The Devil's Calculus: Mathematical Models of Civil War

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Abstract

In spite of the movement to turn political science into a real science, various mathematical methods that are now the staples of physics, biology, and even economics are thoroughly uncommon in political science, especially the study of civil war. This study seeks to apply such methods - specifically, ordinary differential equations (ODEs) - to model civil war based on what one might dub the capabilities school of thought, which roughly states that civil wars end only when one side's ability to make war falls far enough to make peace truly attractive. I construct several different ODE-based models and then test them all to see which best predicts the instantaneous capabilities of both sides of the Sri Lankan civil war in the period from 1990 to 1994 given parameters and initial conditions.

The model that the tests declare most accurate gives very accurate predictions of state military capabilities and reasonable short term predictions of cumulative deaths. Analysis of the model reveals the scale of the importance of rebel finances to the sustainability of insurgency, most notably that the number of troops required to put down the Tamil Tigers is reduced by nearly a full order of magnitude when Tiger foreign funding is stopped. The study thus demonstrates that accurate foresight may come of relatively simple dynamical models, and implies the great potential of advanced and currently unconventional non-statistical mathematical methods in political science.

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Introduction

It was, and perhaps still is, the largest and deadliest war that the world has never heard of. The Second Congolese War, the deadliest war since World War II, nonetheless remains obscure in the public mind (assuming it ever made it there in the first place) simply because it seemed like a Congolese affair rather than an international affair - merely a civil war. But the death count, unparalleled in decades, and the covert but sinister hands of various foreign actors made clear that in this modern age, war is never only a Congolese affair or a Rwandan affair or an Afghan affair.

Political scientists have recognized this fact, and thus the study of civil war is in no ways lacking for literature. Several explanations for their incidence and duration, ranging from the nearly genetic to the wholly economic, have arisen to explain why civil wars begin and, to a lesser extent, why they end. Unfortunately, unlike some of the generals and rebel leaders studied, the field of political science has not gone to all lengths or sought all means to achieve its goals. An entire field of techniques - mathematical methods based largely on calculus - has gone neglected.

Aside from their common tendency to stir unpleasant feelings in the general public, civil wars and calculus seem largely unrelated. In spite of the movement to turn political science into a real science, various mathematical methods that are now the staples of physics, biology, and even economics are thoroughly uncommon in political science, especially the study of civil war. This study seeks to apply such methods - specifically, ordinary differential equations - to model what I dub the capabilities hypothesis of civil war, which roughly states that civil wars end only when one side's ability to make war falls far enough to make peace truly attractive. I construct several different ODE-based models and then test them all to see which best predicts the capabilities of both sides of a civil war at any given time point. Aside from producing a tool that could prove invaluable in the study of civil war, this study seeks more importantly to demonstrate the usefulness of more advanced and currently unconventional non-statistical mathematical methods in political science.

After testing the different models, I find that the best of them makes surprisingly accurate extrapolations of government troop levels and decent predictions of total cumulative death counts while overestimating rebel numbers. Analysis of the model reveals the scale of the importance of foreign funding and weapon costs to the rebel cause. The end conclusion is that the methods explored here have great potential and merit further investigation.

This paper is divided into several parts. After reviewing the literature, the criterion by which the model candidates will be evaluated will be explained in the research design.

The different models are presented and justified in the next section. Then I detail the results of assessing the various different models against actual data from a period of the Sri Lankan civil war. I then analyze the most successful of these models to wring from it whatever counterfactual insight about Sri Lanka's war is possible. Finally, I discuss the limitations of the results and what broader conclusions one may draw.

Chapter 2

Literature Review: Why Men Think Men Rebel

2.1 The Incidence of Civil War

2.1.1 Grievance

The common theme across the many different schools falling under the category of "grievance" is not far removed from common sense: people fight because they dislike or are angry with one another. From this most basic of principles, a vast variety of theories has sprung up to contend for the ultimate prize of causal explanatory power.

The primordialist school of thought puts primary emphasis on culture and ethnicity. Essentially constructivist in its analysis, this school argues that strife, both internal and external, are almost inevitable because the process of creating a sense of ethnicity and belonging toward one's own group inevitably entails creating a sense of foreignness and un-belonging to other groups. This mistrust of the other, part of one's "primordial attachments," is very difficult for the individual to shake and "will provide ethnic conflicts with their seemingly intractable nature" (Stack, 1997, 17). Primordialists are quick to add that "Differences do not necessarily mean conflict," but this nonetheless does not prevent cultural differences from becoming "the battle lines of the future" (Huntington, 1993, 22). Aside from the primordialists' highly deterministic prescription of war, and also Huntington's arbitrary delineation of "civilizations" that may or may not actually exist (not to mention his uncertainty about the existence of African civilization), one can question the actual explanatory power of fractionalization. For example, while Huntington paints the India-Pakistan conflict as a battle between "Hindu" and "Muslim" civilizations, he does not account for why once removed from the subcontinent to, say, the United States, former Pakistanis and former Indians do not spontaneously erupt into violent conflict. Furthermore, statistical analyses have found that ethnic fractionalization is only associated with greater risk of civil war when one has a dominant group versus a relatively large minority (Fearon and Laitin, 2003; Collier and Hoeffler, 2004). Such implies that ethnicity alone is not the overarching cause of violence.

For reasons similar to these, other scholars of grievance have chosen instead to examine grievances generated through more current government or majority actions. The most famous of these theories is Ted Gurr's model of relative deprivation, which states that civil strife occurs when the values alloted to people by government fall short of their expectations. Stated most precisely in (Gurr, 1968), the operationalized version of the theory is that the dependent variable, namely the magnitude of civil strife, is a function of the independent variable relative deprivation as altered by several intermediate variables. Thus, while deprivation is not the sole variable governing the outbreak of violence, it is the original cause that makes other variables such as the coercive potential of authorities and the legitimacy of the regime relevant.

Other scholars have looked at grievance on a more personal level. Miranda Alison, for example, while interviewing women in the Liberation Tigers of Tamil Eelam (Alison, 2003), did indeed find "conventional" grievances liberally strewn in the rhetoric of rebel leaders, most notably nationalism. However, she notes that these national sentiments are stated almost to hide the real and far more personal reasons for enlistment and insurrection: personal losses at the hands of overzealous/indiscriminate government counterinsurgency and the disruption of one's education. While the example of India's actions in Kashmir, for example, certainly seem to vouch for the importance of personal grievance in analyzing civil war, one rarely has vicious counterinsurgency operations unless one has an insurgency to begin with, so the question of why civil war breaks out in the first place remains unanswered.

Psychologists studying terrorism have theorized that not just any grievance, but humiliation specifically, is the root cause of civil war. Evelin Lindner writes that considering the devastating effect humiliation can have on personal relationships, mass humiliation of entire ethnic, racial, religious, or class groups can destroy the bonds that keep society cohesive and peaceful (Lindner, 2001). Noting that deprivation alone is not enough to explain rebellion because deprivation is common while rebellion is not, she theorizes that humiliation is the catalyst that transmutes deprivation into violence. Though this theory does account for the presence of more educated and wealthy individuals in the ranks of rebel organizations, it does not address the question of why then the overwhelming majority of militants in the world are from the lower classes (Stern, 2003). Furthermore, considering that most civil wars occur in the poorest parts of the developing world (Collier et al., 2003), one is forced to wonder why oppressed minorities in the less developed countries (LDCs) feel so much more humiliated than their post-industrialized counterparts.

2.1.2 Greed/Opportunity

What is still often called the greed explanation for civil war is now generally divided into two markedly different hypotheses for why civil war occurs. Both are still distinguishable from the grievance theories because of their emphasis on civil war as an option rather than a last resort, and both are more concerned with material factors than psychological and social factors. Furthermore, both state that since grievances, some very severe, exist in every society but civil wars do not, that discontent cannot be the most important explanatory variable in civil war incidence. However, even when they cite the same variables as those most important in predicting the outbreak of civil war, they give very different reasons for the importance of these variables.

The greed school of thought was quite novel when it first appeared because it focused on the lucrativeness of civil war for a select and brutal few. Civil war, much like the sale of a trendy new product, is more likely to occur wherever and whenever it is most profitable. Deeply rooted in the rational choice model as the theory is, one should not be surprised to see that it employs the language and techniques of economics to explain why civil wars break out. Indeed, Collier and Hoeffler actually write utility functions examining the difference between the potential gain from control of tax revenues and/or from natural resources and the inevitable cost of war (Collier and Hoeffler, 1998). Thus, while for example the presence of concentrated natural resources in an area creates a very strong incentive for the inhabitants of said area to rebel, a high gross domestic product (GDP) per capita raises the opportunity cost of war (because living peacefully is more profitable) and thus decreases the incentive to rebel. The problem with this thesis, however, is that aside from the possibility that most of the variables analyzed might support the opportunity hypothesis over the greed hypothesis (and some the grievance hypothesis as well), more recent evidence has cast doubt on several of the variables analyzed here. Indeed, it is a clear indication that the authors' N was not large enough that six years later, they cast serious doubt on their own natural resource hypothesis using larger data sets and different methods (Collier and Hoeffler, 2004).

Some consider the opportunity school of thought as simply a more watered-down version of greed, and the fact that the same people who invented the greed theory then years later introduced the opportunity theory in response to criticism does not help this impression. While I concede freely that opportunity was derived from the same basic principles (and perhaps the same basic statistical regressions) as greed, I still classify it separately because it provides different causal explanations using those principles. Stated tersely, the opportunity hypothesis theorizes that rebellions are most likely to break out wherever rebellion is most possible. In this analytic perspective, per capita GDP is still a central variable as in the greed theory, but now rather than representing an opportunity cost as rebels' lost income, it is instead an actual cost made inevitable because of the need to feed and equip soldiers. New variables take on importance, such as total GDP, which is a measure of the government's financial base in fighting rebels. Perhaps most indicative of this school's results is the consistent and statistically significant correlation between mountainous terrain and civil war incidence (Fearon and Laitin, 2003; Collier and Hoeffler, 2004). But while statistics certainly do show the importance of the variables named in this theory, simple logic casts doubt on their causal power. Certainly, a rebellion is more likely continue and/or succeed if the rebels have access to the resources they need to fight, but this hardly implies that the reason for their seeking resources is their very existence. Indeed, the proposition that people risk their lives taking up arms simply because they have access to those arms is questionable, to say the least. Thus, separated from its greed roots, the opportunity theory carries predictive power without any real explanatory power. Ergo, one is forced to conclude that the undeniable importance of opportunity only becomes relevant after the outbreak of war.

2.2 The Dynamics of Civil War

Much thought and ink have been invested in the analysis of the outbreak of civil war, yet until recently, surprisingly little was spent on the study of civil war duration, outcome, and intensity. Perhaps due to the obvious policy relevance of the study of civil war, especially for organizations like the World Bank and the United Nations, researchers understandably reasoned that the durations of civil wars would be irrelevant if precise means of preventing their outbreak could be identified. Unfortunately, even events as brutally spectacular as wars carry their subtleties, and thus none of the explanations outlined above have been definitively substantiated or even definitively debunked.

However, the analysis of civil war dynamics, or at least civil war duration, intensity, and outcome provides those concerned with civil war policy a new approach. Taking the incidence of civil war as given for whatever reason, be it humiliation or greedy ringleaders, one can then focus on what exactly makes for a Black September (a month long uprising in Jordan) versus a Second Congolese War (the deadliest war since World War II). If the duration and intensity of civil war can be minimized, then the damage they do to economics, infrastructure, and innocent bystanders can also be minimized.

Delineating broad schools of thought on the dynamics of civil war is more awkward than doing so for civil war incidence, but I feel that because the factors that cause a civil war in the first place must have some link to the factors that prolong it, the easiest way to classify different ideas is by simply extending the schools of civil war incidence into the realm of dynamics. At times, the fit of thought to school is uncomfortable at best, but the only alternative is to list every individual hypothesis separately.

2.2.1 Grievance

Grievance theories are difficult to translate from incidence to dynamics and even harder to analyze, which may be why so few appear in the most current literature. However, several of the incidence theories themselves did not apply exclusively to war incidence, so it is worthwhile to consider how they thus relate to civil war duration and intensity. The argument about humiliation (Lindner, 2001) implies that as long as the source of humiliation persists, or as long as new sources of humiliation (such as a repressive government response to violent discontent) present themselves, insurgency will continue.

Meanwhile, the idea of personal grievances summarized above (Alison, 2003) - that gung ho government responses to rebellion that leave too many innocents dead or too many lives disrupted will only create new militants for the government to fight - seems naturally to relate better to civil war duration than to civil war incidence. Examples such as the Indian government's perceived brutality in Kashmir and that conflict's altered nature over the past few decades certainly lend credence to this idea. However, while both grievance arguments explain why people might be motivated to continue fighting the government after war has begun, they are silent on exactly how these individuals convert rage into something beyond thrown stones and soiled public bathrooms.

2.2.2 Greed

For this reason, much of recent civil war dynamics literature has focused on more economic factors in hopes of discovering the secret of civil war duration in what one might call the stock market of war. The arguments that I classify as greed-based all relate to the literal profitability of war - war will continue as long as certain interests, be they the principal actors or other influential parties, either gain or expect to gain. Winslow and Woost describe how in the case of Sri Lanka, a new economy has evolved around the decades-old war and has created a new breed of entrepreneur that profits off of strife (Winslow and Woost, 2004). A perfect example is the actions of the marketing divisions of various clever firms, which have put down money to sponsor military checkpoints in return for advertising space in a spot where people will be forced to wait for hours (Richardson, 2005).

A theory called war-as-investment (Collier et al., 2004) also falls into this category. Very similar to the incidence version of greed, it states that rebels fight for future gain - be it in the form of cash or freedom from repressive/discriminatory laws - and are willing to suffer costs in the short term as a result. Thus, as long as the costs of war do not come to overwhelm the benefits of victory, the rebels will continue to fight. However, aside from the lack of evidence Collier et al. found to support this hypothesis (2004), all of the hypotheses suffer from the same problem as the grievance explanations: explaining motives without explaining means. Since the overwhelming majority of civil wars do not end in settlement, but rather when one side crushes the other, this seems like a rather awkward omission.

2.2.3 Capabilities

The dynamical version of the opportunity school is the capabilities school. All theories in this school imply that civil war will continue as long as both sides are capable of continuing it. Since the occurrence of a civil war is now a given, the main problem that plagued the opportunity school of thought - namely, that the ability to rebel cannot be the only reason why people rebel - is no longer an issue. Indeed, Stern (2003) finds in her interviews with terrorists and other militants that while most of them were drawn to their respective causes due to personal grievances, nationalist sentiments, or other such psychological issues, that once involved in the struggle, they became career militants. Indeed, often after fighting in one uprising, a militant will simply move on to a different conflict about which he or she feels much less passionately in the pursuit of a paycheck (Stern, 2003), bringing new meaning to the expression rebel without a cause.

