Walrasian Solutions Without Utility Functions

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SUMMARY:

This note reviews consumers' preference orderings in economics and shows that irrationality is a poor explanation for apparent violations of some axioms of order. Apparent violations seem to be better explained by the fact that consumers' utility functions, if they exist at all, might not even belong to the class of quasi-concave functions. However, the main task of markets is the determination of equilibrium price vectors. The note shows in addition that, in Walrasian structures, quasi-concave utility functions are unnecessary for the determination of equilibrium price vectors.

INTRODUCTION

The "construct" known as Neoclassical Economics rests on two fundamental assumptions, namely, rational consumers and technologically efficient producers. Rational behavior and infinite foresightedness were like the *explanans* to the early pioneers of the "construct", while utility maximization was the *explanandum.* Not everyone saw it as such, however, and even today, many still disagree. Cognitive psychologists such as Alberton (1976), Nisbett and Ross (1980), etc. would object to the premise, and a number of economists, in particular the followers of Herbert Simon (1979, 1982), would question the conclusion, probably seeing it as unfounded. There is evidence that very early on, scientists such Henri Poincaré suggested to Léon Walras, one of the forefathers of the construct, to avoid exaggerated claims and unobservables (see, Jaffé, 1965). But, he, Walras, paid no heed to such advices at the end; for him, the economic agent was rational, therefore, he or she had to maximize his or her utility. That is pretty much were things remained until the 1940s and 1950s when theorists such as Wold (1943), Arrow (1951, 1963), among others, decided to revisit one of the pillars of the construct. However, instead of asking whether or not utility maximization was necessary to the determination of a competitive general equilibrium price vector, they formalized the concept of "rationality" in order to put the utility function on a more solid footing, thus turning neoclassical economics into nothing but a maximizing discipline.

As is now well understood, all behaviors are rational according to some normative criteria. Therefore, the modern theorists had to begin by spelling out the orderings as the axiomatic criteria by which rationality was to be defined. Following that effort, the substance of "rationality" in economics came to be expressed as agents' preference orderings. In turn, preference orderings gave substance to utility functions that must be characterized by open and strictly convex upper contour sets, necessary for utility maximization. Instead of pushing the construct to a higher level of generality, as was expected, this foray into rationality seems to give rise to new troubles. In many experimental designs, experimenters claim to have observed violations of some basic axioms of rationality or outright falsifications of the concept of utility maximization in some cases (sic). The main reported violations are: preference reversals, inconsistent choices, and the conclusions of some experiments such as the so-called "Ultimatum Game"(1), etc..

Preference reversal (see note 1) is intuitively obvious, but for the present purpose, inconsistency and irrationality need definitions. Inconsistency is present when a choice is declared desirable and for no apparent reasons it is not selected, while irrationality points to behaviors for which there are no predictive rules. It is then by these that economists judge the observed violations. Whether or not agents maximize utility is never examined. In this paper, I will try to leave no stone unturned.

To begin with, let us recall that attempts to substantiate the existence of well-defined utility functions are fraught with a number of mathematical and biological potential pitfalls. The mathematical pitfalls center on the followings. To maximize, one must turn to the family of concave functions and their indifference maps, but many members of the family, whether concave or not, produce the same indifference maps. As economists claim to be 'ordinalists', they need quasi-concavity and monotonicity over the whole ranges of such functions. This means that they need strictly quasi-concave utility functions. Strictly quasi-concave functions are the class of quasi-concave functions whose contours contain no linear segments, and their monotone increasing transformations remain strictly quasi-concave. But the trouble is that strictly quasi-concave functions need not be continuous. If the opportunity set of the individual agent is, X (defined more formally below), then the utility function, u, is the mapping: u (.): $X \rightarrow R$ (R is a real number) which, on the one hand, is strictly quasi-concave if X is convex. On the other, u (.) is not necessarily continuous if X is not an open set. In other words, if the domain of u (.) is convex, continuity cannot be inferred from concavity; it must be assumed separately. In order to have convex and open upper contour sets (UC), necessary for maximization, X must be finite and open, and u (.) must be continuous. More generally, if $X \subset R^n$ is open relative to R^n (where n is the number of goods) and u (.): $X \rightarrow R$ is continuous, then $\{\mathbf{x}\ \varepsilon\ X |\ u(\mathbf{x}) > y\} = UC(y)$ is an open set for all y in the range of u (.).

