) = 0, \tag{2.22}$$
where
$$A = \frac{\beta a_1 a_2}{D^2}, \qquad V = \frac{\alpha c_1^2}{D^2},$$
and
$$C = \frac{1}{D}\left(c_1 \bar{L} - a_1 \bar{K} + \frac{\alpha c_1 c_2 - \beta a_1 a_2}{D}\right),$$
and where expressions A, V, and C contain parameter values for the South, and $A(\mathrm{N})$, $V(\mathrm{N})$, and $C(\mathrm{N})$ for the North. Solving equation (2.22) yields an equilibrium level of terms of trade p_B^* as a function of all eighteen exogenous parameters of the system.

It is easy to check that equation (2.22) has at most one positive root p_B^* because the constant term is negative and the quadratic term is positive. From this and (2.21), one obtains the equilibrium values of w^* and r^* for each region; from (2.3) and (2.4), L^* and K^* for each region; from (2.20), $(B^S)^*$ and $(I^S)^*$ for each region. From (2.9), (2.10), and (2.11) we then obtain $(B^D)^*$, $(X_B^S)^*$, and $(X_I^D)^*$ for each region. All endogenous variables have been computed, and the solution is complete. ■

In the following, we shall consider different specifications of demand. Equation (2.18) in the South is substituted by one of three different specifications of equilibrium levels of demand:

$$X_B^S(\mathrm{S}) = \bar{X}_B^S, \tag{2.18a}$$

$$B^D(\mathrm{S}) = \frac{wL}{p_B}, \tag{2.18b}$$

and finally,

$$B^D(\mathrm{S}) = \gamma \frac{wL}{p_B} + \lambda rK, \tag{2.18c}$$

for $\gamma, \lambda < 1$ and at least one positive.

A different specification of the North's demand will also be considered. Equation (2.17) is substituted by

$$B^D(\mathrm{N}) = \bar{B}^D, \tag{2.17a}$$

that is, an exogenous specification of the North's demand for basics at the equilibrium.

In the following section, we show that all these different specifications of demand lead to similar results. The specification with equation (2.18a) is useful to parameterize the solutions of the model by one real variable denoting the volume of exports $X_B^S(\mathrm{S})$: As $\bar{X}_B^S$ varies [and $I^D(\mathrm{N}) = \bar{I}^D(\mathrm{N})$ remains fixed], we obtain a one-dimensional path of equilibria across which we may carry out comparative statics exercises about the effects of changes in the volume of exports of the South on the equilibrium levels of endogenous variables. This was done in Chichilnisky (1981) and is also done in the following section. Equation (2.18b) is useful to derive necessary and sufficient conditions for increases or decreases in terms of trade as exports change in cases where labor is not necessarily abundant. Equation (2.18c) shows that the results obtain even when the demand for basics comes also from capital income. Equation (2.17a) is used to show that an increase in the North's demand for basics may lead to increased exports of basics from the South and simultaneously to a lower price of basics. This is also done in the following section.

The model's specification with equations (2.17) and (2.18) was used in Chichilnisky (1981) to study the impact of an industrial expansion in the North (an increase in the exogenous value of $\bar{I}^D(\mathrm{N})$) [Proposition 2, Chichilnisky (1981)].

In all cases, an exogenous change in $\bar{I}^D$, or $\bar{X}_B^S$, is simply a change in a number that is exogenously given to the model. Such an exogenous change can be interpreted in many different fashions, thus making the results rather general. For instance, an increase in $\bar{I}^D(\mathrm{N})$ could be a result of a shift in underlying preferences in the North leading to a new equilibrium level of demand for industrial goods; see Section f of the appendix. We have therefore indexed the utility function in the North by $\bar{I}$ and considered a change in this utility index. Other different possibilities are that industrial demand $\bar{I}^D(\mathrm{N})$ is fixed by a quota, or that it is random. Whatever the reason, any model in which all equations but (2.17) are satisfied, and in which a change in $I^D(\mathrm{N})$ has taken place, will have the same properties predicted in our theorems. The theorems are therefore rather general: They apply to a large class of models all of which share the equilibrium equations (2.1)–(2.11) in each region, and (2.12)–(2.14), (2.16), and (2.18). Similarly, an exogenous change in X_B^S could be regarded as a change in preferences in the North that led to a change in the equilibrium volume of exports of the South; or as a similar change in the South leading to a change in its exports; or finally, as a change in an import quota of the North. In all these cases, the comparative statics results of all exogenous change in X_B^S remain unchanged.

Note, finally, that no information has been given so far about (disequilibrium) excess demand or supply functions: Outside of the equilibrium, demands are not defined so far. Supply functions are not defined outside of an equilibrium either, because we have constant returns to scale. Therefore, the above information does not suffice to discuss stability: This task is taken up in a subsequent section.

3 Further results on export-led strategies

This section concentrates on the comparative statics results. The appendix provides a program for the model, as well as numerical simulations that reproduce the propositions.

The first two propositions deal with the 1981 version of the model, which we call the North–South model. The model consists of twenty-eight equations, or equations (2.1)–(2.11) in each region; (2.12)–(2.14); and (2.16), (2.17), and (2.18). There are twenty-eight variables: In each region, these are p_B, p_I, r, w, B^S, B^D, I^S, I^D, X_I^S, X_I^D, K^S, K^D, L^S, L^D. We now parameterize the solutions of the North–South model by varying the equilibrium level of industrial demand in the North, $I^D(\mathrm{N})$, in equation (2.17). For each value of $\bar{I}^D(\mathrm{N})$, there is (at most) a unique solution to the model, by Proposition 1. Therefore, as $\bar{I}^D(\mathrm{N})$ varies, we obtain a one-dimensional set of equilibria (under regularity conditions, a one-dimensional manifold) contained in the space of all endogenous variables, R^{28}. Comparative statics results arise from studying the relationships between two or more endogenous variables along this set of equilibria. The following propositions study the relationship between the export level X_B^S, the terms of trade p_B, and the real wages w/p_B of the South across different equilibria of the North–South model.

Proposition 2. Consider the North–South economy, where the South exports basic goods, has abundant labor, α large, and dual technologies, that is, $c_2/D < 2w/p_B$. Then a move to an equilibrium with a higher level of exports of basics leads to lower terms of trade and to lower real wages in the South. [Proposition 1 in Chichilnisky (1981).]

When labor is abundant and real wages are low, or else technologies are homogeneous, that is, $c_2/D > 2w/p_B$, then a move to an equilibrium with a higher level of exports X_B^S leads to better terms of trade and higher real wages in the South. [Proposition 2 in Chichilnisky (1981).]

Proof:

$$X_B^S(\mathrm{S}) = B^S - B^D;$$

by equation (2.20),

$$B^S = \frac{c_2 L - a_2 K}{D};$$

and by Walras' law and equation (2.17),

$$B^D = \frac{wL + rK - \bar{I}^D(\mathrm{S})}{p_B}.$$

Therefore,

$$\begin{aligned} X_B^S &= \frac{c_2 L - a_2 K}{D} - \frac{wL + rK - \bar{I}^D(\mathrm{S})}{p_B} \\ &= \frac{\alpha c_1}{D^2 p_B}\left(c_2 - \frac{c_1}{p_B}\right) + \frac{\beta a_1}{D^2}\left(a_2 - \frac{a_1}{p_B}\right) + \frac{c_1 \bar{L} - a_1 \bar{K}}{D p_B} + \frac{\bar{I}^D(\mathrm{S})}{p_B}, \end{aligned} \tag{3.1}$$

and the derivative of X_B^S with respect to p_B across equilibria is

$$\frac{dX_B^S}{dp_B} = \frac{\alpha c_1}{D^2 p_B^2}\left(\frac{2c_1}{p_B} - c_2\right) + \frac{\beta a_1^2}{D^2 p_B^2} + \frac{a_1 \bar{K} - c_1 \bar{L}}{p_B^2} - \frac{\bar{I}^D(\mathrm{S})}{p_B^2}. \tag{3.2}$$

Therefore, when $\alpha(\mathrm{S})$ is large, dX_B^S/dp_B has the sign of $(2c_1/p_B) - c_2$, which equals that of $c_2/D - 2w/p_B$ by equation (2.21). Finally note that by (2.21), across equilibria, $d(w/p_B)/dp_B > 0$. ■

An intuitive explanation of this result is in Arrow (1981):

> Very loosely the argument is the following. Suppose the rise in export demand for the B commodity were followed by an increase in its price. Since its production is highly labor intensive, there would be a rise in real wages and since labor supply is higher responsive to the real wage,

> a considerable increase in labor supply. The rise in both real wage and labor supply increases even more rapidly the domestic demand for the B commodity, since it is all directed to the B commodity. Hence supply available for export would *decrease* and therefore would not match the increased demand for exports. It follows that the only way the (rise in) export demand could be met, under these conditions, would be a decrease in the price of commodity B and of real wages.

In this light, it seems useful to explain the role of the assumption $c_2/D < 2w/p_B$. The term c_2/D represents a supply response and the term $2w/p_B$ a demand response across equilibrium. The inequality $<$ indicates that the demand response exceeds the supply response, as discussed in Arrow (1981).

Remarks: The expression $c_2/D - 2w/p_B$ is simple and has a ready economic interpretation: It describes a relationship between real wages and technological parameters. Intuitively speaking, when technologies are "dual," D is large and c_2/D is small; thus, we call the condition $c_2/D < 2w/p_B$ *technological duality.* This condition has also the advantage of being relatively easy to test econometrically since real wages and input–output coefficients are relatively accessible data (see Chichilnisky et al. 1983).

However, from a theoretical viewpoint, this condition is not presented in a standard fashion because it mixes parameters (c_2/D) with endogenous variables ($2w/p_B$). It seems therefore useful to express $c_2/D - 2w/p_B$ as a function of exogenous parameters only. We can then check *inter alia* its consistency with the other condition in Proposition 2, that is, $\alpha(\mathrm{S})$ large. From equation (2.21), $c_2/D < 2w/p_B$ is equivalent to $p_B > 2(c_1/c_2)$. Since p_B is a function of exogenous parameters only, by equation (2.22), we obtain that $c_2/D < 2w/p_B$ is equivalent to an expression that depends solely on exogenous parameters:

$$\frac{\gamma + (\gamma - 4(V + V(\mathrm{N}))(A + A(\mathrm{N}))^{1/2}}{2(A + A(\mathrm{N}))} > 2\frac{c_1}{c_2} \tag{3.3}$$

where A and V were defined in Proposition 1, and

$$\gamma = C + C(\mathrm{N}) + \bar{I}^D(\mathrm{S}) + \bar{I}^D(\mathrm{N}).$$

It is easy to check that inequality (3.3) is indeed a plausible combination of parameters, when $\alpha(\mathrm{S})$ is large. For example, the simulations in Section a of the appendix have numerical values of the exogenous parameter values where both $\alpha(\mathrm{S})$ large and (3.3) are satisfied simultaneously.

Sufficient conditions can also be given for (3.3) to be satisfied along with $\alpha(\mathrm{S})$ large in terms of an interval of values of $D(\mathrm{S})$ (note that $\bar{L}$ is generally negative; otherwise there would be positive labor supply at zero real wages): One proves this by showing that when α and D are rather large in the South, by (2.22), p_B is approximated by

$$\frac{c_1^2}{(D/\alpha)(c_1\bar{L}-a_1\bar{K})+c_1c_2},$$

where all parameters are for the South. This expression exceeds $2c_1/c_2$ when the denominator is positive and small. This will occur within an interval for the exogenous parameter D.

Proposition 3 [Proposition 3 in Chichilnisky (1981)]. Assume the South has abundant labor and dual technologies: α large and $c_2/D<2w/p_B$. Then a move to a new equilibrium with a higher level of industrial demand in the North leads to a higher level of exports of basics from the South, but to lower terms of trade, real wages, and domestic consumption in the South. This occurs in Walrasian stable markets.

Proof: From equation (2.22) and the implicit function theorem,

$$\frac{dp_B}{d\bar{I}^D(\mathrm{N})}=\frac{p_B}{2p_B(A+A(\mathrm{N}))+C+C(\mathrm{N})+\bar{I}^D(\mathrm{S})+\bar{I}^D(\mathrm{N})}, \tag{3.4}$$

where A and C are defined in (2.22).

When α is large in the South, the sign of the term in α determines the sign of $C+C(\mathrm{N})$. Since the term in α within $C+C(\mathrm{N})$ is $\alpha c_1c_2/D^2$, a positive number, $C+C(\mathrm{N})$ is positive in this case. Furthermore, A and $A(\mathrm{N})$ are always positive. It follows that

$$dp_B/d\bar{I}^D(\mathrm{N})<0 \tag{3.5}$$

when α in the South is large. Therefore when α is large, an increase in the equilibrium level of industrial demand in the North leads to a drop in the equilibrium price of basics.

Now, consider the equation relating exports of basics with their price across equilibria:

$$X_B^S=\frac{\alpha c_1}{D^2p_B}\left(c_2-\frac{c_1}{p_B}\right)+\frac{\beta a_1}{D^2}\left(a_2-\frac{a_1}{p_B}\right)+\frac{c_1\bar{L}-a_1\bar{K}}{Dp_B}+\frac{\bar{I}^D(\mathrm{S})}{p_B}.$$

As seen in Proposition 2, $dX_B^S/dp_B<0$ when $c_2/D<2w/p_B$. Added to inequality (3.5), this implies $dX_B^S/d\bar{I}^D(\mathrm{N})>0$.

To summarize: A move to an equilibrium with a higher level of industrial demand in the North [i.e., an increase in the parameter $\bar{I}^D(\mathrm{N})$] leads both to a larger volume of imports of basic goods by the North (higher X_B^S) and to lower terms of trade for the South (lower p_B).

To complete the proof, it suffices now to point out that real wages w/p_B are always positively associated with the price of basics [by (2.21)] and that in equilibrium the consumption of basics is also positively associated with their price in the South when α is large. This is because $B^D = [wL + rK - \bar{I}^D(\mathrm{S})]/p_B$, and this expression is dominated by the term in α, that is, by $\alpha(w/p_B)^2$, which is an increasing function of p_B. Stability was established in the appendix of Chichilnisky (1981) and is discussed further in the following section. ■

Our next step is to extend the 1981 results. The next proposition sharpens Proposition 1 of Chichilnisky (1981): It obtains results not only on terms of trade but also on *total export revenues* following an export expansion. These results are obtained without any assumptions on the international elasticities of demand.

