Negative net products with positive profits

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ABSTRACT

This unpublished paper was developed out of a numerical example posted on the PEN-L list in June 1995 and submitted to ROPE September 1996. It demonstrates that, contrary to the 'Fundamental Marxian Theorem' (FMT) and the claims of the surplus (post-Sraffian) school, a positive net product is not a necessary condition for a positive profit. That is to say, a positive profit may arise even though society produces less than it consumes of at least one good.

Given technical progress, negative net products are not a pathological exception but a perfectly normal and indeed, rather usual state of affairs. Technical progress implies that society replaces one product with another –steam with internal combustion, mechanical transmission with electrical power, the abacus with the computer, or simply the 286 chip with the 386, the 586, the Pentium, and so on. The principal consequence of technological obsolescence, in short, is that as a product is phased out, society will cease producing it but continue consuming it. That is, capitalism, as a normal aspect of its functioning, produces negative net products.

For the surplus school this makes exploitation inexplicable, since this school relies on the FMT to derive exploitation – the 'theorem' that in Steedman's (1997:53) words

> The conditions for profitability, production of physical surplus and 'production' of surplus labour are … identical.

This paper established for the first time that this statement is false.

This subsequently became central to the 2007-2008 controversy between Mohun, Veneziani, Kliman and Freeman, cf 'Replicating Marx: a reply to Mohun', *Capital and Class* No. 88, Spring 2006, pp 117-123, and 'Simultaneous Valuation vs. the Exploitation Theory of Profit: A summing up', Capital and Class No. 94, Spring 2008.

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INTRODUCTION: WHY A POSITIVE NET PRODUCT?

In the economic literature, the idea that society must produce a surplus of everything is widespread enough to be accounted an axiom. It has supplied the name of an entire body of thought, the 'surplus school'. It is the stated or unstated premise behind the idea that surplus labour is reducible to a surplus physical product. More broadly, the analysis of society in terms of the distribution of this product is widely perceived as a generic alternative to marginalism and the subjectivist view in economics.

Under the assumptions usual in linear production theory[1] a positive net product is a sufficient condition for a positive equilibrium profit rate. This has spawned, almost unnoticed, the complementary idea that it is also necessary, so that the two ideas of a positive net product and positive profits can safely be identified. Thus for example Ian Steedman (1977:53):

> The conditions for profitability, production of physical surplus and 'production' of surplus labour are thus identical.

This is an error. A positive net product is not necessary for positive profits and these conditions are therefore not identical. A simple numerical example shows that systems with negative net products can and do yield positive profits.

Given past excitement concerning negative surplus value and positive profits, it would hardly be just to pass over this lapse without comment. Two questions arise: what condition *is* necessary for positive profits? And what economic meaning attaches to it?

A NUMERICAL EXAMPLE

Consider the following economy in which two producers, P_I and P_{II}, produce and consume the following amounts of two commodities, C_I and C_{II} as follows in 1995.

IN	P_I	P_{II}	Total Inputs	
C_I	4	1	5	
C_{II}	4	1	5	
OUT			Total Outputs	Net Product
C_I	8	0	8	+3
C_{II}	0	3	3	−2

During 1995 a physical surplus of 3 units of C_I is produced and there is a physical shortfall of 2 units of C_{II}. Society has converted 2 units of C_{II} into 3 units of C_I.

The standard linear production model simultaneous price equations for the maximum profit rate R are given by the requirement that the price p_i, ($i = 1,2$) of each output should equal the price of its inputs plus a uniform profit markup, as follows:

$$(4p_1 + 4p_2)(1+R) = 8p_1$$

$$(p_1 + p_2)(1+R) = 3p_2$$

This yields R= 20% with prices as follows:

$$(4p_1 + 4p_2) \times 1.2 = 8p_1$$

$$(p_1 + p_2) \times 1.2 = 3p_2$$

which is satisfied, in units of C_{II}, by

$$p_1 = 1.5, \quad p_2 = 1$$

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[1] See for example Manara (1977:5), Pasinetti (1977:62)

Both profits and prices are thus positive. We have not specified labour inputs but it should be clear that this does not substantially alter the basic facts. Should P_I and P_{II} each employ one worker consuming a real wage of ½ unit of C_{II}, for example, then these workers could purchase their C_{II} when production is complete, with a profit rate of about 7% and approximate price $p_1 =$ 1.3, again with C_{II} as numéraire.

THE FEASIBLE AND THE REAL IN ECONOMIC THEORY

The assumption of a positive net product in all commodities is usually introduced in a remarkably cavalier manner. One of the more circumspect is Manara (1977:5) who gives it as an 'unstated assumption' defined thus

> The overall quantity of every commodity used as a means of production is less than the total quantity of the same commodity produced in the whole economic system

and comments

> we intend to accept the opinion of the economists, who are better able to evaluate the soundness of the hypotheses themselves and of their economic implications. We shall limit ourselves to pointing out that without these hypotheses (or equivalents) the [standard linear production model]…would not be 'viable'.