The mathematical requirements are unambiguous. If anomalies are detected in experimental designs, do they arise from too strong order assumptions, or are they due to the fact that agents are not really maximizers? To address that question, I will start by taking a fresher look at the axioms to see if axioms' violations are the culprits. If they are not, I will then examine the utility maximization concept itself for any trace of empirical contents. The first part will be devoted to symbolic definitions and to clearer statements of the axioms to which I will refer throughout. In the second part, I will examine the so-called violations in order to see why they might appear as such at first sight. And in the last part, I will tackle the pertinence of the hypothetical utility function itself right where it was first formalized, i. e., within the Walrasian construct.

I- SYMBOLS, SETS AND AXIOMS

As already alluded to, the individual agent's utility function is unobservable. Nevertheless, a strong belief in its existence has led economists to pay close attention to the permissible orderings of X, a two-place relation on X that is endowed with certain properties. A voluminous literature ensued in which the properties defined by the axioms of continuity, connexity and transitivity are singled out for close examination. In that connection, some theorists have argued that inconsistent choices must be due to a violation of the transitivity axiom. But others, Sonnenschein (1965), for example, remarked that continuity and connexity imply transitivity in a two-place relation. Later on, Smeidler (1971) stated that continuity and transitivity imply connexity. Both may be right, because it would be hard to define a binary relation on a set which is not well-ordered. And it would be as hard to find a well-ordered set which is not connexe and transitive. But, as regards the observed anomalies, I suspect others causes, as I will show after reviewing the whole set of pertinent axioms. But beforehand, let us define our symbols and terms more precisely.

Symbols, Sets and Subsets

Certain expressions and propositional constants appear frequently in mathematics; to avoid repetitions, it is always useful to introduce symbols for them at the outset. Additionally, present-day logic too is not yet uniform, hence Table 1 shows both the symbols and a few set-theoretic concepts used in the text; the reader will then know in what precise sense I am using them in this discussion.

The sets and subsets used in the specification of the axioms are the following. The set X is often referred to as the *ground set*; in economics, it is the non-empty consumption set, itself a proper subset of a countably infinite set of consumption goods and services available in the market at any instant of time. To simplify, however, let the elements of X be x, y, and z. These are considered as bundles over which the agent expresses a choice. A binary relation $\preccurlyeq$ on X ={x, y, z} gives the set s of ordered pairs <x, y >. Equivalently, one may say that x, say, is in relation s to y, abbreviated x s y, or simply ⇨ (x, y) ε s; either one of these definitions may at times be used in the text. The elements of the pairs belong to the ground set on which relations are defined, and s, x, y, z are variables for them. For example, if x s y, then <x, y> belongs to the relation $\preccurlyeq \leftrightarrow y \leq x$. The set s is also the union of two subsets, s_1 (the first domain or the "yes" set), and s_2 (the converse or the "no" set) which arises by reversing the elements, and s is the domain of the relation $\preccurlyeq$ if $y \leq x$. More explicitly,

Table 1: Symbols and Set Theoretic Concepts

R	= the set of real numbers
$\cup$	= the union of sets
$\cap$	= the intersection of sets
$\subseteq$	= subset of…; e. g. x ε a → x ε b, then a ⊆ b
$\subset$	= proper subset of…; e. g. a ≠ b, then a ⊂ b
ε	= expresses the fact that x, say, is an element of a set; e. g. x ε X
$\mapsto$	= if…..
$\rightarrow$	= then….
$\leftrightarrow$	= if and only if….
⇨	= definition
$\wedge$	= and
$\vee$	= or
$\neg$	= not
$\approx$	= identical to
Λ_x	= for all (the universal quantifier); e. g. for all x
V_x	= there is (the existential quantifier); e. g. there exists an x

⇨ $s = \{s_1 \cup s_2\} = \{X \times X\} \supset \preccurlyeq$, where

⇨ $s_1 = \{z\ s_1\ z,\ y\ s_1\ y,\ x\ s_1\ x,\ y\ s_1\ z,\ x\ s_1\ y,\ x\ s_1\ z\}$.

⇨ $s_2 = \{z\ s_2\ z,\ y\ s_2\ y,\ x\ s_2\ x,\ y\ s_2\ z,\ z\ s_2\ x,\ z\ s_2\ y$, and,

⇨ $a \subseteq s_1 = \{z\ s_1\ z,\ y\ s_1\ y,\ x\ s_1\ x\}$.

⇨ $b \subseteq s_1 = \{x\ s_1\ y,\}$.

⇨ $c \subseteq s_1 = \{x\ s_1\ y,\ y\ s_1\ z,\ x\ s_1\ z\}$.