Collier et al. (2004) outline a war-as-business argument which states that a war, much like a daring entrepreneurial venture, will last only so long as it remains profitable. This is different from the war-as-investment theory because it focuses on present profit rather than expectancy of future profit. Phrased differently, a rebel organization must pay its bills somehow, and the rebellion will continue as long as it can continue to do so. Though this can take a form very similar to the greed-based theories outlined above when formulated such that rebels are literally reaping profit from the war itself, its authors treat it more as though rebellions span as long as their finances (Collier et al., 2004). Findings such as the existence of higher risk of civil war in countries with low per capita GDPs may support this theory (Fearon and Laitin, 2003; Collier and Hoeffler, 2004). Furthermore, the discovery that periods of low primary commodity prices (often a source of rebel funds) coincide with periods of peace-making grant it more certain substantiation (Collier et al., 2004). Collier et al. suggest in the same paper a theory called war-by-mistake, which states that civil wars occur because both the state and the rebels are overly optimistic about their hopes of success and civil wars will continue as long as such delusions persist (Collier et al., 2004). While they pose this as a completely different theory, I believe that since the two sides' misperceptions are inevitably about their own or enemy capabilities that it best belongs as an addition to the war-as-business model.

Other scholars have studied the role of foreign funding with respect to the rebels' capabilities. Various studies have found that external intervention's effect on civil war duration is rarely good (Elbadawi and Sambanis, 2000; Balch-Lindsay and Enterline, 2000), especially when the intervener takes the side of the rebels and thus reduces the costs of waging war. Akcinaroglu and Radziszewski (2005) actually disaggregate interventions into neutral third parties and third parties who are rivals of the war-torn state's government, and find that even the expectation of a rival's intervention (especially economic intervention) will increase the duration of a civil war because the rebels will fight on even as their wallets grow hollow in the hopes of eventual foreign backing (Akcinaroglu and Radziszewski, 2005).

Finally, DeRouen and Sobek (2004) look at the capabilities issue from the other side, examining state capabilities. They theorize that capable state apparatus is most important in deciding a war on the side of the government, and find among other things that one of the deadliest blows to a rebellion's hopes of success is an effective state bureaucracy (DeRouen Jr. and Sobek, 2004).

2.3 Dynamical Models

The literature does not exactly abound with models based on ordinary differential equation - indeed, the study of civil war, like most of political science scholarship, relies largely on statistics rather than dynamics. The difference between the two approaches is that statistical methods are generally inductive in their approach whereas dynamical methods are deductive. Both are important, and neither alone will provide a complete picture of civil war and peace; thus, the present scarcity of dynamical models is troubling. However, a dearth is not a void, and the one model I have found - that of Allan and Stahel (1983) - is certainly worth discussing.

Unlike the model I propose here, Allan and Stahel's model was custom tailored to fit exactly one case - the Soviet invasion of Afghanistan - and was created for predictive rather than analytic purposes. In some ways, the model is more a reflection of Cold War-era beliefs about superpowers than anything. For example, the model is constructed such that the Soviet Union cannot possibly lose. Indeed, considering the rather cataclysmic consequences of the war, some would probably say Allan and Stahel's model was a complete failure. However, one should recall that the CIA itself could not predict the fall of the Soviet Union and thus it seems rather absurd to expect five differential equations to somehow do just that.

Leaving aside the rather unpredictable outcome of the whole affair, the model revealed despite its heavy Soviet-tilt that, contrary to expectations that the U.S.S.R. and its Afghan allies would crush the mujahidiin with little difficulty, the Soviets would take huge losses even using the most conservative of parameter values. Indeed, their exact words about Soviet losses are “Numbering 48,000 over the six years, they are comparable to U.S. losses in Vietnam” (Allan and Stahel, 1983, 600). They go on to emphasize the potential longevity of the struggle, the difficulty of outright Soviet victory, and the inevitably huge cost that will be exacted upon the Afghan people. Considering that this article was published in 1983, and considering the eventual outcome of the war, one can see that even when imperfectly formulated, dynamical models have an eerie power all their own.

Chapter 3

Research Design: Experiments in War and Peace

Short of being on the battlefield and/or within the ranks of the rebels, something that requires a dedication to science slightly beyond what most analysts possess, testing the capabilities hypothesis of civil war is difficult. Certainly, fine work has been done in spite of the difficulties of obtaining all such data that one could want, but dealing with counter-factuality requires instantaneous knowledge of classified information. How can one know precisely how many troops the state has at a given moment? How can one know precisely how many troops the rebels have at *any* time point, much less at every time point?

Mindful of how most political scientists will share my own unwillingness to wander a battlefield in search of data, I thus set out to construct the next best thing: a mathematical model that when given accurate initial conditions and parameter values will predict the troop counts of both the state and the rebels at any given time point (with certain restrictions to be outlined). To do so, I will present several potential models and test all of them against data from a segment of the Sri Lankan civil war to see which is the best predictor of outcomes. However, before testing anything, it would be prudent to set down concretely the criteria for evaluation, the case to be examined, and the sources for all data.

3.1 Criteria

To evaluate the performance of each model in each case, I employ five measures of accuracy.

1. *Troop Levels and GDP* ($\frac{SS_{err}}{SS_{tot}}$) - Using (where available) actual data for yearly state and rebel troop levels and GDP, I take a sum of differences squared between observed and expected values for each variable and divide by the variance of the the data for said variable. One should avoid the temptation to think of the result as $1 - R^2$, where R^2 is the coefficient of determination, because these models are not fitted (in the sense of least square regressions) to the data, so one could very well have error terms larger than one, resulting in an "R^2" that is negative, which makes absolutely no sense. Note that both the individual error terms that result and their sum will be compared between models.

2. *Yearly Combat Deaths* ($\frac{SS_{err}}{SS_{tot}}$) - Much as for yearly troop levels, yearly combat deaths, both total and disaggregated into state, rebel, and civilian, will be sub-

jected to an observed-expected sum of squares of differences over variance.

3. *Cumulative Combat Deaths* ($\frac{SS_{err}}{SS_{tot}}$) - This measure is calculated exactly as the yearly combat deaths measure is, except that cumulative combat deaths up to a given year are used instead. This measure is included mainly because cumulative deaths, an increasing function, is expected to be less variable than yearly deaths and thus a more relaxed indicator of performance.

4. *Outcome* - The model's predicted ending state will be qualitatively scored according to the following rubric:

   4 - Relative difference in capabilities and general dynamics accurate
   GDP is higher or lower as expected

   3 - Relative difference in capabilities accurate and general dynamics accurate
   GDP behavior incorrect

   2 - Relative difference in capabilities and general dynamics inaccurate
   GDP is higher or lower as expected

   1 - Nothing accurate

   See Appendix C for more information on what the actual relative difference in capabilities was scored to be.

5. *Civilian Death Estimation* - This measure is simply the difference of the observed and expected total number of civilians killed during the timespan examined divided by the observed number. A smaller absolute value indicates a better estimator.

3.2 The Case

Due to the limitations of the models (see Chapter 4), rather than studying an entire civil war, I instead examine just a segment of a civil war. More specifically, I will use the Sri Lankan civil war from the departure of the Indian Peacekeeping Force to the peace talks at the end of 1994. This war during this period fits the assumptions of the model well because by 1990, the Tamil Tigers have largely eliminated rival militant separatist groups and the uprising of the communist Janatha Vimukthi Peramuna party has been put down. Civilian deaths, though grim for a tiny island with a population smaller than that of New York City, are not overwhelming. Furthermore, as the war is largely concentrated in the northern and eastern parts of the country, the effects of civilian displacement are less extreme than otherwise might be the case. Thus, the assumptions outlined in section 4.1 are largely unviolated. The period studied naturally ends with the ceasefire and peace negotiations that begin with the arrival of the Kumaratunga administration, a blatant violation of Model Assumption 3.

3.3 Data

The data and methods used in parameter calculations, as well as to verify the models' individual accuracies, can be found in Appendices B and C, respectively.

Chapter 4

Model Development: Constructing Apocalypse

In this section, I will systematically develop several candidates for the title of ultimate dynamical model of civil war military capabilities.

4.1 Basic Assumptions

All of the general models that follow will be built upon the following basic principles:

1. (Materialism) A side's military capabilities are determined solely by material factors, as opposed to grievance-type issues (morale, etc.)

2. (Dualism) A civil war contains only two opposing sides - those fighting for the state and those fighting for the rebels

3. (Zealotry) The two opposing sides will continue to fight until one of them is completely destroyed

4. (Professionalism) Neither side will kill large numbers of civilians in a manner wholly independent of combat

5. (Demographic Constancy) Civilian casualties, internally displaced persons, and refugees do not amount to a sizable proportion of the population

   Though the models are technically of military capabilities, because of their inherent structure (most notably, military capabilities being affected by casualties), they are to some extent models of civil war intensity as well. Thus, one may as well make the following implicit assumption explicit:

6. (Determined Intensity) The intensity of a civil war is determined solely by the two sides' military capabilities.

The first three of these assumptions are largely necessitated by the need to keep the model simple. Modeling the effects of policy grievances and morale, not to mention how they change over time, would be difficult if not impossible, so materialism must be assumed. Dualism simply precludes the inevitable complications of additional sides, namely additional equations and more complicated kill terms that reflect the many combinations of killer and killed. Finally, as modeling the decisions of leaders to sue for peace or lay down their arms is well beyond the scope of this model (and perhaps even ordinary differential equations in general), one must assume zealotry so the model will run until one side can fight no more. By observing the sides' capabilities as predicted by the model, future research can assign "probabilities of peace" to different relative levels of capabilities and/or death counts to account for this flaw.

The next two assumptions, however, are somewhat less tenable. The term "professional" probably does not apply in any form whatsoever to the average civil war participant. Indeed, while one might expect soldiers to leave civilians in peace because they have no weapons, it is in fact for precisely this reason that civilians are almost never left alone. However, the mere killing of civilians in the course of a war is not an issue as much as the systematic killing of civilians *instead of* rather than *in addition to* enemy fighters. In other words, terrorist-style attacks or even small massacres would be acceptable (to use the word loosely), but large-scale and protracted genocide in the absences of actual combat would not. Meanwhile, the Second Congolese War mentioned in the introduction was/is not the only civil war where a huge disruption of demographics was the most obvious facet of the war. It bears noting that at least the assumption of Demographic Constancy must be dropped and the models altered accordingly should one wish to adapt them to more cataclysmic civil wars.

The final assumption is to some extent the embodiment of the capabilities hypothesis itself. As outlined in Chapter 3, to have this hypothesis underpinning all of the models means that if none of the models carry any validity whatsoever, then the validity of the capabilities hypothesis itself must be called into question.

4.2 Developing the Models

The models will be divided into five levels of complexity, where level 1 consists of the most basic of model equations, and subsequent levels progressively add more depth to this baseline. The models are labeled in the form [Lineage][Complexity Level].[Version], where lineage refers to the handling of how rebels kill state troops (see below). As other terms (most notably, the way the state kills rebels) can lead to further variation, version number will serve to distinguish these variants.

4.2.1 Level 1: Foundation

The following bare bones model, which will be called A1, is as good a starting point as any:

$$\begin{aligned}\frac{dS}{dt} &= k_S c_S^{-1}(f_S + i_S G - x_S S) - d_{SR1} SR \\ \frac{dR}{dt} &= k_R c_R^{-1}(f_R + i_{R1} G - x_R R) - d_{RS1} RS \\ \frac{dG}{dt} &= rG\end{aligned}$$

Figure 4.1 summarizes the workings of this model in the form of a component diagram. In the equation for $\frac{dS}{dt}$, the expression $(f_S + i_S G - x_S S)$ represents the state's net income. The revenue is simply the amount of foreign aid the state receives plus the product of the gross domestic product and the percent of the GDP spent on the armed forces, and from this the upkeep expense per soldier times the number of soldiers is subtracted. Of this income, a certain amount inevitably winds up in pockets for which it

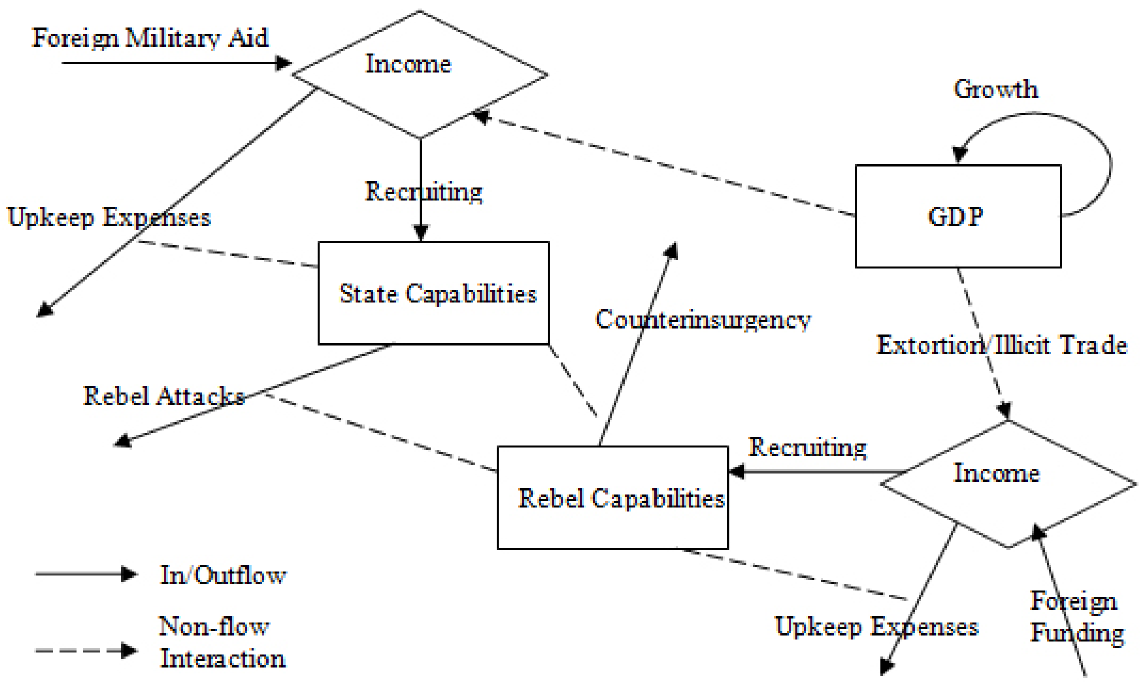


Figure 4.1: Component diagram for level 1 complexity

was never destined, and thus due to this corruption only a k_S proportion of it is actually available for outfitting the army. The assumptions of materialism and zealotry imply that all of this remaining income will be spent on war-making, so the corruption-scaled net income is divided by c_S, the cost to train a new soldier, to yield the number of new troops produced each year. From this, the number of soldiers killed in a year $d_{SR}SR$ is subtracted. Note that in model A1, the rate at which rebels kill soldiers is modeled with a simple nonlinear interaction term, implying the number of soldiers killed increases as the number of soldiers available to be killed and the number of rebels partaking in killing increases. $\frac{dR}{dt}$ follows exactly the same reasoning.