The economists themselves are less given to caution. Steedman's views, in a work which attaches some importance to logical coherence, have been cited. Pasinetti (1977:62) declares

> Negative commodity components of the net production or negative production coefficients would have no economic meaning.

Even more strongly (p97) he asserts

> If the condition were not satisfied, it would mean that the economic system considered is so technically backward as to yield a negative rate of surplus, i.e. to use as means of production greater quantities of commodities than it would then be able to produce. Such a system could obviously [sic] not survive; it would not be viable.

Roemer (1981:19) simply says

> The Marxian notion of reproduction means that the system should create institutions and ideology that enable it to continue existing,

which though reasonable hardly justifies what follows:

> We do not try to capture this deep idea, but rather the simple economic prerequisite that the economy should not operate in such a way as to run down some necessary stock to zero, in which case further production would be impossible

A short period of unprejudiced reflection shows that a physical surplus of all goods is neither obvious nor a requirement of survival, and in no sense a prerequisite for society to create institutions and ideology that enable it to continue existing. Moreover, it does happen. To assume it is, in the coarse language of the street, wrong.

Reproduction, replacement, and change

An organism can reproduce in many ways without reproducing every part of itself identically and this is the norm rather than the exception. If some good, necessary at one time, becomes unnecessary at another time and is replaced by something else then its stock is indeed run down to zero. That is what replacement consists of.

The general philosophical problem of social analysis is not to explain how something can remain identical to itself in every respect at all times, a phenomenon which is hardly ever observed in raw nature let alone civilised society, but to understand how it can persist though every part of it changes. How can it be that the economies of the industrial revolution and the dawn of the third millenium are in vital respects the same society, even though we have no practical use for almost anything the Victorians did or made, except to collect it, regret its passing, or build careers on refuting it?

Modern society with growing abandon substitutes one thing for another. New machines ceaselessly drive out the old. Even more rapidly do new consumer goods follow one on the other through the dictates of fashion and the insatiable search for enjoyments. The negative net production of nonbasics has become almost a definition of what they are.

Moreover, the deeper we penetrate the actual structure of the commodity, the more intractable this problem becomes. If we examine the real range of options exercised by consumers both productive and unproductive in purchasing any actual product whether wines, cars, trainers, computers, factories or even humble corn, we find the reproduction of the product by society in identical form is extremely unusual; what we actually buy is not the same thing but the latest model, which all too soon replaces the outdated. It might be argued this level of detail is too fine. But the difference between, say, a 486 and a Pentium computer is a lot greater than the difference between a new 486 and a year-old 486. Yet according to many the issue of fixed capital has been solved precisely by this second distinction.

Finally and by no means least, market societies periodically experience not only general gluts but their converse, general shortages in periods of expansion. Surpluses are expressed as additions to stocks, and these same stocks are reduced in booms. This corresponds to a shortfall of production. Stocks are the objective expression of supply and demand imbalances, and were it not for negative net production many would pile up without limit.

If a theory inconveniently fails to capture this vibrant reality, the recommended scientific procedure is allegedly to adjust the theory. There is no obvious basis for ruling out negative net production *a priori*; it is on the contrary the normal and natural state of society as we know it for most of what we consume and produce.

Real viability and the consequences of stocks

The real problem is both simpler and well-known to physics: society cannot consume what it has not yet produced. Therefore first, we are obliged to make things before we consume them, and second if we consume more than we currently produce, we use up the results of past production. Correctly stated, the viability condition is hence that consumption be no greater than the total of production and stocks. No further constraint can legitimately be deduced from the physical facts alone, and this condition does not demand a current physical surplus.

In the early history of linear production systems this was known, discussed and formalised, for example in the dynamic model proposed by Leontieff (1953). In the early history of the 'viability condition' it appears as the Hawkins-Simons condition[2] which is not a prerequisite for long-term equilibrium, but tests when an economy is running down its stocks – that is, whether it is accumulating or decumulating. This original dynamic conception has been lost, or more accurately discarded, and an altogether poorer static conception has been substituted. In this particular case, older is better.

We are drawn to the conclusion that the assumption of positive net production is not a description of reality but a convenient way of setting aside an extremely awkward logical problem.

NOTATION

Our notation can be found in many standard textbooks on the question. For simplicity we deal with the maximum profit rate unless otherwise stated so that wages do not figure as costs; readers who introduce wages will find the results no different.