As I will show next, the last three subsets of s describe its topology. But for now, what I want to stress is that these definitions also show that these subsets represent the properties of s, according to Table 2 below. But before commenting the axioms, it might be useful to recall that $x\ s_1\ y$, say, means that x is in relation with y in s_1. Or that means that the thing-like object or consumption bundle x is at least as good as the bundle y; in economics, that relation is referred to as “weak preference”. The second, $(x, y) \varepsilon \succ$ (or $x > y$) means that “x is strictly better than y” from the point of view of the individual agent, of course. Finally, when both $(x \succcurlyeq y) \wedge (y \succcurlyeq x)$ hold at the same time, one writes the identity relation $I \Rightarrow x \approx y$; that is, “x is identical to y”, or “x is identical to y”. It then follows that $(I \subset \succcurlyeq) \wedge (I \cap \succ) = \{0\}$, namely the empty set.

Table 2: The Axioms of Order

(i)	*Reflexivity*	$: \Lambda_x\, (x\ \varepsilon\ a = s_1 \cap s_2) \quad \rightarrow x\ s_1\ x.$
(ii)	*Identitivity*	$: \Lambda_x\ \Lambda_y\, ((x, y)\ \varepsilon\ b\!: x\ s_1\ y \ \wedge\ y\ s_1\ x) \quad \rightarrow x = y.$
(iii)	*Symmetry*	$: \Lambda_x\ \Lambda_y\, ((x, y)\ \varepsilon\ b\!: x\ s_1\ y \quad \rightarrow y\ s_2\ x).$
(iv)	*Connexity*	$: \Lambda_x\ \Lambda_y\, ((x, y)\ \varepsilon\ b : x\ s_1\ y\ \vee\ y\ s_1\ x) \rightarrow x \geq y.$
(iv’)	*Comparability*	$: \Lambda_x\ \Lambda_y\ \Lambda_z\, (x, y, z)\ \varepsilon\ c\!:\mapsto x\ s_1\ y \rightarrow x\ s_1\ z\ \vee\ z\ s_1\ y$, where z is a third object.
(iv’’)	*Comparativity*	$: \Lambda_x\ \Lambda_y\ \Lambda_z\, (x, y, z)\ \varepsilon\ c\!: x \approx z\ \wedge\ y \approx z \rightarrow x \approx y$, if (i) obtains.
(v)	*Transitivity*	$: \Lambda_x\, \Lambda_y\ \Lambda_z\, (x, y, z)\ \varepsilon\ c\!: \mapsto x\ s_1\ y\ \wedge y\ s_1\ z\ \rightarrow x\ s_1\ z.$

Beside these axioms, I will be referring to the result of a theorem due to Aliprantis *et al*. (1989, 4), namely:

Continuity: $\Lambda_x\ \Lambda_y\!: \mapsto x \succ y$ holds on X, $\mathrm{V}_x\ \mathrm{V}_y :(N_x, N_y, (N_x \cap N_y) = \{0\}))|\ x'\ \varepsilon\ N_x \wedge y'\ \varepsilon\ N_y \rightarrow x' \succ y'$.

Put differently, if $m\ \varepsilon\ N_x$ and $n\ \varepsilon\ N_y$, where N_x and N_y are two disjoint neighborhoods of x and y, respectively, then the preference relation is continuous in both senses $\succcurlyeq$ and $\succ$ if X is a topological space.

As indicated before, the nature of the two-place relation defined on X determines the properties to be had, because relations may be considered properties of ordered pairs. Again, as usage and language are not yet uniform, it is of some importance to minimize confusion with clear definitions. For example, Roberts and Schulze, (1973, 19) use a language that leads one to think that a *total order* is irreflexive. Whereas, many other authors, including this one, claim that only the addition of the property of connexity to a reflexive semi-ordered set suffices to define a total or linearly ordered set. Moreover, what is here called Identitivity and Connexity are termed Antisymmetry and total, respectively, in Katzner (1983). Here then, Axiom (i), say, reads “for all x, y, and z in {a}, x is in relation s_1 to x”; while Axiom (v) reads “for all x, y, and z in {c}, if x is in relation s_1 to y and y is in relation s_1 to z, then x is in relation s_1 to z”, etc.. With this understanding, we can now move to spell out the consequences of these axioms. To wit:

Totally Ordered Set: A set X is totally or linearly ordered in the sense $\succcurlyeq$, if it satisfies axioms (i), (ii), (iv) (or (iv’)) and (v). With these properties, the totally ordered set is said to be a *chain*.

Semi-Ordered Set: A set X is semi-ordered in the sense of $\succcurlyeq$, and in the sense of $\succ$, if it fails to satisfy (iv)

Equivalence Relation: The Equivalence relation is $I \subset \succcurlyeq$, then it satisfies axioms (i), (iii), and (v). However, if (i) obtains, (iii), and (v) may be replaced by (iv’’).