Meanwhile, $\frac{dG}{dt}$ consists of simple proportional growth, implying the state's GDP will increase exponentially over time.

These equations yield the following Jacobian matrix:

$$\mathbf{J} = \begin{pmatrix} \frac{k_S x_S}{c_S} - d_{SR1}R & -d_{SR1}S & \frac{k_S i_S}{c_S} \\ -d_{RS1}R & -\frac{k_R x_R}{c_R} - d_{RS1}S & \frac{k_R i_{R1}}{c_R} \\ 0 & 0 & r \end{pmatrix}$$

If the limitations of this model are not obvious from the equations themselves, then the Jacobian obligingly drives them home. Other points aside, the country's gross domestic product is completely unaffected by the war - indeed, $\frac{\partial \dot{G}}{\partial S} = \frac{\partial \dot{G}}{\partial R} = 0$. Also note that the state and rebel equations are basically mirror images of one another - in other words, structurally speaking, the state and the rebels have parallel capabilities. In practice, different parameters would ensure the two sides's capabilities would not change identically, but this nonetheless assumes that state troops fight and kill in much the same way as rebel fighters. The most important point is the kill term $d_{SR}SR$ from $\frac{dS}{dt}$. This basic interaction term is the most intuitive means of describing the rebels' killing potential, and thus models employing it will fall under the A lineage.

However, an interaction term is not the only means of modeling the rebels' killing skills. Indeed, Allan and Stahel's (1983) model uses a simple linear term to model the mujahideen's killing of Soviet and Afghan troops on the justification that such best models guerrilla hit-and-run tactics (one can imagine on a micro basis that the rebels emerge from hiding only long enough to each get off a single shot before hiding again). Such an assumption gives rise to the B lineage of models.

Unfortunately, a direct application of Allan and Stahel's simple linear formulation results in a term of the form $d_{SR}R$. While in their model, such was not a problem as the number of Soviet troops could never really become very small, in the current model, it is very possible that S can be small (if the government is on the verge of collapse) and thus this linear term could force S to be negative, which makes no sense. Thus, I add a Michaelis-Menten factor $\frac{S}{S+1}$. This term is close to 1 for normal values of S, but becomes substantially less than 1 for $S < 5$ and rapidly approaches 0 as $S \to 0$. Thus, this factor ensures soldiers cannot be killed when no soldiers exist to be killed. Model B1 uses this alternative formulation for rebel killing and thus differs from A1 in its equation for $\frac{dS}{dt}$:

$$\frac{dS}{dt} = c_S^{-1}(f_S + i_S G - x_S S) - d_{SR2} R \frac{S}{S+1}$$

Note the appearance of the new parameter d_{SR2}, which must be calculated differently from d_{SR1} due to the structural differences in the two kill terms.

4.2.2 Level 2: Rebel Expense Approximation

Unless a government raid yields meticulously-kept rebel account books and such books are then released to the public, the exact upkeep expense a rebel organization faces per fighter can only be ascertained by infiltrating said organization. Preferring not to risk disembowelment for the sake of a parameter value, I therefore must resort to approximations.

According to Stern (2003), fighters in a commander-cadre organization (which is essentially what a coherent rebel organization of the form studied here is) come to treat militancy as a profession, and thus only fight as long as long as their leaders make it worth their while. Thus, it seems logical that they will fight only as long as their leaders at least meet their opportunity cost, as approximated by the state's median income. As median income can change throughout the course of a war as GDP is affected, it seems logical to represent median income as a function of GDP. From this conceptualization, one can construct models A2 and B2 by updating A1 and B1 respectively with the following $\frac{dR}{dt}$ equation:

$$\frac{dR}{dt} = c_R^{-1}(f_R + i_{R1}G - q_i\frac{G}{p}R) - d_{RS1}RS$$

where p is the population and q_i is the average ratio between per capita GDP and median income.

4.2.3 Level 3: Economic Effects of War

In all of the models presented so far, the most serious flaw of A1 - namely, the non-effect of war on the country's economy - has remained unaddressed. If one assumes that the rate at which a country's economic growth is stymied by war is proportional to the number of civilians killed, then a scaling of an approximation of civilian deaths per year will give an approximation of the amount of growth the country loses per year.

By the assumption of professionalism, one can assert that civilians will only die when caught in the crossfire of battles between the state and the rebels or at least only in spillover aggression from such battles. Thus, it follows that the number of civilians killed in a year is proportional to the number of government and rebel troops killed. Ergo, one can replace the equation for $\frac{dG}{dt}$ in both A2 and B2 with the following:

$$\frac{dG}{dt} = rG - c_C G[d_C \kappa(S, R)]$$

where $\kappa(S, R)$ is the sum of the state and rebel death terms. Thus, $\kappa(S, R) = d_{SR1}SR + d_{RS1}RS$ in model A3, and $\kappa(S, R) = d_{SR2}R\frac{S}{S+1} + d_{RS1}RS$ in model B3.

4.2.4 Level 4: Dynamic State Killing Ability

Aside from the difficulty of calculating the parameter d_{RS1} (because data on the total number of rebels or the number of rebels killed over a long enough span of time needed to create a truly averaged number are scarce), one can further doubt the validity of a simple interaction term because the ease with which the state finds and kills rebels will not be constant throughout the war. As Guevara writes, a rebel organization must make survival its main goal in its formative years; however, once it has grown larger and stronger, it can follow more aggressive tactics and eventually face the state in open battles (Guevara, 2006).

With these considerations in mind, it seems the best approach may be to think of the state's ability to kill rebels throughout the war (not just during battles, but also during lulls when the rebels are not showing themselves) as a fraction of its ability to kill the rebels when they do show themselves. Thus, one can think of the term $d_{RS2}RS$ as the state's full killing ability (note the new parameter d_{RS2}, which is calculated from the most intense part of the war), and in each of the variations below, this term is multiplied by a term reflective of rebel tactics and/or hiding ability.

$$
\begin{aligned}
(1)\quad \frac{dR}{dt} &= c_R^{-1}(i_{R1}G - q_i\frac{G}{p}R) - d_{RS2}RS\frac{R^{es}}{R^{es} + (\frac{S}{h_{R1}/p})^{es}} \\
(2)\quad \frac{dR}{dt} &= c_R^{-1}(i_{R1}G - q_i\frac{G}{p}R) - d_{RS2}RS \times h_{R2}\frac{S}{p} \\
(3)\quad \frac{dR}{dt} &= c_R^{-1}(i_{R1}G - q_i\frac{G}{p}R) - d_{RS2}RS\left[\frac{R^{es}}{R^{es} + (\frac{S}{2})^{es}} + h_{R3}\frac{S}{p}\right]
\end{aligned}
$$

In equation (1), rebel combat preference and rebel hiding ability are aggregated into one term. Thus, as the number of state troops increases (hit-and-run seems more appealing) and as the country's population size increases (hit-and-run is easier), the Michaelis-Menten term becomes smaller to reflect more evasive tactics and thus fewer rebel losses.

In equation (2), it is assumed that the rebels will always prefer guerrilla tactics where possible and thus the only factor that matters is their ability to practice such tactics.

Thus, in this formulation, a greater state soldier density ($\frac{S}{p}$) leads to more rebel losses as hiding among the civilian population becomes more difficult.

Finally, equation (3) disaggregates rebel preference from rebel hiding ability by using separate terms to represent each, the sum of which yields rebel visibility. The first term $\frac{R^{e_S}}{R^{e_S}+(\frac{S}{2})^{e_S}}$, which represents rebel preference, is Michaelis-Menten and is designed to simply equal 1/2 when rebel forces are half the size of the state's and to approach 1 tangentially as R increases. Meanwhile, the hiding ability term $h_{R3}\frac{S}{p}$ is much the same as in (2).

In all of these equations, it bears emphasis that the term h_{Rx} has no simple real-world meaning (an unfortunate inevitability arising from modeling something as complicated as guerrilla tactics with so few terms). Thus, the term must be calculated from actual data from the war under study, and hopefully additional test cases will confirm that the resulting value is either relatively invariant across cases or follows a pattern that corresponds to real world conditions.

4.2.5 Level 5: Balance of Power

As a civil war progresses and the rebels become stronger, the territory they protect/terrorize will expand and thus their income potential will increase while the income potential of the state decreases. To model this, I use what I dub balance of power terms of the form $\frac{S}{S+R}$, which simply reflect the percentage of the country's fighters that are loyal to one side or the other (and thus reflect the relative amount of force either side has with which to extract taxes or protection fees, etc.). This struggle for the country's tax base is best

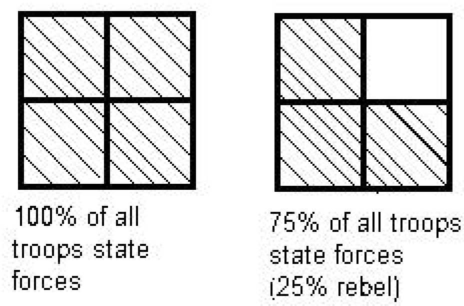


Figure 4.2: State control of tax base in case of no war (state has monopoly on force) and intense war (25% of force is rebel)

depicted in diagram 4.2.

The result is several new level 5 models, all exactly like their level 4 equivalents but where the income terms $i_S G$ and $i_R G$ are replaced with $i_S G \frac{S}{S+R}$ and $i_{R2} G \frac{R}{S+R}$, respectively. In this new formulation, i_S and i_{R2} come to represent the desired (perhaps wishful is the best word) rate at which either side can extract resources from the population - in other words, the rate at which either side would milk the populace if it did indeed have a monopoly on force in the country. It is important to note the new parameter i_{R2}, the proportion the rebels want, and i_{R1}, what they actually get, are vastly different quantities. Meanwhile, in the case of the state, such would probably not change immensely because i_S is calculated from data where the state is largely in control.

Part II

Results

Chapter 5

Model Evaluation: Model Warfare

5.1 Criteria Scores

The results, as gauged by the criteria set out in Section 3.1, are outlined in Tables 5.1 and 5.2, with the "winning" value in each category bolded. Note that models A5.1, A5.2, A5.3, and B5.2 are immediately disqualified from further consideration because of their substandard Outcome scores, and models B5.1 and B5.3 are similarly disqualified for the even more blatant failing of causing Matlab to crash when I attempted to run them (hence the blank entries for all of their criteria scores).

Looking at the criteria, one can see that no model exactly swept the polls, so to speak (except when compared to the level 5 models, which did not make it onto the ballot). Indeed, the battle appears to be a tie between model A3 and model B4.2, the latter taking the gold in estimating troop levels and the former claiming victory in predicting deaths. However, considering that A3's error in predicting troop levels is somewhat less than B4.2's error in predicting death counts (either yearly or cumulative), one can tentatively declare A3 the winner.

This victory is surprising for many reasons, not the least of which is that A3 is much simpler in logic and form than most of the others. However, it should not be too triumphantly celebrated. As plates a and b of Figure 5.1 makes clear, the qualitative behaviors of the levels 2 to 4 models are not very different. Furthermore, one should not put too much stock in the numerical criteria, as Figure 5.1c demonstrates. Though the yearly death counts predicted by model A5.2 are eerily similar to the actual recorded deaths data, one can safely assert that the 1990 to 1994 period did not witness the LTTE absolutely crushing the government while amassing an army of 1.8 million fighters in the process. Furthermore, the data itself should not be trusted too much, as both sides undoubtedly underreport their dead. Indeed, if both the yearly death counts and the civilian death measure are to be believed, then only 4005 fighters died in the entire period, as compared to 20000 civilians (see Appendix C.5).

5.2 Shortcomings

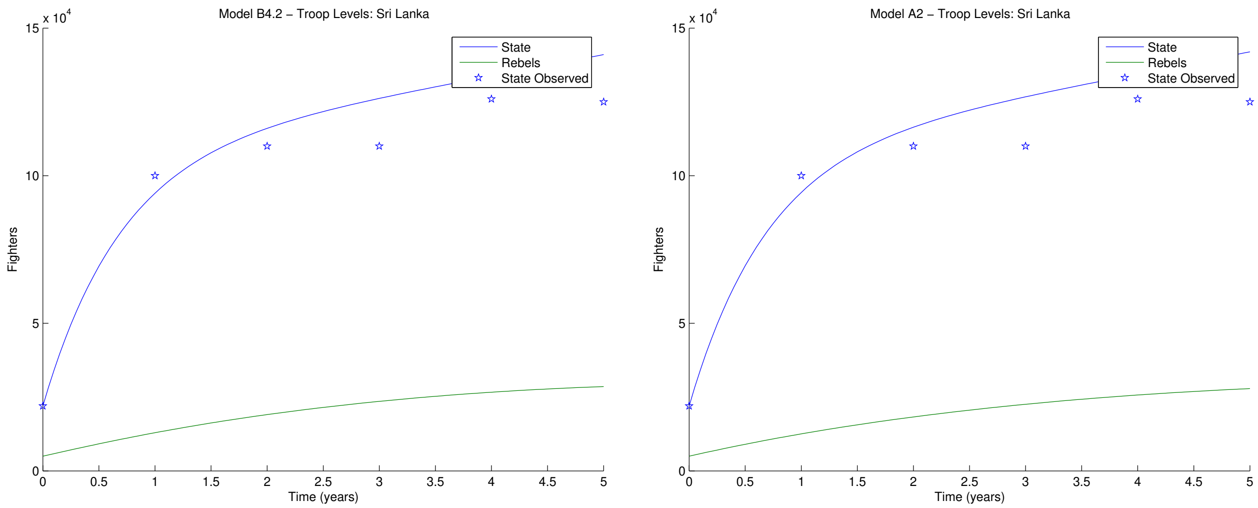
Though surprisingly accurate considering their simplicity, all of these models have serious shortcomings that must be addressed.