Proposition 4. In the North–South economy, assume that the South has abundant labor, α large, and dual technologies, $c_2/D < 2w/p_B$. Then a move to a new equilibrium with a higher volume of exports leads not only to lower terms of trade but also to lower export revenues in the South. However, when $c_2/D > 2w/p_B$, terms of trade and export revenues increase following the expansion in exports.

Proof: When $c_2/D < 2w/p_B$ and α is large, by Proposition 2, as the level of exports X_B^S increases, the South's terms of trade p_B drop at the new equilibrium. By (2.21), wages decrease and the rate of profit increases. This implies from (2.3) and (2.4) that total capital available increases, and total labor employed decreases. Therefore, the domestic supply of industrial goods I^S increases, since $I^S = (a_1 K - c_1 L)/D$. Since the industrial demand in the South is constant by (2.18), and the supply I^S has increased, the volume of imports of industrial goods $X_I^D(\mathrm{S}) = I^D(\mathrm{S}) - I^S(\mathrm{S})$ must therefore decrease when the price of basics drops. By the balance-of-payments condition $p_B X_B^S = X_I^D$, the total revenue from exports, $p_B X_B^S$, decreases when $c_2/D > 2w/p_B$ by Proposition 2. When $c_2/D > 2w/p_B$, a rise in exports leads to better terms of trade so that export revenues increase as well. ■

The next proposition studies changes in *both* regions, following either an increase of exports X_B^S or an industrial expansion in the North, that is, an increase in $\bar{I}^D(\mathrm{N})$; or an increase in demand for basics $\bar{B}^D(\mathrm{N})$. These are exogenous changes in either of three numbers: $I^D(\mathrm{N})$, $X_B^S(\mathrm{S})$, or $B^D(\mathrm{N})$. Cases with exogenous changes in X_B^S refer to the version of the model where equation (2.18) of the North–South model is replaced by (2.18a); all other equations remain unchanged. Cases where $B^D(\mathrm{N})$ increases exogenously refer to the North–South model where equation (2.17) is replaced by equation (2.17a): All others remain unchanged.

Proposition 5. Assume that technologies are dual, $c_2/D < 2w/p_B$, and α is large in the South. Furthermore, assume that labor supply in the North is unresponsive to the real wage [$\alpha(\mathrm{N})$ small] and that industrial goods in the North use little labor (a_2 small). Then:

i. A move to a new equilibrium with a higher level of industrial demand in the North, $\bar{I}^D(\mathrm{N})$, leads to higher levels of imports of basics and of consumption of basic goods in the North. The North consumes simultaneously more of both goods and is in this sense strictly better off. The South exports more basics, at lower prices, and receives lower export revenues. In the South, real wages and consumption decrease. The South consumes fewer basics and the same amount of industrial goods: It is therefore strictly worse off at the new equilibrium.

ii. Identical results obtain when the exogenous parameter is $\bar{X}_B^S(\mathrm{S})$. A move to a new equilibrium with an increased level of exports of the South, $X_B^S(\mathrm{S})$, or of import quotas in the North, $X_B^D(\mathrm{N})$ (allowing now either $I^D(\mathrm{N})$ or $I^D(\mathrm{S})$ to adjust), leads to lower terms of trade, export revenues, wages, and consumption in the South.

iii. Finally, identical results occur when the initial exogenous change is in the equilibrium level of demand for basics in the North, $\bar{B}^D(\mathrm{N})$. At the new equilibrium, the North's demand for basics and its imports of basics increase, but the price of basics goes down. The North consumes more of both goods; the South consumes the same amount of industrial goods and less basics. The North is therefore strictly better off, and the South is worse off.

All these results occur in Walrasian stable markets.

Proof: Consider first the case where in the North $\alpha(\mathrm{N}) = 0$ and $a_2(\mathrm{N}) = 0$. The supply of basics in the North is then a constant, since

$$B^S = (c_2 L - a_2 K)/D = c_2 \bar{L}/D \quad \text{when} \quad \alpha = a_2 = 0.$$

(Note that $\bar{L}$ must be positive in this case.) Since the consumption of basics of the North is the sum of domestic supply plus imports, $B^D(\mathrm{N}) = B^S(\mathrm{N}) + X_B^D(\mathrm{N})$, and $B^S(\mathrm{N})$ is a constant, when imports of basics $X_B^D(\mathrm{N})$ increase, consumption of basics in the North, $B^D(\mathrm{N})$, must increase as well.

Proposition 3 shows that, under the conditions, a move to an equilibrium with a higher level of industrial demand in the North, $I^D(\mathrm{N})$, leads to more exports of basics, $X_B^S(\mathrm{S}) = X_B^D(\mathrm{N})$. Therefore, in our case, this leads to increased consumption of basics in the North. The equilibrium levels of demand for industrial goods $I^D(\mathrm{N})$ and for basics $B^D(\mathrm{N})$ have therefore increased simultaneously in the North. For any reasonable welfare measure, the North is strictly better off. By continuity, the same results obtain when $a_2(\mathrm{N})$ and $\alpha(\mathrm{N})$ are close to zero, proving the first part of the theorem.

In the South, export revenues decrease as shown in Proposition 4 above. As the terms of trade p_B decrease, real wages and the consumption of basics decrease in the South, as shown in Proposition 3. Since industrial demand remains constant in the South, the South is strictly worse off.

The statement in ii follows from the fact that parameterizing the solutions by the level of exports X_B^S or by the level of industrial demand in the North, $I^D(\mathrm{N})$, leads to the same comparative statics results. Finally, we prove part iii.

Assume $\bar{B}^D(\mathrm{N})$ is exogenously increased, $\alpha(\mathrm{N}) = 0$, and $a_2(\mathrm{N}) = 0$. Then, as shown above, B^S remains constant, $B^S = c_2\bar{L}/D$. The increase of demand must be met by a higher level of imports of basics, $X_B^D(\mathrm{N}) = X_B^S(\mathrm{S})$. But $c_2/D < 2w/p_B$ and $\alpha(\mathrm{S})$ large imply that at the new equilibrium with a higher level of $X_B^S(\mathrm{S})$, both p_B and $p_B X_B^S$ are lower, as shown in Proposition 4. This means that exports of industrial goods $X_I^S(\mathrm{N})$ are lower $(p_B X_B^S = X_I^S)$. As p_B is lower, r and K are higher, that is, supply of I increases in the North. Therefore $I^D = I^S - X_I^S$ increases too: The North now consumes more basics *and* more industrial goods. The rest of the theorem is an application of Propositions 1–4. Stability is established in Section 4. ■

Having extended the comparative statics results of the 1981 model in Propositions 4 and 5, we now turn to an extension of the model itself. This allows us to obtain sharper and more general results. The new results are applicable to economies that may or may not have abundant labor.

The extension of the model proposed here was formulated in Chichilnisky (1981, p. 179).

An extension of the North–South model

The North–South model presented in Section 2 is altered now in a rather simple fashion already discussed in Section 2. The change is in the specification of demand in the South. Rather than assuming that the equilibrium level of industrial demand in the South is a given constant, we assume instead that, in equilibrium, wage income in the South is spent on the basic good. This entails replacing equation (2.18), that is, $I^D(\mathrm{S}) = \bar{I}^D(\mathrm{S})$, by

$$p_B B^D = wL. \tag{2.18b}$$

This version of the North–South model (denoted II) consists therefore of the same equations as in the North–South model but for equation (2.18), which is replaced by (2.18b). As before, the model has a unique solution. Note that in the following results no assumption is made about labor abundance in the South. Comparative statics is performed by varying exogenously either X_B^S or $I^D(\mathrm{N})$: The comparative statics results obtained under these two exogenous changes are the same.

Proposition 6. Consider a North–South economy II where capital stocks in the South are fixed ($K = \bar{K}$), *and* $L = \alpha w/p_B$ ($\bar{L} = 0$). Then a necessary and sufficient condition for an increase in exports to lower the South's terms of trade, real wages, and consumption is technological duality: $c_2/D < 2w/p_B$. In particular, when the economy is homogeneous, $c_2/D > 2w/p_B$, the South's terms of trade always improve as the South increases its exports, and its real wages and consumption of basics increase as well.

When $\bar{L} \neq 0$, the necessary and sufficient condition is, instead, $c_2/D < 2w/p_B + \bar{L}$.

Proof: Consider the equation for the equilibrium volume of exports, $X_B^S(\mathrm{S}) = B^S(\mathrm{S}) - B^D(\mathrm{S})$. From $B^S = (c_2 L - a_2 K)/D$ and $B^D = wL/p_B$ and substituting for L and K from equations (2.3) and (2.4), we obtain

$$\frac{dX_B^S}{dw/p_B} = \alpha\left(\frac{c_2}{D} - \frac{2w}{p_B}\right) - \bar{L}. \tag{3.6}$$

When $\bar{L} = 0$, the necessary and sufficient condition for $dX_B^S/d(w/p_B)$ to be negative is $c_2/D < 2w/p_B$. When $\bar{L} \neq 0$, we obtain, instead, $c_2/D < 2w/p_B + \bar{L}$.

To complete the proof, note that the real wage is an increasing function of the price of basics across equilibria [from equation (2.21)].

Finally, the consumption of basics is an increasing function of the real wage across equilibria, since $B^D = w/p_B L = \alpha(w/p_B)^2 + w/p_B \bar{L}$ by (2.3) and (2.18b). ■

The following proposition obtains results analogous to those of Proposition 3 but for the North–South model II.

Proposition 7. Consider a North–South economy II where the capital stock in the South is fixed $(K = \bar{K})$ and $L = \alpha w/p_B$ $(\bar{L} = 0)$. Labor in the South need not be abundant. Then an increase in the North's industrial demand leads to an increase in exports and to lower terms of trade, real wages, and consumption of basics in the South if and only if the duality condition holds in the South, $c_2/D < 2w/p_B$. When $\bar{L} \neq 0$, the condition is $c_2/D < 2w/p_B + \bar{L}$. Furthermore, if the rate of profit in the South is sufficiently low that $r < a_1/D$, an increase in exports lowers also total export revenues of the South.

The consumption of basics and of industrial goods increases simultaneously in the North provided industrial goods use little labor [a_2(N) small] and labor is rather unresponsive to the real wage [α(N) small].

When $c_2/D > 2w/p_B$, the results are positive: An increase in the North's industrial demand leads to an increase in exports and to higher terms of trade, real wages, and consumption of basics in the South. The sign of $c_2/D - 2w/p_B$ determines therefore whether the international market transmits or hinders economic expansion across the two regions under the conditions.

Proof: First, we study the relationship between the equilibrium price of basics and the level of industrial demand of the North. Since Walras' law is always satisfied in an equilibrium, $p_B B^D + I^D = wL + rK$, and by assumption (2.18b), $p_B B^D = wL$, it follows that

$$I^D = rK = \beta r^2 + r\bar{K}.$$

Across equilibria, therefore,

$$\frac{dr}{dI^D} = \frac{1}{2r\beta + \bar{K}} > 0. \tag{3.7}$$

Furthermore, from equation (2.3), across equilibria,

$$\frac{dr}{dp_B} = \frac{-a_2}{D} < 0. \tag{3.8}$$

Therefore, from equations (3.7) and (3.8), it follows that

$$\frac{dp_B}{dI^D(\mathrm{N})} < 0,$$

that is, an increase in the industrial demand in the North leads to a lower price of basics at the new equilibrium.

We have already proved in Proposition 6 that (under the conditions) a necessary and sufficient condition for a negative association of export levels and the price of basics is duality in the South: $c_2/D < 2w/p_B$ when $\bar{L}=0$, or $c_2/D < 2w/p_B + \bar{L}$ when $\bar{L} \neq 0$. Therefore, since p_B is negatively associated with $I^D(\mathrm{N})$, these two conditions are also necessary and sufficient for an *increase* in exports and for a simultaneous *decrease* in the terms of trade of the South, as the industrial demand in the North, $I^D(\mathrm{N})$, increases. Since Proposition 6 showed that real wages and consumption of basics in the South all decrease with the price of basics, this completes the first part of the proof.

Next consider the condition on profits $r < a_1/D$. By (2.18b) and Walras' law, imports of the South equal

$$X_I^D(\mathrm{S}) = I^D(\mathrm{S}) - I^S(\mathrm{S}) = rK - \frac{a_1 K - c_1 L}{D} = \left(r - \frac{a_1}{D}\right)K + \frac{c_1}{D}L.$$

It follows that, across equilibria,

$$\frac{dX_I^D(\mathrm{S})}{dp_B} = \left(r - \frac{a_1}{D}\right)\frac{dK}{dp_B} + \frac{c_1}{D}\frac{dL}{dp_B}.$$

By assumption, $r < a_1/D$; since $dK/dp_B < 0$ and $dL/dp_B > 0$, it follows that $dX_I^D(\mathrm{S})/dp_B > 0$. The imports of the South decrease as the price of basics drops, across equilibria.

By the balance-of-payments condition, total export revenues $p_B X_B^S(\mathrm{S})$ equal the value of imports $X_I^D(\mathrm{S})$. Therefore, we have proved that when $c_2/D < 2w/p_B$, export revenues fall with a decrease in the price of basics, across equilibria. The opposite happens when $c_2/D > 2w/p_B$: p_B and export revenues increase.

Finally, under the conditions, the consumption of basics will increase in the North following an expansion in industrial demand whenever $a_2(\mathrm{N})$ is small and $\alpha(\mathrm{N})$ is small, as proved in Proposition 5. ■

The North–South model with fixed factor endowments

The last two propositions in this section consider economies with fixed factor endowments, that is, $K = \bar{K}$ and $L = \bar{L}$ ($\alpha = \beta = 0$). The first proposition refers to the North–South model, and the last two to its versions II and III.