To demonstrate that last assertion, let us define a set m, say, such that $x\ \varepsilon\ m \Rightarrow x \approx m$ as the set of equivalence class generated by m. Then if I satisfies (i), and $x\ \varepsilon\ m \rightarrow (x\ \varepsilon\ m \wedge y\ \varepsilon\ m) \rightarrow x \approx y$. It also follows:

$$(x\ s\ y \wedge y \approx z) \rightarrow x\ s\ z\ \wedge\ (x\ s\ y \wedge x \approx z) \rightarrow z\ s\ y.$$

It should be observed at this juncture that orderings in the sense of $\preccurlyeq$ is more natural but, as discussed, its upper contour sets, necessary for maximization, are closed relative to X. Economists then revert to strict say, cannot be strictly preferred to x, and both $x\ s_1\ y$ and $y\ s_2\ x$ cannot hold. Hence, s_2 vanishes and s_1 loses reflexivity. Then, from here onward, I will refer to $s_0 \subset s_1$. In this case, axioms (iv) and (iv') may be difficult to satisfy when X is large, and therefore, the information set of the agent may be incomplete. However, in cases where (iv') can be satisfied in small ground sets or in repeated purchases, we then have:

Ordered Sets in Economics: The ground set used in economic discourse is ordered in the sense $\succ$, then it satisfies (iv'), (v), and $\Lambda_x\ \neg\ x\ s_0\ x$ [(2)].

Such an ordered set cannot be a chain because (i), (ii) are excluded and (iv) may not obtain. However, for orderings in the sense of $\succ$, the requirements of Identitivity and Connexity are replaced by Irreflexivity and a modified form of Connexity: $\Lambda_x\ \Lambda_y\ ((x > y) \rightarrow x\ s_0\ y \vee y\ s_0\ x)$ or by (iv'). In any event, if (iv) and (iv') do not obtain, the orderings is *partial.*

What should be emphasized at this point is that it is the binary relation, a subset of the Cartesian product of X with itself that orders the set X. If there are no trade-offs between the elements of X, or if Connexity or Comparability is absent, how can a relation be defined on X? For the moment at least, I will proceed on the assumptions that X is not too big and that the agent is able to establish the direction of increasing preference. Then, Connexity and transitivity are both automatic and robust in a two-place relation.

In summary therefore, economists suppose that the set X is open relative to R^n and strictly convex, that the binary relation in the sense of $\succ$ is continuous, that u (.) is continuous, and its upper contour sets are open relative to X. To complete the picture, economists add the assumption of strong monotonicity to account for the biological requirements. Greed is an important factor in human behavior. This last assumption accounts for the fact that the agent will always prefer more to less until satiation; or better put, until consumption or accumulation brings no net benefits. Hence, the utility function appears to rest on a solid foundation, thus vindicating the early pioneers such as Jevons, Menger, Walras, Edgeworth, etc., as well as the modern ones. And from the 1940s onward, the utility paraphernalia became an integral part of neoclassical economics, the necessity of which few dare question.

II- WHAT DO OBSERVED ANOMALIES REVEAL?

In this section, I will examine four types of what appears as abnormal behavior to those who are in utility maximization mode, mainly to see if there exist alternative explanations to agents' irrationality. They are: preference reversals, intransitive choices, and the apparent failure to maximize utility detected in the Ultimatum Game and in the Tversky and Kaneman's (1985) experiment (see, note 1). These anomalies are usually explained away by appealing to irrationality, but I find such explanations somewhat too extemporaneous. Something is amiss, for how the approach can begin by ruling out irrationality and then reverts to irrationality to explain away observed anomalies, without undermining the approach itself?

Before tackling these apparent irrationalities, I would like to add another point that may foster yet a greater understanding. The neoclassical approach considers the thing-like elements of X to be comparable, although the range of comparable things is limited in practice. Logicians and philosophers, on the other hand, prefer to consider the whole *preference horizon* of the decider, which seems more natural to me also. An example of this is when a subject prefers y (when y comes with x and c_k) to z (when x or c_k do not obtain). If k = 1, 2,…, *l*, then just for one of the k's, we have something like a four-place relation. That situation might not have been accounted for in experiments that purport to reveal preference reversals, in particular when the experimenter was unable to control for the whole preference horizon of the subject. To clarify that point, suppose z is a Mercedes Benz sedan, x is car service in a given locality, c_k (the k^{th} conjunction) is high gas price in the same locality, and y is a Toyota Corolla. In the language and symbols adopted above, that situation is written as: $y\ (x \wedge c_k)\ s_0\ z\ ((\neg\ (x) \wedge c_k)$. In other words, the subject prefers the Toyota with good service and high gas price to the Mercedes if service is poor and gas price is high, even though he knows that the Mercedes commands a higher price on the market than the Toyota. What may have happened in cases of preference reversals is that the experimenter is in two-place-relation-mode while the subjects are wired for a multi-place (in this case, four-place) relation. What it all means is that neglecting the whole preference horizon may give the impression that the subject fails to maximize utility or is inconsistent in his choices.