Model	State Error	GDP Error	Total Value Error	Yearly Deaths Error
A2	0.094	0.0072	0.1013	5.7255
A3	0.0888	**0.0058**	0.0946	**5.687**
A4.1	0.0899	0.0061	0.096	11.1217
A4.2	0.0886	0.006	0.0945	6.378
A4.3	0.0887	0.006	0.0947	7.3519
A5.1	7.7106	0.015	7.7256	97.4989
A5.2	7.7629	0.0049	7.7678	1.6103
A5.3	7.7125	0.0161	7.7286	105.6546
B2	0.0918	0.0072	0.0991	5.9785
B3	0.0861	**0.0058**	0.0919	5.9494
B4.1	0.088	0.0061	0.0941	11.7253
B4.2	**0.0858**	0.0059	**0.0917**	6.5946
B4.3	0.086	0.0059	0.092	7.6403
B5.1	-	-	-	-
B5.2	8.1057	0.0031	8.1088	20.7177
B5.3	-	-	-	-

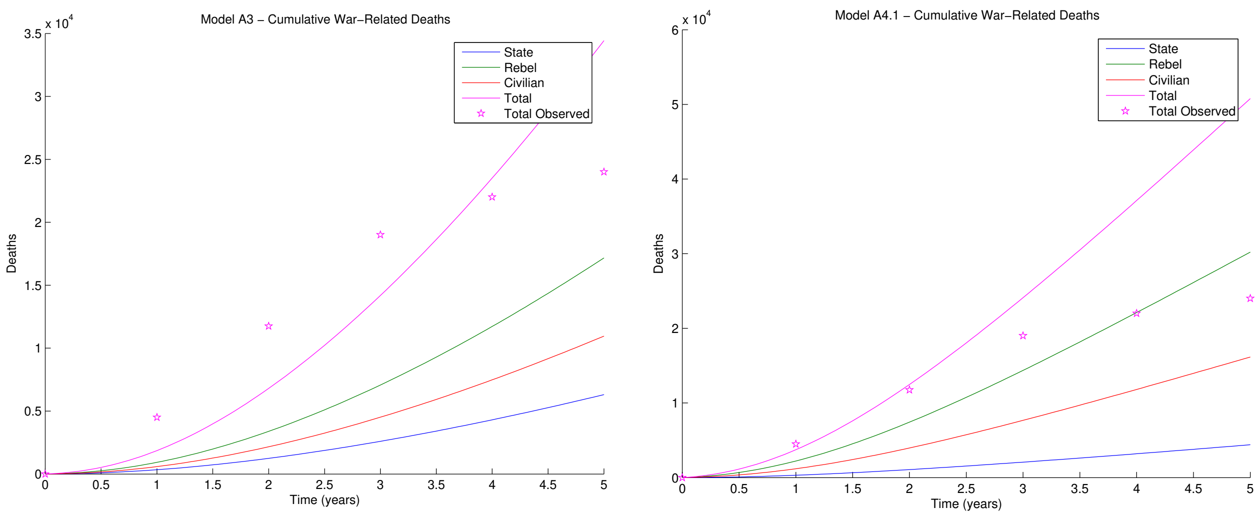
Table 5.1: Scores Based on Critera

Model	**Cumulative Deaths Error**	**Civilian Cost Evaluation**	**Outcome**
A2	0.3486	0.451	4
A3	**0.3458**	0.4521	4
A4.1	2.0287	**0.1921**	4
A4.2	0.3721	0.4647	4
A4.3	0.4564	0.4264	4
A5.1	45.4556	-0.5886	2
A5.2	0.3823	0.7729	2
A5.3	49.2775	-0.6365	2
B2	0.427	0.415	4
B3	0.4242	0.4157	4
B4.1	2.3817	0.1634	4
B4.2	0.4185	0.4276	4
B4.3	0.5467	0.3898	4
B5.1	-	-	-
B5.2	3.7299	0.3753	2
B5.3	-	-	-

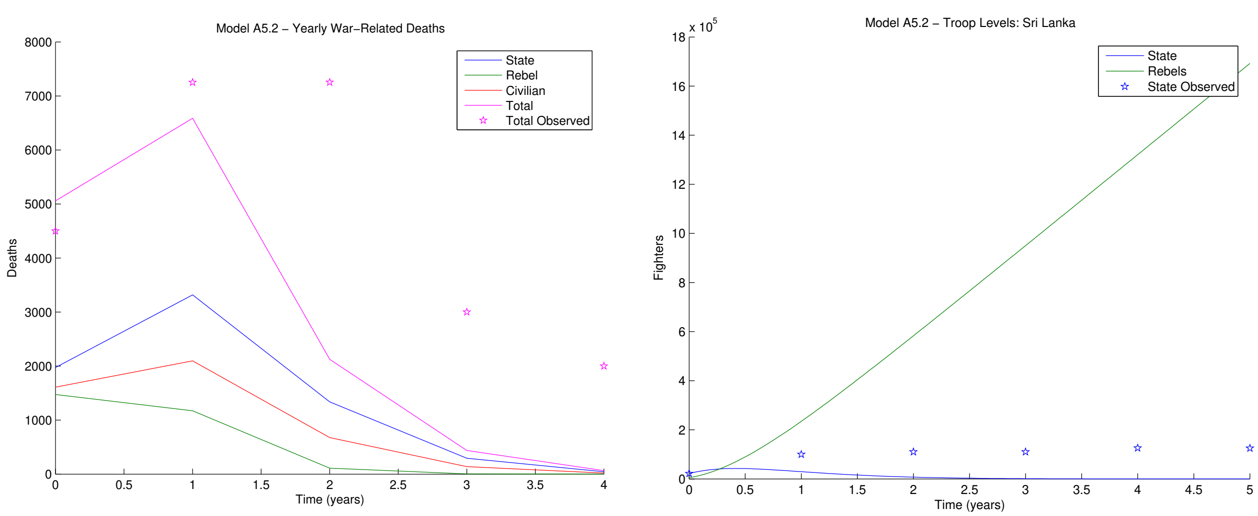
Table 5.2: Scores Based on Critera (cont.)



a



b



c

Figure 5.1: a) Best Performer vs. Worst Performer (Troop Count); b) Best Performer vs. Worst Performer (Cumulative Deaths); c) Actual "Best" Yearly Deaths Performer

5.2.1 Level 5 Models

Though it certainly made finding a winner among the models easier, the dismal failure of all the level 5 models is nevertheless distressing. However, their spectacular inability to make even the most basic predictions about the course of the war is a good example of the main benefit of testing hypotheses with mathematical models: namely, how said hypotheses cannot hide even their tiniest flaws from scrutiny.

In this case, the implicit hypothesis these balance of power models made was that the amount of revenue either side could extract from the GDP was directly proportional to how much power they had. This assumption missed the rather critical fact that the army is not the IRS - in other words, to have control over a territory does not equate to having the proper bureaucratic machinery necessary to govern it (or at least tax it). Indeed, were the entire Sri Lankan government and army to vanish overnight, taking its census statistics and tax payer registration database with it, the Tigers would probably need years to restore the administrative structure to a state where they could finally afford the champagne to celebrate their unexpected victory. Indeed, reports from the eastern province where the Tigers until recently effectively were the government, their method of raising finances involved more along the lines of kidnapping and extortion than standardized income taxation (Nessman, 2007). This seems to support the finding in (DeRouen Jr. and Sobek, 2004) about the importance of the state's bureaucracy in determining the outcome of a civil war.

However, while one can get away with leaving out all consideration of the balance of power and ability to raise revenue in the case of Sri Lanka, where the government was at no point seriously threatened, it would be an issue in other examples. If one were applying the models to the first Liberian civil war, for example, the government's waning power and Charles Taylor's eventual consolidation of control could not be accurately represented in any of the lower level models. At the very least, the balance term in $\frac{dS}{dt}$ should be left in to reflect the inability of the government's tax-collection apparatus to operate in rebel-controlled territory, while the balance term in $\frac{dR}{dt}$ could be removed and the income term be changed to $i_{R1}G$ as it is in the lower level models. However, the ideal scenario is one where both balance terms remain, and additional terms (or even just constant parameters) are added to account for bureaucratic effectiveness.

5.2.2 Other Models

Too Many Rebels - The main flaw of all the more successful models is their overestimation of the number of rebels. Whereas most estimates record the rebel army as between five thousand and fifteen thousand (Sydney Morning Herald, 1990; McCarthy, 1990; International Institute for Strategic Studies, 2008), most of the models show the rebels' numbers to be around twenty-five thousand by the end of the period, with the lowest estimates (models A4.1 and B4.1) being about twenty thousand.

This leads to problems beyond just an overestimation in rebel strength. As all of the models assume the number of people killed is proportional to the sizes of the state's army and the rebels' forces, too many rebels equates with too many deaths, and thus the nearly universal overestimation among models of the final death toll.

One can imagine several possible explanations for this overestimation. The most obvious is actually the easiest to correct (in theory, anyways): bad parameter values. If f_R or i_R is too big, or c_R is too small, then clearly rebel troop counts will be too high. Likewise, if k_R is too large (not difficult to imagine, considering the necessarily haphazardous manner of its estimation), the hypothetical rebels will have more income to use for their hypothetical war effort than their living, breathing, and state-overthrowing counterparts. Unfortunately, the values of some of these numbers are known only to the rebels themselves, and thus may prove impervious to solid estimation.

Moving away from the conveniently easy explanation of deficient parameters, one can also examine possible structural defects in the models. One is the approximated opportunity cost assumption inherent in the term $q_i \frac{G}{p}$ - namely, that the cost of upkeep for a single rebel is the median income. Though this assumption deftly dodges the difficulty in calculating the actual rebel upkeep cost (again, something only the rebels themselves can know for sure), it could be utterly wrong. If so, then one would have to revert back to the level 1 rebel expense term $x_R R$ in all of the models, or at least find a more accurate proportion of GDP/capita to approximate rebel expenses.

The most difficult and unpleasant possibility is that the simplifying assumption that rebel training and outfitting can be accurately modeled by instantaneous inflows is wrong. If this is the case, one would have two options: either to create an additional population to represent rebels-in-training (thus losing some of the models' simplicity), and have incoming rebels flow through this middle man term; or to use delay differential equations rather than ordinary differential equations (thus losing some of the analytic tools one has in studying ODEs). However, if one really cannot represent the training of fighters as an instantaneous inflow, it seems logical that similar problems would arise in the state troop count. But as Table 5.1 makes clear, the size of the state's forces is represented with shocking accuracy in all of the non-level 5 models.

Perpetual Recruitment - Another problem, more abstract because it creates problems mainly over the long term, and yet potentially more dangerous because it leads to inaccuracies in qualitative behavior, is how both sides' numbers increase indefinitely. Though the rate at which both sides forces grow is very slight after their initial recruitment drives, it is nevertheless an upward trend. This is a problem because 1) as noted before, the amount of carnage predicted by the model increases with the number of combatants, so ever increasing combatant pools cause ever increasing death counts; 2) model analysis will be skewed because it assumes both sides are getting larger and larger over time.

Why do the models predict such swelling armies? Mainly because both sides receive some portion of their income from the GDP, which in the case of Sri Lanka has steadily grown despite the war. Thus, it makes sense that the state's numbers are increasing faster than the rebels', since the rebels rely primarily on foreign funding, which is assumed to stay constant in this model (though it might be worth changing this to reflect the growth of the GDPs of the countries from which their funds originate).

Why does an expanding GDP throw off the results like this? After all, the actual Sri Lankan military receives its income from taxes drawn from the GDP as well. And lest one think the reason is that the model does not take inflation into account, note that r, the economic growth rate parameter, was calculated from *real* GDP growth rate data rather than nominal data. In fact, the reason is probably that the model does not acknowledge that military technology, and thus military expenditure, expands over time much like the global economy. Thus, the ever increasing defense budget is invested in more advanced hardware rather than more soldiers, a trend that may well hold for the rebels as well as they throw down their homemade bazookas and pick up Stinger missiles instead.

Solving this problem would probably require a close study of changes in military technology and budget, something well beyond the scope of this study. However, it seems likely the the key to the problem lies in replacing the constants c_S and c_R with functions $c_S(t)$ and $c_R(t)$, thus making the system of ODEs non-autonomous. Perhaps something as simple as $c_S(t) = \alpha t + c_0$, with α and c_0 determined by fitting a trend line to past data on (inflation-adjusted) military hardware costs, would suffice.

Chapter 6

Model Analysis: Examining Apocalypse

Having painstakingly identifying the best model out of these sixteen, we can now extract some insight from it through some analysis. Before continuing, however, it would be useful to define some simple axioms to keep such insight relevant to the real world.

First, it seems reasonable to assume one is examining an actual inhabited country - in other words, as far as our purposes go, economic activity of some kind bounded by borders.

Axiom 1 (State Existence)**.** $G > 0$ for all t.

It also seems reasonable to assume that neither side's army will have a negative number of fighters, and also neither army can get too big as compared to the country's population. In the case of Sri Lanka, if we let α denote the percent of the island's population made up of Tamil's, which was roughly 18% in 1990 (Gamini and Chaudhary, 1990), and assume that neither side's army is larger than 4% of its potential recruiting base, then we arrive at the following axiom:

Axiom 2 (Bounded Armies)**.** $0 \leq S \leq S_{max} = .04p$ and $0 \leq R \leq R_{max} = .04\alpha p$ for all t.

Note that this means $0 \leq S \leq 6.8454 \times 10^5$ and $0 \leq R \leq 1.2322 \times 10^5$, which is hardly too restrictive a bound (if both the army and the LTTE take their maximum values, then 4.72% of the population would be at war, a staggering number).

Finally, as all the parameters for all the models have implicit purposes and the use of addition and subtraction in their formulation follows those purposes(for example, f_S is meant as an inflow of foreign funds, d_{RS1} is meant as a component in a kill rate), it seems reasonable to assume the parameters are non-negative to preserve the signs inherent to their purpose.

Axiom 3 (Parameter Non-Negativity)**.** All parameters are non-negative real numbers.

6.1 Default d_C, c_C, d_{RS}, d_{SR}

To start out, we make the simplifying assumption that the values of d_C, c_C, d_{RS} and d_{SR} are those used to run the model in Chapter 5 (see Appendix B). With this assumption in place, several results arise through simple analysis.

6.1.1 Things Never Get Dull: The Absence of Invariance

Generally, when one has a nonlinear dynamical system in need of analysis, one faces the overwhelming temptation to attempt equilibrium analysis, or at least nullcline analysis. However, the following proposition demonstrates that under the current axioms and assumptions, one should suppress such temptation.