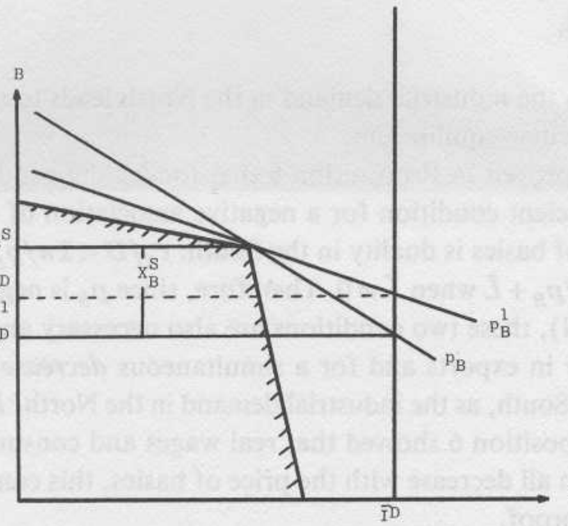


Figure 2. The North–South model with fixed factor endowments: The production possibility frontier has a shaded piecewise linear boundary. Across equilibria, the demand for industrial goods $\bar{I}^D$ is constant and an *increase* in the price of basics from p_B to p_B^1 leads always to a *drop* in the volume of exports X_B^S. Note that domestic supply B^S remains constant, but domestic demand for basics B^D increases to B_1^D when the price of basics increases from p_B to p_B^1.

Proposition 8. Consider a North–South model with fixed factor endowments. In this case, a move to a new equilibrium with higher levels of exports always lowers the terms of trade and export revenues of the South and leads also to lower real wages and consumption of basics in the South. Figure 2 illustrates this result.[8]

Proof: When $\alpha = 0$ and $\beta = 0$, the cross-equilibria relation between exports and their price p_B is

$$X_B^S = \frac{c_2 \bar{L} - a_2 \bar{K}}{D} - \frac{wL + r\bar{K} - \bar{I}^D(\mathrm{S})}{p_B}$$

$$= \left(\frac{c_2}{D} - \frac{w}{p_B}\right)\bar{L} - \left(\frac{a_2}{D} + \frac{r}{p_B}\right)\bar{K} + \frac{\bar{I}^D}{p_B}.$$

Substituting w and r, one obtains

$$X_B^S = \frac{c_1 \bar{L} - a_1 \bar{K}}{D p_B} + \frac{\bar{I}^D(\mathrm{S})}{p_B},$$

so that

$$\frac{dX_B^S}{dp_B} = \frac{a_1\bar{K} - c_1\bar{L}}{Dp_B^2} - \frac{\bar{I}^D(\mathrm{S})}{p_B^2},$$

which is always negative since $a_1\bar{K} > c_1\bar{L}$ [see, e.g., equation (2.20)]. As the price of basics drops, the real wage drops as well. Also, the consumption of basic goods, $B^D = (w\bar{L} + r\bar{K} - \bar{I}^D(\mathrm{S}))/p_B$, decreases when $\bar{L}$ is large since dB^D/dp_B is dominated by the expression $\bar{L}(d(w/p_B)/dp_B)$, which is positive. ■

Proposition 9. Consider a North–South model II with fixed factor endowments in the South. Then a move to an equilibrium with increased exports of the wage good leads always to a drop in the South's terms of trade. It also leads to a drop in real wages and in the consumption of the wage good in the South. However, in the new equilibrium, the South imports more industrial goods.

Proof: In the North–South model II, we have $X_B^S = (c_2\bar{L} - a_2\bar{K})/D - w/p_B\bar{L}$, that is,

$$X_B^S = \left(\frac{c_2}{D} - \frac{w}{p_B}\right)\bar{L} - \frac{a_2\bar{K}}{D}.$$

By substitution from equation (2.21), this equals $X_B^S = (c_1/p_BD)\bar{L} - a_2\bar{K}/D$, so that $dX_B^S/dp_B = -c_1\bar{L}/p_B^2D$, which is always negative. Therefore, a move to an equilibrium with increased exports of the wage good leads always to a decrease in their price, p_B. Since $w/p_B = c_2/D - c_1/p_BD$, the real wage decreases, and domestic demand for wages goods, being $B^D = w\bar{L}/p_B = (c_2/D - c_1/p_BD)\bar{L}$, also decreases as $dB^D/dp_B = c_1\bar{L}/p_B^2D > 0$. Finally, we show that imports of industrial goods increase. Consider the domestic demand for industrial goods in the South: In this case, this is $I^D = r\bar{K}$. Since p_B decreases following the export expansion, the new equilibrium profits r are higher, by equation (2.4). Therefore, industrial demand $I^D(\mathrm{S})$ increases in the South. However, since factor endowments are constant, industrial supply I^S has not changed. Therefore, the higher level demand of industrial goods at the new equilibrium must be due to increased imports of industrial goods. ■

The last proposition in this section substitutes the demand specification (2.18) by (2.18c), that is, demand from basics derives both from wages and from capital income:

$$B^D=\gamma\frac{w}{p_B}L+\lambda rk, \quad \gamma,\lambda<1,$$

and at least one positive in the South. All other equations remain unchanged. We call this the North–South model III.

Proposition 10. Consider the North–South model III where the demand for basics derives both from labor and capital income and from fixed factor endowments ($L=\bar{L}$, $K=\bar{K}$). If all capital income is spent on the basic good B, increasing the volume of exports leads always to better terms of trade but to lower domestic consumption of basics. If demand for basics comes both from labor and capital income, the terms of trade may increase or decrease as exports expand. Their response depends on the sign of the expression

$$\lambda a_2\bar{K}-\gamma\frac{c_1}{p_B^2}\bar{L}.$$

Proof: In the South, across equilibria,

$$\begin{aligned}X_B^S=B^S-B^D&=\frac{c_2}{D}\bar{L}-\frac{a_2}{D}\bar{K}-\left(\gamma\frac{w}{p_B}\bar{L}+\lambda r\bar{K}\right)\\&=\bar{L}\left(\frac{c_2}{D}-\frac{\gamma w}{p_B}\right)-\bar{K}\left(\frac{a_2}{D}+\lambda r\right)\\&=\bar{L}\left(\frac{c_2}{D}(1-\gamma)+\frac{\gamma c_1}{Dp_B}\right)-\bar{K}\left(\frac{a_2}{D}(1-p_B\lambda)+\lambda\frac{a_1}{D}\right),\end{aligned}$$

and

$$\frac{dX_B^S}{dp_B}=-\bar{L}\frac{\gamma c_1}{Dp_B^2}+\frac{a_2}{D}\lambda\bar{K}.$$

When $\gamma=0$, $dX_B^S/Dp_B>0$. When $\lambda=0$, $dX_B^S/dp_B<0$, as in Proposition 9. The sign of dX_B^S/dp_B is that of $a_2\lambda\bar{K}-(\gamma c_1/p_B^2)\bar{L}$. Finally, note that $(dB^D/dp_B)<0$ when $B^D=rK$. ■

4 Stability

This section studies the stability of the North–South model and discusses several comments that address this issue.

It was shown in Chichilnisky (1981) that the North–South model is stable under the conditions of Propositions 2 and 3. Arrow (1981) pointed out that individual equilibria are stable in the usual sense of general equilibrium theory.[9] Heal and McLeod extended and generalized the 1981 stability results to a wide family of adjustment processes that contain, as a special case, the process in Chichilnisky (1981). Other commentators (e.g., Gunning, Findlay, Bhagwati, Srinivasan, Ranney, and Saavedra) proposed an adjustment process quite different from that in Chichilnisky (1981), but these authors nevertheless all agree on the stability of the model.[9]

For a discussion of stability, we must now define our disequilibrium adjustment process. This is necessary because the previous sections studied only moves from one equilibrium to another, and no information has been given so far that could be used to decide whether shocking the system away from one equilibrium would make it return to the equilibrium or not. All equations given so far are equilibrium relations. For instance, each proposition of Sections 2 and 3 assumes that profits are zero, namely, that commodity prices are linear combinations of factor prices, given by the commodity–factor price equations:

$$p_B = a_1 w + c_1 r, \qquad p_I = a_2 w + c_2 r, \tag{4.1}$$

or the equivalent inverse equations, the factor–commodity price relations:

$$w = (p_B c_2 - c_1)/D, \qquad r = (a_1 - p_B a_2)/D.$$

Profits must be zero in equilibrium because the North–South model has constant returns to scale, and the producers are competitive. But profits are typically *not* zero during an adjustment process that leads from a disequilibrium position to an equilibrium. Classical studies of stability in constant returns-to-scale economies such as Samuelson (1949) and Arrow and Hurwicz (1963) use profits as a driving force in the adjustment processes: In disequilibrium, producers increase output when profits are positive, and they decrease it when profits are negative. More recently, Mas-Colell (1974) has studied Walrasian stability in a constant returns-to-scale economy that is very similar, indeed an enlarged version of the model presented here. He too uses profits as a driving force in the adjustment process, a process he attributes to Walras: Profits in his model are only zero at equilibrium. Similarly, the adjustment process of Chichilnisky (1981, 1984) and the more general processes in Heal and McLeod (1984) all have nonzero profits in disequilibrium and, indeed, these profits play an important role in the adjustment process. In all of these adjustment processes, therefore, the commodity–factor price equations (4.1) do *not* hold outside of an equilibrium.

This point is worth noting because some commentators, that is, Findlay, Bhagwati, Srinivasan, and Ranney, use a different process, one in which profits are assumed to be identically zero at every disequilibrium point even while commodity markets adjust. They assume that the *zero profit* commodity–factor price relations (4.1) hold at every disequilibrium position of the commodity markets. Formally, the process used by these commentators cannot be Walrasian because the price equations (4.1) imply that factor prices are continuously varying as functions of good prices, even though their factor markets are continuously at an equilibrium, and excess demand in these markets is always zero: In a Walrasian process, there can be no price changes in markets that remain with zero excess demand. The process used by these commentators, which assumes zero profits at every disequilibrium point, rules out those processes of Samuelson, Arrow and Hurwicz, Mas-Colell, and Heal and McLeod, all of which assume nonzero profits outside of equilibrium. It also rules out the process of Chichilnisky (1981), so that these authors are working on a different model altogether.

We now define the process given in the appendix of Chichilnisky (1981). First, we study one region, and then we study both North and South. The North–South model has four markets in each region, for capital, labor, basics, and industrial goods. A typical Walrasian adjustment requires that market prices be positively associated with the excess demand in that market. Therefore, for each (disequilibrium) price vector $p=(p_B, p_I, w, r)$, we assume

$$\begin{aligned} \dot{p}_b &= \mathrm{DB}(p)-\mathrm{SB}(p), \qquad \dot{p}_I = \mathrm{DI}(p)-\mathrm{SI}(p), \\ \dot{w} &= \mathrm{DL}(p)-\mathrm{SL}(p), \qquad \dot{r} = \mathrm{DK}(p)-\mathrm{SK}(p), \end{aligned} \tag{4.2}$$

where the letters D and S preceding a variable indicate (disequilibrium) demand and supply, respectively. This notation is deliberately different from that for equilibrium supply and demand and also different from the notation in the appendix of Chichilnisky (1981) so as to avoid confusion between, for example, the *equilibrium level of demand* B^D and the *demand function* for basics $\mathrm{DB}(p)$.

To avoid the technicalities of a four-dimensional dynamical system, we assume, in a fashion analogous to Arrow and Hahn (1971, Chapter 12) and to much of the trade literature, that some of the markets are always at an equilibrium and that the burden of adjustment lies on the other markets. Factor markets are assumed always to clear $\mathrm{DL}=\mathrm{SL}$ and $\mathrm{DK}=\mathrm{SK}$; by (4.2), this implies $\dot{w}=\dot{r}=0$. Also, since industrial goods are the *numéraire,* p_I is identically equal to 1, so that $\dot{p}_I = 0$. The market for basics is therefore the only one in which price and quantity adjustments take place along the adjustment process, following the differential equation $\dot{p}_B = \mathrm{DB}(p) - \mathrm{SB}(p)$.

The next step is to define the supply and demand functions $\mathrm{SB}(p)$ and $\mathrm{DB}(p)$ for all disequilibrium prices. This includes, in particular, price vectors $p = (p_B, 1, w, r)$ where profits are not zero, that is, where the commodity–factor price equations (4.1) do not necessarily hold. Such a supply function has not been defined previously in this chapter, and we do so now. Note that as the North–South model has constant returns to scale, profit maximization conditions do not determine standard supply functions independently from demand considerations: Disequilibrium supply functions are not well defined with constant returns to scale [see, e.g., Arrow and Hahn (1971), Chapter 12, Section 10]. Several alternatives are possible, and we follow one that appears reasonable. For any given price vector $p = (p_B, 1, w, r)$, we use the factor supply equations

$$\mathrm{SL}(p) = \alpha w / p_B + \bar{L} \quad \text{and} \quad \mathrm{SK}(p) = \beta r + \bar{K}$$

to determine the level of factors supplied at p. Since factor markets always clear, supplies of labor and capital must match demand,

$$\mathrm{SL}(p) = \mathrm{DL}(p) = L(p) \quad \text{and} \quad \mathrm{SK}(p) = \mathrm{DK}(p) = K(p),$$

and thus we obtain the level of capital and labor employed at price p. If firms use their factors efficiently, this means that at the (disequilibrium) price vector p, supply of B is

$$\mathrm{SB}(p) = \frac{c_2 L(p) - a_2 K(p)}{D} = \frac{c_2}{D}\left(\alpha \frac{w}{p_B} + \bar{L}\right) - \frac{a_2}{D}(\beta r + \bar{K}). \tag{4.3}$$

This defines the *supply function for basics* at any (disequilibrium) price vector p.

We now define the demand function for basics $\mathrm{DB}(p)$ and for industrial goods $\mathrm{DI}(p)$ for any price vector p. These definitions must be consistent with *Walras' law,* which states that at any (disequilibrium) price p, the value of expenditures equals the value of income:

$$p_B \mathrm{DB}(p) + \mathrm{DI}(p) = wL(p) + rK(p) + \Pi(p), \tag{4.4}$$

where $\Pi(p)$ denotes profits at p. Recall that unit profits at the price vector p are given by $p_B - a_1 w - c_1 r$ in the B sector and $1 - a_2 w - c_2 r$ in the I sector. Total profits $\Pi(p)$ are therefore equal to

$$\Pi(p) = \mathrm{SB}(p_B - a_1 w - c_1 r) + \mathrm{SI}(1 - a_2 w - c_2 r) \tag{4.5}$$

and are generally not zero outside an equilibrium.