Basic definitions

A is a square $n \times n$ matrix in which A_{ij} gives the consumption by producer j of commodity i in a year, during which all goods are assumed to turn over once. **X,** a matrix giving gross production, is assumed diagonal (no joint production). Neither simplification alters the qualitative results. The row vector p gives unit prices with p_j = price of good j.

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[2] See Hawkins (1948), Georgescu-Roegen (1951), Leontieff (1951,1953)

Derived definitions

Define **a**, the *coefficient* matrix, as $\mathbf{AX}^{-1}$. With **X** diagonal, $\mathbf{a}_{ij}$ gives the amount of commodity i consumed per unit of output j. It is positive if no input is negative and there is a positive gross product, that is each industry produces something, assumptions I don't challenge. The net product $\mathbf{Y} = \Sigma_j(\mathbf{X}–\mathbf{A})$ may however have negative elements.

PROFITS

The vectors $p\mathbf{A}$, $p\mathbf{X}$ respectively give the total costs and total sales receipts of each industry. Adopting for now the widespread assumptions of constant prices and a uniform (maximum) profit rate r, the vector $p\mathbf{A}(1+r)$ – costs plus profits – must equal sales $p\mathbf{X}$. Hence

$$p\mathbf{A}(1+r) = p\mathbf{X} \tag{1}$$

This can be expressed as an eigenvalue problem as follows: postmultiply by $\mathbf{X}^{-1}$ to give

$$p\mathbf{a}(1+r) = p \tag{2}$$

Dividing through by $1+r$ gives

$$p\mathbf{a} = \lambda\, p \tag{3}$$

where

$$\lambda = \frac{1}{1+r} \tag{4}$$

The conventional problem is to find a unique profit rate and vector of relative prices which make this economic system 'plausible'; that is, permit producers trading at such prices to replenish their inputs and their annual consumption out of sales income. Otherwise put, the problem is to find how the system can reproduce itself identically in kind.

The literature is dominated by the requirement that r and p should be positive. What provides for this? Since **a** is non-negative it has a positive maximal eigenvalue λ, making $1+r$ positive but not necessarily r. It also defines, to within a constant of proportionality, a positive row eigenvector p which can be identified as the equilibrium price, and a positive column eigenvector q, to which we will return. The latter represents a scale on which industries could produce to provide for so-called balanced growth – including negative growth – where every industry shrinks or expands in the same ratio as the proportions of each commodity in the net product.

As is well-known and indicated above, these facts do not guarantee a positive r. The pair [**A**,**X**] could provide a positive λ and p and nevertheless, if λ is greater than 1, the rate of profit (and growth) will be less than zero. A further assumption is therefore required to guarantee a positive profit rate. This is where a positive net product is conventionally introduced.

WHY ASSUME A POSITIVE NET PRODUCT?

This is usually introduced in the following way

(1) The economy must be *viable*, that is, produce a surplus of every good. Otherwise it would run out of something.

(2) Viability thus defined is equivalent to $\mathbf{Y} \geq 0$.

(3) This assumption guarantees positive profits.

(4) Therefore there is nothing more to worry about.

But this argument runs one way only. It establishes that a surplus of everything will yield positive profits. It does not establish that this is the only condition for positive profits. Instead, it simply rules all other options out of order as either embarrassing or ignorable – two oft-confused objections.

In the language of mathematics a positive net product is sufficient but not necessary for positive profits. What does this tell us? Actually, not very much. For example, a sufficient condition for success in almost anything is a brass lamp with a big green genie that does what you ask when you say 'free market'. This is interesting but not very useful.

Much more useful would be to know *minimum* conditions – threshholds under which a market society, according to this treatment, could not sink. Bare minimum conditions for anything are in general hard to establish or even define, but we can at least seek *necessary* conditions, which tradition fails to furnish.

First, however, let us enquire why a physical surplus is in fact a sufficient condition for positive profits. Construct the dual of **a**, a matrix **a*** defined by

$$\mathbf{a}^* = \mathbf{X}^{-1}\mathbf{A} \tag{5}$$

This contrasts with **a**, defined we recall as $\mathbf{AX}^{-1}$. Each element j of row i of **a*** is gotten by dividing the consumption of commodity i in industry j by the total output $\mathbf{X}_i$ of this same commodity. It gives the proportion of the output of each commodity consumed by each industry. Summing across all industries to obtain $\Sigma_j\mathbf{a}^*_{ij}$ gives the proportion of the output of each commodity consumed by society. If this total is greater than 1 for any commodity i, then more i is consumed than produced. Conversely, if there is a surplus of everything then for each i

$$\mathbf{X}_i \geq \Sigma_j\mathbf{A}_{ij} \tag{6}$$

that is

$$\mathbf{1} \geq \Sigma_j\frac{\mathbf{A}_{ij}}{\mathbf{X}_i} \tag{7}$$

where **1** is the unit vector [1,1,…1]. Hence

$$\mathbf{1} \geq \Sigma_j\mathbf{a}_j^* \tag{8}$$

that is, all the row sums of **a*** are less than 1. Matrix norm theory tells us[3] that any upper bound of the row sums of **a*** must be an upper bound for its maximal eigenvalue. Thus the maximal eigenvalue of **a*** must be less than 1. But the eigenvalues of **a** are the same as those of **a***.[4] Hence positive profits. In summary, to repeat, a physical surplus of everything guarantees positive profits. However