The preference relation may change rather quickly in response to new information. In the Ultimatum Game, the observer sees A's decision as (zero s_0 (k M_A)) (see note 1). But, in reality, after observing the behavior of B, an additional information, maybe A's decision is $\neg\ ((1 - k)\ M_B)\ s_0\ k\ M_A$, preferring to punish B's greed to k M. If that is the case, again the experimenter may wrongfully conclude, seeing zero preferred to something, that the subject fails to maximize his utility, without of course knowing what his utility might be.

In the Tversky and Kaneman's experiment, it is observed that 88 percent of the subjects prefer the ticket to \$10. And in the second instance 54 percent prefer \$10 to the ticket; and so they are labeled irrational or inconsistent. Even that, if it is what the 54 percent understood, maybe they considered the lost ticket equivalent to \$10; in that case, they have just obeyed the Axiom of Identitivity. However, I believe that we have here a case that is more complex than it has been assumed. Let P stands for the Play, p is the price of the ticket, and TL is the lost ticket. Then, in the first term of the sequence, for 88 percent: $P + \neg\ (\$10) > p + \neg\ (\$10)$, meaning that seeing the Play is preferred to its price in an environment of \$10 lost. In the second term of the sequence, for the 54 percent: $p + \neg\ (TL) > P + \neg\ (TL)$; if for them p = TL = \$10, then they saw something that 46 percent have missed. If, however, TL > \$10, then for 54 percent saw a change in ground sets from $\bar{A} = \{P, p, \$10\}$ to $\check{A} = \{P, p, TL\}$. Here, we are assuming that \$10 are lost in the first term of a sequence, and a ticket is lost in the second; the preference horizon, a three-place relation, has therefore changed. More explicitly, suppose that the ticket costs \$10, seeing the play after losing \$10 sums up to a \$20 disbursement. If, on the other hand, the ticket costs \$20, buying another one means spending \$40 to see the Play. There is still another possibility. Maybe the 54 percent were just furious at themselves for having lost an expensive ticket, and refuse to buy a second, as an emotional reaction, which the utility maximization approach has ruled out at the outset. What this all means is that if the whole decision frame is not accounted for, one should not be too quick to claim a violation of the Axiom of Transitivity.

There is also the possibility that an observed violation of transitivity could result from a switch in the direction of increasing preference, as it can immediately be seen from the subset s_0 (in the sense $\succ$) defined above. Suppose an individual establishes that his preference runs from z, y, x, where $x > y > z$. As the ground set X has only three elements, it is easy to assume that the set is well-ordered. Axiom (v) is expressed as:

$$\Lambda_z\ \Lambda_y\ \Lambda_x\ ((z, y, x)\ \varepsilon\ c): x\ s_0\ y\ \wedge\ y\ s_0\ z\ \rightarrow x\ s_0\ z.$$

If in some finite time, the subject revises the direction to y, z, x, Axiom (v) now reads:

$$\Lambda_y\ \Lambda_z\ \Lambda_x\ ((y, z, x)\ \varepsilon\ c): x\ s_0\ z\ \wedge\ z\ s_0\ y\ \rightarrow x\ s_0\ y.$$

Reversing the first order to x, y, z (where $x < y < z$), Axiom (v) becomes:

$$\Lambda_x\ \Lambda_y\ \Lambda_z\ ((x, y, z)\ \varepsilon\ c): z\ s_0\ y\ \wedge\ y\ s_0\ x\ \rightarrow z\ s_0\ x.$$

If any pair of these situations is ever observed, it might simply reveal the role of additional information in the determination of the direction of increasing preference rather than inconsistency. Put differently, if a subject does not change the direction of his or her preferences, determined in a multi-place relation, it is unlikely that she would violate the axiom of transitivity, which runs along the direction of increasing preference and is such a robust property in a subsequent two-place relation. In the next section, I will have more to say about the difference between multi and two place relations.