The disequilibrium *demand function for basics* DB(p) was defined on page 190 of Chichilnisky (1981) [denoted $B^D(p)$] as

$$p_B \mathrm{DB}(p) = wL(p) + rK(p) - \bar{I}^D(p), \tag{4.6}$$

that is,

$$\mathrm{DB}(p) = \alpha\left(\frac{w}{p_B}\right)^2 + \frac{w\bar{L} + r\bar{K}}{p_B} + \frac{\beta r^2}{p_B} - \frac{\bar{I}^D}{p_B},$$

where w and r are the equilibrium values and $\bar{I}^D$ is the constant defined in equation (2.17).

By Walras' law, this definition of DB(p) is *equivalent* to defining the (disequilibrium) *demand function for industrial goods* DI(p) as

$$\mathrm{DI}(p) = \bar{I}^D + \Pi(p). \tag{4.7}$$

Therefore, the definition of DB in the appendix of Chichilnisky (1981) implies that profits $\Pi(p)$ are always spent in the industrial sector. In particular, since profits are nonzero and an increasing function of p_B in disequilibrium, DI(p) is *not* a constant out of equilibrium; rather it is an increasing function of p_B, or equivalently DI(p) is downward sloping in the relative price of p_I, as stated in Chichilnisky (1981, p. 168). This is worth noting because some of the commentators (e.g., Findlay, 1984a, b; Srinivasan and Bhagwati, 1984) drew a *vertical* disequilibrium demand function for industrial goods; that is, they postulated that in disequilibrium, industrial demand is always a constant $\bar{I}^D$. This is not correct since, as seen above, a constant demand function for I would contradict Walras' law and the definition of the demand for B in Chichilnisky (1981, p. 190).

We define now the disequilibrium *supply function for industrial goods* in analogy with the supply of basics [equation (4.3)]. At a disequilibrium price $p = (p_B, 1, w, r)$, let

$$\mathrm{SI}(p) = \frac{a_1 K(p) - c_1 L(p)}{D} = \frac{a_1}{D}(\beta r + \bar{K}) - \frac{c_1}{D}\left(\alpha \frac{w}{p_B} + \bar{L}\right). \tag{4.8}$$

By construction, Walras' law is satisfied, that is, the value of excess demand is equal to zero.

Having defined the supply and demand functions for basics and industrial goods, we may now compute whether the system is stable in a neighborhood of one equilibrium. To study stability in the neighborhood of one equilibrium, $p^* = (p_B^*, 1, w^*, r^*)$ under the adjustment process defined above, one considers a shock to p^*. From equation (4.2), the factor prices w and r must remain at the equilibrium values during a Walrasian adjustment process ($\dot{w} = \dot{r} = 0$) because factor markets have been assumed to clear at all times.[10] From here on, I shall therefore assume that $w = w^*$ and $r = r^*$. Stability in a neighborhood of one equilibrium requires that all the eigenvalues of the Jacobian of the system (4.2) have negative real parts. This Jacobian is the 4×4 matrix of the partial derivatives of the four functions $\mathrm{DB}(p) - \mathrm{SB}(p)$, $\mathrm{DI}(p) - \mathrm{SI}(p)$, $\mathrm{DL}(p) - \mathrm{SL}(p)$, and $\mathrm{DK}(p) - \mathrm{SK}(p)$ with respect to the four variables p_B, p_I, w, and r. However, since $\dot{p}_I = \dot{w} = \dot{r} = 0$, the matrix has only one nonzero term, which is the partial derivative of the excess demand for basics with respect to the price of basics, that is, $\partial/\partial p_B[\mathrm{DB}(p) - \mathrm{SB}(p)]$. For stability, this partial derivative must be negative.

From the above definitions of supply and demand [equations (4.3) and (4.6)], we have, for any (disequilibrium) price p, the expression for *excess demand for basics:*

$$\mathrm{DB}(p) - \mathrm{SB}(p) = \frac{wL(p) + rK(p) - \bar{I}^D}{p_B} - \frac{c_2}{D} L(p) + \frac{a_2}{D} K(p), \tag{4.9}$$

which by the definition of supply of labor and capital in disequilibrium is equal to $\alpha(w/p_B)^2 + \beta(r^2/p_B) + w/p_B(\bar{L} - \alpha c_2/D) + r(\beta a_2/D + \bar{K}/p_B) + (\bar{K}a_2 - \bar{L}c_2)/D - \bar{I}^D/p_B$. For stability, the partial derivative of the excess demand function $\mathrm{DB}(p) - \mathrm{SB}(p)$ with respect to p_B must be negative, that is,

$$\frac{\partial}{\partial p_B}(\mathrm{DB}(p) - \mathrm{SB}(p)) = \frac{\alpha w}{p_B^2}(c_2/D - 2w/p_B) - \frac{w\bar{L}}{p_B^2} - \frac{\beta r^2}{p_B^2} - \frac{\bar{K}r}{p_B^2} + \frac{\bar{I}^D}{p_B^2} < 0. \tag{4.10}$$

When α is sufficiently large, the term $\alpha w/p_B^2(c_2/D - 2w/p_B)$ dominates the expression (4.10). Therefore, *the B market is stable when the duality condition $c_2/D < 2w/p_B$ is satisfied.* This is, of course, the stability condition on pages 190–191 of Chichilnisky (1981) and is also the condition obtained by Heal and McLeod (1984) in their adjustment process of Section 4.1.

It is now immediate to show stability in the industrial goods market, for which it is required that the *partial derivative* $\partial/\partial p_B(\mathrm{DI}(p) - \mathrm{SI}(p))$ be positive. This is a direct application of Walras' law. Since the value of excess demand is zero, that is, $p_B(\mathrm{DB}(p) - \mathrm{SB}(p)) + \mathrm{DI}(p) - \mathrm{SI}(p) = 0$, it follows that

$$\frac{\partial}{\partial p_B}(\mathrm{DI}(p)-\mathrm{SI}(p))=\frac{\mathrm{DI}(p)-\mathrm{SI}(p)}{p_B^2}-\frac{\partial}{\partial p_B}(\mathrm{DB}(p)-\mathrm{SB}(p)).$$

Near an equilibrium, $\mathrm{DI}(p)-\mathrm{SI}(p)$ is close to zero, and therefore $(\partial/\partial p_B)(\mathrm{DI}(p)-\mathrm{SI}(p))$ is indeed positive when $\partial/\partial p_B(\mathrm{DB}(p)-\mathrm{SB}(p))<0$. Thus *the market for industrial goods is also stable when α is large and $c_2/D<2w/p_B$*. Figures 3 and 4 reproduce numerically the disequilibrium supply and demand equations for B and for I for a simulation of the model with initial parameter values satisfying the above conditions. These figures illustrate stability of both markets.

We have therefore proved the stability results in Chichilnisky (1981):

Proposition 11. [Appendix 2, Chichilnisky (1981)]. Under the Walrasian adjustment process $\dot{p}=\mathrm{DB}(p)-\mathrm{SB}(p)$ of Chichilnisky (1981), the economy of the South is stable when α is large and $c_2/D<2w/p_B$.

Proof: See the preceding text starting from equation (4.6) to the statement of this proposition. ■

As mentioned above, the process in Chichilnisky (1981) assumes that $I^D=\bar{I}^D+\Pi(p)$: Profits are spent in the I sector. Heal and McLeod (1984) have studied more general adjustment processes for the North-South model, in particular, one where a proportion λ of profits is spent on basics and the rest on industrial goods. For each (disequilibrium) price vector $p=(p_B,1,w,r)$, this changes the demand equation (4.6) for basics into

$$\mathrm{DB}(p)=\frac{wL(p)+rK(p)-\bar{I}^D}{p_B}+\frac{\lambda\Pi(p)}{p_B}$$

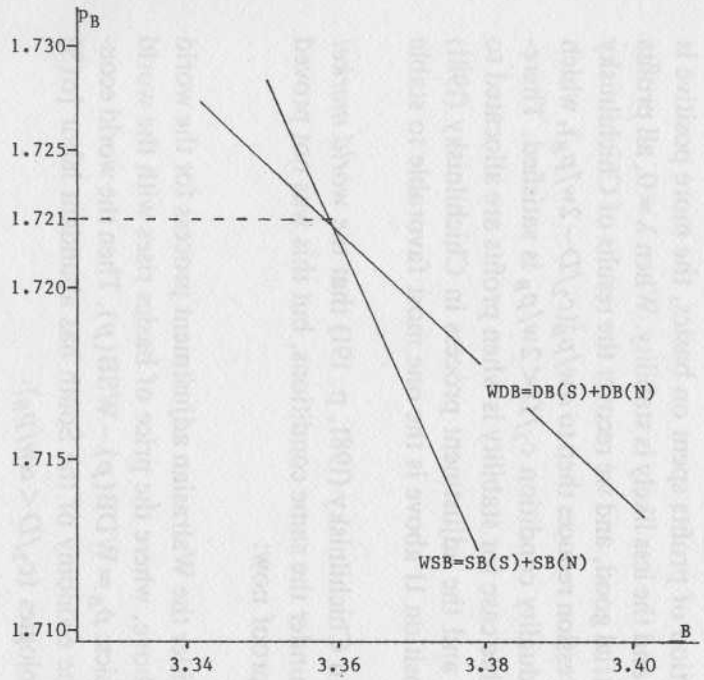
$$=\frac{wL(p)+rK(p)-\bar{I}^D}{p_B}+\frac{\mathrm{SB}(p)(p_B-a_1w-c_1r)\lambda}{p_B}.$$

Since $\mathrm{SB}(p)=(c_2L(p)-a_2K(p))/D$, we obtain

$$\mathrm{DB}(p)=L(p)\left[\frac{w}{p_B}+\frac{\lambda}{D}\left(c_2-\frac{c_2a_1w}{p_B}-\frac{c_2c_1r}{p_B}\right)\right]$$

$$+K(p)\left[\frac{r}{p_B}-\frac{\lambda}{D}\left(a_2-\frac{a_2a_1w}{p_B}-\frac{a_2c_1r}{p_B}\right)\right]-\frac{\bar{I}^D}{p_B}.$$

Therefore, the new *excess demand function for basics* $\mathrm{DB}(p)-\mathrm{SB}(p)$ is



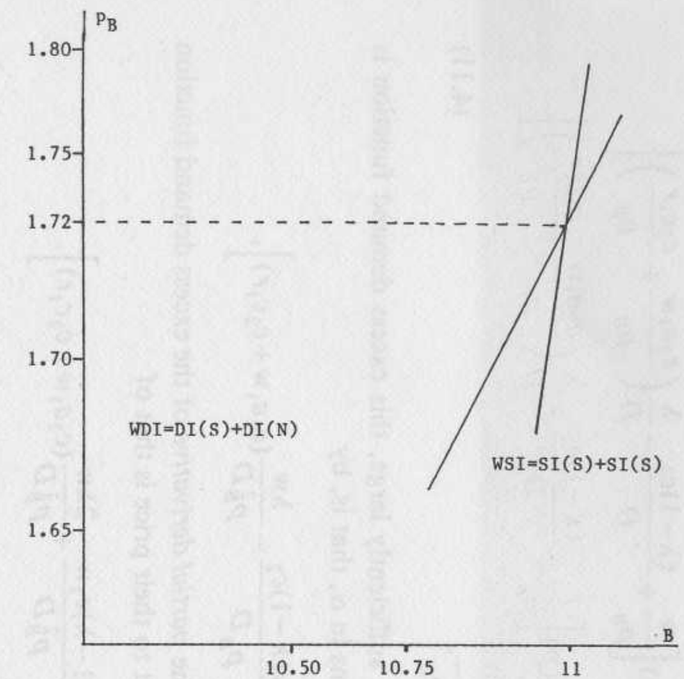


Figure 3. [Left] Simulation of the world's disequilibrium supply and demand curves: world market for basics. Curves were simulated from the given data set at the equilibrium corresponding to $I^D(\mathrm{N}) = 7.0$. WSB = world disequilibrium supply of basics; WDB = world disequilibrium demand for basics. The world market for basics is stable when $\alpha(\mathrm{S})$ is large, and $c_2/D < 2w/p_B$ in the South, Proposition 12, Section 4. WSB = SB(N) + SB(S); WDB = DB(N) + DB(S); slope of WSB = −0.38, slope of WDB = −0.22.

Figure 4. [Right] Simulation of the world's disequilibrium supply and demand curves: world market for industrial goods. Curves were simulated from the given data set at the equilibrium corresponding to $I^D(\mathrm{N}) = 7.0$. WSI = world disequilibrium supply of industrial goods; WDI = world disequilibrium demand for industrial goods. The world market for industrial goods is also stable, under the conditions of Proposition 12, Section 4. WSI = SI(N) + SI(S), WDI = DI(N) + DI(S); slope of WSI = 2.27, slope of WDI = 0.29.

$$\mathrm{DB}(p)-\mathrm{SB}(p)=L(p)\left[\frac{w}{p_B}+\frac{(\lambda-1)c_2}{D}-\frac{\lambda}{D}\left(\frac{c_2a_1w}{p_B}+\frac{c_2c_1r}{p_B}\right)\right]$$
$$+K(p)\left[\frac{r}{p_B}-\frac{(\lambda-1)a_2}{D}-\frac{\lambda}{D}\left(\frac{a_2a_1w}{p_B}+\frac{a_1c_1r}{p_B}\right)\right]$$
$$-\frac{\bar{I}^D}{p_B}. \tag{4.11}$$

Therefore, when α is sufficiently large, this excess demand function is dominated by the terms in α, that is, by

$$\alpha\left[\left(\frac{w}{p_B}\right)^2+\frac{w(\lambda-1)c_2}{p_BD}-\frac{\lambda w}{p_B^2D}(c_2a_1w+c_2c_1r)\right],$$

and thus the sign of the *partial derivative* of the excess demand function for basics with respect to their price is that of

$$\alpha\left[-\frac{2w^2}{p_B^3}+\frac{(1-\lambda)c_2w}{p_B^2D}+\frac{2\lambda w}{p_B^3D}(c_2a_1w+c_2c_1r)\right].$$

Since all the terms in λ are positive, it follows that the larger is λ, that is, the higher the proportion of profits spent on basics, the more positive is the expression above, and the less likely is stability. When $\lambda=0$, all profits are spent on the industrial good, and we recover the results of Chichilnisky (1981): The above expression reduces then to $\alpha w/p_B^2(c_2/D-2w/p_B)$, which is negative when the duality condition $c_2/D<2w/p_B$ is satisfied. Therefore, the most favorable case for stability is when profits are allocated to the industrial sector, and the adjustment process in Chichilnisky (1981) summarized in Proposition 11 above is the one most favorable to stable markets.