1. This assumption does not correspond to observed reality, as we have already discussed. In general there will always be negative net production of something, usually many things;

2. It is not mathematically necessary. Many pairs [**A**,**X**] produce positive profits though $\mathbf{Y} = \Sigma_j(\mathbf{X}-\mathbf{A})$ is not uniformly positive, as we have shown.

This seems to me to leave a fairly large hole in conventional surplus theory. Prior to poking around inside it, let us establish an economically-comprehensible *necessary* condition for positive profits. This is quite well-known; the problem is its interpretation.

THE FORMAL SOLUTION

As von Neumann (1937) indicates, the balanced growth rate is the same as the profit rate. To find out when profits will be positive we can solve the dual problem: under what conditions is positive growth possible in every sector? The world *possible* is important.

Suppose **a** yields positive profits. This implies that a maximum eigenvalue λ less than 1. Now consider the column eigenvector q corresponding to λ;

$$\mathbf{a}q = \lambda q \tag{9}$$

Substituting $\lambda<1$ yields

$$\mathbf{a}q < q \tag{9}$$

An equivalent condition is that *some* vector q exists for which net output is positive. This is a formal necessary condition for positive profits. It is normally interpreted quite glibly as stating that the economy consumes less ($\mathbf{a}q$) than it produces (q).

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[3] The theorem is due to Gershgorin. Seneta (1973) reproduces Gershgorin's simple proof. Householder () is a definitive account of matrix norm theory.

[4] By definition $a = \mathbf{AX}^{-1}$ and $a^*=\mathbf{X}^{-1}\mathbf{A}$. So if p is an eigenvector of a with $pa = \lambda p$, then $p\mathbf{AX}^{-1} = \lambda p$. Now consider the vector $p^* = p\mathbf{X}$. We have $p^*a^* = p^*\mathbf{X}^{-1}\mathbf{A} = p\mathbf{A} = pa\mathbf{X} = \lambda p\mathbf{X}$. Thus $p\mathbf{X}$ is an eigenvector of a^* corresponding to the same eigenvalue λ.

Actually, it tells us nothing of the sort. It states that *were* an economy to produce output q_i, and *were* it to consume inputs in the same proportion as the actual economy, then such an economy *would* produce more of everything than it consumed. In claiming that the sufficient condition **Y**≥0 is also necessary, tradition commits the cardinal error of confusing the ideal with the real; it identifies hypothetical output q with actual output $\Sigma_j\mathbf{X}$.

We can now see how our simple numerical example allowed for positive profits. Instead of what 'actually happened' in our example, we can construct a different economy with the same coefficient matrix but different output scale. Since for our economy

$$\mathbf{a} = \begin{pmatrix} ^1/_2 & ^1/_3 \\ ^1/_2 & ^1/_3 \end{pmatrix}$$

this is given by the following equations:

$$q_1 = 1.2 \times (^1/_2 q_1 + ^1/_3 q_2) \tag{11}$$

$$q_2 = 1.2 \times (^1/_2 q_1 + ^1/_3 q_2) \tag{12}$$

where q_1 and q_2 are the outputs of C_I and C_{II} respectively. This is satisfied for any $q_1=q_2$; for example $q_1=q_2=8$. The economy would then look like this:

IN	P_I	P_{II}	Total Inputs	
C_I	4	8/3	20/3	
C_{II}	4	8/3	20/3	
OUT			Total Outputs	Net Product
C_I	8	0	8	4/3
C_{II}	0	8	8	4/3

This yields positive profits and a positive net product. But what kind of economy is it? P_{II} has grown by a factor of nearly three – marginally less likely than a big green genie. Moreover is precisely P_{II}'s products that are falling into disuse. In a different context this ideal economy is supposed to represent what is 'socially necessary' – an image of necessity best described as 'quaint'. Should we rush out and restore the lost production of handlooms, Edsels and flared trousers before society collapses under the weight of modernity?

But the underlying problem is even more serious: the economy concerned is imaginary, and not only imaginary but impossible.

COMMENTARY

We can summarise the outcome of the previous section as follows: the correct statement of the 'viability' condition is not that the economy actually produces a positive net product, but that it could in principle do so if reorganised subject to constant returns to scale.