By manipulating the ground set X to meet the mathematical requirements, it seems that neoclassical economists just assume that biology follows suit; this is of course pure wishful thinking. I have given reasons why the orderings used in neoclassical economics is most likely partial due to the absence of connexity. Incidentally, the orderings cannot account for the biological property of ambivalence, which is different from equivalence. We have defined indifference as, say, $x \succcurlyeq y \wedge y \succcurlyeq x$ holding at the same time, whereas ambivalence $x \succcurlyeq y \vee y \succcurlyeq x$ holding at the same time; but there is no room in the two-place relation for ambivalence, which is quite frequent in practice. Another example is when an agent claims that he prefers x to y and he also prefers z to y, but he may not be too sure as to whether x is preferred to z or if z is preferred to x. Psychologists often refer to this last case, but it is ignored in economics. The ranking of a particular individual may become connected with repeated purchases and feedbacks from market experience, and in the absence of the introduction of new products in the market. However, the latter possibility is highly unlikely in a modern and dynamic market. Thus, if the agent is unable to well-order the set, all kind of apparent pathologies may subsequently be observed.

The property of continuity does raise eyebrows in some quarters (see Section I), although here it is a restatement of strict preference. But, as is well known, in mathematics, continuity is simply axiomatic, but whether or not anything in nature can be continuous is a moot question in both philosophy and physics; however, this is beyond the scope of this discussion, except to stress the role of imperfectly divisible entities, and the difference between similarity and closeness in preference orderings. More pertinently is the difficulty posed by the Axiom of Comparability. Can it be ascertained that the elements of X are really comparable? We all know that there is no trade-off between, say, oranges and music. What happens then to the two-place binary relation when X consists of non-comparable elements? In such cases, the orderings is said to be lexicographic, but the notion of continuity rules out lexical orderings. Can they then be represented by a real-valued utility function? (3) In my view, the properties the elements of X are sure to share are that they are consumption objects and their market values represent shares of the consumer's budget; I will have more to say about that point below.

The role of emotions is also ignored in the neoclassical approach. I can understand why, but this runs contrary to the findings of neuroscience. It seems (see Dominique, 2006, 2007) that preferences in humans arise on a bimodal equilibrium manifold, expressed outwardly as "acceptance" and "negation" with indifference being an unstable state. This would mean that in the brain, preference is formed on a *multi-place relation*, i. e. the whole horizon, and its output is a *two-place relation*: 'yes' and 'no', while "ambivalence" and "indifference" are rather unstable. This means that goods are selected one at the time to form a selected bundle. And such a bundle is a "points set" which is necessarily nonconvex and closed, thus casting doubt on the existence of strictly quasi-concave utility function. If that is the case, the selection consists of goods that satisfy needs and wants, regardless as to whether they are substitutes or otherwise, comparable or not, and divisible or not. To repeat, if the ground set $X = \{x_1, x_2, \ldots, x_m\}$ is a points set which is nonconvex and closed, then the continuity of u (.) is not only questionable but it precludes strictly convex and open upper contours sets for all $\bar{U}$ in the range of u (.). In fact, if a utility function exists at all, it might not even belong to the class of monotone increasing quasi-concave functions, as it is well-known that the satisfaction derived from possession decreases over time until a default level determined by the biology of a person.

Despite these difficulties though, one indisputable fact remains. That is, the market spews out an equilibrium price vector fairly rapidly when there are no impediments to that effect. How is this possible? I will now demonstrate that the utility paraphernalia was never needed in the first place.

III-THE EQUILIBRIUM PRICE VECTOR

We have seen above that the Axiom of connexity (due to imperfect information), non-comparability and imperfect divisibility, and continuity are strong assumptions in both senses, $\succcurlyeq$ and $\succ$. Also, in empirical settings, there is no way, if all things cannot be held equal, to prevent the agent from taking into account his whole preference horizon. Furthermore, research in neuroscience has cast serious doubt on the belief that the utility function of agents belong to the class of quasi-concave functions. Human decision making, instead, obeys the dynamics of a bimodal fold, an elementary form of Catastrophe Theory; that is, time is a factor even if it is measured in microseconds. The ability of the human brain to solve some complex dynamic problems via simple heuristics is a gift of evolution. And it is that gift that allows the market to arrive at an equilibrium price vector in a very natural way. I will now show that the problem Walras was grapping with could have been solved without the utility paraphernalia.