It was also stated in Chichilnisky (1981, p. 191) that the *world market* for basics was stable under the same conditions, but this was not proved there. We provide a proof now:

Proposition 12. Consider the Walrasian adjustment process for the world economy described above, where the price of basics rises with the world excess demand for basics: $\dot{p}_B=\mathrm{WDB}(p)-\mathrm{WSB}(p)$. Then the world economy is stable when the economy of the South has abundant labor [$\alpha(\mathrm{S})$ large] and dual technologies ($c_2/D<\alpha w/p_B$).

Proof: Consider a trade equilibrium in the North–South model where the equilibrium exports of the South, $X_B^S(\mathrm{S})$, are matched by equilibrium imports of the North, $X_B^D(\mathrm{N})$, and the equilibrium exports of the North, $S_I^S(\mathrm{N})$, are matched by equilibrium imports of the South, $S_I^D(\mathrm{S})$.

There are now six markets: Two markets for commodities, basics and industrial goods, and four markets for factors, capital and labor in each region. Note that since factors are not traded internationally, they constitute different markets in each region: A price vector is now $p=(p_B, p_I, w(\mathrm{S}), w(\mathrm{N}), r(\mathrm{S}), r(\mathrm{N}))$. A Walrasian adjustment process is now described by prices increasing with the *world excess demand:*

$$\dot{p}_B=\mathrm{DB}(\mathrm{S})(p)-\mathrm{SB}(\mathrm{S})(p)+\mathrm{DB}(\mathrm{N})(p)-\mathrm{SB}(\mathrm{N})(p)=\mathrm{WEDB}(p),$$

$$\dot{p}_I=\mathrm{DI}(\mathrm{S})(p)-\mathrm{SI}(\mathrm{S})(p)+\mathrm{DI}(\mathrm{N})(p)-\mathrm{SI}(\mathrm{N})(p)=\mathrm{WEDB}(p),$$

$$\dot{w}(\mathrm{S})=\mathrm{DL}(\mathrm{S})(p)-\mathrm{SL}(\mathrm{S})(p), \qquad \dot{r}(\mathrm{S})=\mathrm{DK}(\mathrm{S})(p)-\mathrm{SK}(\mathrm{S})(p),$$

$$\dot{w}(\mathrm{N})=\mathrm{DL}(\mathrm{N})(p)-\mathrm{SL}(\mathrm{N})(p), \qquad \dot{r}(\mathrm{N})=\mathrm{DK}(\mathrm{N})(p)-\mathrm{SK}(\mathrm{N})(p).$$

As before, we assume that all factors markets clear and that industrial goods are the *numéraire,* so that $\dot{p}_I=\dot{w}(\mathrm{S})=\dot{r}(\mathrm{S})=\dot{r}(\mathrm{N})=0$. Therefore, wages and profits always remain at their equilibrium levels, and we only need to prove that the first differential equation in $\dot{p}_B$ leads to stability. Supply of basics is defined in the same way as before. For any disequilibrium price vector, we obtain one supply function for each region:

$$\mathrm{DB}(\mathrm{S})(p)=(c_2(\mathrm{S})L(\mathrm{S})(p)-a_2(\mathrm{S})K(\mathrm{S})(p))/D(\mathrm{S})$$

and

$$\mathrm{SB}(\mathrm{N})(p)=(c_2(\mathrm{N})L(\mathrm{N})(p)-a_2(\mathrm{N})K(\mathrm{N})(p))/D(\mathrm{N}),$$

where in each region

$$K(p)=\mathrm{SK}(p)=\beta r+\bar{K} \quad \text{and} \quad L(p)=\mathrm{SL}(p)=\alpha w/p_B+\bar{L}$$

for each (disequilibrium) price vector $p=(p_B, 1, w(\mathrm{S}), w(\mathrm{N}), r(\mathrm{S}), r(\mathrm{N}))$.

Next we define the world demand for basics at any (disequilibrium) price p. In each region Walras' law is now

$$p_B\mathrm{DB}(p)+\mathrm{DI}(p)=wL(p)+rK(p)+\Pi(p)+\mathrm{NX},$$

where NX denotes net export revenues at price p. This implies that at any price vector p, the demand function for basics in each region is now

$$\mathrm{DB}(p)=[wL(p)+rK(p)-\bar{I}^D(p)]/p_B+\mu(\mathrm{NX}/p_B),$$

where μ is the proportion of net export revenues allocated to the B sector. Here we have assumed as before that $\mathrm{DI}(p)=\bar{I}^D+\Pi(p)$, so that profits are spent in the industrial goods.

At an equilibrium, net export revenues NX are, of course, zero: This is the balance-of-payments condition. Outside of an equilibrium, however, NX need not be zero. However, the world's net export revenues, which are the sum of the North's and the South's, NX(N)+NX(S), must be zero. In particular, when $\mu(S)=\mu(N)$, that is, when the same proportion of export revenues goes to basics in both regions,

$$\mu(\mathrm{NX(N)}/p_B)+\mu(\mathrm{NX(S)}/p_B)=0.$$

Therefore, the world excess demand for basics (WEDB), which is the sum of the North's and the South's, does not contain any term in NX. We obtain

$$\begin{aligned}
\mathrm{WEDB}(p) &= (\mathrm{DB(N)}(p)+\mathrm{DB(S)}(p))-(\mathrm{SB(N)}(p)+\mathrm{SB(S)}(p)) \\
&= \alpha\left(\frac{w}{p_B}\right)^2+\frac{\beta r^2}{p_B}+\frac{w}{p_B}\left(\frac{\bar{L}-\alpha c_2}{D}\right)+r\left(\frac{\beta a_2}{D}+\frac{\bar{K}}{p_B}\right) \\
&\quad +\frac{\bar{K}a_2-\bar{L}c_2}{D^2}-\frac{\bar{I}^D}{p_B}+\alpha(\mathrm{N})\left(\frac{w(\mathrm{N})}{p_B}\right)^2+\frac{\beta(\mathrm{N})r(\mathrm{N})^2}{p_B} \\
&\quad +\frac{w(\mathrm{N})}{p_B}\left(\bar{L}(\mathrm{N})-\frac{\alpha(\mathrm{N})c_2(\mathrm{N})}{D(\mathrm{N})}\right)+r(\mathrm{N})\left(\frac{\beta(\mathrm{N})a_2(\mathrm{N})}{D(\mathrm{N})}+\frac{\bar{K}(\mathrm{N})}{p_B}\right) \\
&\quad +\frac{\bar{K}(\mathrm{N})a_2(\mathrm{N})-\bar{L}(\mathrm{N})c_2(\mathrm{N})}{D(\mathrm{N})^2}-\frac{\bar{I}^D(\mathrm{N})}{p_B},
\end{aligned} \tag{4.12}$$

where all the parameters are for the South unless otherwise indicated.

We may now study the stability of the world market for basics, that is, the sign of the partial derivative of WEDB(p) with respect to the price p_B:

$$\begin{aligned}
\frac{\partial}{\partial p_B}(\mathrm{WEDB}(p)) &= \alpha\frac{w}{p_B^2}\left(\frac{c_2}{D}-\frac{2w}{p_B}\right)-\frac{\beta r^2}{p_B^2}-\left(\frac{\bar{L}w+r\bar{K}}{p_B^2}\right)+\frac{\bar{I}}{p_B^2} \\
&\quad +\alpha(\mathrm{N})\frac{w(\mathrm{N})}{p_B^2}\frac{c_2(\mathrm{N})}{D(\mathrm{N})}-\frac{2w(\mathrm{N})}{p_B}-\frac{\beta(\mathrm{N})(r(\mathrm{N}))^2}{p_B^2} \\
&\quad -\frac{\bar{L}(\mathrm{N})w(\mathrm{N})+r(\mathrm{N})\bar{K}(\mathrm{N})}{p_B^2}+\frac{\bar{I}(\mathrm{N})}{p_B^2},
\end{aligned} \tag{4.13}$$

where all parameters, unless otherwise indicated, are for the South. Equation (4.13) shows that when $\alpha(S)$ is sufficiently large in the South, so that the terms in $\alpha(S)$ dominate, $(\partial/\partial p_B)\mathrm{WEDB}(p)$ has the sign of the expression $\alpha w/p_B(c_2/D-2w/p_B)$ in the South. Therefore, when $c_2/D<2w/p_B$ in the South, the world market for basics is stable. As in the one-region case, the world market for industrial goods is also stable from Walras' law. We have thus proved Proposition 12. ■

To summarize: the adjustment process defined in Chichilnisky (1981) was analyzed in detail and extended to the two-region economy. It was shown to yield Walrasian stability under the conditions postulated in Chichilnisky (1981). This process has an element in common with the process defined in Arrow and Hahn (1971) for constant returns-to-scale economies: Some of the markets (in our case, factor markets) are always in equilibrium. It also has an element in common with the processes defined in Samuelson (1949), Arrow and Hurwicz (1963), Mas-Colell (1974), and Heal and McLeod (1984): There are nonzero profits outside of an equilibrium, and these have indeed a nontrivial role in determining the stability of the model.

A final task is to compare the stability process in Chichilnisky (1981) with the comments of Findlay, Bhagwati, Srinivasan, Ranney, and Saavedra. All these comments are based on a particular adjustment process initially proposed by Findlay, as pointed out by Srinivasan and Bhagwati (1984), a process that is quite different from that in Chichilnisky (1981). Yet, with this different process, these authors still obtain stability of the model (for details, see note 9).

Figures 2 and 3 of Bhagwati and Srinivasan [first proposed by Findlay (1984)] are useful for this task. They state that (disequilibrium) demand for industrial goods is constant, that is, $\mathrm{DI}(p)=\bar{I}^D$, and therefore that the (disequilibrium) demand function for industrial goods is a vertical line. Yet we know from equation (4.7) that the disequilibrium demand for industrial goods must necessarily be $\mathrm{DI}=\bar{I}^D+\Pi(p)$ (otherwise Walras' law is violated) and therefore $\mathrm{DI}(p)$ is definitely not a constant function because profits $\Pi(p)$ are not zero outside an equilibrium and indeed vary with p. The conclusions drawn by these authors, derived from their assumption that DI is a constant, are therefore inapplicable to the North–South model because they violate Walras' law. Their assumptions require that profits be always zero outside an equilibrium, that is, equations (4.1).

The same problem appears in Findlay and Srinivasan and Bhagwati's analysis of the market for basics. The downward-sloping cross-equilibria curve X_B^S is confused there with a disequilibrium excess supply curve in the usual sense, that is, with the curve $\mathrm{SB(S)}-\mathrm{DB(S)}$, defined in (4.9) and in page 190 of Chichilnisky (1981). Yet the curves X_B^S and $\mathrm{SB(S)}-\mathrm{DB(S)}$ are very different indeed: In fact, one is downward sloping precisely when the other is upward sloping: When $c_2/D<2w/p_B$, as shown in Proposition 2, dX_B^S/dp_B is *negative,* while $\partial/\partial p_B(\mathrm{SB(S)}-\mathrm{DB(S)})$ is *positive,* as shown in Proposition 11. Obviously X_B^S is *not* an excess supply curve in the usual sense: This was pointed out in Arrow (1981) as well.[11] The correct disequilibrium excess demand for basics in (4.9) has nonzero profits outside an equilibrium. However, since in equilibrium profits are necessarily zero and the two approaches differ only in the value of profits, the two approaches give exactly the same set of equilibria. This explains why all these comments agree on the whole with the comparative statics results of Chichilnisky (1981), which study properties of the set of equilibria. That is, there is agreement on the facts that the export volumes of the South are negatively associated to their price and that higher values of industrial demand in the North lead to lower prices and higher volumes of exports of basic goods from the South (when α is large and $c_2/D<2w/p_B$ in the South). There is also agreement on the fact that the model is stable, even though stability is defined differently by some of the commentators (see note 9).

The differences that arise are therefore rather minor. Neither the stability nor the comparative statics results are questioned. The only point at stake is the interpretation of how the comparative statics results arise, which is only natural since the adjustment process has been changed.

The difference of interpretation is most acute when some of these authors state that a drop in the South's terms of trade following an expansion of the industrial demand in the North, "must follow from a decrease in the North's demand for basics."[12] This is incorrect. Both Propositions 3 and 5 of Section 3 and Theorem 1 in Heal and McLeod (1984) show that the terms of trade of the South drop even with an increase in (domestic or international) demand for basics in the North. This point has also been substantiated by the simulation reported in Heal and McLeod, Section 3, Table 2, where the demand for basics in the North rises and the equilibrium price of basics drops. Furthermore, the appendix to this essay also shows numerically that an increase in the demand for basics in the North, DB(N), accompanied by a positive shift in the excess demand for basics of the North, WD (which intersects X_B^S from above), lead to a *drop* in the price of basics, p_B, and in the purchasing power of the South. This is due to the fact that there is also a positive shift in the excess supply curve of the South when $\bar{I}^D$ increases in the North. The point is simple: In a general equilibrium context, a change in an exogenous parameter leads typically to simultaneous changes in excess supply and excess demand curves, so that even in stable markets, an outward shift in excess demand may be accompanied by a lower new equilibrium price.

It is easy to trace the source of the erroneous conclusion that a drop in the price of basics must be due to a drop in the North's demands for basics. It derives from the confusion of the cross-equilibrium curve X_B^S defined in Proposition 2 with the actual excess supply curve of basics of the

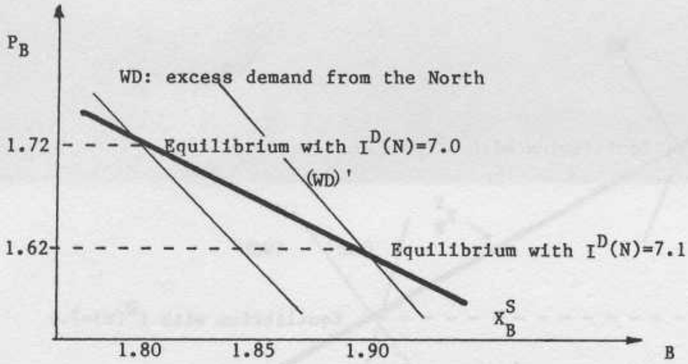


Figure 5. International market for basic goods. This figure reproduces Figure 2(b) in Chichilnisky (1981), where the curves have been computed numerically from the basic data set. For $I^D(\mathrm{N})=7.0$, we obtain WD, and for $I^D(\mathrm{N})=7.1$, we obtain (WD)′. Slope of $X_B^S=-0.9$, slope of WD $=-1.1$, slope of (WD)′ $=-1.1$.