It is not clear that this is a tremendously useful thing to know, though much has been made of it. The British economy could in principle take over the world armed with a brass farthing, subject to the assumption that the brass farthing contains a big green genie. Is this information of any practical use? Expressed thus, the viability condition is more problematical than normally supposed. Let us consider some of its implications.

First, the *actual* net product is *not* positive. Ian Steedman should more accurately have said:

> The condition for profitability of the actual economy is thus identical to the condition for the production of a physical surplus in a hypothetical economy which, however, does not and could not exist.

This makes the general case to say the least somewhat weaker. What is the real relevance of an imaginary economy? It sounds perilously close to a *subjective* definition of price and/or value; it is a purely ideal, purely mental construction which can only be conceived of as an option in the minds of agents.

Second, there is no guarantee that if the scale of output of any industry did change, the proportions of its inputs would remain the same – a highly unrealistic assumption. Sraffa rightly stated that he himself did not need to assume constant returns, but his project was limited to a

critique of neoclassical economics; far less prudent modern writers deploy, as a practical test for reality, an economy so unreal that it cannot even exist.

A struggle for what?

Finally, since the world of the feasibility condition is neither real nor possible, it casts a serious pall over the widespread idea that exploitation is reducible to the outcome of a struggle for a share in some putative physical collection of objects. *What* physical product do the workers and capitalists fight over, if no real economy makes it?

Only two possible interpretations exist: either people fight to get hold of negative products, which is absurd, or they fight for the possession of a product which is not actually produced, for an *imaginary* physical product – one which is, moreover, stuffed full of the very things society is trying to jettison.

But then what is the objective character of this dispute? Why should I fight anyone for a share in a load of clapped-out old kipple which could only be gotten from a zombie production system? The notion of a struggle for distribution of a physical product which does not actually exist is every bit as subjective as marginalism, from which the surplus approach was supposed to deliver us.[5] And marginalism at least lets us fight for dreams rather than dross.

The two faces of equilibrium

The unquestionable fact that we must produce what we consume has mutated into an idea which sounds the same but means something utterly different – a condition on the structure of the economy, instead of the way it moves. It will doubtless be argued that this idea structure affords insight into real movement. This is precisely what we question.

First, the feasibility condition is every bit as subjective as the ideal abstractions of marginal economics. There are no grounds for treating it as in any sense 'more real' than a psychological preference.

Second, the idea has become ingrained in heterodox thinking that value is the outcome of, or a synonym for, a process of reproduction. This emphasis has a deeply apologetic tinge. It turns a contingent outcome – the successful and identical return of an economy into itself – into an inviolable axiom. It depends on the *same* assumption as marginalism – that supply matches demand. Equation 1 cannot hold on any other basis.

Linear production theory is revealed, not as an alternative to neoclassical general equilibrium theory but a variant of it. The basis of its long-standing dispute with subjective marginalism is its insistence that physicality renders it objective. This claim is false. The physical world to which it refers has no objective character.

THEORISING CHANGE

It never occurs to those mesmerised by dazzling insights into structure that the cost is to be blinded to process. The structural requirement imposed by linear production systems is the complete absence of change. Since change is fundamental to actuality, this leaves us without a concept of surplus that correspond to anything actual in a changing economy. We ought to enquire, therefore, whether there is a real and actual property of a changing economy which corresponds to this concept.

In doing so we must wrench the focus from structure, which abstracts from change, to transition which takes change as its starting point. The real world is driven by change. If theory is derived from structural assumptions, it presupposes that these are brought about by the process they are used to study. But process continuously modifies structure. We cannot deduce any given structure to be an outcome or a determinant of a process without first studying this process

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[5] This has particularly strange implications for the surplus approach to fixed capital, which requires that each machine should each year jointly produce its outputs and an older copy of itself. But reproduction then requires that earlier vintages be replaced from current production along with everything else, as if by a society of museum curators, fastidiously renewing everything ever made for eternity.

independently in its own right. A dispassionate enquiry into the market cannot take as given the claims it makes for itself.

This suggests a different treatment of the whole issue of viability – not as the maximum condition for a hypothetical market to reproduce for ever, but as a minimum condition for an actual economy to survive from one moment to the next. What we really need to know is what a viable *transition* consist of. The question now appears thus: on the one hand what combinations of flows and stocks, in one period, can result from a different combination of stocks and flows in the previous period? And on the other, in terms of what unchanging feature of the economy may we express what has changed?

I would argue that this is the real place of the concept of value in political economy. Value, as Mirowski (1989) concedes, corresponds precisely to the requirement that something measurable in an economy persists through change, without which I find it hard to understand how it there can be an economic theory at all.

Viability cannot be formulated in merely physical terms precisely because the consumption of physical goods depends on their distribution through exchange. A society which prices food beyond the reach of its citizens is just as unviable as one which fails to produce it. Equilibrium theory of all types escapes this annoying little difficulty by declaring price to be the child of immaculate reproduction. With the divorce of supply from demand this misbegotten waif is orphaned.