The findings of neuroscience suggest that the agent does not order goods he intends to acquire as neoclassical economists suppose. He simply lists the items that he needs and next matches their market value at t-1 with his shares of his expected budget at time t. He may even index them to some ordinal scale before matching them with the rational scale of budget shares. Whether or not goods are divisible, whether or not goods are comparable, whether or not goods come in bunches, it does not matter. Once values are equated with market shares, the agent's demand is completely determined. As the market approaches equilibrium, the agent has the option of adjusting quantities so as to keep market shares constant. In other words, changes in initial endowments and / or market share distributions from t-1 to t create a gradient of unrealized gains that are minimized by quantity adjustments, which is the law of motion of the system. This process gives the close-form solution for the equilibrium price vector at time t in the Walrasian structure. Thus, the quantity of initial endowments and the budget shares distributions of consumers are all that is needed to determine the equilibrium price vector, as I will now demonstrate.

But beforehand, let me address a very legitimate question. That is, what is the difference between this Walrasian procedure and the construct known as "Reveal Preference"? They are more or less opposites. The main objective of Reveal Preference is to ask: Starting with a set of demand relations (which obey symmetry and negative semi-definiteness of substitution terms), is it possible to infer the utility functions that threw off these demand functions? In other words, Reveal Preference attempts to recover the supposed utility functions from observed demand functions. Whereas, the present modified Walrasian approach attempts to show that utility functions as unnecessary appendages, as the following simple case shows.

Consider a pure exchange economy consisting of a non-empty set of goods X and a set C of agents. Goods are indexed by $j \in X \in R^n_+$ and agents are indexed by $i \in C \in R^m_+$. The budget share that i devotes to goods j is $\alpha^i_j \in (0, 1]$, and i's initial endowment of j is $\omega^i_j \in R^n_+$. The demand of i for good j is:

$$x^i_j = \alpha^i_j B^i / p_j, \text{ and}$$
$$B^i = \textstyle\sum_j p_j \omega^i_j ,$$

where the budget of i comes from trading his initial endowments in the market. Hence, the total demand for j is $\Sigma_i x^i_j$. Applying these relations to the present case, the total demand for j =1, say, is Q_1:

$$(1) \quad Q_1 = (1/p_1)[(\alpha^1{}_1 \omega^1{}_1 + \alpha^2{}_1 \omega^2{}_1 +, \ldots,+ \alpha^m{}_1 \omega^m{}_1) p_1 + (\alpha^1{}_1 \omega^1{}_2 + \alpha^2{}_1 \omega^2{}_2 +,\ldots,+ \alpha^m{}_1 \omega^m{}_2) p_2 +,\ldots,+ (\alpha^1{}_1 \omega^1{}_n + \alpha^2{}_1 \omega^2{}_n +, \ldots,+ \alpha^m{}_n \omega^m{}_n) p_n],$$

and similarly for all $j \in X$. Since the supply of good j entering the market is $\sum_i \omega^i_j$, the equilibrium price vector $\mathbf{P}^*$ is given by:

(2) $\dot{\mathbf{p}} = \mathbf{dg}\,(1/p_j)\,[\,\mathbf{M} - \mathbf{dg}\,(\sum_i \omega^i_j)]\,\mathbf{P}$,

where the dot refers to differentiation with respect to time, **dg** $(1/p_j)$ and **dg** $(\sum_i \omega^i_j)$ are diagonal matrices, and $\mathbf{M}_{(n \times n)}$ is written as :

$$
\mathbf{M} = \begin{pmatrix}
(\alpha^1{}_1\,\omega^1{}_1 + \alpha^2{}_1\,\omega^2{}_1 +,\ldots,+ \alpha^m{}_1\,\omega^m{}_1) & (\alpha^1{}_1\,\omega^1{}_2 +,\ldots,+ \alpha^m{}_1\,\omega^m{}_2) & \ldots & (\alpha^1{}_1\,\omega^1{}_n +,\ldots,+ \alpha^m{}_1\,\omega^m{}_n) \\
(\alpha^1{}_2\,\omega^1{}_1 +,\ldots, + \alpha^m{}_2\,\omega^m{}_2) & (\alpha^1{}_2\,\omega^1{}_2 +, \ldots, + \alpha^m{}_2\,\omega^m{}_2) & \ldots & (\alpha^1{}_2\,\omega^1{}_n +,\ldots, + \alpha^m{}_2\,\omega^m{}_n) \\
\vdots & \vdots & & \vdots \\
(\alpha^1{}_n\,\omega^1{}_1 +,\ldots, + \alpha^m{}_n\,\omega^m{}_1) & (\alpha^1{}_n\,\omega^1{}_2 +,\ldots, + \alpha^m{}_n\,\omega^m{}_2) & \ldots & (\alpha^1{}_n\,\omega^1{}_n +,\ldots, + \alpha^m{}_n\,\omega^m{}_n)
\end{pmatrix}
$$

For given budget shares' and initial endowments' distributions, there exists a **P*** that equates supply and demand. That is, starting from any $\mathbf{P} \neq \mathbf{P}^*$, the system converges to the singular point $\mathbf{P}^*$ as time runs forward. The Jacobian matrix of (2), whose elements $J_{jk} < 0$ for $j = k$ and $J_{jk} > 0$ for $j \neq k$, is Metzlerian. Then starting from any nonnegative initial price vector, the system is guaranteed to preserve the non-negativity of the state variable. Drawing on the direct consequence that exists between existence and stability in positive linear systems, it can be inferred that regardless of the dimension of **M**, $\mathbf{P}^*$ is stable; for a demonstration, see note 4.