South, defined in the appendix of Chichilnisky (1981) and in equation (4.9). The erroneous conclusion that a drop in the price of basics must be due to a drop in the North's demand for basics derives from a partial equilibrium view in which only one curve shifts at a time (e.g., excess demand for the North) *and* in which the zero-profit cross-equilibrium relation X_B^S is confused with an (unmovable) excess supply curve. Walrasian stability of the model, in which we all agree, would in this erroneous partial equilibrium view lead to an excess demand curve that meets X_B^S from above and must shift downward to reach a lower new equilibrium price. However, when the curve X_B^S is seen properly as an equilibrium locus, and it is understood that the excess supply and demand curves both shift with $\bar{I}^D(\mathrm{N})$, this partial equilibrium interpretation collapses.

Propositions 3 and 5 and the numerical simulations in the appendix show clearly that the terms of trade of the South drop after an increase $I^D(\mathrm{N})$ even when the North's demand for basics is higher (i.e., shifts outward) at the new equilibrium. This occurs within a Walrasian stable market because both demand and supply curves vary simultaneously when $\bar{I}^D(\mathrm{N})$ increases. Figure 2b in Chichilnisky (1981) illustrates this fact. This figure is reproduced from a numerical simulation of the model in the appendix (Figs. 5 and 6). In a Walrasian stable market, a drop in the price of basics will clearly follow an upward shift in demand for basics when the supply curve for basics shifts sufficiently. This is precisely what was shown in Chichilnisky (1981). When the equilibrium value of industrial

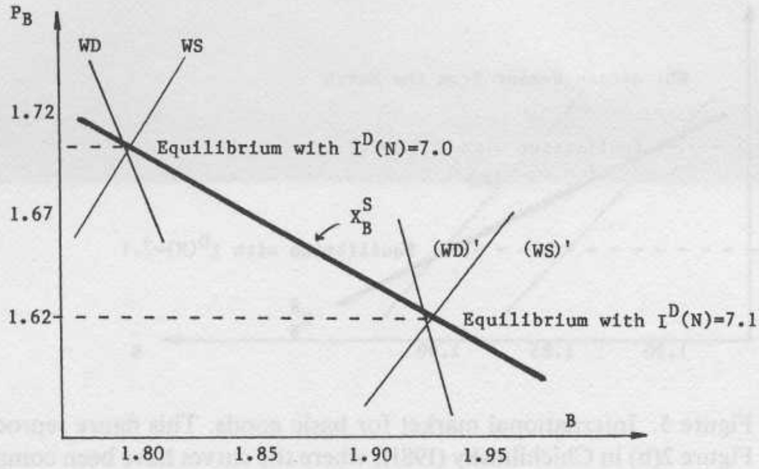


Figure 6. Simulation of X_B^S as a locus of equilibria in the international market for basic goods. WD = DB(N) − SB(N), WS = SB(S) − DB(S), WD and WS correspond to I^D(N) = 7.0. (WD)′ and (WS)′ correspond to I^D(N) = 7.1. The curve X_B^S is computed from a range of different values of I^D(N), which are of course associated to different equilibrium prices p_B. Slope of WD = −1.1, slope of WS = 1, slope of (WD)′ = −1.1, slope of (WS)′ = 1.6, slope of $X_B^S = -0.9$.

demand in the North $\bar{I}^D$(N) increases, both supply and demand curves for basics shift. This is because at the new $\bar{I}^D$(N), a new equilibrium set of prices emerges (p_B^*, 1, w^*, r^*), the only set of prices compatible with the new $\bar{I}^D$(N). From the definitions of the supply and demand functions for basics [equations (4.3) and (4.6)], it is clear that both of these functions shift at the new equilibrium. Obviously, when both demand and supply shift simultaneously, an increase in demand may be accompanied by lower prices, within a Walrasian stable market. This is what was shown to happen in Chichilnisky (1981, Propositions 3 and 5) and in the appendix to this chapter.

In sum: the curve X_B^S is a cross-equilibrium locus of export volumes and export prices and not a disequilibrium excess supply curve valid for testing Walrasian stability. This was pointed out clearly in Chichilnisky (1981) and in Arrow (1981) (see note 11). The use of X_B^S as an excess supply curve is not appropriate for testing Walrasian stability because it requires profits to be zero outside of an equilibrium, that is, the use of price equations (4.1) outside an equilibrium. In addition, when price equations are assumed to be satisfied at all times, factor prices are continuously varying as functions of goods prices, even though the factor markets are continuously at an equilibrium and the excess demand in these markets is always zero, whereas in a Walrasian adjustment process, there can be no price changes in a market that remains with zero excess demand. The disequilibrium excess supply of the South, SB(S) − DB(S), was defined in Appendix 2 of Chichilnisky (1981) and in equation (4.9) above and is clearly very different from X_B^S.

A numerical simulation of the model produces Figure 2(b) of Chichilnisky (1981) with the proper excess supply function of the South, $S =$ SB(S) − DB(S) (see Figs. 5 and 6). These figures show X_B^S as the locus of the intersections of two curves: the (disequilibrium) excess demand curve of the north, WD, and the (disequilibrium) excess supply curve of the South, WS, at different equilibria. Stability is therefore perfectly consistent with a downward-sloping cross-equilibrium relation X_B^S met from above by the North's excess demand curve, WD, that is, with Figure 2(b) in Chichilnisky (1981). How the North's excess demand curve WD meets X_B^S is totally irrelevant for stability; what matters for stability is only how each excess demand of the North, WD, crosses the corresponding excess supply of the South, WS. Since each excess demand WD is downward sloping and the corresponding excess supply WS is upward sloping, an equilibrium is obviously stable (see Figs. 5 and 6).

5 Conclusions and empirical studies

This chapter summarizes and extends the model and results of Chichilnisky (1981, 1984).

Section 2 establishes uniqueness of the solutions and computes an equilibrium explicitly.

Section 3 gives straightforward proofs of Propositions 1, 2, and 3 in Chichilnisky (1981). It also obtains new, more general propositions about the effects of an export expansion. Necessary and sufficient conditions are given for positive and negative outcomes of an export expansion for the South. These conditions are on the internal structures of the economies, technologies, factor endowments, and initial levels of trade and do not depend on international elasticities of demand. Outcomes are in general negative when

(a) labor is abundant and there is "duality," that is, $c_2/D < 2w/p_B$;
(b) the country exports a wage good and there is duality; or
(c) factor endowments are fixed and the country exports a wage good.

We also considered cases where demand for the export good comes from capital income or from a combination of capital and wage incomes. Similar results obtain.

Under certain conditions, following an export expansion of the South, the North consumes simultaneously more of both goods. In contrast, the South's terms of trade, export revenues, and consumption all decrease. There is therefore a transfer of welfare from the South to the North, working its way through the operation of competitive international markets.

The effects of an export expansion typically become positive when the South is more "homogeneous," $c_2/D > 2w/p_B$. The conditions for positive outcomes depend on the initial volume of exports because the sign of the expression $c_2/D - 2w/p_B$ changes as real wages adjust to the new export levels. Therefore, the results address the problem of whether to increase or decrease exports rather than a choice between trade and autarchy. Domestic policies may be used to alter the sign of this expression, $c_2/D - 2w/p_B$, or the responsiveness of labor to real wage, leading to a more positive environment for export-led strategies.

Section 4 discussed stability. It summarized the adjustment process in Chichilnisky (1981) and generalized it to a two-region economy. This section shows the role of profits in the stability of the model and discusses in this context several comments on the 1981 results.

The model introduced in 1981 and developed further above shows that under certain conditions an expansion of labor-intensive exports leads to disadvantageous outcomes for the South, even when this is associated to an expansion of the North's demand for basics. The critical conditions concern the domestic structure of the South's economy as reflected by the inequality $c_2/D \lesseqgtr 2w/p_B$, and the responsiveness of labor supplies to real wages. To evaluate the basic structure of the model and to test its implications for particular cases, a sequence of econometric case studies has been undertaken. The first of these is reported in Chichilnisky, Heal, and Podivinsky (1983). This deals with trade between Sri Lanka and the United Kingdom, which is characterized by the exchange of primary products (mainly tea) for industrial goods.

For this empirical implementation, the equilibrium equations of the North–South model in Section 2 were treated (in reduced form) as describing the long-run steady state of a dynamic system. The system was then assumed to adjust toward this steady-state configuration by a partial adjustment process, in which it adjusts in each period to remove a fraction of the deviations of its variables from their equilibrium values. This is a standard time series implementation of an equilibrium or steady-state model.

The resulting system of nonlinear simultaneous equations was estimated using a twenty-five-year data series by both nonlinear FIML and three-stage least squares. Full details of the results are contained in Chichilnisky et al. (1983): These results confirmed that a dynamic adaptation of the model defined by the equations in Section 2 can provide a good statistical explanation of patterns of trade between Sri Lanka and the United Kingdom and their relationship to technologies and factor prices. The case study also established that in Sri Lanka, where labor is certainly abundant, the inequality $c_2/D < 2w/p_B$ held for every year but one in the sample period 1952–80. In view of Propositions 2 and 3 above, this implies that during the sample period a change in demand in the United Kingdom that led to an expansion of Sri Lanka's exports would lead in statistical terms to a reduction in Sri Lanka's real wages and terms of trade. Statistical analysis of the data confirms this result. The case study thus confirms both the appropriateness of the general structure of the model and the potential importance of the domestic structural issues that it highlights.

A second empirical study was undertaken in Chichilnisky and McLeod (1984) for the case of Argentina and the United States. Here the econometric analysis was restricted to the estimation of the exogenous parameters of the model, in particular the factor supply equations. The model was then simulated on the basis of the estimations of these parameters. An empirical study of export levels and of terms of trade and real wages was undertaken simultaneously. The results showed that when the inequality $c_2/D < 2w/p_B$ was satisfied (plus or minus a standard deviation), terms of trade were negatively associated with export volumes; otherwise, the association was statistically positive. Similar results were obtained with respect to changes in the real wages. The model used for Argentina and the United States was North–South II since Argentina exports a wage good, which is labor intensive (agricultural products).

Appendix

The first few sections of this appendix contain a program in BASIC for solving the model, and the results of several computer runs that reproduce numerically the comparative statics propositions in Section 3 and the stability results in Section 4. These were produced by Eduardo-José Chichilnisky.

a *Computer code names for the variables and parameters*

Variables and parameters (North and South)	Computer code South	North
α	MS	MN
β	NS	NN
a_1	A1	A3
a_2	A2	A4
c_1	C1	C3
c_2	C2	C4
$\bar{L}$	LS	L (*not* LN)
$\bar{K}$	KS	KN
D	DS	DN
A	AS	AN
V	VS	VN
C	CS	CN
W	WS	WN
r	RS	RN
L	L1	L2
K^S	K1	K2
B^D	B3	B4
B^S	B1	B2
I^D	I3	I4
I^S	I1	I2
X_B^S	X1	−X1
X_I^D	X2	−X2

b *Program for the North–South model*

```
1000: INPUT "SOUTH:ALPHA,BETA,A1,A2?";MS,NS,A1,A2
1010: INPUT "SOUTH:C1,C2,L*,K*?";C1,C2,LS,KS
1020: INPUT "NORTH:ALPHA,BETA,A1,A2?";MN,NN,A3,A4
1030: INPUT "NORTH:C1,C2,L*,K*?";C3,C4,L,KN
1040: LF 2:TEXT :CSIZE 1:LPRINT "PARAMETERS":LF 1
1050: LPRINT "SOUTH:";MS;",";NS;",";A1;".";A2
1060. LPRINT TAB 6;C1;",";C2,",",LS;",";KS;LF 1
1070. LPRINT "NORTH:";MN,",";NN;",";A3:",";A4
1080. LPRINT TAB 6;C3;",";C4;",";L;","KN:LF 1
1090. INPUT "ID(S)?";I1
1100. INPUT "ID(N)?";I2.LF 2
1110: DS=A1*C2-A2*C1:DN=A3*C4-A4*C3
1115: IF DS=0THEN1500: 1F DN=0THEN 1500
1120: AS=NS*A1*A2/(DS*DS):AN=NN*A3*A4/(DN*DN)
1130: VS=MS*C1*C1/(DS*DS):VN=MN*C3*C3/(DN*DN)
1140: CS=(1/ (DS/*DS))*(DS*(C1*LS-A1*KS)+MS*C1*C2-NS*A1*A1)
1150: CN=(1/DN*DN))*(DN*(C3*L-A3*KN)+MN*C3*C4-NN*A3*A3)
1160: J=CS+CN+I1+I2:K=J*J+4*(VS+VN)*(AS+AN)
1170: IF K<0THENGOTO 1500
1180: PB=(-J+ K)/(2*(AS+AN))
1190: WS=(PB*C2-C1)/DS:WN=(PB*C4-C3)/DN
1200: RS=(A1-PB*A2)/DS:RN=(A3-PB*A4)/DN
1210: L1=MS*WS/PB+LS:L2=MN*WN/PB+L
1220: KI=NS*RS+KS:K2=NN*RN+KN
1230: I3=(A1*K1-C1*L1)/DS:I4=(A3*K2-C3*L2)/DN
1240: B1=(WS*L1+RS*K1-I1)/PB:B2=(WN*L2+RN*K2-I2)/PB
1250: B3=(C2*L1-A2*K1)/DS:B4=(C4*L2-A4*K2)/DN
1260: X1=B3-B1:X2=PB*X1
1265: LPRINT TAB 6;"PB  =";PB:LF 2
1270: LPRINT "SOUTH:";TAB 18;"NORTH:":LF 1
1272: LPRINT TAB 16;"W":LF 1
1274: LPRINT WS;TAB 18;WN:LF1
1280: LPRINT TAB 15;"W/PB";LF1
1290: LPRINT WS/PB;TAB 18;WN/PB:LF 1
1300: LPRINT TAB 16;"R":LF 1
1310: LPRINT TAB RS;TAB 18;RN:LF1
1320: LPRINT TAB 16;"L":LF 1
1330: LPRINT L1;TAB 18,L2:LF1
1340: LPRINT TAB 16;"K":LF 1
1350: LPRINT K1; TAB 18;K2:LF1
1360: LPRINT TAB 16;"BS":LF 1
1370: LPRINT B3;TAB 18;B4:LF1
1380: LPRINT TAB 16;"BS":LF 1
1390: LPRINT B1;TAB 18;B2:LF1
1400: LPRINT TAB 16;"IS":LF 1
1410: LPRINT I3;TAB 18;14:LF
1420: LPRINT TAB 16;"ID":LF 1
1430: LPRINT I1;TAB 18;I2:LF2
1440: LPRINT "XSB(S) =";X1:LF1
1450: LPRINT "XDI(S) =";X2
1460: GOTO 1100
1500: LPRINT :THIS IS NO CUP OF TEA":GOTO1999
1999: END
```

c *Computer runs*

Data set: initial parameters

	α	β	a_1	a_2	c_1	c_2	$\bar{L}$	$\bar{K}$	D
South:	75	0.025	4.5	0.02	0.01	3	−2	2.7	13.5
North:	6	9.7	2	0.15	1.8	1.7	0.5	12	3.13
				$I^D(S)=4.00$					
Run 1, $I^D(N)=6.00$							Run 2, $I^D(N)=7.00$		