In reaction, schools such as the Post-Keynesians tend to act as if the loss of this category does not matter – as if dynamic motion can be more or less directly studied in terms of the traded money prices of goods. I dissent.

First, no science can operate without logically grounded concepts, and if we discard one source of them, then we require another. Second, there are quantitative issues which cannot be expressed using only observed prices. What is the meaning of the term 'price level' if there is nothing relative to which prices are measured? If the physical composition of output is continuously changing, price indices do not fill this gap. Moreover since production happens before consumption – that is, before traded prices are formed – we require a common measure of some exchangeable property of objects which can be quantified as the outcome of production independent of these traded prices. Otherwise we cannot assess any aggregate properties of the outcome of production such as, for example, the size of the product which is to be distributed in exchange. And indeed a concept of a surplus of some kind is, it seems to me, indispensable. What actually determines the average profit rate? We know from direct observation that it is not arbitrarily large, and does not fluctuate without limit. What sets the limits? A concept of surplus supplies this information and in so doing informs us on one of the most important questions of political economy: what sets the real practical limits on society's potential for growth and change? For all these reasons a concept of value – some measurable quantity in terms of which price can be expressed – is hard to do without.

Real persistence as the source of valid abstractions

This poses the perennial question: where to begin? On the basis of what initial abstractions may a changing thing be analysed, since by making any abstraction we presuppose the persistence of some unchanged thing? Our manifesto is to begin from elements actually present and observably persistent in the economy as it really exists.

This counters many of the most common objections to the value abstraction. It does not say an abstraction tells us all there is to know about an organism. It does not refer to some Kantian unknowable. Nor does it take sides in the long-running dispute between individualism and holism – for example, we may determine that the brain is the organ of personal identity. The approach suggests which *aspects* of both whole and part must be studied in order to theorise change, namely, those which are actually observed to persist, and to be the actual precondition of the coming to be and passing away of all contingent things which are nevertheless aspects of the organism.

A physical surplus is neither a feature of any real economy, nor a precondition of any other feature. Nor is any particular physical good. If on the contrary for one month the human race

either ceased to labour or found itself unable to purchase the results on the market, the majority of it would die. This seems to me a perfectly sensible reason for considering the relation of labour to exchange a suitable starting point for the analysis of a real market economy.

Physical reproduction is the outcome of the value process, that is, of the social relations which this represents. In a market society, these are the real preconditions of the physical relations of humans to the world.

Why, for example, is there crisis and why do profits fluctuate through the business cycle? Not 'because' of any particular relation between technical or physical compositions, or 'because' of any particular structure of physical scrapping or use, but 'because' capital accumulates. Accumulation denominated in money is precisely what is invariant amid the wild profusion of technique. Capitalists have a certain sum of money in their possession after selling their product – their profit – and a certain stock of assets denominated in money – their advanced capital. They have no option but to add to the first to the second in one form or another and create a new stock whose historical money value is greater than before. If they fail to do this, their money ceases to function as capital because it no longer earns further profit. The 'primary cause' of accumulation is the requirement for money to function as capital and earn profits, not the technical exigencies of production.

This is every bit as 'objective' as the structure of production but it is not reducible to anything purely physical. Mathematically the whole obscure discussion about the falling rate of profit reduces to the equation $K' = I$, where K is capital stock and I is investment. If a positive proportion of the surplus, measured in money, is invested then the capital stock measured in money *must* grow. This slumbering fact is known to every trainee bookkeeper, let alone any banker, but academia has done nothing but conceal it behind an ever-taller hedge of mathematical thorns.[6]

Conversely crisis, which expresses the unsustainability of indefinite accumulation, is irreducible to a physical event. Its onset is characterised by a general fall in all prices without any physical change at all. At this moment accumulation is reversed and capitalists are obliged either to cease production or use up productive assets without renewing them, so that I becomes negative in money terms. But the causal sequence is as given; it is the fall in prices and earnings which provokes the decline in production, not the other way around.

This overrides the physical configuration in which these money relations – in fact value relations – happen to be expressed. One must of course ask how these value relations affect and constrain physical structure, but if one demands that they be determined by it, the point gets missed. The organism is constructed as a mummy with only the outward form preserved.

THE FORMAL REQUIREMENTS OF DYNAMIC VALUE THEORY

If we can quantify a viable transition, we at least set a name on the face of the golem. As Harrod, Leontieff and others indicate, the bare equation

$$Y = X - A$$

is inadequate; we must add the consumption or increment of stocks of all kinds; that is

$$Y + \Delta K = X - A \tag{10}$$

where $\mathbf{K}$ is a matrix representing stocks of all types.