As shown, the equilibrium point is where competing forces are balanced. Hence, what is feasible is not necessarily what brings maximum satisfaction. Markets may well be better than known alternatives, but they are not synonymous with maximum satisfaction. From this result, one can see how the theorizing on preference orderings and on utility maximization has encouraged the fiction regarding economic agents' farsightedness and rationality. Infinite farsightedness has led to the assumption of connected ground sets, while utility maximization is due to the early marginalists', in particular, Walras' insistence on such a metaphysical concept despite the advice of Henri Poincaré to just drop it (Jaffé, 1965). Even some forty years later, Walras would write:

> "Individual behavior is totally described starting with utility functions and initial endowments of goods held by each individual…." Leçon 13 (1909).

Obviously, one can always assign a convenient utility function such as $u(.) = K \log (\Pi x^{i\,\alpha})$ to the agent. Letting $K = 1$, then maximal utility gives the total market demand as exactly as in equation (1). The problem here though is that such a function was not observed, it was invented for a convenient extra-scientific interpretation. The present approach just shows that maximizing a supposed utility function is an unnecessary appendage.

NOTES

(1) Preference reversal is an experimentally observed phenomenon in which subjects place lower market values on their preferred alternatives (Litchenstein and Slovic, 1971). Inconsistent choices, whenever observed, are said to be violations of the Axiom of Transitivity ((Axiom (v) in the text). In the Ultimatum Game, two subjects, A and B, are involved. A certain sum, M, is given to, say, B and B must give a proportion k M of his choosing to A. If A accepts k M then B may keep (1-k) M. However, if A rejects k M, as he may deem it too small a proportion, then both subjects end up with nothing. Experiments regularly show that k < 20 percent causes rejection (see Thaler, 2000). Tverski and Kaneman (1985) have observed that 88 percent of subjects in their sample say if they arrived at a theater to buy a ticket for a play and then discovered that they had lost $10 on the way, they would buy the ticket anyway. But if the ticket was bought in advance and discovered when they had arrived at the theater that they had lost the ticket, 54 percent say they would not buy another ticket. In the conventional approach, the 54 percent would have been labeled 'irrational'.

(2) $\bigwedge_x \quad \neg x \; s \; x$ stands for irreflexitivity.

(3) In Social Choice Theory and Welfare Economics, theorists appeal to the so-called 'leximin rule' to deal with non-comparability. Comparing agents' utility levels seems to by-pass the problem.

(4) In a pure exchange economy with i = 1, 2, 3 agents and j = 1, 2, 3 goods, let the budget shares set of i and the initial endowments of i of good j be as follows:

	α				ω			
j i \	1	2	3		1	2	3	
1	.5	.3	.2		50	20	30	100
2	.3	.5	.2		40	25	15	80
3	.4	.3	.3		10	45	35	90

That is, agent 1 establishes his budget in such a way that 50 percent goes to good 1, 30 percent to good 2, and 20 percent to good 3. He also intends to exchange 50 units of good 1, 20 units of good 2, and 30 units of good 3. The The above table may be read in a similar manner for agents 2 and 3. Applying Equations (1) and (2) in the text, we have:

$$\dot{\mathbf{P}} = \begin{array}{cccc} 1/p_1 & -59.0 & 35.5 & 33.5 \\ 1/p_2 & 38.5 & -58.0 & 27.0 \\ 1/p_3 & 21.0 & 22.5 & -60.5 \end{array} \quad \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} .$$

The price vector is $\dot{\mathbf{P}} = (1.3996, 1.3825, 1.0000)^T$. The Jacobian matrix is:

$$J = \begin{array}{ccc} -42.1530 & 25.3632 & 23.9343 \\ 27.4856 & -41.9518 & 19.5293 \\ 21.0000 & 22.5000 & -60.5000 \end{array} .$$

Walras (1909) may have thought that utility functions and initial endowments totally describe individual behave-ior. The above demonstration shows that this only partially true. No appeal to utility functions is warranted.

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