Solutions: endogenous variables

	South		North	
	Run 1	Run 2	Run 1	Run 2
p_B	3.252	1.721	3.252	1.721
w	0.7232	0.3818	1.194	0.3598
w/p_B	0.2220	0.2218	0.3666	0.2090
r	0.3285	0.3308	0.4829	0.5565
L	14.65	14.63	2.7000	1.754
K	2.70822	2.70826	16.683	17.398
B^S	3.352	3.248	0.6667	0.1190
B^D	2.297	1.443	1.621	1.925
X_B^S	0.9541	1.806	−0.9541	−1.806
I^S	0.89189	0.8913	9.108	10.108
I^D	4.00	4.00	6.00	7.00
X_I^D	3.10810	3.10807	−3.1081	−3.10807
$c_2/D-2w/p_B$	−0.2218	−0.2214	−0.1901	0.1251

d *Simulation of comparative statics results*

Runs 1 and 2 reproduce numerically the results of Propositions 2–5 in Section 3, and Propositions 1 and 3 in Chichilnisky (1981).

The initial data show that labor is abundant in the South [$\alpha(S)=75$] and much less abundant in the North [$\alpha(N)=6$]. The duality condition

$c_2/D < 2w/p_B$ is satisfied in both runs of the South. The North has more abundant capital than the South [$\beta(\mathrm{N}) = 9.7$ whereas $\beta(\mathrm{S}) = 0.025$, and $\bar{K}(\mathrm{N}) = 12$ whereas $\bar{K}(\mathrm{S}) = 2.7$]. The level of duality is much higher in the South, $D(\mathrm{S}) = 13.5$, whereas in the North, $D(\mathrm{N}) = 3.13$.

In both runs, the industrial demand in the South, $I^D(\mathrm{S})$, is equal to 4.00. In the first run, the industrial demand in the North is 6.00 and is increased to 7.00 in the second run.

As proved in Proposition 3 of Chichilnisky (1981) and Proposition 3 of Section 3, this increase in the value of $I^D(\mathrm{N})$ has the following general equilibrium effects: Exports of basic goods in the South, X_B^S, increase from 0.9541 to 1.806; the price of basics p_B decreases from 3.252 to 1.721; wages in the South decrease from 0.7232 to 0.3818; and consumption of basics in the South decreases from 2.297 to 1.443. As proved in Proposition 4 of Section 3, total export revenues of the South decrease also (even though export volume has increased), from 3.10810 to 3.10807.

Finally, these runs illustrate the results of Proposition 4 in Section 3: Following an exogenous increase in industrial demand $I^D(\mathrm{N})$, the North's demand for basics increases as well, from 1.621 to 1.925. Thus, an industrial expansion in the North [a higher $I^D(\mathrm{N})$] leads it to consume more of both goods simultaneously, so that the North's welfare strictly increases. The South, instead, exports more basics at lower prices and consumes fewer basics at home. Real wages decrease in the South. Since $I^D(\mathrm{S})$ remains constant, the welfare of the South strictly decreases.

e *Stability*

The world's market for basics. Using always the same exogenous data, we compute now numerically the world's (disequilibrium) supply and demand curves for basics, corresponding to a neighborhood of the equilibrium given by $I^D(\mathrm{N}) = 7.00$, where the equilibrium price is $p_B^* = 1.721$. From equation (4.6), the world demand function for basics is $\mathrm{WDB}(p) = \mathrm{DB(N)}(p) + \mathrm{DB(S)}(p)$, which equals

$$\alpha(\mathrm{N})\left(\frac{w(\mathrm{N})}{p_B}\right)^2 + \frac{w(\mathrm{N})\bar{L}(\mathrm{N}) + r(\mathrm{N})\bar{K}(\mathrm{N})}{p_B} + \frac{\beta(\mathrm{N})(r(\mathrm{N}))^2}{p_B} - \frac{\bar{I}^D(\mathrm{N})}{p_B}$$

$$+\alpha\left(\frac{w}{p_B}\right)^2 + \frac{w\bar{L} + r\bar{K}}{p_B} + \frac{\beta r^2}{p_B} - \frac{\bar{I}^D(\mathrm{S})}{p_B},$$

where as usual all parameters and variables are from the South unless

Table 1. *Numerical values for Figure 3, simulated at the equilibrium with $I^D(\mathrm{N}) = 7.0$.*

p_B	World supply function for basics, WSB[a]	World demand function for basics, WDB[b]
1.60	3.6982	3.9433
1.61	3.6690	3.8906
1.62	3.6401	3.8388
1.63	3.6115	3.7881
1.64	3.5833	3.7383
1.65	3.5555	3.6894
1.66	3.5280	3.6415
1.67	3.5008	3.5944
1.68	3.4740	3.5482
1.69	3.4474	3.5028
1.70	3.4215	3.4582
1.71	3.3953	3.4144
1.721	3.367	3.367
1.73	3.3444	3.3292
1.74	3.3193	3.2877
1.75	3.2946	3.2470
1.76	3.2701	3.2070
1.77	3.2459	3.1676
1.78	3.2220	3.1289
1.79	3.1984	3.0909
1.80	3.1750	3.0536

[a] Slope of WSB = −0.38. [b] Slope of WDB = −0.22.

otherwise indicated. The world supply function for basics is

$$\mathrm{WSB}(p) = \mathrm{SB(N)}(p) + \mathrm{SB(S)}(p)$$

$$= \frac{c_2(\mathrm{N})}{D(\mathrm{N})}\left(\frac{\alpha(\mathrm{N})w(\mathrm{N})}{p_B} + \bar{L}(\mathrm{N})\right) + \frac{a_2(\mathrm{N})}{D(\mathrm{N})}(\beta(\mathrm{N})r(\mathrm{N}) + \bar{K}(\mathrm{N}))$$

$$+ \frac{c_2}{D}\left(\frac{\alpha w}{p_B} + \bar{L}\right) - \frac{a_2}{D}(\beta r + \bar{K}).$$

Figure 3 shows that the world's market for basics is stable when $\alpha(\mathrm{S})$ is large and $c_2/D < 2w/p_B$ in the South, as proved in Proposition 12 of Section 4 and noted on page 191 of the Appendix of Chichilnisky (1981). Table 1 gives the numerical values from the computer runs of WSB and WDB, depicted in Figure 3.

The world's market for industrial goods. We study next, for the same data, the stability of the industrial market at the same equilibrium, where $I^D(\mathrm{N}) = 7.0$ and $p_B^* = 1.721$. We compute numerically the world's (disequilibrium) supply and demand functions for industrial goods, WSI and WDI, respectively. From equations (4.7) and (4.8),

$$\begin{aligned}
\mathrm{WSI}(p) &= \mathrm{SI}(\mathrm{S})(p) + \mathrm{SI}(\mathrm{N})(p) \\
&= (a_1/D)(\beta r + \bar{K}) - (c_1/D)(\alpha w/p_B + \bar{L}) \\
&\quad + (a_1(\mathrm{N})/D(\mathrm{N}))(\beta(\mathrm{N})r(\mathrm{N}) + \bar{K}(\mathrm{N})) \\
&\quad - (c_1(\mathrm{N})/D(\mathrm{N}))(\alpha(\mathrm{N})w(\mathrm{N})/p_B + \bar{L}(\mathrm{N}))
\end{aligned}$$

and

$$\begin{aligned}
\mathrm{WDI}(p) &= \mathrm{DI}(\mathrm{S})(p) + \mathrm{DI}(\mathrm{N})(p) \\
&= \bar{I}^D(\mathrm{S}) + \bar{I}^D(\mathrm{N}) + \Pi(\mathrm{S})(p) + \Pi(\mathrm{N})(p) \\
&= \bar{I}^D(\mathrm{S}) + (p_B - a_1 w - c_1 r)[(c_2/D)(\alpha w/p_B + \bar{L}) \\
&\qquad - (a_2/D)(\beta r + \bar{K})] \\
&\quad + \bar{I}^D(\mathrm{N}) + (p_B - a_1(\mathrm{N})w(\mathrm{N}) - c_1(\mathrm{N})r(\mathrm{N})) \\
&\quad \times [(c_2(\mathrm{N})/D(\mathrm{N}))(\alpha(\mathrm{N})w(\mathrm{N})/p_B) + \bar{L}(\mathrm{N}) \\
&\qquad - (a_2(\mathrm{N})/D(\mathrm{N}))(\beta(\mathrm{N})r(\mathrm{N}) + \bar{K}(\mathrm{N}))].
\end{aligned}$$

Table 2 gives the numerical values of simulating WDI and WSI in a neighborhood of the equilibrium with $I^D(\mathrm{N}) = 7.0$ and $p_B^* = 1.721$.

Figure 2b from Chichilnisky (1981). Next we reproduce, for the same data base, Figure 2b of Chichilnisky (1981). As discussed in Section 4, this figure depicts two curves: The cross-equilibria relation X_B^S relating the exports of the South and their price:

$$X_B^S = \frac{\alpha c_1}{D^2 p_B}\left(c_2 - \frac{c_1}{p_B}\right) + \frac{\beta a_1}{D^2}\left(a_2 - \frac{a_1}{p_B}\right) + \frac{c_1 \bar{L} - a_1 \bar{K}}{D p_B} + \frac{\bar{I}^D(\mathrm{S})}{p_B},$$

where all parameters are for the South; and the North's disequilibrium excess demand for basics, denoted WD in Chichilnisky (1981), which from equation (4.9), is

$$\begin{aligned}
\mathrm{WD} &= \mathrm{DB}(\mathrm{N})(p) - \mathrm{SB}(\mathrm{N})(p) \\
&= \alpha\left(\frac{w}{p_B}\right)^2 + \frac{\beta r^2}{p_B} + \frac{w}{p_B}\left(\bar{L} - \frac{\alpha c_2}{D}\right) + r\left(\frac{\beta a_2}{D} + \frac{\bar{K}}{p_B}\right) + \frac{\bar{K}a_2 - \bar{L}c_2}{D} - \frac{\bar{I}^D}{p_B},
\end{aligned}$$

Table 2. *Numerical values for Figure 4, simulated at the equilibrium with $I^D(\mathrm{N}) = 7.0$.*

p_B	World supply of industrial goods, WSI[a]	World demand for industrial goods, WDI[b]
1.60	10.944	10.553
1.61	10.949	10.593
1.62	10.954	10.632
1.63	10.959	10.671
1.64	10.964	10.710
1.65	10.968	10.748
1.66	10.973	10.785
1.67	10.978	10.822
1.68	10.982	10.858
1.69	10.987	10.893
1.70	10.991	10.928
1.71	10.995	10.963
1.721	10.999	10.999
1.73	11.004	11.030
1.74	11.008	11.063
1.75	11.012	11.096
1.76	11.016	11.128
1.77	11.020	11.159
1.78	11.024	11.190
1.79	11.028	11.221
1.80	11.032	11.250

[a] Slope of WSI = 2.27. [b] Slope of WDI = 0.29.

where all parameters and variables are for the North. Table 3 gives the numerical values of X_B^S at different world equilibria corresponding to different values of $I^D(\mathrm{N})$ (and thus to different prices p_B) and of two disequilibrium excess demand curves of the North, WD and (WD)′. WD is computed in a neighborhood of the equilibrium given by $I^D(\mathrm{N}) = 7.0$ and (WD)′ is computed at the equilibrium given by $I^D(\mathrm{N}) = 7.1$.

Finally, we consider a full version of Figure 2b in Chichilnisky (1981), where X_B^S is plotted together with two other curves at two nearby equilibria. These curves are excess demand of the North, WD = DB(N) − SB(N), and excess supply of the South, WS = SB(S) − DB(S), both of which are disequilibrium curves. Both WD and WS are computed from the same basic data set; WD and WS are computed in a neighborhood of the equi-

Table 3. *Numerical values for Figure 5.*[a]

p_B	X_B^S	WD = DB(N) − SB(N) at $I^D(\mathrm{N}) = 7.0$	(WD)′ = DB(N) − SB(N) at $I^D(\mathrm{N}) = 7.1$
1.56	1.992	1.963	1.978
1.58	1.967	1.941	1.957
1.60	1.943	1.920	1.937
1.62	1.915	1.900	1.915
1.64	1.895	1.880	1.898
1.66	1.872	1.860	1.880
1.68	1.850	1.842	1.862
1.70	1.828	1.823	1.844
1.721	1.807	1.807	1.827
1.74	1.786	1.788	1.810
1.76	1.766	1.772	1.794
1.78	1.746	1.755	1.778
1.80	1.727	1.739	1.763

[a] Slope of $X_B^S = -0.9$; slope of WD $= -1.1$; slope of (WD)′ $= -1.1$.

librium determined by $I^D(\mathrm{N}) = 7.0$, and (WD)′ and (WS)′ are computed for the equilibrium corresponding to $I^D(\mathrm{N}) = 7.1$. The equations for WD and WS were given in Chichilnisky (1981) and in Table 3 of this appendix. Table 4 gives the values of the simulations of WS and WD depicted in Figure 6.

f *The North–South model and the Arrow–Debreu model*

We now exhibit a version of the North–South model as a particular case of the Arrow–Debreu model. This version of the model appears in Propositions 8 and 9.