Important though the quantitative question is, a more subtle causal issue is involved. If current production is axiomatically assumed to match current consumption, the requirement becomes through the mathematics a *structural constraint on production*, as if the producers of today were in some sense caused to produce in the correct proportions by the consumers of tomorrow.

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[6] If we suppose S fixed, and I' $=\alpha$S where α is a proportion given by the distance of S/K from some 'target' rate R, we obtain a business cycle driven *purely* by endogenous profit rate changes. This is not meant to indicate the actual course of the business cycle but it does suggest the literature has overlooked some simple possibilities.

The axiom forms the conduit for a teleological (and apologetic) conception of the determinants of production.

Once stocks are introduced, causal relation follows temporal order: first goods are made, then they are consumed. If consumption fails to match production, the result is a change in stock levels. There is no causal relation between the consumption decisions of tomorrow and the production decisions of today. This was I think the principal reason for the defeat of the Austrian view, which sought both to preserve the subjective determination of value and the temporal order of production, two requirements which Marshall rightly understood to be incompatible.[7]

This causal notion is blurred or absent in many dynamic models such as Leontieff's own, as well as the Harrod-Domar literature. These generally try to write down conditions under which the consumption provoked by investment can match the output from this same investment, restoring teleology by the back door.

In Keynes and Marx, stocks clearly represent the discrepancy between supply as determined by investment decisions in the past, and effective demand as determined by consumption decisions in the present. It then follows that the change in stock levels cannot be deduced from the requirements of production alone but also from the outcome of consumption. Actual stocks move in response to prices.

The issue of stocks properly treated therefore poses a dual question absent from growth dynamics. In the real world the mechanism of stock adjustment is capital movement provoked by price change. The simultaneous equation

$$p\mathbf{A}(1+r) = p\mathbf{X}$$

is profoundly inadequate to this purpose. First and not least, the actual motor force of both market adjustment and technical change is not the formation of an equal profit rate but divergences in profit rates. Otherwise, why should capital move? The idea of an actually equalised profit rate is an idealisation to make the equations work, derived from the apologetic idea that 'capital' is a commodity like any other with its 'price' the rate of profit.[8]

But second, it expresses the absurd idea that the price paid for a commodity as input at one time is equal to the price paid for the same commodity at a later time as output. The price vector then cannot change between the start and the end of production; indeed it cannot change at all. The superstitious fear of margins has exorcised the mechanism of the market. The traditional equations are unreal in a further vital sense; they allow no process by which the market can function.

Once we allow that prices change from one period to another and that profits may differ from capital to capital we are driven to seek a dynamic relation of the form

$$(p^{t+1}, r^{t+1}) = f(p^t, \mathbf{A}^t, \mathbf{K}^t) \tag{11}$$

where there as many rates of profit as there are independent capitals and in which a vector of tomorrow's prices and profits arises when yesterday's supplies meet today's consumption.

Such formalisations produce *numerically different outcomes*. We might pardon a metaphysical lapse if the end result was the same. But it isn't; there is a practical difference. Work long in the public domain[9] clearly shows that for one of the most central issues of economics – the

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[7] See Dobb (1989:185)

[8] An idealisation not adopted by Marx, contrary to popular prejudice: 'In actual fact, demand and supply never coincide, or, if they do so, it is only by chance and not to be taken into account for scientific purposes; it should be considered as not having happened. (Marx 1981:291) 'Between the spheres more or less approximating the average there is again a tendency towards equalisation, seeking the *ideal* average, i.e. an average that *does not really exist*. (ibid p173: my emphasis), 'The industrial rates of profit in various spheres of production are themselves more or less uncertain; but in so far as they appear, it is not their uniformity but their differences which are perceptible. The general rate of profit, however, appears only as the lowest limit of profit, not as an empirical, directly visible form of the actual rate of profit' (ibid p367)

[9] See Kliman (1988), Freeman and Carchedi (1995)

movement of the profit rate – equilibrium and dynamic calculations produce diametrically opposite results.

We are now in a very different world, though closer to the one we live in. What *causal relations* govern the transition from p^t to p^{t+1}? This is in my view the real object of political economy. This idea is not new and is not only held in common with Post-Keynesians[10] but implicit in revived Austrian theory.[11] I also argue it is a far more acceptable reading of Marx.[12] The discussion which the future holds is, I would argue, a debate between these currents, outside the general framework of neoclassical general equilibrium either in its marginal or its linear production variant, about the best way to formalise causal relations in such dynamic systems.

IN PRAISE OF VALUE

To anyone schooled in the simultaneous equation approach, state transition equations like 10 and 11 are riotously indeterminate. Every element of today's vector depends on every single element of yesterday's.