Proposition 1 (Nonexistence of Equilibria)**.** *Under the assumptions and axioms given, the model has no equilibria.*

Before beginning the proof, it is worthwhile to make the following substitutions to simplify the algebra involved:

$$\begin{aligned} m_1 &= k_S f_s c_S^{-1} \\ m_2 &= k_S i_S c_S^{-1} \\ m_3 &= k_S x_S c_S^{-1} \\ m_4 &= d_{SR1} \\ m_5 &= k_R f_R c_R^{-1} \end{aligned} \qquad \begin{aligned} m_6 &= k_R i_{R1} c_R^{-1} \\ m_7 &= k_R q_i (c_R p)^{-1} \\ m_8 &= d_{RS1} \\ m_9 &= r \\ m_{10} &= c_C d_C \end{aligned}$$

This results in the following simplified system of equations:

$$\frac{dS}{dt} = m_1 + m_2 G - m_3 S - m_4 SR \tag{6.1}$$

$$\frac{dR}{dt} = m_5 + m_6 G - m_7 GR - m_8 SR \tag{6.2}$$

$$\frac{dG}{dt} = m_9 G - m_{10} GRS(m_4 + m_8) \tag{6.3}$$

Proof. To find equilibria, one can set the system equal to $[000]^T$. However, note that if $\frac{dG}{dt} = 0$, then either $G = 0$ or $m_9 - m_{10}RS(m_4 + m_8) = 0$. By Axiom 1, $G \neq 0$. Then it must be that $m_9 - m_{10}RS(m_4 + m_8) = 0$. But then $RS = \frac{m_9}{m_{10}(m_4+m_8)} = \frac{r}{c_C d_C(d_{SR1}+d_{RS1})} = 9.7338 \times 10^{10}$ by the assumptions of this section. Even if $R = R_{max} = 1.2322 \times 10^5$, then S must equal $7.8995 \times 10^5 > S_{max} = 6.8454 \times 10^5$ in violation of Axiom 2. Thus, $\frac{dG}{dt} > 0$ and so the system is never at equilibrium. □

This last statement immediately gives rise to the following corollary.

Corollary 1. *G is a strictly increasing function of t.*

However, one may very well wonder if in spite of the GDP's refusal to settle, the size of the state and rebel armies might not eventually come to a still point, perhaps along the stable manifold of some distant saddle point beyond the bounds of variable space cordoned off by the axioms taken. However, the following series of propositions demonstrates that given some reasonable assumptions about the parameters, such is impossible.

Proposition 2 (Non-Invariance of S)**.** *If $m_2 \neq 0$, then S does not remain constant.*

Proof. Assume to the contrary that some S^* exists such that for appropriate values of G, R, and the parameters, $S(t) = S^*$ for a nondegenerate interval of t. Then

$$\frac{dS}{dt} = m_1 + m_2G - m_3S^* - m_4S^*R = 0$$

If either R is constant or $m_4 = 0$, then $\frac{dS}{dt}$ is a function of solely G, which is strictly increasing by Corollary 1. Thus, it must be that $m_4 \neq 0$ and R varies. As $\frac{dS}{dt} = 0$ for a non-degenerate interval of t and thus is constant, it must be that

$$\frac{d^2S}{dt^2} = m_2G' - m_4S^*R' = 0$$

Then $G' \propto R'$. But $G' = \frac{dG}{dt}$ is clearly not proportional to $R' = \frac{dR}{dt}$ for varying G, R. This contradicts the assumptions about S^*, so S^* must not exist.

□

Proposition 3 (Non-Invariance of R)**.** *If $m_2 \neq 0$, and one of the following holds:*

1. $m_5 \neq 0$

2. $m_6 \neq 0$

3. $R \neq 0$

then R does not remain constant.

Proof. Assume to the contrary that some R^* exists such that for appropriate values of G, S, and the parameters, $R(t) = R^*$ for a nondegenerate interval of t. Then

$$\frac{dR}{dt} = m_5 + m_6G - m_7GR^* - m_*SR^* = 0$$

First, note that if condition 1 or 2 is true but 3 is not, then $0 = m_5 + m_6G > 0$, a contradiction. Thus, either condition 1 or 2 implies condition 3.

As $m_2 \neq 0$, S is not constant by Proposition 2. Further, as $\frac{dR}{dt} = 0$ for a non-degenerate interval of t and thus is constant, it must be that

$$\frac{d^2R}{dt^2} = (m_6 - m_7R^*)G' - m_8R^*S' = 0$$

If $m_6 - m_7R^* = 0$, then $\frac{d^2R}{dt^2}$ is a function of $S' \neq 0$ (recall that $m_8 \neq 0$ by the assumptions of the section). Thus, $m_6 - m_7R^* \neq 0$ and so it must be that $G' \propto S'$. But $G' = \frac{dG}{dt}$ is clearly not proportional to $S' = \frac{dS}{dt}$ for varying G, S. This contradicts the assumptions about R^*, so R^* must not exist.

□

For m_2 to be zero, it must be that either $i_S = 0$ or $k_S = 0$. However, as one cannot imagine any situation where the state cuts all military spending (during a civil war, no less), and as it seems unlikely that not a single penny of military spending makes it past rent-seekers, it is probably safe to assume that $m_2 > 0$ in any realistic situation.

Furthermore, the possibility that the rebels funds, both foreign and otherwise, will be cut off completely seems equally unlikely. However, even given that possibility, it so happens that the hypothesis on R^* is stronger than it need be - given any combination of parameter values and R_0 (the initial number of rebels) $\neq 0$, R will never equal zero and thus will remain invariant.

However, this is not to say that for the right parameter values and values of S and G that R will not become very close to zero - which is the motivation of the next section.

6.1.2 Money Matters: Rebel Finances

As Propositions 1 - 3 prove, equilibrium analysis and blind nullcline analysis will not yield much insight. However, using the parameters already defined and more careful examination of the zeros of the individual equations, one can still extract some useful counterfactual insights about this phase of the Sri Lankan civil war.

One may well wonder if the conflict would have ended sooner given different conditions when the Indian Peace-Keeping Force left. More precisely, one can ask how the situation would have had to have differed for the government to have quickly defeated the rebels and ended the insurgency. If one considers a quick end to the situation to entail the rebels' losing from the very start of the period, then one need only find the points when $\frac{dR}{dt}$ is negative keeping all other factors constant. Perhaps the most profitable enterprise would be to find how many state security personnel would be necessary to crush the insurgency from its very inception given different parameters relating to the rebels' finances. More specifically, we can examine c_R (cost of producing new troops) and f_R (amount of foreign funding). Furthermore, as the method of calculating rebel expenses is questionable (especially given the model's massive overestimation of rebel numbers), one can also treat qi as an expense parameter. I leave i_{R1} (proportion of GDP diverted to rebel cause through extortion, etc.) out of this discussion because the rebels are probably extorting income from areas they control and thus which the government cannot affect.

If one considers the time points when $\frac{dR}{dt} < 0$ to be those at which the rebels are losing, then one can check the zero of $\frac{dR}{dt}$ for different values of f_R, c_R, and qi at the beginning of the period studied to find the critical S^* value at which the rebels are neither winning nor losing. Thus, for any $S > S^*$, the rebels will be losing. One can thus form the following function:

$$S^* = \phi(f_R, c_R, qi) = \frac{k_R(f_R + i_{R1}G + q_i\frac{G}{p}R)}{d_{RS1}c_R R}$$

Using the standard values of f_R, i_{R1}, and q_i, one finds that the Sri Lankan government would have needed no fewer than about 1.1716 million soldiers to have been defeating the Tigers from the very moment the IPKF left. Even if one assumes the rebel-associated costs and expenses are erroneously too cheap and takes half the funding and twice both the outfitting cost and the expense proportion of GDP/Capita, one has that the state would have needed at least about three hundred thousand soldiers, a slap in the face of those who say just a little more military might is all the state needs to put down the insurgency.

If one examines only f_R and leaves the other two parameters constant, then one can set $\phi = S_0$, where S_0 is the number of troops the Sri Lankan government had at the start of the observed period, Solving for f_R, one finds that the government could have been defeating the rebels from the start if only the rebels' foreign funding amounted to -5.3784×10^5 - in other words, if only foreigners were somehow stealing over five hundred thousand dollars a year from LTTE coffers, a not-quite realistic hope. What this means is that the rebels are bringing in enough money from extortion, etc. ($i_{R1}G$) that they could have fought on against an army of $S_0 = 22000$ government soldiers without a penny of foreign funds. Depressingly, this result is qualitatively resilient - indeed, once again doubling the rebel's costs only reduces the required pilfering to about 1.2 hundred thousand dollars.

However, if one assumes the government somehow cuts off all foreign funds ($f_R = 0$), then one finds that the government would have needed only about 86,000 troops to have been defeating the rebels - a number well under how many soldiers the government had even by the beginning of 1991. Doubling the assumed rebel costs here brings that number down to only about 30,000 soldiers. Thus, the importance of finances to the rebel cause, especially foreign funding, become starkly obvious. Though of course anyone could guess that less funding would have an adverse impact on an army's functionality, the model now makes quantitatively clear just how important - indeed, the difference between needing 86,000 versus 1.7 million soldiers to put down an insurgency is difficult to ignore.

One can thus vary any of the three parameters while holding the other two constant to calculate values for ϕ. Graphs of the results can be found in Figure 6.1. Furthermore, one can vary all three parameters simultaneously and graph the level plane $\phi = S_0$, as was done in Figure 6.2. Note that due to the limitations of the technology, neither of the graphs here is perfectly accurate but instead depicts $\phi = S_0 \pm S_{err}$ for different values S_{err} within acceptable bounds.

Observing the cornrows of Figure 6.2a and the more narrow trails of Figure 6.2b, one is mainly impressed by the overwhelming power of c_R - the cost of outfitting a new rebel fighter - in determining the number of soldiers needed. Indeed, whereas wide variation in the other two parameters leads to relatively little change, variations in c_R are mainly responsible for the sheathes of parameter space where ϕ drops well *below* the amount of troops the government had to start out the period examined. A quick glance at Figure 6.1b confirms this - whereas the other two parameters show linear relationships with S^*,

c_R relates rationally. Furthermore, it may not be entirely feasible for the government to increase q_i, as such would involve increasing food prices and other expenses of life, at least in the areas where rebels buy such goods. Thus, the marginal gain to the government is maximized by increasing c_R, at least up until about \$1500/rebel. One might consider this a mathematical argument to add to the long list of humanitarian arguments in favor of cracking down on the international arms trade.

6.2 Altered d_C

One need not hold to the assumptions of Section 6.1. However, it seems unlikely that either side could ever easily change cC (neither side has much power over how much economic damage a civilian death has) or either d_{SR1} or d_{RS1} (if either side could do something to kill more efficiently, then it would probably have already been done). But something over which both sides have some control is d_C, the proportion of civilians killed to fighters killed. Unfortunately, it might cross the strategic minds of either side to kill more civilians in the hopes of crippling the enemy's source of funding, and thus it might be worthwhile to vary this parameter.

Strangely, even after increasing d_C to a very large number (such that $\frac{dG}{dt}$ can be made zero with troop counts well within those allowed by Axiom 2), preliminary analysis reveals no reasonable equilibria. However, this could conceivably be due to flaws in the numerical methods used to find such equilibria (modified Newton's method). Nevertheless, it seems that equilibrium analysis again fails to be of use.

However, much like one can analyze the conditions under which the rebel forces

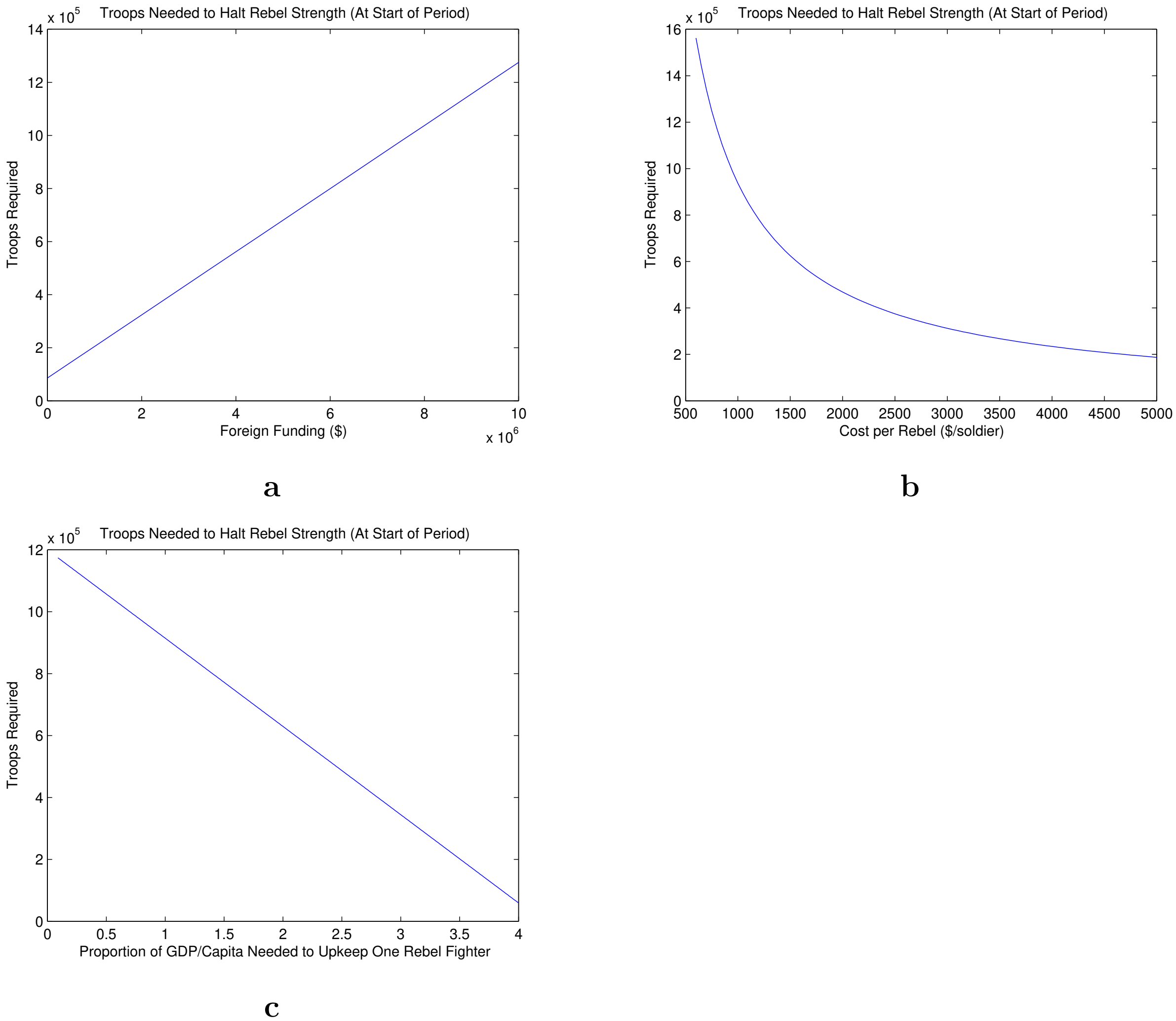


Figure 6.1: Troops Needed for Rebels to be Losing from the Start, Varying a) Foreign Funding (f_R); b) Rebel Creation Cost (c_R); c) Proportion of GDP/Capita Needed to Upkeep One Rebel (q_i)