Consider an Arrow–Debreu economy with six commodities, two agents, and four firms. The commodities are B, I, $K(\mathrm{N})$, $L(\mathrm{N})$, $K(\mathrm{S})$, and $L(\mathrm{S})$. Firms 1 and 2 produce B and I using $L(\mathrm{N})$ and $K(\mathrm{N})$ according to the production functions

$$B^S = \min(L(\mathrm{N})^B/a_1,\ K(\mathrm{N})^B/c_1), \qquad I^S = \min(L(\mathrm{N})^I/a_2,\ K(\mathrm{N})^I/c_2),$$

where the superscripts B and I indicate the sector in which $L(\mathrm{N})$ and $K(\mathrm{N})$ are used. Firms 3 and 4 produce B and I using similar production functions, but with $L(\mathrm{S})$ and $K(\mathrm{S})$ as inputs. All four firms are perfectly competitive.

Table 4. *Numerical values for Figure 6.*[a]

p_B	WD[b]	WS	X_B^S	(WD)′ [c]	(WS)′
1.56	1.963	1.619	1.992	1.978	1.858
1.58	1.941	1.649	1.967	1.957	1.879
1.60	1.920	1.677	1.943	1.937	1.897
1.62	1.900	1.702	1.915	1.915	1.915
1.64	1.880	1.726	1.895	1.898	1.930
1.66	1.860	1.748	1.872	1.880	1.944
1.68	1.842	1.769	1.850	1.862	1.958
1.70	1.823	1.788	1.828	1.844	1.969
1.721	1.806	1.806	1.806	1.827	1.980
1.74	1.788	1.821	1.786	1.810	1.989
1.76	1.772	1.836	1.766	1.794	1.997
1.78	1.755	1.850	1.746	1.778	2.005
1.80	1.739	1.862	1.727	1.763	2.011

[a] Slope of $X_B^S = -0.91$; slope of WD $= -1.1$; slope of WS $= 1$; slope of (WD)′ $= -1.1$; slope of (WS)′ $= 1.6$.

[b] WD is the North's excess demand for basics, computed near the equilibrium determined by $I^D(\mathrm{N}) = 7.0$ and $p_B^* = 1.721$. WS is the excess supply curve of the South computed near the same world equilibrium. Note that X_B^S, WD, and WS all meet at $p_B = 1.721$.

[c] (WD)′ is the North's excess demand for basics computed near the equilibrium determined by $I^D(\mathrm{N}) = 7.1$, and (WS)′ is the excess supply of the South computed near the same equilibrium. X_B^S, (WD)′, and (WS)′ all meet at $p_B = 1.62$.

Agent 1 has the initial endowment vector (in terms of the commodities defined above)

$$(0, 0, 0, 0, \bar{K}(\mathrm{S}), \bar{L}(\mathrm{S})),$$

and *agent 2* has the endowment vector

$$(0, 0, \bar{K}(\mathrm{N}), \bar{L}(\mathrm{N}), 0, 0).$$

For any values of $\bar{K}(\mathrm{S})$, $\bar{L}(\mathrm{S})$, $\bar{K}(\mathrm{N})$, and $\bar{L}(\mathrm{N})$, agent 1's preferences are of the form indicated in Figure 7. This implies that there exists an $\epsilon > 0$ such that for any price $p_B > \epsilon$ this agent consumes the fixed amount $\bar{I}^D(\mathrm{S})$ of good I, and the rest of his or her income is used to consume good B. This agent will consume zero amounts of all other commodities. Agent 2 has similar preferences, but the "kink" occurs at $I = \bar{I}^D(\mathrm{N})$.

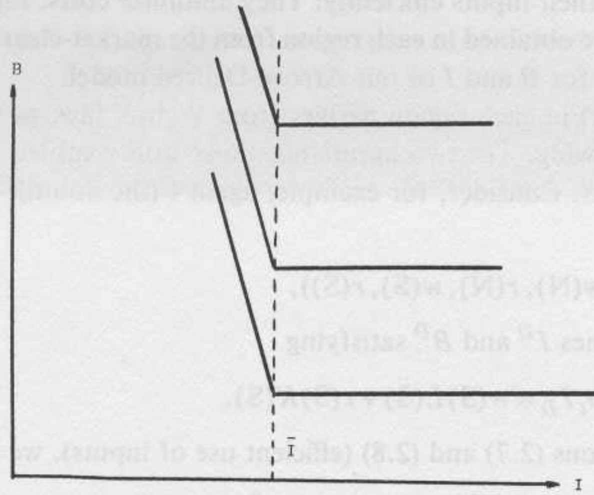


Figure 7. An agent's preference curves.

We thus have all the components of an Arrow–Debreu model: six commodities, two agents with their endowments and preferences, and four firms and their technologies. The next step is to show how the equilibrium equations of our North–South model emerge from the equilibrium equations of the Arrow–Debreu model. The North–South model is defined by the twenty-eight equations (2.1)–(2.11) for each country and (2.12)–(2.18); recall that one of these latter seven equations is always satisfied where all others are.

Equations (2.1) and (2.2) for the North, that is,

$$p_B = a_1 w(\mathrm{N}) + c_1 r(\mathrm{N}), \qquad p_I = a_2 w(\mathrm{N}) + c_2 r(\mathrm{N}),$$

are the zero-profit conditions for firms 1 and 2, where $w(\mathrm{N})$ is the equilibrium price of commodity $L(\mathrm{N})$ and $r(\mathrm{N})$ that of $K(\mathrm{N})$. These zero-profit conditions are satisfied at an equilibrium in our Arrow–Debreu model because it has constant returns to scale.

Similarly, we obtain equations (2.1) and (2.2) for the South; they are the zero-profit conditions of firms 3 and 4 in the South. Each region uses specific inputs for production.

Equations (2.3) and (2.4) in each region are given by the initial endowments of our Arrow–Debreu model. Equations (2.5) and (2.6) are the equilibrium conditions in the markets for $L(\mathrm{N})$ and $K(\mathrm{N})$ and for $L(\mathrm{S})$ and $K(\mathrm{S})$, respectively. Equations (2.7) and (2.8) for each region simply state that all firms use their inputs efficiently: They minimize costs. Equations (2.9) and (2.10) are obtained in each region from the market-clearing condition in markets for B and I of our Arrow–Debreu model.

Equation (2.11) in each region derives from Walras' law, as we shall show in the following. The two agents maximize utility subject to their budget constraints. Consider, for example, agent 1 (the South). Then at the price vector

$$(p_B, p_I, w(\mathrm{N}), r(\mathrm{N}), w(\mathrm{S}), r(\mathrm{S})),$$

this agent consumes I^D and B^D satisfying

$$p_B B^D + p_I I_D = w(\mathrm{S})L(\mathrm{S}) + r(\mathrm{S})K(\mathrm{S}).$$

In view of equations (2.7) and (2.8) (efficient use of inputs), we have

$$w(\mathrm{S})L(\mathrm{S}) + r(\mathrm{S})K(\mathrm{S}) = (a_1 w(\mathrm{S}) + c_1 r(\mathrm{S}))B^S + (a_2 w(\mathrm{S}) + c_2 r(\mathrm{S}))I^S.$$

By the zero-profit conditions (2.1) and (2.2), this implies

$$w(\mathrm{S})L(\mathrm{S}) + r(\mathrm{S})K(\mathrm{S}) = p_B B^S + p_I I^S.$$

Therefore,

$$p_B B^D + p_I I^D = p_B B^S + p_I I^S.$$

Since

$$B^D = B^S - X_B^S \quad \text{and} \quad I^D = I^S + X_I^D,$$

it follows that

$$p_B(B^S - X_B^S) + p_I(I^S + X_I^D) = p_B B^S + p_I I^S,$$

so that

$$p_B X_B^S = p_I X_I^D,$$

which is equation (2.11). Thus, equation (2.11) follows in our Arrow–Debreu model from utility maximization under a budget constraint, zero profits, and cost-minimizing use of inputs. Equations (2.12)–(2.15) are the standard equilibrium conditions in the Arrow–Debreu model, and equations (2.17) and (2.18) follow from utility maximization, with the agents' preferences defined in Figure 7 provided the equilibrium price p_B exceeds some $\epsilon > 0$. Therefore, for any (nonzero) equilibrium price p_B^* of the North–South model, we can find a set of preferences (and thus an $\epsilon > 0$) for which equations (2.17) and (2.18) emerge as the usual utility maximization conditions under a budget constraint of the Arrow–Debreu model. This shows that all the equations of the North–South model are equilibrium equations of the Arrow–Debreu model.

The North–South model utilized so far has fixed factor endowments $[L(\mathrm{S})=\bar{L}(\mathrm{S}),\ K(\mathrm{S})=\bar{K}(\mathrm{S}),\ L(\mathrm{N})=\bar{L}(\mathrm{N}),$ and $K(\mathrm{N})=\bar{K}(\mathrm{N})]$. We have shown that this model is a special case of the Arrow–Debreu model.

It is also possible to consider the North–South model with "variable endowments" (a notation used in the literature on international trade), where, for example, in one region

$$L^S = \alpha w/p_B + \bar{L}.$$

In this case, the result is as follows: At each equilibrium, the equilibrium equations of this North–South model will be those of an Arrow–Debreu model where in addition to the preferences and technologies defined above, the supply of commodity L is provided by utility-maximizing behavior under budget constraints. The supply equation for L defined above may not emerge as a supply function of the Arrow–Debreu model. However, at each equilibrium, the supply L^S is fixed, and it can be described as the equilibrium level L^S that emerges from utility-maximizing behavior in a standard Arrow–Debreu model.

An equilibrium of the North–South model is thus also an equilibrium of a standard Arrow–Debreu model.

NOTES

1 "Practical men and economic theorists have always known that trade may help some people and hurt others." "What in the way of policy can we conclude from the fact that trade is a potential boon?. . . Very little" (Samuelson 1962).

2 Equations (2.1) and (2.2) are equivalent to the production functions when firms are competitive. In equilibrium, profits are zero so that revenues equal costs, i.e., $p_B B^S = wL^B + rK^B$. Now from the production functions $B^S = L^B/a_1 = K^B/c_1$, so that $p_B B^S = a_1 wB^S + c_1 rB^S$, or $p_B = a_1 w + c_1 r$, equation (2.1). Similarly, one derives equation (2.2).

3 Note that when all markets clear, the value of domestic demand $p_B B^D + p_I I^D$ equals the value of domestic income $wL + rK$. This is Walras' law or the national income identity at the equilibrium. From equations (2.1)–(2.11), one obtains

$$p_B B^D + p_I I^D = p_B(B^S - X_B^S) + p_I(I^S + X_I^D) = p_B B^S + p_I I^S$$
$$= (a_1 w + c_1 r)B^S + (a_2 w + c_2 r)I^S = wL + rK.$$

Section f in the appendix exhibits preferences, endowments, and technologies that turn this model into a standard Arrow–Debreu model. The results are unchanged.

4 All relations in this model are homogeneous of degree 0 in prices, i.e., only relative prices matter. Therefore, one may normalize the model by fixing the price of one good (the *numéraire*) equal to 1.
5 Equation (2.15) is not independent: It is always satisfied when (2.11) is satisfied in each region and (2.12)–(2.14) hold.
6 This is not surprising, since demand behavior, or preferences, have not been specified so far.
7 Existence requires standard restrictions on the exogenous parameters to ensure that the equilibrium prices are all positive, e.g., $a_1/a_2 > p_B > c_1/c_2$, where p_B satisfies equation (2.22).
8 This observation and Figure 2, which illustrates it, were suggested by Ron Jones.
9 Arrow (1981, p. 2) states: "Individual equilibria are stable in the usual sense of general equilibrium theory." Gunning (1984) states, in the paragraph after equation (14): "Hence equilibrium is stable in the Walrasian sense." Heal and McLeod (1984, Section 4) state: "It will be shown that under either of these approaches Chichilnisky's model is stable under the conditions assumed in her paper." Findlay's (1982) last section states: "Examination of the structure of the model shows that it possesses a unique equilibrium that is Walras stable." Ranney (1984), in the paragraph after equation (6), states: "Thus an increase in p_B results in a decline in the production of I goods in both countries, and the model is Walrasian stable." Saavedra (1984) states in Section 2: "We know from the preceding section that Walrasian stability of equilibrium always holds in this model." Finally, Srinivasan and Bhagwati (1984) state: "One could not therefore get a stronger result; the equilibrium is unique and evidently Walras-stable."
10 Arrow and Hahn (1971, Chapter 12, p. 317) define an adjustment process where the price equations (4.1) hold, but this is because their commodity markets are assumed to remain at an equilibrium throughout, and their factor markets adjust (see their p. 317, lines 21–3). Since commodity prices are in equilibrium, profits are zero. In our case, instead, factor markets remain in equilibrium and commodity markets adjust; profits are not zero.
11 The difference between X_B^S and an excess supply curve valid for stability analysis was clearly pointed out in Chichilnisky (1981, footnote 10, p. 175, and p. 189, lines 15–31). A similar point is made by Arrow (1981, pp. 1, 2): "Methodologically, the papers are exemplary applications of general equilibrium analysis. *A clear distinction is made between the downward sloping response of the economy as a whole and supply curves in the strict sense. The reaction curve (X_B^S) links alternative equilibria of the economy and is not a curve relevant to any one equilibrium.* It is shown, in fact, that the individual equilibria are stable in the usual sense of the general equilibrium theory." (italics added)
12 A clear version of this error appears in Findlay (1984), who states: "A shift in the demand of the North towards the South's exports in her model actually can only produce the completely standard result that it would improve the terms of trade of the latter." This comment, as already noted, is incorrect.

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