These systems are riotous because life is. The equilibrium approach represents, in a certain sense, a neurotic compulsion to shut out the noise of existence. This is uncalled for; indeterminate is not the same as indeterministic. The fact that we cannot capture in a neat formula all relations between present and past, does not mean these relations have no real existence.

A system is not lawless by virtue of its degrees of freedom. The 'law' of gravity is not an equation, a prediction that a given object will move up or definitely down. It is an invariant relation which holds whatever objects do. In the same way the problem of political economy is to identify those relations which capture such laws of motion as society exhibits, without stifling the movements which pose awkward difficulties for the maths.

Once the notion of physical reproduction is abandoned for the myth it is, the real living being emerges. Its motions have to be understood in the language proper to them. Who can understand limbs who has not seen dancers? Money is the body language of the market; value, however, is its choreographer.

My purpose in this paper has been to take an axe to the root, to borrow Connolly's phrase, of the idea that this can be done by treating things as the ciphers of people, otherwise known as fetishism. I have attacked the sacred link between equilibrium theory and the modern reconstruction of the concept of value; the idea that it is no more than the expression of a 'physical' surplus. I think I have shown there is no such thing.

Axiomatic value dynamics

I am sure it will be said that by removing the physical underpinning I have removed the basis for all valid abstractions, leaving value theory with no foundation and economics with no surplus.

One aspect of my response is to point to persistently ignored work which shows that if the interpretation of Marx as an equilibrium theorist is abandoned utterly, a dynamic foundation of value and surplus is not only clear in his work but coherent within itself and with reality. But this by no means the only such theory. To ascertain what various dynamic theories have in common, and what distinguishes them apart, we can begin by defining a dynamic value theory *axiomatically*:

(1) the price of any commodity entering production must represent a definite amount of value from the past, given independent of the production process in which it is used,

(2) it must be possible to calculate the new value added to each commodity when it is produced, independent of the price for which that product then sells.

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[10] Among many too numerous to mention see Weintraub (1991)

[11] See O'Driscoll and Rizzo (1996)

[12] See Freeman and Carchedi (1995)

It seems to me just as reasonable and necessary to set dynamic value theory on an axiomatic basis, as it was for Arrow, Debreu and Hahn to set equilibrium price theory on an axiomatic footing, and that is the enterprise I propose. Any system satisfying these two axioms will be dynamic. It will distinguish value from price (price is the value which a commodity realises in exchange, as opposed to the value it receives from production). It will formulate the idea of economic surplus as the difference between the value that has been produced and the value that has been consumed in producing it. It will impose no constraint on physical structure. It will possess precisely the characteristics required of a truly general scientific abstraction, which is not to assume away all contingent factors but express their most general properties.

Economics is in one of those crises that occur during prolonged instability in the world. These usually call into question its most abiding dogma, that of equilibrium, though generally not for long. New approaches are surfacing and now is the time to interact; we need critical instruments that can assess the relations of different dynamic theories but also safeguard them against the next apologetic reconstruction. As has been remarked, every determination is a negation. If economics wishes to know where to go, its first task is to understand what it leaves behind. The axioms are proposed with this aim in view.

The objective and the subjective in value dynamics

The last point we wish to consider is this: where, within the spectrum of dynamic theory, does the vexed distinction between subjective and objective in fact lie?

The surplus approach sought to distinguish itself from marginalism by identifying objectivity with something independent of humans. But the economy is made up of humans. The real problem is to ascertain on the one hand how how subjective activity humanises objects and how, on the other, human activity becomes objective.

It is in my opinion only through genuine dynamics that a proper distinction between the objective and the subjective in social theory can be drawn. Equilibrium theory contains neither actually existing objectivity nor really possible subjectivity. Its excision of the past deletes the objective, and its postponement of the future shuts out the subjective. The second half of this century has therefore witnessed a battle of Dickensian ghosts. The marginal school reduces subjectivity to psychology while the surplus approach attributes objectivity to mere physicality.

History has no cancel button, and therefore our most elevated ideas become objective facts once we have thought them. On the other hand the future is unknown neither because of the blindness of the universe or the malice of its gods but because it is *our* future, so that even the most certain of nature's acts remain conjectures until we have lived them, and our most brutally physical creations remain mere ideas until we have brought them into the world.

Nevertheless, our past acts confront us not just as dead objects but as representatives of our present relations to each other. Our view of the world depends, in different ways in different theories, on the way human volition is conceived in the light of these facts; on how it is asserted that we may exploit the possibilities bequeathed to us by our past actions, in short, on the range of possible relations to each other between which we are alleged to choose when we interact with the objects of our own creation.

Becoming conscious of our choices, it has been remarked, is inseparable from making them. The task of rational social analysis is to ensure that we make them knowing what they really are. This idea, in my view, is what needs to be restored to the science of political economy.

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