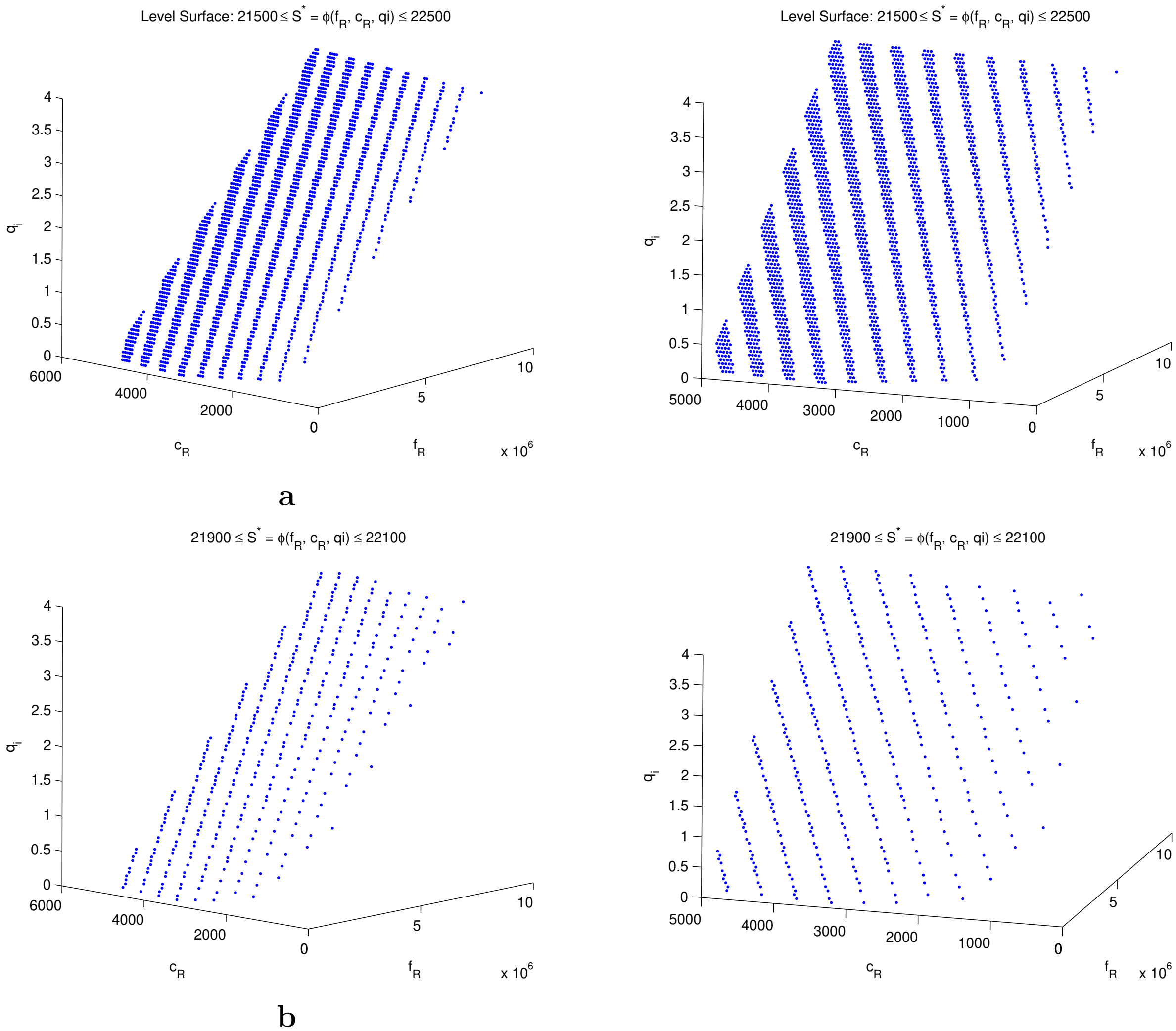


Figure 6.2: Loci of Points Satisfying Conditions: a) Lower Precision; b) Higher Precision

f_R ($\times 10^6$)	c_R	q_i	f_R ($\times 10^6$)	c_R	q_i
0.4545	3800	0.223	4.7727	4550	1.952
0.5682	4750	0.1787	5.3409	4250	2.218
1.0227	1200	0.7107	5.9091	3950	2.484
1.1364	4450	0.4447	6.0227	4900	2.4397
1.5909	900	0.9767	6.4773	3650	2.75
1.7045	4150	0.7107	6.5909	4600	2.7057
2.1591	600	1.2427	7.0455	3350	3.016
2.2727	3850	0.9767	7.1591	4300	2.9717
2.3864	4800	0.9323	7.7273	4000	3.2377
2.8409	3550	1.2427	7.8409	4950	3.1933
2.9545	4500	1.1983	8.2955	3700	3.5037
3.5227	4200	1.4643	8.4091	4650	3.4593
3.9773	650	1.9963	8.8636	3400	3.7697
4.0909	3900	1.7303	8.9773	4350	3.7253
4.2045	4850	1.686	9.5455	4050	3.9913
4.6591	3600	1.9963	9.6591	5000	3.947

Table 6.1: Loci of Points: $21990 \leq S^* = \phi(f_R, c_R, qi) \leq 22010$

decline, one can likewise analyze the conditions under which the national GDP declines. Setting $\frac{dG}{dt} = 0$, one finds that the number of civilian deaths in a year ($d_C[d_{SR1} + d_{RS1}]SR$) needed to completely counter all real economic growth is r/c_C, or about 91,426 for the parameter values calculated. Needless to say, this number cannot be taken as divine revelation or absolute truth - after all, it is based solely on averages derived from small numbers of deaths. Thus, one can easily imagine that as the number of deaths in a given year approaches something large (certainly less than 91,426, or .5% of the population), mass emmigration and the economic implosions that come with it would follow. However, one can take this number as a best case scenario and mentally adjust accordingly.

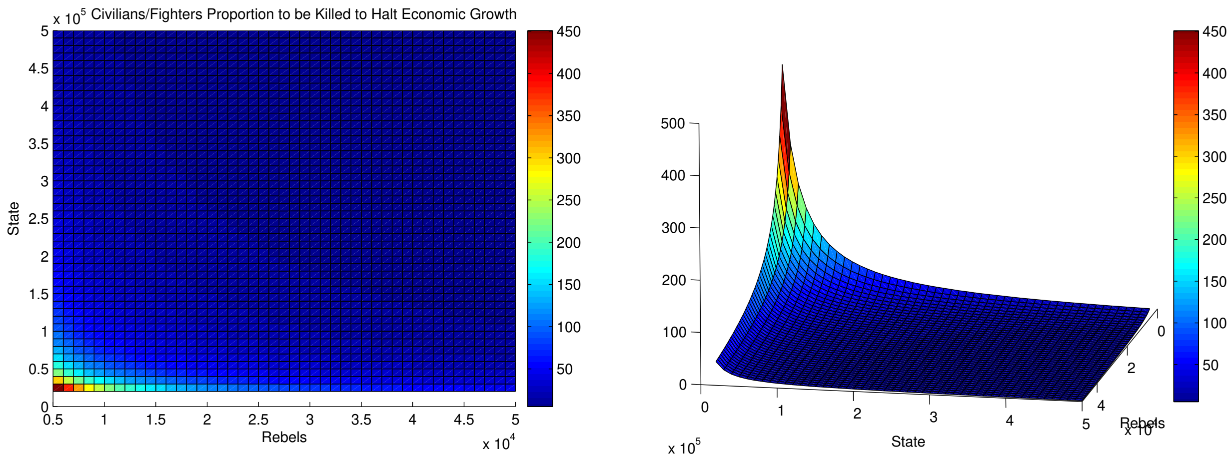
Figure 6.3a and Table 6.2 give some indication of just how little carnage is needed even in this best case scenario to create an economic halt. Even for the relatively modest values of 120,000 soldiers and 15,000 rebels, one need only about 25 civilians to die for each fighter killed for the economy to halt. Though this civilian/fighter proportion is far higher than that calculated for the model tests in Chapter 5, it is probably not too impressive (to use the word very loosely) compared to something like the Second Congolese War, especially if one factors in the indirect deaths resulting from disease and famine that inevitably accompany brutal civil wars. According to Figure 6.2b and Table 6.3, this means somewhere between 3,000 and 4,500 fighter deaths, a higher-than-usual but hardly inconceivable number.

The consequences of such an economic halt (or, if the targeting of civilians is higher than prescribed here, an economic recession) would be dire. For the state, it would

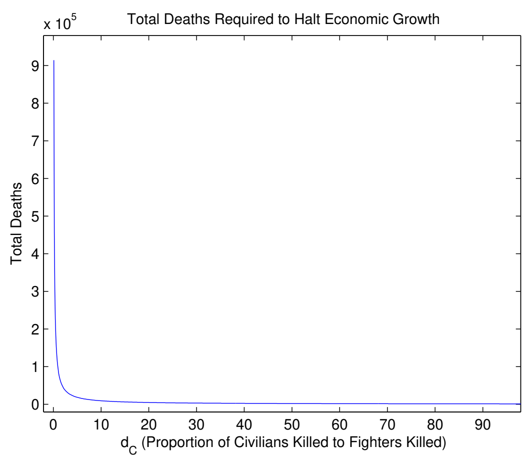
		R				
		5000	**15000**	**25000**	**35000**	**45000**
S	**20000**	454.4	151.5	90.9	64.9	50.5
	120000	75.7	25.2	15.1	10.8	8.4
	220000	41.3	13.8	8.3	5.9	4.6
	320000	28.4	9.5	5.7	4.1	3.2
	420000	21.6	7.2	4.3	3.1	2.4

Table 6.2: Values of dC for different S, R such that economic growth is halted (rounded)

mean first that its own main source of revenue (taxes) would be harmed even as the rebels' main source (foreign funds, at least in the period examined) remains reliable. Worse, the median income ($q_i \frac{G}{p}$) would stagnate or even decline, making it easier for the rebels to entice the poor and/or jobless into their ranks. The rebels, meanwhile, would have to deal with issues not addressed in this model but poignant nevertheless: namely, the possibility of a decline in their funds from diasporic Tamils if they become too closely associated with the killing of civilians, and the prospect of gaining a starving and impoverished Eelam even in the event of victory.



a



b

Figure 6.3: a) Number of civilians to be killed for every fighter killed for given number of soldiers and rebels to halt economic growth; b) Number of fighters to be killed for a given value of d_C to halt economic growth

d_C	**Fighter Deaths**
0.1	914259
10.1	9052
20.1	4549
30.1	3037
40.1	2280
50.1	1825
60.1	1521
70.1	1304
80.1	1141
90.1	1015

Table 6.3: Fighter deaths needed to halt economic growth (rounded)

Part III

Conclusions

Chapter 7

Conclusions

7.1 Summary of Results

7.1.1 Model Testing

After comparing all the models in Appendix A to actual data from the 1990 to 1994 period of the Sri Lankan civil war, I conclude that model A3 barely takes the title of most accurate. The Level 5 models surprisingly turned out to be spectacular failures and thus require reworking. All of the other models show reasonably good qualitative and quantitative results in most respects.

However, they all have the shortcomings of predicting an overabundance of rebel forces and also predicting ongoing (if slight) troop buildups even when the actual data shows a distinct leveling-off. Furthermore, as one would expect from a model based on differential equations, the models are only as good as the parameters they are fed.

Further cases must be tested to find which models are more or less susceptible to the damage caused by poor parameter values.

Nevertheless, if one can calculate decent parameter values, the models may prove very useful in making short-term inferences of civil war course and military capabilities. Such may prove invaluable to future research in either testing hypotheses about the relationship between military capabilities and other variables (such as decisions to sue for peace) or even making predictions about wars in progress.

7.1.2 Model Analysis

By implementing some basic axioms about S, R, G, and the parameters, and further assuming that d_C, c_C, d_{RS}, and d_{SR} are the values as calculated in Appendix B (a reasonable assumption, considering how difficult changing all but d_C would be for either side), one can prove that the model A3 has no equilibria and in fact lacks invariance of any kind. This means that equilibrium analysis within this restricted parameter and variable space is futile.

However, I demonstrated that one can nonetheless find crucial counterfactual information about how the war might have gone given different circumstances. More specifically, it seems that mere troop numbers are not enough to suppress the LTTE insurgency unless the government stops rebel finances.

If one allows d_C to vary, equilibria within the axiomatically defined range for S, R, and G are still scarce. However, analysis of the $\frac{dG}{dt}$ equation reveals that a sharp increase in the number of civilians killed could bring economic growth to a standstill.

7.2 Policy Recommendations

- Casually though terms such as "war on terror" may be thrown about, a mere massive application of troops would not have been enough to suppress the LTTE insurgency in 1990 and probably would not be enough now, either. The less glorious but more effective parts of counter-terrorism - namely, freezing terrorist assets and working with foreign governments to halt the flow of "charitable" donations - could be what really win the war.

- As raising the cost of outfitting new rebel fighters seems the most plausible and most effective means of cutting down rebel growth potential, governments should do more to stop the flow of black market weaponry throughout the world. By making weapons more scarce, governments can make weapons more expensive, and as a result make civil war more expensive.

- Both sides (but especially the government, considering its reliance on tax revenue) must take great pains to reign in the scale of civilian deaths. If civilian deaths get out of hand (as is inevitable if either or both sides begin targeting civilians out of demagoguery or even unprofessional conduct by individual soldiers), the result is inevitably a noticeable decline in economic performance. The analysis here shows that even in a best case scenario, such a decline would happen even with battles of moderate intensity if civilian deaths become a rule and not an exception.

7.3 Future Research

This paper has demonstrated that relatively simple mathematical models - mere three-dimensional systems of ordinary differential equations - can nonetheless make surprisingly accurate predictions about military capabilities and civil war casualties when given accurate parameter values. However, the limitations of the models suggest that adding a little complexity could be beneficial.

At the very least, my recommendations in Section 5.2 - namely, fixing the Level 5 models, improving parameter values, and modeling the ever-increasing prices of weaponry - must be implemented. Furthermore, it would behoove future researchers to test the improved models' accuracies in predicting wars other than Sri Lanka's to verify their universality and find their limits given the assumptions upon which they were built (see Section 4.1). The greatest reward of such work would be a vast body of parameter values across different cases from which analysts can infer the nature of the values parameters take in general. With this understanding, it may be possible to approximate parameters in cases where accurate data is hard to find - such as with wars that have not yet happened or have only just begun.

But why stop with only mild improvements to the models presented here? After all, ordinary differential equations are but one of the advanced mathematical modeling methods available to the intrepid international relations analyst, and indeed are perhaps the simplest. Though less well-understood than their ordinary counterparts, partial differential equations offer a powerful means of breaking the problem into its component causes and adding independent variables other than just time. Delay differential equations, meanwhile, can reduce the instantaneousness that renders other calculus-based models questionable in their accuracy in politics. Finally, one need not restrict oneself to merely deterministic models, as stochastic modeling methods such as Poisson processes, continuous-time Markov chains, and stochastic differential equations should be explored as well.

Indeed, all possible methods of understanding how wars can be stopped and casualties minimized must be explored. After all, the makers of war use all possible methods to ply their trade - can those who wish to make peace afford to do any less?

Part IV

Appendices

Appendix A

Models Tested

Here are the complete equations of all the models tested in this study. Note that parameters of similar function but different derivation and/or dimensions have similar names and are differentiated by numbers.

A2

$$
\begin{aligned}
\frac{dS}{dt} &= c_{S1}^{-1} k_S (f_S + i_S G - x_S S) - d_{SR1} SR \\
\frac{dR}{dt} &= c_R^{-1} k_R (f_R + i_{R1} G - q_i \frac{G}{p} R) - d_{RS1} RS \\
\frac{dG}{dt} &= rG
\end{aligned}
$$

B2

$$\begin{aligned}
\frac{dS}{dt} &= c_{S1}^{-1}k_S(f_S + i_S G - x_S S) - d_{SR2}R\frac{S}{S+1}\\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R1}G - q_i\frac{G}{p}R) - d_{RS1}RS\\
\frac{dG}{dt} &= rG
\end{aligned}$$

A3

$$\begin{aligned}
\frac{dS}{dt} &= c_{S1}^{-1}k_S(f_S + i_S G - x_S S) - d_{SR1}SR\\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R1}G - q_i\frac{G}{p}R) - d_{RS1}RS\\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR1}SR + d_{RS1}RS\right)\right]
\end{aligned}$$

B3

$$\begin{aligned}
\frac{dS}{dt} &= c_{S1}^{-1}k_S(f_S + i_S G - x_S S) - d_{SR2}R\frac{S}{S+1}\\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R1}G - q_i\frac{G}{p}R) - d_{RS1}RS\\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR2}R\frac{S}{S+1} + d_{RS1}RS\right)\right]
\end{aligned}$$

A4.1

$$\begin{aligned}
\frac{dS}{dt} &= c_{S1}^{-1}k_S(f_S + i_S G - x_S S) - d_{SR1}SR\\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R1}G - q_i\frac{G}{p}R) - d_{RS2}RS\frac{R^{es}}{R^{es} + (\frac{S}{h_{R1}/p})^{es}}\\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR1}SR + d_{RS2}RS\frac{R^{es}}{R^{es} + (\frac{S}{h_{R1}/p})^{es}}\right)\right]
\end{aligned}$$

A4.2

$$
\begin{aligned}
\frac{dS}{dt} &= c_{S1}^{-1}k_S(f_S + i_S G - x_S S) - d_{SR1}SR \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R1}G - q_i\frac{G}{p}R) - d_{RS2}RS \times h_{R2}\frac{S}{p} \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR1}SR + d_{RS2}RS \times h_{R2}\frac{S}{p}\right)\right]
\end{aligned}
$$

A4.3

$$
\begin{aligned}
\frac{dS}{dt} &= c_{S1}^{-1}k_S(f_S + i_S G - x_S S) - d_{SR1}SR \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R1}G - q_i\frac{G}{p}R) - d_{RS2}RS\left[\frac{R^{es}}{R^{es} + (\frac{S}{2})^{es}} + h_{R3}\frac{S}{p}\right] \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR1}SR + d_{RS2}RS\left\{\frac{R^{es}}{R^{es} + (\frac{S}{2})^{es}} + h_{R3}\frac{S}{p}\right\}\right)\right]
\end{aligned}
$$

B4.1

$$
\begin{aligned}
\frac{dS}{dt} &= c_{S1}^{-1}k_S(f_S + i_S G - x_S S) - d_{SR2}R\frac{S}{S+1} \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R1}G - q_i\frac{G}{p}R) - d_{RS2}RS\frac{R^{es}}{R^{es} + (\frac{S}{h_{R1}/p})^{es}} \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR2}R\frac{S}{S+1} + d_{RS2}RS\frac{R^{es}}{R^{es} + (\frac{S}{h_{R1}/p})^{es}}\right)\right]
\end{aligned}
$$

B4.2

$$
\begin{aligned}
\frac{dS}{dt} &= c_{S1}^{-1}k_S(f_S + i_S G - x_S S) - d_{SR2}R\frac{S}{S+1} \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R1}G - q_i\frac{G}{p}R) - d_{RS2}RS \times h_{R2}\frac{S}{p} \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR2}R\frac{S}{S+1} + d_{RS2}RS \times h_{R2}\frac{S}{p}\right)\right]
\end{aligned}
$$

B4.3

$$\begin{aligned}
\frac{dS}{dt} &= c_{S1}^{-1}k_S(f_S + i_S G - x_S S) - d_{SR2}R\frac{S}{S+1} \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R1}G - q_i\frac{G}{p}R) - d_{RS2}RS\left[\frac{R^{es}}{R^{es} + (\frac{S}{2})^{es}} + h_{R3}\frac{S}{p}\right] \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR2}R\frac{S}{S+1} + d_{RS2}RS\left\{\frac{R^{es}}{R^{es} + (\frac{S}{2})^{es}} + h_{R3}\frac{S}{p}\right\}\right)\right]
\end{aligned}$$

A5.1

$$\begin{aligned}
\frac{dS}{dt} &= c_{S2}^{-1}k_S(f_S + i_S\frac{S}{S+R}G - x_S S) - d_{SR1}SR \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R2}\frac{R}{S+R}G - q_i\frac{G}{p}R) - d_{RS2}RS\frac{R^{es}}{R^{es} + (\frac{S}{h_{R1}/p})^{es}} \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR1}SR + d_{RS2}RS\frac{R^{es}}{R^{es} + (\frac{S}{h_{R1}/p})^{es}}\right)\right]
\end{aligned}$$

A5.2

$$\begin{aligned}
\frac{dS}{dt} &= c_{S2}^{-1}k_S(f_S + i_S\frac{S}{S+R}G - x_S S) - d_{SR1}SR \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R2}\frac{R}{S+R}G - q_i\frac{G}{p}R) - d_{RS2}RS \times h_{R2}\frac{S}{p} \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR1}SR + d_{RS2}RS \times h_{R2}\frac{S}{p}\right)\right]
\end{aligned}$$

A5.3

$$\begin{aligned}
\frac{dS}{dt} &= c_{S2}^{-1}k_S(f_S + i_S\frac{S}{S+R}G - x_S S) - d_{SR1}SR \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R2}\frac{R}{S+R}G - q_i\frac{G}{p}R) - d_{RS2}RS\left[\frac{R^{es}}{R^{es} + (\frac{S}{2})^{es}} + h_{R3}\frac{S}{p}\right] \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR1}SR + d_{RS2}RS\left\{\frac{R^{es}}{R^{es} + (\frac{S}{2})^{es}} + h_{R3}\frac{S}{p}\right\}\right)\right]
\end{aligned}$$

B5.1

$$
\begin{aligned}
\frac{dS}{dt} &= c_{S2}^{-1}k_S(f_S + i_S\frac{S}{S+R}G - x_S S) - d_{SR2}R\frac{S}{S+1} \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R2}\frac{R}{S+R}G - q_i\frac{G}{p}R) - d_{RS2}RS\frac{R^{es}}{R^{es} + (\frac{S}{h_{R1}/p})^{es}} \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR2}R\frac{S}{S+1} + d_{RS2}RS\frac{R^{es}}{R^{es} + (\frac{S}{h_{R1}/p})^{es}}\right)\right]
\end{aligned}
$$

B5.2

$$
\begin{aligned}
\frac{dS}{dt} &= c_{S2}^{-1}k_S(f_S + i_S\frac{S}{S+R}G - x_S S) - d_{SR2}R\frac{S}{S+1} \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R2}\frac{R}{S+R}G - q_i\frac{G}{p}R) - d_{RS2}RS \times h_{R2}\frac{S}{p} \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR2}R\frac{S}{S+1} + d_{RS2}RS \times h_{R2}\frac{S}{p}\right)\right]
\end{aligned}
$$

B5.3

$$
\begin{aligned}
\frac{dS}{dt} &= c_{S2}^{-1}k_S(f_S + i_S\frac{S}{S+R}G - x_S S) - d_{SR2}R\frac{S}{S+1} \\
\frac{dR}{dt} &= c_R^{-1}k_R(f_R + i_{R2}\frac{R}{S+R}G - q_i\frac{G}{p}R) - d_{RS2}RS\left[\frac{R^{es}}{R^{es} + (\frac{S}{2})^{es}} + h_{R3}\frac{S}{p}\right] \\
\frac{dG}{dt} &= rG - c_C G\left[d_C\left(d_{SR2}R\frac{S}{S+1} + d_{RS2}RS\left\{\frac{R^{es}}{R^{es} + (\frac{S}{2})^{es}} + h_{R3}\frac{S}{p}\right\}\right)\right]
\end{aligned}
$$

Appendix B

Parameters

Paramter	Value	Parameter (cont.)	Value
c_C	5.87908E-07	h_{R1}	2.1493e+008
c_R	800	h_{R2}	38.9550
c_{S1}	9946.399441	h_{R3}	38.6043
c_{S2}	1759.1	i_{R1}	0.000116812
d_C	0.466787853	i_{R2}	0.048507224
d_{RS1}	1.47155E-06	i_S	0.0340625
d_{RS2}	4.78154E-06	k_R	0.7
d_{SR1}	5.40636E-07	k_S	1
d_{SR2}	0.082038835	p	17113532
e_S	2.947448552	q_i	0.098261546
f_R	9128789.842	r	0.05375
f_S	0	x_S	2456.77539

B.1 c_C

[Dimensions] $= \frac{GDP\ lost/Total\ GDP}{year} / Civilians\ Killed = \frac{1}{people \cdot time}$

Constant price GDP growth rates for Sri Lanka for the years 1986 to 2005 were drawn from the GDP annual growth rate, 1990 prices, US$ (UN estimates) series of the U.N. Common Database (United Nations Statistics Division, 2008). Using the intensity indicators from the Uppsalla Conflict Data Project (Department of Peace and Conflict Research, Uppsala University, 2008), each year was classified as either a war year or a peace year (either "No" or "Minor" in UCDP parlance was coded as peace). The difference of the average of the peace years and the average of the war years was then divided by the total number of civilians killed in this time span, as calculated by taking the product of the total number of people killed (Farrell, 2001) and the average proportion of civilian deaths to total deaths as calculated using similar data and methods as d_C was calculated.

B.2 c_R

[Dimensions] $= \frac{cost}{new\ rebel\ fighter} = \frac{dollars}{people}$

According to (Bazzi, 2001), AK-47s can be imported from China for $400. It is assumed for lack of better information that inflation is countered by decreases in gun production prices (especially on such a low-tech model) and thus the price in 1990 was the same. Lacking any better information on the cost of training a new soldier, it was also assumed that it costs as much to learn the use of a weapon as it does to buy the weapon. Thus, the total cost of outfitting a new rebel is $2 \cdot \$400 = \$800$.

B.3 c_{S1}

[Dimensions] $= \frac{cost}{new\ soldier} = \frac{dollars}{people}$

Taking any model of level less than five, and one can solve a slightly more generalized form of the $\frac{dS}{dt}$ equation for: $c_{S1} = \frac{f_S + i_S G - x_S S}{[change\ in\ troop\ levels] - [troops\ killed]}$. Take f_S, i_S, and x_S as calculated below. Calculate the change in troop levels by subtracting the 1990 value from the 1991 value of troop levels from the Armed forces, weapons holdings and employment in arms production table of the Facts on International Relations and Security Trends database (International Relations and Security Network and Stockholm International Peace Research Institute, 2008). The number of troops killed in 1990 was approximated from information in (Morris, 1991). The midpoint method was used to find values for S and G: thus, S was set as the change in troop levels divided by two, and using the value for r as calculated below, one can approximate the midpoint value of G as $G_0 \cdot \exp(.5r)$. Note that the military expenditure is approximated with $i_S G$ rather than the actual 1990 value because the goal is to find an average expenditure number across *all years* rather than simply 1990, and the percentage military expenditure number i_S captures this cross-year average better than the actual expenditure in 1990 (which is somewhat smaller than any of the years after it).

B.4 c_{S2}

[Dimensions] $= \frac{cost}{new\ soldier} = \frac{dollars}{people}$

This was calculated exactly as c_{S1} was, except $i_S G$ is now multiplied by $\frac{S}{S+R}$. The value for R was taken to be simply 5000, which was first found in the Syndey Morning Herald (Sydney Morning Herald, 1990) and corroborated by The Independent (McCarthy, 1990). As both of these articles came out fairly late in the year, this number was taken as the general midpoint value despite that it was also used as the initial value (so we assume rebel numbers did not change dramatically in 1990).

B.5 d_C

[Dimensions] $= \frac{civilians\ killed}{fighter\ killed} = 1$

Data for the number of civilians killed and the total number of people killed were drawn from the Armed Conflict Database (International Institute for Strategic Studies, 2008) for 2005, 2006, and the periods January to March, April to June, and July to September from the year 2007. The number of combatants killed was duly calculated, and for each period the ratio of civilians killed to fighters killed was calculated. These ratios were averaged for the final value of d_C.

B.6 d_{RS1}

[Dimensions] $= \frac{rebels\ killed/total\ rebels}{soldiers}/year = \frac{1}{people \cdot time}$

Unfortunately, data for the number of soldiers and rebels killed in addition to the total number of soldiers and rebels involved was only available for two time periods: six months of 1990 and the entire year of 2006. This parameter was calculated using data from the latter drawn from the Armed Conflict Database (International Institute for Strategic Studies, 2008) using the following formula: $d_{RS1} = \frac{rebels\ killed/total\ rebels}{total\ soldiers}/1\ year$, where the total number of rebels was the number reporting in at year's end plus the number killed and likewise for the total number of soldiers.

B.7 d_{RS2}

[Dimensions] $= \frac{rebels\ killed/total\ rebels}{soldiers}/year = \frac{1}{people \cdot time}$

Calculated similarly to d_{RS1} using soldier counts data from the Armed forces, weapons holdings and employment in arms production table of the Facts on International Relations and Security Trends database (International Relations and Security Network and Stockholm International Peace Research Institute, 2008), rebel counts data from The Sydney Morning Herald (Sydney Morning Herald, 1990) that was corroborated by The Independent (McCarthy, 1990), and soldier and rebel deaths data from The Guardian (Morris, 1991). Note that the formula from d_{RS1} is slightly different here, as everything is divided by 0.5 years rather than one year. This intense period at the resumption of hostilities after the departure of the Indian Peacekeeping Force is taken as an upper bound for the amount of carnage possible, and thus serves as the coefficient of the modified formulas of the level 4 and 5 $\frac{dR}{dt}$ equations.

B.8 d_{SR1}

[Dimensions] $= \frac{soldiers\ killed/total\ rebels}{soldiers}/year = \frac{1}{people \cdot time}$

This was calculated exactly like d_{RS1} was, except with the number of soldiers killed in the numerator rather than the number of rebels killed.

B.9 d_{SR2}

[Dimensions] $= \frac{soldiers\ killed}{rebel}/year = \frac{1}{time}$

This was calculated using the same data as d_{SR1}, and used the same formula except that the total number of soldiers was excluded from the denominator.

B.10 e_S

[Dimensions] $= 1$

As this term is a general indicator of how well outfitted an army is (1 = the United States military, 3 or greater = a marching band carrying water guns), it was calculated using the following formula: $e_S = 3 - 2 \cdot \frac{Sri\ Lankan\ military\ expenditure/total\ troop\ count}{U.S.\ military\ expenditure/U.S.\ troop\ count}$. Data for both Sri Lankan and U.S. expenditures were drawn from the Armed forces, weapons holdings and employment in arms production table of the Facts on International Relations and Security Trends database (International Relations and Security Network and Stockholm International Peace Research Institute, 2008), and both countries' military expenditures were pulled from the Stockholm International Peace Research Institute's military expenditure database (Stockholm Institute of Peace Research, 2008). Data for the each of the years 1990 to 1994 were fed into the formula, and the results were averaged for the final value of e_S.

B.11 f_R

[Dimensions] = $\frac{outside\ funding\ collected}{year} = \frac{dollars}{time}$

The Tamil Tigers fund their war largely through financial support from the various Tamil diasporic communities throughout the world. According to the World Policy Journal, the Tigers collect on average 3 million 2003 U.S. dollars from 45000 Swedish Tamils. Using consumer price index numbers from the Consumer Price Index, 2000 = 100 (IMF) table of the United Nations Common Database (United Nations Statistics Division, 2008), one can find an average contribution of 1990 U.S. dollars per person. Using GDP and population data from the Population, official mid-year estimates series and the GDP at market prices, 1990 prices, US$ series of the UN Common Database (United Nations Statistics Division, 2008), one can calculate the 1990 GDP/capita of Sweden in 1990 U.S. dollars. Using this, one can calculate the average percent of GDP per capita contributed per Swedish Tamil. Assuming this number to be invariant across all countries, one can similarly calculate 1990 GDP per capita numbers for Canada and India (the two largest diasporic destinations) and multiply by the percentage just calculated to find the average contribution in 1990 U.S. dollars per Canadian and Indian Tamil.

These three country-specific average contributions per Tamil can be multiplied by numbers for the sizes of Tamil diasporic communities in the early 1990s in Canada (Morris and Cruez, 1995), Switzerland (Swiss Review of World Affairs, 1995), and India (The Economist, 1993) to find the foreign funding coming to the LTTE from each of those countries. Adding these totals to the (deflated) estimated total coming from Britain (Morris and Cruez, 1995), one has a rough estimate of the total diasporic contribution to the LTTE war chest.

B.12 f_S

[Dimensions] $= \frac{outside\ funding\ collected}{year} = \frac{dollars}{time}$

Although the Sri Lankan government was receiving outside funding from various organizations and governments (Edirisinghe, 1994), most of this was earmarked for specific projects (dams, etc.) and thus could not be directly invested in the army. Any shifting of other budgetary money into the military would be reflected in the military expenditure numbers used to calculate i_S and thus would only be double-counted here. Thus, f_S was assumed to be 0.

B.13 h_{R1}

[Dimensions] = people

This mysterious "hiding" parameter does not yield up its real world meaning easily. Nevertheless, one can calculate its value by solving the kill term in the $\frac{dR}{dt}$ equation of any of the XX.1 models for h_{R1} and substituting Armed Conflict Database (International

Institute for Strategic Studies, 2008) numbers in for the various terms. Thus, one has $h_{R1} = \frac{Sp}{R}(\frac{d_{RS2}SR}{[killed]} - 1)^{-1/e_S}$ where S is the number of troops at year's end plus the number of troops killed in 2006 from the ACD, R is similarly determined from ACD rebel data, p is the population of Sri Lanka in 2006 as drawn from the official mid-year estimates series of the U.N. Common Database (United Nations Statistics Division, 2008) and $[killed]$ is taken to be the number of rebels reported killed in 2006 by the ACD.

B.14 h_{R2}

[Dimensions] = 1

Another "hiding" parameter, this was calculated much as h_{R1} was but with a different formula, as results from solving the rebel kill term in any of the XX.2 models: $h_{R2} = \frac{p \cdot [killed]}{d_{RS2}RS^2}$.

B.15 h_{R3}

[Dimensions] = 1

The third "hiding" parameter was calculated much like the others, this time solving the rebel kill term in any of the XX.3 models: $h_{R3} = \frac{p}{S}(\frac{[killed]}{d_{RS2RS}} - \frac{R^{e_S}}{R^{e_S} + (S/2)^{e_S}})$.

B.16 i_{R1}

[Dimensions] = $\frac{dollars\ extorted}{GDP}/year = \frac{1}{time}$

Lacking any better information on the LTTE's ability to extract money from the national economy via extortion and drug trafficking, I relied largely on a BBC summary of a Sri Lankan radio broadcast (British Broadcasting Corporation, 1995). According to the report, eighty million Sri Lankan rupees were found to have been extorted from businesses in Colombo in 1995. Dividing this by the GDP at current prices in local currency for 1995 from the GDP at market prices, national currency, current prices (UN estimates) series of the U.N. Common Database (United Nations Statistics Division, 2008), one has a very rough estimate of i_{R1}.

B.17 i_{R2}

[Dimensions] $= \frac{dollars\ extorted}{GDP}/year = \frac{1}{time}$

As this term represents the ideal amount the Tamil Tigers would extort if they controlled the entire country, it was taken to be simply equal to what the Sri Lankan army does take (i.e. i_S).

B.18 i_S

[Dimensions] $= \frac{military\ expenditure}{GDP}/year = \frac{1}{time}$

Numbers for Sri Lankan military expenditure as a percentage of GDP as drawn from Stockholm International Peace Research Institute's military expenditure database (Stockholm Institute of Peace Research, 2008) for the years 1988 to 2003 were averaged.

B.19 k_R

$$[\text{Dimensions}] = \frac{\textit{funds that reach intended destination}}{\textit{total funds}} = 1$$

Lacking any substantive information about terrorist funding, one can imagine just how much is known about terrorist corruption. One can make some approximation as follows: in normal militaries, roughly 15% of military spending is lost to corruption (Gupta et al., 2000). However, normal militaries are subject to budgetary review by some higher authority, whereas a guerrilla terrorist group like the LTTE is not. Indeed, Stern (2003) notes that terrorist leaders often live in mansions, sparing no expense for their own creature comforts even as their soldiers live from hand to mouth. Thus, one might assume that corruption is twice as bad in a terrorist group as in a military. Thus, $k_R = 1 - .15 \cdot 2$.

B.20 k_S

$$[\text{Dimensions}] = \frac{\textit{funds that reach intended destination}}{\textit{total funds}} = 1$$

Although corruption is undoubtedly rampant in the Sri Lankan military, as it is in militaries around the world, since the figures for c_S and x_S were calculated using actual expenditure numbers and troop counts, it seems likely that the costs of corruption are incorporated into the costs of outfitting and upkeep. Thus, $k_S = 1$ to avoid double-counting corruption.

B.21 p

[Dimensions] = people

It was assumed that over the short term, the birth rate and death rate (most notably, deaths from the war) would not amount to a very large overall change in the population. Thus, p was taken as simply the 1990 population figure for Sri Lanka from the UN Population Division's Annual Estimates series of the U.N. Common Database (United Nations Statistics Division, 2008).

B.22 q_i

[Dimensions] = $\frac{median\ income}{GDP/capita} = 1$

It was assumed that the median income of Sri Lanka did not vary overmuch from 1990 to 2006, an assumption probably foolish but nonetheless necessitated by the lack of timely data. The median income earner's income for 2006 was deflated to 1990 U.S. dollars using consumer price index numbers for the Sri Lankan Rupee from the Colombo Consumer Price Index (Department of Census and Statistics, Sri Lanka, 2008), and this in turn was converted to U.S. dollars using the 1990 exchange rate from the Exchange rate, national currency per US$, end of period (IMF) series from the U.N. Common Database. Finally, this number was divided by Sri Lanka's per capita GDP in 1990 U.S. dollars as taken from the GDP per capita, current prices, US$ (UN estimates) series from the U.N. Common Database.

B.23 r

$[\text{Dimensions}] = \frac{increase\ in\ GDP}{current\ GDP}/year = \frac{1}{time}$

Constant price GDP growth rates for Sri Lanka for the years 1986 to 2005 were drawn from the GDP annual growth rate, 1990 prices, US$ (UN estimates) series of the U.N. Common Database (United Nations Statistics Division, 2008). Using the intensity indicators from the Uppsalla Conflict Data Project (Department of Peace and Conflict Research, Uppsala University, 2008), each year was classified as either a war year or a peace year (either "No" or "Minor" in UCDP parlance was coded as peace). The averages for the peace years was taken as r.

B.24 x_S

$[\text{Dimensions}] = \frac{upkeep\ cost}{one\ existing\ soldier} = \frac{dollars}{person}$

To calculate x_S, a distinction had to be made somehow between the amount of money spent on creating new units and maintaining units already in service. As specific data on the Sri Lankan military's expenditure allocations could not be found, such had to be constructed using data on the United States military and the Chinese military. Using a 2007 military expenditure budget report (Office of the Under Secretary of Defense (Comptroller), 2006), expenditures on unit creation were distinguished from expenditures on unit maintenance as follows: spending on the Reserve and the National Guard was assumed to be training and thus creation, while all other personnel spending assumed to be salary and thus maintenance; Operations and Management, Military

Construction, Family Housing, and Working Capital Funds were all classified as maintenance while procurement and RDT&E was classified as creation. From this, one can calculate the percentage spent on creation versus maintenance.

Meanwhile, GlobalSecurity.org provides percentages of Chinese defense budget spending. Using the percentage of personnel spending that went into training in the U.S. case, one can split the Chinese personnel spending up and combine the part approximated as training expenses with the percent spent on Equipment for the total percentage spent on creation, and similarly combine the rest of personnel spending with Maintenance of Activities spending to approximate the total percentage spent on maintenance.

Averaging out the percentages calculated in the U.S. case with those calculated for China (neither of which varied greatly from the other), one can calculate numbers for percentage spent on maintenance versus percentage spent on creation hopefully general to militaries across the world.

The approximated percentage spent on maintenance was multiplied by the total military expenditure for each year from the SIPRI military expenditure database as scaled to 1990 U.S. dollars by consumer price index numbers from the Consumer Price Index, 2000 = 100 (IMF) table of the United Nations Common Database (United Nations Statistics Division, 2008). The resulting amount spent on maintenance for each year was divided by the number of troops in the Sri Lankan military for that year as reported in the Armed forces, weapons holdings and employment in arms production table of the Facts on International Relations and Security Trends database (International Relations and Security Network and Stockholm International Peace Research Institute, 2008).

The resulting average upkeep costs per soldier were averaged out for the final value of x_S.

Appendix C

Testing Data

C.1 Armed Forces Personnel

Data for the size of Sri Lanka's military for the beginning of each year from 1990 to 1995 were drawn from the Armed forces, weapons holdings and employment in arms production table of the Facts on International Relations and Security Trends database (International Relations and Security Network and Stockholm International Peace Research Institute, 2008).

C.2 Gross Domestic Product

Data for Sri Lanka's Gross Domestic Product for the beginning of each year from 1990 to 1995 were drawn from the GDP at market prices, 1990 prices, US$ (UN estimates) series of the United Nations Common Database (United Nations Statistics Division,

Year	Armed Forces Personnel
1990	22000
1991	100000
1992	110000
1993	110000
1994	126000
1995	125000

Table C.1: State Troop Values

Year	GDP (1990 US$)
1990	8,204,356,700
1991	8,599,884,900
1992	8,976,470,200
1993	9,597,247,500
1994	10,139,414,700
1995	10,700,060,200

Table C.2: GDP Values

Year	**Deaths**
1990	4500
1991	7252
1992	7253
1993	3000
1994	2000

Table C.3: Deaths (Yearly)

Year	**Cumulative Deaths**
1990	0
1991	4500
1992	11752
1993	19005
1994	22005
1995	24005

Table C.4: Deaths (Cumulative)

2008).

C.3 Deaths (Yearly)

Data for the number of people killed in the government versus Liberation Tigers of Tamil Eelam struggle for each year from 1990 to 1994 were drawn from the yearly conflict deaths series of the Center for the Study of Civil War Battle Deaths Dataset version 2.0 (Lacina and Gleditsch, 2005).

C.4 Deaths (Cumulative)

Data for the cumulative number of people killed at the beginning of each year during the 1990-1995 period were calculated from the numbers for yearly battle deaths (see section C.3).

C.5 Civilian Cost Evaluation

This was calculated to be 20000 for the 1990-1994 period as follows: according to the IPS-Inter Press Service (IPS-Inter Press Service, 1994), 30000 civilians were recorded killed throughout the war at the end of 1994; according to the Sunday Herald (Skelton, 1990), 6000 civilians were killed during the tenure of the Indian Peace Keeping Force; finally, according to United Press International (Athas, 1987), 4000 civilians were killed before the Indian Peace Keeping Force. Thus, the number of civilians killed from 1900 to 1994 equals 30000 - (6000 + 4000) = 20000. Note that this number is quite possibly skewed rather high, as taking this number in combination with the total number killed as calculated in Table C.4 implies that only 4005 combatants were killed during the entire four years.

C.6 Outcome

Drawing on descriptions from (Keerawella and Samarajiva, 1994), (CanagaRetna, 1996), and (Schaffer, 1995), one can form a qualitative conception of the war's status from the middle to the end of 1994.

Though the numbers above indicate that the Sri Lankan military grows steadily during this period, one can conclude from Chandrika Kumaratunga's electoral sweep on a genuine peace platform indicates that the war had no end in site at the end of 1994. Indeed, considering that the words "peace" or "reconciliation" with respect to Tamils were always the rhetorical equivalent of self-impalement in Sri Lankan political history, the war must have become truly brutal after the IPKF departed.

For their part, though the rebels had recently driven the army to its "most devastating defeat" in 1993 (Keerawella and Samarajiva, 1994), they must have been facing hard times and a prospect of war without end as well. Otherwise, they would not have agreed to peace talks with the Kumaratunga administration - indeed, even if one assumes they only acquiesced to negotiations so as to buy time to reload their weapons (as certain elements in the military claimed from the very start of negotiations), that still acknowledges that they needed time and were not exactly poised on the brink of overthrowing the government.

Thus, one would expect the following to be the qualitative state of the war at the end of the period studied:

- No great change in relative strength
- No particularly “exciting” dynamics
- A long period of stagnation in the strength of both sides.

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